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UTILITARIAN COLLECTIVE CHOICE AND VOTING

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ABSTRACT

In his seminal *Social Choice and Individual Values*, Kenneth Arrow stated that his theory applies to voting. Many voting theorists have been convinced that, on account of Arrow's theorem, all voting methods must be seriously flawed. Arrow's theory is strictly ordinal, the cardinal aggregation of preferences being explicitly rejected. In this paper I point out that all voting methods are cardinal and therefore outside the reach of Arrow's result.

Parallel to Arrow's ordinal approach, there evolved a consistent cardinal theory of collective choice. This theory, most prominently associated with the work of Harsanyi, continued the older utilitarian tradition in a more formal style. The purpose of this paper is to show that various derivations of utilitarian SWFs can also be used to derive utilitarian voting (UV). By this I mean a voting rule that allows the voter to score each alternative in accordance with a given scale. UV-*k* indicates a scale with *k* distinct values. The general theory leaves *k* to be determined on pragmatic grounds. A (1,0) scale gives approval voting. I prefer the scale (1,0,-1) and refer to the resulting voting rule as *evaluative voting*.

A conclusion of the paper is that the defects of conventional voting methods result not from Arrow's theorem, but rather from restrictions imposed on voters' expression of their preferences.

The analysis is extended to strategic voting, utilizing a novel set of assumptions regarding voter behavior.

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1. INTRODUCTION

The impossibility theorem[1] of Arrow (1951) has shaped the subsequent evolution of both abstract collective choice and voting theories. As Levin and Nalebuff (1995, p. 4), in their introduction to a symposium on voting, put it:

> Arrow (1951) demonstrates that any voting system applied to an unrestricted set of voter preferences must have some serious defect; we must always choose between flawed alternatives.

The originator of this belief is Arrow (1951, p.6) himself:

> The chief relevant point here is that virtually every particular scheme proposed for election from single-member constituencies has been shown to have certain arbitrary features. The problem of choosing by election one among a number of candidates for a single position, such as the Presidency of the United States or membership in a legislative body when each district returns only a single member, is clearly of the same character as choosing one out of a number of alternative social policies; indeed, selection among candidates is presumably a device for achieving selection among policies.

Neither Arrow himself, nor subsequent writers appear to have noticed the contradiction involved. In his theory, Arrow allows the representation of preferences only as *orderings*:[2]

> We will therefore assume throughout this book that the behavior of an individual in making choices is describable by means of a preference scale without any cardinal significance, either individual or interpersonal. (p. 11).

Arrow's second statement invalidates the first. The reason is that every voting procedure, with the sole exception of the unanimity rule, is cardinal. Voters may be asked to express their preferences in different ways. For example, by making a mark next to the name of a candidate in a plurality vote (PV), by being required to state a strict ordering as in a Borda count (BC), by marking the names of candidates whom they approve, as in approval voting (AV), or in any way whatever. Regardless of how the individual votes are recorded, when it comes to aggregating them in order to determine a winner, they are first converted to cardinal numbers that obey the rules of ordinary arithmetic. Usually, the only arithmetic operation required is addition.[3] The use of cardinal arithmetic is explicitly ruled out in the Arrowian framework; hence his theory does not apply to voting.

The continued dominance of the Arrowian framework in voting and social choice theories, is puzzling for another reason. Starting at about the same time as Arrow's contribution, a number of authors derived utilitarian social welfare functions (SWF) that are free from

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[1] The result is usually referred to under this name, even though Arrow (1951) called it 'general possibility theorem'. Sometimes it is referred to as 'Arrow's paradox'.

[2] In Ch. 3, Section 6, he discusses the contrary utilitarian position at some length and rejects it.

[3] The only exception of which I am aware is maximum likelihood voting, where aggregation is done by means of linear programming. For a description see Young (1995).

Arrowian type contradictions. In his review of collective choice theory, Sen (1986, p. 1074) put the difference between ordinal and cardinal SWFs succinctly:

> It appears that some conditions that look mild – and are indeed satisfied comfortably by utilitarianism when translated into its cardinal interpersonally comparable framework – cannot be fulfilled by *any* rule whatsoever that has to base the social ordering on n-tuples of individual orderings.

Given Arrow's proof that no reasonable social choice mechanism based on preference orderings exists, and given a number of consistent and reasonable derivations of cardinal SWFs, the question arises as to why so much additional effort was invested in the Arrowian approach, rather than going from utilitarian social choice theory to applications, especially to voting. In Hillinger (2004) I suggested some tentative answers to this 'paradox of voting theory'. The purpose of this paper is to show that various approaches that have been used to justify abstract utilitarian collective choice theory can be specialized to derive utilitarian voting (UV).

