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Estimating a War of Attrition: The Case of the U.S. Movie Theater Industry

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Estimating a War of Attrition: The Case of the U.S. Movie Theater Industry*

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Abstract

This paper provides a tractable empirical framework to analyze firm behavior in a dynamic oligopoly when demand is declining over time. I modify Fudenberg and Tirole (1986)'s model of exit in a duopoly with incomplete information to a model that can be used in an oligopoly, and combine this with an auxiliary entry model to address the initial conditions problem. I estimate this model with panel data on the U.S. movie theater industry from 1949 to 1955, using variations in TV diffusion rates across households, market structure before the exit game starts, and other market characteristics to identify the parameters in the theater's payoff function and the distribution of unobservable fixed costs. Using the estimated model, I measure strategic delays in the exit process due to oligopolistic competition and incomplete information. The delay in exit that arises from strategic interaction is 2.7 years on average. Out of these years, 3.7% of this delay is accounted for by incomplete information, while the remaining 96.3% is explained by oligopolistic competition.

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1 Introduction

Industries decline for various reasons. Some sectors are left behind by a new technology, while others are pressured by severe competition from abroad. The U.S. steel industry during the 1960's and 1970's went through a severe consolidation phase. In the U.K. steel castings industry, demand declined by 42% between 1975 and 1981. Current examples are also abundant. In the last ten years, several industries experienced a sharp decline in industry revenue: wired telecommunications carriers (-54.9%), mills (-50.2%), newspaper publishing (-35.9%), and apparel manufacturing (-77.1%).[1] Despite their declining trend, declining industries still have an important role in the economy. Industries whose real output had shrunk more than 10% during the period 2000-2010 accounted for approximately 27% of U.S. manufacturing output in 2010.[2]

In response to declining demand, firms sequentially divest, merge into a bigger entity, or exit from the market, causing the industry capacity to continue to decrease. For several reasons, such a consolidation process could cause substantial inefficiencies, especially in concentrated industries. First, capacity reduction is a public good that should be provided privately, so firms have an incentive to free-ride on competitors' divestment. Second, because firms do not have exact information about competitors' profitability or about the future demand process, they have an incentive to wait to acquire more information before they act. These factors can create an inefficiently slow divestment process. Borders' bankruptcy in 2011 may be a good example of this. In the brick-and-mortar book retail industry, two big firms, Barnes & Noble and Borders, faced severe competition from online book stores (especially Amazon) since the early 2000's. These physical book retailers started to steal customers from each other. Despite such a costly form of competition, their speed of divestment (i.e., closing stores) was very slow.[3][4] They tried to outlast each other, as when one of them exited/divested, the profit of the other would increase at least for some period of time.[5] As a

[1] See *IBISWorld*, May 2011, available at http://www.ibisworld.com/Common/MediaCenter/Dying%20Industries.pdf.

[2] Employment Projections Program by U.S. Bureau of Labor Statistics. I use the 2007 NAICS 3 digit-level.

[3] See *US News*, January 10, 2011, available at http://money.usnews.com/money/blogs/flowchart/2011/01/10/why-barnes–noble-is-thumping-borders.

[4] Borders' profit turned from positive to negative in 2006. Since then, it incurred losses until its bankruptcy in 2011. The number of Borders' superstores, however, decreased only marginally, from 528 in 2005 to 511 in 2009. See *The Ann Arbor News*, February 16, 2011, available at http://www.annarbor.com/business-review/borders-bankruptcy-ann-arbor-books/.

[5] "Wall Street firm Credit Suisse estimates that Barnes & Noble will take more than 50% of the business that is up for grabs from Borders' store closings. Almost 70% of the two retailers' stores overlap, good news for Barnes & Noble as it tries to pick up former Borders customers." (See *Time*, March 8, 2011, available at http://www.time.com/time/business/article/0,8599,2057760,00.html)

result, the divestment process imposed costs on the whole industry.

While these divestment and consolidation processes are very common, the economic costs of such processes are not well understood. In particular, the economic costs due to the strategic interactions mentioned above have not been studied empirically in the literature, mostly because of technical difficulties including non-stationary environments, the prevalence of equilibrium multiplicity in dynamic games, and the initial conditions problem. This paper addresses these difficulties and methodologically contributes to the literature. The main innovation is a tractable empirical framework for analyzing the economic costs of consolidation that arise due to strategic interaction. I focus on the case in which firms make a binary exit-stay decision.

As an example, I study the U.S. movie theater industry in the 1950's, when a drop in demand caused the sequential exits of theaters from local oligopolistic markets. Figure 1 shows the total yearly theater attendance and the total number of indoor theaters from 1947 to 1960. The average attendance for the average theater declined severely during the period. This decline in demand was mostly due to exogenous forces, such as the nationwide penetration of TVs (Lev (2003) and Stuart (1976)). Figure 2 shows growth of the TV penetration rate in the U.S. Accordingly, the total number of indoor movie theaters decreased by 35% in ten years from the end of the 1940's.

Competition in the U.S. movie theater industry in the 1950's resembled a "war of attrition." In those days, costs were mostly fixed and capacity adjustments were usually infeasible. Hence, theaters responded to declining demand by leaving the market. Given localized demand and a small number of movie theaters in each market, theaters considered their opponents' behavior when choosing an optimal exit time. Since each theater does not internalize increased profits received by its competitors when exiting the market, oligopolistic competition leads to slower sequential exits when compared to the coordinated exits that maximize the industry's profit. In addition, theaters were unlikely to have exact information about competitors' profitability, because the major part of a theater's profit depends on sales/costs from concession stands and rent payments, which differ widely and in idiosyncratic ways across theaters and are private information. Therefore, theaters learn about their competitors' profitability over time, and may have incurred a loss while competing, in the hope that they could outlast their competitors. Thus, strategic interaction could generate a significant delay in the exit process.[6]

To quantify the delay in exit (and the resulting cost) due to theaters' motives to outlast

[6] There is another dimension in which strategic interaction could have non-trivial impacts on the consolidation process. The non-cooperative nature of the game could result in an inefficient order of exits; less efficient firms outlast more efficient competitors. Ghemawat and Nalebuff (1985) demonstrate such a possibility theoretically.

each other, the effect of competition among theaters as well as their beliefs about competitors needs to be inferred separately from the effect of exogenous demand. For this purpose, I modify Fudenberg and Tirole (1986)'s model of exit in duopoly with incomplete information to one that can be used in an oligopoly setting. At each instant, theaters choose whether to exit or stay in the market. I assume each theater knows its own time-invariant fixed cost but not that of its competitors. Thus, from a theater's perspective there is benefit of not exiting the market, as there is some chance their competitors will exit instead, which increases their profit. They compare this benefit of waiting with the cost of waiting, which increases over time. In equilibrium, theaters get discouraged and exit if competitors stay open long enough. When one of the theaters exits, the remaining theaters revise their planned exit time and wait until then. As a result, theaters exit sequentially, with the theater with the highest fixed cost leaving first.

One major advantage of this framework is that the uniqueness result in Fudenberg and Tirole (1986) is preserved in the N player game. Thus, for any set of parameters of the model, there is a unique distribution of equilibrium exit times. Furthermore, as demonstrated in Section 3, the cost of computing the equilibrium is low. As a consequence, I can take the full-solution approach, which allows me to explicitly take theaters' expectations and unobservable market-level heterogeneity into account.

I apply the proposed framework to theater-level panel data from the U.S. movie theater industry, estimating theaters' payoff functions and the distribution of fixed costs. I use TV penetration rates, which vary across locations and time, to measure changes in demand. By imposing the equilibrium condition, the model predicts the distribution of theaters' exit times for a given set of parameters and unobservables. I estimate the parameters by matching the distribution predicted by the model with the observed distribution of exit times.

Identification of the model is possible because, in equilibrium, exit times are determined by theaters' expectations about their competitors' behavior as well as demand decline and product market competition. Therefore, exit times are informative about all these factors. In addition, I observe the number of theaters in each market before the war of attrition started. Intuitively, the strategic delay is measured as follows. The market structure before the decline in demand helps me to infer how theaters interact in the product market. Using exit behavior in monopoly markets, I can infer how demand declines with TV penetration. With these components, exit behaviors in a strategic environment (markets with more than one theater) are implied from the model without asymmetric information. Then, the difference between these implied exit behaviors and data is attributed to the strategic delay in exit and the extent of asymmetric information.

Using the estimated model, I quantify the effect of strategic interaction on the consolidation process. To do so, I define two benchmarks. First, if fixed costs are common knowledge, then in equilibrium, a theater exits the market at the exact time that its operating profit becomes lower than its fixed costs, implying that no theater incurs a loss. This is called the complete information benchmark. The difference in cumulative market profits between the war of attrition and the complete information benchmark is defined as the cost of asymmetric information. Second, I shut down the incentive to free-ride on competitors' exit and calculate the path of theater exits that maximizes the industry's profit. I call this the regulator benchmark, and the difference in cumulative industry profits under the complete information benchmark and the industry regulator case is defined as the cost of oligopolistic competition.

The delay in exit that arises from strategic interaction is 2.676 years on average. From these years, 3.7% of this delay is accounted for by asymmetric information, while the remaining 96.3% is explained by oligopolistic competition. The resulting cost, measured by the percentage difference in cumulative market profits, is 4.9% in the median market. The cost of oligopolistic competition accounts for 95.5% of this total cost, while the cost of asymmetric information accounts for 4.5%.

The extent of delay due to asymmetric information differs across different market structures. Specifically, such delays are longer in markets with fewer competitors. For example, the average delay in duopoly markets is 0.154 years (6.5% of the total delay in duopoly markets), while the average across all markets is 0.099 years. For a given player, the probability of winning the war of attrition, i.e., the probability of being a monopolist, is highest in a duopoly, and therefore theaters have the greatest incentive to wait. Moreover, the increase in profits when one competitor exits is highest in a duopoly, which also partly explains the difference in delay between the two cases. As the initial number of competitors gets large, competition becomes closer to perfect competition, and hence motives to outlast competitors become less significant.

The cost of asymmetric information is larger in markets with a slow decline in demand. Splitting the sample into markets with slow and fast decline based on the speed of TV diffusion, the median cost in slowly declining markets, measured by the percentage difference in cumulative market profits, is 0.83%, while the corresponding number for markets with fast decline is 0.57%. The intuition is as follows. In markets with slow decline, the cost of waiting increases slowly. On the other hand, the benefit of waiting is still large because a winner of the game can enjoy a higher profit over a longer period of time. These two factors prolong the war of attrition. For example, in a counterfactual scenario in which demand is fixed over time, the average delay in exit due to asymmetric information becomes 1.059 years, which is more than ten times as long as the original case. An example of such a situation would be battles to control new technologies discussed by Bulow and Klemperer (1999), as demand in those industries is not declining. Consequently, large losses accumulate over time.

Related Literature

I use a full-solution approach to estimate the dynamic game with learning, exploiting the uniqueness property of the game and simplicity of computation. In contrast, most papers estimate a dynamic game using a two-step estimation method. Early papers that proposed two-step estimation methods for dynamic Markov games include Jofre-Bonet and Pesendorfer (2003), Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008). Recent empirical applications using a two-step method include Ryan (2012), Collard-Wexler (2013), and Sweeting (2013). In the first stage of the two-step method, a policy function is calculated for every possible state, which is difficult in a non-stationary environment. Moreover, unobservable variables (market-level heterogeneity and theaters' profitability) play an important role in my model, so the first-stage estimation in the two-step method would not be consistent.

Schmidt-Dengler (2006) analyzes the timing of new technology adoption, separately estimating how it is affected by business stealing and preemption. In the environment he considers, the cost of adopting a new technology declines over time. One important difference is the source of inefficiency in oligopolistic markets. In Schmidt-Dengler (2006), players can delay competitors' adoption times by adopting before they do, even though such an adoption time is earlier than the stand-alone incentive would suggest as optimal. Thus, this preemption motive hastens the industry's adoption of new technology. In the current study, there is asymmetric information, so players have an incentive to delay their exit, hoping that they can outlast their competitors, even if they are currently making a negative profit.

Klepper and Simons (2000) and Jovanovic and MacDonald (1994) investigate the U.S. tire industry, in which a large number of firms exited within a relatively short period of time. They assume this market is competitive. In their model, innovation opportunities encourage entry in the early stage of the industry's development. As the price decreases due to the new technology, firms that fail to innovate exit. Competition affects the devolution of the industry through the market price. In comparison, in the movie theater industry during the relevant period, competition was local, and hence strategic interactions among theaters should be taken into account. Another important difference is that the shakeout in the U.S. tire industry was not due to declining demand.

A number of papers analyze firm exit (Fudenberg and Tirole (1986), Ghemawat and Nalebuff (1985, 1990)).[7] I estimate a modified version of Fudenberg and Tirole (1986), which has

[7] Several papers, including Baden-Fuller (1989), Deily (1991), and Lieberman (1990) analyze empirically the relationship between a firm's characteristics and its exit (plant closing) behavior.

asymmetric information between players that delays their exits. A model similar to Ghemawat and Nalebuff (1985), one without asymmetric information, is used for my counterfactual analysis to evaluate the significance of asymmetric information. Ghemawat and Nalebuff (1990) consider a case in which firms can continuously divest their capacity in a declining industry. While such a case is more sensible in many settings, as Section 2 will discuss, the current application fits better into a case of binary exit/stay decisions.

Several recent papers analyze consolidation processes using a dynamic structural model. Stahl (2012) uses the deregulation in the U.S. broadcast TV industry as an exogenous event that led to significant consolidation to estimate firms' benefits (increased revenue) and costs of purchasing competitors' stations. Jeziorski (2013) develops a dynamic model of endogenous mergers to estimate fixed-cost efficiencies of mergers in the U.S. radio industry. These two papers quantify the cost reduction the merging firm achieves. On the other hand, Nishiwaki (2010) develops and estimates an oligopolistic model of divestment using data from the Japanese cement industry. With the estimated demand and cost parameters, he asks the hypothetical question of what would have happened to social welfare if a merger in the data had not been approved. An important difference is that he considers a case without learning and focuses on firms' divestment processes and business stealing effects.

This paper is related to the literature on all-pay auctions with incomplete information. Krishna and Morgan (1997) analyze auction settings in which losing bidders also have to pay positive amounts and examine the performance of these settings in terms of expected revenues. Moldovanu and Sela (2001) study a contest with multiple unequal prizes with asymmetric bidding costs. Bulow and Klemperer (1999) analyze a general game in which there are $N + K$ players competing for N prizes. My model can be considered as one variant of this class of models with heterogenous costs/prizes of bidders, but it is different because the value of the prize (operating profits) changes over time and is affected by the number of surviving players, which is endogenous. In addition, this paper is one of few empirical applications of such models.

To the best of my knowledge, almost no paper in the literature estimates a dynamic game with serially-correlated private values (a notable exception is Fershtman and Pakes (2010)). Two difficulties arise in estimating such models. First, to account for theaters' expectations, the entire history of the game should be included in the state space. It is difficult to do so in the framework of Ericson and Pakes (1995), which is commonly used in the literature. Second, the initial conditions problem is more significant with serially-correlated private values, as players at the beginning of the sample period are selected samples. Because I account for these factors, I can estimate a game with serially-correlated private values, in comparison to much of the literature.

The remainder of the paper is organized as follows. Section 2 briefly summarizes the U.S. movie theater industry in the 1940's and 1950's. Section 3 modifies the model of Fudenberg and Tirole (1986) to be used in an oligopoly. Section 4 describes the data. Section 5 discusses my estimation strategy. Section 6 presents estimation results and simulation analysis. Section 7 concludes. All proofs are shown in the appendices.

2 Case Study: The U.S. Movie Theater Industry

The U.S. movie theater industry in the late 1940's and 1950's is a relevant case study for the economic costs of consolidation. This subsection discusses the industry background and underlying factors behind its declining demand. I focus on demand and exit behavior in the classic single-screen movie theater industry.

After a big boom starting in the 1920's, the U.S. movie theater industry faced a severe decrease in demand in the 1950's and the 1960's, primarily due to the growth of TV broadcasting. In 1950, fewer than one out of ten households in the U.S. owned a TV set. By 1960, however, almost 90% of households had a TV. In response, demand for theaters decreased. Movie attendance declined most quickly in places where TVs were first available, implying that TV penetration caused a decline in demand. According to Stuart (1976), the addition of a broadcast channel in the market caused an acceleration in the decline in movie theater attendance. There were other factors that contributed to the decline in demand. Suburb growth and motorization facilitated the growth of drive-in theaters, which in turn further decreased demand for classic single-screen movie theaters.

Changes in government policy at the end of the 1940's also contributed to the downturn in demand.[8] Vertical integration among producers, distributors, and exhibitors had been widespread until the late 1940's. The major movie producers (called the "Hollywood majors"[9]) formed an oligopoly, and they had control over theaters through exclusive contracts and explicit price management. They owned 3,137 of 18,076 movie theaters (70% of the first-run theatres in the 92 largest cities). The Paramount Decree (1946), however, put an end to this vertical integration, resulting in the separation of those producers from their vertical chains of distributors and exhibitors. For example, explicit price management by distributors was prohibited. The government also mandated that the spun-off theater chains would have to further divest themselves of between 25% and 50% of their theater holdings.

The Paramount Decree created a more unstable and risky business environment for movie

[8] The figures and facts in this paragraph are from Chapter 6 of Melnick and Fuchs (2004).

