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ON SUBDIVISION OF SEMISIMPLICIAL SETS

§ 1 Introduction

The regular subdivision $\Delta' X$ of a semisimplicial set X can be easily defined in a purely combinatorial way; this has been done by Barratt [1] and Kan [7]. To investigate the geometrical meaning of this let us first consider especially the two degeneracy maps from a 2-simplex to an 1-simplex and their subdivisions. We find out that there can't be a natural homeomorphism between the geometric realizations of a semisimplicial p -simplex and its regular subdivision, although the underlying spaces are nothing but geometric p -simplices. As the category \underline{S} of semisimplicial sets and semisimplicial maps is the completion of the category of semisimplicial simplices and semisimplicial maps with respect to colimits, such a natural homeomorphism would be necessary and sufficient for the existence of a natural homeomorphism between $|X|$ and $|\Delta' X|$, the geometric realizations of a semisimplicial set X and its regular subdivision $\Delta' X$. So we have

Theorem 1. *There exists no natural equivalence between the functors $|\Delta' ?| : \underline{S} \rightarrow \underline{CW}$ and $|\Delta' ?| : \underline{S} \rightarrow \underline{CW}$ ("CW" denotes the category of CW-complexes and continuous maps).*

Now the question arises if there is any homeomorphism between $|X|$ and $|\Delta' X|$. The answer to this question is in my mind far away from being trivial—as many people believed a long time—and is first given in my paper [3] as a special case of a general result on a class of various subdivisions. Subsequent to this Puppe has found an explicit formula, which—as we proved in [4]—gives a homeomorphism in the regular case.

The question mentioned above has also suggested my paper [2] and my aim here is to outline the content of [2] and [3].

2 Standard division functors

Standard division functors are introduced in [2].

Definition 1. *A "standard division functor" is a pair (U, u) consisting of a functor*

$$U : \underline{S} \rightarrow \underline{S}$$

and a family

$$u = (u_p / p \text{ non-negative integer})$$

such that the following conditions are satisfied:

(i) u_p is a homeomorphism $|U[p]| \rightarrow \Delta_p$ for each non-negative integer p

$$(ii) \quad |\Delta \beta| \circ u_p = u_q \circ |U \beta|$$

for each injective map $\beta: [p] \rightarrow [q]$ of $\underline{\Delta}$. (Here the notation must be explained: $\underline{\Delta}$ denotes the category of non-empty finite ordered sets and weak order preserving maps and $[p]$ the set of the numbers $0, 1, 2, \dots, p$, that means the ordered set of $p+1$ elements, for each non-negative integer p ; the maps of $\underline{\Delta}$ are symbolized by small greek letters; we shall briefly write " $\beta \in \underline{\Delta}$ " to indicate that β is a map of $\underline{\Delta}$. $\underline{\Delta}: \underline{\Delta} \rightarrow \underline{S}$ means the functor which assigns to each object $[p]$ of $\underline{\Delta}$ the semisimplicial p -simplex $\Delta[p]$ and to each $\beta \in \underline{\Delta}$ the semisimplicial map $\Delta \beta$; it is the simplest example for a standard division functor. Finally Δ_p denotes the geometric p -simplex.)

Given a standard division functor (U, u) one can identify each $|U[p]|$ with Δ_p by means of (i). Then we have two CW-structures on Δ_p , the one is induced by the simplices of $\Delta[p]$, the other by the simplices of $U[p]$; the same is to say that the underlying spaces of the CW-complexes $|\Delta[p]|$ and $|U[p]|$ coincide. From this point of view (ii) assures that $|U[p]|$ is a CW-subdivision of $|\Delta[p]|$, that means that each cell of $|U[p]|$ lies in a cell of $|\Delta[p]|$.

Each standard division functor (U, u) can be extended uniquely to a continuous functor from \underline{S} to itself, which we denote—by abuse of notation—also by " U ". Such a so-called "division functor" has the following properties:

Proposition 1. U preserves the fundamental group and the homology groups up to natural equivalence.

