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Fertility Assimilation of Immigrants: A Varying Coefficient Count Data Model

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Abstract. This study presents the first econometric application of the Poisson varying coefficient (PVC) model. This count data model is applied to investigate immigrant fertility adjustment after migration. Data on completed fertility are taken from the 1996 wave of the German Socioeconomic Panel (GSOEP). We find evidence in favor of the assimilation model according to which immigrant fertility converges to native levels over time. Other determinants of completed fertility are marital history and female human capital, well in accordance with theoretical predictions.

JEL Category: C14, C11, C25, J13, J61.
Keywords: Count Data Estimation, Varying Coefficients, Immigrant Fertility, Assimilation, Disruption.

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1 Introduction

The assimilation of immigrants to destination country standards is discussed in a wide and growing literature. Past assimilation research has focused on labor market aspects such as earnings, unemployment[1], or transfer program participation[2]. With continuously accelerating flows of international migration (cf. Segal, 1993) issues of *demographic* assimilation, increasingly gain in importance as well.

This paper contributes to the literature on immigrant fertility assimilation, applying a newly developed econometric method. Since the early contribution of Ben-Porath (1973) the literature on immigrant fertility has been debating whether immigrant fertility adjustment should be explained in a framework of fertility assimilation or in a model of fertility disruption. The assimilation model predicts that immigrant fertility converges to native levels, whereas the disruption model suggests increasing fertility following the disruptive effect of migration itself. With rising population shares of immigrants in western countries[3], and given ongoing debates of appropriate immigration policies, this is an important issue to investigate. Also, immigrant fertility has direct implications for the labor market involvement of the first generation, and - due to tradeoffs between the demand for child quantity and child quality - indirect effects on the human capital of second generation immigrants.

While almost all fertility adjustment studies investigate the case of immigrants to the United States (using decennial census data) this analysis focuses on migration to Europe. The selection and attraction mechanisms causing migration to Europe may differ considerably from those relevant for the United States, which may affect subsequent immigrant behavior.

In the literature on immigrant fertility adjustment over the duration of stay in the destination country it is accepted that duration and immiration year effects cannot be separately identified on the basis of cross-section data. We argue that the fertility literature, which unanimously controls for years since migration, has applied an inappropriate duration measure: When one is interested in fertility outcomes it is not the total duration of stay which should affect the number of births but the duration of stay in the receiving country which occurs *during a woman's reproductive phase*. In other words, whether a woman who migrated at age 35 has been in the country for 10 or 20 years will hardly make a difference

[1]See e.g. Schmidt 1995, Bauer and Zimmermann 1997, Schoeni 1998, or Chiswick et al. 1997.

[2]See e.g. Baker and Benjamin 1995, Hu 1998, Borjas and Hilton 1996, Riphahn 1999.

[3]The population share of immigrants in Germany grew from 1 percent in the 1950s to about 10 percent today; similarly, immigrants made up more than 10 percent of the 1990 population in countries such as Canada, Australia, or France (Segal, 1993).

for her completed fertility. What matters is the number of fertile years spent in the receiving country. This issue has been overlooked in the existing literature on fertility. An interesting consequence of this correction in variable definitions is that now cross-section data are sufficient to separately identify the effects of the number of fertile years in the host country and the year of immigration.

The investigation of the determinants of completed fertility, with an integer number of births as the outcome measure, warrants the application of count data estimation techniques. Since we are interested in the effect of fertile time spent in the host country on immigrants' completed fertility, we apply the recently developed Poisson varying coefficient (PVC) model in which the coefficient estimates themselves are modelled as functions of the number of fertile years a woman spent in the host country. The core advantage of the PVC model is that it combines the flexibility of a non-parametric estimation approach with the transparency of a fully parametric model. In this framework we estimate the *functional form* of the impact of 'fertile years spent in Germany' on completed fertility, where typically linear effects are imposed. The adjustment process in immigrant fertlity is reflected directly and in the most flexible way in the coefficient estimates.

The paper proceeds as follows: After a discussion of the literature on models of immigrant fertility adjustment and a review of past findings in section two, we provide a brief description of our data, which are taken from the German Socioeconomic Panel (GSOEP). Section four explains in some detail the estimation approach. The results are discussed and interpreted in part five of the paper before we conclude in section six.

2 The Assimilation of Immigrant Fertility

While many studies have analysed immigrant fertility adjustments, few justify their hypotheses using economic arguments. This is surprising because the economic theory of fertility provides convincing rationales for fertility adjustments after migration (for a survey see Hotz et al. 1997). Following e.g. Becker (1981), couples' demand for children can be modelled as a function of prices and income: Among the relevant prices are the (potential) wage of the wife, which is frequently approximated by her human capital, the cost of child care, and the cost of fertility regulation. Husbands' earnings are the source of income effects. The model predicts that the demand for children declines if the opportunity cost of the wife's time, her potential wage, increases. Thus one reason for fertility adjustment after migration may be that potential wages in the destination country differ from women's earnings potential at home. The effects of husbands' income on fertility demand predicted by theory are ambiguous. On the one hand a higher income may increase the demand for child quantity, because the costs of children become affordable. On the other hand higher incomes increase the demand for child quality. Child quality raises the cost per child and thus justifies a negative correlation between income and the demand for children. Again, with different incomes in the origin and host countries, couples may adjust their fertility plans after migration.

This demand focused model of fertility ('Chicago-Columbia model') contrasts with the 'Pennsylvania' model, which also considers supply side factors of fertility determination, in particular a couples' fecundity and the cost of fertility regulation (e.g. Easterlin 1987, or Rosenzweig and Schultz 1985). This perspective provides another justification for the adjustment of immigrant fertility from origin to destination country levels: not only may potential incomes converge to the receiving country's standards, also cost and availability of contraception may differ from those in the country of origin.

Thus economic fertility theory yields three immediate arguments for fertility adjustments of immigrants: Changes in female wages, in male incomes, and the price of fertility regulation. The relevant demographic and economic literature [4], however, has focused on a separate line of argument in the analysis of immigrant behavior, and juxtaposes two models of fertility adjustment neglecting the above given arguments. The assimilation model suggests that couples, who migrate from a high fertility country to a low fertility country, initially follow traditional high fertility patterns, and over time adjust to the lower fertility in the destination country. Therefore it is hypothesized that the difference in completed fertility between natives and immigrants falls, the earlier in a woman's reproductive career migration to the destination country occurs. In constrast, the disruption model stresses that migration itself causes an initial drop in couples' fertility and that, subsequently, fertility will rise again. This model does not explain the level of initial or final immigrant fertility relative to the native population, but argues in terms of the direction of adjustments in period-specific - though not necessarily completed - fertility.

The two models lead to different conclusions with respect to two aspects of immigrant fertility: First, they differ with respect to the direction of short-term fertility adjustment. The assimilation model considers a slow decline in fertility and the disruption model expects an increase in fertility after the disruptive migration event. Second, the migration effect on completed fertility may differ in the two scenarios: Since in the assimilation framework migrants generally have above native level fertility until assimilation is completed, they will have higher levels of completed fertility. This "excess fertility" beyond the native level should decline, the longer a couple spent during its fertile years in the receiving country.

[4] See e.g. Blau 1992, Schoorl 1990, Gorwaney et al. 1990, Kahn 1994, or Ford 1990.

This is not clear in the case of the disruption model. Here completed fertility may fall below the levels of the country of origin, due to temporary disruption. However, the U.S. literature suggests that fertility may well be postponed to later years, in which case the level of home country fertility may be maintained. In neither scenario do we expect to see a decline in completed fertility as a function of the time spent in the receiving country. [5]

Therefore a test between the two models has to evaluate first the total difference in cumulative fertility for natives and immigrants. If immigrants from high fertility countries have below native level cumulative fertility the assimilation model can be rejected. Second, the direction of fertility adjustment can be investigated. If cumulative fertility falls with the fertile years spent in the destination country this is suggestive of assimilation effects.

Overall it appears that the literature on the fertility adjustment of immigrants supports the disruption model more than the assimilation explanation. In comparing the observed fertility rates over the last decades (available studies use data from the 1960, 1970, and 1980 census) e.g. Blau (1992) and Jasso and Rosenzweig (1990) are careful to control for the effects of declining fertility in the native U.S. reference population as well as the effect of a changing composition of immigrant origin countries.

Besides the assimilation vs. disruption issue, the literature explicitly analyses the additional effects of (1) different countries of origin, (2) self-selection among immigrants, and (3) emigration bias. Both, Blau (1992) and Jasso and Rosenzweig (1990) find that immigrants from high fertility source countries have higher fertility in the destination country and Kahn (1988) shows the pervading influence of home country fertility. Secondly, relative to their home country population self-selected migrants are more prone to undertake long-term (e.g. human-capital) investments and to have low fertility rates. Blau (1992) shows that immigrant women are among the best educated in their native countries, which indicates high opportunity costs of child bearing. She also provides evidence that immigrants have higher tastes for child quality than natives, suggesting low fertility in the destination country. This taste for child quality is confirmed by Jasso and Rosenzweig (1990) who show that immigrants school their children at higher rate than natives. Finally, these authors point out that selective emigration of immigrants may cause an upward bias in measured immigrant fertility, since couples with many children are less likely to return to their home country.

