Labour Turnover, Wages Structure, and Natural Unemployment

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Introduction and Summary

A firm which pays wages above the average will experience a less than average rate of labour turnover. If this is taken into account by the firms when fixing their wages, there will result a certain wage structure even if the supply of labour is absolutely homogeneous, and equally qualified workers will get different pay.

Simultaneously, an equilibrium will be established which clears the labour market but accidentally. In general, however, either a certain excess demand or a certain stable "natural rate of unemployment" will emerge. A general rate of labour turnover will be generated thereby which makes the decisions of the individual firms compatible with each other.

Contrary to an established view, natural unemployment is caused by too much rather than insufficient labour mobility in the presence of high turnover costs on the side of the firms.

The analysis suggests that technologies and forms of organization with high turnover costs might create labour market difficulties under capitalism which are comparable to those emerging in a labour managed economy where the performance is tied to a sufficiently low degree of labour turnover as well, as has been shown elsewhere (WEIZSÄCKER and SCHLICHT [1977]).

1. Labour Turnover and Productivity

The *rate of labour turnover* of a given kind of labour for a given firm is defined as the ratio between annual job separations (quits plus other separations) and the amount of employment.

Typically, the rate of labour turnover exceeds net employment changes by far.[1] Most of labour hiring is done for replacement purposes, therefore.

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[1] In 1961, the rate of labour turnover has been between 11 and 35 per cent with an average of 20 per cent. The figures for the U.S.A. are even higher (DE WOLF et al. [1965], p. 53).

The larger the rate of labour turnover, the larger will be the fraction of relatively unexperienced workers in the work force. This gives rise to a relationship between average labour productivity and the rate of labour turnover in case of considerable on-the-job training and considerable difficulties to get acquainted with the new technique and organization.[2] If the productivity of a worker who is fully acquainted with this job is normed to unity, relationships as depicted in Fig. 1 might result, which relate average productivity α to the rate of labour turnover.

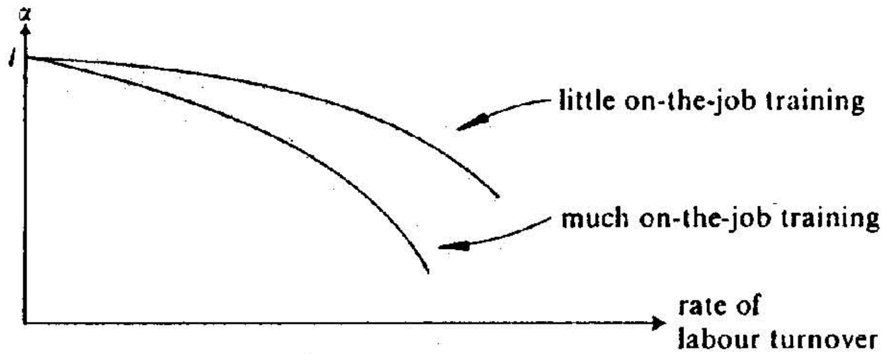

Fig. 1 The relationship between average productivity α and the rate of labour turnover.

A firm employing a technique which requires much on-the-job training will face a more negatively inclined relationship than a firm requiring only a slight amount of on-the-job training.

2. *Labour Turnover and Relative Wages*

Consider the frequency distribution of the pay which is attached to a given kind of labour (Fig. 2). The larger the wage rate which a given worker receives, the smaller will be his inclination to quit the job since the probability of finding an even better paid job declines with an increasing wage rate.

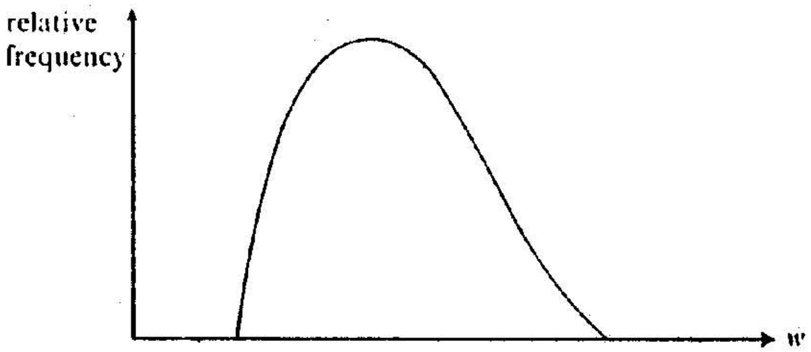

Fig. 2 Frequency distribution of wages *w*.

