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**Simultaneous inter- and intra-
group conflicts**

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Simultaneous inter- and intra-group conflicts

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Abstract

This paper models the trade-off between production and appropriation in the presence of simultaneous inter- and intra-group conflicts. The model exhibits a ‘group cohesion effect’: if the contest between the groups becomes more decisive, or contractual incompleteness between groups becomes more serious, the players devote fewer resources to the intra-group conflict. Moreover, there is also a ‘reversed group cohesion effect’: if the intra-group contests become less decisive, or contractual incompleteness within groups becomes less serious, the players devote more resources to the inter-group contest. The model also sheds new light on normative questions. I derive exact conditions for when dividing individuals in more groups leads to more productive and less appropriative activities. Further, I show that there is an optimal size of the organization which is determined by a trade-off between increasing returns to scale in production and increasing costs of appropriative activities.

Key-words: Conflict, rent-seeking, federalism, hierarchy

JEL codes: D72, D74, H11, H74

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1 Introduction

Vilfredo Pareto famously remarked that men utilize their efforts in two different ways: to produce economic goods, or to appropriate goods produced by others. The corresponding trade off between productive and appropriative activities has been studied extensively in the recent literature on endogenous property rights (for a survey, see Skaperdas 2003).

Appropriative activities take place at different levels. For example, within the EU, the member countries compete for subsidies. At the same time, there is a contest over the allocation of subsidies within the countries. Or, within a firm, several departments compete for resources, and in addition there is a contest about the allocation of resources within the departments. The common structure is that there are appropriative conflicts between certain groups (states, departments, etc.), and appropriative conflicts within these groups. The aim of this paper is to provide a model of situations like these. What determines the amount of conflict within groups and between groups, respectively? And what is the optimal design of an organization (be it a federal state or a firm), taking into account that organizational structure has an effect on appropriative activities?

In order to study these questions, I set up a model which is related to the conflict models of Hirshleifer (1988, 2001) and Skaperdas (1992). These models are motivated by some kind of contractual incompleteness which leads to the absence of well defined and enforced property rights. Thus, individuals can engage into appropriative activities and face a trade-off between production and appropriation. The novel feature of my model is that I explicitly study simultaneous inter- and intra-group conflicts. Individuals are partitioned in groups, and utilize their resources in three different ways: for production, for appropriation in a contest between groups, and for appropriation in a contest within their own group. The contractual incompleteness problems may be different between groups and within groups. I model this in a stylized way, taking contractual incompleteness as an exogenous parameter, and being agnostic as to whether it is smaller or greater between groups or within groups. Moreover, the technology of conflict (see Hirshleifer 1991) may be different in the intra-group contest from that in the inter-group contest, since these contests are usually fought with different instruments. Again I model this by a parameterization, taking an agnostic point of view about which of the contests is the more decisive.

Situations of inter-group conflict have long been studied in the social sci-

ences, especially in sociology, psychology, and anthropology. One seemingly robust finding is that inter-group competition leads to increased cohesiveness within a group (see Fisher 1990, chapter 4, for a survey). My model provides an economic explanation of this ‘group cohesion effect’: if the inter-group conflict becomes more decisive, or more is at stake in this conflict, then intra-group rent-seeking declines. Moreover, there is also an ‘reversed group cohesion effect’: if the intra-group contest becomes less decisive, or contractual incompleteness problems within groups less severe, then appropriative activities in the inter-group contest increase.

In addition, my paper has normative implications concerning the optimal design of organizations. In this respect, it is related to a series of papers by Karl Wärneryd and coauthors. They point out that, while traditionally it has been thought that rent-seeking activities will increase if an organization acquires more layers in a hierarchy, such multitiered structures can actually *reduce* the costs of appropriative activities. For example, Wärneryd claims that “the institutional framework of federalism, such as that of the EU may be seen as an efficient response (...) to rent-seeking activities, since it lowers the dead weight losses from such activities” (Wärneryd 1998, 436). Other applications include the allocation of free cash flow inside organizations (Inderst, Müller and Wärneryd 2002), and distributional conflict between shareholders of corporations (Müller and Wärneryd 2001). The analysis in these papers rests on three key assumptions. First, it is assumed that there is a temporal order in which the contests take place: the inter-group contest is fought first, and only when it is resolved, do the intra-group contests begin.¹ Here, while deciding how much to spend on the contest between groups, an individual will anticipate that if his group gets a bigger share, his fellow group members will fight harder in the following intra-group contest. This dampens incentives to engage into appropriative activities between groups in the first stage. This is an important reason behind the result of Wärneryd. In many real world examples, however, it seems more natural to assume that the distributional conflicts take place simultaneously. I study

¹There are several other papers that study this kind of two stage contest game. The earliest paper I am aware of is Katz and Tokatlidu (1996). Stein and Rapoport (2004) study asymmetries and the reversed order of timing where the intra-group contest comes first and the inter-group contest second. Konrad (2004) considers a perfectly discriminating contest and heterogeneous contestants. Garfinkel (2004) explores the endogenous formation of groups when there is conflict both within and between groups. However, none of these papers studies simultaneous inter- and intra-group conflicts.

this case. Then the effect described above is not present, and this changes the results. Second, Wärneryd models the contests in a way which follows the literature on rent-seeking started by Tullock (1980): he uses a partial equilibrium approach where the size of the contested rent is exogenous, and does not depend on the amount of rent-seeking. This is realistic in some contexts, but less so in others, where the allocation decision of the players are likely to have a discernible impact on the size of the contested rent. Hence a general equilibrium approach, in the spirit of the conflict models of Hirshleifer (1988, 2001) and Skaperdas (1992) might be appropriate, since in these models output is endogenous. Wärneryd himself pointed out that explicitly modelling the trade-off between production and appropriation is an fruitful direction for further research (1998, p. 448; see Neary 1997 for a comparison of rent-seeking and conflict models). Third, Wärneryd assumes that the technology of conflict is the same in the intra-group contest as in the inter-group contests. He relies on axiomatic foundations of contest technologies given by Skaperdas (1996), which can be generalized for inter-group contests. However, the axioms pin down the functional form of the contest success functions only up to a parameter, known as the decisiveness of the contest, which is a major influence on the marginal benefits of rent-seeking activities. As argued above, since inter- and intra-group contests are often fought with different “weapons”, they may differ in their decisiveness. Moreover, the contractual problems that lead to rent-seeking activities might be more or less severe within groups than between groups.

I study the trade off between production and appropriation in a general equilibrium conflict model, where there are simultaneous inter- and intra-group conflicts, taking into account possible differences in the technology of conflict. For comparison I also study a partial equilibrium rent-seeking model (with simultaneous inter- and intra-group contests and possibly different contest technologies).² My normative findings are as follows. First, consider the optimal number of groups for a given number of players. Whether splitting up individuals into (more) groups leads to more or less rent-seeking depends on the difference between the nature of the inter-group conflict and that of the intra-group conflict. If these conflicts are equally decisive, and the con-

²In addition to the papers discussed above, this is also related to Nitzan (1991), who studied rent-seeking between groups. The main difference is that in Nitzan (1991) there is no intra-group rent-seeking and the distribution of rents both within groups and between groups depends on the *inter*-group rent-seeking efforts. For an excellent survey of rent-seeking theory, see Nitzan (1994).

tractual problems are equally severe, the amount of rent-seeking does *not* depend on the number of groups. If the conflict is more decisive between the groups than within groups, and if the degree of contractual incompleteness is higher between the groups than within groups, then a flat structure where all individuals belong to one group is optimal. And vice versa: if the intra-group conflict is sharper, then one should split up the individuals in as many groups as possible. These findings contrast starkly with the results in the literature discussed above. The difference is due to the different assumption on the timing of the conflicts. Thus, these results show the importance of the simultaneity assumption. Second, I show that there is an optimal size of the organization which is determined by a trade-off between increasing returns to scale in production on the one hand, and increasing costs of conflict on the other.

The paper proceeds as follows. The model of simultaneous inter- and intra-group conflicts is laid out in section 2. Section 3 derives the basic predictions of the model. Section 4 considers the question of optimal design. Section 5 discusses extensions of the basic model to different production technologies, conflict technologies, and unequal group size. Section 6 gives a comparison with a partial equilibrium rent-seeking model of simultaneous inter- and intra-group conflicts. Section 7 concludes.