[9] Majors in this era include the "Big Five" (Loew's/MGM, Paramount, 20th Century-Fox, Warner Bros., and RKO) and the "Little Three" (Universal, Columbia, and United Artists).

theaters. For example, movie producers no longer had a strong incentive to produce movies year round. Furthermore, according to Lev (2003), the production companies started to regard TV as an important outlet for their movies. In the era of vertical integration, producers had an incentive to withhold their movies from TVs in the interests of their exhibitor-partners. After divestment, however, movie theaters became just one of the customers for producers, along with the TV companies.

Because of all of these factors, demand for incumbent movie theaters shrunk in an arguably exogenous way. As shown in Figure 1, theater attendance started to decrease in 1949, and kept declining mostly monotonically afterwards. Almost all theaters had only a single screen in those days (e.g., the first twin theater in the Chicago area opened in 1964), and their fixed investments were often heavily mortgaged. Therefore, they could not adjust capacity to deal with declining demand. They could only bear the loss and stay open or exit the market. Thus, the number of indoor movie theaters decreased with demand.

Figure 1 also shows that the decline in the number of indoor theaters was slower than the decline in theater attendance. This slow divestment process is consistent with the argument that capacity reduction is a public good so the industry tends to maintain excess capacity in a declining environment. One way to further explore this difference is to look at the relationship between exits of incumbent theaters and market structure. Figure 3 shows the exit rate during the sample period averaged by the initial number of competitors. As is clear from the figure, the exit rate increases with the number of competitors. One possible explanation is that theaters were trying to outlast their competitors in the declining environment. Since a few theaters could still operate profitably, each theater preferred to stay open as long as it expected some competitors to exit early enough. If there are many competitors, it is highly unlikely that a theater will be one of the few survivors at the end, so the theater may give up and exit early. This situation fits nicely into the framework of a war of attrition. Thus, in this paper, I use the framework of a war of attrition, exploiting the relationship between the number of competitors and exit probability to analyze the exit behavior of movie theaters.

The structure of the U.S. movie theater industry changed significantly in the 1960's, when multiplex theaters emerged. This arguably changed the nature of competition. Once a theater has multiple screens, it can potentially respond to a change in demand by, for example, closing several screens. The structure of the industry became even more different and complicated after the 1980's because of the advent of home videos/DVD and horizontal integrations by big theater chains. Horizontal integration has become more prevalent over time. Nowadays, ten nation-wide movie theater chains own 34% of indoor movie theaters and 58% of screens in the U.S.[10] In the 1940's and 1950's, however, such horizontal integration was much less

[10] See the website of the National Association of Theatre Owners at http://www.natoonline.org/.

common.[11] Considering these changes in the industry structure since the 1960's, I focus on the late 1940's and 1950's.

It is difficult to evaluate how much of firms information is privately observed. Theater-specific demand is most likely common information between theaters, as the number of admissions is easily observed. Profits from concession sales, however, which account for an important part of total profit, are much harder to observe. Information about costs would also tend to be private. First, the outside option or opportunity cost of a theater's owner is difficult to observe. Second, the theater's fixed costs mainly come from rent payments, which vary widely across theaters and are not easily observed by competitors. With unobserved competitors' costs and sales, in an environment in which the industry demand is declining, theaters could keep updating their beliefs about competitors' profitability over time.

3 The Model

In this section, I modify Fudenberg and Tirole (1986)'s model of exit in a duopoly with incomplete information to be used in an oligopoly.

3.1 Setup

There are N theaters, $i = 1, ..., N$, which play a game of exit in a market. The game starts at $t = 0$ and time is continuous. At each instant, theaters decide whether to stay in or exit the market. If more than one theater chooses to exit at $t = 0$, one of these theaters is randomly chosen with equal probability and exits, and then the remaining $N - 1$ theaters restart the game at $t = 0$.[12]

While staying, theaters earn a common instantaneous profit of $\Pi_n(t)$, where $n \in \{1, ..., N\}$ is the number of currently active theaters in the market. Once a theater exits the market, it cannot re-enter. When theater i exits, it receives an exit value (scrap value) of θ_i, which is privately observed by theater i at the beginning of the game. Note that θ_i incorporates both the value of exit (opportunity cost) that the theater would forgo by staying in and the fixed cost of production. The values θ_i are drawn independently from the common distribution $G : [\bar{\theta}, \underline{\theta}] \to [0, 1]$, where $0 < \underline{\theta} < \bar{\theta} < \infty$. Theaters discount the future at a common rate of r.

[11] Online Appendix D analyzes how important movie theater chains are in determining the exit process.

[12] This particular rule at $t = 0$ is introduced to avoid the problem of the non-existence of a pure strategy equilibrium. It is possible in the case of N-player games that exiting immediately given opponents not doing so is optimal for more than one player. In this case, a symmetric equilibrium does not exit. For the necessity of this randomization device in N-player games and discussion, see Haigh and Cannings (1989), footnote 31 of Bulow and Klemperer (1999), and Argenziano and Schmidt-Dengler (2013).

I use "theater θ_i" to denote a theater with exit value θ_i. I focus on symmetric perfect-Bayesian equilibria.

I impose several restrictions on the payoff function and the distribution of θ.

Assumption 1 (i) $\Pi_n(t)$ *decreases over time and converges to* $\overline{\Pi}_n$ *for all* n. (ii) *For each* $n = 2, ..., N$, $\Pi_n(t) < \Pi_{n-1}(t)$ *for all* t. (iii) $\lim_{t\to\infty}\Pi_N(t) > \underline{\theta}$. (iv) $\Pi_1(0) < \bar{\theta}$. (v) G *has a density* g *everywhere positive and absolutely continuous with respect to Lebesgue measure.*

The game proceeds as follows. At $t = 0$, each theater decides when to exit, conditional on none of the competitors having exited by then. In Online Appendix A, I show that this decision is equivalent to choosing to exit or to remain open at every instant. When one theater exits, other surviving theaters revise their planned exit times based on the currently available information, since now the instantaneous profit is higher and there is one less active theater in the game. Thus, a strategy is a mapping from the state space to the real line (exiting time), $T_i : \Omega \to [0, \infty]$, where Ω denotes the set of states detailed below.

Suppose that one theater exits at $s > 0$. The remaining theaters play a subgame. The only information that each theater has obtained about its remaining opponents is the fact that they have survived up until s. As I will show in Section 3.2, there exists the highest possible value of θ of surviving opponents, denoted by $\widetilde{\theta}$, which is a sufficient statistic for the history of the game up until s. For ease of exposition, I keep dependence of $\widetilde{\theta}$ on other factors implicit. The state space consists of the number of survivors, the current time, the highest possible exit value of surviving opponents, and the theater's own exit value. The domain of the strategy is therefore given by $\Omega = \mathcal{N} \times \mathbb{R}_+ \times \left[\underline{\theta}, \overline{\theta}\right] \times \left[\underline{\theta}, \overline{\theta}\right]$, where $\mathcal{N} = \{1, ..., N\}$. Let T_{-i} be a set of strategies for i's opponents and let $T = (T_i, T_{-i})$.

To characterize the value of theaters, define $t^*(\theta_{-i}) = \min_{j\neq i}\{T_j(n, s, \widetilde{\theta}, \theta_j)\}$. That is, $t^*(\theta_{-i})$ gives the earliest exit time of i's competitors. Note that I make the dependence of t^* on state variables and strategy implicit. If theater i chooses stopping time τ when the state variables are given by $(n, s, \widetilde{\theta}, \theta_i)$ and the other theaters follow strategy T_{-i}, the present discounted value of i's expected payoff is

$$
\begin{aligned}
V(\tau, T_{-i}, n, s, \widetilde{\theta}, \theta_i) = {} & \Pr(t^*(\theta_{-i}) \geq \tau)\left[\int_s^{\tau} \Pi_n(t)\, e^{-r(t-s)}dt + \frac{\theta_i}{r}e^{-r(\tau-s)}\right] \\
& + \int_{\{\theta_{-i}|t^*(\theta_{-i})<\tau\}}\left[\int_s^{t^*(\theta_{-i})} \Pi_n(t)\, e^{-r(t-s)}dt\right. \\
& \left. + e^{-r(t^*(\theta_{-i})-s)}V(T, n-1, t^*(\theta_{-i}), \widetilde{\theta}', \theta_i)\right] g(\theta_{-i}|\theta \leq \widetilde{\theta})d\theta_{-i}.
\end{aligned}
$$

where $\widetilde{\theta}'$ is the highest possible exit value of surviving theaters at time $t^*(\theta_{-i})$ and $g(\theta_{-i}|\theta \leq \widetilde{\theta}) = \prod_{j\neq i} g(\theta_j|\theta_j \leq \widetilde{\theta})$.

In other words, an $n-1$ player game is nested into an n player game, which is further nested into an $n+1$ player game, and so on.

Definition 1 *A set of symmetric strategies* $\{\hat{T}_i(n, s, \widetilde{\theta}, \theta_i)\}_{i=1}^N$ *with posterior beliefs* $g(\theta|\theta \leq \widetilde{\theta})$ *is a symmetric perfect Bayesian equilibrium if for all* $n \in \mathcal{N}$, $s \in \mathbb{R}_+$, $\widetilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, *and* $\theta_i \in \left[\underline{\theta}, \overline{\theta}\right]$,

1. *For all* i *and any strategy* T_i,

$$V(\hat{T}_i, \hat{T}_{-i}, n, s, \widetilde{\theta}, \theta_i) \geq V(T_i, \hat{T}_{-i}, n, s, \widetilde{\theta}, \theta_i),$$

and

2. *For any opponent* j, $g(\theta_j|\theta_j \leq \widetilde{\theta})$ *is given by*

$$g(\cdot|\theta_j \leq \widetilde{\theta}) = \begin{cases} \frac{g(\theta_j)}{\Pr(\theta_j \in \Theta)}, & \theta_j \in \Theta \\ 0, & \theta_j \notin \Theta \end{cases}$$

where Θ *is the set of types* θ *that can survive on-equilibrium path.*

3.2 The General Solution

This subsection characterizes the Bayesian equilibrium in an N-player game. The major difference between this game and Fudenberg and Tirole (1986)'s duopoly game is that when one player drops out, the game still continues. The following lemmata fully characterize the necessary conditions of the Bayesian equilibrium in an N-player game. Since I focus on symmetric equilibria, firm subscripts in strategy functions are omitted. All proofs are presented in Online Appendix A.

Lemma 2 *Assume* $n = N$. *Under Assumption 1, if* $T(n, 0, \overline{\theta}, \cdot)$ *is the equilibrium strategy in a perfect Bayesian equilibrium, then (i) For all* $\theta \in \left[\Pi_{n-1}(0), \overline{\theta}\right]$, $T(n, 0, \overline{\theta}, \theta) = 0$. *(ii)* $T(n, 0, \overline{\theta}, \theta)$ *is continuous and strictly decreasing on* $(\underline{\theta}, \Pi_{n-1}(0))$. *(iii) The inverse function of strategy in terms of* θ, *denoted as* $\Phi(t; n, 0, \overline{\theta}) \equiv T^{-1}(t; n, 0, \overline{\theta})$, *is differentiable on* $(0, \infty)$. *(iv) Its derivative is given by*

$$\Phi'(t; n, 0, \overline{\theta}) = -\frac{G(\Phi(t; n, 0, \overline{\theta}))}{(n-1)\, g(\Phi(t; n, 0, \overline{\theta}))} \left[\frac{\Phi(t; n, 0, \overline{\theta}) - \Pi_n(t)}{V(T, n-1, t, \widetilde{\theta}', \widetilde{\theta}') - \Phi(t; n, 0, \overline{\theta})/r}\right], \tag{1}$$

where $\widetilde{\theta}' = \Phi(t; n, 0, \overline{\theta})$, *with the boundary conditions*

$$\Phi(0; n, 0, \overline{\theta}) = \Pi_{n-1}(0) \tag{2}$$

$$\lim_{t \to \infty} \Phi(t; n, 0, \overline{\theta}) = \lim_{t \to \infty} \Pi_n(t). \tag{3}$$

The next lemma characterizes the equilibrium strategy for $n < N$. Let s_n denote the time when the n-player subgame starts. There are two cases to consider. First, the subgame still starts at time zero; i.e., $s_n = 0$. This is the case when one or more competitors exit at time zero in the preceding subgame. Second, there is already selection in the preceding subgame; i.e., $s_n > 0$.

Lemma 3 *Suppose Assumption 1 holds. First, assume* $1 < n < N$*. If* $s_n = 0$*, properties (i)-(iv) in Lemma 2 hold. Next, assume* $s_n > 0$ *and let* $\theta^* = \Phi(s_n; n+1, s_{n+1}, \widetilde{\theta}_{n+1})$ *and* $t^* = \Pi_n^{-1}(\theta^*)$*. Then, (i') There is no exit at* $t \in (s_n, t^*]$*. In addition, given that* $T(n, t^*, \theta^*, \theta)$ *is the equilibrium strategy, (ii')* $T(n, t^*, \theta^*, \theta)$ *is continuous and strictly decreasing in* θ *on* $(\underline{\theta}, \theta^*)$ *. (iii')* $\Phi(t; n, t^*, \theta^*) \equiv T^{-1}(t; n, t^*, \theta^*)$ *is differentiable on* (t^*, ∞)*. (iv') Its derivative is given by (1) and the boundary conditions are*

$$\Phi(t^*; n, t^*, \theta^*) = \theta^* \tag{4}$$

$$\lim_{t \to \infty} \Phi(t; n, t^*, \theta^*) = \lim_{t \to \infty} \Pi_n(t). \tag{5}$$

Finally, assume $n = 1$*. The exit time of the surviving theater is given by the solution to the following single-agent problem:*

$$T(1, s_1, \widetilde{\theta}_1, \theta) \in \arg \max_{\tau \in [s_1, \infty]} \left[\int_{s_1}^{\tau} \Pi_1(t) e^{-r(t-s_1)} dt + \frac{\theta}{r} e^{-r(\tau - s_1)} \right].$$

Proposition 4 *Under Assumption 1, equations (1)-(5) constitute a symmetric Bayesian equilibrium of the entire game.*

I do not have a formal proof for existence. In the estimation, however, for any set of parameters, I numerically find the $\Phi(t; n, s_n, \widetilde{\theta}_n)$ that satisfies equations (1), (2), and (3).[13] Moreover, as Proposition 5 shows, if I find a symmetric equilibrium, it is the unique symmetric equilibrium.

Proposition 5 *The symmetric equilibrium, if it exists, is unique.*

The logic behind this result is the same as that of Fudenberg and Tirole (1986). Introducing a positive probability that no theater has to exit brings the uniqueness. This is an attractive feature of the model and is extremely important for the full-solution approach in estimation. Finally, the following proposition bounds the policy function from below and above.

Proposition 6 $0 < \Pi_n(t) < \Phi(t; n, s_n, \widetilde{\theta}_n) < \Pi_{n-1}(t)$ *for all* $n > 1$ *and* $t > s_n$.

[13] Online Appendix E provides the details for computing the solution to the differential equation.

3.3 Intuition and Example

The intuition behind the solution can be easily understood by considering the *marginal* player. For the marginal player i who is indifferent between dropping out at time t and staying until time $(t+dt)$ and then dropping out, it follows that

$$\Pr\left(\begin{array}{c}\text{one of } n-1 \text{ competitors}\\ \text{drops in } [t,t+dt]\\ \text{conditional on survival until } t\end{array}\right)\underbrace{\left[V(T,n-1,t,\widetilde{\theta}',\widetilde{\theta}')-\frac{\widetilde{\theta}'}{r}\right]dt}_{i\text{'s payoff when one of competitors drops}} = \underbrace{\left[\widetilde{\theta}'-\Pi_n(t)\right]}_{i\text{'s cost of staying in}}dt, \tag{6}$$

where $\widetilde{\theta}' = \Phi(t;n,s_n,\widetilde{\theta}_n)$. Since the conditional probability that one of the $n-1$ competitors drops out in $[t,t+dt]$ is given by $\{(-(n-1)\,g(\Phi(t;n,s_n,\widetilde{\theta}_n))\Phi'(t;n,s_n,\widetilde{\theta}_n)/G(\Phi(t;n,s_n,\widetilde{\theta}_n)))\ dt\}$, rearranging (6) gives the differential equation (1).

Thus, the differential equation with boundary conditions fully characterizes the time path of the marginal type and serves as a policy function. Figure 4 shows a typical duopoly case. The policy function $\Phi(t;2,0,\Pi_1(0))$ gives a one-to-one mapping between the type space $[\underline{\theta},\overline{\theta}]$ and the space of exit time $[0,\infty]$. For example, starting the game from $t=0$, a theater with θ_k makes a negative profit (or equivalently, the opportunity cost is higher than the operating profit) from the beginning, which is represented by the vertical distance between $\Pi_2(0)$ and θ_k. Despite this being the case, theater θ_k chooses to remain in the market in the hope that its competitor will exit soon, because at that point the theater would earn $\Pi_1(t)$. If the competitor has not dropped out by $T(2,0,\Pi_1(0),\theta_k)$, however, then theater θ_k gives up competing and exits.

