Ortalo-Magné, François und Rady, Sven:

Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints (Revised Version)

Housing Market Dynamics:
On the Contribution of Income Shocks and Credit Constraints[*]

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Abstract

We propose a life-cycle model of the housing market with a property ladder and a credit constraint. We focus on equilibria which replicate the facts that credit constraints delay some households' first home purchase and force other households to buy a home smaller than they would like. The model helps us identify a powerful driver of the housing market: the ability of young households to afford the down payment on a starter home, and in particular their income. The model also highlights a channel whereby changes in income may yield housing price overshooting, with prices of trade-up homes displaying the most volatility, and a positive correlation between housing prices and transactions. This channel relies on the capital gains or losses on starter homes incurred by credit-constrained owners. We provide empirical support for our arguments with evidence from both the U.K. and the U.S.

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1 Introduction

Buying a home requires a substantial amount of cash up front. The down-payment requirement limits the value of many first home purchases, primarily for younger households with little savings. Once a household owns its first home, it must accumulate further wealth if it wants to move up the property ladder. Any gains or losses on the first home have a major impact on the household's net worth and hence the timing of its next move.

We argue that understanding the equilibrium consequences of these features of housing consumption is key to understanding the volatility of housing prices, their tendency to display overshooting patterns, fluctuations in their cross-sectional variance, and the relationship between housing prices and transactions.

We propose a life-cycle model of the housing market with two types of homes available in limited supply, "starter" homes and "trade-up" homes, and a down-payment constraint on borrowing. While all households enjoy living in a house of their own, they differ in the utility premium they derive from a trade-up home compared to a starter home. Households also differ in their income streams.

The first contribution of the model is to highlight the critical role of marginal first-time buyers in housing market fluctuations. That is, any factor that affects the ability of potential first-time buyers to afford the down payment on a starter home can have a dramatic impact on the overall housing market. This points to the volatility in the income of young households as a factor in some of the "excess" volatility of housing prices. The same insight helps us rationalize large housing market swings following national institutional reforms in financing availability.

The second contribution of the model is to shed new light on the mechanism whereby down-payment constraints affect the transmission of income shocks to housing prices and transactions. We show that this mechanism offers a rationale for the positive correlation between housing prices and transactions observed in the U.S. and the U.K., given empirical evidence on the response of housing prices to income shocks and on fluctuations in the cross-sectional variance of housing prices.

Lamont and Stein (1999) and Malpezzi (1999) show in the U.S. and Miles and Andrew (1997) in the U.K. that housing prices overreact to income shocks. Poterba (1991), Smith and Tesarek (1991), Mayer (1993) and Earley (1996) provide evidence that the prices of properties at the upper end appreciate more than the prices of cheaper properties during a boom and depreciate at a higher rate during downturns. In the model, when both these phenomena occur in response to a change in income, there are more housing transactions when housing prices are on the rise and fewer when housing prices are on the decline. In

both the U.S. and the U.K., there is evidence that this relationship between housing prices and transactions has been holding over time; see Stein (1995) and Ortalo-Magné and Rady (2004).

Young households are the marginal first-time buyers of starter homes in our model, and the price of starter homes regulates entry into the housing market. Any household that can afford the down payment on a starter home buys it. Thus the equilibrium price of starter homes must be such that the number of first-time buyers who can afford the down payment equals the number of homeowners who are leaving the housing market. This establishes a direct link between the price of starter homes and the wealth of the youngest households.

A large proportion of households in our model have sufficient wealth to choose their home without any concern for the down-payment constraint. Changes in the price of starter homes shift their demand for trade-up homes in the obvious way. This establishes a first link between the price of starter homes (and hence the ability of younger households to afford down payments) and the price of trade-up homes.

There is a second way the price of starter homes affects the demand for trade-up homes: capital gains. In response to an income increase, gains on starter homes make their owners more able to afford the down payment on a trade-up home. To the extent that some of the owners of starter homes are keen to move up the property ladder, this again shifts the demand for trade-up homes upward. Once income stabilizes, gains on starter homes disappear and the demand for trade-up homes shifts back down. The symmetric reasoning applies for a negative change in incomes. The capital gains mechanism can explain both why housing prices may display overshooting and why they depend on past incomes and prices; see, for example, Case and Shiller (1989), Meese and Wallace (1994), Cho (1996) and Capozza et al. (2002).

In the case of an income increase, gains on starter homes may affect enough owners that the price of trade-up homes will overreact to the change in income, i.e., increase at a higher rate than income. We show that whenever the capital gains channel is strong enough to cause this overreaction in the price of trade-up homes, moves up the property ladder by owners of starter homes generate an overall increase in transactions. The symmetric result obtains for an income decrease.

In summary, if the effect of capital gains or losses on the housing demand of constrained repeat buyers is strong enough to generate price overreaction, the level of prices, the cross-sectional variance of prices and the number of transactions move with income. This is likely to happen when: (1) few first-time buyers of starter homes pay cash for them, (2) there are few first-time purchases of trade-up homes, and (3) changes in the relative price of homes

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have only a limited impact on the willingness of unconstrained households to move along the property ladder.

Our approach is different from the traditional approach to modelling housing prices, which treats homes like any other financial asset. Poterba (1991), for example, assumes that a home provides units of housing services whose price is forward-looking, determined by arbitrage in relation to other financial assets. This approach cannot account for the observed time series properties of housing prices without some form of departure from rationality; see Wheaton (1999). By design, this approach is silent with regard to fluctuations in transactions and assumes constant relative prices for homes of different sizes.

The search for alternative frameworks to rationalize housing market dynamics has taken primarily two directions. One line of research focuses on the search and matching features of the housing market; see, for example, Arnott (1989), Wheaton (1990), Williams (1995), Krainer (2001) and Krainer and LeRoy (2002). This approach has yielded a number of insights that we see as complementary to our findings.[1]

A second line of research focuses on the role of credit market imperfections and households' consumption demand for housing. Our work is closest to that of Stein (1995) in its focus on down-payment constraints but differs in terms of model design and predictions. Stein's static model demonstrates how extreme credit distress may result in lower housing prices and fewer transactions because negative equity prevents some households from moving. Assuming many households have too much debt to meet the down-payment requirement on their current homes, he finds a lower equilibrium housing price than if there were no down-payment constraint. The lower the price of housing, the more households that find themselves with too much debt to move, hence the fewer transactions.

None of our results require such extreme joint distributions of housing and debt, and our reasoning applies to booms as well as busts. In our dynamic framework, transactions arise out of changes in the equilibrium allocation of properties from one period to the next; the allocation of debt at the start of each period is the result of households' optimizing behaviour in the period before. Furthermore, our model incorporates first-time buyers, and we allow the relative prices of homes of different sizes to fluctuate.

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[1]Genesove and Mayer (2001) add another factor that could amplify the depth of housing market downturns. They provide evidence that nominal loss aversion on the part of sellers may contribute to depressing housing transactions when prices decline. See also Engelhardt (2003).

2 Empirical Evidence

We describe evidence on the relation between housing prices and the income of young households, and between housing prices and transactions. We then draw on the empirical literature to provide a background for our modelling choices.

2.1 Income of the young

Many empirical studies identify income as one of the drivers of housing prices; see, for example, Poterba (1991), Englund and Ioannides (1997), Muellbauer and Murphy (1997), Malpezzi (1999) and Sutton (2002). Researchers usually rely on average income measures such as per capita disposable income. These average measure are meant to capture the fact that when households are richer, they demand more of everything and thus more housing. We abstract from this effect of income on housing demand. In our model, income drives housing demand through a different and complementary mechanism: Changes in income affect a household's housing demand whenever they free the household from a binding credit constraint. Most broadly, the model suggests that any factor that affects the housing demand from potential first-time buyers has a direct impact on the housing market.

Figure 1 graphs real U.S. housing prices, per capita disposable income and the median income of 25- to 34-year-old households, the time series closest to the income of the marginal first-time buyers in our model.[2] A simple linear regression using the yearly data from 1970 to 2003 confirms the visual impression; both variables have positive and highly significant effects on housing prices.[3]

In Ortalo-Magné and Rady (1999), we document that down-payment requirements in England and Wales dropped from 25% to 15% following the credit market liberalization of the early 1980s. In response to such a change, the price of starter homes would rise by 66% in our model because, with the same savings, first-time buyers could afford the down payment on a home 66% more expensive ($0.66 = 0.25/0.15 - 1$). The income of young households in England and Wales grew by 27.5% over 1982-89.[4] When we combine the effects of this credit market liberalization and the growth in income, our model accounts for the 88% housing price growth over the period.

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[2]Data sources: FHLMC national conventional mortgage home price index, disposable personal income from the U.S. Bureau of Economic Analysis, and median 25- to 34-year-old household income from the U.S. Census Bureau. All variables are converted into real terms with the deflator used by the B.E.A. to deflate the disposable personal income series.

[3]The estimated coefficients for income per capita and the median income of 25- to 34-year-old households are 0.50 (0.06) and 0.78 (0.18), respectively; standard errors in parentheses. Together, the two income variables explain 90 percent of housing price variations.

[4]According to Family Expenditure Survey and U.K. Economic Accounts, as compiled by Simons (1996).
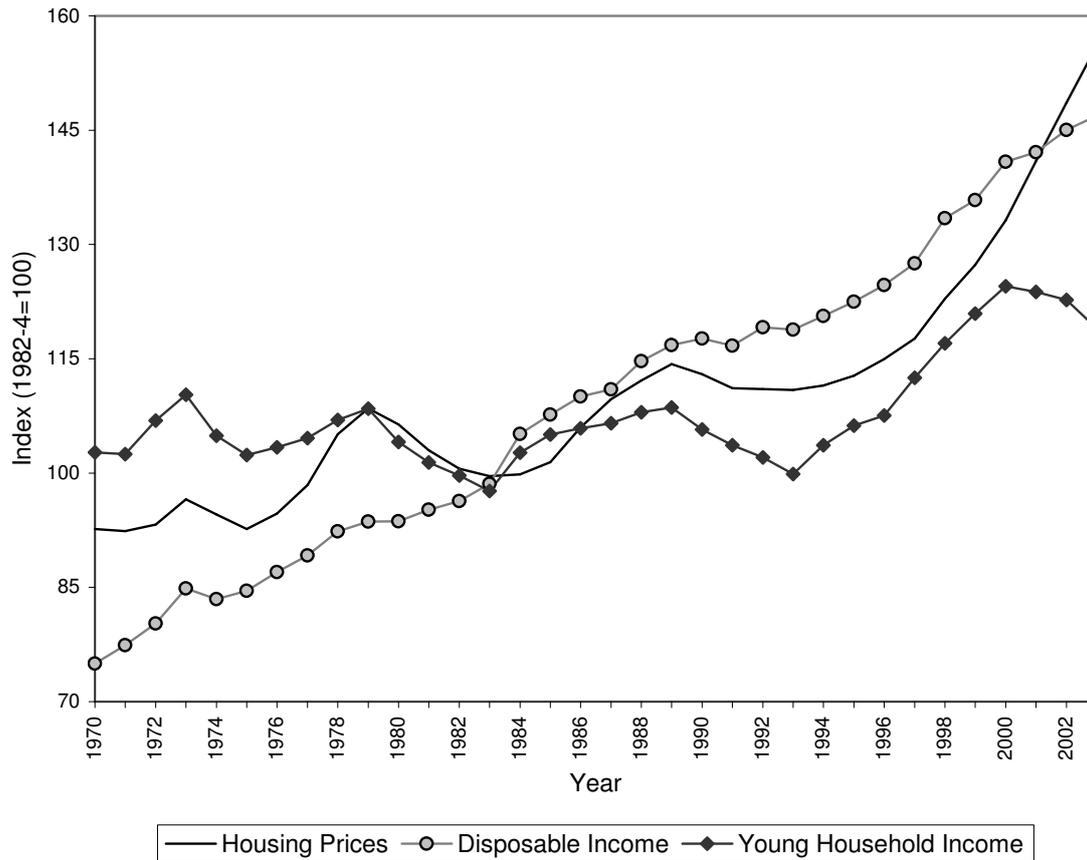
Figure 1: Real US housing prices and incomes

With regards to the subsequent housing market bust in England and Wales, Andrew and Meen (2003b) find that changes in the income of young cohorts were a critical factor of the declines in both transactions and prices.