2 The model

There are n identical individuals and G groups of equal size $m = n/G$. Each individual is endowed with one unit of time and has three choice variables: productive effort e_{ig} , intra-group rent-seeking effort x_{ig} , and inter-group rent-seeking effort y_{ig} . The first subscript refers to the individual, the second to the group he is a member of. The budget constraints are given by

$$e_{ig} + x_{ig} + y_{ig} = 1$$

for all $i = 1, \dots, m$ and all $g = 1, \dots, G$.

For simplicity, I assume that output is given by the constant elasticity production function

$$q = \left(\sum_{g=1}^G \sum_{i=1}^m e_{ig} \right)^h. \quad (1)$$

The parameter $h > 0$ determines returns to scale: if $h > 1$ we have increasing returns to scale, if $h = 1$ constant, and if $h < 1$ decreasing returns to scale. Equation (1) assumes that the complementarities within the organizations are independent of the number of groups. This seems a natural benchmark for studying the effects of the number and size of groups *from a rent-seeking perspective*.

The output is distributed among the groups. Denote the share that goes to group g by p_g . Hence group g gets $p_g q$. This amount is distributed within the group; player i gets the share r_{ig} . Thus the payoff of player i in group g is

$$u_{ig} = p_g r_{ig} q.$$

Let us first turn to the allocation of output within groups. It depends on the intra-group rent-seeking activities. I will assume that

$$r_{ig} = \begin{cases} \gamma \frac{x_{ig}^a}{\sum_j x_{jg}^a} + (1 - \gamma) \frac{1}{m}, & \text{if } \sum_j x_{jg}^a > 0, \\ \frac{1}{m}, & \text{if } \sum_j x_{jg}^a = 0. \end{cases} \quad (2)$$

Here, $\gamma \in [0, 1]$ is a measure of the contractual incompleteness within groups: one part $(1 - \gamma)$ of the group's share is allocated by a simple equal division rule, and the other part (γ) is allocated according to the intra-group rent-seeking activities.

The specific functional form of the intra-group contest success function,

$$\frac{x_{ig}^a}{\sum_j x_{jg}^a},$$

has been used widely in the literature. There is an axiomatic foundation by Skaperdas (1996). The parameter a describes the decisiveness of the intra-group contest. If $a \rightarrow 0$, rent-seeking effort has little influence on the division of the gains, whereas if $a \rightarrow \infty$, tiny differences in rent-seeking effort are decisive.

The allocation of output to groups depends on the inter-group rent-seeking efforts. Group k gets the fraction

$$p_k = \begin{cases} \delta \frac{(\sum_j y_{jk})^b}{\sum_g (\sum_j y_{jg})^b} + (1 - \delta) \frac{1}{G}, & \text{if } \sum_g (\sum_j y_{jg})^b > 0, \\ \frac{1}{G}, & \text{if } \sum_g (\sum_j y_{jg})^b = 0, \end{cases} \quad (3)$$

of the output. Here $\delta \in [0, 1]$ is a parameter that measures how important rent-seeking activities are in the inter-group contest. It measures the contractual incompleteness between the groups. As in the intra-group contest, in the inter-group contest only a part (δ) is allocated according to rent-seeking activities. Contractual incompleteness problems may be more or less severe between groups than within groups. Hence δ may be bigger or smaller than γ .

If the axioms of Skaperdas (1996) hold for the inter-group contest as well, and the rent-seeking efforts of a group are aggregated efficiently, then one can show that

$$\frac{\left(\sum_j y_{jk}\right)^b}{\sum_g \left(\sum_j y_{jg}\right)^b}$$

is the only possible functional form of the inter-group contest success function (see the appendix for the details). However, the axioms do not pin down the decisiveness of the contest. Here, the parameter b describes the decisiveness of the contest between the groups. It may, or may not, be equal to the decisiveness of the contest within the groups a . Since contests between groups are usually fought with instruments different from those in contests between groups, they might well have a different decisiveness.³

Note that two pairs of parameters describe the different layers of conflict: the decisiveness parameters a and b , and the parameters γ and δ that indicate the importance of rent-seeking.

I will assume that $0 < a \leq 1$ and $0 < b \leq 1$. The assumption that a and b are positive means that the a player's share of the output increases in his rent-seeking activities. The upper bounds are imposed to make the model tractable. As we will see, they are sufficient to make all the optimization problems well behaved. Let me point out that, if $\gamma = \delta = 1$, no upper bounds on a and b are necessary. In this case one can easily find the equilibria even if $a \rightarrow \infty$ and $b \rightarrow \infty$ (this is the case of discontinuous contest success functions, as in all pay auctions) (see section 5).

³To give an analogy, consider chess and backgammon. These games are certainly governed by different contest success functions, since luck plays a much more important role in backgammon.

3 The group cohesion effect

Using the budget constraints to express u_{ik} as a function of rent-seeking efforts alone, we can write

$$u_{ik} = p_k r_{ik} \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right)^h. \quad (4)$$

I show in the appendix that the log of u_{ik} is strictly concave in (x_{ik}, y_{ik}) . Hence any critical point of $\ln u_{ik}$ is a strict global maximum. Since the log is a strictly monotone function, it follows that any critical point of u_{ik} is a strict global maximum, too. This means that we can solve the maximization problem of an individual by looking at the first order conditions. We will ignore the non-negativity constraints temporarily, and check afterwards that all the constraints hold.

Differentiating equation (4) with respect to y_{ik} and setting the result equal to zero, we get

$$\frac{\partial p_k}{\partial y_{ik}} q = p_k h \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right)^{h-1}. \quad (5)$$

In a symmetric equilibrium, all individuals choose the same allocation of their budget: $y_{ig} = y$ and $x_{ig} = x$ for all i and g . Conjecturing that a symmetric equilibrium exists, we get

$$(G - 1) b \delta m (1 - x - y) = (m y) h$$

Differentiating equation (4) with respect to x_{ik} , and setting the result equal to zero yields

$$\frac{\partial r_{ik}}{\partial x_{ik}} q = r_{ik} h \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right)^{h-1}. \quad (6)$$

In a symmetric equilibrium,

$$(m - 1) a \gamma G (1 - x - y) = x h$$

Solving, we finally get (recall $n = mG$)

$$y = \frac{b \delta (G - 1)}{a \gamma (n - G) + b \delta (G - 1) + h} \quad (7)$$

and

$$x = \frac{a\gamma(n-G)}{a\gamma(n-G) + b\delta(G-1) + h}. \quad (8)$$

Note that $0 < x < 1$ and $0 < y < 1$. Productive effort per person equals

$$e = 1 - x - y = \frac{h}{a\gamma(n-G) + b\delta(G-1) + h} > 0. \quad (9)$$

This is positive, therefore no constraint is violated. We can conclude that a symmetric equilibrium does in fact exist.

However, the equilibrium is not unique. The first order conditions pin down only the total amount of inter-group rent-seeking done by a group, and the total amount of productive effort put in by the group's members. How the members of the group coordinate in supplying productive and inter-group rent-seeking effort is not determined. The following lemma sums up this discussion.

Lemma 1 *There is a continuum of equilibria, where*

a) *all contestants choose the same intra-group rent-seeking effort x given in equation (8),*

b) *for all groups g , the total amount of inter-group rent-seeking chosen by g 's members equals*

$$\sum_i y_{ig} = my \quad (10)$$

where y is given in equation (7),

c) *for all groups g , the total amount of productive effort of the members of group g equals*

$$\sum_i e_{ig} = me, \quad (11)$$

where e is given in equation (9),

d) *the utility of an individual is*

$$u = \frac{1}{n} \left(n \frac{h}{a\gamma(n-G) + b\delta(G-1) + h} \right)^h. \quad (12)$$

Proof. Parts a, b, and c follow from the discussion above; part d follows by inserting equilibrium choices into the utility function. ■

In these equilibria, the average amount of inter-group rent-seeking is y given in equations (7) above. Similarly, e given in equation (9) is the average amount of productive effort. We will make use of this in some of the comparative static exercises below.

What determines the allocation of effort to production and inter- and intra-group rent-seeking? The following proposition studies the influence of the technology of conflict, contractual incompleteness, and the production technology.

Proposition 1 *a) If the contest between the groups becomes more decisive (i.e. b increases) and/or contractual incompleteness between groups becomes more serious (i.e. δ increases), then rent-seeking within groups and productive effort decline, while inter-group rent-seeking increases.*

b) If the intra-group contests become more decisive (i.e. a increases) and/or contractual incompleteness within groups becomes more serious (i.e. γ increases), then rent-seeking between groups and productive effort decline, while intra-group rent-seeking increases.

c) An increase in the returns to scale in production h increases productive effort, and decreases rent-seeking both within and between groups.