To see how the game transitions from an n player to an $n-1$ player game, consider the case of triopoly. Figure 5 shows a typical example. When no competitors have dropped out, theaters follow the policy function $\Phi(t;3,0,\Pi_2(0))$. Assume that three theaters have $(\theta_i,\theta_j,\theta_k)$ and that $\theta_k = \max\{\theta_i,\theta_j,\theta_k\}$. Following the policy function, theater θ_k waits until $T(3,0,\Pi_2(0),\theta_k)$ and then drops out. At this moment, the highest possible exit value in the two-player game, denoted by $\widetilde{\theta}$ in the previous subsection, is given by θ_k. Any theaters with a higher exit value should have exited earlier in equilibrium. Now that there is one less competitor, the instantaneous payoff jumps from $\Pi_3(t)$ to $\Pi_2(t)$, so any surviving players are not making a negative profit. Thus, there will be no selection until the marginal player $\widetilde{\theta}$ gets hit by $\Pi_2(t)$; i.e., until $t=\Pi_2^{-1}(\widetilde{\theta})$, when the marginal player just breaks even. Then, selection restarts again. The time path of the marginal player in the two-player game is given by $\Phi(t;2,\Pi_2^{-1}(\theta_k),\theta_k)$ and serves as a policy function.

3.4 Computing the Equilibrium of the Model

For a given payoff function and exit values of theaters, I can simulate the game and compute the equilibrium exit times. As the three-player example above illustrates, a general N player game can also be solved sequentially, starting from solving $\Phi(t; N, 0, \Pi_{N-1}(0))$. The key to the tractability is that the evaluation of $V(T, N-1, t, \widetilde{\theta}', \widetilde{\theta}')$ with $\widetilde{\theta}' = \Phi(t; N, 0, \Pi_{N-1}(0))$ in equation (1) is computationally simple. Since this is the value of entering the $N-1$ player subgame for the "worst" type implied by the equilibrium, it can be written as

$$V(T, N-1, t, \widetilde{\theta}', \widetilde{\theta}') = \int_t^{\Pi_{N-1}^{-1}(\widetilde{\theta}')} \Pi_{N-1}(t') e^{-r(t'-t)} dt' + \frac{\widetilde{\theta}'}{r} e^{-r(\Pi_{N-1}^{-1}(\widetilde{\theta}') - t')}.$$

That is, if a player is the worst type at the moment, then the value of entering the subgame is simply the sum of the following two terms: the discounted sum of profits earned until $\Pi_{N-1}(t)$ declines down to $\widetilde{\theta}$ and the discounted sum of exit values from time $\Pi_{N-1}^{-1}(\widetilde{\theta}')$ on. Note that these terms consist of the model's primitives only. Thus, computing an equilibrium repeatedly is feasible in my framework.[14]

Finally, I investigate the model's predictions and the role played by asymmetric information. In a duopoly, as demonstrated in Figure 4, the first exit is delayed while the second exit is not, compared to the case of complete information in which theaters exit as soon as their profits become negative. Thus, exit times tend to cluster in the war of attrition relative to the case of complete information. This holds for a general N player game when it comes to the interval between the N-th and $N-1$th exit, since the last firm simply solves the monopoly problem and never delays its exit.

What is less clear is about other intervals. In a triopoly, for example, consider the first and second exit times. In Figure 5, the delay of theater θ_k is given by $T(3, 0, \Pi_2(0), \theta_k)$. On the other hand, assuming that theater θ_j is the next one to exit, its delay is measured by $T\left(2, \Pi_2^{-1}(\theta_k), \theta_k, \theta_j\right) - \Pi_2^{-1}(\theta_j)$. If this is smaller than $T(3, 0, \Pi_2(0), \theta_k)$, then the two exit times are closer to each other in a war of attrition than under complete information. Intuitively, as time goes on, theaters learn more about their competitors. This reduces the incentive to learn more about competitors. On the contrary, the increase in profit is larger when the market changes from duopoly to monopoly, compared to when it changes from triopoly to duopoly. Thus, whether the length of delay tends to be shorter as the game proceeds is not determinate.

[14] Other models such as Ericson and Pakes (1995) are hard to compute, and hence are difficult to use for a full solution approach.

4 Data

4.1 Data Source and Selection Criteria

The main data for this study come from *The Film Daily Yearbook of Motion Pictures* (1949, 1950, 1951, 1952, 1954, and 1955), which contains information on every theater that has ever existed in the U.S.[15] The dataset includes the name, location, number of seats, and type (indoor, drive-in, etc.) of each theater.[16] The data do not show the exact date of exit, so I constructed the exit year in the following way. If a theater was observed in year t but not year $t+1$, I assume that the theater exited sometime between years t and $t+1$.[17]

I assume that wars of attrition started in 1949, when demand started to shrink rapidly in an exogenous way. I define all the non-drive-in theaters that were open in 1949 as players in the exit game. Theaters that entered after 1949 are treated as exogenous demand shifters. While the focus of this analysis is on single screen theaters, theaters that entered after 1949 were brand-new, and sometimes equipped with luxurious concession stands and nicer seats. There was certainly competition between classic single-screen theaters and these new theaters. It is not unreasonable, however, to assume that the game I developed was played among old theaters, and the entry/exit of new theaters was exogenous from the viewpoint of the old theaters.

Movie theaters compete in local markets (Davis, 2005). In this paper, I define a market as a county. One big advantage of doing this is that data on demand shifters, such as TV penetration and demographics, are at the county-level. One drawback of this market definition is that the geographical area of some markets may be too large, because customers would not drive for long distances to go to a movie theater. Another problem is that some counties extend over many cities and contain hundreds of theaters (e.g., San Francisco county). To alleviate these problems, I focus on markets (counties) with fewer than or equal to ten theaters in 1949. Because of this selection, 313 markets out of 3,020 markets were dropped.

[15] The yearbook in 1953, unlike other years, does not provide a list of all movie theaters. Movie theaters in Alaska are listed only in the 1949 Yearbook, so I exclude Alaska from the analysis.

[16] For location variables, the exact address is often missing. We do, however, know the name of the city where the theater is/was located. The number of seats is often missing, too.

[17] Occasionally, a theater is observed in t, not observed in $t+1$, and observed again in $t+2$. In this example, it could be the case that the theater did not exit between years t and $t+1$, but it was simply missed in the Yearbook for year $t+1$. To deal with such spurious exits, I use the following criterion. If a theater is observed in year t but not in year $t+1$, I also check whether the theater is observed in years $t+2$ or $t+3$. If the theater is not observed in both years, the theater is considered to have exited the market. If the theater is observed again in either year $t+2$ or $t+3$ with exactly the same name and location as in year t, then it is considered not to have exited in year t. Note that I also augment the data using the Yearbooks of 1956 and 1957 to deal with spurious exits between 1954 and 1955.

I assume that the diffusion of TVs was the main driving force behind the decline in demand for classic single-screen movie theaters. Gentzkow and Shapiro (2008a) provide TV penetration rates by county and year. The TV penetration rate is defined as the share of households which have at least one TV set. This data are available for 1950, 1953, 1954, and 1955. To interpolate and extrapolate TV diffusion rates, for each market, I fit the cumulative distribution function (CDF) of the Weibull distribution to finite data points and minimize the distance between these points and the interpolated series by choosing two parameters. Thus, the TV penetration rates are obtained for all $t \in [1949, \infty)$, and they are smooth and monotonically increasing everywhere. Since I specify the theater's profit as a decreasing function of TV penetration, this approach guarantees that the profit function satisfies Assumption 1-(i). Online Appendix C provides the details of the interpolation method. 57 markets are dropped from the sample because TV penetration rates are not observed in multiple years.

Basic demographic/market variables, obtained from the U.S. Census, also provide across-market variations that will help to identify the theaters' payoff functions. Population determines the potential market size of a county. I assume that the median age, family income, urban share, and employment share also shift demand. Counties are substantially different in terms of geographic sizes, which may affect the profitability of theaters. To account for this, I also include land area in the theater's payoff function. As I discuss in the next section, I assume that these variables determine the base demand for theaters, which is market-specific and constant over time.[18] I also discard markets with missing covariates. Because of this, 44 markets were dropped from the sample.

Thus, for estimation, I am left with 2,606 markets, which have a total of 9,768 theaters in 1949.

4.2 Data Description

4.2.1 Market-Level State Variables

Table 1 shows the frequency of markets according to the initial number of competitors. There are a lot of monopoly markets, which helps identify the theater's payoff function, as decisions in such markets are a single-agent optimal stopping problem. The majority of markets have few competitors in 1949, there are many duopoly and triopoly markets, and almost 80% of all markets have five competitors or fewer. Table 2 shows summary statistics for the market-level variables that determine the base demand for theaters.

While these demographic variables shift the base demand, I assume that TV diffusion,

[18] A regression analysis shows that these demographic variables can explain a substantial portion of the cross-sectional variation in the number of theaters in 1949.

population changes, and the entry of theaters affect how demand for incumbent theaters declines over time. The diffusion process of TVs varies across markets. In 1950, in 87% of the markets, TV penetration rates were lower than 10%. In 1955, however, the 5th and 95th percentiles of the TV penetration rate across markets were 14% and 86%, respectively, indicating a wide variation in the diffusion process across counties. This rich cross-section and time-series variation in the TV penetration rate is the main source of identification of theaters' payoff functions.[19] In the estimation, I specify the decline in demand as a function of the TV penetration rates.

The change in population during the sample period may affect the decline in demand. The 5th, 25th, 75th, and 95th percentiles of the population change are -23.7%, -11.2%, 10.0%, and 40.7%, respectively. Because of these large population changes, it is important to control for population growth when measuring declines in demand.

New theater entry is also assumed to affect the decline in demand for incumbent theaters. In 1,611 markets (61.8%), there was no entry in any year studied. In 641 markets (24.6%), there was one entry. In the remaining 354 markets (13.6%), there was more than one entry in the sample period. If I focus on markets with four competitors or fewer, in 90% of the markets, the number of entries is one or fewer.

4.2.2 Exit Behavior

As shown in Figure 1, the number of indoor movie theaters decreased from 17,367 in 1949 to 11,335 in 1959 (a 34.7% decrease). During the sample period, 1,836 theaters (18.8% of the sample) exited the market. In 1949, there were 3.75 theaters in the average market. Out of these theaters, 3.66 theaters survived in 1950, 3.64 in 1951, 3.58 in 1952, 3.24 in 1954, and 3.04 in 1955. The standard deviation across counties decreased gradually and monotonically from 2.34 in 1949 to 1.95 in 1955. This implies that markets with more competitors have higher exit rates. Demand for movie theaters declined significantly during this time period. Looking at the aggregate statistics in Figure 1, attendance in the average movie theater per year was 261,991 in 1949. This figure dropped to 184,571 in 1950, and it gradually decreased to 163,689 in 1955.

Exit behaviors appear to be correlated with the initial market structure. Figure 3 plots the exit rate in the sample period against the initial number of theaters. The exit rate increases with the initial number of theaters, and is concave. About 9.5% of the sample exited in monopoly markets, 13.7% in duopoly markets, and 15.1% in triopoly markets.

[19] I assume the diffusion of TVs across households is exogenous. Alternatively, Gentzkow and Shapiro (2008b) use the year in which each geographical market began receiving TV broadcasts as an instrument for TV diffusion across households.

To further investigate the determinants of theaters' exits, I regress the market-level exit share (the share of theaters in each market that exited during the sample period) on the change in the TV rate, the change in population, and the number of new entrants in the market. The first two columns of Table 4 show the result. The coefficient of TV penetration is positive, implying that the faster is the TV diffusion in a county, the higher is the exit rate. The negative coefficient of change in population means that the inflow of population slows down the decline in demand, although it is not statistically significant. The result also shows that new entrants hasten theaters' exits.

Strategic elements appear to be important in theaters' exits. The second regression includes the number of theaters in 1949. To capture the non-linear effect of market structure, I also add its squared term. The linear term is positive, while the quadratic term is negative. This implies that a market with more competitors has a higher exit rate, and that the increment in the exit rate becomes smaller as the number of theaters increases. This is consistent with what I found in Figure 3.

One potential problem of this regression analysis is that the market structure may be endogenous. If unobservable demand shifters that affect the initial number of competitors are correlated with unobservable declines in demand, then the coefficients of the number of competitors and its squared term would be inconsistent. To alleviate the endogeneity problem, I run an instrumental variable (IV) regression. I use the demographic variables reported in Table 2 as instruments for the number of competitors and its squared term, assuming that these demographic variables are uncorrelated with unobservable decline in demand. The results are reported in the last column of Table 4. Importantly, the signs of the number of competitors and its squared term remain the same and statistically significant. To conclude, the market structure seems to have an important impact on theaters' exit behaviors.

Theaters' exits tend to be clustered within a market. Focusing on markets that experienced at least two exits during the sample period, I calculated the interval between the first and second exit. In approximately 74% of these markets, the interval is shorter than two years. On the contrary, only in 16% of these markets was the interval longer than three years. Such clusters are observed even after controlling for observable variables.[20] This observation points to two things. First, there could be unobservable heterogeneity in the decline in demand. In a market with severe decline in demand, theaters could exit shortly one after another. Second, in a duopoly with asymmetric information, the first exit is delayed while the second exit is not, implying shorter intervals on average. In an N player game, this corresponds to the case

[20] For a simple check, I divide these markets into four groups based on the change in the TV penetration rate. The share of markets within each group in which the interval between the first and second exit is shorter than two years ranges between 69% and 75%.

in which the learning process is not fast, as discussed in Section 3.4. Thus, clustered exits in the data are consistent with the theoretical model.

In the empirical analysis below, I assume that theaters are homogeneous in terms of observable variables and are different from one another only in terms of unobservable and privately known exit values for the following reasons. First, the capacity variable (the number of seats) and the name of the street where a theater is located are frequently missing in my dataset. In addition, information on chain stores is only partially observed. Second, the differential equation (1) that I use for estimation will be very complicated once I abandon the symmetry assumption. This would make computation highly demanding. Thus, rather than a firm-level analysis, this study may also be considered as a market-level analysis; e.g., how the initial market structure and the exit process in the market are related.

5 Estimation Strategy

The theoretical model in Section 3 has two important advantages for estimation. First, the uniqueness result in Fudenberg and Tirole (1986) is preserved in the N player game. Second, the model is numerically tractable and easy to compute, as Section 3.4 illustrates. Thus, I can use the full-solution approach (nested fixed-point approach), as potential multiplicity of equilibria and complexity of the game are two main obstacles to using the full-solution approach in estimating multi-agent dynamic models.[21] This enables me to explicitly take theaters' expectations, game history, and unobservable market-level heterogeneity into account. Furthermore, given that the industry is still in a transition process during the sample period (i.e., observed states are in the transient class), using the full-solution method is even more important.[22]

I observe M independent markets and from this section on, I use a market subscript $m \in \{1, ..., M\}$ for all variables that differ across markets. For each market, I observe the initial number of theaters n_m, their exit times $\{t_1, ..., t_{n_m}\}$ with right censoring, the TV penetration rate over time, and the set of market-level time-invariant variables that affect market profitability and decline in demand. Note that, because of relatively short time periods in the data, many independent repetitions of the game (markets) with the uniqueness property are essential for inference.

[21] See Aguirregabiria and Mira (2010) for a discussion of these difficulties.

[22] To apply two-step methods, one should be able to estimate the conditional choice probability for every state that the game could visit at some point in the future. This implies that, in the current application, one has to extrapolate choice probabilities for states that have not been observed in the data. In addition, due to learning, the current market structure is not sufficient to determine theaters' strategies. Therefore, using a two-step method is practically infeasible in the current context.

5.1 Specification

To solve the model numerically, I parameterize the payoff function. The specification and selection of variables are guided by the data analysis in Section 4 and theoretical requirements. Let $\Pi_n(t, m)$ be a theater's instantaneous profit in market m at time t, when n theaters stay in the market. I assume that

$$\Pi_n(t, m) = \pi_n(m) \cdot d(t, m),$$

where $\pi_n(m)$ is the base demand and $d(t, m)$ is a function showing the decline in demand (called "decay function" hereafter).

The base demand is specified as

$$\pi_n(m) = \frac{\alpha_m + \boldsymbol{\beta}' \mathbf{X}_m}{n_m^{\delta}}, \tag{7}$$

where α_m is unobservable (to the econometrician) market-level heterogeneity, and $\mathbf{X}_m$ is a vector of observable demographic variables, including a constant, population size, the median age of the population, the median income, the share of the population living in urban areas, the employment share of the population, and land size. $\delta > 0$ is a parameter that captures the effect of competition and guarantees that $\pi_n(m)$ decreases in n.[23] This specification captures the idea that the change in profit when an additional theater is added depends on the original size of demand. Since the market size differs significantly across markets, this dependence is reasonable.[24]

Note that α_m plays an important role in theaters' profit. Not all the determinants of movie demand are likely observed by the econometrician. To see this, Table 3 splits markets into several groups according to population size in 1950 and calculates the summary statistics of the number of theaters in 1949. There is wide variation in the initial number of theaters among similarly sized markets. Furthermore, even when I control for other observable covariates, there still is variation in the initial number of theaters. Thus, it is important to account for unobservable market-level heterogeneity.

I assume that $d(t, m)$ decreases over time to satisfy Assumption 1 in Section 3 and is specified as

$$d(t, m) = \{1 - \lambda_1 TV_{tm}\}^{\exp(\lambda_m + \lambda_2 \Delta POP_m + \lambda_3 NEW_m)},$$

[23] I use a reduced-form profit function instead of fully specifying demand and cost functions. Demand for differentiated products is implicitly considered.