Some definitions and clarifications will be useful before proceeding. The essential difference in collective choice theory is between ordinal and cardinal approaches. Instead of 'cardinal', I mostly use the term 'utilitarian' on the basis of its long tradition both in classical economics and in modern social choice theory. In that tradition numbers referred to as 'utilities' are added. In this paper the numbers relevant for voting are referred to as 'scores'. The interpretation is the same: the scores are the cardinal numbers that express voters' preferences regarding the alternatives at issue; they are the relevant cardinal utilities.

In this connection there arises the central issue of *measurement* that was the focus of my previous paper. This issue did not arise in collective choice theory because no empirical applications were proposed. It is clear though that the 'utilities' involved where viewed as 'belonging' to the individuals involved and that no external restriction on permissible values were involved.

Measurement universally proceeds by assigning a number to the object that is to be measured and, in the case of several objects, by adding the individual measurements to obtain an aggregate. Measurement, in fact, evolved alongside arithmetic and is, almost without exception, cardinal. An aspect of measurement that is so basic as to be hardly ever explicitly noticed is that the measure assigned to an object depends on that object alone and not on other objects. This is the basic difference between measurement and ranking. A ranking places objects in relation to each other and results in an ordering. A cardinal number assigned to an object indicates its place on a *scale* that is independent of other objects. This description

applies not only to the natural sciences, but also to the social sciences and economics, as I discussed in some detail in the previous paper.

The sole field that restricts the expression of preferences is voting. All voting systems that have been employed, or proposed, limit voters' freedom to assign the numbers of the voting scale to the alternatives. The sole exception is AV. An example, of a restriction is plurality voting (PV), the most popular of all voting procedures. Under PV, the voting scale is (1,0) and the restriction is that only one alternative can receive the score 1; all others must receive 0. If the restriction is lifted, so that either score can be given to any alternative, the result is AV. Another example is BR with the scale $(T, T-1, ..., 1)$, where T is the number of alternatives and the restriction is that every score must be used once. I will use the term restricted voting (RV) to refer to all such voting procedures. That common voting procedures limit voters' freedom to express their preferences appears not to have been explicitly discussed.

Define utilitarian voting (UV) as a voting rule that allows the voter to freely score each alternative in accordance with a given voting scale. In my previous paper I advanced two arguments in favor of UV. The first is an appeal to the universal practice of measurement, coupled with the belief that no plausible argument for restricting voters' preferences can be made. The second argument is somewhat more formal. I show that all voting methods obey two postulates, one on the voting scale and the other on aggregation. Add a postulate on *voter sovereignty* and UV is implied. This argument is taken up in Section 2 and introduces the formalism that will be used subsequently. Section 3 is a related but less formal argument. I point out that if the utilitarian principle of adding individual utilities to obtain the social outcome is accepted as the relevant ethical principle, and if the ballot scores are regarded as the relevant individual utilities, then UV is the immediate consequence. In the following two sections I use arguments that have been used to derive cardinal SWFs in order to derive UV. In Section 4 the idea is that electors at a constitutional convention choose the voting rule from behind a veil of ignorance regarding the alternatives to be decided on in future elections. Section 5 shows that axioms provided by May (1952) for majority rule and by d'Aspremont and Gevers (1977) for SWFs are all satisfied by UV, when suitable interpreted. In Section 6, I discuss strategic behavior under two assumptions that I believe are novel in this context: Voters prefer to vote sincerely and they like to vote for a winner. Under these assumptions, UV is shown to be largely immune to strategic voting. Section 7, relates the Condorcet criterion to UV. I argue that information is lost when a multi-candidate election is reduced to pair wise comparisons. In Section 8 I urge more empirical research on voter behavior under

alternative voting rules, as well as on voter preferences concerning the rules. The paper concludes with Section 9.

2. THREE CONDITIONS THAT IMPLY UV

2.1 Voting scales

A fundamental concept on which much of the paper is based is the voting scale.

Definition: Voting Scales

a. A voting scale is a set of numbers, $S_k = (s_k, s_{k-1}, ..., s_1)$, called scores, such that $s_j = s_{j-1} + d, \quad d > 0$.

b. Given a voting scale S_k, we can define another valid voting scale $S'_k = (s'_k, s'_{k-1}, ..., s'_1)$, with $s'_j = a + bs_j, \quad b > 0$.

In the following I will assume $d = 1$, which corresponds to the voting scales in actual use.

That scales are arbitrary up to a positive linear transformation is a universal property. For example, scales expressed in centimeters, meters, or feet, are related to each other by simple ratio transformations. Relating the centigrade and Fahrenheit scales requires in addition a shift of origin.

