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VOTING AND THE CARDINAL AGGREGATION OF JUDGMENTS

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ABSTRACT

The paper elaborates the idea that voting is an instance of the aggregation of *judgments*, this being a more general concept than the aggregation of *preferences*. To aggregate judgments one must first measure them. I show that such aggregation has been unproblematic whenever it has been based on an *independent* and *unrestricted* scale. The scales analyzed in voting theory are either *context dependent* or subject to unreasonable restrictions. This is the real source of the diverse ‘paradoxes of voting’ that would better be termed ‘voting pathologies’. The theory leads me to advocate what I term *evaluative voting*. It can also be called *utilitarian* voting as it is based on having voters express their cardinal preferences. The alternative that maximizes the sum wins. This proposal operationalizes, in an election context, the abstract cardinal theories of collective choice due to Fleming and Harsanyi. On pragmatic grounds, I argue for a three valued scale for general elections.

Systems of voting and of counting votes are the mechanism by means of which the country records and measures its reactions to the political issues of the day. As with all recording and measuring devices, therefore, it is important that these systems should be as accurate, as reliable, and as impartial as we can make them, To tamper with them – or to tolerate the continuance of their known defects – is on par with using false weight and measures. (Lakeman, 1970, 9)

1. INTRODUCTION

Voting theory that deals with the properties of specific voting rules, as well as the more abstract theory of social choice, in the mold established by Arrow (1951), are remarkable for the negativity of their results. Arrow appears to have proven that no good method exists, while the record of voting theory suggests that we cannot even establish a ranking of relative merit among the available procedures. Thus, Tullock (2004), after contemplating the paradoxes of voting concludes:

All of this leads me to something I tentatively call “Tullock’s generalization”. It is: All voting methods are unsatisfactory when there are more than two possible alternatives”. Since there are always more than two alternatives to be chosen among this implies that the voting methods are always unsatisfactory. Lewis Carroll, an early student of these matters, recommended that whenever you run into one of these problems voting be suspended and the various people think about the matter more. If the difficulty remains, or for that matter, a new Paradox develops, he offered no solution. That is also my problem.

. Turning from theory to practice, *plurality voting (PV)* is still the far and away most used method, particularly in general elections. This in spite of the fact that it can lead to the election of a candidate supported only by a minority, or even of one strongly disliked by a large majority.

I argue that the intractable nature of social choice theory is not a reflection of a corresponding intractability of the relevant reality, but rather of a poor conceptualization of the required measurement operation. To aggregate preferences one must first measure them. Measurement requires a scale. The scales that have been used in social choice and voting theories can be classified along two dimensions. One is the familiar dichotomy *ordinal/cardinal*. The second dimension involves a dichotomy that I refer to as *context dependent/independent*. Context dependent means that the scale, depends on the objects being measured, in choice theory the alternatives. Independent means not context dependent. This dimension has not been considered before. The reason appears to be that context dependent scales do not exist outside of social choice theory and have therefore not been studied there. In social choice theory the conceptualization of the problem has not been along the lines of measurement and scales so that the concept was not analyzed here either.

The paradoxes of social choice arise from the fact that the ordinal scales that are taken as the starting point are also context dependent. On such a scale, the distance between two alternatives is given by the number of intermediate alternatives and changes as these are added or subtracted. This is the reason for the violation of Arrow’s axiom of independence of irrelevant alternatives. Both Fleming (1952) and Harsanyi 1955, 1977) have shown that social choice procedures that satisfy reasonable conditions are possible, as long as preferences are expressed by means of an independent cardinal scale. For a given normalization, each alternative has a unique value on this scale and the problem of irrelevant alternatives does not arise. The present paper may be interpreted as an operationalization, in the electoral context, of their abstract theories.

An independent ordinal scale does away with the paradoxes, but still does not tell us how to aggregate the votes in order to arrive at a decision in the absence of unanimity. The only aggregation method that has been used in practice is cardinal aggregation, i.e. the simple addition of votes, irrespective of the rules under which they were cast. Under traditional voting rules the voters' expression of cardinal preference is severely restricted. I define the *evaluative voting (EV)* rule as that which allows voters to express their cardinal preferences without restriction on a uniform, standardized scale. I propose that in a general election the scale should be *EV-3* with the scale (-1 (against), 0 (neutral), +1 (for)). In a committee of experts a more differentiated rule, *EV-5*, with the scale (-2,-1,0,+1,+2) may be appropriate. *EV* could also be referred to as the *utilitarian voting* rule.

I learned about two related papers after completing the present one. Felsenthal (1989) provides a game theoretic analysis of *EV-3* in small group voting. Smith (2000) advocates utilitarian voting with a *continuous* scale, which he terms *range voting (RV)*. In extensive computer simulations *RV* topped a long list of other methods. This outcome was largely induced by assigning random continuous utilities over the outcomes to the voters. In my opinion, *RV* is neither desirable nor feasible for actual elections. Both papers employ entirely different frameworks from the present one and are complementary to it.

The structure of the paper is as follows: The deficiencies of *PV* are the subject of Section 2. The problem of aggregating judgments is discussed in Section 3 with examples given from a variety of fields. I conclude that a scale with three to five different values is sufficient in virtually all instances. Section 4 shows that all conventional voting systems can be interpreted as being cardinal, but with severe and arbitrary restrictions on the expression of preference. Section 5 discusses the scale from a more theoretical point of view including the argument of Brams and Fishburn in favor of dichotomous preferences. Section 6, discusses the *interpretations* that have been given of the central concepts of social choice theory, *preference* and *utility* and tries to fathom theorists' preference for ordinal over cardinal measures. Section 7 discusses Harsanyi's second derivation of his utilitarian *SWF* and shows that the same argument can be used to derive the *EV* rule. Strategic voting is discussed in Section 8. Section 9 examines the motivation for voting. I argue that an expressive component of the motivation to vote is better satisfied if voters are allowed to vote against as well as for candidates. Criticisms that have been advanced in the literature against cardinal voting in general and approval voting in particular are examined in Section 10. Section 11 examines the voting paradoxes in the light of cardinal theory. Section 12 deals with a currently popular proposal for electoral reform, *Instant Runoff Voting (IRV)*. Standard runoff voting in multi-candidate elections is the subject of Section 13. Section 14 deals with the lack of impact from voting theory to voting practice and asks if the theory is 'abnormal science'. The conclusions are in Section 15

2. WHAT'S WRONG WITH PLURALITY VOTING?

For those who have studied voting mechanisms in the hope of improving them, the principal motivation has been to cure the defect of *PV*, that it may lead to the election of a minority candidate. This was true of Borda, of Brams and Fishburn (1983) and is also true of the present writer. Various examples of such pathological outcomes of *PV* are given by Brams and Fishburn (1983). Tabarrok and Spector (1999) argue that Lincoln was a minority candidate.

Discussing the issue raises a definitional problem: what defines a minority and what a majority candidate? With several candidates, it will be hard for anyone to win an arithmetic majority and the winner will necessarily be a minority candidate in this sense. The favorite definition of a majority winner in the literature is the Condorcet winner, i.e., the candidate who could win every pair wise contest. As Condorcet was himself aware, such a contest may be circular so that a Condorcet winner may not exist. Moreover, I argue below that the winner of an *EV* election is superior to the Condorcet winner if they differ. Fortunately, we need not resolve these subtle issues to agree on the inferiority of possible outcomes of *PV*.