[24] Another possible specification would be

$$\pi_n(m) = \alpha_m + \boldsymbol{\beta}' \mathbf{X}_m + \delta \log(n_m),$$

where δ is negative and the logarithm generates a decreasing effect of competition. One problem for this specification is that the amount of profit eroded by additional competitors is independent of market size.

where TV_{tm} is the TV penetration rate in market m at time t, ΔPOP_m is the growth rate of the population in market m from 1950 to 1960,[25] and NEW_m is the total number of new entrants in market m during the sample period. I restrict $0 \leq \lambda_1 \leq 1$. Note that $d(t,m)$ is between zero and one, and decreases over time as long as TV_{tm} is increasing in time. To rationalize the observation that exit times are sometimes clustered, unobservable heterogeneity λ_m is introduced in the decay function. This specification is flexible and potentially captures various types of declines in demand. In later sections, the set of λ is denoted by $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$.

I assume that theaters' exit values identically and independently follow a truncated normal distribution with mean μ_θ, variance σ_θ^2, and lower and upper bounds of l_θ and h_θ, respectively. I also assume that α_m (λ_m) follows a normal distribution with mean μ_α (μ_λ) and variance σ_α^2 (σ_λ^2). To allow for the possibility that a market with high unobservable demand shrinks more slowly or quickly due to unobservable factors, α_m and λ_m may be correlated with the correlation coefficient $\rho_{\alpha\lambda}$. For tractability, (α_m, λ_m) are assumed to be *ex-ante* independent of θ, and all unobservable variables are *ex-ante* independent of $\mathbf{X}_m$.

5.2 An Auxiliary Entry Model

I construct an auxiliary entry model and assume that potential entrants make their entry decisions right before the war of attrition starts. I jointly estimate the entry model and the exit game. There are two main advantages of doing so. First, as Table 1 shows, there are substantial variations in the number of theaters in 1949. Since firm entry reveals profitability, I utilize the variation in the initial number of theaters to infer the parameters in the base demand function (7).

Second, the entry model helps solve the initial conditions problem. If unobservable market heterogeneity α_m affected theaters' profit before 1949, the number of theaters in 1949 and market heterogeneity would be correlated through selection. Furthermore, firm-level unobservables, if they are serially correlated, introduce an additional source of endogeneity.[26] In other words, the initial competitors (incumbent firms in 1949) are a selected sample so the distribution of exit values of incumbents is different from the population distribution. To solve this initial conditions problem, I approximate the joint distribution of α_m and $\boldsymbol{\theta}$ conditional on n_m using the restrictions implied by the entry model. Then, I use this joint distribution to solve the dynamic exit game. Online Appendix B provides a detailed discussion and procedure of the proposed method.

The auxiliary entry model is based on Seim (2006). Specifically, I assume that $\bar{N}$ potential

[25] I use demographic data from 1950 and 1960 because census data are available only every ten years.

[26] Unobservable time-invariant exit values in this paper are an extreme example of this with perfect serial correlation.

theaters decide simultaneously whether or not to enter the market at the beginning of 1949, right before the war of attrition starts. Before making a decision, each player draws and privately observes its own value of exit, θ_i. If n theaters enter the market as a result of their decisions, each entrant earns

$$\pi_n^e(m) = \frac{\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m}{n_m^{\delta_e}}.$$

On the other hand, if theater i does not enter, it earns θ_i. Although I use the same form of base demand as in (7), I allow the effect of competition, δ_e, to be different from δ in the base demand of the exit stage to capture the effect of the Paramount Decree on competition.

Let $D_i = 1$ if theater i enters the market and zero otherwise. Theater i enters if the expected profit $E[\pi_n^e(m)]$ is higher than θ_i, so its optimal choice D_i^* is given by

$$D_i^* = \mathbf{1}\left\{(\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m) E\left[n_m^{-\delta_e}\right] \geq \theta_i\right\}.$$

Letting P denote a theater's belief about the probability that an opponent enters the market, it can be shown easily that $E\left[n_m^{-\delta_e}\right]$ is simply a decreasing function of P, which is denoted by $K(P)$. The symmetric equilibrium belief $P^*(X_m, \alpha_m)$ is thus given by a unique fixed point of the following equation:

$$P = G\left((\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m) K(P)\right), \tag{8}$$

where G is the CDF of θ_i.

The number of entrants predicted by this entry model equals the number of $\theta_i s$ in $\{\theta_1, ..., \theta_{\bar{N}}\}$ that satisfy

$$(\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m) K(P^*(\mathbf{X}_m, \alpha_m)) - \theta_i \geq 0. \tag{9}$$

Thus, for any pair $(\alpha_m, \mathbf{X}_m)$ and realization of $\boldsymbol{\theta} = \{\theta_1, ..., \theta_{\bar{N}}\}$, I can compute the equilibrium number of entrants. Arguing backwards, for any pair of $(\mathbf{X}_m, n_m)$, the entry model implies the set of $(\alpha_m, \boldsymbol{\theta})$ that is consistent with the pair. I use simulation to approximate the joint distribution of $(\alpha_m, \boldsymbol{\theta})$ conditional on $(\mathbf{X}_m, n_m)$.

5.3 Identification and Estimator

The theoretical model was constructed in continuous time in order to exploit several convenient properties of the model for estimation (uniqueness and ease of computation). On the other hand, the data are discrete. Therefore, when aggregating over time, one wants to keep as much of the power of the model's identifying restrictions as possible.[27]

[27] For a discussion of several estimation issues in continuous-time models, see Arcidiacono, Bayer, Blevins, and Ellickson (2012). See also Doraszelski and Judd (2010) for a discussion of tractability of continuous time models.

The intuition behind the identification of strategic delay is given by the following argument. First, for the sake of argument, suppose that the effect of competition is the same between the exit and entry game; i.e., $\delta = \delta_e$. The market structure before the war of attrition helps me to infer how theaters interact in the product market. That is, the base demand is identified from the entry stage. Next, using information on exits in monopoly markets in which strategic interactions are absent, the decay function is identified. Then, with these components, exit behaviors in markets with more than one theater are implied from the model without strategic delays. In theory, any difference between these implied exit behaviors and their empirical counterparts is attributed to the strategic delay of exit. Next, assume that $\delta \neq \delta_e$. In this case, I need another source of information, as the difference between the implied exit and data mentioned above could also come from a different value of δ. Note that for a given rate of decline in demand and exit rate during the entire sample period, how many exits took place in the first several years would be informative about the extent of strategic delay. Therefore, by adding such information, I can separate the strategic delay in exit from the effect of competition (δ).

For estimation, I use indirect inference. I choose several moments that seemingly capture the relevant features of the data. I simulate moments from the model and minimize the distance between the simulated moments and data moments. I use moments jointly from the dynamic exit game and from the entry game.

Let $\boldsymbol{\gamma}$ be a set of structural parameters and $\boldsymbol{\rho}$ be a set of auxiliary parameters (moments) that summarize certain features of the data. For any arbitrary moment x, I use x and $\hat{x}$ to denote the empirical and computed (from the model) moments, respectively. Thus, $\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})$ denotes the set of auxiliary parameters estimated from the simulated data. Note that I keep the dependence of $\hat{\boldsymbol{\rho}}$ on $\boldsymbol{\gamma}$ explicit.

Several normalizations are necessary to identify the parameters of the model. First, location normalization for profit is achieved by setting the means of unobservable market heterogeneity and exit values to zero, $\mu_\alpha = \mu_\theta = 0$. The constant in $\boldsymbol{\beta}$ pins down the mean profit. Next, I normalize the mean of unobservable heterogeneity in the decay function to zero, $\mu_\lambda = 0$, because in practice it is difficult to identify it separately from the coefficient of the TV penetration rate λ_1. Finally, I set the variance of unobservable heterogeneity in the decay function to one, $\sigma_\lambda^2 = 1$, since it is not well identified empirically given that there are a number of markets with no exit during the sample period. In sum, the set of structural parameters to be estimated is $\boldsymbol{\gamma} = (\delta, \delta_e, \boldsymbol{\beta}, \boldsymbol{\lambda}, \sigma_\theta, \sigma_\alpha, \rho_{\alpha\lambda})$.

Moment selection is guided by the theoretical model and data analysis in Section 4.2. The restrictions implied by the entry model identify δ_e, $\boldsymbol{\beta}$, and σ_α. I use the average number of entrants $E(n_m)$ and the average of interactions between n_m and each of the demographic variables and its squared term in $\mathbf{X}$. As was discussed above, there is additional variation in the number of entrants after controlling for demographic variables. To capture this, $Var(n_m)$ is also added. The parameters in the decay function, $\boldsymbol{\lambda}$, are identified mainly by the average of market-level exit rates interacted with the TV penetration rate in 1955, the growth rate of the population during the sample period, and the total number of new entrants.

The relationship between exit and the market structure at the beginning of the exit game is informative about δ and $\rho_{\alpha\lambda}$. The effect of competition δ determines how quickly profits decrease in the number of active theaters, and hence affects theaters' exit. As is seen in Figure 3, the market-level exit rate and the initial number of theaters are positively correlated. This aids identification of δ. On the other hand, a large market (which typically has large α_m and n_m) may have a different rate of decline in demand, which could slow down or speed up theaters' exit. This is captured by $\rho_{\alpha\lambda}$. Intuitively, δ affects the slope of the line in Figure 3, while $\rho_{\alpha\lambda}$ affects the curvature of the line. I add the following ten moments: the average of market-level exit rates in monopoly markets, in duopoly markets, and in markets with $n_m = 3$, etc.

To capture the magnitude of asymmetric information and distinguish the current model from the complete information benchmark, I use one additional moment. The rate of exit in the first three years of the sample period is calculated for each market and is denoted as the rate of *early* exit. This variable, for given values of total exit rates and the decay function, is expected to capture the magnitude of strategic waiting. I use the average of the rate of *early* exit as an additional moment.

I have 28 moments to estimate 15 parameters. The elements in $\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})$ consist of the same 20 moments with the exit rates and the number of entrants being replaced by their simulated counterparts. The indirect inference estimator $\hat{\boldsymbol{\gamma}}$ is given by

$$\hat{\boldsymbol{\gamma}} = \arg\min_{\boldsymbol{\gamma}} \left(\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})\right)' \Omega \left(\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})\right), \tag{10}$$

where Ω is a positive definite weighting matrix.

The procedure to calculate the value of the objective function is as follows:

Step 1: Take a guess of structural parameters $\boldsymbol{\gamma}$.

Step 2: Draw $\{\boldsymbol{\theta}^{ns}\}_{ns=1}^{NS}$ and $\{\alpha^{ns}\}_{ns=1}^{NS}$ independently from their distributions. Use (8) and (9) to solve the entry game to calculate $\hat{n}_m^{ns}$ for $ns = 1, ..., NS$ and form $\hat{n}_m = \frac{1}{NS}\sum_{ns=1}^{NS}\hat{n}_m^{ns}$. To solve the entry game, I set $\bar{N} = 11$.[28]

Step 3 For $\mathbf{X}_m$ and n_m, simulate $\hat{F}_{\boldsymbol{\theta},\alpha|\mathbf{X}_m,n_m}$, following the procedure in Online Appendix B.

[28] I set $\bar{N}$ at 11 arbitrarily, since the maximum number of actual entrants is ten in my sample.

Step 4: Draw $(\boldsymbol{\theta}^{ns}, \alpha^{ns})_{ns=1}^{NS}$ randomly from $\hat{F}_{\boldsymbol{\theta},\alpha|\mathbf{X}_m,n_m}$. For each simulation draw, calculate the equilibrium of the dynamic game of exit: $\left\{\left(t_1^{ns}, ..., t_{n_m}^{ns}\right)\right\}_{ns=1}^{NS}$.

Step 5: Calculate the rate of theaters' exit for each market, denoted by $\hat{e}_m^{ns}$, and form $\hat{e}_m = \frac{1}{NS}\sum_{ns=1}^{NS} \hat{e}_m^{ns}$.

Step 6: Calculate moments $\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})$ and obtain the value of the criterion function

$$J(\boldsymbol{\gamma}) = (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma}))' \, \Omega \, (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})).$$

Then, repeat Steps 1-6 to minimize $J(\boldsymbol{\gamma})$.

The estimator $\hat{\boldsymbol{\gamma}}$ is consistent and the asymptotic distribution is

$$\sqrt{M}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \xrightarrow{d} \mathcal{N}(0, W), \tag{11}$$

where W is given by

$$W = \left(1 + \frac{1}{NS}\right)[H'\Omega H]^{-1} H'\Omega (E\boldsymbol{\rho}\boldsymbol{\rho}') \Omega H [H'\Omega H]^{-1},$$

with $H = \partial\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})/\partial\boldsymbol{\gamma}'$. An optimal weight matrix $\Omega = (E\boldsymbol{\rho}\boldsymbol{\rho}')^{-1}$ is used so I have $W = \left(1 + \frac{1}{NS}\right)\left[H'(E\boldsymbol{\rho}\boldsymbol{\rho}')^{-1} H\right]^{-1}$. For implementation, I bootstrap the data 1,000 times to get $\{\boldsymbol{\rho}_b\}_{b=1}^{1{,}000}$, and then calculate its variance-covariance matrix Σ_M. Then, I replace $(E\boldsymbol{\rho}\boldsymbol{\rho}')^{-1}$ and H with Σ_M^{-1} and H_M, respectively.

6 Estimation Results

This section first presents parameter estimates. Using these estimated parameters, I then perform several counterfactual analyses.

6.1 Parameter Estimates

6.1.1 Base Demand

Table 5 presents estimates of the structural parameters. The coefficient of population (β_1) implies that theaters earn higher profits in bigger markets. The coefficients of median age (β_2), income (β_3), and land area (β_6) are all positive and significant. One possible interpretation for the coefficient of urban share (β_4) is that once I control for other observable and unobservable (to the econometrician) market characteristics, people living in urban areas are exposed to various types of other entertainment. An interpretation of the coefficient of employment share (β_5) could be that employed people have less time to watch movies. The parameters that capture competition (δ and δ_e) suggest that a theater's profit is eroded by competition. To see the relative sizes of these estimates, I calculate the value of base demand (7) at the sample mean of $\mathbf{X}$. Then, $\hat{\boldsymbol{\beta}}'\bar{\mathbf{X}} = 1.89$. Duopoly and triopoly profits are 1.59 and 1.45, which are about 15% and 23% lower than monopoly profits, respectively. In the entry stage, using the same mean $\hat{\boldsymbol{\beta}}'\bar{\mathbf{X}}$ and a different competition effect $\hat{\delta}_e$, duopoly and triopoly profits are 19% and 29% lower than monopoly profits, respectively.

6.1.2 Decay Function

Table 5 reports the parameters in the decay function. The coefficient of the TV rate (λ_1) is significantly different from zero and is around 0.37. Since $1 - \lambda_1 TV_{tm}$ lies between zero and one, a larger value in the exponential function means that the rate of decline in demand is more severe. The coefficient of population growth (λ_2) is consistent with the intuition that in a county with an outflow of people, the decline in demand is faster. The coefficient is not, however, statistically significant. The coefficient of the number of theaters that entered after 1949 (λ_3) is positive, implying that entry of a new competitor hastens the decline in demand for incumbent theaters.

6.1.3 Estimates of Standard Deviations

Estimates of σ_θ and σ_α are reported in Table 5. The standard deviation of exit values is 2.413 and is statistically significant. This implies that 95% of theaters have an exit value below 4.729. Meanwhile, the standard deviation of market-level heterogeneity is 0.035, which means that 95% of the value of unobservable market heterogeneity is between -0.071 and 0.071. Compared with the value of base demand (7) evaluated at the sample mean of $\mathbf{X}$ and the estimated parameters (i.e., $\hat{\boldsymbol{\beta}}'\bar{\mathbf{X}} = 1.89$), this variation explains a relatively minor proportion of the variation in initial numbers of competitors among similarly sized markets.

The variance of exit values can be interpreted as the extent of asymmetric information. If the variance is small, a theater's assessment about its competitors' exit values is more precise. Hence, if a theater's value of exit is significantly higher than the mean, the theater would give up and exit relatively earlier. As the previous paragraph suggests, the value of exit varies widely, implying that theaters should stay in the market in the hope of outlasting their competitors.

The estimate of $\rho_{\alpha\lambda}$ is imprecisely estimated and not significantly different from zero. This implies that the base demand and rate of decline in demand may still be correlated, but the correlation can be captured by observable market-level variables. Indeed, the correlation between $\hat{\boldsymbol{\beta}}'\mathbf{X}_m$ and $\hat{\lambda}_2 \Delta POP_m + \hat{\lambda}_3 NEW_m$ is 0.15 and is statistically different from zero. That is, given the TV penetration rate, a larger market would have a faster rate of decline in demand on average.

6.2 Model Fit

To investigate the model fit, I simulate the model ten times and for each simulation count the number of markets in which the model predicts correctly the number of exits during the sample period. Then, I take average over simulation draws for different market structures. The model predicts the number of exits correctly in 83.3% of the monopoly markets. In 59.2% of duopoly or triopoly markets, the number of exits predicted by the model and the one observed in the data match exactly. The corresponding number for markets with four or five theaters is 35.5%. Finally, in markets with more than five theaters, the share of markets in which the model predicts the number of exits correctly is 23.2%.