2.2 Transactions

Stein (1995) provides evidence of a positive relationship between the number of transactions and the percentage of change in prices in the U.S. over the period 1968-1992. Berkovec and Goodman (1996) find a positive relationship between the percent changes in transactions and prices over 1968-1993. Follain and Velz (1995), however, report a negative relationship between the number of transactions and the housing price level in a panel of 22 U.S. cities over 1986-1992.

With the benefit of an extra decade of data since these studies, we run the regressions reported in Stein (1995) using the same data source. We find a significant positive relationship between transactions and price changes.[5]

In Ortalo-Magné and Rady (2004), we find the same positive relationship in data from England and Wales over 1970-2002. Andrew and Meen (2003a) and Benito (2004) report similar results.[6]

Holmans (1995) and Benito (2004) find that fluctuations in the overall number of transactions are attributable mainly to fluctuations in the number of repeat buyers moving up the property ladder, which accords with our model.

2.3 Model background

Caplin et al. (1997, p. 31) argue "it is almost impossible for a household to buy a home without available liquid assets of at least 10% of the home's value." It is this effective wealth requirement that we want to capture with the credit constraint in our model.

Studies of household-level data confirm that credit constraints restrict the housing consumption of a significant proportion of households; see, for example, Ioannides (1989), Jones (1989), Zorn (1989), Duca and Rosenthal (1994), Engelhardt (1996), Haurin, Hendershott and Wachter (1996, 1997) and Engelhardt and Mayer (1998). Linneman and Wachter (1989) find that the down-payment requirement in U.S. mortgages restricts households' access to credit more than the income constraint that precludes monthly payments above a given fraction of income. Even when lending standards allow some households to buy property without much initial wealth, the poorest buyers cannot borrow beyond the liquidation value of their collateral. And buying a home without a reasonable down payment remains very expensive.

According to the Annual National Survey of Recent Home Buyers in Major Metropolitan Areas by the Chicago Title and Trust Company and the English Housing Survey, buyers' own savings and housing equity are by far the two major sources of funds for repeat buyers; down payments of first-time buyers come primarily from their savings. Engelhardt (1996)

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[5]For example, for the U.S. as a whole, using annual price and transaction data from the National Association of Realtors over 1968-2003, we obtain Transactions = 153686 % Price change + 122202.9 Time trend + 289121.2 with standard errors equal to 18990, 6232 and 208105, respectively. The adjusted R^2 is 0.92. The estimate for the effect of price changes is very similar to the one obtained by Stein: a 10 percent drop in prices is associated with a reduction of the number of transactions by about 1.5 million units relative to an average of 4 million transactions per year in the 1990s.

[6]Leung et al. (2002) report evidence of a positive relationship between housing prices and transactions for Hong Kong. Hort (2000) does not find any robust pattern in Swedish data. Bardhan et al. (2003) report evidence of a positive relationship between capital gains at the bottom of the property ladder and transactions at the top end of the ladder in Singapore.

reports that only one-fifth of U.S. first-time buyers receive some help from relatives in accumulating their down payment, and only 4 percent receive their whole down payment from relatives. The vast majority of first-time buyers pay down payments out of their own savings. Engelhardt and Mayer (1998) find that only 4 percent of repeat buyers receive help from family and friends for their down payments.

The life-cycle pattern of housing consumption of a significant proportion of households involves lumpy adjustments along the property ladder with jumps toward larger dwellings when buyers are young. Evidence from housing surveys both in the U.S. and the U.K. indicates that some households move to a more expensive property within a few years of their first purchase. Fernández-Villaverde and Krueger (2002) report evidence consistent with the view that households in the U.S. cannot move into their target home early in their life cycle because of financial constraints and that they work their way up the property ladder. Clark et al. (2003) provide support for this view. Households with the strongest income growth tend to climb the ladder progressively except for the richest households, who appear to move right away into a home at the upper end of the property ladder. Banks et al. (2002) find that households in the U.K. also tend to climb the property ladder progressively. They report that first-time buyers in the U.K. tend to be younger than first-time buyers in the U.S. They attribute this difference in part to the lower down-payment requirement in the U.K.

We assume housing preferences such that some households move up the ladder for preference reasons, and some down. There is some evidence that housing consumption declines with age for the elderly, but this remains a debated issue in the literature; see, for example, Mankiw and Weil (1989), Venti and Wise (1990, 1991, 2001), Green and Hendershott (1996), Jones (1997) and Megbolugbe et al. (1997). What is critical in our model is that in every period, *some* households move to a home cheaper than the one that they sell. This is supported by evidence in both the U.S. and the U.K.; cf. Statistical Abstract of the U.S. and English Housing Survey.

3 Model

The model must be rich enough to capture the interaction of households eager to climb the property ladder but credit constrained, and wealthier households who choose their home according to preferences. The concept of a property ladder requires at least two types of dwelling. Climbing this minimal property ladder requires at least three periods: one period to buy, one to trade up, and one to sell. For income shocks to affect the pace at which some agents climb the property ladder, agents must differ in terms of wealth.

There are many options available to characterize agents who trade without restrictions imposed by wealth. We choose to add an extra period of life, a period when wealth is high enough so that credit constraints no longer bind, no matter the type of property. Of course, if not all wealthy agents are to hold the same dwelling, they must have heterogeneous preferences. The challenge is to design a model that incorporates this double-heterogeneity of wealth and preferences and still permits a tractable determination of equilibrium prices and transaction volume.

3.1 Economic environment

Population. A measure one of agents is born at the start of each period. Each agent lives for four periods, so the total population always has measure four. Within each cohort, agents are distributed uniformly over the unit square. Each agent is identified by the indices $(i, m) \in [0, 1] \times [0, 1]$ which determine her endowment stream and her preference for houses relative to flats at age 4, respectively. While all agents learn their index i at the beginning of life, they learn their index m at age 3 only.

Commodities. The commodities are a numeraire consumption good and two types of dwellings: starter homes, called "flats" hereafter, and trade-up homes, called "houses." Each dwelling can accommodate only one agent, who must be an owner-occupier. Thus, the set of possible housing choices is $\mathcal{H} = \{\emptyset, F, H\}$, where $\emptyset$ stands for no housing consumption, F for a flat, and H for a house.

Endowments. Agents are born without any initial wealth. At age $j = 1, \ldots, 4$, agent (i, m) receives an endowment of $e_j(i)$ units of the numeraire good, where the functions $e_j : [0, 1] \to I\!\!R_+$ are continuous and strictly increasing.

Preferences. The preferences of agent (i, m) are described by the utility function

$$\sum_{j=1}^{4} c_j + U(h_2, \tfrac{1}{2}) + U(h_3, \tfrac{1}{2}) + U(h_4, m),$$

where c_j is the non-negative amount of the numeraire good consumed in the jth period of life; $h_j \in \mathcal{H}$ is the type of housing enjoyed at the beginning of the jth period of life; and the utility of housing is given by

$$U(h, m) = \begin{cases} -\Delta & \text{if} \quad h = \emptyset, \\ 0 & \text{if} \quad h = F, \\ u(m) & \text{if} \quad h = H \end{cases}$$

where $\Delta > 0$, and $u \colon [0, 1] \to I\!\!R$ is strictly increasing and continuous. Thus, all agents have the same utility premium Δ for a flat relative to no housing. Up to age 3, all agents' utility premium for a house relative to a flat is $u(\tfrac{1}{2})$. At age 3, they learn their preference index

m before they trade in the housing market. Subsequently, all agents with preference index m have the utility premium $u(m)$ for a house relative to a flat.

Technology. Flats and houses are in fixed supply at measures S_F and S_H, respectively. Agents have access to a storage technology for the numeraire good that allows them to save at the exogenously given rate of interest r.

Credit constraints. Agents are allowed to borrow at the rate of interest r, but face a borrowing constraint. An agent's end-of-period non-housing wealth is not allowed to fall below $\gamma - 1$ times the value of any property he owns, where $0 < \gamma < 1$. This constraint implies that in order to acquire property $h \in \{F, H\}$, an agent must have a total wealth of at least γ times the price of the property. We will refer to this amount as the required "down payment."

Markets. In each period, there are competitive markets for flats and houses, with prices denoted p_F and p_H. We use the notation $p_\emptyset$ for the "price" of the no-housing option; by definition, $p_\emptyset = 0$ at all times. There are no rental markets for dwellings and no other asset markets.

Timing. Within each period, agents first derive utility from housing. Second, they receive their endowment of the numeraire good; third, they trade in the housing market; and fourth, they consume the numeraire good.

3.2 Comments

The age 1 and age 2 cohorts represent households whose housing consumption may be limited by their wealth because of the credit constraint. First-time buyers build their down payments solely from their own endowments; if need be, repeat buyers build their down payments from their own endowments and any gains on their first purchase.

The age 3 cohort represents households whose housing choices are not restricted by the credit constraint. The shock at age 3 to a household's relative preference for houses and flats (that is, the revelation of the preference index m) implies that not all unconstrained households attach the same utility premium to houses relative to flats and that some households move due to preference reasons at age 3. The direction of these moves can be up the property ladder as well as down.

The model allows for an arbitrary continuous income distribution within each cohort, but requires the ranking of agents according to income to be invariant over the life cycle for tractability.

The linear utility of non-housing consumption will imply that all such consumption is postponed until the last period of life. This feature keeps the model analytically tractable,

particularly with respect to the equilibrium law of motion of the distribution of dwellings and savings.

We assume a fixed rate of return r on the storage technology in order to capture a small open economy where the interest rate is set exogenously. We abstract from the effects of shocks to interest rates (or equity returns) on the demand for housing.[7]

The assumption of a perfectly inelastic supply of flats and houses is not critical to our results; the results obtain as long as supply is not perfectly elastic, which will hold as long as the supply of land is upward sloping. Of course, everything else equal, prices would respond less to changes in endowments, the higher the price elasticity of supply.

3.3 Parameter assumptions

We impose assumptions on the parameters of the model so that the steady-state equilibrium of the model captures the interaction of the three groups of agents: constrained first-time buyers, constrained repeat buyers, and unconstrained repeat buyers.

First, the supplies of flats and houses are assumed to satisfy

$$\frac{5}{2} < S_F + S_H < 3 \quad \text{and} \quad \frac{1}{2} < S_H < 1. \tag{1}$$

The first condition implies that not all households can own property. The second condition implies that within each cohort there must be some households that do not own a house.

Second, we assume that the down payment is greater than the discounted user cost of a property when its price is constant; i.e.,

$$\gamma > \frac{r}{1+r}. \tag{2}$$

This ensures that a household who can afford the down payment on a property can also afford to live in this property for one period when prices are constant.

Third, the endowment profiles $e_j : [0,1] \to I\!R_+$ $(j = 1, \ldots, 3)$ are assumed to satisfy:

$$
\begin{align}
e_1(0) &= 0, \tag{3}\\
e_2(0) &> e_1(3 - S_F - S_H), \tag{4}\\
e_2(i) &> e_1(i) \quad \text{for all} \ \ i \in [0,1], \tag{5}\\
e_3(0) &> e(1), \tag{6}\\
e(1) &> \max\{e_1(1), e(3 - S_F - S_H)\} + r\gamma^{-1}e_1(3 - S_F - S_H), \tag{7}
\end{align}
$$

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[7]Flavin and Yamashita (2002), Flavin and Nakagawa (2003), Cocco (forthcoming) and Yao and Zhang (forthcoming) study the interaction between housing demand and investments in other assets. Housing returns are set exogenously in these models. Lustig and Van Nieuwerburgh (2004) analyze the impact of housing price fluctuations on the pricing of stocks in a model where housing serves as collateral.

where

$$e(i) = (1 + r) e_1(i) + e_2(i)$$

denotes the accumulated value of the endowments agents receive at ages 1 and 2.