PV elections that result in undesirable outcomes are of two types: the first is illustrated by US presidential primaries with many candidates, the second by US presidential elections when there is a third party candidate.

My interest in the subject was aroused by thinking about the US presidential elections in 1952 and 1956. The Democratic candidate in both elections was Adlai Stevenson, an intellectual with a sharp self-deprecating wit, the darling of the college crowd and rather disliked by the American mainstream. In the primaries Stevenson was opposed by bland mainstream candidates. Stevenson was elected because the mainstream vote was divided among his opponents. No Democratic candidate could have won against ‘Ike’ in these elections, but the fact remains that the Democrats probably chose their weakest candidate.

The other pattern is illustrated by third part candidates. A recent example is the candidacy of Ralph Nader in the presidential election of 2000. Most Nader supporters would probably have voted for Gore rather than Bush. In this very close and contested election, in which Gore actually received more popular votes than Bush, Nader’s candidacy probably tilted the scale in Bush’s favor. It is typical of such elections that the third part candidate takes votes away from the candidate that he is ideologically closest to and perversely favors the election of the candidate that he and his supporters most oppose.

From a more formal point of view, the problem with *PV* is that only the voters’ most favored alternatives are considered. A possible consequence is that an alternative that is the worst for a majority of voters may actually be chosen. A simple example is given in Table 1, where *abc* means *a* is preferred to *b* is preferred to *c*.

Table 1: Most Disliked Candidate Wins Under *PV*

Preference	<i>abc</i>	<i>cba</i>	<i>bca</i>
No. of Voters	3	2	2

Here *a* wins the plurality vote and is at the same time the worst alternative for a majority.

For anyone committed to the proper functioning of democracy, these examples and the many others that are discussed in the literature provide a strong incentive to look for a superior alternative to *PV*.

3. THE MEASUREMENT AND AGGREGATION OF JUDGMENTS¹

In the modern theory of social choice, the problem is generally taken to be the aggregation of *preferences*. The precise meaning of this term is left undefined, but it is clearly related to how a person expects a given outcome to effect him. This basic thrust is not negated by the possible presence of altruistic motives. When voting theory began to emerge first in medieval and then in early modern Europe, the conception was different. It was assumed that one candidate was objectively the best, the problem being that of finding her. To this end voters, for example the cardinals gathered to elect a new pope, would pray for divine guidance. Later, Condorcet and Borda, inspired by Bernoulli's work on probability, assumed that there was a probability distribution over the candidates that peaked at the correct choice. Voting was thus akin to sampling.

A preference ranking over alternatives is, in my view, a ranking of expected utilities. We can never be sure how the alternative we choose will ultimately turn out and how we will actually like the realization when it occurs. Preferences thus involve two kinds of judgments, about utilities and probabilities of outcomes.

There are other kinds of judgments that are regularly collected and aggregated. Firms and other organizations often ask to be quality ranked by their customers. Typically the customer is asked to express his views on either a numerical or a verbal scale. The verbal scale is usually analogous to (excellent, good, average, poor, unacceptable). These terms are then converted to numbers, for example (2,1,0,-1,-2). The numbers obtained over all respondents are averaged to obtain respondents' collective evaluation. The interpretation depends on the context. In a consumer context the numbers might be interpreted as cardinal utilities experienced as a result of the consumption of the good or service involved. In the case of commercial buyers, the numbers reflect expert judgments regarding quality.

The most common example of judgments expressed on a cardinal scale, familiar to all of us since early childhood, are the grades given in schools, colleges, universities, and other contexts that involve the evaluation of individual knowledge or capabilities. Typically, these grades are aggregated by means of repeated weighted averaging. For example, a short quiz will enter the final grade for a course with less weight than a longer examination. Course grades are again averaged, possibly subject to some restrictions, to obtain the final grade for the course of study. Final grades are expressed on a scale of five, maximally six values. For example, at German universities the grades are (1,2,3,4,5) with 1 standing for excellent and 5 for failure. In addition, a note of 0.5 is sometimes awarded for an exceptional performance.

Closely related to political elections, in fact often used to predict their outcomes, are political opinion surveys. These are of various kinds, but one kind involves the evaluation of politicians on a cardinal scale. An example is the 'Politbarometer' constructed by the German political survey institute *Forschungsgruppe Wahlen*. Respondents are asked to rate prominent politicians on a continuous scale from +5 (very positive) to -5 (very

¹ A fascinating history of social, to some extent also of physical, measurement is Duncan (1984). Interestingly, the first chapter deals with the history of voting, regarded by Duncan, as by the present author, as an instance of social measurement. The scales discussed in this paper are referred to in social science research as *rating scales*, and are a special case of *attitude scales*. A good survey of these scales and their history is Dawes (1972).

negative). The values are averaged to give a value for each politician.² The “feeling thermometer” of the University of Michigan Survey Research Center assesses voters’ feelings about candidates on a scale in the interval (0,100).

These examples show that the aggregation of different kinds of judgments on the basis of averaging cardinal scales is common. They also suggest that when a discrete scale is used, it is generally made to have 5-6 different values. This appears to be a limit to our ability to differentiate judgments along a cardinal scale.

In general, the fineness of a scale is a pragmatic issue. It depends on such factors as the ability to discriminate as well as a cost/benefit analysis related to the task at hand. No one would want to employ a precision scale to weigh the load of a freighter. My judgment is that, at least pending more experience and evaluation, a three valued scale (-1,0,+1) is fine for general elections and that a five point scale (-2,-1,0,+1,+2) should be used for votes in committees of experts.

4. THE CARDINAL INTERPRETATION OF VOTING MECHANISMS

In the following I limit myself to a discussion of voting rules that have been prominent in the literature or in practice, or that are relevant from the perspective of the present paper. I begin the discussion with the Borda count, because though it is cardinal like all voting procedures, it corresponds most closely to the idea of an ordinal scale.

a Borda count (BC)

It is assumed that the voter can establish a strict ranking over the alternatives that will be denoted by $abc\dots$, meaning that a is preferred to b , b is preferred to c , etc. Without further assumptions, the only decision that can be made on the basis of these rankings is the unanimity choice, if it exists. To reach a decision in the general case, the BC specifies the following rule: Given N alternatives, each voter assigns $N-1$ to the most preferred, $N-2$ to the next best, and so on, so that the least preferred receives 0. The candidate receiving the largest sum wins. It was clear to Borda that this amounted to a cardinal choice in which the intensity of preference, referred to by Borda as “degree of superiority” increased in equal steps in going from one choice to the next better.³

A complete statement of the Borda rule from a cardinal point of view is as follows: The scale has as many equidistant values as there are candidates, say N . These values must be assigned in the sequence $N-1, N-2, \dots, 0$.

b. Plurality voting (PV)

This can be defined as involving a cardinal scale with just two values (0,1), with the additional restriction that 1 can be given to only one candidate, with the rest receiving 0.

c. Cumulative voting (CV)

In this method, also known as bucket voting, each voter is given a fixed number of votes, usually equal to the number of candidates, and is free to distribute these over the candidates in any manner he chooses. Assume the number of candidates and votes given

² The institute has informed me that they chose the continuous scale to enable the respondents to rank each politician differently from the others. This does not seem to me like a cogent reason, but neither is there any harm in doing so in this case.