Proposition 2. U preserves coverings.

A semisimplicial map f is said to be a "weak homotopy equivalence" if f induces an isomorphism of the fundamental groups and \tilde{f} , the universal covering of f , induces isomorphisms of the homology groups. This definition is justified by the fact that the geometric realization of a semisimplicial weak homotopy equivalence is indeed a homotopy equivalence. From the propositions 1 and 2 now it follows at once

Proposition 3. U preserves weak homotopy equivalences.

Much deeper is the following result:

Theorem 2. If X is a semisimplicial set, then the CW-complexes $|X|$ and $|U X|$ have the same homotopy type; more precisely: if $[?]: \underline{CW} \rightarrow \underline{CWh}$ denotes the projection onto the homotopy category of CW-complexes, then the functors $[|U ?|]$ and $[|?|]$ are naturally equivalent.

To obtain this result we need an interesting device, which is explained in the following section.

§ 3 Non-degenerate semisimplicial sets

Definition 2. A semisimplicial set X is "non-degenerate" if no non-degenerate simplex of X has a degenerate face. If X and Y are non-degenerate semisimplicial sets, a semisimplicial map $f: X \rightarrow Y$ is "non-degenerate" if f maps non-degenerate simplices of X on non-degenerate simplices of Y .

Non-degenerate semisimplicial sets and non-degenerate semisimplicial maps from a subcategory \underline{P} of \underline{S} and one can prove:

Proposition 4. \underline{P} is a reflective subcategory of \underline{S} .

That means that the embedding functor $E: \underline{P} \rightarrow \underline{S}$ is left adjoint to a functor $\underline{S} \rightarrow \underline{P}$, the reflector R . To prove this one has to define to each semisimplicial set X a non-degenerate semisimplicial set RX and a semisimplicial map $X: R X \rightarrow X$ such that for each semisimplicial map $f: Y \rightarrow X$ with Y non-degenerate there exists a unique non-degenerate semisimplicial map $f': Y \rightarrow RX$ with $f = r X \circ f'$.¹⁾

Having done this, a simple straightforward computation yields

Proposition 5. rX induces in a natural way isomorphisms of the fundamental groups and all homology groups, and

Proposition 6. R preserves coverings.

From this two propositions it follows at once

Theorem 3. rX is a weak homotopy equivalence²⁾.

In our context the meaning of the category \underline{P} is due to

Proposition 7. If X is a non-degenerate semisimplicial set, then the spaces $|X|$ and $|U X|$ are homeomorphic; more precisely: the functors $|E?|: \underline{P} \rightarrow \underline{CW}$ and $|U E?|: \underline{P} \rightarrow \underline{CW}$ are naturally equivalent.

We omit the proof of this proposition; now theorem 2 is an easy consequence of theorem 3, proposition 3 and proposition 7.

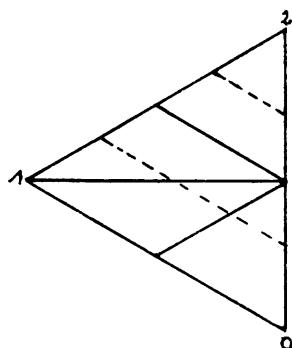
§ 4 Examples

1) The "regular" or "barycentric" subdivision of the semisimplicial simplices induces a standard division functor. It was already described by Kan [7]; he uses the symbol " Δ' " for the functor $\Delta \rightarrow \underline{S}$, but the symbol " Sd " for the extended division functor, which we according to our conventions denote also by " Δ' "; we mentioned it in § 1.

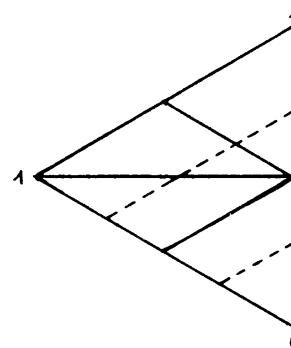
¹⁾ For this situation the terminology "coreflective" seems to become standard; but obeying the demand for logical consistency we use "reflective" in accordance with the book of Mitchell [9].