[2] It is guessed that "the costs caused by replacing one white collar worker amount, as a rule of thumb, to an annual salary." (Marr [1975], p. 849).

Denote the wage rate by w and denote the average wage rate by W. The relative wage level $v := w/W$ will be inversely related to the rate of labour turnover, therefore, as depicted in Fig. 3.[3]

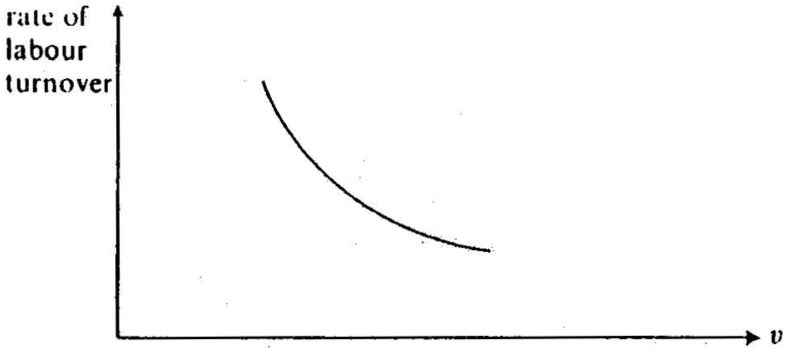

Fig. 3 The relationship between the rate of labour turnover and the relative wage level v.

3. The Choice of the Optimal Relative Wage Level

From Figs. 1 and 3, a relationship between the relative wage level v and average labour productivity α can be deduced. This is called the productivity curve (Fig. 4).

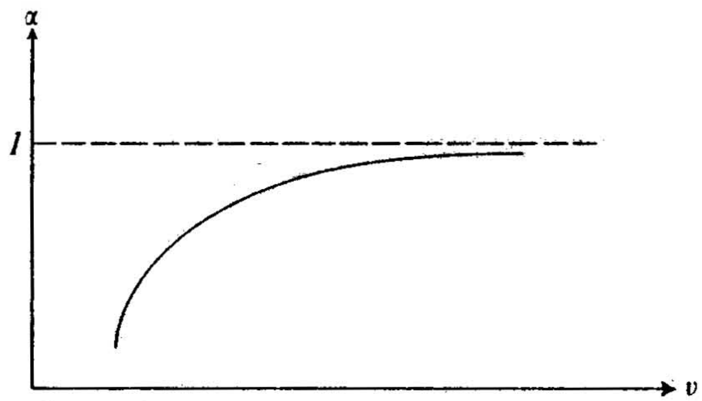

Fig. 4 The productivity curve depicting the relationship between average labour productivity α and the relative wage level v.

Algebraically, this translates into

$$(1) \qquad \alpha = \alpha(v), \quad \alpha' > 0, \quad \alpha'' < 0, \quad \lim_{v \to \infty} \alpha(v) = 1.$$

[3] The OECD study (DE WOLF et al. [1965]) summarizes: "One of the most significant findings of this study is that when the association between earnings levels and labour turnover is examined, it turns out to be consistently of negative sign." (p. 52) Although this relates to an inter-industry study, it lends support to the above hypothesis beyond mere theoretical plausibility.

The following considerations start from this relationship which has been derived here from the productivity effects of alternative rates of labour turnover. Other considerations might lead to a similar relationship, e.g. direct productivity effects of wage incentives or the possibility to get workers with better background characteristics for a higher wage. The particular derivation of the productivity curve would not affect the main argument. For the sake of simplicity, however, and because of its great practical significance, the argument is restricted to the turnover interpretation.