Assumption (3) guarantees that some age 1 households cannot pay the down payment on any dwelling. Thanks to assumption (4), however, all age 2 households will be able to afford the down payment on a flat in steady state.

Assumption (5) specifies that households earn more at age 2 than at age 1. Combined with assumption (2), this ensures that at constant housing prices, a household that can pay the down payment on a given dwelling at age 1 can do so again at age 2, taking into account the cost of holding that property from one period to the next.[8]

Assumption (6) gives age 3 households an endowment large enough to render both housing alternatives affordable in the sense that no age 3 household faces a binding credit constraint.

Assumption (7) will ensure that in steady-state equilibrium, some households trade up from a flat to a house at age 2; this will allow capitals gains on flats to have an effect on transitional dynamics.

Our last set of assumptions concern the parameters of the agents' utility functions:

$$0 \; > \; u(1 - S_H), \tag{8}$$
$$u(\tfrac{1}{2}) \; > \; (1+r)^2 r\gamma^{-1} \left[e(1) - e_1(3 - S_F - S_H) \right], \tag{9}$$
$$\Delta \; > \; u(\tfrac{1}{2}) + (1+r)^2 r\gamma^{-1} e_1(3 - S_F - S_H). \tag{10}$$

Assumption (8) ensures that not all houses are held by agents of age 3 when housing prices are constant. Assumptions (9) and (10) will ensure that in steady state, each agent of age 2, 3 or 4 achieves a better trade-off between a dwelling's utility and its effective cost when they own a flat rather than no property, and all members of cohorts 2 and 3 achieve a still better trade-off when they own a house.

In fact, the right-hand side of (9) will turn out to be an upper bound, at steady-state prices, on the cost of choosing a house rather than a flat at age 1, 2, or 3, evaluated in terms of numeraire consumption at age 4. Similarly, $(1 + r)^2 r\gamma^{-1} e_1(3 - S_F - S_H)$ is an upper bound on the cost of acquiring a flat, evaluated in terms of numeraire consumption at age 4.

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[8] Formally, under (2) and (5), $e_1(i) \geq \gamma p_h$ implies $e(i) - r p_h > (2+r)\gamma p_h - r p_h = \gamma p_h + [(1+r)\gamma - r] p_h > \gamma p_h$.

4 Recursive Equilibrium

The state of the economy at the beginning of a period is given by the collection of distribution functions $\mathbf{x} = (M_{\emptyset,2}, M_{F,2}, M_{H,2}, M_{\emptyset,3}, M_{F,3}, M_{H,3}, M_{\emptyset,4}, M_{F,4}, M_{H,4})$ defined on $[0,1] \times \mathbb{R}$, where $M_{h,j}(i,w)$ is the measure of households of age j who own a property of type h, and have an endowment index lower than or equal to i and non-housing wealth lower than or equal to w. We do not need to include cohort 1 here as it is born without any wealth or property.

The state of an individual household of age 1 is simply given by its endowment index i. At age 2, the state of a household further comprises the dwelling h and the non-housing wealth w with which it enters the given period. At ages 3 and 4, the preference index m also becomes part of the state of the household.

In a recursive equilibrium, prices are deterministic time-invariant functions of the state of the economy. This implies that household decisions depend on their individual state variables and, through prices, on the state of the economy as a whole.

Definition 1. A recursive competitive equilibrium consists of

- a state space X, that is, a set of states $\mathbf{x}$ as defined above,

- decision rules $c_1(i,\mathbf{x})$, $c_2(i,h,w,\mathbf{x})$, $c_3(i,m,h,w,\mathbf{x})$ and $c_4(i,m,h,w,\mathbf{x})$ for numeraire consumption,

- decision rules $h_1(i,\mathbf{x})$, $h_2(i,h,w,\mathbf{x})$, $h_3(i,m,h,w,\mathbf{x})$ and $h_4(i,m,h,w,\mathbf{x})$ for housing purchases,

- decision rules $w_1(i,\mathbf{x})$, $w_2(i,h,w,\mathbf{x})$, $w_3(i,m,h,w,\mathbf{x})$ and $w_4(i,m,h,w,\mathbf{x})$ for next-period non-housing wealth,

- value functions $v_1(i,\mathbf{x})$, $v_2(i,h,w,\mathbf{x})$, $v_3(i,m,h,w,\mathbf{x})$ and $v_4(i,m,h,w,\mathbf{x})$,

- property price functions $p_F(\mathbf{x})$ and $p_H(\mathbf{x})$, and

- a law of motion $\mathbf{x}' = \phi(\mathbf{x})$ for the state of the economy

such that the following conditions hold:

(a) Given the law of motion for the state of the economy and the property price functions, the decision rules solve agents' maximization problems and generate the respective value functions. That is, for all $\mathbf{x} \in X$, and with $\Gamma_j(i,h,w,\mathbf{x})$ denoting the set of all

$(c, h', w') \in I\!R_+ \times \mathcal{H} \times I\!R$ such that:

$$c + p_{h'}(\mathbf{x}) + \frac{w'}{1+r} \;\leq\; e_j(i) + p_h(\mathbf{x}) + w, \qquad (11)$$

$$\frac{w'}{1+r} \;\geq\; (\gamma - 1)p_{h'}(\mathbf{x}), \qquad (12)$$

(a.1) $c = c_1(i, \mathbf{x})$, $h' = h_1(i, \mathbf{x})$ and $w' = w_1(i, \mathbf{x})$ solve

$$v_1(i, \mathbf{x}) \;=\; \max_{(c, h', w') \in \Gamma_1(i, \emptyset, 0, \mathbf{x})} \left\{ c + v_2(i, h', w', \phi(\mathbf{x})) \right\};$$

(a.2) $c = c_2(i, h, w, \mathbf{x})$, $h' = h_2(i, h, w, \mathbf{x})$ and $w' = w_2(i, h, w, \mathbf{x})$ solve

$$v_2(i, h, w, \mathbf{x}) \;=\; \max_{(c, h', w') \in \Gamma_2(i, h, w, \mathbf{x})} \left\{ c + U(h, \tfrac{1}{2}) + \int_0^1 v_3(i, m, h', w', \phi(\mathbf{x})) \, dm \right\};$$

(a.3) $c = c_3(i, m, h, w, \mathbf{x})$, $h' = h_3(i, m, h, w, \mathbf{x})$ and $w' = w_3(i, m, h, w, \mathbf{x})$ solve

$$v_3(i, m, h, w, \mathbf{x}) \;=\; \max_{(c, h', w') \in \Gamma_3(i, h, w, \mathbf{x})} \left\{ c + U(h, \tfrac{1}{2}) + v_4(i, m, h', w', \phi(\mathbf{x})) \right\};$$

(a.4) $c = c_4(i, m, h, w, \mathbf{x})$, $h' = h_4(i, m, h, w, \mathbf{x})$ and $w' = w_4(i, m, h, w, \mathbf{x})$ solve

$$v_4(i, m, h, w, \mathbf{x}) \;=\; \max_{(c, h', w') \in \Gamma_4(i, h, w, \mathbf{x})} \left\{ c + U(h, m) \right\};$$

where it is understood that the right-hand side equals $-\infty$ if $\Gamma_4(i, h, w, \mathbf{x})$ is empty.[9]

(b) Housing markets clear. That is, for all $\mathbf{x} \in X$:

$$S_F \;=\; \int_0^1 1_{h_1(i, \mathbf{x}) = F} \, di + \sum_{h \in \mathcal{H}} \int_{[0,1] \times I\!R} 1_{h_2(i, h, w, \mathbf{x}) = F} \, dM_{h,2}(i, w)$$

$$+ \sum_{h \in \mathcal{H}} \int_0^1 \int_{[0,1] \times I\!R} 1_{h_3(i, m, h, w, \mathbf{x}) = F} \, dM_{h,3}(i, w) \, dm, \qquad (13)$$

$$S_H \;=\; \int_0^1 1_{h_1(i, \mathbf{x}) = H} \, di + \sum_{h \in \mathcal{H}} \int_{[0,1] \times I\!R} 1_{h_2(i, h, w, \mathbf{x}) = H} \, dM_{h,2}(i, w)$$

$$+ \sum_{h \in \mathcal{H}} \int_0^1 \int_{[0,1] \times I\!R} 1_{h_3(i, m, h, w, \mathbf{x}) = H} \, dM_{h,3}(i, w) \, dm, \qquad (14)$$

where 1_ξ is the usual indicator function for statement ξ.

---

[9]Clearly, $h_4(i, m, h, w, \mathbf{x}) = \emptyset$ and $w_4(i, m, h, w, \mathbf{x}) = 0$ are optimal. This is taken into account in the formulation of the market clearing conditions in (b).

(c) The law of motion of the state of the economy is generated by agents' decision rules. That is:

$$
\phi \begin{pmatrix} M_{\emptyset,2} \\ M_{F,2} \\ M_{H,2} \\ M_{\emptyset,3} \\ M_{F,3} \\ M_{H,3} \\ M_{\emptyset,4} \\ M_{F,4} \\ M_{H,4} \end{pmatrix} (i,w) = \begin{pmatrix} \int_0^i 1_{h_1(y,\mathbf{x})=\emptyset} 1_{w_1(y,\mathbf{x}) \leq w} \, dy \\ \int_0^i 1_{h_1(y,\mathbf{x})=F} 1_{w_1(y,\mathbf{x}) \leq w} \, dy \\ \int_0^i 1_{h_1(y,\mathbf{x})=H} 1_{w_1(y,\mathbf{x}) \leq w} \, dy \\ \sum_{h \in \mathcal{H}} \int_{[0,i] \times \mathbb{R}} 1_{h_2(y,h,z,\mathbf{x})=\emptyset} \, 1_{w_2(y,h,z,\mathbf{x}) \leq w} \, dM_{h,2}(y,z) \\ \sum_{h \in \mathcal{H}} \int_{[0,i] \times \mathbb{R}} 1_{h_2(y,h,z,\mathbf{x})=F} \, 1_{w_2(y,h,z,\mathbf{x}) \leq w} \, dM_{h,2}(y,z) \\ \sum_{h \in \mathcal{H}} \int_{[0,i] \times \mathbb{R}} 1_{h_2(y,h,z,\mathbf{x})=H} \, 1_{w_2(y,h,z,\mathbf{x}) \leq w} \, dM_{h,2}(y,z) \\ \sum_{h \in \mathcal{H}} \int_0^1 \int_{[0,i] \times \mathbb{R}} 1_{h_3(y,m,h,z,\mathbf{x})=\emptyset} \, 1_{w_3(y,m,h,z,\mathbf{x}) \leq w} \, dM_{h,3}(y,z) \, dm \\ \sum_{h \in \mathcal{H}} \int_0^1 \int_{[0,i] \times \mathbb{R}} 1_{h_3(y,m,h,z,\mathbf{x})=F} \, 1_{w_3(y,m,h,z,\mathbf{x}) \leq w} \, dM_{h,3}(y,z) \, dm \\ \sum_{h \in \mathcal{H}} \int_0^1 \int_{[0,i] \times \mathbb{R}} 1_{h_3(y,m,h,z,\mathbf{x})=H} \, 1_{w_3(y,m,h,z,\mathbf{x}) \leq w} \, dM_{h,3}(y,z) \, dm \end{pmatrix}.
$$

5 Steady-State Equilibrium

Definition 2. A steady-state equilibrium is a recursive competitive equilibrium with a singleton state space $\mathrm{X} = \{\mathbf{x}^*\}$.

In particular, every fixed point of the law of motion of a recursive competitive equilibrium gives rise to a steady-state equilibrium.

In the remainder of the paper, we adopt the following convention. Given a continuous and strictly increasing function $f : [0,1] \to \mathbb{R}$, we set $f^{-1}(x) = 1$ for $x > f(1)$, and $f^{-1}(x) = 0$ for $x < f(0)$.