²⁾ In some papers Giever [5] and Hu [6] have studied the space $|RX|$; they denoted it by " PX'' " and called it "geometric realization of X'' ", but they had not defined the semisimplicial set RX explicitly. Then Kodama [8] has constructed the map $|rX|$ —he denoted it by " pX'' "—and a homotopy inverse to it, but by using the fact that $|\Delta' X|$ can be interpreted as CW —subdivision of $|X|$, which was not proved at that time.

2) The "natural" subdivision. We denote the corresponding functor $\underline{\Delta} \rightarrow S$ by " Δ^v ". Its effect on [2] and the two degeneracy maps $\sigma_e: [2] \rightarrow [1]$ ($e=0,1$) can be illustrated by the following pictures:



$|\Delta^v \sigma_0|$

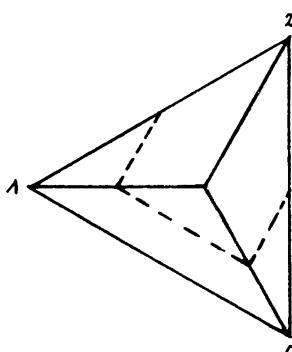


$|\Delta^v \sigma_1|$

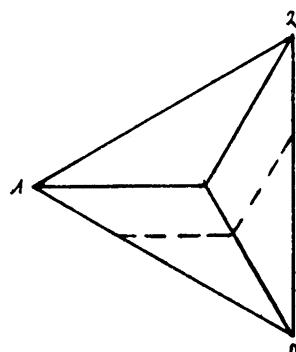
(The fully traced line segments indicate the cell structure of $\Delta_2 = |\Delta'[2]|$; $|\Delta^v \sigma_0$ resp. $|\Delta^v \sigma_1|$ identifies the dotted line segments and just so their parallels to a point.)

In this case the condition (ii) of definition 1 is satisfied for all $\beta \in \underline{\Delta}$, not only for the injective ones. The name "natural" is justified by the fact that for all semisimplicial sets X the CW-complexes $|\Delta' X|$ and $|X|$ are naturally homeomorphic. The disadvantage of this functor is that we know no method to approximate continuous maps by semisimplicial maps by means of it. The approximation constructed by Kan [7] can't be transferred.

3) The "r-skeleton-preserving" subdivision (r non-negative integer) has got its name from the fact that for all semisimplicial sets X there is a natural semisimplicial isomorphism between $U_r(X^r)$ and X^r , where U_r denotes the corresponding division functor and X^r the r -skeleton of X . The effect of U_1 on [2] and the two degeneracy maps $\sigma_e: [2] \rightarrow [1]$ ($e=0,1$) can be illustrated by the following pictures:



$|U_1 \sigma_0|$



$|U_1 \sigma_1|$

(Again the fully traced line segments indicate the cell structure, here that of $\Delta_2 = |U_1[2]|$; $|U_1\sigma_0|$ resp. $|U_1\sigma_1|$ identifies the dotted line segments and just so their parallels to a point; moreover $|U_1\sigma_1|$ identifies the whole left upper triangle to one point.)

§ 5 Natural transformations

One can ask now if the natural equivalence in the category CWh of theorem 2 is induced by a natural transformation in CW . We are not able to give a general answer to this question. Here we list the partial results we have obtained.

Proposition 8. *Each natural transformation between $|U ?|$ and $|\Delta ?|$ induces natural equivalence in the homotopy category CWh .*

Proposition 9. *The natural transformations between $|U ?|$ and $|\Delta ?|$ and between $|U\Delta ?|$ and $|\Delta ?|$ correspond in an one-to-one fashion.*

So it suffices to consider natural transformations between the functors $|U\Delta ?|$ and $|\Delta ?|$. We know almost nothing about natural transformations $|\Delta ?| \rightarrow |U\Delta ?|$, therefore let us deal with natural transformations $|U\Delta ?| \rightarrow |\Delta ?|$. Such a natural transformation can be given by a sequence t_0, t_1, t_2, \dots of maps $t_i: \Delta_i \rightarrow \Delta_i$ such that certain commutativities hold. Then one can prove:

Proposition 10. *Each natural transformation t_0, t_1, t_2, \dots is uniquely determined by t_1 .*

The essential device for proving this is the following almost trivial

Lemma. *Let V be a topological space, $f, g: V \rightarrow \Delta_n$ continuous maps, i_0, i_1 distinct elements of $[n-1]$ and*

$$|\Delta\sigma_{i_e}| \circ f = |\Delta\sigma_{i_e}| \circ g, \text{ for } e = 0, 1$$

$(\sigma_{i_e}: [n] \rightarrow [n-1]$ denotes the i_e -th degeneracy map). Then holds: $f = g$.

This lemma also yields

Proposition 11. *Each triple t_0, t_1, t_2 with*

$$|\Delta\beta| \circ t_m = t_n \circ |U\beta|$$

for $0 < m, n < 2$ and $\beta \in \Delta$ such that $|\Delta\beta| \circ t_m$ and $t_n \circ |U\beta|$ are defined can be extended to a natural transformation $|U\Delta ?| \rightarrow |\Delta ?|$.

We can also give necessary and sufficient conditions that a continuous map t_1 induces a natural transformation. But we do not know if for each standard division functor (U, u) there exists a map t_1 , which satisfies these conditions. This may at most depend on certain hypotheses about $U\sigma_e$ in case $\dim \sigma_e = 2$ ($e = 0, 1$). In our example the existence of such natural transformations can be easily established; there are even natural transformations $U \rightarrow \Delta$. To end this section we mention that there is an infinite number of natural transformations $|U\Delta ?| \rightarrow |\Delta ?|$, if there is one.

§ 6 Standard homotopies

Now we turn to the main problem: We want to show that under certain further assumptions on a given division functor U the CW -complexes $|UX|$ and $|X|$ are homeomorphic X being any semisimplicial set. Such a homeomorphism will be constructed inductively, so we arrive at the problem to continue a given map of the boundary of Δ_n onto itself over the whole geometric simplex Δ_n such that the interior of Δ_n is mapped homeomorphically onto itself: We have to stuff holes. To this end we need

Definition 3. Let V be a topological space. A homotopy $h_t: V \rightarrow V$ is "stuffing" if $h_0 = \text{id}_V$ and h_t is a homeomorphism for all $t < 1$

By means of a stuffing homotopy one can stuff holes:

Proposition 12. Given a stuffing homotopy $h_t: S^n \rightarrow S^n$ there exists an extension $h: B^{n+1} \rightarrow B^{n+1}$ of h_1 such that the interior of B^{n+1} is mapped homeomorphically onto itself. Moreover there is a stuffing homotopy $H_t: B^{n+1} \rightarrow B^{n+1}$ such that $H_1 = h$ and $H_t v = h_t v$ for all $v \in S^n$ and $t \in [0, 1]$. (" S^n " denotes the n -sphere and " B^{n+1} " the $(n+1)$ -ball.)

Now let be given a fixed division functor U ; we describe the additional condition:

Definition 4. A "standard homotopy (for U)" is a family $(l_t \beta / \beta \in \underline{\Delta})$ of stuffing homotopies $l_t \beta: \Delta_{\dim \beta} \rightarrow \Delta_{\dim \beta}$ such that the following conditions are satisfied:

$$(10) \quad l_t(i d) = i d,$$

$$(11) \quad |\Delta \beta| \circ l_t(\alpha \beta) = l_t \alpha \circ |\Delta \beta| \text{ for injective } \beta \in \underline{\Delta},$$

$$(12) \quad l_t(\alpha \beta) \circ (l_t \beta)^{-1} \text{ single-valued (and therefore a continuous map)?}$$

$$(13) \quad |\Delta \beta| \circ l_1(\alpha \beta) = l_1 \alpha \circ |U \beta|;$$