Both, Blau (1992) and Jasso and Rosenzweig (1990), conclude that their

[5] If the net disruption effect varies by age at migration, e.g. due to varying age-specific fertility, we would expect something like a U-shaped function of completed fertility over the number of fertile years spent in the host country: Disruption affects total fertility the most if it occurs in the main child-bearing years.

evidence is consistent with the model of fertility disruption. They do not find assimilation to the destination country fertility levels but show that after initially low birth rates immigrants added to their family sizes at faster rates than the native population. This finding is confirmed by Ford (1990) and Kahn (1994). Only Gorwaney et al. (1990) detect disruption effects for immigrants from developed countries but conclude in favor of the assimilation model for immigrants from developing countries.

All of these studies use data from the United States (U.S.) censuses and build their analyses on the results of cross-tabulations and least squares regressions. None of the papers discusses whether the period of residence in the United States is the appropriate variable to describe immigrants' exposure to U.S. culture and labor markets for the purpose of describing its effects on fertility. Also, none of these studies chose the count data estimation approach, which is compatible with the positive integer valued outcome measure. Since King (1988) we know that least squares regression may yield inconsistent estimates if applied to count data outcomes. Therefore our analysis extends the extant literature in a number of important dimensions.

While we are not aware of past studies on immigrant fertility adjustment for Germany, a related literature analyses the fertility effect of German unification. Between 1989 and 1994 East German births fell by sixty percent. Conrad et al. 1996 and Lechner 1998 investigate the East German fertility transition and conclude that fertility adjusts to West German patterns. They suggest that the strong fertility disruption immediately after unification was only a temporary adjustment phenomenon: Fertility rates of older women suddenly dropped, since their completed fertility already exceeded Western patterns, and young women postponed births adhering to the West German pattern of late first births. Here a situation which looked like disruption masks the first signs of assimilation.

From the above discussion we derive four hypotheses which we test below:

(H1) The higher a woman's (potential) labor market income, the lower her completed fertility.

(H2) High husband income is likely to be correlated with a higher demand for child quality and thus with a reduced number of births.

(H3) The assimilation hypothesis suggests that immigrants' completed fertility exceeds that of natives and that it falls with the number of fertile years spent in Germany.

(H4) Country of origin fertility differences are reflected in immigrant fertility abroad.

The specification of our empirical model is described in detail in the next section.

3 The Data and Specification of the Empirical Model

Our data are taken from the 1996 wave of the German Socioeconomic Panel (GSOEP). The GSOEP is a representative survey of households and individuals which has been administered annually since 1984. It oversamples the guest-worker population in Germany with Turkish, Spanish, Greek, Italian and what was Yugoslavian origin. The original 1984 sample consisted of about 4,500 native German and 1,400 foreigner households with a total of more than 12,000 respondents.

Since guest-worker immigration to Germany commenced in the late 1950s, [6] some of the foreign respondents of the 1996 GSOEP survey are already second generation immigrants, and born in Germany. To generate a homogenous sample we consider only those female respondents, who are either of German nationality and born in Germany (our native sample), or of foreign nationality and born abroad (the immigrant sample). Additionally, we restrict attention to immigrants from the five oversampled sending countries. Since we are interested only in completed fertility, we selected observations of women age 45 and above, and coded the number of their past births as our dependent variable. After omitting observations with missing values on core variables (such as the immigration year or marital status) our native sample consisted of 1,232 and the immigrant sample of 268 observations.

In the immigrant sample one third of the women are Turkish, 26 percent originated in former Yugoslavia, 16 percent each came from Greece and Italy, and 7.5 percent are of Spanish decent. Table 1 describes the fertility developments in these countries and in Germany over the last seven decades. It is apparent, first, that German fertility up to the late 1980s has been below that of the five sending countries. Second, fertility in Turkey has at any time exceeded that of any other country. Third, we find secular fertility declines in all countries over time and, finally, since the mid eighties fertility in Greece, Italy, Spain, and Germany has converged at a low level.

These trends are also apparent in the distribution of the dependent variable of our analysis. Table 2 describes the dependent and explanatory variables of our

[6] The first guest-worker treaty was signed in 1955 with Italy.

analysis by subsample. German completed fertility with 1.96 births per woman is below that of the immigrant population with an average of 2.97 births. However, this average across immigrant groups hides substantial nationality differences: The average number of 3.9 births for Turkish women far exceeds the immigrant average. Next in rank are women of Italian and Spanish nationality with 2.9 and 2.6 births, respectively. The women from Greece and from former Yugoslavia in our sample average 2.3 births.

Figure 1 gives an impression of the correlation between completed fertility and the number of fertile years an immigrant woman spent in Germany. The overall negative trend is obvious and even clearer when we consider average completed fertility summarized by fertile year groups in Table 3: Women who spent five or fewer of their fertile years in Germany have on average 3.9 births, those who spent almost all their fertile time in Germany averge 2.07 births, close to the native average of 1.96. Table 3 presents average fertility also by the standard measure of duration in this literature, years since migration. The tabulation by fertile years in Germany shows a smoother development in the average number of births than years since migration, which confounds age and immigration year effects in its depiction of the assimilation process.

Based on the discussion in section 2, the specification of our empirical model for completed fertility considers five groups of explanatory variables. First, we control for overall "demographic effects" consisting first of a woman's age, to account for cohort differences in fertility (cf. Table 1). To control for her health - assuming that health in 1996 is indicative of health during the reproductive phase - the specification controls for her handicap status. The health effect is not clear a priori: Through biological mechanisms poor health may reduce fertility, but reduced earnings potentials of those in poor health reduce the opportunity costs of fertility.

In group two, a woman's "marital history" is measured controlling for whether she was ever married, the age at first marriage, and whether a spouse is currently present. Third, we approximate the effect a woman's "earnings potential", using four variables: Her years of education, indicators of school and vocational training degrees, where the omitted categories are 'no degree', and a measure of average regional unemployment in the woman's current state of residence [7]. "Husbands' income" is similarly approximated using school and vocational degree indicators in variable group four. Fifth, we control for the women's "immigrant status:" In a first immigrant model we consider only an indicator variable for whether the woman is an immigrant. In a second nationality model we evaluate independent

[7]Over the period of available data, 1984 through 1996, more than 96 percent of all women stayed in the same state. This confirms that the state's average unemployment over the past 25 years indicates relevant past labor market opportunities for its female population.

nationality effects for the source countries represented in our sample.

Finally and as explained below, the effect of fertile years in Germany on completed fertility is captured through flexible estimation of the coefficients describing the effect of immigrant status: In principle, our estimation method allows the effect of each control variable to vary over the number of fertile years a woman spent in Germany. We focus, however, only on the immigrant and nationality coefficients. Thus, the effects of adjustment mechanisms, be it assimilation or disruption, are specified most flexibly, avoiding the imposition of undue structure.

When comparing the descriptive statistics of the explanatory variables in Table 2 across the two samples we notice a number of differences. First, the average age of the German sample is clearly above that above the immigrant population. Further, the immigrant sample is characterized by a slightly higher probability of being ever married, and a significantly higher probability of having a partner present. In terms of human capital, immigrant women have on average about two years less education than their native counterparts. This difference is more strongly reflected in the distribution of schooling degrees, where more than half of all immigrant women fall in the omitted category of no schooling degree compared to less than one percent of the natives. Whereas only 43 percent of all native females in our sample have no vocational degree, this holds for 85 percent of the immigrant women. Immigrants appear to settle in states with slightly less unemployment than the native population. The human capital disadvantage of immigrant spouses is not quite as pronounced, though still substantial. In the sample with spouses present (figures not presented in Table 2) less than one percent of native husbands have no schooling degree and about 10 percent have no vocational degree. The figures are at 38 and 67 percent for the immigrant sample, respectively. By the arguments of female opportunity cost, these statistics suggest that immigrant families are likely to have higher rates of fertility than natives.

4 The Econometric Method

In this article we present a very flexible way of count data modelling: Bayesian type models with varying coefficients. The parameters are assumed to be functions of a metrical reference variable which could be time, age, etc. Values of these parameter functions at a certain realization of the reference variable are then treated as a state vector allowing a state space interpretation of the model.

Bayesian type models with varying coefficients combine the flexibility of a non-parametric approach with the advantage of transparency of a fully parametric approach. Non-parametric models allow to estimate the functional form of the impact of a certain regressor without imposing a particular—in general linear—structure *a priori*. The main advantage of such an approach is obvious (see Härdle 1990): Imposing a particular functional form always involves the danger of masking important features. At least as a first step, a non-parametric approach therefore seems to be preferable. The counterargument usually is that a great range of functions can be approximated by a series expansion providing a means of modelling them by including polynomials of higher order. While this is true in general, there are still some problems to deal with: First, if the real functional form is not differentiable this does not hold. Functions possessing a kink cannot be identified by including polynomials of any order. Second, the estimates of parameteric models involving high order polynomials are often distorted by outliers. Non-parametric methods provide a tool to detect outliers and to give a correct interpretation of the data. Finally, parametric models involving high order interaction terms of one reference variable with different regressors often result in problems of multicollinearity as the high order polynomials become dominant.