Proposition 1 *There is a unique steady-state equilibrium. The price of flats in this equilibrium is*

$$
p_F^* = \gamma^{-1} e_1 (3 - S_F - S_H). \tag{15}
$$

The price of houses, p_H^*, solves

$$
\begin{aligned}
3 - S_H &= e_1^{-1}(\gamma p_H^*) + \min \left\{ e_1^{-1}(\gamma p_F^*), e^{-1}(\gamma p_H^*) \right\} \\
&\quad + e^{-1}(r p_F^* + \gamma p_H^*) - e_1^{-1}(\gamma p_F^*) + u^{-1}(r[p_H^* - p_F^*]).
\end{aligned} \tag{16}
$$

The steady-state allocation of properties at the beginning of each period is determined by the critical endowment indices

$$
\begin{aligned}
i_F^* &= e_1^{-1}(\gamma p_F^*) &&= 3 - S_F - S_H, & (17) \\
i_H^* &= e_1^{-1}(\gamma p_H^*), &&& (18) \\
i_{\emptyset H}^* &= e^{-1}(\gamma p_H^*), &&& (19) \\
i_{FH}^* &= e^{-1}(r p_F^* + \gamma p_H^*) &&& (20)
\end{aligned}
$$

and the critical preference index

$$m_H^* = u^{-1}(r[p_H^* - p_F^*]), \tag{21}$$

where $0 < i_F^* < \frac{1}{2} < i_{FH}^* < i_H^* \le 1$, $0 < i_{\emptyset H}^* < i_{FH}^*$, and $0 < m_H^* < \frac{1}{2}$:

- *Age 2 agents with endowment index $i < i_F^*$ hold no property; those with $i_F^* < i < i_H^*$ hold a flat; and those with $i > i_H^*$ hold a house.*

- *Age 3 agents with endowment index $i < \min\{i_F^*, i_{\emptyset H}^*\}$ or $i_F^* < i < i_{FH}^*$ hold a flat; those with $\min\{i_F^*, i_{\emptyset H}^*\} < i < i_F^*$ or $i > i_{FH}^*$ hold a house.*

- *Age 4 agents with preference index $m < m_H^*$ hold a flat; those with $m > m_H^*$ hold a house.*

The steady-state measures of flats and houses bought and sold each period are

$$n_F^* = i_H^* - i_F^* + \min\{i_F^*, i_{\emptyset H}^*\} + (i_F^* - \min\{i_F^*, i_{\emptyset H}^*\} + 1 - i_{FH}^*)\, m_H^*, \tag{22}$$

$$n_H^* = 1 + i_F^* - \min\{i_F^*, i_{\emptyset H}^*\} - i_{FH}^* + (\min\{i_F^*, i_{\emptyset H}^*\} + i_{FH}^* - i_F^*)\,(1 - m_H^*). \tag{23}$$

The critical endowment and preference indices identified in Proposition 1 are sufficient to compute the state of the economy $\mathbf{x}^*$ associated with the steady-state equilibrium. First, as all non-housing consumption is postponed to age 4, a household's state variable w at ages 2, 3 and 4 is determined fully by the history of endowments and housing choices. Second, the critical indices are enough to determine almost every household's history of housing choices from its endowment index i and preference index m.[10]

Figure 2 depicts the steady-state allocation of properties to households at the beginning of any period. The endowment index i increases from 0 to 1 as we move right. The preference index m increases as we move up. This allocation is such that households complete up to three housing transactions over the life cycle. Some buy their first property at age 1 ($i \ge i_F^*$), and all others at age 2. Repeat buyers are either of age 2 ($i \ge i_{FH}^*$) or age 3 ($i \ge i_{FH}^*$ and $m < m_H^*$, or $i < i_{FH}^*$ and $m > m_H^*$). Some households purchase only one home over the course of their lives (for example, $i_F^* \le i < i_{FH}^*$ and $m < m_H^*$), some purchase three ($i_{FH}^* \le i < i_H^*$ and $m < m_H^*$), all others two.

These patterns of life-cycle behaviour match the empirical observations in both the U.S. and the U.K. That is, (1) first-time buyers tend to be younger than repeat buyers; (2) first-time buyers tend to buy cheaper properties than repeat buyers; (3) some households move to a more expensive property within a few years of their first purchase; and (4) some

---

[10]The allocation of dwellings to the null set of age 4 households with preference index $m = m^H$ is arbitrary.
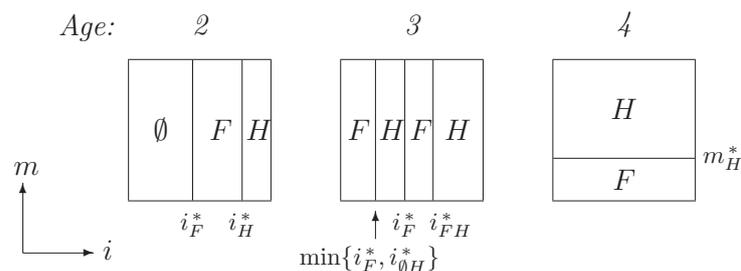
Figure 2: Steady-state equilibrium allocation of properties

households move to a home cheaper than the one they sell. Computing the measure of households that buy flats and houses in each cohort and adding up across cohorts yields the expressions (22)–(23) for the steady-state transaction volumes.

Proposition 1 allows for $i_H^* = 1$ or $i_{\emptyset H} \geq i_F^*$, both leading to a simpler allocation of properties than depicted in Figure 2. If $i_H^* = 1$, there are no house purchases at age 1; if $i_{\emptyset H} \geq i_F^*$, there are no first-time purchases of houses at age 2. As $i_F^* < i_{FH}^* < 1$, however, there are always some house purchases by households that owned a flat before. Given the down-payment constraint, this creates a channel for capital gains or losses on flats to affect the transition dynamics of our model economy.

The proof of Proposition 1 establishes that three statements must hold in any steady-state equilibrium: (1) When weighing housing utility against user costs, all age 1 and 2 households find it optimal to acquire as much housing as possible, given their current wealth and the down-payment constraint; (2) all age 2 households can afford the down payment on a flat; and (3) when weighing housing utility against user costs, all age 3 households find it optimal to acquire either a flat or a house, and in this choice they are not restricted by the down-payment constraint.

These statements imply that the households that do not acquire any property must be the poorest households in the economy, and hence households of age 1. Since there must be a measure $3 - S_H - S_F$ of them by market clearing, the steady-state price of flats must be such that the households of age 1 with endowment index $3 - S_H - S_F$ are just able to afford the down payment on a flat. This yields the steady-state flat price p_F^* defined in (15).

The three statements also imply that in any steady-state equilibrium, housing purchases at age 1 and 2 are driven entirely by the credit constraint, while housing purchases at age 3 are determined entirely by preferences. This is why the beginning-of-period steady-state allocation of properties to cohorts 2 and 3 is determined entirely by the endowment indices i_F and i_H of the agents who are just able to finance a flat or a house, respectively, at age 1 and by the endowment indices $i_{\emptyset H}$ and i_{FH} of the agents who are just able to finance a house

16

at age 2, having acquired no property or a flat, respectively, at age 1. The allocation of properties to cohort 4 is determined entirely by the preference index m_H of the households who are indifferent between holding a flat and holding a house, given the steady-state user costs of each.

Using these critical indices to compute the total demand for houses and imposing market clearing yields equation (16) for the steady-state price of houses. Lemma A.1 in the Appendix shows that this equation has a unique solution p_H^* and in particular that $p_H^* > p_F^*$. It is then straightforward to verify that the prices p_F^* and p_H^* together with the housing decisions derived from the critical indices (17)–(21) give rise to a unique steady-state equilibrium.

To interpret equation (16), note that its left-hand side is the equilibrium measure of agents of age 1 to 3 who do not acquire a house. On the right-hand side, e_1^{-1} is the distribution function of age 1 endowments; e^{-1} is the distribution function of accumulated age 1 and 2 endowments; and u^{-1} is the distribution function of utility premiums of houses relative to flats after age 3.

The first term on the right-hand side is thus the measure of age 1 agents who cannot afford the down payment on a house. The second term is the measure of age 2 agents who at age 1 could not afford the down payment on a flat and now cannot afford the down payment on a house. The difference $e^{-1}(rp_F^* + \gamma p_H^*) - e_1^{-1}(\gamma p_F^*)$, which is shown to be positive in Lemma A.1, is the measure of age 2 agents who at age 1 could afford the down payment on a flat but, having held the flat from the last until the current period and having incurred the user cost rp_F^*, cannot afford the down payment on a house now. The last term on the right-hand side of (16) is the measure of age 3 agents who, at constant prices p_F^* and p_H^*, prefer a flat to a house.

6   Permanent Changes in Endowments

Starting from the steady state described in the previous section, we want to investigate the dynamic response of equilibrium prices and numbers of transactions to a small unanticipated permanent change in agents' endowments. We focus on proportional changes that multiply each agent's endowment by the same factor. We first discuss how steady-state prices change with endowments. Then, we establish that for sufficiently small changes in endowments, there is a recursive competitive equilibrium that reaches the new steady state within five periods, with prices and the housing allocation settling down within two periods.

6.1 Comparison of steady states

Consider endowment profiles $\{ze_j\}_{j=1}^4$ with a constant $z > 0$ such that all the conditions set out in Section 3.3 continue to hold. By Proposition 1, there is again a unique steady-state equilibrium, featuring an allocation of properties as in Figure 2. We denote variables pertaining to this new steady state by the superscript $**$. Thus, we have $p_F^{**} = \gamma^{-1} z e_1 (3 - S_F - S_H)$ by the analogue of equation (15), while the price of houses p_H^{**} is uniquely determined by the analogue of equation (16):

$$
\begin{aligned}
3 - S_H &= e_1^{-1}(z^{-1}\gamma p_H^{**}) + \min\left\{e_1^{-1}(z^{-1}\gamma p_H^{**}), e^{-1}(z^{-1}\gamma p_H^{**})\right\} \\
&\quad + e^{-1}(z^{-1}[rp_F^{**} + \gamma p_H^{**}]) - e_1^{-1}(z^{-1}\gamma p_F^{**}) + u^{-1}(r[p_H^{**} - p_F^{**}]). \quad (24)
\end{aligned}
$$

The corresponding critical endowment and preference indices are

$$
\begin{aligned}
i_F^{**} &= e_1^{-1}(z^{-1}\gamma p_F^{**}) = 3 - S_F - S_H, & (25) \\
i_H^{**} &= e_1^{-1}(z^{-1}\gamma p_H^{**}), & (26) \\
i_{\emptyset H}^{**} &= e^{-1}(z^{-1}\gamma p_H^{**}), & (27) \\
i_{FH}^{**} &= e^{-1}(z^{-1}[rp_F^{**} + \gamma p_H^{**}]), & (28) \\
m_H^{**} &= u^{-1}(r[p_H^{**} - p_F^{**}]). & (29)
\end{aligned}
$$

The following proposition compares steady-state prices:

Proposition 2 *The steady-state price of flats is proportional to endowments:*

$$
p_F^{**} = z p_F^*.
$$

The steady-state price of houses changes less than proportionally with endowments, but more, in absolute terms, than the price of flats:

$$
\begin{aligned}
p_H^* + (z-1)p_F^* < p_H^{**} < z p_H^* \quad \text{if} \quad z > 1, \\
z p_H^* < p_H^{**} < p_H^* + (z-1)p_F^* \quad \text{if} \quad z < 1.
\end{aligned}
$$

Proportionality of the steady-state price of flats to z follows because this price is proportional to the endowment of the age 1 households with endowment index $3 - S_F - S_H$. If the steady-state price of houses were to change proportionally to z as well, we would see unchanged demand for houses by households of age 1 and 2, but, because of a proportional change in the user cost difference between houses and flats, a decline (for $z > 1$) or increase (for $z < 1$) in the demand by age 3 households. If the steady-state price of houses were to change by the same amount as the steady-state price of flats, keeping the user cost difference unchanged, we would see unchanged demand for houses by age 3 households, but an

increase (for $z > 1$) or decline (for $z < 1$) in the demand by households of age 1 and 2. As both these scenarios are incompatible with market clearing in the new steady state, we obtain the second part of Proposition 2.