Proof. a) From equations (7)- (11) it is obvious that as δb increases, x and $\sum_i e_{ig}$ decrease, while $\sum_i y_{ig}$ increases.

b) Again from equations (7)- (11), if γa increases, $\sum_i y_{ig}$ and $\sum_i e_{ig}$ decrease, while x increases.

c) Differentiate equations (7)- (11) to get

$$\begin{aligned}\frac{\partial x}{\partial h} &= -\frac{a\gamma(n-G)}{(a\gamma(n-G) + b\delta(G-1) + h)^2} < 0, \\ \frac{\partial}{\partial h} \left(\sum_i y_{ig} \right) &= -m \frac{b\delta(G-1)}{(a\gamma(n-G) + b\delta(G-1) + h)^2} < 0, \\ \frac{\partial}{\partial h} \left(\sum_i e_{ig} \right) &= m \frac{a\gamma(n-G) + (G-1)b\delta}{(a\gamma(n-G) + b\delta(G-1) + h)^2} > 0.\end{aligned}$$

■

Part a) says that an increase in the contractual incompleteness between groups, or an increase in the decisiveness of the inter-group contest, leads

to less intra-group rent-seeking. This is reminiscent of the “group cohesion effect” documented in psychology and anthropology: increased competition between groups leads to more cohesion within the groups. In the model, we can interpret more competition between groups as an increase in δ and/or γ , and more cohesion as lower x . In the model a group cohesion effect arises by individual, noncooperative utility maximization, without any need for centralized leadership of the group. An increase in δ means that more is at stake in the conflict between groups, and leads to less intra-group hostility. Further, an increase in b means that the inter-group contest gets more decisive, and leads to less intra-group hostility, too. The intuition is simply that the marginal benefit of inter-group rent-seeking activities is proportional to b and δ . Hence an increase in b and δ makes more inter-group rent-seeking more attractive compared to intra-group rent-seeking and production.

Sometimes a reverse of the group cohesion effect is postulated as well: “heightened in-group cohesion is itself a condition for out-group hostility” (Fisher 1990, p. 68).⁴ As part b) of the proposition shows, this effect holds in the model studied here, too. A lower decisiveness a of the intra-group contest success technology leads to more inter-group conflict. Also, a decrease in γ - which means that the group is more ‘egalitarian’ and the distribution within the group is less dependent on rent-seeking - leads to more conflict between groups.

Part c) of proposition 1 says that an increase in h leads to more productive effort, and less rent-seeking. This is due to the fact that the marginal benefit of working productively is proportional to h . Usually, an increase in h will increase utility. Utility can decline only if the sum of all productive efforts is smaller than one. In this range, an increase in h corresponds to a *decrease* in productivity, and may lead to lower equilibrium utility.

4 The optimal number and size of the groups

The next exercise is to describe the influence of the number and size of the groups on equilibrium behavior.

Proposition 2 *a) For a given number of individuals n , productive effort increases in the number of groups G if, and only if, $\delta b < \gamma a$. Intra-group rent-*

⁴However, empirically the existence of such an effect is much more in doubt than the original cohesion effect, see Fisher (1990).

seeking effort x declines in G , and average inter-group rent-seeking effort y increases.

b) Increasing the size m of the groups while holding constant their number G results in an unambiguous decrease in average productive effort and average inter-group rent-seeking, while it increases intra-group rent-seeking.

c) Increasing the number of groups G while holding constant the size m of the groups leads to a decline in productive effort and an increase in inter-group rent-seeking. The effect on the intra-group conflict depends on the parameters: it increases if, and only if, $h > b\delta$.

Proof. a) By differentiating equation (9) we find that

$$\frac{\partial e}{\partial G} \Big|_{n=\text{const}} = h \frac{a\gamma - b\delta}{(a\gamma(n - G) + b\delta(G - 1) + h)^2} \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} 0$$

if, and only if,

$$a\gamma \left\{ \begin{array}{l} > \\ = \\ < \end{array} \right\} b\delta.$$

Further, we get

$$\begin{aligned} \frac{\partial x}{\partial G} \Big|_{n=\text{const}} &= -a\gamma \frac{h + (n - 1)b\delta}{(a\gamma(n - G) + b\delta(G - 1) + h)^2} < 0, \\ \frac{\partial y}{\partial G} \Big|_{n=\text{const}} &= b\delta \frac{a\gamma(n - 1) + h}{(a\gamma(n - G) + b\delta(G - 1) + h)^2} > 0. \end{aligned}$$

b) Use $n = Gm$ to eliminate n in equations (9), (8), and (7):

$$\begin{aligned} e &= \frac{h}{a\gamma G(m - 1) + b\delta(G - 1) + h} \\ x &= \frac{a\gamma G(m - 1)}{a\gamma G(m - 1) + b\delta(G - 1) + h} \\ y &= \frac{b\delta(G - 1)}{a\gamma G(m - 1) + b\delta(G - 1) + h} \end{aligned}$$

Obviously, e and y decrease in m for a given G , whereas x increases.

c) Suppose G increases while m is constant. Clearly, e goes down, while y goes up. Further,

$$\frac{\partial x}{\partial G} \Big|_{m=const.} = (m-1) a\gamma \frac{h - b\delta}{(a\gamma G(m-1) + b\delta(G-1) + h)^2},$$

hence x increases if, and only if, $h > b\delta$. ■

Now we can turn to the normative implications of the model. The aim is to understand what a rent-seeking perspective can contribute to the question of an optimal design of an organization that is ridden by simultaneous inter- and intra-group conflict. Especially, what are the optimal number and size of groups? That is, which m and G maximize equilibrium utility as given in equation (12) above?

As noted in the introduction, there is a discussion in the literature on the effect of additional levels of hierarchy in an organization on rent-seeking activities. The message of the present model is that the different technologies of conflict, and the amount of contractual incompleteness, are of the paramount importance.

Proposition 3 *If $a\gamma < b\delta$, then a flat structure where all individuals belong to the same group is optimal. On the other hand, if $a\gamma > b\delta$, then one should split the individuals up in as many groups as possible.*

Proof. This maximizes productive effort by proposition 2 above. Output depends by assumption only on the sum of the individual productive efforts, and not directly on the number of groups. Each individual gets the share $1/n$ of the output. Hence the result follows. ■

Having studied the optimal number of groups, I turn now to the question of optimal group size. Consider first the case that $a\gamma < b\delta$. Here, we have seen that having only one group is optimal. Setting $G = 1$ in equation (12) we get

$$u = \frac{1}{n} \left(n \frac{h}{a\gamma(n-1) + h} \right)^h.$$

Maximizing this over n (ignoring integer constraints for convenience) leads to an optimal size of the organization which is given by

$$n^* \Big|_{a\gamma < b\delta} = 1 + h \frac{h - 1 - a\gamma}{a\gamma}.$$

Several features are worth noting. First, the production technology must exhibit sufficiently increasing returns to scale for it to be worthwhile forming a partnership. If $h \leq 1 + a\gamma$, the optimal ‘organization’ consists of only one person. Second, the optimal size is increasing in returns to scale in production h , decreasing in the decisiveness of the contest a , and decreasing in the severity of contractual incompleteness γ .

We have here a trade of between increasing returns and rent-seeking. If $h > 1$ production exhibits increasing returns to scale. Therefore it would be optimal to have as many people as possible working together, if one could distribute the gains without rent-seeking activities (formally, as $a\gamma \rightarrow 0$, $n^* \rightarrow \infty$). But if the output is distributed by rent-seeking, the optimal size of the partnership is limited by the increasing rent-seeking cost. An increase in the number of individuals leads to unambiguously lower productive effort per person.

A similar analysis applies for the case $a\gamma > b\delta$. Here, it is optimal to have $G = n$, and the optimal size of the organization is

$$n^* |_{a\gamma > b\delta} = 1 + h \frac{h - 1 - b\delta}{b\delta}.$$

In the remaining case where $a\gamma = b\delta$, the number of groups plays no role for welfare, and the optimal size of the organization is given by either of the equations above. The following proposition sums up the findings concerning the optimal size of the organization.