The literature on count data models with flexible parameter functions is rather limited so far. There are some articles, however, that treat changes of the parameters in a dynamic context, i.e. they allow for changes in the parameters over time. This can be seen as a special case of our objective, as we are interested in changes in the parameters depending on any metrical reference variable. Zehnwirth (1988), Harvey and Fernandez (1989), Shonkwiler and Harris (1993), Jorgensen et al. (1996), Jorgensen et al. (1997) and Bolstad (1995) construct count data state space models with an unobserved heterogeity component changing over time. However, they provide no means to include more general varying parameters. Fahrmeir (1992) and Fahrmeir and Wagenpfeil (1997) estimate generalized linear state space models. While their models allow general parameter changes over time, they do not treat the problem of over- or underdispersion, which is an important feature in count data modelling. In this paper we elaborate on the procedures proposed by Fahrmeir and Wagenpfeil (1997) aiming at two points: First we generalize their dynamic model by using any metrical reference variable. Second we explicitly take underdispersion into account by introducing a Penalized Quasi Maximum Likelihood (PQML) procedure.

In the following we will first present the basic count data varying coefficient model, which we call the Poisson varying coefficient model (PVC). Then we discuss the problem of underdispersion. For presentation of algorithmic details we refer to Appendix A.

4.1 The Poisson Model with Varying Coefficients (PVC)

Like the standard Poisson model the PVC links each observed univariate count variable $\tilde{y}_i$ to a predictor $\tilde{\eta}_{\tilde{\imath}}$:

$$\tilde{y}_{\tilde{\imath}}|\tilde{\eta}_{\tilde{\imath}} \sim Po(\tilde{\lambda}_{\tilde{\imath}}), \quad \tilde{\lambda}_{\tilde{\imath}} = exp(\tilde{\eta}_{\tilde{\imath}}), \quad (\tilde{\imath} = 1, ..., \tilde{I}), \tag{1}$$

where $\tilde{\imath}$ denotes the observations. The dependent univariate variable is supposed to be Poisson distributed with mean $\tilde{\lambda}_{\tilde{\imath}}$ given $\tilde{\eta}_{\tilde{\imath}}$. For the first two conditional moments we get

$$\text{Conditional expectation:} \quad \tilde{\lambda}_{\tilde{\imath}}(\tilde{\eta}_{\tilde{\imath}}) := E[\tilde{y}_{\tilde{\imath}}|\tilde{\eta}_{\tilde{\imath}}] = exp(\tilde{\eta}_{\tilde{\imath}}) \tag{2}$$

$$\text{Conditional variance:} \quad \tilde{\Sigma}_{\tilde{\imath}}(\tilde{\eta}_{\tilde{\imath}}) := V[\tilde{y}_{\tilde{\imath}}|\tilde{\eta}_{\tilde{\imath}}] = exp(\tilde{\eta}_{\tilde{\imath}}). \tag{3}$$

From (2) and (3) the main restriction of the Poisson model becomes obvious: The conditional expectation equals the conditional variance. We will come back to this point later.

Following Hastie and Tibshirani (1993) the standard Poisson model is generalized by redefining the predictor $\tilde{\eta}_{\tilde{\imath}}$. $\tilde{\eta}_{\tilde{\imath}}$ is not simply a linear function of the deterministic regressors but can capture fairly general interactions between one special covariable, ν, and the other regressors. The predictor is of the form

$$\tilde{\eta}_{\tilde{\imath}} = \psi_0(\nu_{\tilde{\imath}}) + \psi_1(\nu_{\tilde{\imath}})\ \tilde{z}_{\tilde{\imath},1} + ... + \psi_k(\nu_{\tilde{\imath}})\ \tilde{z}_{\tilde{\imath},k} \quad (\tilde{\imath} = 1, ..., \tilde{I}), \tag{4}$$

where ν can be any metric variable. $\psi_j(.)$ are functions of this univariate reference variable and $\tilde{z}_{\tilde{\imath},j}$ $(j = 1, \ldots, k)$ are the covariates, whose effects can change depending on ν.

The standard Poisson model is obtained as a special case of the above if the $\psi_j(.)$ $(j = 0, \ldots, k)$ are constant functions. While we do not want to impose this very strong restriction it is obvious that without any restrictions on the smoothness of the functions, there is not enough structure in this model to estimate it.

To impose some additional structure on the model we force the parameter functions $\psi(.)$ to change smoothly depending on the value of ν. Changes in the impact of the regressors $\tilde{Z}_{\tilde{\imath}} = (1, \tilde{z}_{\tilde{\imath},1}, \cdots, \tilde{z}_{\tilde{\imath},k}) \in \mathbb{R}^{1\times k+1}$ are still possible, but it is ruled out that the parameter functions show erratic jumps. To be more precise we have to define $\nu_1 < \nu_2 < \ldots < \nu_I$ as the ordered sequence of different realizations of ν and $\delta_i := \nu_i - \nu_{i-1} > 0$ $(i = 2, \ldots, I)$ as the distances between successive values [8]. Additionally, let $\alpha_i = (\psi_0(\nu_i), \psi_1(\nu_i), \ldots, \psi_k(\nu_i))' \in \mathbb{R}^{k+1}$ be

[8] In the following we will use $\tilde{\imath} = 1, \ldots, \tilde{I}$ for the single observations, while $i = 1, \ldots, I$ denotes the groups of observations belonging to a particular realization of the reference variable.

the vectors containing the values of the functions $\psi_j(.)$ $(j = 0, \ldots, k)$ at ν_i. As a so-called smoothing prior (see e.g. Kohn and Ansley 1988) we assume that the parameter functions progress following independent first order random walks [9]

$$\alpha_i = \alpha_{i-1} + \xi_i, \quad \xi_i \sim N(0, \delta_i Q). \tag{5}$$

where Q is a diagonal matrix. The initial condition of this random walk is modelled as $\alpha_0 \sim N(a_0, Q_0)$ and $\delta_1 := 1$. Q, a_0 and Q_0 are fixed hyperparameters, which in a first step are assumed to be known. Later on we will demonstrate how to estimate them simultaneously.

Since in the so-called 'transition equation' (5)—equation (1) is called the 'observation equation' —the parameters α_i are supposed to be random variables, we are in a Bayesian setting[10]. The model becomes a dynamic Poisson state space model if ν equals time. In analogy to these models the vectors α_i of the values of the parameter functions at ν_i are called 'states'. The matrix Q contains the so-called smoothing hyperparameters. The smaller the elements of Q the smoother the parameter functions are. If one smoothing hyperparameter is equal to zero the corresponding parameter function is constant. If all smoothing hyperparameters are equal to zero we get the special case of the standard Poisson model.

For clarification let us summarize the n_i observations belonging to a certain realization ν_i of the reference variable[11] in defining

$$\begin{aligned} y_i &:= (\tilde{y}_{l_i+1}, \ldots, \tilde{y}_{l_i+n_i})' \in \mathbb{R}^{n_i} \quad &(6)\\ Z_i &:= (\tilde{Z}'_{l_i+1}, \ldots, \tilde{Z}'_{l_i+n_i})' \in \mathbb{R}^{n_i \times (k+1)}, \quad &(7) \end{aligned}$$

$l_i := \sum_{j=1}^{i-1} n_{\tilde{i}}$ is the number of observations with reference variable realizations smaller than ν_i. For the estimation of the states the joint density of the states $\alpha := (\alpha_0, \alpha_1, \cdots, \alpha_I) \in \mathbb{R}^{(k+1)\times(I+1)}$ and the dependent variable $y := (y_1, \cdots, y_I)' \in \mathbb{R}^{\tilde{I}}$ given $Z := (Z_1', \cdots, Z_I')' \in \mathbb{R}^{\tilde{I}\times(k+1)}$ will be needed. To facilitate the calculation of this density we assume

Assumption 1 $f(y_i|\alpha_i, \alpha_{i-1}, \cdots, \alpha_0, Z_i) = f(y_i|\alpha_i, Z_i)$, *i.e. given the corresponding states* α_i*, preceding states do not contain any additional information on* y_i*.*

Assumption 2 $f(\alpha_i|\alpha_{i-1}, \cdots, \alpha_0, Z_i) = f(\alpha_i|\alpha_{i-1}, Z_i)$, *i.e. the sequence of the states is a Markov process.*

[9]Random walks of higher order can also be assumed. However we will focus on the first order random walk transition model for simplicity.

[10]Using a complete Bayesian approach would involve modelling the hyperparameters as random variables as well. Our appoach therefore is a so-called empirical Bayes approach.