³ See *On Elections by Ballot*, in McLean and Urken (1995, p.85).

to each voter is N . Then there is an implied cardinal scale, $(0,1,\dots,N)$, with $N+1$ equally spaced values. The restriction is that N must also be the sum of votes. Thus, if one candidate gets the maximum N , all others must get 0.

d. Approval voting (AV)

As in plurality voting, there is a two-valued $(1, 0)$ scale. The plurality rule's restriction on the allocation of these values to the alternatives is lifted so that either value can be given to any alternative.

e. Evaluative voting (EV)

This is the only method that regards the issue of how many different values the scale should have as a pragmatic one. The number should be determined in relation to voters ability to make meaningful differentiations among the alternatives. I suggested above three for a general election, five for expert decisions.

Some conclusions:

No method for aggregating judgments can be better than its starting point, the determination of the individual judgments. Looked at from the perspective of measurement, the traditional methods *a* to *c* are severely defective. Having agreed upon a scale, it makes no sense to restrict the possible results that one might come up with when using that scale. Only *d* and *e* are free of such restrictions making them the only truly cardinal methods. There remains the issue of scale divisions. All methods except *e* and *g* have either 2 values or N , the number of alternatives. No discussion of these choices has been given; N as the number of values is completely arbitrary and 2 appears to be too restrictive.

Some additional insight into the desirable number of values can be gained by considering the number of different votes that are possible given each voting method. This information is given in Table 2. The cases in which all alternatives are given the same vote, equivalent to abstention, have been subtracted out.

Table 2: Number of Possible Votes

Alternatives	<i>PV</i>	<i>AV</i>	<i>EV-3</i>	<i>EV-5</i>
N	N	$2^N - 2$	$3^N - 3$	$5^N - 5$
5	5	30	240	3120

The table shows that from left to right the number of possible different votes by which a voter can express his judgment increases dramatically. From *PV* to *AV*, the increase is six fold and from there to *EV-3* 8 fold. The opportunity of choice given to voters by *EV-3* appears to be abundant for the purpose of a general election.

5. DERIVING THE SCALE

Voting theorists almost invariably take a context dependent, ordinal scale as their starting point. *EV* uses an independent cardinal scale. Usually, scales are simply postulated. The purpose of this section is to proceed in steps in order to see under which elementary assumptions the different cardinal scales that are in use can be arrived at.

The simplest way to arrive at a cardinal scale is to posit that the measured values must lie on a defined segment of the real line. The *Forschungsgruppe Wahlen* scale described above allows the respondent to state any number in the interval (+5,-5). The University Michigan Survey Research Centers “feeling thermometer” scale has the interval (0,100) and is for practical purposes continuous even if only integer values are given.

Continuous scales are a rare exception in practice. Their only advantage is that no decision on a specific discrete scale is required. The disadvantage is that reading and processing the scales becomes much more laborious and error prone. I do not regard them as a viable alternative, particularly in general elections.

Is there a plausible way of going, one step at a time, from a conventional context dependent/ordinal scale to one that is independent/cardinal. The answer is in the affirmative and given by Brams and Fishburn (1983). I limit myself here to a summary of their definitions and terminology.

Let P be the strict preference relation and assume that, for a given voter, P assigns the set of all candidates into disjoint subsets $A_1, A_2, \dots, A_K, K \geq 1$, such that the voter is indifferent between candidates in the same subset and strictly prefers a candidate $a_i \in A_i$ to a candidate $a_j \in A_j$ if and only if $i > j$. The number K being given independently of the number of candidates, this is an independent ordering. If $K = 1$, i.e., the voter is indifferent among all candidates, P is *unconcerned*; for $K = 2$, P is *dichotomous*; for $K = 3$, P is *trichotomous*; and for $K \geq 4$, P is *multichotomous*. The examples given in Section 2 strongly suggest that preferences are tri- or multichotomous, as does ordinary language with its many valuing terms such as ‘good’, ‘bad’, ‘indifferent’.

Having an independent ordinal scale still does not solve the problem of aggregation. The only reasonable solution that suggests itself is to construct a simple cardinal scale. This is accomplished for a given K by letting the voter assign to each alternative one of the numbers $x_0, x_0 + 1, \dots, x_0 + K - 1$, a higher number denoting a higher preference. The choice of x_0 does not effect the outcome. My suggestions for *EV-3* and *EV-5* imply respectively $x_0 = -1, x_0 = -2$.

Brams and Fishburn choose $K = 2$ and arrive at *AV* as their preferred voting rule. In view of the preceding discussions, this is a surprising choice. Their justification, given in Ch.3, will be informally described here. It is based on the Condorcet criterion which requires that the winner of an election should also be able to defeat any other candidate in a binary contest. Two problems associated with this criterion have been prominent in voting theory: *a.* No Condorcet winner may exist. *b.* Voting systems do not necessarily pick the Condorcet winner when he exists. Brams and Fishburn base their analysis on a result by Inada who had shown that a Condorcet winner always exists if preferences are dichotomous. They go on to show that in this case approval voting is the unique method that will necessarily pick the Condorcet winner.

I find this argument for approval voting unconvincing for two reasons:

- a. It is an instance of argumentation on the basis of unrealistic assumptions. If $K > 2$ is the correct description of voter preferences, then it must be incorporated in any theory that claims to empirically relevant. A theory that assumes $K = 2$ may have some neat mathematical properties; from an empirical point of view it is irrelevant.
- b. Inada’s result is just the statement for $K = 2$ of a property that hold quite generally for the cardinal aggregation of preferences as explained in Section 11. Inada focused on

$K = 2$ because majority decisions are traditionally made by voting with a two-valued scale. If $K = N$, and if voters are always sincere, including in a binary election, the winner of the multi-candidate election would also win any binary contest. Admittedly though, for $K > 2$ the incentive to vote strategically is present.

From the point of view of this paper, the fact that an *EV* winner may not be a Condorcet winner need cause no concern. It is due to the fact that *EV* allows a fuller expression of voter preferences.

The formalism developed by Brams and Fishburn is of independent interest in that it represents an *independent ordinal scale*. The formalism makes it immediately clear that Arrow's paradox is a consequence of the *situational* scale he chose. On the independent scale, the assignment of an alternative a_j to a subset A_j of the scale depends on a_j alone, and not on any other alternative. The condition *Independence of Irrelevant Alternatives* is always satisfied with a independent scale. This matter is taken up again in Section 11.

6. UTILITY AND PREFERENCE: CONCEPTS AND HISTORY

'Utility' and 'preference' are the most fundamental concepts of economics and, coming from there, have assumed a similar importance in political science, particularly in voting theory. The meaning of these concepts has, in spite of their centrality, by no means been satisfactorily clarified. On the contrary, I will argue that there are deep seated confusions both regarding the relevance of alternative concepts and about how to operationalizes them.

Both terms have a wide range of meanings, with considerable overlap. In economics and social choice theory 'preference' is linked directly to choice; the preferred alternative, if available will be chosen. The associated mathematical concept is that of a *preference ordering*, either of the *weak* form $a \geq b$, ' a is not worse than b ', or of the *strict* form $a > b$, ' a greater than b '. 'Utility', in a theoretical context, is usually defined as a numerical value given by a function $u = f(a)$, where a may be a discrete alternative, as in voting theory, or a vector of continuous variables as in consumer demand theory. Choice is then determined *indirectly* by assuming that the alternative chosen from the available set is the one that maximizes u . If the choice is invariant under a monotone transformation of $f(\cdot)$, as is the case in the static theory of consumer choice, utility is said to be *ordinal*. If choice remains invariant only under a positive linear transformation of $f(\cdot)$, we speak of *cardinal* utility. A monotone transformation leaves the ordering of the alternatives unchanged. A linear transformation leaves in addition the ordering of the *utility differences* of the alternatives unchanged. These utility differences are usually interpreted as *preference intensities*. Cardinal utility has been central to the theory of choice under uncertainty. According to Harsanyi, it is also central to the theory of social choice. Harsanyi's theory and its extension to voting will be the subject of the next section.