Proposition 4 *There exists an optimal size of the organization which is determined by a trade off between increasing returns to scale in production on the one hand and increasing costs of conflict on the other. The optimal size increases in the returns-to-scale parameter h . It decreases in the decisiveness of the contest and in the amount of contractual incompleteness.*

5 Extensions: production technology, contest technology, unequal group size

Production technology. As a robustness check I generalize the model by considering a more general production function

$$q = f \left(\sum_{i=1}^n e_i \right). \quad (13)$$

I assume that f is strictly increasing and twice differentiable. In addition, I will assume that the function f is ‘not too convex’. A sufficient condition is that the log of f be concave. Given this assumption, the objective functions of the players are log-concave, and we can rely on first order conditions in order to characterize equilibria.

After imposing symmetry the first order conditions boil down to

$$my = \delta b \frac{(G-1) f(n(1-x-y))}{G f'(n(1-x-y))} \quad (14)$$

and

$$x = \gamma a \frac{m-1}{m} \frac{f(n(1-x-y))}{f'(n(1-x-y))} \quad (15)$$

Since f is log-concave by assumption, the right hand sides of the previous equations are decreasing in x and y . Hence the equations determine x and y uniquely.

Summing over equations (14) and (15), it follows that the total amount of rent-seeking

$$R := n(y+x)$$

equals (use $(m-1)/m = (n-G)/n$)

$$R = ((G-1)(\delta b - \gamma a) + \gamma a(n-1)) \frac{f(n-R)}{f'(n-R)} \quad (16)$$

I show in the appendix that the comparative statics given in proposition 1 above holds true also with the more general production technology considered here, except part c concerning the productivity parameter h , which does not appear in (13). In particular, both the group cohesion effect and the reversed group cohesion effect still hold.

Moreover, as we found above, the total amount of rent-seeking is increasing in the number of groups if, and only if, $\delta b > \gamma a$. This can be seen easily from equation (16). Since f is by assumption log-concave, the right hand side is decreasing in R . Suppose $\delta b > \gamma a$, and G increases. Then R has to increase in order that (16) holds. Hence it is clear that the findings are not an artifact of the production function (1) considered above.

Contest technology. So far, the analysis was based on the assumption that $a \leq 1$ and $b \leq 1$. That is, the decisiveness of the contest success functions

was assumed to be bounded from above. Here I add some brief considerations on the case where the contest technologies are very decisive. Then it is no longer clear whether the first order conditions in fact do describe equilibria. However, the benchmark case where $\gamma = \delta = 1$ turns out to be easy to solve. Here the objective functions are log-concave for all $a, b \in (0, \infty)$ (see appendix). Hence the analysis above holds without alteration for all $a, b \in (0, \infty)$. We can use this case to study a situation where the environment is very conflictual: the whole output is distributed according to the appropriative activities, and the contests are very decisive. For example, it is interesting to note what happens in the limiting case $a = b = \infty$. Here the contest success functions are discontinuous: the group that puts in the most inter-group rent-seeking effort gets the whole output, and, within groups, the player who chooses the highest intra-group effort gets everything (in case of a tie the groups or persons involved share equally). There is an equilibrium where all players devote all their energy to inter-group rent-seeking ($y_{ig} = 1$). In this equilibrium, output and utility equals zero. No one has an incentive to deviate, since then the inter-group rent-seeking efforts of his group would be smaller than those of the other groups, and thus his group will get nothing. Similarly, there is an equilibrium where all devote their energy solely to intra-group rent-seeking ($x_{ig} = 1$).

Unequal group size. When groups are of equal size, equilibrium utility is increasing in the number of groups if, and only if, $a\gamma > b\delta$. However, when the groups are of unequal size, utility of the individuals also depends on the size of the groups.

Suppose there are two groups. Define ‘symmetric interior equilibrium’ as an equilibrium where all individuals in a given group $g = 1, 2$ choose the same amounts of inter-group rent-seeking, intra-group rent-seeking, and productive effort, and all the choice variables are strictly positive (no corner solution). In contrast to the case of equal group size, a symmetric interior equilibrium does not always exist here. However, if the technology is sufficiently productive (h sufficiently large), a symmetric interior equilibrium exists.

One can show that in a symmetric interior equilibrium, each group gets $1/2$ of the output. Thus an individual in group g gets the share $1/(2m_g)$ of the output, where m_g is the number of individuals in group g .

To compare this with a situation where all individuals belong to only one group, consider first the case where $a\gamma = b\delta$. Here, output is the same irrespective of the number of groups. But its allocation to the individuals

differs. With only one group, each individual gets the share $1/n$ of the output. Hence, members of the smaller group prefer the situation with two groups, but members of the bigger group would rather have a unified organization. If $a\gamma \neq b\delta$, we have the additional effect that total output is different between the two structures. See the appendix for a formal derivation of these remarks.

6 Comparison with a partial equilibrium rent-seeking model

It is interesting to compare the model derived above with a rent-seeking model where production is exogenous. The model considered so far is a general equilibrium model, whereas rent-seeking models typically assume that the size of the contested rent is constant and are thus partial equilibrium models. Which of the two types of models is more appropriate depends on the application one has in mind (see Neary 1997 for a discussion).

Consider the following rent-seeking model. As above, there are n individuals in G groups of equal size $m = G/n$. They compete over some exogenously given prize of value v . Each individual simultaneously chooses some inter-group rent-seeking effort y_{ig} and some intra-group rent-seeking effort x_{ig} . The payoff of individual i in group g is

$$v_{ig} = p_g r_{ig} v - x_{ig} - y_{ig}$$

where r_{ig} and p_g are as given in equations (2) and (3) above. There are no budget constraints.

For simplicity, I will only consider the case where a symmetric equilibrium exists. Contrary to the conflict model above, symmetric equilibria do not always exist. Mathematically, this difference is due to the fact that $\ln u_{ig}$ is concave and hence a critical point of u_{ig} is a maximum, whereas $\ln v_{ig}$ is not necessarily concave. In fact, critical points of v_{ig} are not always maxima. Implicitly, the assumption of a symmetric equilibrium imposes restrictions on the parameters of the model.⁵

In a symmetric equilibrium, it is clear that x and y must be positive. Hence the derivative of u_{ig} with respect to x_{ig} and y_{ig} has to be zero. Using

⁵For example, suppose $a\gamma = b\delta = 1$ and $G = 2$. Then a symmetric equilibrium exists if $n = 4$, but no symmetric equilibrium exists if $n = 10$ (see appendix).

symmetry, these first order conditions boil down to

$$\begin{aligned} x &= \frac{(m-1)a\gamma}{m^2 G} v = \frac{(n-G)a\gamma}{n^2} v, \\ y &= \frac{(G-1)b\delta}{G^2 m^2} v = \frac{(G-1)b\delta}{n^2} v. \end{aligned}$$

These equations show that, in the partial equilibrium rent-seeking model, neither the group cohesion effect, nor its converse holds: x is independent of b and δ , and y is independent of a and γ . Therefore, the group cohesion effect and its reverse are general equilibrium effects in nature.

However, the effect of dividing contestants into subgroups is qualitatively similar in the rent-seeking and the conflict models. In the rent-seeking model, the total amount of rent-seeking equals

$$x + y = \frac{(n-G)a\gamma + (G-1)b\delta}{n^2} v.$$

Clearly, this is increasing in G if, and only if, $a\gamma < b\delta$. Therefore, proposition 3 above holds in the rent-seeking model, too, and is not due to a general equilibrium effect.

As noted in the introduction, Wärneryd (1998) argues that dividing the contestants in groups leads to less rent-dissipation, although it induces an additional layer of (inter-group) conflict. The model studied here differs from Wärneryd's in two respects. First, in Wärneryd's model the contest between groups takes place first, and only when it is resolved do the contests within groups start. Second, Wärneryd assumes that the contest technology is the same in the intra- and inter-group contests. For a comparison, consider the case where $b\delta = a\gamma$. If inter- and intra-group conflicts take place simultaneously, then total rent-seeking expenditure does not depend on G . Therefore, it is mainly the sequential timing of choices that drives Wärneryd's results.⁶ The intuition for this is clear. If the contests take place sequentially, an individual will reason as follows while deciding how much to spend on the contest between groups: if my group gets a bigger share, my fellow group members will fight harder in the following intra-group contest. This dampens incentives for inter-group rent-seeking. However, this effect is not present in the case of simultaneous inter- and intra-group conflicts, where, in a Nash equilibrium, each player takes the actions of the other players as given.

⁶This corrects the erroneous claim that, in Wärneryd's model, it is of no importance whether the conflicts take place simultaneously or sequentially (see Wärneryd 1998, p. 444 footnote 7, Müller and Wärneryd 2001 p. 531, who attribute this claim to a referee).

