INTERMEDIATE DOSIMETRIC QUANTITIES

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Abstract

The transfer of energy from ionizing radiation to matter involves a series of steps. In wide ranges of their energy spectra photons and neutrons transfer energy to an irradiated medium almost exclusively by the production of charged particles which ionize and thereby produce electrons that can ionize in turn. A systematic examination of these processes leads to conversion diagrams which identify a series of intermediate quantities. One of these is kerma which has long been employed as a measure of the energy imparted in the first of the interactions. It depends only on the fluence of uncharged particles and is therefore - unlike absorbed dose and δ-ray fluence - insensitive to local differences of receptor geometry and composition. An analogous quantity for charged particle fields, cema (converted energy per unit mass), is defined which quantifies the energy imparted in terms of the interactions of charged particles, disregarding energy dissipation by δ-rays.

Cema can be expressed as an integral over the fluence of ions times their stopping power. However, complications arise when the charged particles are electrons, and when their fluence cannot be separated from that of the δ-rays. The resulting difficulty can be circumvented by the definition of a reduced cema which corresponds largely to the concept employed in the cavity theory of Spencer and Attix. In reduced cema not all δ-rays but all electrons below a chosen cut-off energy are considered to be locally absorbed. As the cut-off energy is reduced, cema approaches absorbed dose and thereby becomes sensitive to highly local differences in geometry or composition. Reduced cema is a function of the fluence at the specified location at and

above the chosen cut-off energy. Its definition requires a modification of restricted LET, and it is recommended that the definition of restricted LET be so changed.

The various dosimetric quantities can be seen as different realizations of a common concept, kinetic energy of various categories of ionizing particles converted per unit mass at a point in an irradiated material. Disregarding energy dissipation by low energy electrons, all electrons, or all charged particles one moves from absorbed dose, to reduced cema, to ion cema, and to kerma.

1. Introduction

The term 'dosimetry' can be taken to refer solely to the determinations of absorbed dose (1), i.e. the energy absorbed per unit mass in the vicinity of a point in a medium exposed to ionizing radiations. However, in its wider sense dosimetry deals with the processes that link the energy transferred to matter with the radiation fluence (1), and in this wider sense one can consider certain intermediate quantities that correspond to successive phases of energy transfer. Apart from their conceptual significance intermediate dosimetric quantities appear in various steps of absorbed dose calculations and they can often serve as useful approximations to the absorbed dose.

The first major radiological quantity, the exposure (1), with its (now obsolete) unit, the roentgen, was formulated nearly a century ago. It served for many years as the only quantification of radia-

tion 'dose', although it refers to the amount of ionization which the electrons, generated by x- or γ-rays in a specified mass of air located at the point of interest, would produce in air.

A quantity that is both more general and fundamental is the **kerma**, originally formulated by Roesch (2), which refers to the first step in the interaction between uncharged particles (e.g. photons or neutrons) and irradiated matter.*)

In the subsequent considerations similar quantities will be defined that concern further steps in the transfer of radiation energy to matter.

Particles of a given kind and energy produce fluences of particles of different kind and/or energy at rates that are determined by the appropriate interaction coefficients. The process can be described in terms of flow diagrams. But the diagrams can be complex, because a given kind of radiation can appear repeatedly; thus photons produce electrons which in turn can produce photons by fluorescence or bremsstrahlung.

In dose calculations any of the steps must be taken into account provided they are considered to be of importance, i.e., affect the stated value of the absorbed dose beyond the accuracy claimed. In calculations the numerical value of the absorbed dose is taken to be

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*) Roesch proposed the acronym KERM (kinetic energy released per unit mass); accepting the concept the ICRU added an A to obviate confusion with the German word Kern (nucleus).

equal to that of kerma at the same location, when the energy transport by charged particles can be neglected; the condition is, that the distances involved are small compared to the attenuation lengths of the uncharged particles. In certain cases, however, the energy transport after the interaction can be more far-ranging than that by the incoming radiation; an example is bremsstrahlung. There can also be an increase of the energy transported when rest-mass energy is converted into kinetic energy of charged particles or into photon energy; low energy neutrons in tissue are a case where both processes are important.

However, in two important cases, those of photons and of neutrons of moderate energy the situation is simpler; the bulk of the energy is transported by uncharged particles, charged secondaries, and δ-radiation over successively shorter distances, and the energies transferred per unit mass at each step can serve as an increasingly accurate approximation to the absorbed dose at the point of energy transfer. If δ-radiation produced by electrons or successive generations of δ-radiation produced by any charged particles cannot be distinguished, complications arise. However, it is possible to formulate a quantity that can then be applied and that permits approximations to absorbed dose with chosen accuracy.

The quantities defined below are non-stochastic, i.e., they are the expectation values of quantities that are subject to statistical distributions. The definitions of the stochastic quantities would be largely analogous to those of their expectation values.

2. The Energy-Degradation Process

The interrelations between the fluences of various ionizing particles can be expressed by field equations that contain the interaction coefficients (4). They can also, as in earlier work by Hubbell (5), be represented by diagrams, which permit a more direct synopsis of the various channels of energy degradation. The specific considerations presented here are initially simplified by reference to a uniform isotropic radiation source in a homogeneous medium; this is termed the condition of complete equilibrium (3). In this case there are no geometric complications, because the distribution of fluence in particle type and energy is independent of location and depends merely on the source and on the interaction coefficients, and at each point the energy transferred is equal to the energy absorbed.

The major modes of energy conversion from various radiations to matter will be considered in detail, but less important routes will be disregarded. Thus, processes in which ions recoiling from neutrons produce additional energetic ions or photons in elastic or inelastic collisions will be ignored as well as nuclear reactions initiated by photons or electrons. There will also be no reference to modifications of the energy balance by changes in rest mass. Omission of these interactions simplifies the discussion, and it will become apparent that they can be readily accommodated in diagrams that are more complex but do not introduce additional features.

2.1 The Example of a Neutron Field

Fig.1 is a diagram illustrating the major modes of energy degradation when a field of fast neutrons interacts with matter. Each arrow represents energy conversion between the different forms of energy; the term energy conversion denotes energy transformed per unit mass during the time of interest. It must be noted that the connecting lines do not refer to spatial transport of energy, but to energy conversions taking place in interactions at a point. The pointed enclosures symbolize kinetic energy of neutrons (n), ions (i), and electrons (e). The rectangle (D) represents energy removed from the field of ionizing radiation; this is energy expended against binding energy, but it includes also energy of particles or photons that is insufficient to cause ionization and that is, therefore, considered as energy imparted.