[11]Note that $\tilde{I} = \sum_{i=1}^{I} n_i$.

f stands for the different densities with $f(y_i| \,.\, , Z_i) := \prod_{\tilde{\imath}=l_i+1}^{l_i+n_i} f(\tilde{y}_{\tilde{\imath}}| \,.\, , \tilde{Z}_{\tilde{\imath}})$ and $f(\alpha_i| \,.\, , Z_i) := \prod_{\tilde{\imath}=l_i+1}^{l_i+n_i} f(\tilde{\alpha}_{\tilde{\imath}}| \,.\, , \tilde{Z}_{\tilde{\imath}})$.

The standard Bayesian estimator for stochastic parameters is the posterior expectation given the data. However in our context this standard approach involves the determination of high dimensional integrals, which are not tractable analytically.[12] Therefore we use a simpler method which is in some respects closer to the usual econometric approaches.

To estimate the states α_i we maximize the posterior density of the stochastic parameters given the data, i.e. we will determine the most likely values of the parameters given the information contained in the data. For a linear Gaussian observation model this so-called posterior mode would coincide with the posterior mean which is usually reported in Bayesian estimation procedures. In the nonlinear setting this is not the case. Nevertheless simulation studies (see Lang 1996) show that the posterior distribution of the states given the data is quite symmetric, indicating that the posterior mode is a useful approximation of the posterior mean (see also Fahrmeir and Wagenpfeil 1997). Moreover the posterior mode is also an important characteristic of the posterior density in its own right.

Using assumptions 1 and 2 the posterior density of the states given the data is

$$f(\alpha|y, Z) = \frac{\prod_{i=1}^{I} f(y_i|\alpha_i, Z_i) \prod_{i=1}^{I} f(\alpha_i|\alpha_{i-1}, Z_i) f(\alpha_0)}{f(y|Z)}. \tag{8}$$

Taking logarithms and ignoring all terms that do not depend on α leads to the following objective function

$$\begin{aligned} PL(\alpha) = \sum_{i=1}^{I} ln f(y_i|\alpha_i, Z_i) - \frac{1}{2}(\alpha_0 - a_0)' Q_0^{-1} (\alpha_0 - a_0) \\ - \frac{1}{2} \sum_{i=1}^{I} \frac{1}{\delta_i} (\alpha_i - F_i \alpha_{i-1})' Q^{-1} (\alpha_i - F_i \alpha_{i-1}). \end{aligned} \tag{9}$$

Equation (9) is a penalized likelihood criterion. Maximizing this criterion with respect to α gives the posterior mode smoother which in the following will be denoted $a := (a_{0|I}, a_{1|I}, \cdots, a_{I|I}) \in \mathrm{IR}^{(k+1)\times(I+1)}$.

[12] Recently Markov Chain Monte Carlo (MCMC) procedures have been proposed to cope with this problem (see e.g. Besag et al. 1995). The basic idea of this method is to construct a Markov chain that converges to the posterior distribution. As soon as convergence has been achieved the posterior expectation can be estimated by drawing samples from the limiting distribution. However, there are still some open questions concerning the convergence of the method and the computational effort is very high.

While the first term of $PL(\alpha)$ measures the goodness of fit the second term penalizes the roughness of the estimated functions. If an element Q_{jj}^{-1} is 'very high' (i.e. Q_{jj} 'very low') the PL-criterium forces the corresponding function to be especially smooth, if it is 'rather small' (i.e. Q_{jj} 'rather high') changes in the parameter functions are penalized less. Starting directly from the PL-criterion would be possible as the above interpretation indicates. Nevertheless the Bayesian interpretation of equation (9) provides us with an additional and very robust method of estimating the hyperparameters.

To get the posterior mode estimator we will solve the penalized likelihood equation

$$\frac{\partial PL}{\partial \alpha} = s(\alpha) + p(\alpha) = 0, \tag{10}$$

where $s(\alpha)$ is the score function of $\sum_{i=1}^{I} lnf(y_i|\alpha_i, Z_i)$ and $p(\alpha)$ represents the penalization part. To do so in principle every non-linear optimization algorithm could be used. To get an efficient procedure, however, the special structure of the model should be taken into account. Fahrmeir and Wagenpfeil (1997) derive an algorithm which performs a Fisher scoring procedure for generalized linear time series models making efficient use of this structure. Every Fisher Scoring step corresponds to a linear Kalman filtering and smoothing applied to a so-called 'working observation' resulting in a so-called Iterated Working Kalman Filter and Smoother (IWKFS).[13] In Appendix A we describe our adaptation of their proposal to the present situation.

Up to now we have treated the hyperparameters a_0, Q_0 and Q as known constants. As in most applications, we do not have any information concerning the starting point of the random walk and on the variability of the parameter functions *a priori*. To deal with this problem we first, as proposed in Harvey (1989), set $Q_0 = k * I$ where I is the appropriate unity matrix and k is a large positive number[14]. This reflects our ignorance concerning a_0. For k going to infinity we would get a so-called diffuse prior which is approximated by our choice of Q_0. Additionally, we estimate a_0 and Q in an outer loop as described in the following.

There are several methods to estimate the hyperparameters including the maximization of a generalized cross-validation criterion (see e.g. Kohn and Ansley 1989 and Fahrmeir and Tutz 1994) and the determination of an approximative likelihood function (see Durbin and Koopman (1992)). Some authors even leave it to the statistician to choose the values of the hyperparameters without any standardized procedure. As we have given a Bayesian interpretation for equation

[13]For the derivation of this algorithm see Fahrmeir und Wagenpfeil (1997).

[14]In the application we choose $k = 1000$.

(9), using an EM-type algorithm is an additional possibility for estimating the smoothing parameters. Unlike the other methods the EM algorithm has proved to be numerically stable (see Fahrmeir and Wagenpfeil 1997).

To apply an EM algorithm the states α are treated as unobserved variables. Both the observed variable y and the 'unobserved variables α' depend on the fixed hyperparameters. The EM algorithm is a popular way to perform a ML-estimation in such a setting. As the likelihood function $L(y, \alpha | a_0, Q)$ cannot be calculated, an indirect approach is necessary to maximize it with respect to the hyperparameters. The EM algorithm is such an indirect maximization algorithm. Its basic idea is to eliminate the unobservables in the joint density by integrating them out (E-step). The maximization of the resulting expression then gives updated values of the hyperparameters (M-step). Iterating these two steps the procedure converges to the ML-estimator (see Dempster, Laird and Rubin 1977).

As proposed in Fahrmeir (1992), we can apply an EM-type algorithm replacing conditional expectations by conditional (posterior) modes, which are given from the IWKFS procedure. An implementation following Fahrmeir and Wagenpfeil (1997) is shown in Appendix A.

4.2 The Case of Underdispersion

In the last section we studied the Poisson varying coefficient model. We assumed that the observations are Poisson distributed given the realizations of the parameter functions. However, in many applications the underlying assumption of equidispersion is not fulfilled. While typically overdispersion is found, count data based studies of completed fertility often (see e.g. Winkelmann and Zimmermann 1994) find underdispersion in their samples. In the following we focus on this second case where the conditional variance exeeds the conditional expectation.

To cope with this problem two approaches are possible: First one could use an observation model, which can account for underdispersion. Models of that kind are the hurdle models (Mullahy 1986), the generalized event count model (Winkelmann and Zimmermann 1994) or the generalized poisson model (Consul 1989).

Alternatively, following McCullagh and Nelder (1989), we adapt a quasi-likelihood approach to generalize the Poisson observation model. On the basis of the specifications of the first two moments

$$E[y_i|\eta_i] \quad = \quad \mu_i = exp(Z_i\beta) \tag{11}$$

$$V[y_i|\eta_i] \quad = \quad \phi * \mu_i, \tag{12}$$

McCullagh and Nelder (1989) construct the following quasi-score function

$$qs = \sum_i \phi^{-1} * Z_i'(y_i - \mu_i). \tag{13}$$

Setting (13) equal to zero and solving numerically gives the quasi-maximum likelihood estimator $\hat{\beta}$. Under the assumptions (11) and (12) $\hat{\beta}$ is consistent with the asymptotic covariance matrix

$$cov(\hat{\beta}) = \phi * \sum_i exp(Z_i\beta)Z_i'Z_i$$

For $\phi = 1$ this is in consistence with the limiting distribution of the Poisson ML estimator.

For estimation of ϕ McCullagh and Nelder (1989) propose the moment estimator

$$\hat{\phi} = \frac{1}{n}\sum_i \frac{(y_i - \hat{\mu}_i)^2}{\hat{\mu}_i}, \tag{14}$$

making use of the fact that $\hat{\beta}$ is consistent.

We use this approach in the context of varying coefficient models replacing $s(\alpha)$ in the penalized score function (10) by a quasi-score function. This quasi-score function differs from (13) only in that μ_i has to be changed in $\lambda_i = exp(Z_i\alpha_i)$, where the parameters can change depending on the reference variable. This results in penalized quasi-maximum-likelihood estimation (PQML).

As the IWKFS only relies on the first two moments of the observation model, it can again be used to perform the estimation by setting

$$\tilde{\Sigma}_{\tilde{i}}^{PQML} = \phi * \tilde{\Sigma}_{\tilde{i}}^{PML}.$$

For given ϕ the algorithm directly provides adjusted approximate error covariances $V_{i|I}$.