7 Conclusion

This paper studied the trade off between production and appropriation in the presence of simultaneous inter- and intra-group conflicts. It gave an economic model of the ‘group cohesion effect’: if the contest between groups becomes more decisive, or the degree of contractual incompleteness between groups increases, this leads to less conflict within groups. Moreover, in the model there is also an ‘reversed group cohesion effect’: if the intra-group contest becomes less decisive, or the degree of contractual incompleteness within groups decreases, this leads to more inter-group conflict.

The model has two normative implications. First, whether a multitiered structure with several groups leads to less rent-seeking activities depends on the decisiveness of the inter- and intra-group contest success functions, and on the amount of contractual incompleteness within groups and between groups. If the inter-group contest is less decisive than the intra-group contest, and contractual problems are less severe between groups than within groups, a multi-tiered structure is beneficial - it leads to less rent-seeking and more production. On the other hand, if the inter-group contest is more decisive, and contractual problems are more severe between groups, a multi-tiered structure leads to more rent-seeking.

Second, there is an optimal size of the organization which is determined by a trade-off between increasing returns to scale in production and increasing costs of rent-seeking.

8 Appendix

8.1 Axiomatic foundation for the inter-group contest success function

In an important paper, Skaperdas (1996) gave an axiomatic characterization of contest success functions. The purpose of this appendix is to extend his work to the case of contests between groups.

Suppose there are n individuals divided into G groups of $m = n/G$ members each. (Extensions to unequal group size are straightforward.) Denote the set of the groups by $\Gamma = \{1, \dots, G\}$. Each individual chooses some inter-group contest effort. Denote the effort chosen by individual i in group g by

x_{ig} . Let $x_g = (x_{1g}, \dots, x_{mg})$ be the m -vector of efforts of the members of group g . Further, define $x_{-g} = (x_1, \dots, x_{g-1}, x_{g+1}, \dots, x_n)$. This is the vector of the efforts of all individuals except those in group g . Finally, let $x = (x_1, \dots, x_n)$ be the n -vector collecting all the individual efforts. I sometimes write (x_g, x_{-g}) to denote x .

For any group $g = 1, \dots, G$, it is assumed that there exists some function $p_g(x)$ which can be interpreted as the probability that group g wins the contest. Alternatively, $p_g(x)$ can also be interpreted as the share of the cake that group g gets. The following axioms adapt the axioms Skaperdas (1996) laid out for the case of a contest within a group to the case of inter-group conflict.

Assumption 1) (Probability) $\sum_{g=1}^G p_g(x) = 1$; $p_g(x) \geq 0$ for all x and all $g \in \Gamma$; if $x_i > 0$ for some $i \in g$, then $p_g(x) > 0$.

Assumption 2) (Monotonicity) For all $i = 1, \dots, m$ and all $g, k \in \Gamma$ with $g \neq k$, $p_g(x)$ is increasing in x_{ig} and decreasing in x_{ik} .

Assumption 3) (Group Anonymity) For any bijection $\psi : \Gamma \rightarrow \Gamma$ we have $p_{\psi(g)} = p_g(x_{\psi(1)}, x_{\psi(2)}, \dots, x_{\psi(G)})$.

Assumption 1 says that p_g is a probability, and that any group has a positive chance to win if at least one of the group's members puts in positive effort. Assumption 2 says that the winning probability is increasing in the expenditure of the group members, and is decreasing in the expenditure of the other players. Assumption 3 says that the labeling of the groups does not matter: the winning probabilities are determined by the vector of efforts alone. This means that the contest is 'fair' between the groups.

Assumption 4) (Subcontest consistency) Let $p_g^M(x)$ be group g 's probability of winning a subcontest played by a subset M of the groups. Then

$$p_g^M(x) = \frac{p_g(x)}{\sum_{k \in M} p_k(x)}$$

for all $g \in M$ and all $M \subset \Gamma$.

Assumption 5) (Subcontest independence) $p_g^M(x)$ is independent of expenditure of individuals belonging to groups not in M . That is, $p_g^M(x)$ may be written as $p_g^M(x^M)$, where $x^M = \{x_g \mid g \in M\}$.

Assumption 6) (Homogeneity) $p_g(\lambda x) = p_g(x)$ for all $\lambda > 0$ and all $g \in \Gamma$.

Assumptions 1-6 simply reformulate the assumptions in Skaperdas (1996) for an inter-group contest. In particular, if there is only one individual in each group, Assumptions 1-7 are identical to the corresponding assumptions in Skaperdas (1996).

In addition, I will make the following

Assumption 7) (Efficient aggregation)

$$p_g^M((x_{1g}, \dots, x_{ig} - \Delta, x_{jg} + \Delta, \dots, x_{mg}), x_{-g}) = p_g^M(x_g, x_{-g})$$

for all $\Delta \leq x_{ig}$, all x , and all $i, j \in \{1, \dots, m\}$, $g \in M \subseteq \Gamma$.

Let $\hat{x}_g = (\sum_{i=1}^m x_{ig}, 0, \dots, 0)$ and $\hat{x}_{-g} = (\hat{x}_1, \dots, \hat{x}_{g-1}, \hat{x}_{g+1}, \dots, \hat{x}_n)$. Now we can show the following.

Lemma 2 $p_g(\hat{x}_g, \hat{x}_{-g}) = p_g(x_g, x_{-g})$.

Proof. From assumption 7, taking $M = \Gamma$, it follows that $p_g(\hat{x}_g, x_{-g}) = p_g(x_g, x_{-g})$.

Next we will show that

$$p_g(x_1, \dots, x_{h-1}, x_h, x_{h+1}, \dots, x_n) = p_g(x_1, \dots, x_{h-1}, \hat{x}_h, x_{h+1}, \dots, x_n).$$

I will show this for groups 1 and 2. In words, if we replace x_2 by \hat{x}_2 , this does not change the winning probability group 1.

To see this, consider the subcontest among the groups 1 and 2. By assumption 1 and subcontest consistency, we get

$$\begin{aligned} p_1^{\{1,2\}}(x) &= 1 - p_2^{\{1,2\}}(x) = \\ &= 1 - p_2^{\{1,2\}}(x_1, \hat{x}_2, x_3, \dots, x_n) = \\ &= p_1^{\{1,2\}}(x_1, \hat{x}_2, x_3, \dots, x_n) \end{aligned}$$

where the second equality uses assumption 7, taking $M = \{1, 2\}$.

Using subcontest consistency, we get

$$\frac{p_1(x)}{p_1(x) + p_2(x)} = \frac{p_1(x_1, \hat{x}_2, x_3, \dots, x_n)}{p_1(x_1, \hat{x}_2, x_3, \dots, x_n) + p_2(x_1, \hat{x}_2, x_3, \dots, x_n)}$$

Since by assumption 7 (taking $M = \Gamma$) we have

$$p_2(x) = p_2(x_1, \hat{x}_2, x_3, \dots, x_n)$$

we can conclude that

$$\frac{p_1(x)}{p_1(x) + p_2(x)} = \frac{p_1(x_1, \hat{x}_2, x_3, \dots, x_n)}{p_1(x_1, \hat{x}_2, x_3, \dots, x_n) + p_2(x)}$$

and hence

$$p_1(x) = p_1(x_1, \hat{x}_2, x_3, \dots, x_n).$$

This completes the proof. ■

The lemma shows that the winning probabilities depend only on the sum of each group's expenditures. Hence we can view an inter-group contest as a contest between individuals where, for each group, one individual puts in the whole expenditure of the group. Denote the contest success function for this contest by $\hat{p}_g : R^G \rightarrow [0, 1]$ (note this is a function of G arguments only):

$$\hat{p}_g \left(\sum_{i=1}^m x_{i1}, \dots, \sum_{i=1}^m x_{iG} \right) := p_g(\hat{x}_g, \hat{x}_{-g}).$$

By the previous lemma,

$$\hat{p}_g \left(\sum_{i=1}^m x_{i1}, \dots, \sum_{i=1}^m x_{iG} \right) = p_g(x) \quad \forall x.$$

Next, we have to show that the function \hat{p}_g satisfies the assumptions of Skaperdas (1996). But this is obvious because $p_g(\hat{x}_g, \hat{x}_{-g})$ satisfies the corresponding assumptions for an inter-group contest. Therefore it is clear that the proofs in Skaperdas (1996) apply to \hat{p}_g as well, and we can conclude with

Remark 1 *The only continuous contest success function satisfying Assumptions 1-7 is*

$$p_g(x) = \frac{\left(\sum_{j=1}^m x_{ig} \right)^b}{\sum_{k=1}^G \left(\sum_{j=1}^m x_{ik} \right)^b}$$

for some $b > 0$.