The long and complex history of these terms is beyond the scope of this paper. However, one important tendency should be mentioned: The ordering formalism has tended to dominate abstract theorizing, while cardinal approaches, based on utility, or more generally on judgments have dominated empirical work. This has led to an unhealthy estrangement of theory from empirical reality.

A watershed between theoretical and empirical realms was demarcated by the modern social welfare function (*SWF*) approaches. In the Bergson/Samuelson formulation, The *SWF* is defined on the *ordinal* utilities derived by a suitably defined population from a given state of the world. Arrow's *SWF* is defined on a similarly derived set of preference orderings. The two *SWFs* are quite similar, but they have contrasting fatal flaws that make them inapplicable for empirical applications. The Arrowian *SWF*'s fatal flaw, and at the same time its claim to fame, is that it does not exist. The Bergson/Samuelson *SWF* has the opposite problem: the number of function that satisfy the requirements of a Bergson/Samuelson *SWF* is infinite and no way has been found to show that a particular one is the correct representation of social preference. Moreover, neither approach gives a hint as to how *individual* preferences, the foundation of all else, are to be measured.

To fully understand the commitment of theorists to preference orderings, or ordinal utility, would require a full-blown history of these concepts. One aspect stands out: A long traditions regards subjective phenomena in general, and cardinal utility in particular, as suspect from a scientific point of view. The adherents of this tradition argue that scientific inquiry should concentrate on directly observable behavior. In economics this view began with the *marginalists* who claimed a superior scientific status relative to the classical economists adherence to the "metaphysical" doctrine of utilitarianism. Much later, Paul Samuelson, influenced by the doctrine of *operationalism* expounded by the physicist Bridgeman, developed his theory of *revealed preference*, based solely on observable consumer choices. *Behaviorism* in psychology exemplifies the same psychological stance.

Writers on cardinal utility have generally not been explicit about exactly what they mean by it. It seems clear though that what has generally been meant are our subjective *feelings* with which we *evaluate* aspects of the world and, in particular, the *intensity* of these feelings. The difficulty is seen in obtaining *objective measurements* of these feelings, rather than in their *relevance*. A current paper by Tullock (2004) clearly takes this position. After discussing the conventional paradoxes of voting, he writes:

Let me, however, introduced a little science fiction. We do not know now what will be discovered in the next hundred years. As we learn more about the brain it may be possible for us, with the aid of supercomputers, to determine how strong various people's feelings are. Probably the rating of the alternatives in the Borda method is to some extent an indication of the strength of people's feelings on the various alternatives. It is of course a very poor measure, but there's no reason to believe that we may not have, 100 years in the future, a very good measure.⁴

If we had such a measure we might prefer not that alternative, which has the highest number of first preferences, but that alternative which maximizes the total satisfaction received by the voters. Maximizing total satisfaction, if we could do it, would seem to be better than simply selecting the alternative, which is preferred by the most voters even if in some cases that preference is very weak.

Both my agreements and disagreements with the conventional view can be related to the foregoing passage. Of course, I agree that voters' preference intensities over the alternatives should be reflected in the vote. That is the central message of this paper and of all proponents of cardinal aggregation. What I fundamentally disagree with is the relevance of the psycho/physical intensity of preferences. I propose measures that are *arbitrarily normed by specifying the admissible utility values* that a voter is allowed to

⁴ Tullock is evidently unaware of the fact that the measurement of emotional responses to political advertisements is already being practiced. Cf. Johnston (2004).

report. These scales are uniform for all voters and give to each voter the same *power* to effect the election outcome. Some voters will feel much more passionately about the alternatives than others. I believe, and I think that almost all voters would agree, that such passions should not be rewarded by a proportional increase in voting power. Like all of the major scales that are used in the physical sciences, the voting scales that I propose are arbitrary up to a linear transformation. Only, when adding different measurements, we must use the same scale!

My conclusion is that the long tradition of skepticism about the relevance of cardinal utility as the basis of welfare economics and social choice is based on a counterproductive interpretation of the concept.

7. UTILITARIANISM, HARSANYI'S SWF AND VOTING THEORY

Utilitarianism

Utilitarianism was the philosophical underpinning of classical economics. It postulated that the ultimate purpose of social and economic arrangements is the maximization of the sum of individual utilities. The transition from mercantilism, with its emphasis on accumulating treasure for the sovereign, to modern economics was thereby marked. The constructive effect of utilitarianism on the development of economics and social philosophy cannot be over estimated. It did remain an abstract philosophy because it made no progress towards its stated goal of actually measuring individual utility. As mentioned above, this led to the counterattack of marginalism and the ultimate domination of ordinalism in welfare economics.

Harsanyi's SWF

A modern utilitarian theory of social choice was provided by Harsanyi (1955), based on an earlier contribution by Fleming (1952). In Harsanyi (1977, Ch. 4), titled 'Morality and social welfare', he gives an alternative derivation that I find to be conceptually superior as well as analytically simpler. His 1977 approach will be briefly discussed and then applied to voting theory.

Harsanyi's general theme is the derivation of a *SWF* based on an individualistic ethics. This means that the *SWF* should be based on individual preferences and should therefore be acceptable to the individuals involved. In order to make this idea more precise, Harsanyi uses two assumptions, of which the first is basic to individual decision making, and the second to social, or equivalently ethical decisions.

Assumption on I: Individual Decision Making

Individuals decide under uncertainty by maximizing the expected utility of the outcome.

Assumption II: Social Decision Making

In making an ethical decision regarding a desirable social state, individuals abstract from their own circumstances. Specifically, they assume that they could with equal probability be in the position of *any* member of society, including that person's external circumstances, as well as his tastes.

The Utilitarian SWF

Under the assumptions made, the expected utility of a social outcome a , to the j th individual, $w_j(a)$ is given by

$$(7.1) \quad w_j(a) = \frac{1}{N} \sum u_i(a) \quad ,$$

with u_i the utility of the i th. Further, the same condition holds for everyone, so that $w_1 = \dots = w_N = w$. It follows from *Assumption I*, that the optimal *SWF* is of the utilitarian form

$$(7.2) \quad w = \max(k) \frac{1}{N} \sum_i u_i(a_k) \quad ,$$

since thereby the expected utility of every individual is maximized.

Discussion

Assumption I is the standard assumption made in modern decision theory, usually based on a derivation from the axioms of von Neumann/Morgenstern or an equivalent formulation. The fact that the decisions people actually make under uncertainty often deviate from the expected utility maximization rule does not reduce its normative significance. Analogously, the fact that people often argue illogically does not reduce the normative power of logic.

Harsanyi's use of the assumption does involve a conceptual problem: It is not clear what it *means* to assume the preferences of another person and to use that preference in order to arrive at an expected utility related to one's own preferences. Harsanyi discusses this problem at length, but at a 'philosophical' level that leaves it open how the relevant utilities are to be determined and the theory operationalized in a concrete instance. In the application to voting that I make below, the problem does not arise, because the utilities in question are not subjective, they are the numbers entered by voters on a given *EV* scale.