While the estimation of α using IWKFS is straightforward, the estimation of the underdispersion parameter ϕ involves the problem that the posterior mode estimator a now depends on the used estimate of ϕ. Therefore an estimation of ϕ using (14) needs not to be consistent. To get a consistent combination $(a, \hat{\phi})$ we have to iterate the entire procedure until convergence. For the algorithm itself we again refer to Appendix A.

The iteration usually will break up after very few iteration steps. The reason is that the asymptotic bias of a given a wrong $\hat{\phi}$ arises only in terms of the variability of the parameter functions. Note that this bias vanishes if we assume constant parameter functions. The effect of this variability bias on the estimate $\hat{\phi}$, which depends on $a(\phi)$ is usually very small as it is shown for an estimation with our data set in table 4.

5 Estimation Results

This section interprets the estimation results with respect to the hypotheses formulated above, and provides the first empirical evaluation of the Poisson model with varying parameters. As outlined above, our estimation strategy proceeds in two steps: First, the immigrant model is estimated which controls for an indicator variable of immigrant status. The coefficient of this indicator variable is allowed to vary with the number of fertile years an immigrant has spent in Germany. In the nationality model, separate variables for each nationality group are considered, where again each may vary with the number of fertile years spent in Germany. The data for our native and immigrant samples are pooled in the estimations. Therefore the immigrant and nationality indicators are to be interpreted relative to the native average.

Table 5 presents the estimation results of the immigrant model in the first column. As always in the framework of Poisson regressions, the estimated coefficients can be interpreted as semi-elasticities. Thus e.g. a one unit increase in age causes here an increase in completed fertility by two percent. Given the scaling of the age variable, the one unit change represents a ten year difference in birth cohorts. Going back to Table 1, which indicated continuously falling fertility rates over time, the estimated age effect, though not significantly different from zero, corresponds to expectations. The effect of a handicap on completed fertility is positive, indicating that women who suffer a health problem today have a cumulative fertility about two percent above average.

The group of factors most strongly affecting completed fertility are those describing a woman's marital history. Having ever been married has a large and highly significant coefficient estimate which suggests that this characteristic alone increases cumulative fertility by 240 percent for our sample. Similarly important is the effect of age at marriage which confirms findings of the general fertility literature. This effect is again highly precisely estimated and indicates that *ceteris paribus* an increase in the age of marriage by ten years reduces the total number of births by about 20 percent. Finally, the effect of having a spouse present is somewhat surprising: Those women who have a spouse present average an about 10 percent lower total number of births. While the result would be completely counterintuitive for contemporary fertility outcomes, the negative correlation with completed fertility is likely due to confounding effects, such as the interplay of cohort fertility and survival probabilities of men, the correlation of low husband incomes and mortality rates, or historic effects on the development of divorce and separation rates.

Six separate variables are considered in our model to reflect female labor market opportunities and only one indicator variable yields a statistically significant result. The number of years of schooling has the expected negative - though insignificant - coefficient, confirming our hypothesis (H1) above. The variables describing the degree of schooling and the woman's vocational training similarly confirm H1: Relative to those women without a schooling or vocational degree those who received an education have fewer births over their reproductive career. The only surprising finding is that women with only an apprenticeship degree reduced their births by more than those with an advanced degree. Finally, we controlled for the unemployment history in the women's state of residence as an indicator of past labor market opportunities. There appears to be no measurable effect. We would have expected a positve coefficient indicating that high average unemployment reduces potential earnings, and thus lowers the opportunity costs of fertility (effects like these were confirmed for East Germany by Witte and Wagner 1995).

In contrast to female human capital, human capital of the spouse (as of 1996) appears to be correlated overall with higher completed fertility. In particular having exactly a mandatory degree of schooling yields a statistically significantly positive effect of about 15 percent on completed fertility. This, as well as the positive effect of having an apprenticeship degree can be interpreted as a simple positive income effect on the demand for children, refuting our hypothesis two (H2) above. The fact that advanced partner schooling and partner vocational degrees have smaller and even negative effects on completed fertility may indicate the onset of an increased demand for child quality at higher income levels, also confirming the findings of the fertility literature.[15]

We are most interested in the set of variable immigrant coefficients which is depicted in Figure 2. The un-dotted bands represent the coefficient estimates corresponding to the PML column of the immigrant model in Table 5. Figure 2 contains three interpretable pieces of information: First, over the entire range of fertile years that are possibly spent in Germany the immigrant effect is positive. Based on this we cannot reject the assimilation model out of hand. Second, it is clear that the impact of being an immigrant in Germany falls over the entire spectrum of fertile years, and third based on the confidence bands we see that immigrant status is correlated with a statistically significant positive effect on completed fertility for those women who spent less than twenty years, or two thirds, of their fertile years in Germany. For those who came to Germany say by age 25 we no longer observe a significant difference in completed fertility relative to the native sample.

[15]In interpreting these indicators it is important to keep in mind that the husband variables could only be coded for women with partner present. Therefore the reference group consists not only of spouses without a degree but also of those women without a spouse present. The findings, however, were confirmed in a test regression on only the sample of women, with a spouse present.

This analysis does not permit conclusions on the existence of disruption effects. However, it is apparent that cumulative fertility approaches that of the native population "from above", the longer a woman's exposure in Germany during her fertile period. Therefore we conclude with respect to hypothesis three (H3) above that our evidence favors the assimilation model of fertility adjustment for the sample of immigrants to Germany, who had completed their reproductive years by 1996.

A limitation of the standard Poisson model, which needs to be tested because it may also affect our estimation, is the assumption of equidispersion. However, a sound asymptotic theory for such a test in the context of varying coefficient models is not available in the literature. Therefore we test for underdispersion in the context of the corresponding fixed parameter model, by replacing the varying parameter for the immigrant indicator by a fixed parameter. Performing a regression based test following Cameron and Trivedi (1990) we find clear evidence for underdispersion[16]. While the hypothesis rejected in this test is not exactly what we wanted to test—note that the conditional moments in our model are not exactly identical to the conditional moments in the fixed parameter model—this is nevertheless a strong indication for underdispersion in our model.

As described in the methodology section above we developed an estimator that provides correct estimates if the equidispersion assumption is violated. The results based on this penalized quasi-maximum-likelihood estimation (PQML) are presented in the second coefficient column in Table 5. A comparison of the coefficient estimates yields that they are basically not affected. An observable difference between the two columns is that the asymptotic t-values under the PQML estimation are larger, since the correction for underdispersion now yields smaller standard errors. This effect is also depicted in the dotted lines in Figure 2.The confidence bands of the PQML estimation are within those derived by the PML estimation.

Finally, Table 5 provides some information on starting values as well as the final estimates of the hyperparameters and shows some characteristics of the algorithm. The variability parameter Q is estimated slightly higher using the PQML estimation which corresponds to a slightly steeper decline in the immigrant effect in Figure 2. The additional effort for this estimation method is approximately one third which is due to the moment estimation of ϕ.

In step two of our empirical analysis we generalize the immigrant model to allow for nation-specific fertility adjustments. Given the limited effect of the underdispersion control for the immigrant model, the estimation results presented in the last column of Table 5 are derived using the PML estimation. The estimates

[16]The equidispersion hypotheses is rejected even at a level of 0.5%.

of the parameters, which do not vary over the number of fertile years a woman spent in Germany, hardly differ from those presented in the first two columns of Table 5. The magnitudes of the coefficient for age increases and those for female schooling degrees fall slightly. The significance levels of the coefficient estimates are basically not affected by the specification change.

The more interesting fertility adjustment effect is depicted by nationality in Figures 3 through 7. It is apparent that the nationality effect across fertile years in Germany follows different transition paths for the different origin countries, thus supporting hypothesis four (H4) above. For Greek and Spanish women completed fertility does not differ significantly from the native sample. Significant differences in the number of births relative to natives are found for women from Italy and from former Yugoslavia only, when they spent most of their reproductive careers outside of Germany. It is predominantly the Turkish sample which seems to drive the finding of significant positive immigrant effects on overall fertility. However, abstracting from statistical significance we find for all but the Greek women that the nationality effect is sizable (with up to 60 percent higher fertility) for those who spent only few fertile years in Germany, and that it declines and converges to the German level, the more fertile years the women spent in Germany. This clearly supports the assimilaton model.

There are two special cases deserving attention. The first is the humpshaped development of Turkish fertility over time (Figure 3), and the second is the decline below zero for women from former Yugoslavia (Figure 4). Figure 4 is the only occasion where the average nationality effect on completed fertility falls below zero. This suggests that women from former Yugoslavia who passed between fifteen and twenty years of their fertile time in Germany on average had fewer children than their German counterparts. For these cases the assimilation model is not appropriate. Given higher fertility in Yugoslavia compared to Germany these cases actually might represent some mixture of the effects of fertility disruption due to migration and the effect of self-selection of low fertility individuals into the pool of migrants.