8.2 Utility is log-concave if $0 < a \leq 1$ and $0 < b \leq 1$

From equation (4), we get

$$\begin{aligned} \ln u_{ik} = & \underbrace{\ln \left(\gamma \frac{\left(\sum_{j=1}^m y_{jk} \right)^b}{\sum_{g=1}^G \left(\sum_{j=1}^n y_{jg} \right)^b} + \frac{(1-\delta)}{G} \right)}_{A(y_{ik})} + \\ & + \underbrace{\ln \left(\gamma \frac{x_{ik}^a}{\sum_j x_{jk}^a} + \frac{(1-\gamma)}{m} \right)}_{B(x_{ik})} + \\ & + \underbrace{h \ln \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right)}_{C(x_{ik}, y_{ik})}. \end{aligned}$$

We want to show that this is strictly concave in (x_{ik}, y_{ik}) .

Let us look at the terms in turn. For notational convenience, let $\sum_{j \neq i}^m y_{jk} =: Y$ and $\sum_{g \neq k}^G \left(\sum_{j=1}^n y_{jg} \right)^b =: Z$. Then

$$A(y_{ik}) = \ln \left(\gamma \frac{(y_{ik} + Y)^b}{(y_{ik} + Y)^b + Z} + \frac{(1-\delta)}{G} \right).$$

By differentiation, we get

$$A'(y_{ik}) = \frac{1}{\left(\gamma \frac{(y_{ik} + Y)^b}{(y_{ik} + Y)^b + Z} + \frac{(1-\delta)}{G} \right)} \gamma \frac{bZ (y_{ik} + Y)^{b-1}}{\left((y_{ik} + Y)^b + Z \right)^2}.$$

Since $b \leq 1$, the numerator is decreasing in y_{ik} . The denominator is increasing in y_{ik} . Hence $A''(y_{ik}) < 0$.

Now to the second term. Let $\sum_{j \neq i} x_{j1}^a =: X$ for notational convenience. Then

$$B(x_{ik}) = \ln \left(\gamma \frac{x_{ik}^a}{x_{ik}^a + X} + \frac{(1-\gamma)}{m} \right).$$

Differentiating,

$$B'(x_{ik}) = \frac{1}{\left(\gamma \frac{x_{ik}^a}{x_{ik}^a + X} + \frac{(1-\gamma)}{m}\right)} \gamma \frac{aX x_{ik}^{a-1}}{(x_{ik}^a + X)^2}.$$

Since $a \leq 1$, the numerator is decreasing in x_{ik} . The denominator is increasing in x_{ik} . Hence $B''(x_{ik}) < 0$.

Now consider the third term. For notational convenience, define

$$W := \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right) + x_{ik} + y_{ik}.$$

Then $C(x_{ik}, y_{ik}) = h \ln(W - x_{i1} - y_{i1})$. Differentiating, we find

$$\frac{\partial^2 C}{\partial x_{ik}^2} = \frac{\partial^2 C}{\partial y_{ik}^2} = \frac{\partial^2 C}{\partial x_{ik} \partial y_{ik}} = -\frac{h}{(W - x_{ik} - y_{ik})^2} < 0$$

The determinant of the Hessian matrix is zero. Hence the Hessian matrix is negative semidefinite, and $C(x_{ik}, y_{ik})$ is concave in (x_{ik}, y_{ik}) .

Finally, we can put things together and show that $\ln u_{ik}$ is strictly concave in (x_{i1}, y_{i1}) .

Write $\ln(u_{ik}(x, y)) = A(y) + B(x) + C(x, y)$. For any $(x, y), (x', y') \in R_+^2$ and any $t \in (0, 1)$, we have

$$\begin{aligned} & \ln(u_{ik}(tx + (1-t)x', ty + (1-t)y')) = \\ & = A(ty + (1-t)y') + B(tx + (1-t)x') + C(tx + (1-t)x', ty + (1-t)y') < \\ & < tA(y) + (1-t)A(y') + tB(x) + (1-t)B(x') + tC(x, y) + (1-t)C(x', y') = \\ & = t \ln(u_{ik}(x, y)) + (1-t) \ln(u_{ik}(x', y')). \end{aligned}$$

Hence $\ln u_{ik}(x_{ik}, y_{ik})$ is strictly concave in (x_{ik}, y_{ik}) .

8.3 Utility is log-concave if $\gamma = \delta = 1$

If $\gamma = \delta = 1$, then we find that

$$A'(y_{ik}) = \frac{b}{y_{ik} + Y} - \frac{b(y_{ik} + Y)^{b-1}}{(y_{ik} + Y)^b + Z}$$

and

$$A''(y_{ik}) = -\frac{b}{(y_{ik} + Y)^2} - \frac{\left((y_{ik} + Y)^b + Z\right) b(b-1)(y_{ik} + Y)^{b-2} - \left(b(y_{ik} + Y)^{b-1}\right)^2}{\left(\left((y_{ik} + Y)^b + Z\right)^2\right)}.$$

Simplifying, we get

$$A''(y_{ik}) = -\frac{b}{(y_{ik} + Y)^2} Z \frac{(y_{ik} + Y)^b + Z + (y_{ik} + Y)^b b}{\left(\left((y_{ik} + Y)^b + Z\right)^2\right)} < 0.$$

Further,

$$B'(x_{ik}) = \frac{a}{x_{ik}} - \frac{ax_{ik}^{a-1}}{(x_{ik}^a + X)}$$

and

$$B''(x_{ik}) = -\frac{a}{(x_{ik})^2} - \frac{(x_{ik}^a + X) a(a-1)x_{ik}^{a-2} - (ax_{ik}^{a-1})^2}{(x_{ik}^a + X)^2}.$$

Simplifying, we get

$$B''(x_{ik}) = -aX \frac{x_{ik}^a + X + ax_{ik}^a}{x_{ik}^2 (x_{ik}^a + X)^2} < 0.$$

As above, it follows that $\ln u_{ik}$ is strictly concave in (x_{ik}, y_{ik}) . Note that we needed no upper bounds on a and b .

8.4 Comparative statics with a general production function

In this section I derive the comparative statics with the more general production technology given in equation (13) $q = f\left(\sum_{g=1}^G \sum_{i=1}^m e_{ig}\right)$, where the function f is strictly positive, twice differentiable and strictly increasing.

The following proposition 1* generalizes proposition 1 from the main text.

Proposition 1* *Suppose that output is given in equation (13) and f is log-concave.*

a) If the contest between the groups becomes more decisive (i.e. b increases) and/or contractual incompleteness between groups becomes more serious (i.e. δ increases), then rent-seeking within groups x and productive effort e decline, while inter-group rent-seeking y increases.

b) If the intra-group contests become more decisive (i.e. a increases) and/or contractual incompleteness within groups becomes more serious (i.e. γ increases), then rent-seeking between groups y and productive effort e decline, while intra-group rent-seeking x increases.

Proof. As argued in the main text, after imposing symmetry the first order conditions imply equations (14) and (15), that I reproduce here for convenience:

$$\begin{aligned} my &= \delta b \frac{(G-1)}{G} \frac{f(n(1-x-y))}{f'(n(1-x-y))} \\ x &= \gamma a \frac{m-1}{m} \frac{f(n(1-x-y))}{f'(n(1-x-y))} \end{aligned}$$

Define

$$\phi(z) := \frac{f(z)}{f'(z)}$$

Note that $\phi(z) > 0$, and $\phi'(z) > 0$ since f is assumed to be log-concave. Total differentiation of equations (14) and (15) leads to

$$M \begin{pmatrix} dy \\ dx \end{pmatrix} = N \begin{pmatrix} a \\ b \\ \gamma \\ \delta \end{pmatrix}$$

where

$$M = \begin{bmatrix} m + \delta b \frac{(G-1)}{G} n \phi' & \delta b \frac{(G-1)}{G} n \phi' \\ \gamma a \frac{m-1}{m} n \phi' & 1 + \gamma a \frac{m-1}{m} n \phi' \end{bmatrix},$$

and

$$N = \begin{bmatrix} 0 & \delta \frac{(G-1)}{G} \phi & 0 & b \frac{(G-1)}{G} \phi \\ \gamma \frac{m-1}{m} \phi & 0 & a \frac{m-1}{m} \phi & 0 \end{bmatrix}$$

(For brevity, I omit the arguments of the functions ϕ and ϕ').