6.2 Transition to the new steady state

Let p_F^* and $\mathbf{x}^*$ be the price of flats and the state of the economy in the steady-state equilibrium for endowment profiles $\{e_j\}_{j=1}^4$. Take this state as the initial condition of an economy with endowment profiles $\{ze_j\}_{j=1}^4$. For z sufficiently close to 1, we construct a recursive competitive equilibrium that reaches the steady state for endowment profiles $\{ze_j\}_{j=1}^4$ within five periods, with housing prices and the housing allocation settling down within two periods. As before, we write $\mathbf{x}^{**}$, p_F^{**} and p_H^{**} for the state and the property prices in the steady-state equilibrium for endowment profiles $\{ze_j\}_{j=1}^4$.

Proposition 3 *For z sufficiently close to 1, an economy with endowment profiles $\{ze_j\}_{j=1}^4$ and initial condition $\mathbf{x}^*$ admits a recursive competitive equilibrium with*

- *law of motion $\phi(\mathbf{x}^*) = \mathbf{x}^1$, $\phi(\mathbf{x}^1) = \mathbf{x}^2$, $\phi(\mathbf{x}^2) = \mathbf{x}^3$, $\phi(\mathbf{x}^3) = \mathbf{x}^{**}$ and $\phi(\mathbf{x}^{**}) = \mathbf{x}^{**}$,*

- *flat prices $p_F(\mathbf{x}) = p_F^{**} = zp_F^*$ for all $\mathbf{x} \in \{\mathbf{x}^*, \mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \mathbf{x}^{**}\}$,*

- *house prices $p_H(\mathbf{x}^*) = p_H^+$ and $p_H(\mathbf{x}) = p_H^{**}$ for all $\mathbf{x} \in \{\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \mathbf{x}^{**}\}$, where p_H^+ is the unique solution of*

$$
\begin{aligned}
3 - S_H &= e_1^{-1}(z^{-1}\gamma p_H^+) + \min\left\{e_1^{-1}(\gamma p_F^*), e_+^{-1}(\gamma p_H^+)\right\} \\
&\quad + e_+^{-1}((1+r)p_F^* - p_F^{**} + \gamma p_H^+) - e_1^{-1}(\gamma p_F^*) \\
&\quad + u^{-1}((1+r)p_H^+ - p_H^{**} - rp_F^{**}),
\end{aligned} \tag{30}
$$

and $e_+(i) = (1+r)\,e_1(i) + ze_2(i)$.

The idea behind the construction of this equilibrium is that for sufficiently small changes in endowments, the allocation of properties in all states along the transition should be of the same type as in the initial state $\mathbf{x}^*$; that is, allocations should be fully determined by critical endowment and preference indices as shown in Figure 2. Flat prices should continue to be determined through the requirement that a measure $3 - S_F - S_H$ of age 1 households are unable to acquire any property because of a binding credit constraint. This means that in all states along the transition, flat prices should be proportional to the endowment of the age 1 households with endowment index $3 - S_F - S_H$, and thus adjust within one period, changing proportionally with endowments.

With flat prices determined by contemporaneous age 1 endowments, the only lagged variables needed to compute the critical indices that determine the demand for houses are once-lagged age 1 endowments. These enter directly into $i_{\emptyset H}$, the endowment index of the households that are just able to acquire a house at age 2 as their first property. In addition, once-lagged age 1 endowments enter directly and through the lagged price of flats into i_{FH}, the endowment index of the households that are just able to move from a flat to a house at age 2. Thus, the price of houses and these critical indices should reach their new steady-state levels within two periods – the time it takes for once-lagged age 1 endowments to reach their new level.

It takes up to five periods for the state of the economy to settle on the new steady state because the price of houses at the time endowments change, $p_H(\mathbf{x}^*)$, is different from the new steady-state price of houses. If some households of age 1 buy a house at that price, the joint income and wealth distribution keeps adjusting until these households leave the economy four periods later.

We can interpret equation (30) much like equation (16). Note that $e_+(i)$ corresponds to the endowments accumulated after two periods of life by a household that is age 1 under endowment profiles $\{e_j\}_{j=1}^4$ and age 2 under endowment profiles $\{ze_j\}_{j=1}^4$. The difference $e_+^{-1}((1+r)p_F^* - p_F^{**} + \gamma p_H^+) - e_1^{-1}(\gamma p_F^*)$ is the measure of age 2 agents who own a flat in state $\mathbf{x}^*$ and cannot afford the down payment on a house at price p_H^+. Here, we take advantage of the fact that in state $\mathbf{x}^*$, an age 2 household's endowment index i and the prices p_F^* and p_H^* determine the household's dwelling h and non-housing wealth w at the beginning of the period. In the case of an age 2 flat owner, we know that its endowment index satisfies $e_1(i) \geq \gamma p_F^*$ and that its non-housing wealth is $w = (1+r)[e_1(i) - p_F^*]$. With the changed endowments and property prices, this household can afford the down payment on a house if and only if $w + ze_2(i) + p_F^{**} \geq \gamma p_H^+$, or $e_+(i) \geq (1+r)p_F^* - p_F^{**} + \gamma p_H^+$. The last term on the right-hand side of (30) is the measure of age 3 agents who, anticipating the user costs $(1+r)p_H^+ - p_H^{**}$ and rp_F^{**} for houses and flats, respectively, prefer a flat to a house.

The critical endowment and preference indices that characterize the allocation of properties in state $\mathbf{x}^1$ are

$$i_F^+ = e_1^{-1}(z^{-1}\gamma p_F^{**}) = 3 - S_F - S_H, \tag{31}$$

$$i_H^+ = e_1^{-1}(z^{-1}\gamma p_H^+), \tag{32}$$

$$i_{\emptyset H}^+ = e_+^{-1}(\gamma p_H^+), \tag{33}$$

$$i_{FH}^+ = e_+^{-1}((1+r)p_F^* - p_F^{**} + \gamma p_H^+), \tag{34}$$

$$m_H^+ = u^{-1}((1+r)p_H^+ - p_H^{**} - rp_F^{**}). \tag{35}$$

20

As $i_F^+ = i_F^* = i_F^{**} = 3 - S_F - S_H$, we henceforth write simply i_F for this cut-off. The transaction volumes arising in the transition from $\mathbf{x}^*$ to $\mathbf{x}^1$ are

$$n_F^+ = i_H^+ - i_F + \min\{i_F, i_{\emptyset H}^+\} + (i_F - \min\{i_F, i_{\emptyset H}^*\} + 1 - i_{FH}^*)\, m_H^+, \tag{36}$$

$$n_H^+ = 1 - i_H^+ + i_F - \min\{i_F, i_{\emptyset H}^+\} + i_H^* - i_{FH}^+ + (\min\{i_F, i_{\emptyset H}^*\} + i_{FH}^* - i_F)(1 - m_H^+). \tag{37}$$

These numbers can be directly read off Figure 2 and its counterpart in state $\mathbf{x}^1$.

7   Price and Volume Dynamics

We can now analyse the reaction of property prices and transaction volumes to a small change in endowments. Suppose the economy is in steady-state equilibrium with endowment profiles $\{e_j\}_{j=1}^4$. At time $t = 0$, endowment profiles shift permanently to $\{ze_j\}_{j=1}^4$, with z close enough to 1 that Proposition 3 applies.[11] From $t = 0$ on, the economy evolves according to the recursive competitive equilibrium constructed above. The price of flats changes proportionally with endowments at $t = 0$, reaching its new steady-state level $p_F^{**} = zp_F^*$ immediately.

As to the price of houses at $t = 0$, it is straightforward to see that $p_H^+ > p_H^*$ if $z > 1$, and $p_H^+ < p_H^*$ if $z < 1$. Suppose, for example, that endowments rise ($z > 1$). First, the demand for houses from first-time buyers of age 1 and 2 shifts upward. Second, flat owners of age 2 have a dual advantage over their counterparts in the initial steady state: They enjoy both higher endowments in the second period of their lives and capital gains on their flats. This causes the demand for houses from repeat buyers of age 2 to shift upward. Third, the higher price of flats and the expectation of a higher price differential between houses and flats in the future (implied by the second part of Proposition 2) cause the demand for houses from age 3 households to shift upward. Thus the price of houses must rise.

Note that the change in the price of houses depends on the change in age 1 endowments in two ways: first, through the demand from credit-constrained age 1 buyers, and second, through the change in the price of flats. Age 1 endowments are thus critical to both flat and house price dynamics in our model.

How much does the price of houses change, and what does this mean for the concomitant change in the number of transactions? We start the analysis with the case where capital gains and losses on flats have the strongest possible effect on the dynamics of prices and transaction volumes.

---

[11]In other words, we analyze the response of the economy to an unexpected change in endowments. By continuity, the results obtained here remain if households attach a low positive probability to the change in endowments.

Definition 3. We say that all house purchases are repeat purchases if $i_H^* = i_H^+ = i_H^{**} = 1$ and $\min\{i_{\emptyset H}^*, i_{\emptyset H}^+, i_{\emptyset H}^{**}\} \geq i_F$.[12] We say that the price of houses overshoots its new steady state level if $|p_H^+ - p_H^*| > |p_H^{**} - p_H^*|$. Finally, we say that transaction volumes move with prices if the differences $n_H^+ - n_H^*$, $n_F^+ - n_F^*$, $p_H^+ - p_H^*$ and $p_F^+ - p_F^*$ all have the same sign.

Proposition 4 *Suppose that all house purchases are repeat purchases. Then, the price of houses overshoots its new steady-state level if and only if*

$$z e_1(i_{FH}^{**}) < p_F^{**}, \tag{38}$$

that is, if in the new steady state the price of flats exceeds the age 1 wealth of those households that are the marginal house buyers at age 2. If the price of houses overshoots, transaction volumes move with prices.

A sufficient condition for (38) in terms of the primitives of the model is the inequality $e_1(1) < \gamma^{-1} e_1(3 - S_F - S_H)$.

To provide some intuition for this result, we first note that in the absence of first-time buyers of houses, the demand for houses comes entirely from flat owners of age 2 and households of age 3; market clearing for houses simply means $i_{FH}^* + m_H^* = i_{FH}^+ + m_H^+ = i_{FH}^{**} + m_H^{**} = 2 - S_H$. What is more, the change in the price of houses from $p_H(\mathbf{x}^*) = p_H^+$ to $p_H(\mathbf{x}^1) = p_H^{**}$ is driven entirely by the wealth of the age 2 owners of flats. This is because age 3 households' willingness to pay for a house (which depends on the current price of flats and predicted future property prices) is the same in the two states $\mathbf{x}^*$ and $\mathbf{x}^1$, and hence in periods $t = 0$ and $t = 1$.

Now assume $z > 1$ for concreteness, and consider an age 2 household that owns a flat at the beginning of period 1. One period earlier, this household earned $z e_1(i)$ and bought its flat at the price p_F^{**}. Contrast this with an age 2 household born one period earlier that has the same income index and owns a flat at the beginning of period 0.

When this household was age 1 (and the economy was still in the old steady state), it earned $e_1(i)$ and bought its flat at the price $p_F^* < p_F^{**}$; that is, it earned less than the first household, but bought its flat more cheaply. Under condition (38), the endowment disadvantage is more than compensated by the price advantage; as both income and the price of flats rise by the same *proportion*, and the income is lower than the price of flats, the price rises by more than the income *in absolute terms*. Given that the resale value of a flat is the same in periods 0 and 1, the household that is age 2 in period 0 is thus wealthier.

---

[12]With z close to 1, a sufficient condition for this in terms of the primitives of the model is $e(\frac{3}{2} - S_H) \geq \max\{e_1(1), e(3 - S_F - S_H)\} + r\gamma^{-1} e_1(3 - S_F - S_H)$, which is stronger than assumption (6). In fact, arguing as in the proof of Lemma A.1, one sees easily that this inequality implies $p_H^* > \gamma^{-1} \max\{e_1(1), e(3 - S_F - S_H)\}$. By continuity, z close to 1 then implies the same lower bound for p_H^+ and p_H^{**}.

As this translates into a greater ability to pay for a house, the equilibrium price of houses must be higher in period 0, so $p_H^+ > p_H^{**}$.