If N denotes the employment of the type of labour considered in a given firm, and if Y denotes the output net of all costs save wages for the labour input considered we assume a production function $F(\cdot)$ relating output Y, labour input N, and productivity α:

(2) $$Y = F(\alpha \cdot N), \quad F' > 0, \quad F'' < 0.$$

For a given average wage level W, the firm chooses a level of employment N and a relative wage level v such that profits are maximized:

(3) $$F\{\alpha(v) \cdot N\} - v \cdot W \cdot N = \max!$$

This gives rise to the necessary conditions[4]

(4) $$F'\alpha - v \cdot W = 0$$

(5) $$F'\alpha' N - W \cdot N = 0.$$

Since $v := w/W$, (4) is the usual marginal productivity condition $\partial Y/\partial N - w = 0$. From (4) and (5), a condition for the optimal relative wage level v^+ is derived:

(6) $$\alpha'(v^+) = \alpha(v^+)/v^+.$$

The optimal relative wage level is given by the point of the productivity curve where the tangency passes the origin (Fig. 5).

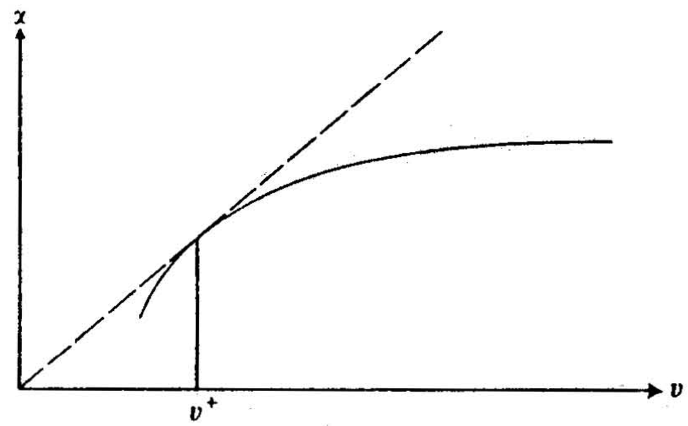

Fig. 5 Determination of the optimal relative wage level v^+.

[4] The second order conditions are satisfied.

It is independent of the absolute wage level.[5] If v^+ is inserted into (4), the corresponding optimal demand for labour N^+ is determined in the usual fashion. It depends inversely on the average wage level W, rather than on the particular wage level w, which is subject to the individual firm's discretion: $N^+(W)$ is implicitely defined by

$$(7) \qquad F'\{a^+ \cdot N^+(W)\} \cdot \alpha^+ - v^+ \cdot W = 0, \quad \text{where } \alpha^+ := \alpha(v^+)$$

and is depicted in Fig. 6.

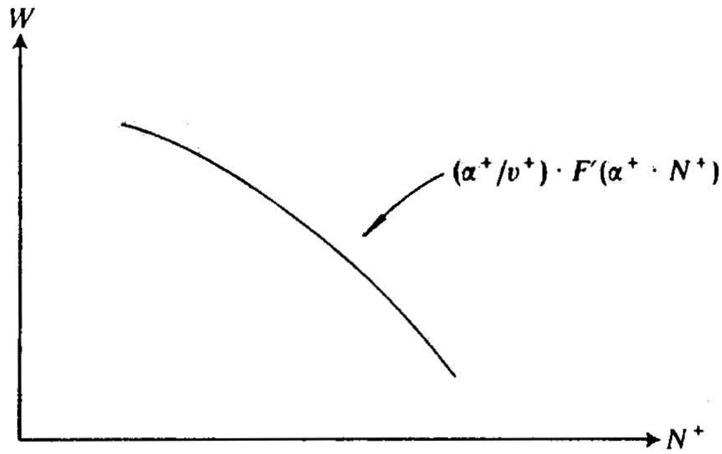

Fig. 6 The optimal demand for labour N^+ as a function of the average wage level W.

Depending on the particular shape of the productivity curve, the optimal relative wage level v^+ will be different for different firms. *Wage differences will emerge even if the supply of labour is absolutely homogeneous, therefore.*

4. *Unemployment and Labour Turnover*

Up to this point, equilibrium conditions for individual firms have been explored while assuming tacitly that an industry equilibrium is compatible with those conditions. But this compatibility poses some problems, as can be seen if it is assumed that all firms are alike: Each firm will choose the same optimal relative wage level v^+. If $v^+ \neq 1$, no equilibrium can emerge since it is impossible that all firms set their wage levels above or below the average.