A crucial features of voting scales is that they are *truncated* and that the scale is the same for all voters under a given voting method. This assures that all voters have the same *power* to influence the outcome.

The definition leaves open the number of different scores, or *values* of the scale. These differ between voting methods. In my view they must be determined on pragmatic grounds. The choice should depend on voters' ability to differentiate between alternatives, as well as costs and benefits associated with a finer differentiation. To give some examples: PV and AV have *k*=2, EV has *k*=3, the Borda rule has *k*= the number of alternatives.

Condition VS: A voting method must be based on a voting scale as defined above.

2.2 Cardinal aggregation

Voting rules determine the outcome by adding the scores. In sequential voting systems this applies to the individual rounds of voting

Condition CA: The outcome of an election must be based on cardinal aggregation ,i.e., the simple sums of scores for the various alternatives.

2.3 Voter sovereignty

The restrictions imposed by the various RV rules are in my view a consequence of the fact that voting has not been conceptualized in relation to measurement. I can see no justification for such restrictions and therefore propose

Condition VS: A voting rule must allow for voter sovereignty, so that each voter is free to assign to any alternative any of the scores provided by the voting scale.

2.4 The implication: UV

The three postulates taken together define UV, with the number of possible scores left open. For a given *k*, the result can be written as UV-k, so that AV=UV-2, EV=UV-3.

3. UTILITARIANISM AND VOTING

Beginning with the early Utilitarians, particularly Bentham and J. S. Mill, utilitarianism provided the generally accepted ethical foundation of economics and much of social thought. The basic utilitarian position is that the aim of social policy should be the maximization of the populations total utility, defined as the sum of individual utilities. Utilitarians were convinced of the measurability uf utility in principle, but did not progress in operationalizing this view. In economics, utilitarianism was first challenged by the Marginalists in relation to consumer demand theory. An influential frontal attack on utilitarianism as the foundation of social and economic policy was launched by Robbins (1932). A central theme was the alleged impossibility of interpersonal comparisons of welfare. In the 1950s, Arrow and Samuelson cemented the ordinal approach to collective choice theory and welfare economics. In the 1950s, a counter movement began, most prominently associated with Harsanyi (1955, 1976), that aims at providing an analytical foundation for utilitarianism.

It is not my purpose here to review the vast literature pro and con utilitarianism, instead I limit myself to a few comments that are directly related to the purpose of this paper. A commitment to ordinalism and the associated view that interpersonal comparisons of utility are either impossible, or lack an ethical foundation, implies that only unanimous collective actions can be justified. Since unanimity in a sizeable population can never be achieved, this position is nihilistic. Voting methods may in fact be defined as devices that allow decisions to be reached in the absence of unanimity. They are therefore in fact, and out of necessity cardinal.

The connection between voting and cardinal social choice theory depends on

Condition CU: The scores by means of which voters express their preferences on an appropriate UV-k scale can be interpreted as their cardinal utilities regarding the relevant alternatives.

Given condition CU, it follows that UV is the utilitarian solution to the voting problem. Let $(a_1,...,a_T)$ be the set of alternatives and s_{ij} the score of the *i*th voter for the *j*th alternative. If a_h is the winning alternative, then from the definition of UV, $\sum_i s_{ih} = \max(j) \sum_i \sum_j s_{ij}$.

Accepting utilitarianism as the relevant ethical postulate is, in connection with condition CU sufficient to establish UV. In the following two sections I show how modern derivations of utilitarianism can be applied to voting.

4. DERIVING THE VOTING RULE FROM BEHIND THE VEIL OF IGNORANCE

A persistent theme of both religion and ethics throughout the ages has been that ethical behavior is not narrowly self-centered, but involves an emphatic identification with others. This idea was formalized by both Rawls (1958) and Harsanyi (1955, 1976). Rawls has been justifiably criticized for employing the maximin rule, while Harsanyi uses the standard assumption of expected utility maximization. While adopting Harsanyi's formal argument, I interpret it differently.

Harsanyi assumes that when an individual decides behind the veil of ignorance he assumes that in a future situation of collective decision making he could, with equal probability, *be* one of the other members of society involved in that decision. The following quotation (Harsanyi, 1976, Ch. II, p.22, footnote 16) makes this clear:

> Or rather, if he had an equal chance of being 'put in place of' any individual member of the society, with regard not only to his objective social (and economic) conditions, but also to his subjective attitudes and tastes. In other words, he ought to judge the utility of another individual's position not in terms of his own attitudes and tastes but rather in terms of the attitudes and tastes of the individual actually holding this position.