Each symbol for energy conversion is given two indices that identify the forms of energy between which the transition occurs. For example $\eta_{s,n}$ stands for the conversion of energy from an unspecified source to kinetic energy of neutrons. Thus the diagram might refer to a solution containing ^{252}Cf in which the kinetic energy $\eta_{s,n}$ of neutrons has been generated per unit mass. This is predominantly transformed into kinetic energy of ions in conversion $\eta_{n,i}$ and to a small part expended against binding energy in $\eta_{n,D}$. In a further degradation step kinetic energy of ions *) is partly transformed into kinetic energy of electrons in $\eta_{i,e}$ and partly expended against binding

*) The term ions is here used for ionizing heavy charged particle i.e., for energetic protons and heavier charged particles.

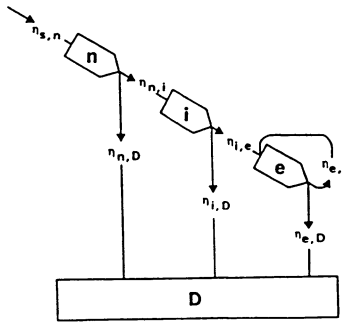

Fig. 1

Energy degradation diagram for neutron radiation.

The pointed enclosures represent kinetic energy of neutrons (n), ions (i), and electrons (e). The rectangle represents absorbed dose. The arrows symbolize energy conversion, i.e. energy converted per unit mass during the time of interest:

$\eta_{s,n}$: neutron energy released from unspecified source

$\eta_{n,D}$: energy expended by neutrons against binding energy

$\eta_{n,i}$: energy transferred from neutrons to kinetic energy of ions

$\eta_{i,D}$: energy expended by ions against binding energy

$\eta_{i,e}$: energy transferred from ions to kinetic energy of electrons

$\eta_{e,D}$: energy expended by electrons against binding energy

$\eta_{e,e}$: energy transferred from electrons to kinetic energy of electrons

energy in $\eta_{i,D}$. A final step of degradation is the expenditure of kinetic energy of electrons against binding energy in $\eta_{e,D}$. The related degradation term $\eta_{e,e}$, the transfer of electron energy to other liberated electrons, leads back to the same energy compartment and could, therefore, be omitted in the diagram. However, it is here included because it needs to be considered in a subsequent analysis in terms of the linear energy transfer of electrons.

As stated, the rectangle represents energy transferred from ionizing radiation to the exposed material, and hence the absorbed dose, D, is equal to the sum of the energy conversions terminating at the rectangle.

Energy conservation requires, that under complete equilibrium $\eta_{s,n}$ equals the absorbed dose. Furthermore the influx equals the eflux for each of the kinetic energy compartments, and the absorbed dose, therefore, also equals $\eta_{n,D}+\eta_{n,i}$ or $\eta_{n,D}+\eta_{i,D}+\eta_{i,e}$. Thus:

$$\eta_{s,n} = D = \eta_{n,D}+\eta_{n,i} = \eta_{n,D}+\eta_{i,D}+\eta_{i,e} = \eta_{n,D}+\eta_{i,D}+\eta_{e,D} \qquad (1)$$

γ-rays due to neutron-induced nuclear reactions have here been disregarded, although they can often be important. The case of photons will, instead, be considered separately.

2.2 The Example of a Photon Field

Fig.2 shows a diagram for photons which is drawn in analogy to Fig.1 and could be linked to Fig.1, to represent the situation of a mixed neutron and photon field. Again, certain complexities - such as changes of rest mass or production of neutrons at high photon energies - are disregarded in the interest of simplicity. However, $\eta_{e,ph}$ denotes the important generation of photons in the liberation and in the deceleration of electrons (fluorescence photons and bremsstrahlung).

Pair production and annihilation radiation would require an added loop that represents the transient change of photon energy into electron-positron rest-mass energy and its subsequent complete reversion to photon energy. But this closed loop which does not interfere with the remaining routes of energy degradation is omitted in the diagram.

One obtains from Fig.2 the analogue to Eq(1):

$$\eta_{s,ph} = D = \eta_{ph,D} + \eta_{ph,e} - \eta_{e,ph} = \eta_{ph,D} + \eta_{e,D} \qquad (2)$$

3. Intermediate Quantities

3.1 Kerma

The kerma, K, is the sum of the initial kinetic energies of charged ionizing particles liberated by uncharged particles per unit mass of irradiated material (1). For example the neutron kerma is equal to the term $\eta_{n,i}$ in Fig.1 and, as each of the flow terms, it can be

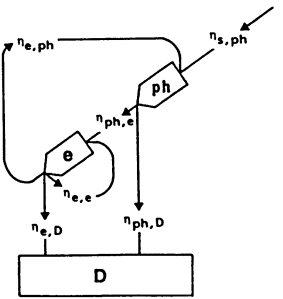

Fig. 2

Energy degradation diagram for photon radiation.

The symbols are analogous to those in Fig.1. The symbol ph represents photon energy. The conversion terms are, apart from those in Fig.1:

$\eta_{s,ph}$: photon energy released from unspecified source

$\eta_{ph,D}$: energy expended by photons against binding energy

$\eta_{ph,e}$: energy transferred from photons to kinetic energy of electrons

$\eta_{e,ph}$: energy transferred from electrons to photons (e.g. bremsstrahlung).

expressed as an integral in kinetic energy, T, over fluence and certain interaction coefficients:

$$K = \eta_{n,i} = \int_0^\infty T \left[\mu_{tr}(T)/\varrho \right] \varphi_n(T)\, dT \qquad (3)$$

$\varphi_n(T)\, dT$ is the fluence due to neutrons of energy between T and T+dT, it will subsequently be termed the fluence spectrum (in energy). The term $\mu_{tr}(T)/\varrho$ is the mass-energy transfer coefficient (1) of the neutrons in the specified material. One concludes that the neutron kerma K is, under the condition of complete equilibrium, slightly less than the absorbed dose, $D = \eta_{n,D} + \eta_{n,i}$.

If the source or the medium are not uniform, kerma and absorbed dose will differ more markedly. The spatial variations of absorbed dose are somewhat smaller than those of kerma, because the distribution of absorbed dose reflects energy dissipation not only by the uncharged particles but in addition also by the charged particles.