Assumption II has come to be known as the 'veil of ignorance' assumption. The specific idea of a veil of ignorance is due to Rawls (1958, 1971) and has been employed by a number of modern authors⁵, but it is a particular realization of the central ethical idea of Western Civilization; that an individual ought to judge the ethical value of his actions not from a narrow selfish viewpoint, but rather from the point of view of society as a whole. Both the golden rule of the Bible: "Do unto others as you would have them do unto you," and the categorical imperative of Immanuel Kant: "Act only on that maxim through which you can at the same time will that it should become universal law." embody this idea.

The Application to Voting Theory

Assume that there is a constitutional convention, one of whose functions it is to determine the voting rule to be used in the convention's further deliberations as well as in future general elections. The members of the convention agree that the voting rule should be decided on from behind a veil of ignorance. Specifically, each member of the

⁵ For a review see Mueller (2003, Chapters 25, 26).

convention should assume that in future elections he could be, with equal probability, in the position of any voter participating in that election.

To apply Harsanyi's *SWF* to voting theory, we need to do no more than to reinterpret the $u_i(a_k)$ in (7.1) as being taken from a voting scale. For example, assuming *EV-3*, $u_i(a_k) \in (-1, 0, 1)$. Denote by s_t the scale with t values. The corresponding to *EV-t* rule is

$$(7.3) \quad \max(k) \frac{1}{N} \sum_i u_i(a_k), \quad u_i \in s_t.$$

The interpretation is, that if an individual thinks he can, with equal probability, be in the position of any voter in a future election, and if an *EV* rule is adopted, then the expected utility of the outcome to him, as measured by his *EV* score over the alternatives, will be as high as possible. Since this argument applies equally to any individual, the members of the constitutional convention would, under the assumptions made, *unanimously* adopt the *EV* rule.

I believe that the above argument, following Harsanyi's later derivation of his *SWF* amounts to a convincing justification of *EV*. It is, of course, not the only justification. In addition to the arguments made in other parts of this paper, it is possible to argue directly that the *SWF* defined by (7.2) and consequently the *EV* rule defined by (7.3) is the most plausible one, particularly given the absence of any serious alternative.

There is a common objection to a utilitarian *SWF* that I will briefly discuss. It is argued that it is insensitive to distributional considerations. The typical example given is a rise in average incomes accompanied by a worsening of the income distribution, so that the poorest members of society are worse off. This objection is based on the false assumption that individual preferences are purely selfish and do not take ethical considerations into account. This assumption is empirically false and in no way implied by utilitarianism. One may question if a collection of purely selfish individuals even deserve to be called a society.

SWFs Welfare Indicators

The concept of a *SWF* was developed by economists and is usually employed in economic analysis. In a series of papers culminating in Hillinger (), I argued that the aim of developing a universal cannot be operationalized and should be replaced by the concept of a *welfare indicator*, defined as a measure of some *impact* on welfare, not the totality of it. I discussed measures of *real expenditure* and *inflation*, but there are many others. A voting rule can also be regarded as such an indicator. It shows how a population's expected future welfare is related to the choice of some alternative.

8. STRATEGIC VOTING

In this section I deal with some aspects of strategic voting under *EV*. My argumentation is more elementary than it would be if I introduced the apparatus of game theory as has become customary in such contexts. I freely admit to lacking the required technical expertise, but in addition also doubt the empirical relevance of such an analysis. The typical assumptions of game theory are that all participants: **a.** are completely selfish; **b.** are completely informed about the preferences of all players; **c.** are completely rational according to a given game theoretic definition and assume all other players to be completely rational also. These assumptions are quite implausible in the case of general

elections. I believe that they are also implausible for voting in committees. Here the issue is generally not so much the aggregation of preferences as the aggregation of expert judgments about what is best for the organization. An example would be an investment committees choice among alternative projects.

In general elections voters receive their information about preferences of other voters largely from opinion polls. Regarding the reaction of voters to this information, Brams and Fishburn (1983, 110) write:

...there exist almost no theoretical models that offer insight into how voter reactions to the strategic information that polls provide may effect individual voting decisions, and how these in turn impinge upon election outcomes.

Brams and Fishburn suggest that the voter will chose the candidate he prefers among the two that are leading in the polls. This may be regarded as a bounded rationality choice.

In spite of all these limitations and uncertainties, some important statements about strategic voting can be made. Turning first to *PV*, there is a strong incentive to vote strategically for the candidate most preferred among those thought to have a chance of winning, rather than voting for the most preferred candidate in the entire set. The most favored candidate thus gets the same zero vote as the least preferred. This is a severely distorted expression of the voter's preference. The voter is also put into a considerable quandary if, as will usually be the case, the chances of the different candidates are unclear.

A great advantage of *EV* is that the voter has no strategic incentive to withdraw his vote from the candidates he likes best. To analyze the strategic vote under *EV* further assume that voter's preferences can be expressed *exactly* by means of an *EV-3* vote. A relaxation of this assumption will be discussed below. Let *ABC* denote respectively the sets of candidates evaluated with +1,0,-1. Clearly, the voter never has an incentive to vote insincerely relative to any candidate in *A* or *C*, so that his strategic choices are restricted to the set *B* of indifferent candidates. The choices are to either elevate one or more candidates in *B* to a (+1) vote, or downgrade them to a (-1) vote. The first choice, by increasing the chances of candidates originally in *B*, decreases those of candidates originally in either *A* or *C*. The second has the reverse effect. The choice depends on what the voter fears or desires the most as well as on his estimate of the probability of various outcomes. The choice may be difficult to make and the voter may decide to simply cast her sincere vote. If she does vote strategically, the distortion of her revealed preference will not be great, since the extreme sets *A* and *C* are not effected and only the indifferent candidates are moved by half a scale.

The assumption that the *EV-3* scale allows a totally accurate expression of voter preferences is somewhat unrealistic. Assume that a voter has in his preferred set *A* three candidates among whom he still has a strict preference ordering, *abc*. To enhance the chance of his favorite *a*, the voter could move *c*, or *c* and *b* to a lower ranking in *B*, or *C*. At best, the voter gains little, since the three candidates have the same *EV-3* rating. At worst, he may lose considerably, since he is enhancing the chances of cadidats in his sets *B* and *C*. It is therefore plausible that voters will not diverge much from the appropriate strategy given that *EV-3* is an exact representation of their preferences.

Another possible assumption is to postulate the Brams/Fishburn multichotomous-*k* scale, with set $k > 3$. A plausible strategy is to make a cut at some k' , $1 < k' < k$, such that -1

is assigned to all alternatives numbered $\leq k$ and +1 to all alternatives numbered $>k$. In this case AV reduces to EV .⁶

While it would be desirable to investigate strategic voting under EV more thoroughly, I conclude that, though it cannot be ruled out, it is much less serious than under PV .

9. WHY VOTE?

In the preceding sections I have adhered to the standard assumption of social choice theory that the voter is a perfectly informed and perfectly rational agent ($PIPRA$) seeking to maximize the expected utility of the outcome of his vote. As a description of reality, this assumption is problematic. Nevertheless, $PIPRA$ represents a simple ideal state and it is the plausible starting point for any analysis, though not necessarily the end point. In leaving the $PIPRA$ assumption aside, I will also abandon the formal style of reasoning from a set of assumptions in favor of a less formal, more descriptive style.