In addition, I randomly drew a set of structural parameters $\boldsymbol{\gamma}$ from the estimated asymptotic distribution $\mathcal{N}(0, W)$ given in (11) 200 times, and simulated the survival rate of theaters for each draw. Then, I calculated the 95% confidence interval (top 2.5% and bottom 2.5% of exit rates) for the exit rates for different market structures. Figure 6 shows four graphs. The model fits the data well, except for the survival rate in 1952 for markets with four or more theaters in 1949. For most of the other years for other markets, the model shows a good fit.

6.3 Simulation Analysis

6.3.1 Delay of Exit

To quantify the effect of strategic interaction on the consolidation process, I define two benchmarks in relation to the war of attrition equilibrium. First, if exit values (fixed costs) are common knowledge, in equilibrium, any theater exits the game at the exact moment when its profit becomes lower than its exit value.[29] Thus, no theater incurs a loss.[30] I call this the complete information case. Second, since capacity reduction in a declining environment is a public good that has to be provided privately, the exit process in a non-cooperative equilibrium does not maximize the industry profit. I consider a hypothetical industry regulator that

[29] Without private information, multiple equilibria may exist. I chose an equilibrium in which a theater with the highest exit value exits first.

[30] A loss or a negative profit in this context is in terms of economic profit. That is, if the profit of a theater is lower than its exit value (the value of the outside option), I call this "incur a loss" or "make a negative profit."

maximizes the industry profit. It would hasten the exit process in order to weaken business stealing effects and save fixed costs. I call this the industry regulator case.

Let $T^{*}=\{t_1^{*},...,t_N^{*}\}$ and $T^{**}=\{t_1^{**},...,t_N^{**}\}$ be the vector of exit times in the complete information benchmark and in a war of attrition equilibrium, respectively. The industry regulator chooses a vector of exit times $T=\{t_1,...,t_N\}$ to maximize the industry profit:

$$\int_{1949}^{1955}\sum_{k=1}^{n_t}\left[\Pi_{n_t}(t,m)-\theta_k\right]e^{-rt}dt$$

where n_t is the number of theaters at time t implied by $T=\{t_1,...,t_N\}$. Denote the regulator's solution by $T^{R}=\left\{t_1^{R},...,t_N^{R}\right\}$. For any n, $\left(t_n^{*}-t_n^{R}\right)$ measures the delay in exit that arises from oligopolistic competition. In an oligopoly with declining demand, firms have an incentive to free-ride on competitors' exits/divestments, so the speed of capacity reduction is slower than what the social planner would dictate. Meanwhile, $(t_n^{**}-t_n^{*})$ measures the delay in exit due to asymmetric information. In the presence of asymmetric information about competitors' profitability, firms have an incentive to wait, because when some of the competitors exit, their profit would increase.

I compute (T^{*},T^{**},T^{R}) for each market and calculate the average delay of the first, second, and third exits, as well as the average of all delayed exits. Table 6 summarizes the averages according to the initial number of competitors. Note that some markets do not have any delay during the sample period, so the averages are calculated only using markets in which delays occur by the end of the sample period. The table also reports the share of such markets. Overall, a theater's exit is delayed by 2.577 years due to oligopolistic competition, while the delay in exit created by asymmetric information is 0.099 years. That is, 3.7% of the total delay is accounted for by asymmetric information. The delay in exit differs significantly across different market structures. In the case of duopoly, the exit is delayed by 2.22 years due to oligopolistic competition, while asymmetric information delays exit by 0.154 years, implying that 6.5% of the total delay is accounted for by asymmetric information.

The delay in exit becomes shorter as the game proceeds. For example, in markets with four initial competitors, the first exit is delayed by 3.107 years due to oligopolistic competition, while the third exit is delayed by 0.729 years. The intuition is as follows. When the first exit occurs, three other theaters enjoy a higher profit. On the other hand, when the third exit occurs, only one theater will have a higher profit. In addition, the fixed cost that is saved is higher for the first exit than the third exit under the regulator's solution. The delay due to asymmetric information has the same features: the earlier exit is delayed more than later exits. Consistent with the argument in Section 3.4, as time goes on, theaters learn more about their competitors and the incentive to delay their exit becomes weaker.

6.3.2 Cost of Strategic Interaction

Next, I compute the differences in industry profits and costs of strategic interaction. Let $\{n_t^*\}$, $\{n_t^{**}\}$, and $\{n_t^R\}$ be a sequence of the number of theaters in the market implied by T^*, T^{**}, and T^R, respectively. Using these, define

$$Q_m^* = \int_{\underline{t}}^{1955} \sum_{k=1}^{n_t^*} \left[\Pi_{n_t^*}(t,m) - \theta_k\right] e^{-rt} dt \tag{12}$$

$$Q_m^{**} = \int_{\underline{t}}^{1955} \sum_{k=1}^{n_t^{**}} \left[\Pi_{n_t^{**}}(t,m) - \theta_k\right] e^{-rt} dt \tag{13}$$

$$Q_m^R = \int_{\underline{t}}^{1955} \sum_{k=1}^{n_t^R} \left[\Pi_{n_t^R}(t,m) - \theta_k\right] e^{-rt} dt \tag{14}$$

where $\underline{t}$ denotes the moment when the first exit occurs under the regulator's solution. In other words, these variables measure the cumulative profits that all surviving theaters in market m earn in each scenario. The difference in cumulative industry profits under the complete information benchmark and the industry regulator case can be regarded as the *cost of oligopolistic competition.* I use $(Q_m^R - Q_m^*)/Q_m^{**}$ to measure the cost. Meanwhile, the difference in cumulative industry profits under a war of attrition and complete information can be regarded as the *cost of asymmetric information.* $(Q_m^* - Q_m^{**})/Q_m^{**}$ measures such costs. Note that I use the same denominator to ease comparisons.

Table 7 summarizes these two statistics according to the initial number of competitors. The cost of oligopolistic competition in the median market is 4.68%. The cost of asymmetric information in the median market is relatively small (0.22%), and accounts for 4.5% of the total cost. Overall, the loss of industry profit is larger in markets with fewer competitors. Above all, the variation in the cost of asymmetric information across different market structures is worthy of note. The difference in the median duopoly market is 0.7%, which is more than three times as big as the median market of all samples (0.22%). For a given player, the probability of winning the war of attrition, i.e., the probability of being a monopolist, is highest in a duopoly, and therefore theaters have the greatest incentive to wait. Moreover, the increment of profit when one competitor exits is highest in duopoly, which also partly explains the big difference in the industry profit between the two cases. As the initial number of competitors gets large, competition becomes closer to perfect competition, and hence motives to outlast competitors become less significant.

The cost of strategic interaction also differs across markets with a different rate of decline in demand. To see this, I split the sample into two groups of markets with slow and fast rates of decline in demand according to the TV penetration rate in 1955. Table 8 summarizes the average of each group according to the initial number of competitors. The cost of asymmetric information is largest in markets with slow rates of decline in demand. For example, in duopoly markets, the median of the cost in the group of markets with slow declines in demand is 0.83%, while the corresponding number for markets with fast declines in demand is 0.57%. The intuition is as follows. In markets with slow declines in demand, the cost of waiting increases slowly. On the other hand, the benefit of waiting is still large because a winner of the game can enjoy a higher profit over a longer time period. These two factors prolong the war of attrition. On the contrary, interestingly, there is no clear pattern between markets in which demand declines quickly and markets in which demand declines slowly in terms of the cost of oligopolistic competition.

To further investigate the relationship between the decline in demand and the cost of asymmetric information, I separate the effect of the war of attrition from the effect of declining demand on the exit process. To do so, I fix the TV penetration rate at its initial level in each market so that the decay function is constant over time. As discussed in Section 3, theaters expect a higher profit if they outlast their competitors and thus stay until the expected benefit of waiting becomes lower than the expected cost of waiting. As time goes on, theaters become discouraged and exit if their competitors remain in the market. Notice that this dynamic selection may occur even if demand is not declining.[31] There are three types of theaters in equilibrium. The first set of theaters does not exit. Since demand is constant, their instantaneous profits are forever higher than their values of exit. The second set of theaters exits as soon as a war of attrition starts. They chose to enter the market in the static entry game. Playing the exit game is, however, not profitable for them, so they exit immediately. The third set of theaters stays in the market for a while, in the hope that they will outlast their competitors. Notice that this is due to asymmetric information. With complete information, they would exit the market immediately.

Holding demand constant, I simulate the game 100 times for each market and focus on theaters that exit the market. Table 9 averages the delay in exit due to asymmetric information according to the initial number of competitors. As above, the average delay is larger in markets with fewer competitors. In duopoly, the average delay is 1.841 years, which is significantly larger than the 0.154 years reported in Table 6. The constant demand prolongs the war of attrition the most. An example of such a situation are battles to control new technologies discussed by Bulow and Klemperer (1999), as demand in those industries is not declining. Consequently, large losses accumulate over time.

[31] For example, the original game in Fudenberg and Tirole (1986) is mainly for the case of a growing industry. The case of constant demand may be simply thought of as a special case of either a declining or growing market.

7 Conclusion

Many industries face declining demand and consequently firms sequentially divest and exit from the market. In an oligopolistic environment, strategic interaction plays an important role and has a nontrivial impact on the consolidation process. Despite their importance in the economy, economic costs of consolidation arising from strategic interactions have not been studied sufficiently well. A suitable framework to analyze such a process would entail dynamic and strategic players in a declining phase. This paper provides a tractable empirical framework to achieve this goal.

Specifically, I modify Fudenberg and Tirole (1986)'s model of exit in duopoly with incomplete information to work in an oligopoly. I use data on the U.S. movie theater industry and rich cross-section and time-series variations of TV penetration rates to estimate theaters' payoff functions and the distribution of exit values. By imposing the equilibrium condition, the model predicts the distribution of theaters' exit times for a given set of parameters and unobservables. I use indirect inference and estimate the model parameters by matching the predicted distribution with the observed distribution of exit times.

Using the estimated model, I measure strategic delays in the exit process due to oligopolistic competition and incomplete information. The delay in exit that arises from strategic interaction is 2.7 years on average. Out of these years, 3.7% is accounted for by incomplete information, while the remaining 96.3% is explained by oligopolistic competition. I also find that the delay and its resulting cost are relatively large in markets with few competitors and in markets with slow rates of decline in demand.

The framework in this paper can be applied to analyze other industries in which exogenous decline in demand creates a non-stationary environment in an oligopoly. It should be emphasized that the profit lost in the war of attrition is not necessarily detrimental to society. Due to delays in exit, consumers have access to more varieties if movie theaters are differentiated products. If demand-side data (price and quantity of the product) are available, one could compare the increase in consumer surplus and decreased firms' profit due to the strategic delay in exit. Applying this method to a currently declining industry is a useful exercise.

REFERENCES

Aguirregabiria, Victor, and Pedro Mira. 2007. "Sequential Estimation of Dynamic Discrete Games." *Econometrica,* 75(1): 1-53.

Aguirregabiria, Victor, and Pedro Mira. 2010. "Dynamic Discrete Choice Structural Models: A Survey." *Journal of Econometrics*, 156(1): 38-67.

Arcidiacono, Peter, Patrick Bayer, Jason R. Blevins, and Paul Ellickson. 2012. "Estimation of Dynamic Discrete Choice Models in Continuous Time." Economic Research Initiatives at Duke (ERID) Working Paper No. 50.

Argenziano, Rossella and Philipp Schmidt-Dengler. 2013. "Clustering in N-Player Preemption Games." http://schmidt-dengler.vwl.uni-mannheim.de/2648.0.html.

Baden-Fuller, C.W.F. 1989. "Exit From Declining Industries and the Case of Steel Castings." *Economic Journal*, 99: 949-961.

Bajari, Patrick, C. Lanier Benkard, and Jonathan Levin. 2007. "Estimating Dynamic Models of Imperfect Competition." *Econometrica,* 75(5): 1331-1370.

Bulow, Jeremy I., and Paul D. Klemperer. 1999. "The Generalized War of Attrition." *American Economic Review,* 89(1): 175-189.

Collard-Wexler, Allan. 2013. "Demand Fluctuations in the Ready-Mix Concrete Industry." *Econometrica*, 81(3): 1003–1037.

Davis, Peter J. 2005. "The Effect of Local Competition on Admission Prices in the U.S. Motion Picture Exhibition Market." *Journal of Law and Economics*, 48(2): 677-708.

Deily, Mary E. 1991. "Exit Strategies and Plant-Closing Decisions: The Case of Steel." *RAND Journal of Economics*, 22(2): 250-263

Doraszelski, Ulrich, and Kenneth L. Judd. 2010. "Avoiding the Curse of Dimensionality in Dynamic Stochastic Games." http://www.economics.harvard.edu/faculty/doraszelski.

Ericson, Richard, and Ariel Pakes. 1995. "Markov-Perfect Industry Dynamics: A Framework for Empirical Work." *Review of Economic Studies,* 62(1): 53-85.

Fershtman, Chaim, and Ariel Pakes. 2010. "Oligopolistic Dynamics with Asymmetric Information: A Framework for Empirical Work." http://www.tau.ac.il/~fersht/.

Fudenberg, Drew, and Jean Tirole. 1986. "A Theory of Exit in Duopoly." *Econometrica,* 54(4): 541-568.

Gentzkow, Matthew A., and Jesse M. Shapiro. 2008a. "Introduction of Television to the United States Media Market, 1946-1960." Inter-university Consortium for Political and Social Research, Chicago, IL: University of Chicago/Ann Arbor, MI: University of Michigan. ICPSR22720-v1. doi:10.3886/ICPSR22720 (accessed October 24, 2010).

Gentzkow, Matthew A., and Jesse M. Shapiro. 2008b. “Preschool Television Viewing and Adolescent Test Scores: Historical Evidence from the Coleman Study.” *Quarterly Journal of Economics,* 123(1): 279-323.

Ghemawat, Pankaj, and Barry Nalebuff. 1985. “Exit.” *Rand Journal of Economics,* 16(20): 184-194.

Ghemawat, Pankaj, and Barry Nalebuff. 1990. “The Devolution of Declining Industries.” *Quarterly Journal of Economics,* 105(1): 167-186.

Haigh, John, and Chris Cannings. 1989. “The n-Person War of Attrition.” *Acta Applicandae Mathematicae*, 14(1-2): 59-74.

Heckman, James J. 1981. “The Incidental Parameters Problem and the Problem of Initial Conditions in Estimating a Discrete Time-Discrete Data Stochastic Process and Some Monte Carlo Evidence.” In *Structural Analysis of Discrete Data With Econometric Applications,* ed. Charles F. Manski and Daniel L. McFadden, 179-195. Cambridge: The M.I.T. Press.

Jeziorski, Przemyslaw. 2013. “Estimation of cost efficiencies from mergers: Application to U.S. radio.” http://faculty.haas.berkeley.edu/przemekj/costSynergiesPaper.pdf.

Jofre-Bonet, Mireia and Martin Pesendorfer. 2003. “Estimation of a dynamic auction game.” *Econometrica*, 71 (5): 1443-1489.

Jovanovic, Boyan, and Glenn M. MacDonald. 1994. “The Life Cycle of a Competitive Industry.” *Journal of Political Economy,* 102(2): 322-347.

Klepper, Steven, and Kenneth L. Simons. 2000. “The Making of an Oligopoly: Firm Survival and Technological Change in the Evolution of the U.S. Tire Industry.” *Journal of Political Economy,* 108(4): 728-760.

Krishna, Vijay, and John Morgan. 1997. “An Analysis of the War of Attrition and the All-Pay Auction,” *Journal of Economic Theory*, 72(2): 343-362.

Lev, Peter. 2003. *History of the American Cinema, Volume 7: Transforming the Screen, 1950-1959.* New York: Charles Scribner’s Sons.

Lieberman, Marvin. 1990. “Exit from Declining Industries: "Shakeout" or "Stakeout"?” *RAND Journal of Economics*, 21(4): 538-554.

Melnick, Ross, and Adrea Fuchs. 2004. *Cinema Treasures: A New Look at Classic Movie Theaters*. Minnesota: MBI Publishing Company.

Moldovanu, Benny, and Aner Sela. 2001. "The Optimal Allocation of Prizes in Contests." *American Economic Review*, 91(3): 542-558.

Nishiwaki, Masato. 2010. "Horizontal Mergers and Divestment Dynamics in a Sunset Industry." http://www3.grips.ac.jp/~mstnishi/.

Pakes, Ariel, Michael Ostrovsky, and Steve Berry. 2007. "Simple Estimators for the Parameters of Discrete Dynamic Games (with Entry/Exit Examples)." *RAND Journal of Economics*, 38(2): 373-399.

Pesendorfer, Martin, and Philipp Schmidt-Dengler. 2008. "Asymptotic Least Squares Estimators for Dynamic Games." *Review of Economic Studies,* 75(3): 901-928.

Ryan, Stephen. 2012. "The Costs of Environmental Regulation in a Concentrated Industry." *Econometrica*, 80(3): 1019-1061.

Schmidt-Dengler, Philipp. 2006. "The Timing of New Technology Adoption: The Case of MRI." http://schmidt-dengler.vwl.uni-mannheim.de/2648.0.html

Seim, Katja. 2006. "An Empirical Model of Firm Entry with Endogenous Product-Type Choices." *RAND Journal of Economics,* 37(3): 619-640.