In order to discuss this question and to show how a certain wage structure emerges in equilibrium, another important source of influence on the rate of labour turnover has to be taken into consideration which has been neglected up to now for the sake of simplicity: the excess demand for labour.

[5] This results of course from writing $F(\alpha \cdot N)$ rather than $F(N, \alpha)$. In the latter case, v^+ would be dependent on W as well. This would not affect the main argument.

Typically, the rate of labour turnovers is heavily dependent not only on the level of relative wages, but also on the state of the labour market:[6] The higher the rate of unemployment, the less will the workers be inclined to quit and the more they will try to avoid dismissal. Labour turnover will be comparatively low. The productivity curve will shift upward with increasing unemployment, therefore. On the other hand, there will be more opportunities in a tight labour market for workers to find a better paid job, and the rate of labour turnover will be greater. The productivity curve will shift downward with increasing excess demand in the labour market, therefore.

Denote the (positive or negative) excess demand in the labour market by e. Productivity α can be viewed as a function of the relative wage level v and excess demand e:

$$\alpha = \alpha(v, e). \tag{8}$$

The influence of v on α remains as described in (1):

$$\alpha_v > 0, \quad \alpha_{vv} < 0, \quad \lim_{v \to \infty} \alpha(v, e) = 1. \tag{9}$$

Excess demand e will affect productivity inversely:

$$\alpha_e < 0. \tag{10}$$

For a given v and a rather high rate of unemployment ($e \ll 0$), labour turnover will be rather small and the influence of wage increases on labour turnover and productivity will be very slight. On the other hand, very high excess demand ($e \gg 0$) causes high labour turnover and renders the influence of wage increases on turnover and productivity more effective. The productivity effect of wage increases α_v will be an increasing function of excess demand, therefore:

$$\alpha_{ve} > 0. \tag{11}$$

Assumptions (9) – (11) specify the shape of the productivity curve sufficiently for the following purposes. The analysis can proceed analogously to section 3. For a given e, the optimal relative wage level v^+ is obtained according to (6):

$$\alpha_v(v^+, e) = \alpha(v^+, e)/v^+. \tag{12}$$

It is an increasing function of the state of excess demand e (Fig. 7)

$$v^+ = v^+(e), \quad v_e^+ = \frac{\alpha_e - v \cdot \alpha_{ve}}{v \cdot \alpha_{vv}} > 0. \tag{13}$$

[6] De Wolf et al. [1965], p. 65.

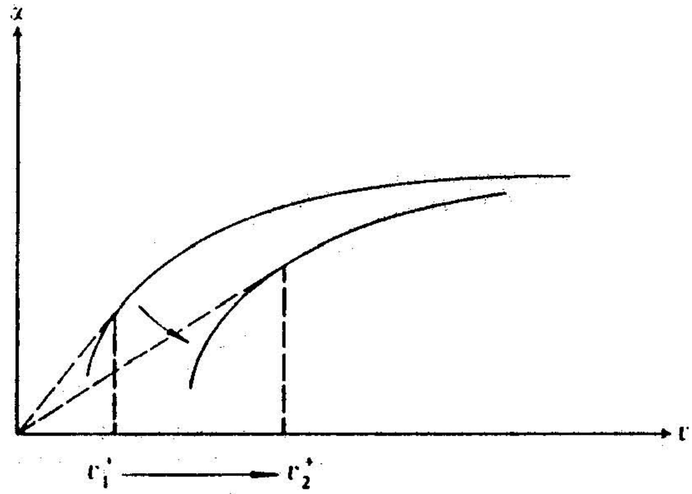

Fig. 7 The productivity curve shifts downward and the optimal wage rate increases with increasing excess demand in the labour market.

5. The Equilibrating Process

Consider an initial state characterized by a certain wage level W_0 and a certain excess demand for labour e_0. Each firm chooses its optimal relative wage level $v^+(e_0)$ and, according to (7), an optimal labour demand $N^+(W_0, e_0)$. This gives rise to a new excess demand e_1. By choosing the optimal relative wage level v^+, each firm fixes its absolute wage level according to $w = v^+ \cdot W_0$. This leads to a new average wage level W_1, and the process continues.