Figure 3 shows that the little hump observable in the overall immigrant effect in Figure 2 above derives from the idiosyncratic effect found for the Turkish sample. The results suggest that Turkish women who spent only say ten years of their reproductive time in Germany, i.e. who immigrated after age 35, have lower completed fertility than those who came to Germany at younger ages and who spent between ten and fifteen fertile years in Germany.[17] The declining curvature of the fertile-year effect after the maximum at eleven years, follows the

[17] One possible explanation of this low fertility is that immigration after age 35 is selective in that only those women come to Germany who have no or few children in Turkey already. Younger women can still easily complete their fertility in Germany.

common pattern and does not require special explanation. Also, the value of the immigrant effect prior to the hump is within the range of other nationality effects. Thus the fact to be explained is the sizeable immigrant effect in the range around eleven fertile years in Germany. We offer a data driven and a substantive explanation: The data driven explanation is that four of the five outliers in our Turkish sample, i.e. women with eight and more children, happen to have spent beween 11 and 14 fertile years in Germany. Thus these four datapoints (out of a total of 89 Turkish observations) explain the hump. Substantively, three of these four women immigrated in or after 1973, when immigration had become possible only for the purpose of family reunification. This indicates that women with many children were possibly selected for immigration on the basis of their fertility outcomes, which provides an exogenous explanation to the surprising hump.

In regression estimations not presented here we tested for the fertile-year effects in the coefficients of other explanatory variables in addition to those found for nationality indicators. However, possibly due to the limited number of 268 immigrant observations these variable coefficient effects did not yield interesting additional insights. We also applied the PQML algorithm in the nationality model and found confirmation for the decrease in standard errors without interesting effects of the coefficients themselves.

6 Conclusions

This study contributes to the literature on fertility adjustment in a number of ways. First, we test the assimilation hypothesis for the case of immigration to Germany. Given that almost the entire literature focuses on the United States and applies the same U.S. population census data, new insights are gained by widenening the perspective to the scenario of European immigration. Second, we suggest that a measurement error has pervaded the extant literature. Since the researcher is interested in the effects of living in the destination country on immigrant behavior, years since migration has been utilized as the relevant duration measure. We argue that this is inappropriate for the issue of fertility, where one should be interested in the number of fertile years spent in the destination country. Third, this is the first study of completed immigrant fertility which applies the appropriate count data estimation technique. In addition, we apply the newly developed Poisson varying coefficient model to evaluate the determinants of completed fertility.

In contrast to the results on U.S. immigrant fertility adjustment, we find evidence supporting the assimilation model of immigrant fertility adjustment.

The assimilation model suggests that immigrant fertility is initially above the native level and over time converges to that of the native population. In the United States recent immigrants entered with very low fertility rates but then added to their family sizes at rates beyond those of natives (e.g. Blau 1992, Jasso and Rosenzweig 1990). The increase in immigrant fertility rates over the duration of stay in the host country is taken as evidence for the disruption and against the assimilation model of fertility adjustment.

Our results suggest that immigrants to Germany enter the country with fertility rates above native levels and that their completed fertility falls the more of their fertile time they spend in Germany. This finding corresponds to the predictions of the assimilation model. Beyond the fertility adjustment effect we confirm the prediction of the standard economic model regarding the negative opportunity cost effect of female human capital on total fertility outcomes and the crucial role of marital history for the number of births in a woman's life.

We can only speculate as to why our findings may differ from those of prior U.S. studies. If assimilation behavior is in fact driven by economic variables then the fertility convergence result which we find for Germany but not for the United States must be explained by the differences in fertility determinants (wages, incomes and cost of contraception) between the countries of origin and destination. These differences must be more pervading for immigrants to Germany, than for immigrants to the United States. In other words, the difference between the Turkish rural standard of living and that in German towns must differ by more than prices and incomes in, say, northern Mexico and southern Texas. To the degree that German society is more homogenous than the American society this argument is plausible. Interestingly, the analysis of Dutch immigrants by Schoorl (1990) also yields an assimilation result.

However, the findings may at least in part be due to the different data and estimation method. While the U.S. studies use decennially available census evidence we apply a representative micro-level dataset with fewer observations. Our estimation method accounted for the discrete nature of the outcome variable and proofed to be highly appropriate for the research question. As is typical for the Poisson count data model we found the equidispersion assumption to be violated by our data. However, estimations which correct for the upward bias in standard errors do not affect the magnitude of the coefficient estimates. Therefore we are confident that our results are reliable and provide an interesting addition to the literature on immigrant fertility adjustment.

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A The Estimation Algorithm

In this appendix we show how to implement the algorithms mentioned in the text. In analogy to the definition of $y_i = (\tilde{y}_{l_i+1}, \ldots, \tilde{y}_{l_i+n_i})'$ and $Z_i = (\tilde{Z}'_{l_i+1}, \ldots, \tilde{Z}'_{l_i+n_i})'$ let us first summarize the conditional expectation and the conditional variance of observations belonging to the same observation group in the following matrices

$$\lambda_i(\alpha_i) := exp(Z_i\alpha_i) = \begin{pmatrix} exp(\tilde{Z}_{l_i+1}\alpha_i) \\ \vdots \\ exp(\tilde{Z}_{l_i+n_i}\alpha_i) \end{pmatrix} \tag{15}$$

$$\Sigma_i(\alpha_i) := \begin{pmatrix} exp(\tilde{Z}_{l_i+1}\alpha_i) & & 0 \\ & \ddots & \\ 0 & & exp(\tilde{Z}_{l_i+n_i}\alpha_i) \end{pmatrix} \tag{16}$$

To solve the penalized likelihood equation (10) a Fisher Scoring algorithm is performed adapting a proposal of Fahrmeir and Wagenpfeil (1997) made in the context of generalized linear time-series models. In their procedure every iteration step corresponds to a linear Kalman filtering and smoothing applied to a working observation.

To describe the resulting algorithm we have to make some further definitions: The derivatives $\frac{\partial h(\tilde{\eta})}{\partial \tilde{\eta}}$ of the response function $h(\tilde{\eta}) = exp(\tilde{\eta})$ at $\tilde{\eta}_{l_i+r} = exp(\tilde{Z}_{l_i+r}\alpha_i)$, $r = 1, \ldots, n_i$ are

$$D_i(\alpha_i) := \begin{pmatrix} \left.\frac{\partial h(\tilde{\eta})}{\partial \tilde{\eta}}\right|_{\tilde{\eta}=\tilde{\eta}_{l_i+1}} & & 0 \\ & \ddots & \\ 0 & & \left.\frac{\partial h(\tilde{\eta})}{\partial \tilde{\eta}}\right|_{\tilde{\eta}=\tilde{\eta}_{l_i+n_i}} \end{pmatrix} = diag(\lambda_i(\alpha_i)) \in \mathrm{IR}^{n_i \times n_i}, \tag{17}$$

the working observation is

$$y_i^*(\alpha_i) := [D_i^{-1}(\alpha_i)]'[y_i - \lambda_i(\alpha_i)] + Z_i\alpha_i \in \mathrm{IR}^{n_i} \tag{18}$$

and the weight matrix is

$$W_i(\alpha_i) := D_i(\alpha_i)\Sigma_i(\alpha_i)D_i'(\alpha_i) \in \mathrm{IR}^{n_i \times n_i}. \tag{19}$$

Finally, following the notation in Fahrmeir and Wagenpfeil (1997) $a_{i|i}$, $a_{i|i-1}$ and $a_{i|I}$ are numerical approximations to filtered, predicted and smoothed values of α_i $(i = 0, \ldots, I)$ and $V_{i|i}, V_{i|i-1}$ and $V_{i|I}$ are corresponding approximate error covariance matrices[18]. Given this notation a Fisher scoring step

[18]It should be mentioned that the notation $a_{i|I}$ refers to the smoothed value of α_i having taken the information of all I observation groups into account. The notation $a_{i|\tilde{I}}$ could be used alternatively.

on $\alpha^k := (\alpha_0^k, \alpha_1^k, \cdots, \alpha_I^k)$ of α can be performed using the following **Working Kalman Filter and Smoother (WKFS)**:

$$
\begin{array}{lll}
\multicolumn{2}{l}{\text{Initialization: } a_{0|0} = a_0, V_{0|0} = Q_0} & (20) \\
\multicolumn{2}{l}{\text{for } i = 1 \text{ to } I \text{ do:}} & \\
\quad\text{Prediction step:} & a_{i|i-1} = a_{i-1|i-1} & (21) \\
 & V_{i|i-1} = V_{i-1|i-1} + \delta_i * Q & (22) \\
\quad\text{Correction step:} & K_i = V_{i|i-1} Z_i' [Z_i V_{i|i-1} Z_i' + W_i^{-1}(\alpha_i^k)]^{-1} & (23) \\
 & a_{i|i} = a_{i|i-1} + K_i [y_i^*(\alpha_i^k) - Z_i a_{i|i-1}] & (24) \\
 & V_{i|i} = V_{i|i-1} - K_i Z_i V_{i|i-1} & (25) \\
\multicolumn{2}{l}{\text{for } i = I \text{ to } 1 \text{ do:}} & \\
\quad\text{Smoothing step:} & B_i = V_{i-1|i-1} V_{i|i-1}^{-1} & (26) \\
 & a_{i-1|I} = a_{i-1|i-1} + B_i (a_{i|I} - a_{i|i-1}) & (27) \\
 & V_{i-1|I} = V_{i-1|i-1} + B_i (V_{i|I} - V_{i|i-1}) B_i' & (28) \\
\multicolumn{2}{l}{\text{set } \alpha^{k+1} := (a_{0|I}, a_{1|I}, \cdots, a_{I|I})} &
\end{array}
$$

Iterating the WKFS gives a complete Fisher Scoring algorithm. We call this algorithm an **Iterated Working Kalman Filter and Smoother (IWKFS)**:

Initialization: $\alpha^0 = a_0 \otimes 1'_{I+1}, V_{0|0} = Q_0, k = 0$.
Step 1: Calculate an update α^{k+1} of α^k using WKFS.
Step 2: If $\|\alpha^{k+1} - \alpha^k\| < \epsilon$ STOP
else set k:=k+1 and go back to step 1.