That is, owners of flats who reach age 2 as endowments rise enjoy capital gains, while those one period later do not. Under condition (38), these capital gains outweigh the endowment disadvantage at age 1 and cause the price of houses to overshoot.

Because of this overshooting, the difference between the costs of holding a house and holding a flat from period 0 to period 1, $(1-r)p_H^+ - p_H^{**} - r p_F^{**}$, exceeds the corresponding difference in the new steady state, $r[p_H^{**} - p_F^{**}]$, which by the second part of Proposition 2 in turn exceeds the corresponding difference in the initial steady state, $r[p_H^* - p_F^*]$. This means that the critical preference index m_H^+ exceeds m_H^*.

Relative to the initial steady state, the measure of age 3 house owners who buy a flat in period 0 thus increases by $(1 - i_{FH}^*)(m_H^+ - m_H^*)$; as the measure of first-time purchases of flats is constant and equal to 1, the transaction volume for flats increases. The measure of age 2 owners of flats who buy a house in period 0 increases by $i_{FH}^* - i_{FH}^+ = m_H^+ - m_H^*$, while the number of age 3 flat owners who buy a house declines by $i_{FH}^* (m_H^* - m_H^+)$.

Figure 3 compares the resulting allocation of dwellings to households in period 1 and the allocation in the initial steady state. The dark shaded area at age 3 results from the increase in house purchases by age 2 households in period 0. The light shaded area at age 4 results from the decline in house purchases by age 3 households, and the dark shaded area at age 4 from the increase in flat purchases by age 3 households. The rise in repeat purchases at age 2 is the dominant force; it causes the transaction volume for houses to rise as well.
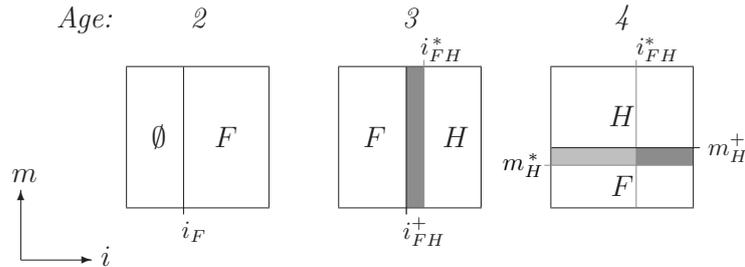

Figure 3: Allocations of dwellings and transactions

The intuition behind the rise in transactions is that for house prices to overshoot, age 2 flat owners' ability to pay for houses must have risen strongly. This increased ability to pay allows more of these households to trade up, thereby causing a surge in house transactions.

At the same time, more age 3 households trade down when the house price rises strongly, which causes a surge in flat transactions.[13]

Note that the increase in the number of houses owned by the young does not mean a one-for-one increase in the number of older households trading down from a house to a flat. Instead, the shift in the distribution of houses in favour of the young is moderated by those age 3 households that were planning to move to a house for preference reasons but now choose to remain in their flat because houses have become more expensive.

The absence of first-time buyers of houses assumed in Proposition 4 makes capital gains or losses on flats as strong a driver of equilibrium dynamics as possible. The impact of these capital gains or losses is weaker in the presence of first-time buyers of houses because their ability to pay for a house does not depend on the flat price.[14]

Still, as the result below shows, it remains true that whenever the house price reacts strongly enough, transaction volumes move with prices.

Definition 4. We say that the price of houses overreacts to the change in endowments if $|p_H^+ - p_H^*| > |z - 1|p_H^*$.

By the second part of Proposition 2, this is a stronger condition than overshooting.

Proposition 5 *If the price of houses overreacts, transaction volumes move with prices.*

The intuition for this result is essentially the same as that given after Proposition 4. If the house price overreacts to rising endowments, capital gains for credit-constrained flat owners must have been large enough to compensate for the reduced demand for houses by credit-constrained first-time buyers and unconstrained repeat buyers. These capital gains allow more credit-constrained flat owners to trade up, which produces a surge in the transaction volume for houses. At the same time, more credit-constrained first-time buyers acquire flats and more unconstrained repeat buyers trade down from a house to a flat, so the transaction volume for flats rises as well.

It remains to identify conditions for overreaction of the house price. If $z > 1$, we have $p_H^+ > zp_H^*$ if and only if the right-hand side of (30) with p_H^* replaced by zp_H^* is smaller

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[13]Note that overshooting is not necessary to generate the co-movement of prices and volumes. In fact, it is trivial to verify from (21) and (35) that $m_H^+ - m_H^*$ has the same sign as $p_H^+ - p_H^*$ and $p_F^+ - p_F^*$ if and only if

$$|p_H^+ - p_H^*| > \frac{1}{1+r} |p_H^{**} - p_H^*| + \frac{r}{1+r} |p_F^{**} - p_F^*|,$$

which is somewhat weaker than overshooting by the second part of Proposition 2.

[14]For example, condition (38) is necessary but no longer sufficient for overshooting of house prices when some house purchases at age 2 are first-time purchases.

than the left-hand side. Combining this with the characterization of p_H^* in (16), we see that $p_H^+ > zp_H^*$ if and only if

$$
\begin{aligned}
e^{-1}(rp_F^* + \gamma p_H^*) &- e_+^{-1}((1+r)p_F^* - p_F^{**} + \gamma zp_H^*) \\
&> \min\left\{i_F, e_+^{-1}(\gamma zp_H^*)\right\} - \min\left\{i_F, e^{-1}(\gamma p_H^*)\right\} \\
&+ u^{-1}((1+r)zp_H^* - p_H^{**} - rp_F^{**}) - u^{-1}(r[p_H^* - p_F^*]).
\end{aligned}
\tag{39}
$$

It is straightforward to verify that the difference in the second line of (39) is non-negative, while the difference in the third line is positive. So, a necessary condition for overreaction of house prices to increasing endowments is that the left-hand side of (39) be strictly positive. Proceeding as in the proof of Proposition 4, one sees easily that this is equivalent to the inequality $e_1(i_{FH}^*) < p_F^*$, which is reminiscent of condition (38) for overshooting when there are no first-time buyers of houses, and has the same intuitive interpretation.

The difference in the second line of (39) is trivially lower than $i_F - \min\{i_F, i_{\emptyset H}^*\}$, which is the measure of age 2 first-time buyers of houses in the initial steady state. Overreaction in the price of houses is more likely when that measure is small.

Finally, the difference in the third line of (39) shows that overreaction in the price of houses is more likely when the density of utility premiums of houses over flats is small around $r[p_H^* - p_F^*]$.

We are overall likely to observe overreaction in prices and a positive correlation between prices and transactions when: (1) few first-time buyers of flats pay cash for them; (2) there are few first-time purchases of houses; and (3) changes in the relative price of homes have only a limited impact on the willingness of unconstrained households to move along the property ladder.

8 Conclusion

We identify a new driver of housing prices: the ability of young households to afford the down payment on a starter home. We present evidence of a strong and positive correlation between housing prices and one of the determinants of this ability, the income of young households.

We identify a new channel whereby changes in income affect housing price levels, the relative prices of properties and housing transactions. Our model explains conditions under which we expect prices to overshoot, to overreact to changes in income, and to display a positive correlation with housing transactions. Critical in determining whether such patterns occur in equilibrium is the extent to which changes in income affect the number of credit-constrained owners moving up the property ladder.

These results suggest avenues for further empirical research on housing consumption and housing market fluctuations. A number of papers consider the effects of housing capital gains or losses on household moving behaviour; see, for example, Genesove and Mayer (1997), Henley (1998), Chan (2001), Engelhardt (2003) and Seslen (2004). None of them considers this question conditional on purchase of the current home under a binding credit constraint. Thus, the empirical literature does not speak to the capital gains channel we identify in our model.

More could be learned from a focus on the factors identified as determinants of the strength of the capital gains channel. For example, Lamont and Stein (1999) find that housing prices are more likely to display overreaction to income shocks in Metropolitan Statistical Areas (MSAs) where a large proportion of households have a high level of debt relative to the value of their home. The distribution of debt is of course determined endogenously. It would be interesting to see whether differences in the distribution of debt across MSAs can be related to exogenous factors that determine the response of prices to income changes in the model.

The model is analytically tractable despite its dynamic nature and the presence of credit constraints, two assets, and two dimensions of household heterogeneity. This allows us to provide a transparent characterization of basic forces that drive the housing market. Our simplifying assumptions deserve some comments, however.

We assume households live for four periods. This does not mean we think of each model period as representing a fourth of a typical household's life span. Rather, four is the minimum number of periods required to capture interactions in the market of first-time buyers, credit-constrained repeat buyers, and unconstrained households that might move due to preference reasons. Taking the model to data would require an increase in the number of periods that households live. This would obviously increase the number of income and price lags relevant to current prices and the current allocation. The relevant number of lags would be determined by how long it takes credit-constrained households to accumulate their down payments and climb the property ladder.

The model abstracts from "horizontal" transactions: moves by repeat buyers between two homes within the same price range. In Ortalo-Magné and Rady (2000), we provide evidence that fluctuations in the number of "vertical" transactions up and down the property ladder accounted for most of the fluctuations in housing transactions in England and Wales over the past three decades.

The model also abstracts from the option to rent a home. If rental properties are different from owner-occupied properties and conversion is costly, our results extend readily. In Ortalo-Magné and Rady (1999), we study the effects of income growth and credit market

liberalization in a simpler variant of the model where starter homes can be either rented or owner-occupied, with households preferring the latter. Chiuri and Jappelli (2003) provide cross-country evidence in support of our predictions with regard to variation in the owner-occupancy rates of different cohorts across countries. If all homes can be either rented or owned and provide the same utility flow either way, then credit constraints are irrelevant and our results disappear.

We assume an exogenous and constant interest rate, and we do not consider a number of features of mortgage contracts. We also focus on the consumption demand for housing, not any investment motive households may have. Relaxing some of our assumptions should not affect the basic forces we identify.

Appendix

Lemma A.1 shows that equation (16) has a unique solution and in particular that $p_H^* > p_F^*$.

Lemma A.1 *Equation* (16) *has a unique solution* p_H^*, *which satisfies the inequalities* $p_F^* < p_H^*$ *and* $\frac{1}{2} < e^{-1}(rp_F^* + \gamma p_H^*) < 1$.

PROOF: The function $f : {I\!\!R} \to {I\!\!R}$ defined by

$$
\begin{aligned}
f(p_H) &= e_1^{-1}(\gamma p_H) + \min\left\{ e_1^{-1}(\gamma p_F^*), e^{-1}(\gamma p_H) \right\} \\
&\quad + e^{-1}(rp_F^* + \gamma p_H) - e_1^{-1}(\gamma p_F^*) + u^{-1}(r[p_H - p_F^*]).
\end{aligned} \tag{A.1}
$$

is continuous and weakly increasing, constant at the negative level $-e_1^{-1}(\gamma p_F^*)$ for small p_H, constant at the level 3 for large p_H, and strictly increasing everywhere else. So there is a unique real number p_H^* such that $f(p_H^*) = 3 - S_H$.

Note that $e_1^{-1}(\gamma p_H) \leq 1$ and $\min\left\{ e_1^{-1}(\gamma p_F^*), e^{-1}(\gamma p_H) \right\} - e_1^{-1}(\gamma p_F^*) \leq 0$ for all p_H. This implies that

$$
f(\gamma^{-1}[e(\tfrac{1}{2}) - rp_F^*]) \leq \tfrac{3}{2} + u^{-1}(r\gamma^{-1}[e(\tfrac{1}{2}) - rp_F^* - \gamma p_F^*]). \tag{A.2}
$$

As $e(\tfrac{1}{2}) - rp_F^* - \gamma p_F^* < e(1) - \gamma p_F^* = e(1) - e_1(3 - S_F - S_H)$, assumption (9) implies that the second term on the right-hand side of (A.2) is strictly smaller than $\frac{1}{2}$. So we have $f(\gamma^{-1}[e(\tfrac{1}{2}) - rp_F^*]) < 2 < 3 - S_H$ by the second part of assumption (1). This proves $p_H^* > \gamma^{-1}[e(\tfrac{1}{2}) - rp_F^*]$ or $e^{-1}(rp_F^* + \gamma p_H^*) > \frac{1}{2}$. As $e(\tfrac{1}{2}) > e(3 - S_F - S_H) > (1 + r\gamma^{-1})e_1(3 - S_F - S_H)$ by assumptions (1)–(2) and (4)–(5), we also have $p_H^* > p_F^*$.