Stahl, Jessica. 2012. **"**Demand-side and Supply-side Advantages of Consolidation in the Broadcast Television Industry." https://sites.google.com/site/jessicacalfeestahl/.

Stuart, Fredric. 1976. *The Effects of Television on the motion Picture and Radio Industries*. New York: Arno Press.

Sweeting, Andrew. 2013. "Dynamic Product Positioning in Differentiated Product Markets: The Effect of Fees for Musical Performance Rights on the Commercial Radio Industry." forthcoming in *Econometrica*.

Number of Theaters in 1949	Frequency	Percent
1	451	17.31
2	520	19.95
3	445	17.08
4	369	14.16
5	245	9.40
6	209	8.02
7	143	5.49
8	87	3.34
9	80	3.07
10	57	2.19
Total	2,606	100.00

Table 1: **Number of Competitors.**

Variable	Obs	Mean	Std. Dev.	Min	Max
Population	2,606	22,791	19,627	1,870	194,182
Median age	2,606	4.09	1.03	1.00	7.00
Median family income	2,606	4.91	1.57	0.00	9.00
Urban share	2,606	0.24	0.23	0.00	0.95
Employment share	2,606	0.96	0.02	0.81	1.00
Land area (square miles)	2,606	967	1,289	25	18,573

Table 2: **Summary Statistics of Demographic Variables in 1950.**

Note: Median age and median family income are categorical variables.

Source: Hanes, Michael R., and the Inter-university Consortium for Political and Social Research HISTORICAL, DEMOGRAPHIC, ECONOMIC, AND SOCIAL DATA: THE UNITED STATES, 1790-2000.

Population of counties	Observations	Mean	Std. Dev.	Minimum	Maximum
0-10,000	648	2.0	1.1	1	7
10,000-20,000	863	3.0	1.6	1	9
20,000-30,000	473	4.2	1.9	1	10
30,000-40,000	258	5.4	2.2	1	10
40,000-50,000	161	6.2	2.1	1	10
50,000-75,000	129	7.2	1.8	2	10
75,000-100,000	49	8.0	1.7	5	10
100,000-150,000	23	8.3	1.2	6	10
150,000+	2	8.0	1.4	7	9

Table 3: **Summary Statistics of Number of Theaters in 1949 by Population Size.**
Source: Author's calculation based on the population data used in Table 2 and theaters' data from The Film Daily Yearbook of Motion Pictures.

	OLS		OLS		IV Regression	
Parameters	Coef.	Std. Err	Coef.	Std. Err	Coef.	Std. Err
Constant	0.0845	0.0114	0.0200	0.0163	-0.2272	0.0541
n_m in 1949	-	-	0.0365	0.0071	0.1907	0.0323
$(n_m$ in $1949)^2$	-	-	-0.0025	0.0007	-0.0193	0.0033
Change TV rate	0.1229	0.0216	0.0813	0.0222	0.0729	0.0262
Change in population	-0.0229	0.0227	-0.0382	0.0226	-0.0369	0.0251
New entrants	0.0706	0.0088	0.0604	0.0089	0.0612	0.0100
Adjusted R^2	0.0350		0.0533		0.0598	

Table 4: **Preliminary Evidence**.
Note: The dependent variable is the share of theaters in each market that exit during the sample period. For the IV regression, I use demographic variables in Table 2 as instruments for the initial number of competitors and its squared term. The p-value for the F statistic in the first stage is 0.0000 for both regressions.

Parameters	Coef.	Std. Err
δ (competition in dynamic game)	0.2416	0.0362
δ_e (competition in entry game)	0.3121	0.0167
β_0 (constant)	0.8744	0.0221
β_1 (population)	7.4122	0.2179
β_2 (median age)	1.9634	0.0997
β_3 (median income)	0.8300	0.0186
β_4 (urban share)	-0.3939	0.0118
β_5 (employment share)	-3.4959	0.0282
β_6 (log of land area)	2.4158	0.1060
λ_1 (TV rate)	0.3715	0.0175
λ_2 (change in population)	-0.2220	0.7917
λ_3 (new entrants)	0.5137	0.0945
σ_θ (std. of exit value)	2.4130	0.0901
σ_α (std. of demand shifter)	0.0354	0.0049
$\rho_{\alpha\lambda}$ (corr coef. b/w α_m and λ_m)	-0.1892	0.1323

Table 5: **Estimates of Structural Parameters**.

	$t^* - t^R$					$t^{**} - t^*$				
Market	s	Mean	1st	2nd	3rd	s	Mean	1st	2nd	3rd
$n_m = 2$	0.39	2.220	2.220	-	-	0.20	0.154	0.154	-	-
$n_m = 3$	0.60	2.587	2.967	1.265	-	0.31	0.120	0.126	0.102	-
$n_m = 4$	0.74	2.483	3.107	1.759	0.729	0.42	0.107	0.118	0.093	0.085
All markets	0.68	2.577	2.984	2.248	1.498	0.38	0.099	0.112	0.084	0.073

Table 6: **Delay in Exit in Years**.

Note: Let t^R, t^*, and t^{**} be the exit time in the the regulator benchmark, in the complete information benchmark, and in a war of attrition equilibrium, respectively. s is the share of markets in which delays occur during the sample period. I calculate the average delay of the first, second, and third exits, as well as the average of all delayed exits by the initial number of competitors.

Market	$\left(Q_m^R - Q_m^*\right)/Q_m^{**}$ Mean	5th	Median	95th	$\left(Q_m^* - Q_m^{**}\right)/Q_m^{**}$ Mean	5%	50%	95%
$n_m = 2$	8.21%	2.23%	7.22%	16.42%	0.99%	0.17%	0.70%	2.83%
$n_m = 3$	5.67%	2.57%	5.34%	10.18%	0.37%	0.07%	0.30%	0.91%
$n_m = 4$	4.76%	2.42%	4.56%	8.19%	0.27%	0.04%	0.23%	0.63%
All markets	5.40%	2.06%	4.68%	11.25%	0.39%	0.05%	0.22%	1.17%

Table 7: **Cost of Oligopolistic Competition and Asymmetric Information**.

Note: Let Q_m^R, Q_m^*, and Q_m^{**} be the total cumulative profit earned by all theaters in market m in the regulator benchmark, in the case of complete information, and in a war of attrition, respectively. This table shows the summary statistics of these variables by the initial number of competitors.

Market	$\left(Q_m^R - Q_m^*\right)/Q_m^{**}$ Slow	Fast	$\left(Q_m^* - Q_m^{**}\right)/Q_m^{**}$ Slow	Fast
$n_m = 2$	7.25%	7.21%	0.83%	0.57%
$n_m = 3$	5.29%	5.35%	0.39%	0.23%
$n_m = 4$	4.61%	4.46%	0.27%	0.17%
All markets	4.44%	5.09%	0.24%	0.20%

Table 8: **Cost of Strategic Interaction and Decline in Demand**.

Note: Q_m^R, Q_m^*, and Q_m^{**} are defined in the same way as in Table 7. The table shows the value of the median market for each market structure and speed of decline in demand.

Market	$t^{**} - t^*$ Mean	5th	Median	95th
$n_m = 2$	1.841	0.144	1.059	5.342
$n_m = 3$	1.593	0.002	0.908	5.412
$n_m = 4$	1.623	0.002	1.232	4.802
All markets	1.059	0.002	0.303	4.568

Table 9: **Delay in Exit in Years when Demand is Constant**.

Note: t^* and t^{**} are defined in the same way as in Table 6. I calculate the model 100 times in each market assuming demand is constant, and average the delay in exit due to asymmetric information by the initial number of competitors. Approximately one-third of the exits occur immediately.

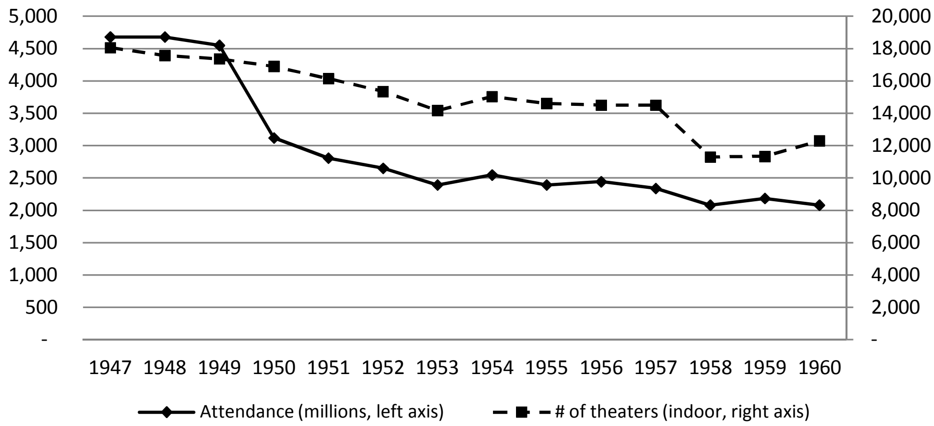

Figure 1: Movie Attendance and Number of Movie Theaters

Note: Movie theater yearly attendance and the total number of indoor movie theaters.

Source: The Film Daily Yearbook of Motion Pictures.

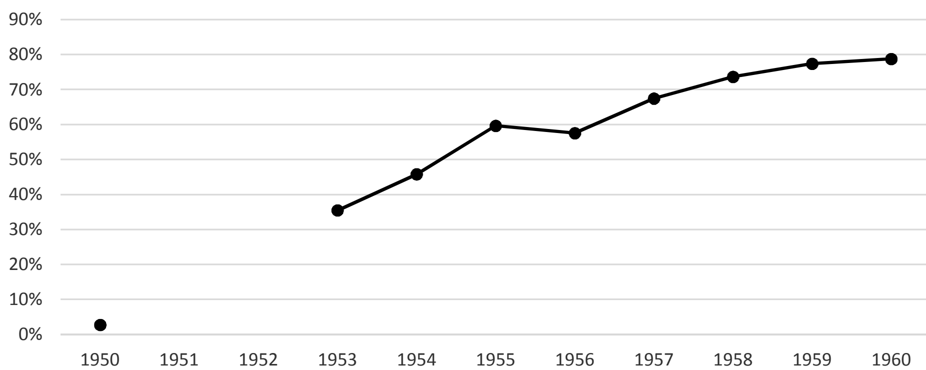

Figure 2: Average of Market-Level TV Penetration Rates

Note: These rates are not reported in 1951 and 1952.

Source: Gentzkow and Shapiro (2008a).

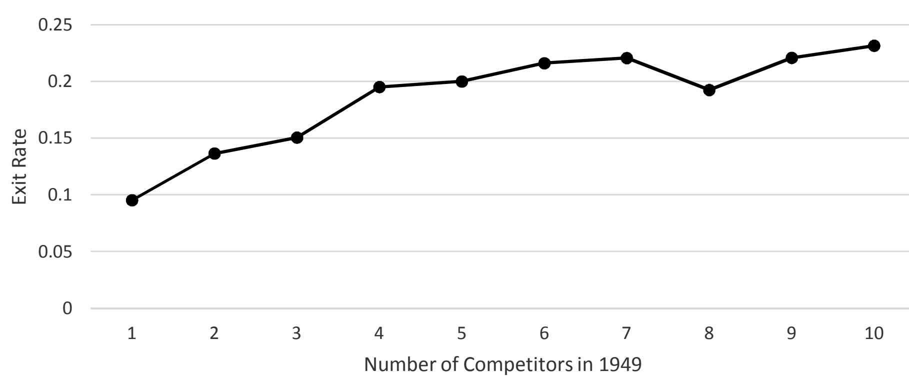

Figure 3: Exit Rate by Number of Competitors in 1949

Source: The Film Daily Yearbook of Motion Pictures and author's calculation.

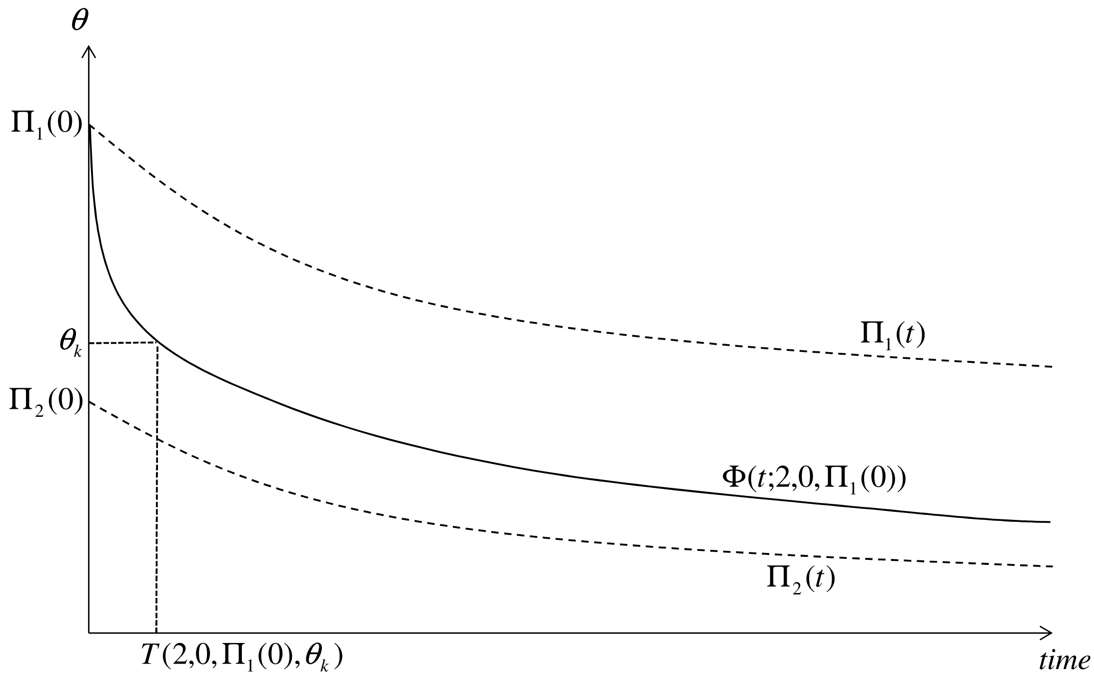

Figure 4: Policy Function in the Case of Duopoly

Note: The policy function in the two-player game is given by $\Phi(t;2,0,\Pi_1(0))$. Following this, Firm θ_k waits until $T(2,0,\Pi_1(0),\theta_k)$, and in case nobody has dropped out, exits.

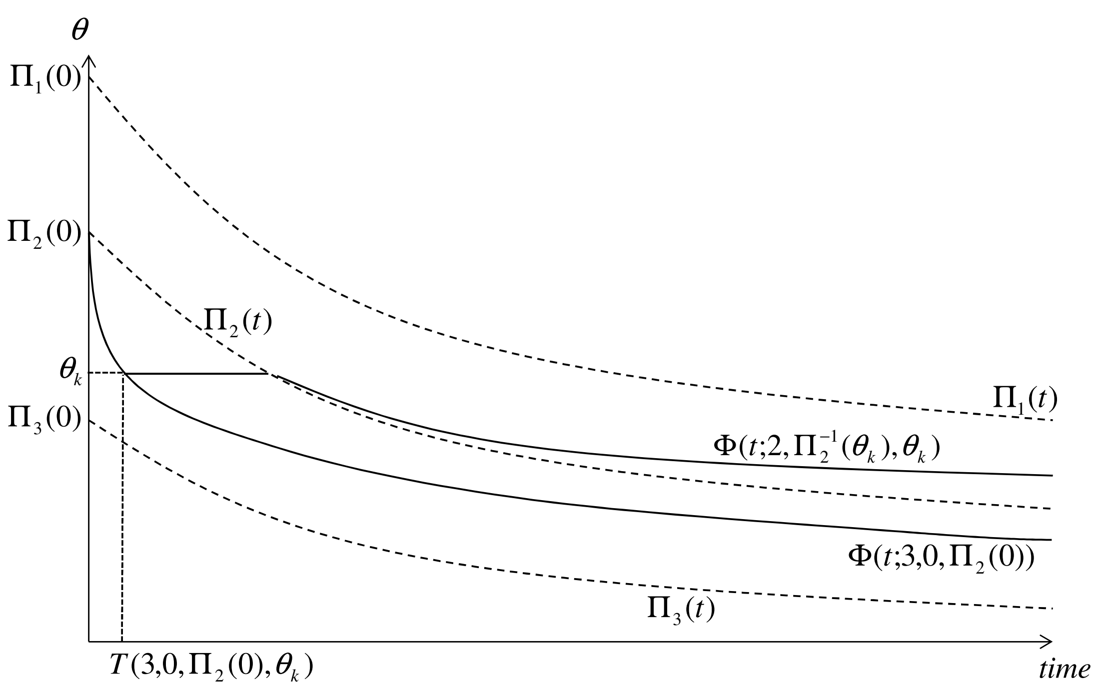

Figure 5: Policy Function in the Case of Oligopoly

Note: The policy function in the three-player game is given by $\Phi(t;3,0,\Pi_2(0))$. After firm θ_k drops out at time $T(3,0,\Pi_2(0),\theta_k)$, the policy function in the two-player subgame is given by $\Phi(t;2,\Pi_2^{-1}(\theta_k),\theta_k)$.