The degradation diagrams in Fig.1 and 2 have been referred to the condition of complete equilibrium, and the flow terms were interpreted as energy densities, i.e., as energies transformed per unit mass. The diagrams can, however, also refer to the non-uniform condition, if the flow terms are understood as the total energies converted in a radiation field, or if they are considered - with suitable scaling - as the energy densities integrated over the exposed medium and divided by the mass of this medium. Eqs(1) and (2) can then be seen as identities between spatial averages of various dosimetric

quantities. For brevity we will say that the quantities are equal on average. Thus one concludes from Eqs(1) and (3) that absorbed dose and ($\eta_{n,D}$+kerma) are equal on average, and that, accordingly, the neutron kerma is somewhat 'smaller on average' than the absorbed dose. However, $\eta_{n,D}$, the energy expended by uncharged particles against the binding energies, can usually be disregarded, and the neutron kerma is then nearly equal on average to absorbed dose.

Precise equality on average would obtain for neutrons if kerma had the slightly different definition:

$$K' = \eta_{n,D}+\eta_{n,i} \tag{4}$$

i.e., if it were defined in terms of energy lost by the neutrons in liberating charged particles, rather than in terms of the energy appearing as kinetic energy of the liberated charged particles.

There is, on the other hand, no strong reason for this modification of kerma, because a difference on average between kerma and absorbed dose could even then persist for energetic photons. This is seen from the degradation diagram for photons in Fig.2.

Photon kerma is defined as $\eta_{ph,e}$ and can, therefore, even on average, be either smaller or larger than absorbed dose. A redefinition of K, to make it equal on average to D, would require inclusion of the binding energy expended in the liberation of electrons ($\eta_{ph,D}$) but exclusion of their generally more important production of photons ($\eta_{e,ph}$). Attix (6) has introduced collision kerma which accounts for the latter process and equals $\eta_{ph,e}-\eta_{e,ph}$, if rest mass changes are

omitted.

Unlike charged particles, uncharged ionizing particles have substantial mean free paths between collisions, and this implies that the fluence of uncharged particles is only gradually changed - due to absorption and scattering - when small receptors are introduced into a radiation field. A dosimetric quantity, such as kerma, that is defined purely in terms of the fluence of uncharged particles and their interaction coefficients has, therefore, values that pertain to small exposed objects without critical dependence on their size or shape. Kerma can thus be specified also for a material other than that at the point of interest (e.g. tissue kerma in free air) and it is defined even in the absence of material (e.g. kerma for any material in outer space).

These simple features make kerma a convenient intermediate quantity, but the actual use of kerma depends, of course, on its relation to absorbed dose. This aspect will be considered in somewhat broader context in Section 6, after an analogous intermediate dosimetric quantity for charged particle fields is introduced.

3.2 Cema

The diagrams in Figs.1 and 2 suggest that, in analogy to kerma which relates to the energy expended by uncharged particles in the production of charged particles, one can define also a quantity relating to the energy expended by these secondaries in turn. Thus in the case of ions the absorbed dose is equal on average to $\eta_{i,D}+\eta_{i,e}$ which can,

therefore, be used, as intermediate dosimetric quantity. One obtains the following equation:

$$C = \eta_{i,D} + \eta_{i,e} = \frac{1}{\varrho} \int_0^\infty L(T)\, \varphi_i(T)\, dT \qquad (5)$$

where $\varphi_i(T)$ is the fluence spectrum in energy of ions and L(T) is the (unrestricted) LET, i.e. the linear collision stopping power, of the ions *). Although this is not indicated here, one must sum over the different types of ions that are present.

The definition in Eq(5) differs from that of kerma in a major aspect: the kinetic energy released in the liberation of electrons is not the sole, dominant component; the energy expended against the binding energy of electrons is of comparable importance, and the inclusion of the term $\eta_{i,D}$ is, therefore, essential.

C equals absorbed dose on average but shows somewhat different spatial variations under non-equilibrium conditions. The kerma, K, disregards the energy transport by the comparatively long ranged charged particles immediately produced by uncharged particles; C disregards merely - as it is common in the continuous slowing down approximation (CSDA) - the energy dissipation by the short ranged δ-rays. The differences between D and C are, therefore, substantially smaller and more local than those between D and K.

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*) In order to simplify notation the more explicit symbol L_∞ is replaced by L.

The importance of the term $\eta_{i,D}$ obviates the use of the name ion kerma for C. One can instead speak of converted energy per unit mass and accordingly utilize the term cema for C.

Cema has evident applicability whenever one deals with radiations such as charged particles emerging from accelerators or charged particles in solar or galactic cosmic radiation. It is a rigorously defined quantity to replace the somewhat ambiguous, but frequently invoked concept of absorbed dose 'under electron equilibrium'.

While there is no ambiguity in the definition of C for ions or other charged particles (such as mesons), complications arise for electrons, because the analogue of Eq(5) for electrons provides a quantity,

$$\eta_{e,D} + \eta_{e,e} = \frac{1}{\varrho} \int_0^{\infty} L(T)\, \varphi_e(T)\, dT \,, \qquad (6)$$

($\varphi_e(T)$: fluence spectrum in energy of electrons)

that can be considerably larger than the absorbed dose which equals $\eta_{e,D}$. To obtain a quantity that equals D on average one needs to integrate not over the entire electron fluence, $\varphi_e(T)$, but over the fluence, $\varphi_\varepsilon(T)$, of primary electrons only *). This results in the cema for electrons:

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*) The term primary electrons denotes all electrons except δ-rays.

$$C_e = \frac{1}{\varrho} \int_0^{\infty} L(T)\, \varphi_\varepsilon(T)\, dT \qquad (7)$$

($\varphi_\varepsilon(T)$: fluence spectrum in energy of primary electrons)

The quantities in Eqs(6) and (7) are equal for an electron beam in vacuo that is not accompanied by δ-rays. However, in matter the expression in Eq(6) is substantially larger on average than the absorbed dose, because some of the energy transmitted by the incident fluence is repeatedly added, as it is dissipated by successive generations of δ-radiation. Fig.3 illustrates, for the example of electrons released by 100 keV photons, the substantial difference between the integrals in Eqs(6) and (7); it also illustrates the broad overlap of the δ-ray and the primary electron-fluence spectrum.