Note that the final covariances matrices $V_{i|I}$, provided as a by-product of IWKFS are indeed the diagonal blocks of $-E[\frac{\partial^2 PL}{\partial\alpha\partial al'}]^{-1}$, the inverse of the expected information matrix (see Fahrmeir and Kaufmann 1991). No addional procedure has to be performed to calculate pointwise confidence bands.

To estimate the hyperparameters a_0 and Q[19] simultaneously, the **IWKFS** can be **combined with an EM-type algorithm** as shown below (see Fahrmeir and Wagenpfeil 1997).

1. Choose $Q_0 = k * I$ and the starting values $\theta^{(0)} = (Q^{(0)}, a_0^{(0)})$; set p=0.
2. IWKFS-iteration: Compute $a_{i|I}^{(p)}, V_{i|I}^{(p)}$ $(i = 0, 1, \cdots, I)$ by IWKFS, replacing the unknown hyperparameters by their current estimates $Q^{(p)}$and $a_0^{(p)}$.

[19] As described in section 4.1 Q_0 is set to $k * I$, with k a large positive integer.

3. EM-step: Compute updated estimates $Q^{(p+1)}$ and $a_0^{(p+1)}$ of the hyperparameters using the results from 2.

$$a_0^{(p+1)} = a_{0|I}^{(p)} \tag{29}$$

$$\begin{aligned} Q^{(p+1)} &= \frac{1}{I}\sum_{i=1}^{I}\frac{1}{\delta_i}[(a_{i|I}^{(p)} - a_{i-1|I}^{(p)})(a_{i|I}^{(p)} - a_{i-1|I}^{(p)})' + V_{i|I}^{(p)} \\ &\quad -B_i^{(p)}V_{i|I}^{(p)} - (B_i^{(p)}V_{i|I}^{(p)})' + V_{i-1|I}] \end{aligned} \tag{30}$$

   with B_i as defined in (26). Diagonalize $Q^{(p+1)}$.

4. If $\|\theta^{(p)} - \theta^{(p-1)}\| < \epsilon$ STOP, else set $p = p + 1$ and go to (2).

Finally to **estimate the hyperparamter and the states simultaneously using the PQML approach** we have to add an iteration for the estimation of ϕ. The resulting algorithm consists of three loops: The inner loop for the PQML, the EM-iteration for the indirect ML estimation of Q and a_0 and the outer loop for the moment estimation of ϕ. It can be implemented as follows:

1. Choose a starting value (usually 1) $\hat{\phi}_{(0)}$; set r=0.

2. Moment-iteration: Compute $a_{i|I}, V_{i|I},\ (i = 0, 1, \cdots, I)$ as well as Q and a_0 by the combined EM-IWKFS procedure described above.

3. Moment estimation: Compute an updated estimate $\hat{\phi}_{(r+1)}$ using the results from 2.

$$\hat{\phi}_{(r+1)}(a) = \frac{1}{n}\sum_i \frac{(y_i - \hat{\lambda}_i)^2}{\hat{\lambda}_i}, \tag{31}$$

   where $\hat{\lambda}_i = exp(Z_i a_{i|I})$.

4. If $\|\hat{\phi}_{(r+1)} - \hat{\phi}_{(r)}\| < \epsilon$ STOP, else set $r = r + 1$ and go to (2).

B Tables and Figures

Table 1: Crude Fertility Rate (CFR)

Year	West Germany	Greece	Italy	Spain	Turkey	Ex-Yugoslavia
1930	18	31	27	28	—	32
1940	20	25	24	24	—	25
1950	16	20	19	20	—	—
1960	17	19	18	22	43	24
1965	18	19	18	22	41	24
1970	13	17	17	20	36	18
1975	10	16	15	19	34	18
1982	10	14	11	15	31	15
1985	10	12	10	12	30	16
1989	11	10	10	11	26	14
1993	11	10	10	10	27	—

Note: CFR: rounded livebirths per 1000 inhabitants;
Source: United Nations Demographic Yearbook, Federal Statistical Office Germany: Statistical Yearbook, World Bank: World Development Report.

Table 2: Descriptive Statistics

Variable	Description	Native Sample	Immigrant Sample
NumBirth	Number of births	1.961	2.966
		(1.360)	(1.802)
Age	Woman's age (*10^{-1})	62.777	54.787
		(11.064)	(6.915)
Handicap	0/1 woman is handicapped	0.192	0.123
		(0.394)	(0.329)
Ever married	0/1 woman was ever married	0.955	0.985
		(0.208)	(0.121)
Age married	Woman's age at marriage (*10^{-1})	23.865	22.004
		(8.543)	(6.253)
Partner	0/1 partner present	0.625	0.866
		(0.484)	(0.342)
Schooling	Years of schooling	10.477	8.705
		(1.850)	(1.894)
S_Mandatory	0/1 Woman completed mandatory schooling	0.735	0.004
		(0.441)	(0.061)
S_Advanced	0/1 Woman completed advanced schooling degree	0.260	0.459
		(0.438)	(0.499)
V_Apprentice	0/1 Woman completed apprenticeship	0.360	0.052
		(0.480)	(0.223)
V_Advanced	0/1 Woman completed advanced vocational degree	0.208	0.093
		(0.406)	(0.291)
Unemployment	Indicator of past regional unemployment	6.052	5.469
		(1.357)	(1.460)
PS_Mandatory	0/1 Partner completed mandatory schooling	0.416	0.007
		(0.493)	(0.086)
PS_ Advanced	0/1 Partner completed advanced schooling degree	0.202	0.533
		(0.402)	(0.500)
PV_Apprentice	0/1 Partner completed apprenticeship	0.310	0.134
		(0.463)	(0.342)
PV_Advanced	0/1 Partner completed advanced vocational degree	0.246	0.153
		(0.431)	(0.361)
Immigrant	0/1 Woman is Immigrant	0.000	1.000
		(0.000)	(0.000)
N_Turkish	0/1 Woman of Turkish Nationality	0	0.332
		(0)	(0.472)
N_Yugoslav	0/1 Woman of Ex-Yugoslavian Citizenship	0	0.265
		(0)	(0.442)
N_Greek	0/1 Woman of Greek Nationality	0	0.168
		(0)	0.374
N_Italian	0/1 Woman of Italian Nationality	0	0.160
		(0)	(0.368)
N_Spanish	0/1 Woman of Spanish Nationality	0	0.075
		(0)	(0.263)
Fertile Years	Number of fertile years spent in Germany	0	16.034
	(coded 0 for Native sample)	(0)	(7.751)
Number of observations		1232	268

Source: German Socio-economic panel

Table 3: Completed Fertility by Alternative 'Duration' Indicator

	Average Completed Fertility by	
Years	Fertile Years in Germany	Years Since Migration
0–5	3.94	3.25
6–10	3.33	6.50
11–15	3.49	2.56
16–20	2.78	4.19
21–25	2.38	3.19
26–30	2.07	2.71
> 30	n.a.	2.34

Source: Own calculations based on German Socioeconomic Panel (GSOEP)

Table 4: Mapping $\phi \longrightarrow \hat{\phi}(a(\phi))$

ϕ	$\hat{\phi}(a(\phi))$
0.5	0.8083
0.6	0.8095
0.7	0.8108
0.8	0.8121
0.9	0.8134
1.0	0.8145
1.1	0.8154

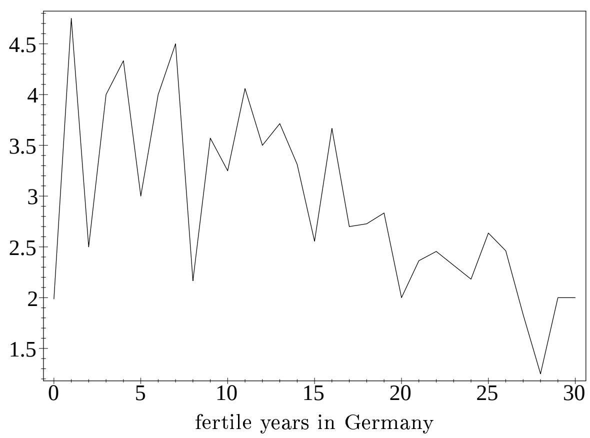
Note: Moment estimation of ϕ using the results of an EM algorithm with different fixed values of ϕ; Specification: Nationality Model (see Section 5).