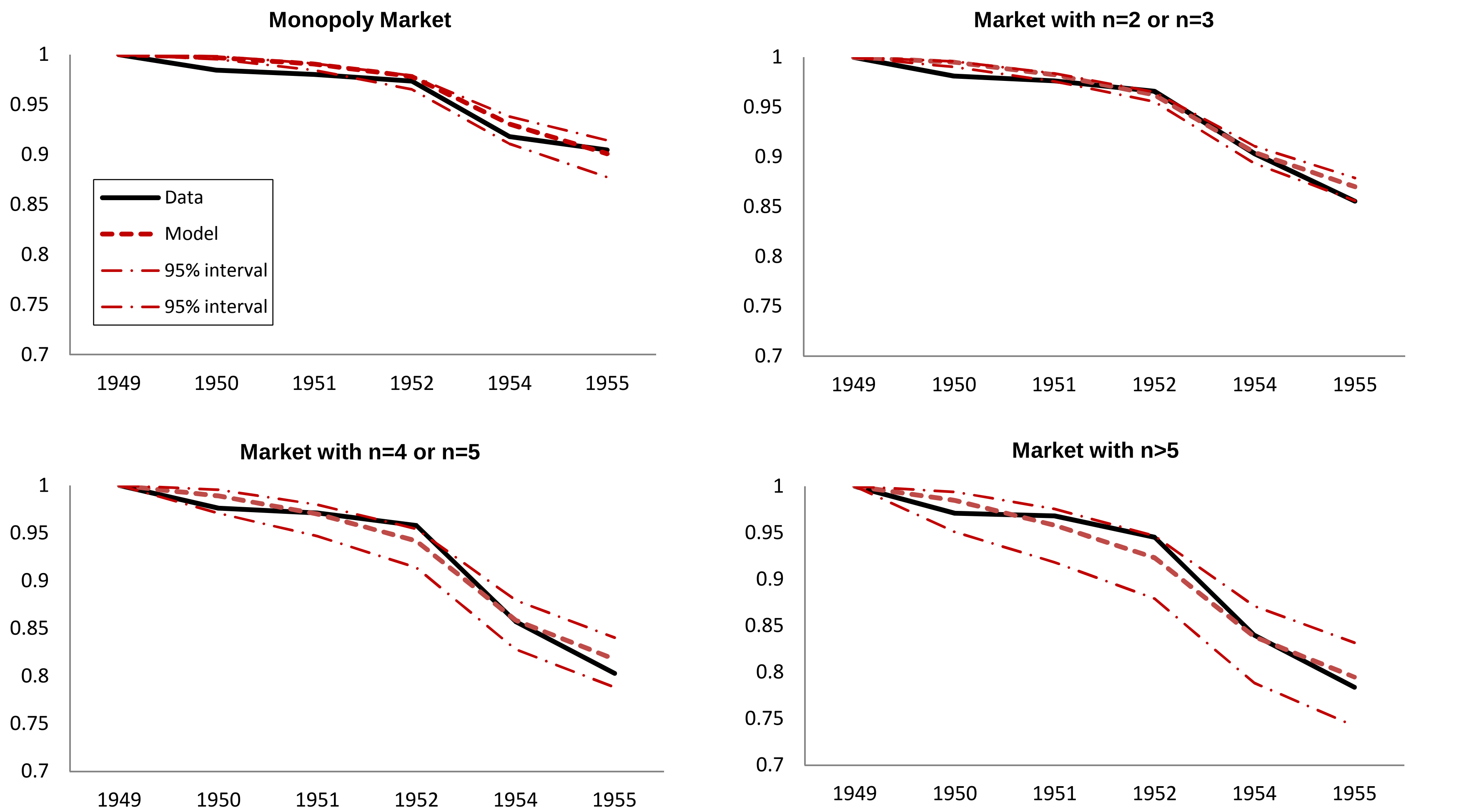

Figure 6: Model Fit

Note: I randomly drew a set of structural parameters γ from the estimated asymptotic distribution N(0,W) 200 times, and simulated the survival rate of theaters for each draw. Then, I calculated the 95% confidence interval (top 2.5% and bottom 2.5% of exit rates) for the exit rate for different market structures.

Appendix A: Proofs (Online Appendix, Not for Publication)

Claim *Consider a two-player game. Pick θ arbitrarily such that $\theta > \lim_{t\to\infty} \Pi_2(t)$. Let $T\left(2,0,\overline{\theta},\cdot\right)$ be the equilibrium strategy of the game if the two-player subgame starts at $t=0$. Let $\Phi\left(t;2,0,\overline{\theta}\right)$ be its inverse. For any t_0 such that $0<t_0<T\left(2,0,\overline{\theta},\theta\right)$, let $\widetilde{T}(2,t_0,\widetilde{\theta},\cdot)$ denote the equilibrium strategy of the game starting from t_0. Also let $\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})$ be its inverse. Then, $\widetilde{\theta}=\Phi\left(t_0;2,0,\overline{\theta}\right)$ and $T\left(2,0,\overline{\theta},\theta\right)=\widetilde{T}(2,t_0,\widetilde{\theta},\theta)$* for all $\theta \le \widetilde{\theta}$.

In other words, the optimal exit time that is planned at time zero equals the one that is planned at some later time, conditional on nobody having exited until then.

Proof of Claim In equilibrium, at time t_0, theater i knows that θ_j is equal to or smaller than $\Phi\left(t_0;2,0,\overline{\theta}\right)$. If not, theater j would have been better off by exiting at $t_0-\varepsilon$, which contradicts the construction of $\Phi\left(t_0;2,0,\overline{\theta}\right)$. Thus, $\widetilde{\theta}=\Phi\left(t_0;2,0,\overline{\theta}\right)$. If theater i chooses stopping time $\tau \ge t_0$, the present discounted sum of its expected profits at t_0 is

$$\begin{aligned} V(\tau,\widetilde{T}_{-i},2,t_0,\widetilde{\theta},\theta_i) &= \Pr(\widetilde{T}(2,t_0,\widetilde{\theta},\theta_j)\ge\tau)\left[\int_{t_0}^{\tau}\Pi_n(t)\,e^{-r(t-t_0)}dt+\frac{\theta_i}{r}e^{-r(\tau-t_0)}\right] \\ &\quad+\int_{\left\{\theta_j|\widetilde{T}(2,t_0,\widetilde{\theta},\theta_j)<\tau\right\}}\left[\int_{t_0}^{\widetilde{T}(2,t_0,\widetilde{\theta},\theta_j)}\Pi_n(t)\,e^{-r(t-t_0)}dt\right. \\ &\quad\left.+e^{-r(\widetilde{T}(2,t_0,\widetilde{\theta},\theta_j)-t_0)}V(\widetilde{T},1,\widetilde{T}(2,t_0,\widetilde{\theta},\theta_j),\widetilde{\theta}',\theta_i)\right]g(\theta_j|\theta_j\le\widetilde{\theta})d\theta_j. \end{aligned}$$

Taking the first-order condition and rearranging gives

$$\begin{aligned} \widetilde{\Phi}'(t;2,t_0,\widetilde{\theta}) &= -\frac{G(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})|\theta_j\le\widetilde{\theta})}{g(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})|\theta_j\le\widetilde{\theta})}\left[\frac{\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})-\Pi_2(t)}{V(\widetilde{T},1,t,\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}),\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}))-\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})/r}\right] \\ &= -\frac{G(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}))}{g(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}))}\left[\frac{\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})-\Pi_2(t)}{V(\widetilde{T},1,t,\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}),\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}))-\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})/r}\right]. \end{aligned} \tag{A1}$$

where the second equality follows from $\frac{G(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})|\theta_j\le\widetilde{\theta})}{g(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})|\theta_j\le\widetilde{\theta})}=\frac{G(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}))}{g(\widetilde{\Phi}(t;2,t_0,\widetilde{\theta}))}$. The boundary condition is

$$\widetilde{\Phi}(t_0;2,t_0,\widetilde{\theta})=\Phi(t_0;2,0,\overline{\theta}). \tag{A2}$$

Since (1) and (A1) are the same, (A2) implies

$$\widetilde{\Phi}(t;2,t_0,\widetilde{\theta})=\Phi(t;2,0,\overline{\theta})\ \forall t\ge t_0.$$

Equivalently, $T\left(2,0,\overline{\theta},\theta\right)=\widetilde{T}(2,t_0,\widetilde{\theta},\theta)$ for all $\theta\le\Phi\left(t_0;2,0,\overline{\theta}\right)$.■

Proof of Lemma 2 (i) Arbitrarily pick $\theta \in \left[\Pi_{n-1}(0), \overline{\theta}\right]$. Suppose $T_i(n, s_n, \widetilde{\theta}_n, \theta) > 0$. Then, θ is the marginal type at $t = T_i(n, s_n, \widetilde{\theta}_n, \theta)$ that is indifferent between exiting at t and exiting at $(t + dt)$. The value of waiting until $(t + dt)$ is the probability that one of i's opponents drops out in $[t, t + dt]$ conditional on it having survived until t, times the value of entering the $n - 1$ player subgame. On the other hand, the cost of waiting until $(t + dt)$ is $\{(\theta - \Pi_n(t))\, dt\}$, which is positive because $\theta \geq \Pi_{n-1}(0) > \Pi_n(0) > \Pi_n(t)$. Since the value should be equal to the cost of waiting by assumption of the marginal type, the value of entering the $n - 1$ player subgame should be positive as well.

If one of the competitors, say j, drops out in the interval, theater θ's value of entering the subgame depends on the exit values of other surviving competitors. Since θ is the marginal type at $T_i(n, s_n, \widetilde{\theta}_n, \theta)$, however, it follows that $\Pr(\theta < \theta_k) = 0$ for any competitor k. Otherwise, player θ_k should have dropped out earlier. This implies that player θ drops out immediately after the two-player subgame starts. Therefore, the value of staying for theater θ should be zero. This is a contradiction.

(ii), (iii), (iv) By the same argument as Lemma 1 of Fudenberg and Tirole (1986).■

Proof of Lemma 3 (i') The value of exit of surviving theaters is θ^* or lower. Therefore, $\Pi_n(t)$ is higher than the value of exit for any theater until t^*, and hence no theater will exit between s_n and t^*.

(ii'), (iii'), (iv') By the same argument as Lemma 2.■

Proof of Proposition 5 In the symmetric case, a slight modification of the proof for Lemma 3 in Fudenberg and Tirole (1986) suffices. In particular, letting $\Phi(t; n, s_n, \widetilde{\theta}_n)$ and $\widetilde{\Phi}(t; n, s_n, \widetilde{\theta}_n)$ be distinct solutions of (1), (2), and (3), I can obtain a contradiction.■

Proof of Proposition 6 A theater with $\theta_i = \Phi(t; n, s_n, \widetilde{\theta}_n)$ does not drop out if $\Pi_n(t)$ is strictly larger than its exit value. If $\Pi_n(t) = \Phi(t; n, s_n, \widetilde{\theta}_n)$, the theater does not drop out either, because there is a positive probability that some of its competitors exit in the next instant; i.e.,

$$\Pr(\Phi(t; n, s_n, \widetilde{\theta}_n) < \max_{j \neq i} \{\theta_j\} < \Phi(t + s; n, s_n, \widetilde{\theta}_n)) > 0$$

for all t and $s > 0$. Thus, $\Pi_n(t) < \Phi(t; n, s_n, \widetilde{\theta}_n)$. Next, to show that $\Phi(t; n, s_n, \widetilde{\theta}_n) < \Pi_{n-1}(t)$, suppose $\Phi(t; n, s_n, \widetilde{\theta}_n) = \Pi_{n-1}(t)$. A theater with $\theta_i = \Pi_{n-1}(t)$ is indifferent between staying in and exiting. By Assumption 1, however, there is always a positive probability that every theater stays in; i.e., $\Pr(\max\{\theta_j\}_{j \neq i} < \lim_{t \to \infty} \Pi_n(t)) > 0$.

Therefore, theater $\theta_i = \Pi_{n-1}(t)$ would be better off by dropping out at $t-\varepsilon$. This contradicts the construction of $\Phi(t; n, s_n, \widetilde{\theta}_n)$. Thus, $\Phi(t; n, s_n, \widetilde{\theta}_n) < \Pi_{n-1}(t)$.■

Appendix B: Initial Conditions Problem (Online Appendix, Not for Publication)

Formally, letting $f_Y(y|z)$ be the joint density of $Y \in \mathbb{R}^K$ conditional on $Z = z$, the initial conditions problem suggests that

$$f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}, n) \neq f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}),$$

where n is the observed number of theaters in 1949 and $\mathbf{X}$ is a set of observable market covariates. In addition, by assumption, we have

$$f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}) = f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathbf{X}) f_\alpha(\alpha|\mathbf{X})$$

but

$$f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}, n) \neq f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathbf{X}, n) f_\alpha(\alpha|\mathbf{X}, n).$$

Thus, I need to obtain $f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}, n)$, which is generated endogenously from the game before 1949 in order to calculate the likelihood function or to simulate moments of the dynamic game of exit. One way to address this issue is to simulate the model starting from the time when the industry was born, and infer the distributions of $\boldsymbol{\theta}$ and α that are consistent with the industry structure in 1949. The movie theater industry, however, had a non-stationary structure in the sense that it experienced a boom in the 1920's and 1930's, and afterwards faced declining demand in the 1950's. Therefore, simulating the entire history of the industry requires one to model the life cycle of the industry, which is beyond the scope of the paper.

Instead, I approximate $f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}, n)$ by simulation as follows. I assume that $\bar{N}$ potential players play an entry game at time zero (in 1949 in my model), and entrants play the exit game afterwards. Specifically, each potential entrant draws its value of exit, and by comparing it with the expected value of entry, it chooses whether or not to enter the market. For a given value of market heterogeneity and set of exit values of all potential entrants, the entry game predicts the number of entrants in equilibrium. Since the initial number of theaters is observed, I use the entry game to restrict the support of market heterogeneity and exit values so that, in equilibrium, the entry game predicts the same number of entrants as is observed. Since it is hard to characterize such a restriction analytically, I use a simulation to approximate the joint distribution of market heterogeneity and exit values, conditional on the observed number of entrants. This simulated distribution can then be used as an input to simulate and solve the dynamic stage of the game.

My approach can be regarded as a reduced form of a full simulation of the entire history of the industry. Heckman (1981) proposes introduction of an additional reduced-form equation to account for the potential correlation between the initial state of a sample and unobservable heterogeneity, and jointly estimates the model with the additional equation. Thus, the current approach can also be regarded as an extended version of Heckman (1981)'s model in the context of games with serially correlated private information. Aguirregabiria and Mira (2007) also account for the initial conditions problem associated with market-level unobservables, but in their context private information is iid over time and thus players are not selected samples in that aspect.

To begin, I make the following assumption:

Assumption *All potential players make an entry decision simultaneously at time zero without knowing that a war of attrition will immediately follow the entry game. That is, every player assumes that, upon entry, it earns the same per-period payoff forever.*

I use $\pi_n^e(m)$ as the payoff from entering the game when n theaters do so. The above assumption justifies this. If theaters expect that a war of attrition will start or payoffs will change sometime in the future, then computing the value of entry is a lot more complicated.

Remember that theater i's optimal choice D_i^* is given by

$$\begin{aligned} D_i^* &= \mathbf{1}\left\{(\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m) E\left[n_m^{-\delta_e}\right] \geq \theta_i\right\} \\ &= \mathbf{1}\left\{(\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m)\left[\sum\nolimits_{k=0}^{\bar{N}-1} \Pr(n_{m,-i} = k)(k+1)^{-\delta}\right] \geq \theta_i\right\} \end{aligned} \tag{1}$$

where $\bar{N}$ is the number of potential entrants and $n_{m,-i}$ is the number of entrants aside from i. Let P denote a theater's subjective probability that a competitor enters the market. By the binomial theorem, we have

$$\Pr(n_{m,-i} = k) = \binom{\bar{N}-1}{k} P^k (1-P)^{\bar{N}-k-1}.$$

Finally, (1) implies

$$\Pr(D_i^* = 1) = G\left((\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m)\left[\sum\nolimits_{k=0}^{\bar{N}-1} \binom{\bar{N}-1}{k} P^k (1-P)^{\bar{N}-k-1} (k+1)^{-\delta}\right]\right).$$

In equilibrium, the beliefs are consistent with the strategies, meaning that

$$P = G\left((\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m) K(P)\right) \tag{2}$$

where

$$K(P) = \sum_{k=0}^{\bar{N}-1} \binom{\bar{N}-1}{k} P^k (1-P)^{\bar{N}-k-1} (k+1)^{-\delta}.$$

Letting $P^*(\mathbf{X}_m, \alpha_m)$ denote the unique fixed point of (2), the number of entrants predicted by this entry model equals the number of $\theta_i s$ in $\{\theta_1, ..., \theta_{\bar{N}}\}$ that satisfy

$$(\alpha_m + \boldsymbol{\beta}'\mathbf{X}_m) K(P^*(\mathbf{X}_m, \alpha_m)) - \theta_i \geq 0. \tag{3}$$

B.1: Simulating the Joint Distributions of α and θs

This subsection explains how the entry model is used to approximate the joint distributions of $\boldsymbol{\theta}$ and α conditional on $\mathbf{X}$ and n. Let $F_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}, n)$ be the CDF of the density $f_{\boldsymbol{\theta},\alpha}(\boldsymbol{\theta}, \alpha|\mathbf{X}, n)$. Based on the entry model, I simulate the joint distribution of $\boldsymbol{\theta}$ and α conditional on $(\mathbf{X}_m, n_m)$ in the following way:

Step 1: Draw α^0 from $N(0, \sigma_\alpha^2)$.