Eq(7) can be employed in calculations, where the primary fluence can generally be separated from the fluence of δ-rays which extends up to one half of the maximum electron energy *); examples are dosimetric calculations such as the extension of the Bragg-Gray principle by Laurence (8) or Spencer and Attix (9) in terms of the CSDA, or modified CSDA-computations that account in terms of averages for the production of δ-rays. However, C can not be evaluated on the basis of the electron-fluence spectrum at a given point and it can, in fact,

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*) The convention that a δ-ray can not have more energy than the parent electron means that the maximum δ-ray energy is $(T-b_{min})/2$, where b_{min} is the minimum binding energy. But here and in the subsequent treatment b_{min} will be neglected.

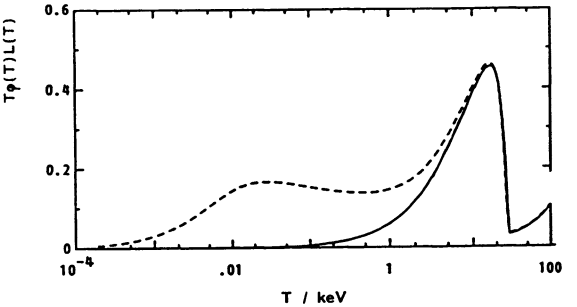


Fig. 3

$T\varphi_e(T)L(T)$ (dashed line) and $T\varphi_g(T)L(T)$ (solid line), the arguments of the integrals in Eqs(6) and (7) multiplied by T to indicate relative contributions in this logarithmic plot that apply to 100 keV photons in water. The area under the solid curve is normalized to unity. (K.Hahn, unpublished data).

have different values for the same fluence and the same energy distribution of fluence, depending on the fraction of fluence that is due to δ-rays. The resulting difficulty in applying cema to electrons can be avoided, but this requires, as will be seen, a modified cema and a changed definition of LET.

3.3 Reduced Cema

Fig.4 is a modification of the segment of the degradation diagram that relates to electrons. Δ represents here a cut-off for the kinetic energy of electrons. One can exclude electrons below this cut-off from the radiation field in the sense that they are assumed to 'dissipate their energy on the spot'(3). This energy is then included with the energy imparted to matter. The approach is an alternative to the concept underlying Eq(7); rather than disregarding energy dissipation by all δ-rays, one disregards energy dissipation by all electrons (including the primaries) below the chosen energy Δ. To indicate the modified convention, the symbol C_Δ is utilized instead of D. A cut-off is, in fact, implied also in the definition of absorbed dose which invokes the notion of 'ionizing particles', even though there has been no numerical specification of a minimum kinetic energy (1). The ambiguity is unavoidable because there can be very low or poorly defined ionization potentials in condensed materials. This difficulty has, however, little practical consequence, because there is only short range energy transport by low energy electrons.

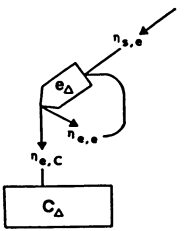


Fig. 4

Modified energy degradation diagram for electrons.

The symbol e_Δ represents kinetic energy of 'fast' electrons, i.e., of electrons with energy larger than Δ. The rectangle stands for reduced cema. The arrows symbolize energy converted per unit mass during the time of interest:

$\eta_{s,e}$: energy of 'fast' electrons released from unspecified source

$\eta_{e,C}$: energy expended by 'fast' electrons against binding energy and kinetic energy of 'slow' electrons emerging from interactions.

$\eta_{e,e}$: energy transferred from 'fast' electrons to kinetic energy of their 'fast' δ-rays.

It is instructive to consider first the limit case of the cut-off $\Delta=0$. The diagram in Fig.4 is then equivalent to the corresponding segments of Figs.1 and 2, and C_0 equals very nearly the absorbed dose:

$$C_0 = \eta_{e,C} = \frac{1}{\varrho} \int_0^{\infty} \lambda_0(T)\, \varphi_e(T)\, dT \approx \eta_{e,D} = D \qquad (8)$$

Here $\varphi_e(T)$ is the fluence spectrum of electrons, and $\lambda_0(T)$ is the mean energy expended by an electron of energy T per unit path length against binding energy in the material of interest. λ_0 equals the collision stopping power of the electron minus the sum of the kinetic energies of electrons released per unit track length.

Eq(8) is, in somewhat simplified form, the exact equation for absorbed dose which has been given by Alm Carlsson (7,10) and which is in agreement also with earlier formulations by Spencer (11).

The absorbed dose is, as exemplified in Fig.5, predominantly imparted by low energy δ-rays, but their fluence depends critically on receptor geometry and cannot, in general, be evaluated with sufficient precision. A suitable intermediate quantity must, therefore, be independent of the fluence of low energy δ-rays. While it is difficult to separate out all δ-rays when electrons are the primary charged particles, one eliminates, according to the above considerations, most of the dependence on δ-ray fluence by disregarding energy dissipation by all electrons below a chosen energy, Δ. The approach corresponds to the convention adopted in the cavity theory of Spencer and Attix (9).

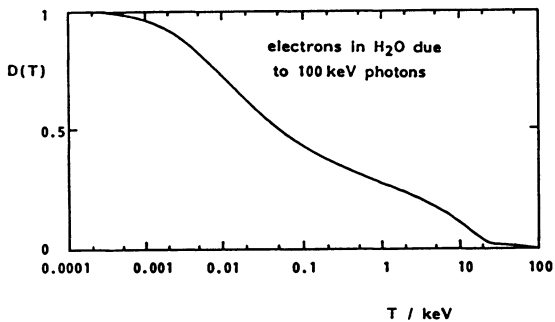


Fig. 5

The fraction, D(T), of absorbed dose contributed, in the ultimate degradation step, $\eta_{e,D}$, by electrons above energy T.

D(T) equals the fractional part of the integral in Eq(8) from T to ∞. The fluence $\varphi_e(T)$, of electrons (including δ-rays) due to 100 keV photons and the quantity $\lambda_\varphi(T)$ in water vapour are computed from the cross-section formulae of Olivero et. al.(12). (K.Hahn, unpublished data).