The determinant of the matrix M equals

$$|M| = \frac{mG + \phi' a n G \gamma (m-1) + \delta b n \phi' (G-1)}{G} > 0$$

Now we can prove part a).

$$\frac{dx}{db} = -\frac{1}{|M|}\gamma a \frac{m-1}{m} n\phi' \delta \frac{(G-1)}{G} \phi < 0.$$

Moreover,

$$\frac{dy}{db} = \frac{1}{|M|}\delta \frac{(G-1)}{G} \phi \left(1 + \gamma a \frac{m-1}{m} n\phi'\right) > 0.$$

Finally,

$$\begin{aligned} \frac{de}{db} &= -\left(\frac{dx}{db} + \frac{dy}{db}\right) = \\ &= -\frac{1}{|M|}\delta \frac{(G-1)}{G} \phi < 0. \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{dx}{d\delta} &= -\frac{1}{|M|}\gamma a \frac{m-1}{m} n\phi' b \frac{(G-1)}{G} \phi < 0, \\ \frac{dy}{d\delta} &= \frac{1}{|M|}b \frac{(G-1)}{G} \phi \left(1 + \gamma a \frac{m-1}{m} n\phi'\right) > 0, \\ \frac{de}{d\delta} &= -\frac{1}{|M|}b \frac{(G-1)}{G} \phi < 0. \end{aligned}$$

Part b).

$$\begin{aligned} \frac{dx}{da} &= \frac{1}{|M|} \left(m + \delta b \frac{(G-1)}{G} n\phi'\right) \gamma \frac{m-1}{m} \phi > 0, \\ \frac{dy}{da} &= -\frac{1}{|M|}\delta b \frac{G-1}{G} n\phi' \gamma \frac{m-1}{m} \phi < 0, \\ \frac{de}{da} &= -\frac{1}{|M|}\gamma (m-1) \phi < 0. \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{dx}{d\gamma} &= \frac{1}{|M|} \left(m + \delta b \frac{(G-1)}{G} n\phi'\right) a \frac{m-1}{m} \phi > 0, \\ \frac{dy}{d\gamma} &= -\frac{1}{|M|}\delta b \frac{G-1}{G} n\phi' a \frac{m-1}{m} \phi < 0, \\ \frac{de}{d\gamma} &= -\frac{1}{|M|}a (m-1) \phi < 0. \end{aligned}$$

■

8.5 Unequal group size

In this section I study the case of $G = 2$ groups of different size, which was briefly discussed in the main text at the end of section 5.

Denote the number of individuals in group g by m_g . The objective function of player $i = 1, \dots, m_g$ in group $g = 1, 2$ can be written as

$$u_{ig} = p_g r_{ig} \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right)^h$$

where

$$p_g = \delta \frac{\left(\sum_{j=1}^{m_g} y_{jg} \right)^b}{\left(\sum_{j=1}^{m_1} y_{j1} \right)^b + \left(\sum_{j=1}^{m_2} y_{j2} \right)^b} + \frac{(1 - \delta)}{2},$$

$$r_{ig} = \gamma \frac{x_{ig}^a}{\sum_{j=1}^{m_g} x_{jg}^a} + (1 - \gamma) \frac{1}{m_g}$$

As mentioned in the main text, I will concentrate on symmetric interior equilibria, where

$$\begin{aligned} (x_{i1}, y_{i1}) &= (x_1, y_1) \forall i = 1, \dots, n_1, \\ (x_{i2}, y_{i2}) &= (x_2, y_2) \forall i = 1, \dots, n_2. \end{aligned}$$

and $0 < x_{ig} < 1, 0 < y_{ig} < 1, 0 < e_{ig} < 1$.

Remark 2 *In a symmetric interior equilibrium, both groups get the same share of the output*

$$p_1 = p_2 = \frac{1}{2},$$

and the share of an individual $i = 1, \dots, m_g$ in group $g = 1, 2$ equals

$$p_i r_{ig} = \frac{1}{2m_g}.$$

Proof. Writing out the first order condition $\frac{\partial u_{i1}}{\partial y_{i1}} = 0$, we get

$$\begin{aligned} & \delta \frac{b \left(\sum_{j=1}^{m_1} y_{j1} \right)^{b-1} \left(\sum_{j=1}^{m_2} y_{j2} \right)^b}{\left(\left(\sum_{j=1}^{m_1} y_{j1} \right)^b + \left(\sum_{j=1}^{m_2} y_{j2} \right)^b \right)^2} \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right) \\ &= \left(\delta \frac{\left(\sum_{j=1}^{m_1} y_{j1} \right)^b}{\left(\sum_{j=1}^{m_1} y_{j1} \right)^b + \left(\sum_{j=1}^{m_2} y_{j2} \right)^b} + \frac{(1 - \delta)}{2} \right) h \end{aligned}$$

Imposing symmetry, we have

$$\begin{aligned} & \delta \frac{b (m_1 y_1)^{b-1} (m_2 y_2)^b}{\left((m_1 y_1)^b + (m_2 y_2)^b \right)^2} (n - m_1 (x_1 + y_1) - m_2 (x_2 + y_2)) \\ &= \left(\delta \frac{(m_1 y_1)^b}{(m_1 y_1)^b + (m_2 y_2)^b} + \frac{(1 - \delta)}{2} \right) h \end{aligned}$$

Analogously

$$\begin{aligned} & \delta \frac{b (m_2 y_2)^{b-1} (m_1 y_1)^b}{\left((m_1 y_1)^b + (m_2 y_2)^b \right)^2} (n - m_1 (x_1 + y_1) - m_2 (x_2 + y_2)) \\ &= \left(\delta \frac{(m_2 y_2)^b}{(m_1 y_1)^b + (m_2 y_2)^b} + \frac{(1 - \delta)}{2} \right) h \end{aligned} \tag{17}$$

Dividing the previous equations, we get

$$\frac{m_2 y_2}{m_1 y_1} = \frac{\delta \frac{(m_1 y_1)^b}{(m_1 y_1)^b + (m_2 y_2)^b} + \frac{(1 - \delta)}{2}}{\delta \frac{(m_2 y_2)^b}{(m_1 y_1)^b + (m_2 y_2)^b} + \frac{(1 - \delta)}{2}}$$

Since the left hand side is increasing in y_2 while the right hand side is decreasing in y_2 , this implies

$$m_1 y_1 = m_2 y_2. \tag{18}$$

Hence we can conclude that the shares of the groups are $p_1 = p_2 = 1/2$. By symmetry, an individual i in group g gets $r_{ig} = 1/m_g$ of what his group gets.

■

To study existence of such an equilibrium, let us derive a candidate equilibrium and then check whether all the constraints hold. Insert equation (18) into equation (17) and solve for

$$m_1 y_1 = m_2 y_2 = \frac{1}{2} \delta b \frac{n - m_1 x_1 - m_2 x_2}{\delta b + h} \quad (19)$$

Writing out the first order condition $\frac{\partial u_{i1}}{\partial x_{i1}} = 0$, we get

$$\gamma \frac{a x_{i1}^{a-1} \left(\sum_{j \neq i} x_{j1}^a \right)}{\left(\sum_j x_{j1}^a \right)^2} \left(\sum_g \sum_j (1 - x_{jg} - y_{jg}) \right) = \left(\gamma \frac{x_{i1}^a}{\sum_j x_{j1}^a} + (1 - \gamma) \frac{1}{m_1} \right) h$$

Imposing symmetry,

$$\gamma \frac{a(m_1 - 1)}{m_1} (n - m_1(x_1 + y_1) - m_2(x_2 + y_2)) = h x_1 \quad (20)$$

Similarly, for group 2

$$\gamma \frac{a(m_2 - 1)}{m_2} (n - m_1(x_1 + y_1) - m_2(x_2 + y_2)) = h x_2 \quad (21)$$

Solving equations (19), (20), and (21) we find

$$x_g = \frac{(m_g - 1)}{m_g} \frac{a \gamma n}{(\gamma a (n - 2) + \delta b + h)} > 0, \quad (22)$$

$$y_g = \frac{n}{2m_g} \frac{b \delta}{\gamma a (n - 2) + \delta b + h} > 0. \quad (23)$$

(Note that if $m_1 = m_2 = n/2$, then $(m_g - 1) n / m_g = n - 2$ and these expressions simplify to the corresponding expressions in the main text.) Using the budget constraint, we get

$$e_g = \frac{(n - 2m_g)(2\gamma a - \delta b) + 2m_g h}{2m_g (\gamma a (n - 2) + \delta b + h)} \quad (24)$$

If the technology is sufficiently productive, then $e_g > 0$ for both groups and no constraints are violated; hence equations (22), (23) and (24) describe a symmetric interior equilibrium. The following remark makes this precise.