Within the theory of social choice, the issue of rationality arose in connection with the *paradox of voting*. Since the probability of any voter in a general election casting a decisive vote is nearly nil, it seems rationally inexplicable why people vote at all. Attempts at explaining this behavior, while maintaining the $PIPRA$ assumption, are unconvincing. Such models predict that participation would be higher in a close election, but this prediction is not supported by the empirical evidence.⁷ The probability of being a decisive voter is so small that, as has been remarked, it is exceeded by the probability of being hit by a car on the way to the polls. Moreover, the concept of a decisive voter is a mathematical rather than operational concept. In a general election, involving millions of voters, the error margin is in the thousands, or hundred thousands. Any close election will be challenged by the nominally defeated party. If a recount is also close, doubts will remain and the issue will be resolved either by a more or less arbitrary court ruling, or a new election. A more elementary, though no less convincing, refutation is that we all know from our experience of ordinary life that no one goes to the polls expecting to cast a decisive vote.

The principal alternative to the assumption that voting is goal directed is that it has an intrinsic value. Several explanations for this intrinsic value have been given. Perhaps the simplest is that people vote out of a sense of civic obligation. Brennan and Lomansky (1993) argue that voting is an *expressive* activity, similar to cheering at a sports event. Frey and Stutzer (1999) discuss evidence from a happiness survey conducted in Switzerland. Above a certain minimum income level, the felt ability of political *participation* turns out to be a key factor in promoting happiness. Participation and expression are related concepts, since to participate in an activity involves expressing ones self in that activity.

The above discussion contains an implication for voting theory. If voting is at least in part an expressive activity, then it is clearly desirable to be able to express, in the most direct manner possible, negative as well as positive feelings. In the current political culture of most democracies negative feelings about many politicians and parties abound. Many voters, lacking an attractive alternative to vote for, prefer staying at home. They

⁶ Brams and Sanver (2003) deal in detail with the game-theoretic properties of AV . Since EV generalizes AV , it may be expected that many of their results will apply with some modification to EV also. Felsenthal that in a voting game with perfect information EV reduces to AV .

⁷ The evidence is reviewed in Mueller (2003, Ch. 14) and in Brennan and Lomansky (1993, Ch. 7).

might be induced to vote if they are given the possibility of voting against their most disliked alternative. I feel that this is a strong argument in favor of *EV*.

10. THE CHOICE OF SCALE ONCE MORE

Saari and van Newenhizen (1988), hereafter *SVN*, advanced a fundamental criticism of a wide range of voting methods including *AV* and more generally cardinal voting. A defense, focusing specifically on *AV* was given by Brams, Fishburn and Merrill (1988), hereafter *BFM*, leading to a rejoinder by *SVN*. The impression conveyed by the exchange is that the two sides talked past each other. Saari (1994, 1995) repeated the criticism of *AV*.

Any criticism that applies to *EV* needs to be examined from the perspective of this paper. Moreover, I view the defense given by *BFM* to have been weak for principally two reasons: One is the restricted scale of *AV*, that makes it difficult to argue that it gives an accurate representation of voter preferences. The other is that *BFM* live in a halfway house, not having completely abandoned the traditional ordinal framework. From the consistent cardinal framework of the present paper, the *SVN* criticism is not only easily met, it is completely turned around.

The substantive discussion of this section also illuminates an important methodological issue: the relationship between mathematical rigor and the amount of care devoted to an examination of the relevance of the assumptions on which the analysis is based.

The central argumentation is as follows: Assume a voter has the strict preference *abc*. Under *AV* he cannot express this preference exactly and must arbitrarily choose to vote either (1,0,0), or (1,1,0). One can construct examples showing that, depending on such arbitrary choices, *any* outcome is possible.

The entire force of this argument is dependent on the assumption that the given strict orderings are exact expressions of the true preferences of the voters. *AV* then has the stated arbitrariness. If instead we assume that the cardinal *AV* values are the correct expressions of voters' preferences, then possible rankings and the corresponding *BC*s become arbitrary.

This is illustrated by the example shown in Table 3.

Table 3: Arbitrariness of Voting Rules

No. of Voters	3	2	2	Score
Order	<i>abc</i>	<i>bca</i>	<i>acb</i>	
<i>BC</i>	2,1,0	0,2,1	2,0,1	10,7,4
<i>AV</i>	1,0,0	0,1,1	1,0,1	5,2,4
Alt. Order	<i>acb</i>	<i>bca</i>	<i>cab</i>	
Alt. <i>BC</i>	2,0,1	0,1,2	1,0,2	8,2,11

The *AV* score is compatible with the given initial order and its *BC* as well as with the alternative order and its *BC* given below. The lower *BC* score reverses the initial one, showing the complete arbitrariness of strict orderings for a given *AV* score.

In terms of empirical relevance, the issue boils down to which formalism allows in general a better expression of voter preferences: a strict preference ordering and the

corresponding *BC*, or *EV*? The case for *EV* has already been made. The case for *BC* depends on there being no alternatives that are indifferent, or that the voter, lacking sufficient information, cannot distinguish. The probability of existence of such pairs increases with the total number of alternatives. Let k be the number of alternatives and p the probability that for any pair chosen at random, one alternative is strictly preferred to the other. For $k = 3$, there are 3 pairs and the probability that no two alternatives are indifferent, $(0.8)^3 = 0.512$. Even with only 3 candidates, the probability is nearly 1/2 that there will not be a strict ordering over all alternatives. For $k = 6$ there are 15 pairs and the probability of a strict order is $(0.8)^{15} = 0.035$. Assuming numbers of candidates that are common in multi-party, or multi-candidate elections, the probability of a strict order is seen to go to zero rapidly.

By way of contrast, in an *EV-k* election, a voter can report a strict order over up to k candidates, *he can, but is not forced*.

Saari has used the assumption of a strict preference ordering not only to attack *AV*, but also to support *BC*. He usually employs the assumption of a strict preference ordering without commenting on it from an empirical point of view. An exception is Saari (2001, 36):

Why strict preferences? This assumption seems to violate reality. All of us have experienced indifference between, say, whether apple or cherry pie is served; we all have stood in the voting booth liking one candidate but indifferent about the rest of them. We assume strict preferences primarily to simplify the analysis. After all, searching for explanations of voting difficulties has proved to be as difficult as searching for that proverbial needle in the haystack. Allowing voters to have all possible preferences is akin to complicating the needle search by dropping another load of hay on the pile. Rather than adding to the, disarray, let's first understand what happens when voters have strict, transitive preferences.

In accordance with the methodological stance outlined in the introduction, this statement sinks Saari's argumentation at least in so far as it is intended to be applicable to reality. The reader may think that I am doing injustice to Saari since he stated that he merely wished to "first understand what happens" under a strict preference ordering. But, this promise only leads to a new problem. Saari regards the alternative as being "all possible preferences". This is confirmed a little further on with the following definition and discussion:

Definition 1 Unrestricted Domain *requires each voter to have a strict, complete, transitive ranking of the candidates. There are no other restrictions on the choice of the ranking.*

This unrestricted domain condition completes our modeling of the rational voter. A voter without rational preferences is traditionally called irrational. As this term is somewhat harsh, occasionally I use more gentle terms of calling such a voter cyclic or unsophisticated.

By posing the choice as being between a strict preference ordering and *irrational* preferences, Saari eliminates all interesting alternatives. Specifically, these are the standard *weak orderings*, The *independent ordering* devised by Brams and Fishburn and discussed above, as well as all forms of *cardinal utility*.