With the adoption of a cut-off, Δ , one obtains the intermediate quantity reduced cema, C_Δ, which equals the absorbed dose on average, but can deviate from it over spatial distances up to the range of electrons with energy Δ. Using the same approximation as in the simplest form of the Spencer and Attix theory (9,Eq(3)) one might wish to approximate reduced cema by the expression:

$$C_\Delta \approx \frac{1}{\rho} \int_\Delta^\infty L_\Delta (T)\, \varphi_e(T)\, dT \qquad (9)$$

However this equation excludes, according to the definition of restricted LET (1,3), the energy expended against binding energy in releasing 'fast' δ-rays in excess of kinetic energy Δ. It disregards, furthermore, the energy of 'track ends', i.e., of primary electrons or 'fast' δ-rays after falling below Δ. The energy of these track ends is, in the same way as that of low energy δ-rays, to be treated as if it were dissipated on the spot, but it is not contained in the integral of Eq(9). In the cavity theory the first inaccuracy has been uncritical, because comparatively large values Δ, substantially in excess of the binding energies, were employed which were equated - in the simplest, initial treatment - to δ-rays with energy 'just sufficient to span the cavity'(9). The exclusion of the binding energy in the infrequent production of the fast δ-rays is then insignificant, and this is reflected in the current, somewhat arbitrary definition of restricted LET. The second inaccuracy, too, is of comparatively minor influence in cavity theory, because it affects equally the two terms in a ratio, i.e., the energy densities

in the gas and in the wall material. However, Spencer and Attix have, even in their initial calculations (9), utilized modified formulations to account for the influence of 'track ends'.

In the present, more general context a rigorous formulation of reduced cema is required. Disregarding electrons with initial energy less than Δ - an approximation that will be retained subsequently, to simplify some of the formulae - reduced cema is given by the equation:

$$C_\Delta = \frac{1}{\varrho} \int_\Delta^\infty \lambda_\Delta(T)\, \varphi_e(T)\, dT \qquad (10)$$

where $\lambda_\Delta(T)$ is , for an electron of energy T, the linear rate of energy conversion to slow electrons and to binding energy. For large values of Δ and for $T>2\Delta$ the quantity $\lambda_\Delta(T)$ is only slightly larger than $L_\Delta(T)$. But substantial differences can occur for smaller values of T or Δ, and it is, therefore, necessary to consider $\lambda_\Delta(T)$ in detail.

The linear energy conversion rate, $\lambda_\Delta(T)$, can be expressed in terms of the cross sections differential in energy loss, W, of the electron and those differential in energy, E, of the released δ-rays. Let $\mu(W;T)\cdot dW\cdot dx$ be the probability of an energy loss between W and W+dW of the electron while traversing dx, and let $\mu'(E;T)\cdot dE\cdot dx$ be the probability of a δ-ray with energy E to E+dE being released along dx. One has then the relation:

$$\lambda_{\Delta}(T) = \int_{0}^{W_{max}} W\,\mu(W;T)dW - \int_{\Delta}^{E_{max}} E\,\mu'(E;T)dE + \int_{T-\Delta}^{W_{max}} (T-W)\,\mu(W;T)dW \qquad (11)$$

The first integral is the linear collision stopping power of the electron, i.e. its total energy loss in collisions per unit path length. The second integral represents that part of the energy loss that reappears as kinetic energy of 'fast' δ-rays. The last integral is the track-end term, it refers to collisions in which the electron energy falls below Δ and is added because it is treated as dissipated on the spot. Eq(11) refers to all values T in excess of Δ, however the requirement of $E>\Delta$ limits the contribution of the second integral to $T>2\Delta$, and the requirement that following a collision $T<\Delta$ limits the last integral to $T<2\Delta+b_{max}$, where b_{max} is the largest binding energy.

The ratio of $\lambda_{\Delta}(T)$ to the linear collision stopping power, L(T), for electrons is depicted in Fig.6. $\lambda_{\Delta}(T)$ is smaller than L(T) when the electron energy is larger than 2Δ. At energies below 2Δ it rises sharply and exceeds L(T); this reflects the discounting of the remaining energy of the electron as it falls below Δ. The dependences in Fig.6 are in general accord with Spencer's work (13) who has treated the problem of the 'track ends' analytically in terms of the transport equation and Møller's cross sections which disregard binding energies.

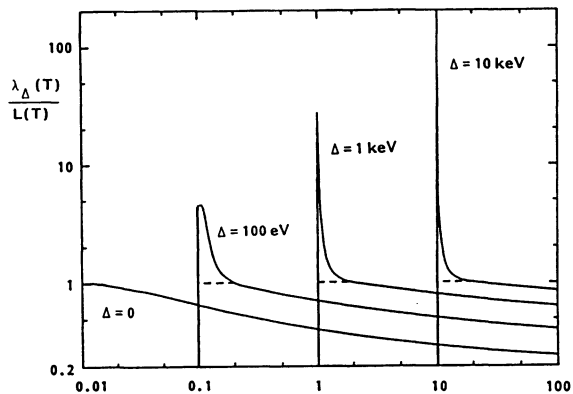


Fig. 6

The ratio, $\lambda_\Delta(T)/L(T)$, for electrons in water for selected cut-off energies.

Eq(11) and the cross sections given by Olivero et al.(12) for water vapor are used in the computations (K.Hahn, unpublished data). The peaks near Δ that account for the energy of the electron when it falls below Δ.

4. A Modification of Linear Energy Transfer

4.1 Reduced LET

$\lambda_\Delta(T)$ takes the place of restricted LET when the energy balance needs to be exact in dosimetric relations, e.g. in the Spencer and Attix cavity theory or in the relation for reduced cema (Eq(11)). In radiobiological considerations the situation is different. $L_\Delta(T)$ is used as a convenient, if only approximative, measure of local energy concentrations that determine the biological effectiveness of a radiation. Experimental findings indicate the important role of highly localised energy concentrations, and this is in line with the frequent reference to the small cut-off value Δ=100eV. The choice of this specific value of Δ is, however, largely arbitrary and it reflects merely the fact that restricted LET is meaningless at even lower cut-off energies, and specifically at Δ=0. The quantity $\lambda_\Delta(T)$ does not suffer from this restriction, and it is therefore a suitable parameter to replace $L_\Delta(T)$. However, the track-end term that applies to electrons, i.e. the sharp peak of $\lambda_\Delta(T)$ at energies T below 2Δ, is not a meaningful measure of local energy concentrations, and it is, therefore, more appropriate to utilize for radiobiological considerations the quantity that excludes the track-end term. This quantity which can also be employed for charged particles other than electrons:

$$\Lambda_\Delta(T) = \int_0^{W_{max}} W\,\mu(W;T)dW - \int_\Delta^{E_{max}} E\,\mu'(E;T)dE \qquad (12)$$

is similar to the restricted LET, $L_\Delta(T)$, but to facilitate the distinction it will here be termed reduced LET.