Table 5: Estimates of Fixed Parameters in the Immigrant Model (Estimated via PML and PQML) and in the Nationality Model (Estimated via PML)

Variable	Immigrant Model (IM) Coeff. PML	Immigrant Model (IM) Coeff. PQML	Nationality Model (NM) Coeff. PML
Constant	-1.021	-1.018	-1.040
	(-2.855)	(-3.092)	(-2.827)
Age	0.020	0.019	0.026
	(0.955)	(1.009)	(1.223)
Handicap	0.023	0.023	0.035
	(0.484)	(0.531)	(0.739)
Ever married	2.377	2.377	2.329
	(9.652)	(10.482)	(9.162)
Age married	-0.213	-0.213	-0.203
	(-6.559)	(-7.120)	(-6.284)
Partner	-0.096	-0.096	-0.098
	(-1.050)	(-1.148)	(-1.034)
Schooling	-0.006	-0.006	-0.008
	(-0.228)	(-0.240)	(-0.318)
S_Mandatory	-0.125	-0.124	-0.084
	(-1.266)	(-1.361)	(-0.832)
S_Advanced	-0.126	-0.124	-0.084
	(-1.160)	(-1.247)	(-0.762)
V_Apprentice	-0.198	-0.198	-0.191
	(-3.439)	(-3.739)	(-3.310)
V_Advanced	-0.083	-0.083	-0.073
	(-0.974)	(-1.057)	(-0.857)
Unemployment	-0.000	-0.000	-0.005
	(-0.019)	(-0.024)	(-0.424)
PS_Mandatory	0.151	0.151	0.146
	(1.614)	(1.750)	(1.534)
PS_ Advanced	0.074	0.074	0.062
	(0.887)	(0.961)	(0.721)
PV_Apprentice	0.039	0.040	0.051
	(0.636)	(0.701)	(0.810)
PV_Advanced	-0.065	-0.064	-0.044
	(-0.955)	(-1.032)	(-0.636)
		starting values for hyperparameter	
$Q_{\text{Immigrant}}$	0.005	0.005	—
ϕ	—	0.8	—
		hyperparamter estimates	
ϕ	—	0.848	—
$Q_{\text{Immigrant}}$	0.0035	0.0036	—
		algorithm details	
# moment estimation steps	—	4	—
total # EM-Steps	44	59	294
total # WKFS-Steps	102	135	633

Note: Approximative t-statistics in parantheses. The parameter functions for the immigrant indicator are shown in Figure 2, those for the NM in Figures 3–7. Starting value for all the elements of Q in the NM was 0.005. The final hyperparameter estimates in the NM are $Q_{\text{Turkish}} = 0.0087, Q_{\text{Yugoslav}} = 0.0098, Q_{\text{Greek}} = 0.0002, Q_{\text{Italian}} = 0.0064$ and $Q_{\text{Spanish}} = 0.0047$.

Figure 1: Average Completed Fertility by Fertile Years in Germany



Source: Own calculations based on German Socioeconomic Panel (GSOEP)

Figure 2: Parameter function of the nationality indicator "Immigrant" estimated in the Immigrant Model via PML and PQML

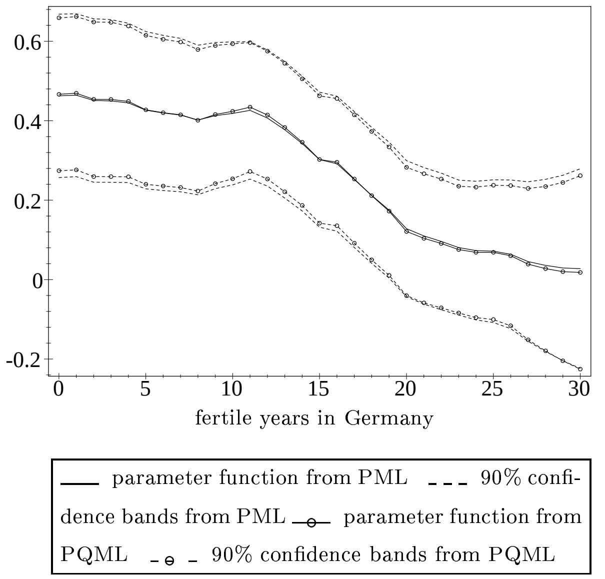


Figure 3: Parameter function of the nationality indicator "Turkish" estimated in the Nationality Model via PML

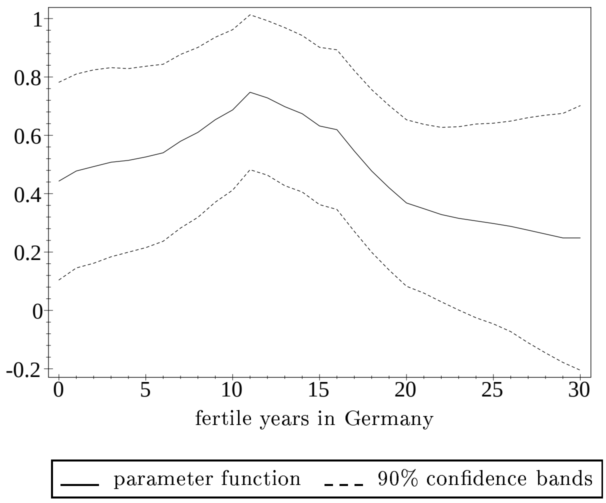


Figure 4: Parameter function of the nationality indicator "Yugoslavian" estimated in the Nationality Model via PML

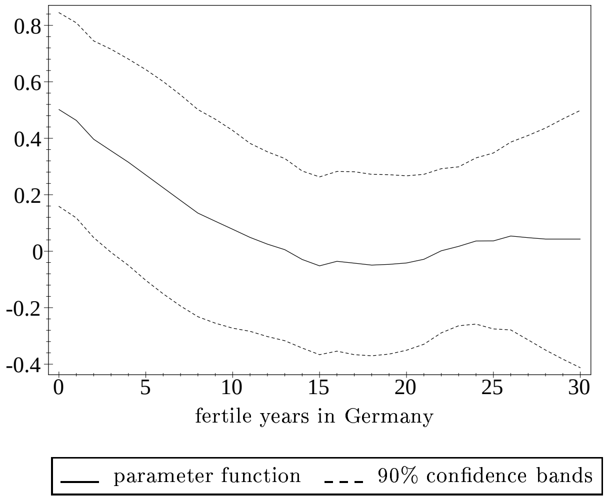


Figure 5: Parameter function of the nationality indicator "Greek" estimated in the Nationality Model via PML

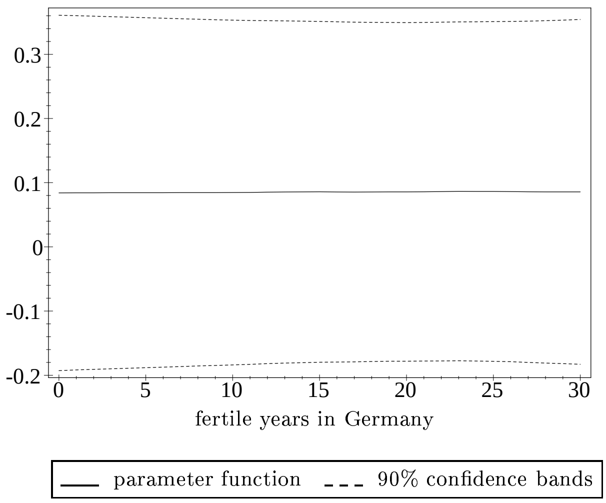


Figure 6: Parameter function of the nationality indicator "Italian" estimated in the Nationality Model via PML

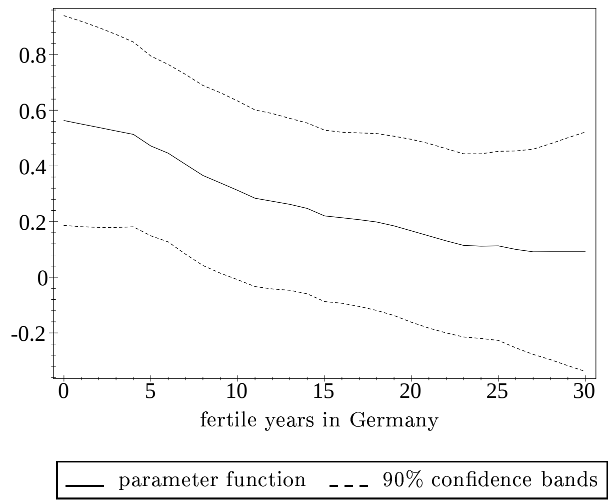


Figure 7: Parameter function of the nationality indicator "Spanish" estimated in the Nationality Model via PML