Step 2: Calculate $P^*(\mathbf{X}_m, \alpha^0)$ using (2). Then, define θ^* as

$$\theta^* = (\alpha^0 + \boldsymbol{\beta}'\mathbf{X}_m) K(P^*(\mathbf{X}_m, \alpha^0)).$$

That is, θ^* is the threshold of exit values below which a theater finds it profitable to enter the game.

Step 3: Draw a value of exit n_m times from $TN(0, \sigma_\theta^2; l_\theta, \theta^*)$ and $\bar{N}-n_m$ times from $TN(0, \sigma_\theta^2; \theta^*, h_\theta)$, where $TN(\mu, \sigma^2; a, b)$ denotes the truncated normal distribution with mean μ, variance σ^2, and lower and upper bounds of a and b. Sort these values in an ascending order. Call them $\boldsymbol{\theta}^0 = (\theta_1^0, ..., \theta_{n_m}^0, \theta_{n_m+1}^0, ..., \theta_{\bar{N}}^0)$.

Step 4: Define α_l and α_h such that

$$\theta_{n_m}^0 = (\alpha_l + \boldsymbol{\beta}'\mathbf{X}_m) K(P^*(\mathbf{X}_m, \alpha_l))$$
$$\theta_{n_m+1}^0 = (\alpha_h + \boldsymbol{\beta}'\mathbf{X}_m) K(P^*(\mathbf{X}_m, \alpha_h)).$$

Step 5: Draw α^1 from $TN(0, \sigma_\alpha^2; \alpha_l, \alpha_h)$.

That is, α^1 is large enough to support entry of n_m theaters but not enough to support entry of $n_m + 1$ theaters.

Step 6: Return to Step 2 and repeat these steps J times to get $(\boldsymbol{\theta}^j, \alpha^j)_{j=1}^J$.[1] Call this $\hat{F}_{\theta,\alpha|\mathbf{X}_m,n_m}$.

Note that this procedure is completed for each market. Any random draw $(\boldsymbol{\theta}, \alpha)$ from $\hat{F}_{\boldsymbol{\theta},\alpha|\mathbf{X}_m,n_m}$ supports exactly n_m entrants in a symmetric equilibrium of the entry game.

[1] I discard the first 30 sets of $(\boldsymbol{\theta}^j, \alpha^j)$ before storing.

B.2: Role of Initial Conditions

If a market has a high unobservable demand shifter, we would observe more firms in the market than otherwise. If one were to ignore the unobservables in such a case, one would underestimate the negative effect of competition, since many firms appear to be able to operate profitably. On the other hand, if the unobservable demand shifter is low, the number of competitors would be small. In such a case, one would infer that the effect of competition is strong. Furthermore, ignoring the initial conditions problem affects estimates of other parameters as well.

To see the role of the initial conditions problem, I estimate the model using the *ex-ante* distributions of market level heterogeneity and exit values; i.e., I use $F_{\boldsymbol{\theta},\alpha|n_m}$ instead of $\hat{F}_{\boldsymbol{\theta},\alpha|n_m}$. Note that the entry stage is used only for identifying the parameters in the payoff function. I have 27 moments to estimate 15 parameters.[2] Table 1 shows the parameter estimates. The effect of competition in the dynamic game, δ, is estimated to be smaller than the full model, while that in the entry game, δ_e, is larger. Thus, ignoring the initial conditions problem significantly changes estimates of competition effects. The estimates of the variance of market-level heterogeneity are also different between the two specifications.

To investigate the model fit, I simulate the model ten times and for each simulation count the number of markets in which the model predicts correctly the number of exits during the sample period. Then, I take average over simulation draws for different market structures. The model predicts the number of exits correctly in 83.3% of the monopoly markets. In 59.5% of duopoly or triopoly markets, the number of exits predicted by the model and the one observed in the data match exactly. The corresponding number for markets with four or five theaters is 34.0%. Finally, in markets with more than five theaters, the share of markets in which the model predicts the number of exits correctly is 23.0%.

It is difficult, however, to evaluate the importance of initial conditions, since parameters in the payoff function are reduced-form parameters.[3] To better understand the role of initial conditions, I first simulate the model starting from 1949 until 1955, using the *ex-ante* distri-

[2] Compared to the full model, the average number of entrants $E(n_m)$ is excluded from the set of moments for the following reason. Since the mean of α_m is *ex-ante* zero, the exit game predicts too many exits at $t = 0$ for markets with many competitors, unless the mean profit is very high. If the mean profit is high, however, the entry stage predicts too many entrants.

[3] Another reason is that I use the entry model not only for addressing the initial condition problem but also for identifying parameters in the base demand. One alternative would be to identify all the parameters from "dynamic" moments only, and to use the entry model only for addressing the initial condition problem. Then, I would be able to compare the estimation results with and without the entry model. I did not use this strategy because the correlations between exit rates and $\mathbf{X}_m$ are weak in the data, and thus I would not identify $\boldsymbol{\beta}$ in the base demand well.

bution of unobservables.[4] To fix observable variables, I only use Steuben county in Indiana for the simulation, as the TV penetration rate in 1955 and the total population in 1950 are close to the median values in the sample.[5] Then, I assume 1955 is the "initial" period of my hypothetical sample. Put differently, I assume that the industry was born in 1949 and the data are available from 1955. Panel (a) in Figure B1 plots the distribution of market-level heterogeneity (α) conditional on the number of surviving theaters in 1955. This result clearly shows that $f_\alpha(\alpha|\mathbf{X}, n) \neq f_\alpha(\alpha|\mathbf{X})$. Next, I simulate $\hat{F}_{\boldsymbol{\theta},\alpha|\mathbf{X}_m,n_m}$ using the simulation method proposed above, and integrate over θ to calculate $\hat{f}_\alpha(\alpha|\mathbf{X}, n)$ for $n = 3$. Panel (b) in Figure B1 plots this and compares it with the true distribution of α conditional on $n = 3$, which I already showed in panel (a), along with the *ex-ante* distribution of α. The "true" distribution of α is closer to the simulated distribution than to the *ex-ante* distribution.

Since unobservable exit values are time-invariant, selection based on exit values may be substantial too. In panel (c) in Figure B1, I plot the distribution of exit values when the industry was born in this exercise (i.e., *ex-ante* distribution) and the distribution of exit values of surviving theaters in 1955. As expected, these two distributions are significantly different from one another. In particular, the distribution of survivors' exit values has a high density at the low end compared to the unconditional distribution. I also calculated $\hat{f}_{\boldsymbol{\theta}}(\boldsymbol{\theta}|\mathbf{X}, n = 3)$ for surviving theaters in 1955, using the proposed simulation method. Panel (c) plots this distribution. Again, the simulated distribution is closer to the true distribution of exit values of surviving theaters. Although these two distributions are still different, it can be seen that the proposed method, to some extent, alleviates the initial conditions problem.

One caveat is that in this exercise the game played by the theaters before the "initial" period (i.e., 1955) is the exit game in which the payoff function is very similar to that of the entry game. As I discussed above, in the current application, the type of game that was played before 1949 may have been different from the exit game. If the game before the initial period were, however, different, the parameter estimates in the entry game would change accordingly, and hence be able to capture the correlation between n, $\boldsymbol{\theta}$, and α.

[4] Throughout this exercise, I fix $\sigma_\alpha^2 = 0.6$ to facilitate comparisons. Other parameters are set at the estimated values.

[5] The TV penetration rate in 1955 and the total population in 1950 of Steuben county are 0.54 and 17,087, respectively. The median value for each variable is 0.54 and 17,150, respectively. The number of theaters in 1949 in this county was three in the data, but I start from four theaters to have more variations in the number of theaters in 1955.

Appendix C: Interpolation of TV Penetration Rates (Online Appendix, Not for Publication)

The TV penetration rate is reported once in 1950 and afterwards every year starting from 1953. I use data from all available years up to 1960 to approximate the TV diffusion process by the CDF of the Weibull distribution. For market m, I observe

$$(TV_{m1950}, TV_{m1953}, TV_{m1954}, ..., TV_{m1960}).$$

Using 1949 as the reference year, let

$$\widehat{TV}_{mt}(k_1, k_2) = 1 - e^{-((t-1949)/k_1)^{k_2}}.$$

Then, I choose (k_1, k_2) to minimize

$$\sum_{t=1950,1953,...,1960} \left(TV_{mt} - \widehat{TV}_{mt}(k_1, k_2)\right)^2$$

for each market. Once the minimizers are obtained, I can calculate $\widehat{TV}_{mt}(\hat{k}_1, \hat{k}_2)$ for all $t \in [1949, 1955]$.

This approach has several advantages. First, the interpolated TV penetration rate is continuous and smooth. Linear interpolation would generate many kinks, which may not be natural in the continuous-time model. Second, this method ensures that the theater's payoff is monotonic, which satisfies Assumption 1 (i). In 84 markets (about 3% of all markets), the TV penetration rate decreases slightly from one year to another. This is most likely due to mis-measurement. I do not need to discard these observations. Third, this method enables me to exploit information after 1955. Given that the TV penetration rate is missing in 1951 and 1952, such additional information is valuable.

Appendix D: Importance of Movie Theater Chains (Online Appendix, Not for Publication)

Movie theater chains existed in the sample period, although their influence and market power were much more limited compared to later periods. Unfortunately, comprehensive information on movie theater chains in the time period of this study is not available. The *Film Daily Yearbook of Motion Pictures*, however, lists all theaters owned by big theater chains (circuits); i.e., theater chains that own at least four theaters in the U.S. I collect markets in which at least two movie theaters are owned by the same theater chain and call them "market with circuit".

The remaining markets are called "market without circuit". In other words, in these markets, any two theaters in the market are not owned by the same theater chain. It could still be the case that two theaters are owned by two different chains. Table 2 shows the frequency of these markets. The majority of markets do not have a circuit that owns more than one theater in the same market.

To further investigate if this may distort my results, I construct a dummy variable defined as whether the market is a "market with circuit" or not, and regress the market-level exit rate on the dummy variable as well as on the initial number of competitors, its squared term, the change in the TV rate, the change in the population, and the number of new entrants in the market. The sign of the coefficient of the dummy variable is positive, but not statistically significant. If theaters under the same ownership coordinate, there would be no strategic delay and the exit rate would be higher. In this sense, the sign of the coefficient is consistent with the argument that competition delays exits. There is no evidence, however, that "market with circuit" significantly distort the result. This should not be taken as evidence that chains are not important, as (i) this additional data contain only big theater chains, and (ii) the explanatory power of the linear regression is limited. Rather, the delay and cost of asymmetric information in this paper can be interpreted as their lower bounds, as coordination would hasten exits.

Appendix E: Numerical Solution (Online Appendix, Not for Publication)

To solve this differential equation, the method of forward shooting cannot be used because the problem is ill-defined at zero. The standard backward-shooting method does not work either, as a finite end point does not exist. To deal with this issue, I apply the following algorithm. I start from an arbitrary point $t' > 0$ and use backward shooting from t' to 0. Then, I choose $\Phi\left(t'; n, 0, \overline{\theta}\right)$ such that $\Phi(0; n, 0, \overline{\theta}) = \Pi_{n-1}(0)$. Then, for an arbitrary $\varepsilon > 0$, I confirm that there exists t'' such that $|\Phi(t''; n, 0, \overline{\theta}) - \Pi_n(t'')| < \varepsilon$.

Parameters	Coef.	Std. Err
δ (competition in dynamic game)	0.2009	0.1050
δ_e (competition in entry game)	2.1173	0.2285
β_0 (constant)	8.7248	1.8764
β_1 (population)	0.0687	4.3860
β_2 (median age)	2.5228	1.5975
β_3 (median income)	0.0206	4.0410
β_4 (urban share)	-2.5481	1.1484
β_5 (employment share)	2.7038	1.0338
β_6 (log of land area)	-2.7572	1.3948
λ_1 (TV rate)	0.5487	0.0596
λ_2 (change in population)	-3.3260	0.8608
λ_3 (new entrants)	0.4326	0.1257
σ_θ (std. of exit value)	2.7545	0.5959
σ_α (std. of demand shifter)	1.8622	0.5057
$\rho_{\alpha\lambda}$ (corr coef. b/w α_m and λ_m)	0.3320	0.0861

Table 1: **Estimates of Structural Parameters without Correcting the Initial Condition Problem**.

Number of Theaters in 1949	Market with Circuit	Market w/o Circuit	Total
1	0	451	451
2	124	396	520
3	160	285	445
4	174	195	369
5	138	107	245
6	123	86	209
7	111	32	143
8	72	15	87
9	64	16	80
10	46	11	57
Total	1,012	1,594	2,606

Table 2: **Number of Markets With and Without Circuit.**

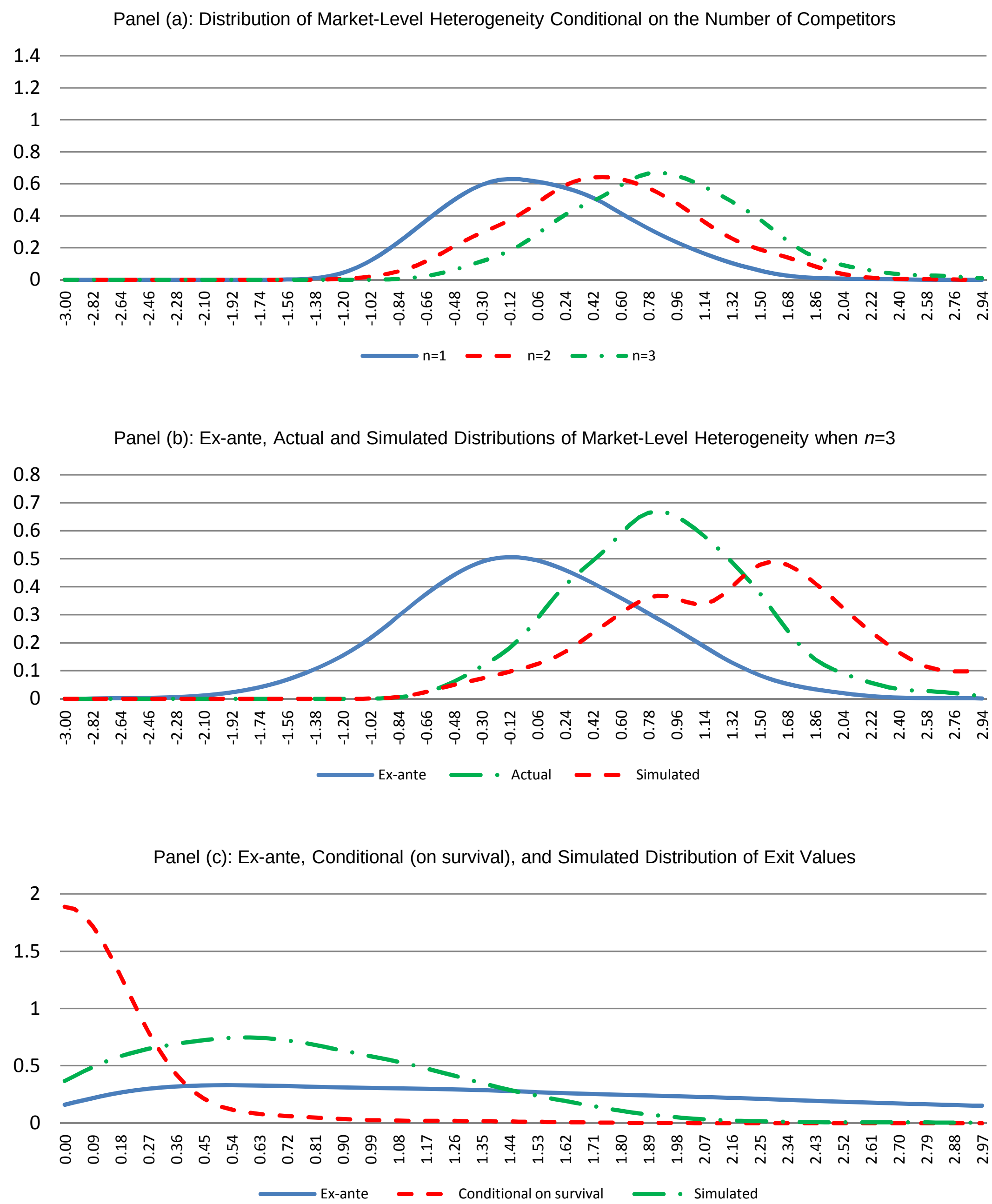

Figure B1 Importance of Initial Condition Problem

Note: I simulate the model many times starting from 1949 until 1955, using the ex-ante distributions of unobservables. I fix observable covariates at the level of the median county. Panel (a) plots the distribution of market-level heterogeneity, splitting the simulated outcomes according to the number of surviving theaters in 1955. Then, assuming that 1955 is the initial period of my hypothetical sample, I simulate the distribution of market-level heterogeneity for $n=3$, using the method I discribed in Section B.1. Penel (b) plots the resulting distribution and the distribution conditional on $n=3$ from panel (a), along with its *ex-ante* distribution. Finally, panel (c) plots the *ex-ante*, conditional (on survival in 1955), and simulated distributions of exit values, which I obtained from the above simulation.