By assumption (7), $e(1) - rp_F^*$ exceeds both $e_1(1)$ and $e(3 - S_F - S_H)$, so we have

$$
f(\gamma^{-1}[e(1) - rp_F^*]) = 2 + u^{-1}(r\gamma^{-1}[e(1) - rp_F^* - \gamma p_F^*]). \tag{A.3}
$$

As $e(1) - rp_F^* > e_1(3 - S_F - S_H) = \gamma p_F^*$ by assumption (7) again, the second term on the right-hand side of (A.3) exceeds $u^{-1}(0)$, which by assumption (8) is strictly larger than $1 - S_H$. So we have $f(\gamma^{-1}[e(1) - rp_F^*]) > 3 - S_H$, which proves $p_H^* < \gamma^{-1}[e(1) - rp_F^*]$ or $e^{-1}(rp_F^* + \gamma p_H^*) < 1$. ∎

Note that this result implies $e^{-1}(rp_F^* + \gamma p_H^*) > e_1^{-1}(\gamma p_F^*)$ as the latter equals $3 - S_F - S_H$, which is smaller than $\frac{1}{2}$ by the first part of assumption (1).

PROOF OF PROPOSITION 1: Consider a steady-state equilibrium with flat price p_F and house price p_H. Clearly, both these prices are strictly positive. The time-invariant ex post costs of holding a property from one period to the next are $\eta_F = rp_F$ and $\eta_H = rp_H$, respectively.

Suppose that $p_F \geq p_H$ and thus $\eta_F \geq \eta_H$. Then, there is no demand for flats from agents of age 1 or 2 since houses yield higher utility to these agents and are not more expensive to hold than flats. Moreover, $\eta_H - \eta_F \leq 0 < u(\tfrac{1}{2})$ by assumption (9), so fewer than half of all age 3 agents demand a flat (whether they are credit-constrained or not). Given that $S_F > 1$ by assumption (1), therefore, the market for flats cannot clear. This proves $p_F < p_H$.

By market clearing, the measure of agents who own no property in equilibrium is $3 - S_F - S_H$, which by assumption (1) lies strictly between 0 and $\frac{1}{2}$. This is an upper bound on the measure of agents of age 1 who cannot afford a flat. As a consequence, the steady-state flat price must satisfy $\gamma p_F \leq e_1(3 - S_F - S_H)$ or $p_F \leq p_F^*$. By assumption (10), this implies $(1 + r)^2 \eta_F < \Delta$.

By assumption (8) and the fact that $\eta_H > \eta_F$, the demand for houses from cohort 3 is strictly lower than S_H. This means that some houses must be held by agents of age 1 or 2. Thus,

$$
p_H < \gamma^{-1} e(1), \tag{A.4}
$$

the maximum price any agent of age 1 or 2 could pay.

As a consequence of the upper bound (A.4), all agents of age 3 are unconstrained in their choice of housing. By an argument similar to the one in footnote 8, in fact, the total beginning-of-period wealth of each age 3 agent amounts to at least $e_3(0)$, whatever the history of housing consumption. By (6) and

(A.4), this is enough to cover the down payment γp_H and, by (2) again, to permit positive consumption at age 4. As the housing option chosen at age 3 is the one that maximizes the difference between the utility of the option and the corresponding user cost, and $\eta_F < \Delta$ by what we saw earlier, owning a flat clearly dominates owning no property for all age 3 agents. So the optimal housing correspondence at age 3 is

$$G_3(m) = \begin{cases} \{F\} & \text{if} \quad m < m_H, \\ \{F, H\} & \text{if} \quad m = m_H, \\ \{H\} & \text{if} \quad m > m_H, \end{cases} \tag{A.5}$$

where the critical preference index is

$$m_H = u^{-1}(r[p_H - p_F]). \tag{A.6}$$

Cohort 3 thus demands a measure $1 - m_H$ of houses.

Since $e_2(0) > e_1(3 - S_F - S_H) \geq \gamma p_F$ by assumption (4), all agents of age 2 can afford a flat, whatever their housing consumption at age 1. (For households that acquired property at age 1, this is established in footnote 8.) As the housing choice at age 2 does not affect the set of feasible housing choices at age 3, the housing option chosen at age 2 is the one that maximizes the difference between the utility of the option and $(1 + r)$ times the corresponding user cost. As $(1 + r)\eta_F < \Delta$ by what we derived earlier, owning a flat dominates owning no property for all age 2 agents.

Thus, agents who do not own a property must be of age 1, and there must be a measure $3 - S_F - S_H$ of them. Yet, we already know that any property affordable at age 1 continues to be so at age 2 (see footnote 8). We also know that $(1+r)^2 \eta_F < \Delta$, while assumption (10) implies $\Delta - (1+r)^2 \eta_F > u(\frac{1}{2}) - (1+r)(\eta_H - \eta_F)$. At steady-state user costs, therefore, consuming a flat at age 2 and 3 dominates the no-property option followed by a house. This means that those households that do not acquire a flat at age 1 want one, but cannot afford the required down payment; as there are a measure $3 - S_F - S_H$ of them, we must have $p_F = p_F^*$. By (A.4) and assumption (9), we thus have $(1+r)^2(\eta_H - \eta_F) < u(\frac{1}{2})$. This implies $\eta_H - \eta_F < u(\frac{1}{2})$; hence $0 < m_H < \frac{1}{2}$.

At age 2, a flat is a feasible choice, and the housing choice at age 2 does not affect the set of feasible housing choices at age 3. We have established that all age 2 agents will buy a property. We also know that $(1 + r)(\eta_H - \eta_F) < u(\frac{1}{2})$, so when the user cost difference is taken into account, a house dominates a flat. Thus, housing choices at age 2 depend entirely on whether an agent can afford the down payment on a house or not. For those agents who did not buy any property at age 1, therefore, the optimal housing correspondence at age 2 is

$$G_2(i, \emptyset) = \begin{cases} \{F\} & \text{if} \quad i < i_{\emptyset H}, \\ \{H\} & \text{if} \quad i \geq i_{\emptyset H}, \end{cases} \tag{A.7}$$

where the critical endowment index,

$$i_{\emptyset H} = e^{-1}(\gamma p_H), \tag{A.8}$$

is strictly smaller than 1 by (A.4). For agents who bought a flat at age 1, the optimal housing correspondence at age 2 is $G_2(i, F) = F$ if $rp_F + \gamma p_H > e(1)$; if $rp_F + \gamma p_H \leq e(1)$, it is

$$G_2(i, F) = \begin{cases} \{F\} & \text{if} \quad i < i_{FH}, \\ \{H\} & \text{if} \quad i \geq i_{FH}, \end{cases} \tag{A.9}$$

where the critical endowment index is

$$i_{FH} = e^{-1}(rp_F + \gamma p_H). \tag{A.10}$$

And for agents who bought a house at age 1, the optimal housing correspondence at age 2 is $G_2(i, H) = \{H\}$.

We already know that any property affordable at age 1 will continue to be so at age 2. We also know that $(1+r)^2 \eta_F < \Delta$ and $(1+r)^2(\eta_H - \eta_F) < u(\frac{1}{2})$, so a flat is better than no property, and a house is better than a flat, when user costs are taken into account. Moreover, consuming a flat at age 2 and 3 dominates the no-property option followed by a house. If $\gamma p_H \leq e_1(1)$, therefore, the optimal housing correspondence at age 1 is

$$G_1(i) = \begin{cases} \{\emptyset\} & \text{if} \quad i < i_F, \\ \{F\} & \text{if} \quad i_F \leq i < i_H, \\ \{H\} & \text{if} \quad i \geq i_H, \end{cases} \tag{A.11}$$

where the critical endowment indices are

$$i_F = e_1^{-1}(\gamma p_F) = 3 - S_F - S_H, \tag{A.12}$$

$$i_H = e_1^{-1}(\gamma p_H). \tag{A.13}$$

If $\gamma p_H > e_1(1)$, the optimal housing correspondence at age 1 is

$$G_1(i) = \begin{cases} \{\emptyset\} & \text{if} \quad i < i_F, \\ \{F\} & \text{if} \quad i \geq i_F. \end{cases} \tag{A.14}$$

Cohort 1 demands a measure $1 - i_H$ of houses.

Note that $i_{FH} < i_H$ if $i_H < 1$ because the equality $e_1(i_H) = \gamma p_H$ implies $e(i_H) > r p_H + \gamma p_H > r p_F + \gamma p_H$ by the argument in footnote 8. As $i_{FH} \leq i_H$ trivially if $i_H = 1$, we can conclude that $i_{FH} \leq i_H$ always. Thus, cohort 2 demands a measure $\max\{i_F - i_{\emptyset H}, 0\} + 1 - \max\{i_F, i_{FH}\}$ of houses.

Since $S_H < 1$ and $1 - m_H > \frac{1}{2}$, cohorts 1 and 2 cannot acquire more than the measure $\frac{1}{2}$ of houses in equilibrium. This implies $i_H > \frac{1}{2}$ and $\max\{i_F, i_{FH}\} > \frac{1}{2}$. As $i_F < \frac{1}{2}$, the latter means $i_{FH} > \frac{1}{2} > i_F$. The market clearing condition for houses thus becomes

$$1 - i_H + \max\{i_F - i_{\emptyset H}, 0\} + 1 - i_{FH} + 1 - m_H = S_H. \tag{A.15}$$

After inserting $p_F = p_F^*$, this is equivalent to the equation $f(p_H) = 3 - S_H$ with the function f defined in (A.1). The only possible steady-state house price is therefore $p_H = p_H^*$.

Setting $p_F = p_F^*$ and $p_H = p_H^*$ in (A.6), (A.8), (A.10), (A.12) and (A.13) yields the cutoff indices (17)–(21). By Lemma (A.1), we have $\frac{1}{2} < e^{-1}(r p_F^* + \gamma p_H^*) < 1$. First, this means $\frac{1}{2} < i_{FH}^* < 1$; in particular, $i_{FH}^* < i_H^*$ by the argument two paragraphs earlier. Second, it implies that $\gamma p_H^* > e(\frac{1}{2}) - r p_F^* > (1 + r)\gamma p_F^* + e_2(\frac{1}{2}) - r p_F^* > e_2(\frac{1}{2}) > e(0)$ by assumptions (1)–(3), hence $i_{\emptyset H}^* > 0$. Third, it implies that p_H^* satisfies the upper bound (A.4), so by assumption (9) we have $(1 + r)^2(\eta_H^* - \eta_F^*) < u(\frac{1}{2})$ and $0 < m_H^* < \frac{1}{2}$.

We also have $(1 + r)^2 \eta_F^* < \Delta$ and $\Delta - (1 + r)^2 \eta_F^* > u(\frac{1}{2}) - (1 + r)(\eta_H^* - \eta_F^*)$ by assumption (10). The same arguments as above show that all agents of age 3 are unconstrained in their choice of housing, all agents of age 2 can afford a flat, and any property affordable at age 1 continues to be affordable at age 2. Thus, the policy correspondences G_j, evaluated for the cutoff indices (17)–(21), describe the optimal housing decisions when households face the time-invariant prices p_F^* and p_H^*. These decisions are the ones described in the statement of the proposition, and as all non-housing consumption is postponed to age 4, they uniquely determine, via the budget constraints, a state $\mathbf{x}^*$ as well as households' savings decisions. That this constitutes a steady-state equilibrium is obvious from what we have shown before.