Remark 3 *A symmetric interior equilibrium exists if and only if*

$$h \geq \left(1 - \frac{n}{2m_g}\right) (2\gamma a - \delta b) \text{ for } g = 1, 2. \quad (25)$$

In the case of equal group size, condition (25) holds trivially since $h \geq 0$. An easy example with unequal group size where existence is ensured is $\gamma a = \delta b = h = 1$. Here, $e_g = y_{ig} = 1/(2m_g)$, $x_g = (m_g - 1)/m_g$. However, with unequal group size, there is no symmetric interior equilibrium if h is too small: if $h \rightarrow 0$, $\text{sgn}(e_g) = \text{sgn}((n - 2m_g)(2\gamma a - \delta b))$, but this is negative for at least one group. For the rest of this section, I will assume that (25) holds.

Finally, we can turn to welfare considerations.

Remark 4 *If $a\gamma = b\delta$, total output is the same when there are two groups and when there is a unified organization (only one group). The members of the smaller group prefer the situation with two groups, but members of the bigger group would rather have a unified organization.*

Proof. With two groups, output equals

$$q^{II} = (m_1 e_1 + m_2 e_2)^h = \left(\frac{nh}{\gamma a(n-1) + \delta b - \gamma a + h} \right)^h$$

With one group (see lemma 1 in the main text), output equals

$$q^I = \left(\frac{nh}{a\gamma(n-1) + h} \right)^h$$

Clearly, $q^I > (=, <) q^{II}$ if and only if $\delta b > (=, <) \gamma a$.

If $a\gamma = b\delta$, equilibrium utility with two groups equals

$$u_{ig}^{II} = \frac{1}{2m_g} \left(\frac{nh}{a\gamma(n-1) + h} \right)^h.$$

Equilibrium utility with one group equals

$$u_{ig}^I = \frac{1}{n} \left(\frac{nh}{a\gamma(n-1) + h} \right)^h.$$

Obviously, $u_{ig}^{II} >^I_{ig}$ if and only if $m_g < \frac{n}{2}$. ■

8.6 Existence of symmetric equilibria in the rent-seeking model: two examples

This section studies two examples for the (non-) existence of symmetric equilibria in the rent-seeking model discussed in section 6.

Example 1 *Let $a\gamma = b\delta = 1$, $G = 2$, and $n = 4$. Then there exists a symmetric equilibrium of the rent-seeking model, where*

$$x = \frac{1}{8}v, \quad y = \frac{1}{16}v. \quad (26)$$

To prove this, suppose that three players behave according to equation (26). The problem of the remaining player is

$$\frac{y + \frac{1}{16}v}{y + \frac{3}{16}v} \frac{x}{x + \frac{1}{8}v} v - x - y \rightarrow \max_{x,y} \text{ subject to } x \geq 0, y \geq 0. \quad (27)$$

We have to show that the solution of this problem is given in equation (26).

1. The player can always ensure zero utility by choosing $x = y = 0$. Hence we can constrain our search to pairs (x, y) with $x + y \leq v$. Now we have a maximization problem of a continuous function over a compact domain. By the Weierstrass theorem, a solution exists.
2. The constraint $x \geq 0$ is not binding in the optimum. Suppose it were. Then utility is at most zero. But the player can get a positive utility by behaving as in equation (26). Contradiction.
3. The constraint $y \geq 0$ is not binding, either. Suppose it were. Then the best the player can do is to choose $x \geq 0$ to solve

$$\frac{1}{3} \frac{x}{x + \frac{1}{8}v} v - x \rightarrow \max_{x \geq 0}$$

The objective function is concave. The optimal choice of x is given by

$$x = -\frac{1}{8}v + \frac{1}{12}v\sqrt{6} > 0.$$

The utility equals

$$\frac{1}{3} \frac{-\frac{1}{8}v + \frac{1}{12}\sqrt{6}v}{-\frac{1}{8}v + \frac{1}{12}\sqrt{6}v + \frac{1}{8}v} v - \left(-\frac{1}{8} + \frac{1}{12}\sqrt{6}\right)v = \left(\frac{11}{24} - \frac{1}{6}\sqrt{6}\right)v$$

But by behaving as in equation (26), the player gets utility

$$\frac{v}{4} - \frac{v}{8} - \frac{v}{16} = \frac{v}{16} > \left(\frac{11}{24} - \frac{1}{6}\sqrt{6} \right) v.$$

Contradiction.

4. Of course, the constraint $x + y \leq v$ is not binding, either.
5. Hence, the first order conditions have to hold at the optimum:

$$\begin{aligned} \frac{\frac{2}{16}v}{\left(y + \frac{3}{16}v\right)^2} \frac{x}{x + \frac{1}{8}v} v &= 1 \\ \frac{y + \frac{1}{16}v}{y + \frac{3}{16}v} \frac{\frac{1}{8}v}{\left(x + \frac{1}{8}v\right)^2} v &= 1 \end{aligned} \tag{28}$$

6. Dividing these equations yields

$$32x \frac{8x + v}{(16y + 3v)(16y + v)} = 1.$$

Solving for x we get two possibilities: $x = -y - \frac{3}{16}v$ or $x = y + \frac{1}{16}v$. Since $x > 0$ and $y > 0$ at the optimum, we can exclude the first possibility. This leaves us with

$$x = y + \frac{1}{16}v. \tag{29}$$

7. Plugging this back into equation (28) gives us

$$\frac{\frac{2}{16}v}{\left(y + \frac{3}{16}v\right)^2} \frac{y + \frac{1}{16}v}{y + \frac{1}{16}v + \frac{1}{8}v} v = 1$$

or

$$32v^2 \frac{16y + v}{(16y + 3v)^3} = 1$$

This equation has three roots, one positive root $y_I = \frac{1}{16}v$, and two negative roots: $y_{II} = \left(-\frac{5}{16} + \frac{1}{8}\sqrt{5}\right)v \approx -3.3 \times 10^{-2}v < 0$ and $y_{III} = \left(-\frac{5}{16} - \frac{1}{8}\sqrt{5}\right)v \approx -0.59v < 0$. Hence we know that $y = \frac{v}{16}$.

8. Plugging this back in equation (29) we get $x = \frac{v}{8}$.

9. The problem (27) has a solution (step 1), the solution satisfies $x > 0$ and $y > 0$ (steps 2 and 3), and it is a critical point (step 5). The only critical point satisfying $x > 0$ and $y > 0$ is $x = \frac{v}{8}$ and $y = \frac{v}{16}$, as in equation (26) (steps 6-8). Hence, the solution is indeed given by equation (26). This completes the proof.

It might be reassuring to check the local second order condition. The Hessian corresponding to the objective function in problem (27) is

$$H(x, y) = \begin{bmatrix} -128 \frac{16y+v}{(16y+3v)(8x+v)^3} v^2 & \frac{256}{(16y+3v)^2 (8x+v)^2} v^3 \\ \frac{256}{(16y+3v)^2 (8x+v)^2} v^3 & -8192 \frac{x}{(16y+3v)^3 (8x+v)} v^2 \end{bmatrix}$$

Hence,

$$H\left(\frac{v}{8}, \frac{v}{16}\right) = \begin{bmatrix} -\frac{8}{v} & \frac{4}{v} \\ \frac{4}{v} & -\frac{8}{v} \end{bmatrix}$$

is negative definite, and the local second order condition holds.

Example 2 Let $a\gamma = b\delta = 1$, $G = 2$, and $n = 10$. Then no symmetric equilibrium exists in the rent-seeking model.

To see this, note that there is a unique candidate for a symmetric equilibrium, where

$$x = \frac{(10-2)}{10^2}v, \quad y = \frac{v}{10^2}.$$

In this candidate equilibrium, a player gets utility $v/100$. However, if a player deviates to $y = 0$ and $x = 6v/(100)$, he gets

$$\frac{\frac{4v}{100}}{\frac{4v}{100} + \frac{5v}{100}} \frac{\frac{6v}{100}}{\frac{6v}{100} + 4\left(\frac{8}{100}v\right)} v - \frac{6v}{100} = \frac{29}{2850}v > \frac{v}{100}.$$

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