When Harsanyi goes on to postulate that an individual maximizes his expected utility under the assumption just described, he is not using the conventional assumption of maximizing *ones own* expected utility. Harsanyi here squeezes the concept of empathy into a narrow mathematical corset taken from individual decision making under uncertainty, but not evidently relevant for the social decision.

My position is that voting theory is distinct from ethics in the sense that there cannot be any control over the votes that are ultimately cast to determine if they are ethical or not. The

aggregation of votes may well be considered from an ethical viewpoint, but the evaluation of individual preferences is beyond the scope of voting theory.

In classical utilitarianism the utilitarian decision rule was advanced as a fundamental ethical principle, for which no further justification was either needed, or available. In my view, if the argument involving the veil of ignorance is to carry conviction beyond postulating utilitarianism directly, it must involve a straight forward application of expected utility maximization based on the deciding individual's own utility function. A similar position was taken by Rae (1969) and Taylor (1969) who also used the veil of ignorance argument to derive a voting rule. They considered the choice between two alternatives on the basis of a 2-valued scale and arrived at majority rule. Had they considered an arbitrary number of alternatives, while retaining the 2-valued scale, they would have discovered AV. The general case, with an UV-k scale and an arbitrary number of alternatives is considered below.

Assume that the members of a constitutional convention, hereafter called *electors*, have to decide, possibly among other issues, on a voting rule. Each elector is perfectly selfish, solely interested in how he himself will fare in future elections, as measured by her expected utility. The utility experienced by the *i*th voter in a future election given that the *j*th alternative is chosen is the score s_{ij} on her ballot, taken from the relevant voting scale.

Let N be the number of voters in a future election. The elector assumes that any of the ballots cast in that election could, with equal probability $1/N$, reflect her preferences in that election, If the *j*th alternative wins, the expected utility of this outcome to the elector is $\frac{1}{N}\sum_i s_{ij}$, the average utility of the *j*th outcome. Let h be the alternative that maximizes this expression:

$$\frac{1}{N}\sum_i s_{ih} = \max(j)\,\frac{1}{N}\sum_i s_{ij}\ .$$

Alternative, h that maximizes the electors expected utility is by definition the UV winner. This argument holds with regard to *any* election in which the elector may participate at some future time. UV is the election method that a rational utility maximizing elector would prefer. If all electors are expected utility maximizers and are able to understand the consequences of their choice, they will unanimously choose UV.

Implicit in this argumentation is not Harsanyi's assumption of an elector's serial *identity* with all voters of a future election, but rather an assumption of *similarity of tastes, or values* among the members of a society. If in a future election, alternative *a* gets twice as many votes as alternative *b*, then by assumption, the elector will be twice as likely to vote for *a* than for *b*.

A similarity of tastes and values may be regarded as a defining characteristic of a *society*, as distinct from a random collection of individuals.

This derivation, based on self interest only, clearly differs from the previous section where the utilitarian rule was simply postulated as a fundamental ethical principle. The argument involving the veil of ignorance is also distinct from Harsanyi's axiomatic derivation of a utilitarian SWF. That derivation gave rise to a literature that focuses on the intermingling of differences in tastes and differences in beliefs in shaping the differences in individual choices.[1] It would be interesting to have an analysis along these lines of the choice from behind the veil of ignorance.

5. AXIOMATIC DERIVATIONS OF UV

In this section I discuss two axiomatic systems that can be easily applied to UV. May (1952) postulated four conditions to derive majority rule (MR) for the case of two alternatives.[2] I reformulate these slightly so that they can be related to UV, which satisfied them easily. The following subsection considers a set of conditions due to d'Aspremont and Gevers (1977) that is necessary and sufficient to establish a utilitarian SWF.

5.1 May's conditions

Decisiveness: The voting rule produces a definite outcome for any pattern of individual preferences.

Anonymity: The outcome depends only on the votes cast, and not on which voter cast which vote.

Neutrality: If two alternatives receive the same total score, they have the same outcome.

Positive responsiveness: If a voter increases his score for some alternative, then the outcome for that alternative cannot be worse.

That these conditions are satisfied by UV is trivial and. I have not extended May's proof of sufficiency to the case of UV.

[1] A recent contribution is Gilboa, Samet and Schmeidler (2004).

[2] An exposition of May's result can also be found in Mueller (2003, 133-136).

5.2 The conditions of d'Aspremont and Gevers

d'Asprement and Gevers (1977) presented a set of conditions that are necessary and sufficient for a cardinal SWF.[1] I reinterpret these to fit the assumption that preferences are expressed as scores on a ballot. There is some overlap between these conditions and those of May.