The use of three different symbols, $\lambda_\Delta(T)$, $\lambda_\Delta(T)$, and $L_\Delta(T)$ for three closely related quantities may appear undesirably complicated, but in the present context it is needed for conceptual clarity, and because the energy in track-ends or the energy expended against binding energies can be essential for a correct energy balance in dosimetric computations. The three quantities are, however, identical for $T>>\Delta$, and the definition of $L_\Delta(E)$ should ultimately be changed to agree with $A_\Delta(T)$. Apart from certain instances, where the track-end term needs to be considered for electrons, $A_\Delta(T)$ is, therefore, the only concept required.

Eqs(11) and (12) are more complicated than the relation for restricted LET:

$$L_\Delta(T) = \int_0^\Delta W\, \mu(W;T)\, dW \qquad (13)$$

but they share with this relation the convenient feature, that no knowledge of the 'double differential' cross sections is required. Let $\mu(W,E;T)\cdot dW\cdot dE\cdot dx$ be the probability of a collision occurring along dx with energy loss between W and W+dW with an emerging δ-ray of energy between E and E+dE. Eqs(11) to (13) require then merely the (marginal) spectra:

$$\mu(W;T) = \int E\, \mu(W,E;T)\, dE \qquad (14)$$

and

$$\mu'(E;T) = \int W\, \mu(W,E;T)\, dW \qquad (15)$$

which are more readily measured or computed than the two-dimensional spectrum $\mu(W,E;T)$.

$A_\Delta(T)$ is the linear rate of energy loss of a charged particle of energy T excluding all kinetic energy transferred to 'fast' δ-rays, i.e. to δ-rays with initial energy in excess of Δ. For electrons of energy less than 2Δ the reduced LET is, in the same way as restricted LET, equal to the unrestricted LET. For larger energies $A_\Delta(T)$ exceeds $L_\Delta(T)$, but the two quantities are, as stated, nearly equal for cut-off energies which are substantially larger than the ionization energies.

Numerical values based on a consistent, if not entirely accurate, set of cross sections can illustrate the relation between $A_\Delta(T)$ and the conventional quantity $L_\Delta(T)$. The solid lines in Fig.7 represent, for selected cut-off energies, the ratio $A_\Delta(T)/L(T)$ derived from the formulae of Olivero et al.(12) for $\mu(W;T)$ and $\mu'(E;T)$. These curves differ from the dependences in Fig.6 merely by the absence of the peaks that are due to the track-end contributions. The broken lines give the corresponding ratios for the restricted LET. For small values of the cut-off energy one recognises substantial differences, and largely similar results would be obtained for the reduced LET of ions.

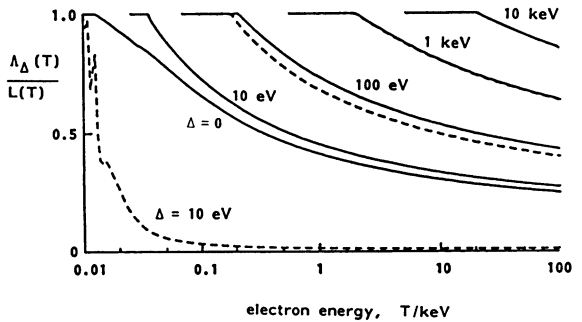


Fig. 7

Ratio, $L_\Delta(T)/L(T)$, of the reduced LET to unrestricted LET of electrons in water (solid lines) and the corresponding ratio, $L_\Delta(T)/L(T)$, of restricted LET to unrestricted LET (broken lines).

Eq(12) and the cross-section formulae of Olivero et.al.(12) are used in the computations (K.Hahn, unpublished data). The solid lines correspond for $T>2\Delta$ to the curves in Fig.6.

The restricted LET vanishes for $\Delta=0$. In contrast one can employ - as exemplified in Eq(8) - the 'completely reduced' LET. Eq(12) takes then the particularly simple form:

$$\Lambda_0(T) = \lambda_0(T) = \int_0^T W\,\mu(W;T)\,dW - \int_0^{E_{max}} E\,\mu'(E;T)\,dT = \bar{W}(T)-\bar{E}(T) \tag{16}$$

where $\bar{W}(T)$ and $\bar{E}(T)$ are the mean energy lost and the mean energy transmitted to δ-rays per unit track length. With regard to highly localised molecular effects of ionizing particles $\Lambda_0(T)$ is a more fundamental parameter than unreduced LET. But $\Lambda_0(T)$ is also a convenient scaling function for reduced LET which can be written in the form:

$$\Lambda_\Delta = \tau_\Delta \cdot \Lambda_0(T) \tag{17}$$

where τ_Δ is, according to the data in Fig.6 and for $T>2\Delta$, nearly independent of T. This approximation - which may partly be an artifact of the cross-section formulae by Olivero et al. (12) - highlights the fundamental importance of $\Lambda_0(T)$; it can be seen as the formal relation behind the assertion (14,15) that radiation quality is adequately, if not fully, described by the distribution of L_{100eV}.

4.2 Continuous Slowing Down Approximation of the Track-End Term

Instead of the rigorous solution for the track-end term one can utilize the CSDA. In the CSDA the electron falls below Δ when it reaches this energy. The contribution to C_Δ of 'track ends' of electrons of initial energy above Δ is then the product of Δ and the number, $n(\Delta)$, per unit mass, of such track ends. In the CSDA the fluence spectrum $\varphi_e(T)$ equals $\varrho \cdot n(T)/L(T)$, where $n(T)$ is the number of electrons, including δ-rays, per unit mass of the material, with initial energy in excess of T. Accordingly one has $n(\Delta)=L(\Delta)\varphi_e(\Delta)/\varrho$, and this provides the following approximation for reduced cema:

$$C_\Delta \approx \frac{1}{\varrho}\left[\int_\Delta^\infty \Lambda_\Delta(T)\,\varphi_e(T)\,dT + \Delta\cdot L(\Delta)\cdot\varphi_e(\Delta)\right] \qquad (18)$$

The comparison of Eqs(11) and (18) shows that the linear rate of energy conversion can be expressed in the simpler form:

$$\lambda_\Delta(T) = \Lambda_\Delta(T) + \Delta\cdot L(\Delta)\cdot\delta(T-\Delta) \qquad (19)$$

The track-end contribution, i.e. the peak in $\lambda_\Delta(T)$, is thus replaced by a Dirac delta function at $T=\Delta$, and this will be an acceptable approximation in most dosimetric computations.