The adherence of social choice theorists to, weak or strong, context dependent preference orderings is clearly not motivated empirically, it is rather an example of the persistence of tradition. To effectively argue for the cardinal alternative one must step out of this tradition.

Another criticism of AV , is due to Lawrence Ford, chair of the mathematics department, Idaho State University, who answered a question about AV in the *Ask the Experts* section of *Scientific American*:

One big flaw [of AV] is that most voters are fairly positive of their favorites and fairly positive of those they hate, but wishy-washy in the middle. If they choose randomly for or against approval in that middle range, the whole election can become random.

Directed against AV , this criticism has some validity because under AV , not to approve a candidate is equivalent to being against him. This puts the voter in a bind of having to be for or against, when in fact he lacks the relevant information for a judgment. Under EV , a zero vote, indicating indifference or abstention is genuinely neutral. The voter is under no compulsion to lump those he is ignorant about with those he either likes or dislikes.

11. CARDINAL THEORY AND VOTING PARADOXES

McLean and Urken (1995), note in their introduction that social choice theory has been very different from a normal progressive science. Essential elements of the theory have been independently discovered four times in the course of history, after prior efforts had been forgotten. There has been little effort at making the theory relevant for the political practice and correspondingly little impact. The rather obvious explanation is the overwhelming negativity of the results. As Tullock (2004) and others have noted, since most people, at least in the western world, are committed to democracy, they turned away from a theory that purports to prove its impossibility.

The central message of the present paper is that the voting paradoxes are the consequence of restrictions on voters' possible expressions of preference. When there are no restrictions, and voters cast sincere, non-strategic votes, then there are no paradoxes, at least not of the kind examined in the traditional voting theory. This result is, its importance notwithstanding, rather trivial from a formal point of view. This is because, in cardinal voting without restrictions involves nothing more than simple addition. The only substantial assumption required is that voters are able to evaluate alternatives by assigning a numerical value on an independent scale. This procedure evidently satisfies the following axioms:

A1. Unrestricted Domain

For any collection of votes, the outcome is defined, in that one or more alternatives will have received the maximum of votes. In case of a tie, the decision can be made by lot.

A2. Responsiveness

If a voter increases his vote for a candidate, the aggregate ranking of that candidate cannot decline.

A3. Equal Power

Each voter has the same power to effect the outcome. This is stronger than *Non Dictatorship*

A.5 Anonymity

Personal characteristics of a voter play no role.

A6. Independence of Irrelevant Alternatives

This follows immediately from the definition of an independent scale as one such that the value attached to an alternative depends on that alternative alone.

A7. Non-circularity

The literature discusses circularity in the context of conventional majority decisions between K alternatives, considered two at a time. This means that voters are restricted to a two-valued scale and can express indifference only by abstention. If they are given an adequate scale for expressing their preferences exactly and if they vote sincerely, then the vote total for each alternative will be unchanging and circularity cannot occur. This is discussed further in Section 13.

12. INSTANT RUNOFF VOTING

Instant Runoff Voting (*IRV*) is perhaps the currently most active proposal for electoral reform in the United States. It is being actively promoted by a public interest group, *The Center for Voting and Democracy*⁸. The basic voting rule is not new, being known in the literature under several names: *Hare system*, after its inventor who published the idea in 1873; *Single transferable vote (STV)*; *Preferential voting*.⁹ In the following I will refer to *STV* when discussing theoretical properties and to *IRV* when discussing the current US proposal, which relies heavily on the use of computers. The *STV*, as originally proposed and still practiced, requires two or more rounds of voting. If no candidate wins a majority in the first round, the candidate with the least votes is eliminated and a second vote over the remaining candidates is taken. If there is still no majority candidate, the process is repeated until a majority winner appears.

The innovation of *IRV* is to eliminate all subsequent voting rounds and instead have computers determine the end result on the basis of information obtained in the first round. Voters are asked to produce a ranking over all the candidates, or alternatively, if no more than three rounds are expected, a ranking over their first three choices. If there is no majority winner, the computers remove the candidate with the fewest votes from the rankings, so that for all voters who had this candidate as their first choice, the second choice advances to first place. On this basis the outcome of the second round is computed. If there is still no majority winner the process is repeated.

IRV is a purely technological innovation, unrelated to voting theory. Before turning to the theoretical aspects, I will discuss some practical ones. The proponents of *IRV* assume that election campaigns are a waste of time and money that could be eliminated by means of their proposal. While it is true that campaign regulations in the US and elsewhere are in dire need of reform, it is not true that they have no function. Campaigns do elicit information about candidates' abilities, plans, ideological biases, and last but not least, their character. Regarding the use of computers in elections, there is currently an intensive critical debate, independently of *IRV*. The 2002 presidential election in the US had shown that, particularly in regard to the disputed Florida vote, the world's most technologically advanced nation was unable to provide a tally that inspired confidence.

⁸ In addition to *IRV*, the *CVD* pushes another terrible, though popular, idea: proportional representation.

⁹ I discuss *IRE* only for the case where a single winner is to be determined. The Hare system was originally proposed, and still is usually employed, in connection with proportional representation where several candidates are to be elected. Winning candidates then require a smaller fraction of the vote than a majority.

Unless the use of computers in elections is very carefully planned, the possibilities for fraud, error, and inability to replicate the result will greatly increase. This is particularly true of *IRE* which is much more complex than a simple tally of votes.

I now turn to the more theoretical defects of *STV*. Fishburn and Brams (1983) list 4 ‘paradoxes’. An alternative and more accurate term would be ‘pathologies’.

NO SHOW PARADOX: the addition of identical ballots with candidate x ranked last may change the winner from another candidate to x.

THWARTED MAJORITIES PARADOX: A candidate who can defeat every other candidate in direct comparison majority votes may lose the election.

MULTIPLE DISTRICTS PARADOX. A candidate can win in each district separately, yet lose the general election in the combined districts.

MORE IS LESS PARADOX. If the winner were ranked higher by some voters, all else unchanged, then another candidate might have won.

At the end of their paper, Fishburn and Brams note that the defenders of *STV* might argue that these pathologies are so infrequent as to present no serious practical problem. They therefore suggest that a study of the probability of such outcomes would be useful. I view this judgment as being much too kind, since I am not aware that any particularly *good* properties have been proven for *STV*. Another reason is, that as the following example will show, *STV* suffers from a *mirror pathology of PV*. The pathology of *PV* is that an unpopular candidate may win. The corresponding pathology of *STV* is that at every round of voting the most popular candidate may be eliminated. This is illustrated in Table 4.

Table 4: Elimination of Most Popular Candidate under *STV*

No. of Voters	9	10	11	Total
<i>Initial PO</i>	<i>abc</i>	<i>bac</i>	<i>cab</i>	
<i>Sec. Round PO</i>	<i>bc</i>	<i>bc</i>	<i>cb</i>	
<i>EV-3</i>	2,1,0	1,2,0	1,0,2	39,29,22

According to the initial preference ordering, every candidate is in first place with one group of voters, but only *a* is never in last place. This suggests that *a* is the most popular overall. The impression is confirmed by the fact that *a* is the Condorcet candidate. In pair wise voting the results would be: *ab*(20/11), *ac*(19/11). A second confirmation comes from *EV*, which is also won by *a*. Under *STV*, *a* is eliminated in the first round.