The change of the general wage level can be formalized therefore as

(14) $$W_{t+1} = W_t \cdot \varphi(e_t), \quad \varphi' > 0.$$

According to (7), labour demand is a decreasing function of the general level of wages. In a slightly simplified manner, this leads to the assumption that the overall excess demand is a decreasing function of the wage level:

(15) $$e_t = e(W_t), \quad e' < 0.$$

Inserting (15) into (14) yields the difference equation

(16) $$\hat{W}_t = \varphi\{e(W_t)\} - 1$$

where

$$\hat{W}_t := \frac{W_{t+1} - W_t}{W_t}$$

and

$$\frac{\partial \hat{W}_t}{\partial W_t} = \varphi' \cdot e' < 0.$$

In other words: The growth rate of the wage level is a decreasing function of the wage level.

If an equilibrium wage level $\bar{W}$ exists, that is, if it can be written

$$\bar{W} := e^{-1}\{\varphi^{-1}(1)\}, \tag{17}$$

then the wage level will decline if it is above $\bar{W}$, and it will increase, if it is below $\bar{W}$, thus tending to its stable equilibrium level $\bar{W}$.[7]

To $\bar{W}$ corresponds a certain state of excess demand $\bar{e}$ which will be termed *natural excess demand.*

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[7] This needs some qualification, however, since undamped oscillations around $\bar{W}$ have to be excluded. But undamped oscillations do not make much sense, from an economic point of view, anyhow and can thus be excluded by interpreting (16) in continuous rather than in discrete terms, i.e. by taking

$$\bar{W}' := \frac{dW}{dt} \cdot \frac{1}{W}.$$

Furthermore, the above analysis is to be understood in an approximative sense only. A more detailed analysis would render the exposition of the economic argument much less transparent; it had to proceed as follows.

Labour demand N^+_{t+1} of the typical firm is given by the marginal productivity condition (7), i.e.

$$F'(\alpha \cdot N^+_{t+1}) \cdot \alpha - v^+(e) \cdot W = 0. \tag{i}$$

Since we are dealing with the typical firm, we have to put

$$\alpha = \phi(e) \tag{ii}$$

which will differ from $\alpha\{v^+(e), e\}$ since changes in v will have different effects in the aggregate than on the firm level: If one firm increases its relative wage level, this increases its own productivity but decreases the productivity of the other firms. This implies

$$\phi' \leq \alpha_v \cdot v^+_e + \alpha_e. \tag{iii}$$

We will require even a little more: By the foregoing argument it can be reasonably supposed that the productivity effect of changes in the relative wage level be smaller than the productivity effect of excess demand on the aggregate level, that is

$$\phi' < 0. \tag{iv}$$

Inserting (ii) into (i) yields a labour demand function

$$N^+_{t+1} = N^+_{t+1}(W_t, e_t) \tag{v}$$

with the following properties

$$\frac{\partial N^+_{t+1}}{\partial W_t} < 0 \tag{vi}$$

$$\frac{\partial N^+_{t+1}}{\partial e_t} = \frac{F'}{\alpha^2 \cdot F''} \cdot \{\alpha_v \cdot v^+_e - \phi'\} - \frac{\phi'}{\phi} \cdot N^+_t. \tag{vii}$$

To derive a difference equation for e therefrom, the definition of e in terms of labour demand N^+ and labour supply N^s is needed, for instance

$$e_t = \sigma_t - 1, \quad \sigma_t = N^+_t / N^s \tag{viii}$$

where σ denotes the demand-supply ratio and e is relative excess demand.

Up to now, the existence of an equilibrium wage rate $\bar{W}$, as defined in (17), has been assumed. This seems to be plausible for the following reasons:

A very high general wage level causes a very high unemployment level. This will reduce labour turnover in such a way that no additional wage incentives are necessary to reduce it: All firms will try to fix their wage level below the average and this will cause a decline of the general level of wages. In other words, there exists a W^1 such that $\varphi\{e(W^1)\} - 1 < 0$.