from (13) it follows that (for all suitable degeneracy maps $\sigma_i \in \underline{\Delta}$) $l_1(\beta \sigma_i) \circ l_1(\sigma_i)^{-1}$ maps each line segment parallel to the line segment between the i -th and $(i+1)$ -st vertex of $\Delta_n = |\Delta[n]|$ on such a line segment. We demand further:

$$(14) \quad \text{for each such line segment this map is weakly monotone.}$$

We do not know if there exists a standard homotopy for each division functor U . If there is a standard homotopy for a given U , then it follows at once that the sequence

$$l_1([0] \rightarrow [0]), \quad l_1([1] \rightarrow [0]), \quad l_1([2] \rightarrow [0]), \dots$$

represents a natural transformation $|U ?| \rightarrow |?|$, which we call the "corresponding natural transformation".

§ 7 The main theorem

Main theorem. *Let U be a division functor with standard homotopy. Then X being any semisimplicial set there is a homeomorphism $|UX| \rightarrow |X|$ which is homotopic to the map $|UX| \rightarrow |X|$ deduced from the corresponding natural transformation.*

Here we can only give the idea of the proof. To do this we need a more explicit description of the spaces $|UX|$ and $|X|$ for a given semisimplicial set X . They are quotient spaces of $FX = \Sigma_p X_p \times \Delta_p$, where X_p denotes the set of p -simplices of X provided with the discrete topology and Σ the topological sum. We obtain $|UX|$ by taking the equivalence relation which is generated by

$$(x\beta, v) \sim (x, |U\beta|v)$$

and $|X|$ by taking that which is generated by

$$(x\beta, v) \sim (x, |\Delta\beta|v)$$

for $x \in X_q$, $\beta: [p] \rightarrow [q]$ in Δ (as X is a semisimplicial set, β induces a map from X_q to X_p and $x\beta$ denotes the image of x under this map).

Therefore a continuous map $|UX| \rightarrow |X|$ can be constructed if there is given a family $(h_x: x \in X)$ of continuous maps $h_x: \Delta_{\dim x} \rightarrow \Delta_{\dim x}$ with $|\Delta\beta| \circ h_{x\beta} = h_x \circ |U\beta|$ for all $x \in X$ and all $\beta \in \Delta$ such that $x\beta$ is defined. It is easy to show that a map $|UX| \rightarrow |X|$ constructed in such a way is a homeomorphism iff h_x maps the interior of Δ_p homeomorphically onto itself for each non-degenerate $x \in X_p$. In order to establish the first part of the main theorem one has to construct such a family $(h_x: x \in X)$ and it is obvious that the given standard homotopy plays an essential part in this construction.

§ 8 Examples of standard homotopies

We indicate how a standard homotopy for the regular subdivision can be given. Let be $\beta \in \Delta$ and $p = \dim \beta$. $I_1\beta$ maps the cells of $|\Delta'[p]| = \Delta_p$ —they are simplices—linearly and $I_t\beta$ is the linear connection between the identity and $I_1\beta$. So it suffices to show as $I_1\beta$ maps the vertices of $|\Delta'[p]|$. Any vertex b of $|\Delta'[p]|$ corresponds to a 0-simplex of $\Delta'[p]$ that is an injective map $\mu: [m] \rightarrow [p]$ of Δ . As $p = \dim \beta$ the composition $\beta\mu$ is defined; $\beta\mu$ can be uniquely decomposed in an injective and a surjective part; let us denote the latter by ϱ . We define a right inverse $\hat{\rho}$ to ϱ by setting

$$\hat{\rho}(i) = \max \varrho^{-1}(i).$$

Then we take

$$I_1\beta(b) = |\mu \hat{\rho}|$$

(we interpret $\mu \hat{\rho}$ to be a 0-simplex of $\Delta'[p]$ and denote by $|\mu \hat{\rho}|$ the corresponding vertex of $|\Delta'[p]|$, which is obviously a point of Δ_p).

For the other examples of standard division functors we have given there exist standard homotopies, too; in the case of natural subdivision it is easy to see that one can take the identity.

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