From $\Delta\cdot L(\Delta)\,\varphi_e(\Delta)/\varrho = n(\Delta)\cdot\Delta$ and Eq(17) one obtains an interesting approximation for reduced cema:

$$C_\Delta \approx \tau_\Delta\cdot D(\Delta) + \Delta\cdot n(\Delta) \qquad (20)$$

where $D(\Delta)$ is the 'dose due to fast electrons'(see values in Fig.5):

$$D(\Delta) = \frac{1}{\varrho} \int_{\Delta}^{\infty} A_0(T)\, \varphi_e(T)\, dT \qquad (21)$$

while the last term in Eq(20) is the contribution of the 'track ends', i.e., of the primaries and fast δ-rays when falling below Δ.

C_Δ has the essential feature that it does not depend on the electron fluence below energy Δ . The magnitude of the fluence at energy Δ is, however, important because the track-end contribution, $\Delta \cdot n(\Delta)$, can be substantial, as is shown in Fig.8 for electrons of different initial energies. As stated earlier, Eqs(9) and (16) fail to account for primary electrons with initial energy below Δ. But with suitably chosen Δ the deficit due to the omitted source term is usually unimportant.

One concludes from Eqs(5), (10), or (17) that cema and reduced cema can - in the same way as kerma - be specified also for a material other than that at the point of interest (e.g. tissue cema in free air or air cema in the wall material of an ionization chamber). The equations must then contain the fluence above Δ that is actually present but the reduced LET of the reference material. The fluence below energy Δ does not appear in the formulae for C_Δ (Eqs(11) or (18)), but the equations reflect the implicit assumption of fluence equilibrium below Δ with respect to the reference material. This stipulated equilibrium pertains to the 'slow' δ-rays and to the 'track ends' of primary electrons and of 'fast' δ-rays.

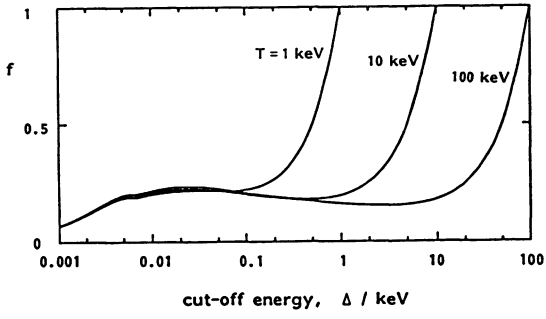


Fig. 8

The fractional contribution, f, of 'track ends' to the energy imparted, i.e. the relative contribution of the last term in Eq(18) or (20). (K.Hahn, unpublished data).

In this context one notes that C_Δ is independent of Δ and equal to the absorbed dose for a material with electronic equilibrium. However C_Δ for an absent material depends on Δ, even if there is electronic equilibrium in the material actually present.

5. The Concept of Reduced Fields

The multiplicity of dosimetric quantities makes a unifying concept desirable. Such a concept and the resulting general definitions will, therefore, be considered.

Energy imparted and related quantities, such as absorbed dose, are defined in terms of energy transferred from the radiation field to the exposed material. Before radiation energy is transferred to the material, it is degraded in successive steps from uncharged to charged, and from high energy to low energy particles. Part of this complexity can be eliminated by disregarding some of the later steps in the degradation process, and this can be achieved by excluding formally certain components of the radiation from the field and by treating their energy as dissipated on the spot. The extent of the exclusion determines the resultant intermediate quantity. By excluding all charged particles, one obtains, in essence, kerma. By excluding δ-rays, one obtains cema. By excluding merely the electrons below a chosen cut-off energy, one obtains reduced cema. Absorbed dose results when there is no exclusion.

The unified point of view suggests certain definitions that are of the same form as the ICRU-definitions of energy imparted and of absorbed dose, and that contain these definitions as special cases:

Category of particles:

A category, K, of ionizing radiation comprises ionizing particles of specified type and energy.

Energy transfer:

The energy transfer, $\varepsilon_{K,i}$, by a particle of category K is the energy converted from category K in a single interaction, i:

$$\varepsilon_{K,i} = T_{in} - T_{out} + Q$$

T_{in} = the energy of the incident particle of category K (exclusive of rest energy)

T_{out} = the sum of the energies of all particles of category K leaving the interaction (exclusive of rest energy)

Q = the conversion of rest mass into kinetic energy of particles of category K ($Q>0$: decrease of rest mass; $Q<0$: increase of rest mass).

Unit: J

Note: a) When the category comprises all ionizing particles, t energy transfer equals the energy deposit ε_i.

b) $\varepsilon_{K,i}$ may be considered as the energy transferred at the point of interaction, if quantum mechanical

uncertainties and collective effects (e.g. plasmons and phonons) are neglected. The point of interaction is also called the transfer point.

Energy converted:

The energy converted, ε_K, by a category of particles to the matter in a volume is:

$$\varepsilon_K = \sum_i \varepsilon_{K,i}$$

where the summation is performed over all energy transfers, $\varepsilon_{K,i}$, in that volume.

Unit: J

Note: When the category of particles comprises all ionizing particles, the energy converted equals energy imparted, ε.

C_K:

C_K is the quotient of $d\bar{\varepsilon}_K$ by dm, where $d\bar{\varepsilon}_K$ is the mean energy converted in matter of mass dm.

$$C_K = \frac{d\bar{\varepsilon}_K}{dm}$$

Unit: J kg^{-1} (special name: gray (Gy))

Note: When the category of particles comprises all ionizing particles, C_K equals absorbed dose. When the category of particles comprises all uncharged ionizing particles, one obtains a quantity that is nearly equal to kerma. When the category of particles excludes δ-rays, C_K equals cema for charged particles. Exclusion of electrons below energy Δ results in reduced cema.

6. The Role of Intermediate Quantities in Dosimetry

Dosimetric measurements generally determine the mean absorbed dose in the sensitive element of a detector (e.g. by ionization, heating, etc.), and one might, therefore, conclude that the evaluation of the intermediate quantities requires a regression, i.e. a backward evaluation, that is of little practical interest. When one deals with the irradiation of specified receptors, this is indeed the case. However, the intermediate quantities are useful, if one is dealing with radiation in free space (i.e., in a vacuum or in free air). On the one hand, they are indications of the dose generating potential of a radiation, on the other hand they can provide useful reference conditions for measurements with small detectors and can, thereby, obviate the need to chose a rigorously specified - but necessarily arbitrary - receptor geometry. Both aspects deserve some consideration.