On the other hand, a very low level of general wages will induce a considerable tightness in the labour market. The rate of labour turnover would be detrimentally high if the typical firm does not pay wages much above the average, which implies an increase of wages in the aggregate. In other words, there exists a W^2 such that $\varphi\{e(W^2)\} - 1 > 0$.

For continuity reasons, it follows that there exists an $\bar{W}$ in between W^1 and W^2 with $\varphi\{e(\bar{W})\} - 1 = 0$ which is unique since $\varphi' e' < 0$.

In case of a fixed supply N^s, (v)–(viii) imply

(ix) $$e_{t+1} = f(W_t, e_t), \quad f_w < 0, \quad f_e < -\frac{\phi'}{\phi} \cdot (e_t + 1).$$

Define

(x) $$z_t = \log W_t \text{ implying } W_t = \exp z_t$$

and rewrite (16) and (ix) in continuous terms, i.e. replace W_{t+1} by $W_t + dW_t/dt$ and e_{t+1} by $e_t + de_t/dt$, and apply transformation (x):

(xi) $$\dot{z} = g(e), \qquad g_e > 0$$

(xii) $$\dot{e} = h(e, z) \qquad h_z < 0, \text{ sign } h_e = \text{sign } \{f_e - 1\}.$$

If the conditions on the Jacobian

(xiii) $$tr = h_e < 0, \ \det = -h_z \cdot g_e > 0$$

are satisfied, the system is globally stable by Olech's theorem (Garcia [1972], p. 541). The determinant condition is satisfied. If the trace is positive, the system is unstable by standard arguments. Neglecting the singular case $h_e = 0$, the stability condition turns out to be $h_e < 0$. From (viii), (ix), and (xii), the following sufficient condition for global stability can be derived

(xiv) $$\varepsilon := -\frac{\sigma}{\alpha} \cdot \frac{d\alpha}{d\sigma} < 1$$

i.e. the elasticity of productivity with respect to changes in the demand-supply ratio is to be less than unity; an increase in the demand-supply ratio by one per cent has to induce productivity losses by less than one per cent.

It is to be noted that stability conditions become even less demanding if the impact of the quantity signal e on labour supply is taken into consideration.

6. *Determinants of the Natural Rate of Unemployment*

If the natural rate of excess demand $\bar{e}$ is negative, it is termed natural unemployment. The wage mechanism does not eliminate this unemployment. In spite of excess supply of labour, no firm will reduce its wages since this would cause too much labour turnover and productivity losses.

The determinants of the natural rate of unemployment are straightforward: Unemployment results because all firms try to reduce labour turnover by increasing their wages up to the point where the presence of sufficient unemployment limits the rate of labour turnover by its mere presence.

In other words: Natural unemployment is caused by too high a degree of labour mobility rather than by insufficient labour mobility; *ceteris paribus the natural rate of unemployment will decrease if labour mobility is reduced.*

"Too high a mobility" refers to the given state of technique and organization, of course. In other words: *Ceteris paribus, natural unemployment will increase if the costs of labour turnover increase.*

Zusammenfassung

Arbeitskräftefluktuation, Lohnstruktur und natürliche Unterbeschäftigung

Je höher die Löhne sind, die eine Unternehmung zahlt, umso geringer wird die Arbeitskräftefluktuation und umso geringer werden mithin die Fluktuationskosten sein. Den gleichen Effekt auf die Fluktuation hat ein Steigen der Arbeitslosigkeit. Das Zusammenspiel dieser beiden Einflüsse führt – bei völlig flexiblen Löhnen – zu einem stabilen Gleichgewicht auf dem Arbeitsmarkt, bei dem nur in Ausnahmefällen der Arbeitsmarkt geräumt wird. Ceteris paribus nimmt die Arbeitslosigkeit zu, wenn die Arbeitskräftemobilität steigt. Auch bei völlig homogenem Arbeitsangebot ergibt sich dabei eine gewisse Lohnstruktur, d.h. in jeder Hinsicht identische Arbeitnehmer werden unterschiedlich entlohnt.

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