There is nothing contrived about this example and we may expect the mirror pathology of *STV* to be just as serious as the well documented original pathology of *PV*. That a voting rule this bad is widely practiced and is seriously being considered for electoral reform in the US demonstrates that there is little influence from voting theory on the market place of ideas regarding electoral reform.

13. RUNOFF VOTING

If voters were perfectly informed about the candidates, a single vote would suffice. A runoff vote with the conventional majority rule would be positively undesirable, because with a 2-valued scale, many voters would be unable to give a precise expression of their true preferences. This could lead to the election of the second best candidate. The reality is quite different. Many voters are ill informed. A runoff election would allow the two remaining candidates to position themselves relative to each other and would give the voters more time and incentive to better inform themselves about the two finalists. I would therefore be in favor of runoff elections under *EV*. I would recommend though that the scales of the primary election be retained. This would give those voters who wish to cast sincere votes in the runoff election the possibility of doing so.

14. SOCIAL CHOICE: ABNORMAL SCIENCE?

My motivation in thinking about voting and in writing the present paper has been to promote election reform, a reform that I think is direly needed in political systems throughout the world. The need for electoral, as well as wider reaching constitutional reforms, is widely recognized throughout society. This is documented by Table 5, which lists the Google hits for some relevant terms:

Table 5: Google Hits on Political Reform

Search Term	Millions of Hits
Political Reform	4.33
Constitutional Reform	1.04
Election Reform	2.64
Approval Voting	1.38
Instant Runoff Voting	0.305

This public interest contrasts sharply with the apparent lack of interest in the implementation of practical reforms on the part of the current generation of voting theorist. This problem has been eloquently discussed by McLean and Urken (1995, Ch. 1) from whom I cite the following passages.

the repeated appearance, disappearance, and reappearance of social choice prompts the question, why is this such an abnormal science? There are other instances of dead ends in science. A famous one is the use of steam power by Hero of Alexandria to move statues in temples. But we know of no other case where a body of work was codified three times (Lull and Cusanus, Condorcet and Daunou, and Dodgson), and then lost every time so that the modern codifiers started from scratch. It cannot be that it is inherently difficult: even Condorcet, the most inaccessible of the codifiers, was using more elementary mathematics than his contemporaries and friends Laplace and Lagrange, whose work in physical science was immediately recognized as classic. (p. 11).

Our authors did not shy away from applications. Indeed, most of them started with applications and moved on to theory. In contrast, modern social choice theorists develop very general models and are consequently reluctant to give advice. To the nonspecialist, the obvious question thrown up by social choice is, What is the best electoral system? This is an urgent practical question all over the world, never more so than since the collapse of communism began in 1989. But whereas theoretical molecular biology has started to play a

large role in curing bodily disease, theoretical social choice has played almost no role in curing constitutional disease. (p. 12).

It is useful to supplement this general evaluation with a look at the history of the one major attempt made by academics during the past decades to reform the electoral process. I am referring to the efforts of Brams and Fishburn to popularize *AV*, beginning with their 1983 book. Fortunately, they recently provided their own evaluation of their experience in Brams and Fishburn (2003). Before turning to their paper, I will indicate how I regard *AV* and *EV* to be related as far as a reform agenda is concerned. The two are of course very closely related since *AV* is just *EV-2*. The step from *PV* to *AV* is the step from a bad voting system to a good one and is therefore huge. To the extent that the arguments offered in this paper are valid, the step from *AV* to *EV-3* is a further significant improvement that makes a good system better.

The history of *AV* in a nutshell is that it was adopted by a number of scientific societies, but hardly at all in general elections. Brams and Fishburn summarize their experience as follows:

The confrontation between theory and practice offers some interesting lessons on "selling" new ideas. The rhetoric of *AV* supporters has been opposed not only by those supporting extant systems like plurality voting (*PV*)-including incumbents elected under *PV* -but also by those with competing ideas, particularly proponents of other voting systems like the Borda count and the Hare system of single transferable vote.

We conclude that academics probably are not the best sales people for two reasons: (1) they lack the skills and resources, including time, to market their ideas, even when they are practicable; and (2) they squabble among themselves. Because few if any ideas in the social sciences are certifiably "right" under all circumstances, squabbles may well be grounded in serious intellectual differences. Sometimes, however, they are not. (p. 5).

Among the lessons we draw from our experience is that the adoption of *AV*, and probably any election reform, requires key support from within an organization. We never received this kind of support from politicians or political parties in our attempts to get *AV* adopted in public elections. (p. 25).

Brams and Fishburn give two principal reasons for the limited success of their efforts at popularizing *AV*: The inability of academics to agree among themselves and the unwillingness of vested interests to support reforms. I will discuss each of these in turn.

If it were true that academics have a fundamental inability to agree on any thing, societies would seem to be well advised to save the resources currently devoted to their maintenance, leaving them to conduct their disputations as private hobbies. The fact is that the natural sciences have been advanced by academics, or by applied scientists trained in the academies and having similar habits of thought. Natural scientists are if anything even more disputatious than those in the social sciences, who often simply go their separate ways.

The philosopher and historian of science Ravetz (1971) in Chapter 14, *Immature and Ineffective Fields of Inquiry*, describes an immature science as one that is unable to agree on facts. The interesting question is why do the natural sciences reach agreements on facts while the social sciences do not. A reason often given is that the natural sciences can experiment, whereas the social sciences cannot. This is hardly correct. It would be much cheaper and easier for society to experiment with alternative voting methods than to build another particle accelerator or orbiting observatory. Societies *choose* to do the latter, but not the former. Besides, not all natural sciences are experimental, viz. Astronomy, the oldest of them all.

The argument, while literally false, does contain an important and insufficiently appreciated germ of truth. Throughout most of the history of the natural sciences crucial observations and experiments could be made by individuals without requiring the cooperation of the larger society. If just a few interested persons repeated the observation or experiment, they could convince themselves of the validity of what had been claimed, and could then transmit the knowledge gained to a larger group. Eventually, the facts at issue would be common knowledge, at least among specialists, or the educated part of the population. Contrast this with the unsuccessful efforts of Brams and Fishburn, over the course of two decades, to run a social experiment in the form of a general election using *AV*.

Social experiments would be pointless if they did not generate information that would suggest some change in existing arrangement. But those who stand to lose the most under any change are precisely those who have benefited the most from the way things are. These are the people who have gained the maximum of power under existing arrangements and are in the best position to prevent change, or even any attempt at gaining knowledge that may, at some point, endanger the status quo. Not surprisingly, politicians elected under *PV* were uninterested in experimentation with *AV*.¹⁰

These objective difficulties should not blind us to the fact that social science generally, and voting theory in particular, have not been well structured, or motivated, to effectively promote social experimentation and intelligent social reform. I don't know if, or when, this will change, but I am willing to make the following prediction: If voting theory does not connect with and improve voting practice, it will once again cease to exist and fade from memory.

15. CONCLUSION

The paper offers a comprehensive argument for the cardinal aggregation of votes. Specifically, I advocate *evaluative voting* that allows a voter with regard to any alternative, to vote for or against it, or to abstain. I argue, that to allow voters to freely express their evaluation, positive or negative, is not only rational, but in addition better meets the emotional needs of voters and is therefore likely to increase participation. The central theoretical argument made for evaluative voting is along the lines of Harsanyi's derivation of a cardinal social welfare function.

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¹⁰ The argumentation of this and the preceding paragraph is also found in Tullock (1966), Ch. VII, *The backwardness of the social sciences*.

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