Charged particles undergo frequent collisions and liberate in these collisions secondary electrons (δ-rays) with a fluence strongly dependent on local differences of the receptor geometry or composition. Absorbed dose is - as stated in section 3.3 and exemplified in Fig.5 - primarily produced by low energy electrons, and it is, therefore, highly sensitive to receptor geometry. Nevertheless, the term 'dose' is occasionally employed with reference to radiation at a point in air, or even in a vacuum, with the intent of specifying potential energy absorption in tissue. This terminology is inappropriate. According to the approach developed here, absorbed dose might be considered to be equal to C_0, but usually it is not feasible to measure this quantity that comprises only energy expended against binding energy, with exclusion of the energy of electrons that ionize in turn.

The reason for the frequent use of the improper terminology is the implicit notion of a transient radiation equilibrium that occurs below the surface of a receptor and is not greatly dependent on the receptor geometry. As Alm Carlsson (7) points out, there are various types of equilibrium which are not always clearly distinguished. One important case of absence of equilibrium occurs when a beam of uncharged particles (without accompanying charged particles) impinges on a block of material. The fluence of charged particles increases then up to a depth which corresponds to their maximal range, and near this depth a region of transient equilibrium is reached where kerma and absorbed dose are nearly equal. Most measurements pertain to this region, and it is instructive to examine this in some further detail.

Following production of charged secondary particles the uncharged particles retain some of their energy (i.e. elastic neutron scattering, Compton scattering), or secondary uncharged particles are produced (e.g. fluorescence radiation). However, unless the primary uncharged particles are very energetic or arrive in a broad spectrum of energies, the ionization in a homogeneous detector with wall thickness just sufficient to establish transient equilibrium is determined almost exclusively by the fluence of uncharged particles that would exist in the absence of the detector. Measurements of the absorbed dose are frequently performed in this arrangement, and usually they are effectively converted to kerma by a correction for attenuation of the uncharged particles in the build-up layer. This conventional linkage between kerma and an absorbed dose measured under a special receptor condition may tend to blur the distinction between the two quantities, but the essential difference is that kerma has a precisely defined value, while absorbed dose 'under charged particle equilibrium' is only loosely defined. Kerma is, therefore, the more suitable reference quantity for calibration purposes. Analogous considerations apply to cema, and this quantity can, therefore, be employed in the case of charged particle fields in free space.

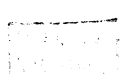
A specific example for the applicability of cema is its relation to cavity theory. When non-homogeneous (usually air-filled) ionization chambers are calibrated in photon fields, it is common to employ an energy cut-off for electron fluence, and the resulting approximations are largely equivalent to the use of reduced cema. The cavity theory of Spencer and Attix (6,3) can, in fact, be conveniently phrased in

terms of reduced cema. Its central statement is, that one measures in the air cavity reduced air cema, $C_{\Delta,\ air}$, in the wall. The cut-off Δ equals the energy of electrons with range comparable to the mean diameter of the cavity. The conversion factor, f, in the Spencer and Attix theory is, thus, equal to $C_{\Delta,air}/C_{\Delta,wall}$. The two quantities $C_{\Delta,air}$ and $C_{\Delta,wall}$ are determined by Eq(18), with the equilibrium electron fluence in the wall material but in the one case with the reduced LET for air and in the other case with the reduced LET for the wall material.

7. Conclusion

Kerma and the related quantity exposure are routinely employed in standardization and calibration of devices for the measurement of uncharged particles. They have also been commonly applied in evaluating radiation environments for purposes of radiation protection. Cema can serve analogous purposes for charged particles.

ICRU report 39 (17) recommends operational quantities that are appropriate in radiation protection and are related to a simple phantom, the ICRU-sphere. But a simple, and often sufficiently accurate, approach for 'free field' measurements is to determine $(Q_u K + Q_c C)$ where Q_u and Q_c are the quality factors for the uncharged and the charged particles, while K is the kerma and C the cema for charged particles that excludes energy transport by δ-rays. In most cases this is an overestimate of $H^*(10)$ and $H'(0.07)$, the ambient dose equivalent (at 10 mm depth) and the directional dose equivalent (at



0.07 m depth), which are the quantities recommended in ICRU report 39. This is so, not only because the maximum values, rather than those under a fixed depth, are involved, but also because partial equilibrium between uncharged particles and their charged secondaries may exist even under 'free field' conditions. In the case in which uncharged particles appear in substantial equilibrium with their charged secondaries, their contribution to the maximum absorbed dose in a phantom could be exaggerated by a factor of about two.

Measurements of the cema which excludes δ-rays are impractical for electrons, because the δ-ray fluence is often inseparable from the primary fluence. In agreement with formulations developed by Spencer and by Alm Carlsson one can express absorbed dose as an integral (see Eq(7)) over electron fluence times the completely reduced LET, $\Lambda_0(E)$. However, this is an abstract concept; the integral depends critically on the fluence at low electron energies which is difficult to measure and is highly dependent on local differences of the receptor geometry. Even in computations one encounters the further difficulty that $\Lambda_0(E)$ is less accurately known than the LET or the reduced LET with a sufficiently large cut-off. To obtain a more stable and easier to use intermediate quantity, one must, therefore, disregard energy transport by electrons below a chosen cut-off energy, Δ, and this leads to reduced cema, C_Δ, which depends only on the electron fluence at and beyond energy Δ. Spatial differences between the reduced cema and the absorbed dose can occur over distances smaller than the range of electrons with kinetic energy Δ.

The integrals over fluence that determine reduced cema require a modified definition of restricted LET, and to avoid confusion with

the present convention a different symbol, A_Δ, and a different name, reduced LET, have here been used for the modified quantity. A_Δ is the energy-loss rate of a charged particle excluding the kinetic energy of the δ-rays released with kinetic energy in excess of Δ. In the familiar definition of L_Δ one excludes the kinetic energy of the δ-ray as well as the binding energy when their sum exceeds Δ; a cut-off $\Delta=0$ is, then meaningless. With the modified definition one can - in line with earlier work by Spencer and by Alm Carlsson - choose zero cut-off energy, and A_0 appears in the integral over fluence that equals absorbed dose. While a distinction has here been made between L_Δ and A_Δ, it may be preferable to change the definition of L_Δ and to make it equal to A_Δ; the symbol L_Δ and the name restricted LET could then be retained. In fact, there appear to be few, if any, applications that require the present definition rather than the modified convention.

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