As to steady-state transaction volumes, the measure of age 1 agents who buy a flat each period is $i_H^* - i_F^*$. In the age 2 cohort, it is $\min\{i_F^*, i_{\emptyset H}^*\}$ (all those agents who could not afford a flat one period earlier and do not move to a house now); and in the age 3 cohort, it is $(i_F^* - \min\{i_F^*, i_{\emptyset H}^*\} + 1 - i_{FH}^*)m_H^*$ (all those agents who lived in a house and now decide to move to a flat). A measure $1 - i_H^*$ of agents buys a house at age 1, a measure $i_F^* - \min\{i_F^*, i_{\emptyset H}^*\} + i_H^* - i_{FH}^*$ does so at age 2, and a measure $(\min\{i_F^*, i_{\emptyset H}^*\} + i_{FH}^* - i_F^*)(1 - m_H^*)$ at age 3. This proves (22)–(23). ∎

PROOF OF PROPOSITION 2: Suppose $z > 1$. Evaluating the right-hand side of (24) with p_H^{**} replaced by $p_H^* + (z-1)p_F^*$ yields an expression that is easily seen to be strictly smaller than the right-hand side of (16), hence smaller than $3 - S_H$. This proves that $p_H^{**} > p_H^* + (z - 1)p_F^*$. Similarly, evaluating the right-hand side of (24) with p_H^{**} replaced by $z p_H^*$ yields an expression that is easily seen to be strictly larger than the right-hand side of (16), and so $p_H^{**} < z p_H^*$. All inequalities are reversed if $z < 1$. ∎

PROOF OF PROPOSITION 3: We rely on the fact that in state $\mathbf{x}^*$, a household's endowment index i fully determines its state variables w and h at age 2 and 3.

We set $p_F(\mathbf{x}^*) = p_F^{**}$ and $p_H(\mathbf{x}^*) = p_H^+$, and define the decision rules

$$h_1(i, \mathbf{x}^*) = \begin{cases} \emptyset & \text{if} \quad i < i_F^+, \\ F & \text{if} \quad \gamma p_H^+ > z e_1(1) \quad \text{and} \quad i \geq i_F^+, \\ & \text{or} \quad \gamma p_H^+ \leq z e_1(1) \quad \text{and} \quad i_F^+ \leq i < i_H^+, \\ H & \text{if} \quad \gamma p_H^+ \leq z e_1(1) \quad \text{and} \quad i \geq i_H^+, \end{cases} \tag{A.16}$$

$$h_2(i,h,w,\mathbf{x}^*) = \begin{cases} F & \text{if} \quad i < \min\{i_F^*, i_{\emptyset H}^+\} \quad \text{or} \quad i_F^* \le i < i_{FH}^+, \\ H & \text{if} \quad \min\{i_F^*, i_{\emptyset H}^+\} \le i < i_F^* \quad \text{or} \quad i \ge i_{FH}^+, \end{cases} \tag{A.17}$$

$$h_3(i,m,h,w,\mathbf{x}^*) = \begin{cases} F & \text{if} \quad m < m_H^+, \\ H & \text{if} \quad m \ge m_H^+, \end{cases} \tag{A.18}$$

$h_4(i,h,w,\mathbf{x}^*) = \emptyset$ for housing consumption, and $c_1(i,\mathbf{x}^*) = c_2(i,h,w,\mathbf{x}^*) = c_3(i,m,h,w,\mathbf{x}^*) = 0$, $c_4(i,m,h,w,\mathbf{x}^*) = e_4(i) + p_h^+ + w$ for non-housing consumption. By the budget equations, they uniquely determine decision rules $w_1(i,\mathbf{x}^*)$, $w_2(i,h,w,\mathbf{x}^*)$, $w_3(i,m,h,w,\mathbf{x}^*)$ and $w_4(i,m,h,w,\mathbf{x}^*) = 0$ for next-period non-housing wealth. By equation (30), the decision rules for housing are market clearing. Via the mapping defined in part (c) of the definition of recursive equilibrium, the decision rules for housing consumption and non-housing wealth yield a state $\phi(\mathbf{x}^*)$ which we denote by $\mathbf{x}^1$.

Next, we set $p_F(\mathbf{x}^1) = p_F^{**}$ and $p_H(\mathbf{x}^1) = p_H^{**}$, and define the decision rules

$$h_1(i,\mathbf{x}^1) = \begin{cases} \emptyset & \text{if} \quad i < i_F^{**}, \\ F & \text{if} \quad \gamma p_H^{**} > z e_1(1) \quad \text{and} \quad i \ge i_F^{**}, \\ & \text{or} \quad \gamma p_H^{**} \le z e_1(1) \quad \text{and} \quad i_F^{**} \le i < i_H^{**}, \\ H & \text{if} \quad \gamma p_H^{**} \le z e_1(1) \quad \text{and} \quad i \ge i_H^{**}, \end{cases} \tag{A.19}$$

$$h_2(i,h,w,\mathbf{x}^1) = \begin{cases} F & \text{if} \quad i < \min\{i_F^+, i_{\emptyset H}^{**}\} \quad \text{or} \quad i_F^+ \le i < i_{FH}^{**}, \\ H & \text{if} \quad \min\{i_F^+, i_{\emptyset H}^{**}\} \le i < i_F^+ \quad \text{or} \quad i \ge i_{FH}^{**}, \end{cases} \tag{A.20}$$

$$h_3(i,m,h,w,\mathbf{x}^1) = \begin{cases} F & \text{if} \quad m < m_H^{**}, \\ H & \text{if} \quad m \ge m_H^{**}, \end{cases} \tag{A.21}$$

$h_4(i,h,w,\mathbf{x}^1) = \emptyset$ for housing consumption, and $c_1(i,\mathbf{x}^1) = c_2(i,h,w,\mathbf{x}^1) = c_3(i,m,h,w,\mathbf{x}^1) = 0$, $c_4(i,m,h,w,\mathbf{x}^1) = e_4(i) + p_h^{**} + w$ for non-housing consumption. By the budget equations, they uniquely determine decision rules for next-period non-housing wealth. As $i_F^+ = i_F^{**}$, equation (24) implies that the decision rules for housing clear the markets. Via the mapping defined in part (c) of the definition of recursive equilibrium, the decision rules for housing consumption and non-housing wealth yield a state $\phi(\mathbf{x}^1)$ which we denote by $\mathbf{x}^2$.

Setting $p_F(\mathbf{x}^2) = p_F^{**}$ and $p_H(\mathbf{x}^2) = p_H^{**}$, imposing the same decision rules in state $\mathbf{x}^2$ as in state $\mathbf{x}^1$ and arguing as above, we obtain a state $\phi(\mathbf{x}^2)$ that we denote by $\mathbf{x}^3$. The distribution functions $M_{\emptyset,2}, M_{F,2}, M_{H,2}, M_{\emptyset,3}, M_{F,3}$ and $M_{H,3}$ in state $\mathbf{x}^3$ are identical to their counterparts in state $\mathbf{x}^{**}$. Therefore, setting $p_F(\mathbf{x}^3) = p_F^{**}$ and $p_H(\mathbf{x}^3) = p_H^{**}$, imposing the same decision rules in state $\mathbf{x}^3$ as in states $\mathbf{x}^1$ and $\mathbf{x}^2$, and proceeding as above, we obtain the state $\phi(\mathbf{x}^3) = \mathbf{x}^{**}$.

For $\mathbf{x} \in X$ and $h \in \{F, H\}$, let $\eta_h(\mathbf{x}) = (1+r)p_h(\mathbf{x}) - p_h(\phi(\mathbf{x}))$. By continuity of the property prices $p_h(\mathbf{x})$ in z, the following statements, that we obtained for $z = 1$ in the proof of Proposition 1, also hold for all z sufficiently close to 1 and all $\mathbf{x} \in X$: $(1+r)^2\eta_F^*(\mathbf{x}) < \Delta$, $(1+r)^2[\eta_H^*(\mathbf{x}) - \eta_F^*(\mathbf{x})] < u(\frac{1}{2})$, and $\Delta - (1+r)^2\eta_F^*(\mathbf{x}) > u(\frac{1}{2}) - (1+r)[\eta_H^*(\phi(\mathbf{x})) - \eta_F^*(\phi(\mathbf{x}))]$; all agents of age 3 are unconstrained in their choice of housing; all agents of age 2 can afford a flat; and any property affordable at age 1 continues to be affordable at age 2. The same arguments as in the proof of Proposition 1 establish that the actions assigned to households via the above decision rules are indeed optimal.

If $z > 1$, finally, the right-hand side of (30) with p_H^+ replaced by p_H^* is smaller than the right-hand side of (16), hence smaller than $3 - S_H$, which establishes $p_H^+ > p_H^*$. In the same way, we see that $p_H^+ < p_H^*$ if $z < 1$. ■

PROOF OF PROPOSITION 4: If all house purchases are repeat purchases, equation (30) for the house price p_H^+ simplifies to

$$2 - S_H = e_+^{-1}\left((1+r)p_F^* - p_F^{**} + \gamma p_H^+\right) + u^{-1}\left((1+r)p_H^+ - p_H^{**} - r p_F^{**}\right), \tag{A.22}$$

and equation (24) for the new steady-state house price p_H^{**} simplifies to

$$2 - S_H = e^{-1}\left(z^{-1}[r p_F^{**} + \gamma p_H^{**}]\right) + u^{-1}(r[p_H^{**} - p_F^{**}]). \tag{A.23}$$

Now suppose $z > 1$, so that $p_H^{**} > p_H^*$. We have $p_H^+ > p_H^{**}$ if and only if the right-hand side of (A.22) with p_H^+ replaced by p_H^{**} is smaller than the left-hand side. By (A.23), this is the case if and only if

$$e_+^{-1} \left((1+r) p_F^* - p_F^{**} + \gamma p_H^{**} \right) < e^{-1} \left(z^{-1} [r p_F^{**} + \gamma p_H^{**}] \right). \tag{A.24}$$

Noting that the term on the right-hand side of this inequality equals i_{FH}^{**}, we see that $p_H^+ > p_H^{**}$ if and only if

$$(1+r) p_F^* - p_F^{**} + \gamma p_H^{**} < (1+r) e_1(i_{FH}^{**}) + z e_2(i_{FH}^{**}). \tag{A.25}$$

As $(1+r) z e_1(i_{FH}^{**}) + z e_2(i_{FH}^{**}) = r p_F^{**} + \gamma p_H^{**}$ and $p_F^{**} = z p_F^*$, we can simplify this inequality to $e_1(i_{FH}^{**}) < p_F^*$, which is the same as (38). If $z < 1$, the result is obtained by the same steps, with the two inequalities (A.24)–(A.25) reversed.

That overshooting implies changes of transaction volumes in the same direction as the changes in prices is shown in the main text. ∎

PROOF OF PROPOSITION 5: Consider the case $z > 1$ and $p_H^+ > z p_H^*$. Comparing transactions taking place in the transition from period 0 to period 1 to those in the initial steady state, we see more purchases of flats at age 1 ($i_H^+ > i_H^*$), age 2 ($\min\{i_F, i_{\emptyset H}^+\} \geq \min\{i_F, i_{\emptyset H}^*\}$), and age 3 ($m_H^+ > m_H^*$). The volume of transactions of flats rises with prices. By the market clearing condition for houses, on the other hand, $i_{FH}^* - i_{FH}^+ - (i_H^+ - i_H^*) - (\min\{i_F, i_{\emptyset H}^+\} - \min\{i_F, i_{\emptyset H}^*\}) = m_H^+ - m_H^*$, so not only are there more house purchases by flat owners of age 2 ($i_{FH}^* > i_{FH}^+$), but the increase $i_{FH}^* - i_{FH}^+$ in these purchases is great enough to outweigh the loss of $i_H^+ - i_H^*$ and $\min\{i_F, i_{\emptyset H}^+\} - \min\{i_F, i_{\emptyset H}^*\}$ first-time house purchases at age 1 and 2, respectively, as well as the loss of $(\min\{i_F, i_{\emptyset H}^*\} + i_{FH}^* - i_F)(m_H^+ - m_H^*)$ repeat purchases at age 3. The volume of transactions of houses rises as well. The case of $z < 1$ and $p_H^+ < z p_H^*$ is dealt with in exactly the same way. ∎

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