Universal domain (UD): This is the same as May's decisiveness.

Independence of irrelevant alternatives (IIA): The choice from any subset of alternatives is independent of the preferences over alternatives outside that subset.

Unanimity (U): If each voter gives a higher score to alternative a than to alternative b, then a must be socially preferred to b.

Anonymity (A): Any permutation of ballots among voters leaves the result unchanged.

Positive linear transformation (PLT): A transformation of individual utilities of the form $a_i + bu_i, \quad b > 0$, *where b is identical over all voters, does not change the outcome.*

d'Aspremont and Grevers demonstrate that in their formulation these assumptions imply a utilitarian SWF of the form $w = \sum u_i$.

Instead of PLT, I assume Condition VS. This assumption is stronger than PLT, since now $a_i = a, \quad \forall i$.

Given the five assumption, the utilitarian SWF is simply UV, since the individual utilities are in this case the scores assigned by voters to the alternatives. That UV satisfies all conditions is easily checked. I omit the proof of necessity, given by d'Aspremont and Grevers for the more general case based on PLT.

6. STRATEGIC VOTING

A major objection to many voting procedures is that they provide strong incentives for strategic voting and hence for a falsification of voters' true preferences. This is true of PV, and consequently of sequential voting methods based on PV. It is also a criticism made from the beginning against BR. I will argue that the incentive to vote strategically is almost entirely due to restrictions on voters expression of their preferences and that under UV, the incentive to vote strategically is either completely absent, or involves only a mild distortion of voters' sincere preferences. I limit the analysis to UV-2, UV-3, equivalent to AV and EV.

Predicting how voters will vote, given their preferences and given the voting rule, requires assumptions about their behavior. The usual assumptions made to analyze strategic voting are

[1] Sen (1986, p.1125) also discusses the result.

a. Voters view themselves as players in a strategic game. *b*. They base their strategy on the perceived probability of casting a decisive vote. In the context of general political elections, both assumptions are implausible.

The following two assumptions appear to be reasonable. I offer them as testable hypotheses, as discussed in the next section.

Condition (SB): Voters prefer to cast a sincere ballot, i.e., one that reflects their actual preferences as closely as is possible with the given voting scale.

Condition (WA): Voters prefer a high probability of voting for the winning alternative. A restriction is that a voter will never elevate the lowest possible sincere score

In other words, she will never embrace what she most dislikes, just to be on the winning side.

Definition strategic voting (SV): Voting is strategic if it is not sincere.

SB appears to be the minimum that one must assume in order for voting to make any sense. WA is a powerful determinant of social attitudes and behavior. This is most strongly evidenced with regard to sports. Fans generally want 'their' team to win, not the better one. In voting, it is common to refer to a vote given to a candidate with little chance of winning as 'wasted'.

Strategic behavior under PV and BR:

Under PV the incentive for strategic voting and the possibly resulting preference distortion are severe. This defect motivated the search for alternatives. One of the first suggested was the BR. When it was shown to be equally susceptible to strategic manipulation, Borda had only the lame response "my method was designed for honest men". For both voting methods the conflict arises when candidates that are high on a voter's preference scale, have little chance of winning. Typically, a voter moves down his preference scale until he finds a candidate with a reasonable chance of winning. When a voter votes in this manner under PV, her most preferred candidates get the same score as the most disliked. Preference distortion is thus severe. Under BR the distortion is less severe. For example, if the candidate thought most likely to win is in the middle of the ranking and is moved to first place, then the most liked candidates of the sincere ranking are only moved down by one place, still receiving support.

Strategic behavior under AV:

Assuming that the two-valued AV scale allows a complete expression of voters' preferences, there is no incentive for strategic voting. A voter would vote for all candidates of whom she approves. Not voting for an approved candidate both distorts preferences and reduces the probability of having voted for the winner. Since all candidates of whom she disapproves are equally at the bottom of her scale, she will by assumption not vote for one of these, even if the

most likely winners are among them. However, in that case she may abstain from voting altogether. I offer this analysis as an empirical hypothesis. If voters differentiates further within each group, they may adopt a different strategy.

Strategic voting under EV:

The basic assumption continues to be that the EV scale of (1,0,-1) allows the voter an exact expression of her preferences. In accordance with assumption WA, the voter will not change any alternative receiving a sincere negative score. She has no incentive to reduce any +1 score since that would make her both less sincere and less likely to have voted for the winner. She has no incentive to reduce a 0 score either, since that makes her insincere and does not increase the probability of voting for a winner. She may change a 0 score to +1, if she feels that the increased probability of having a winner outweighs the loss in sincerity.

The conclusion must be that while strategic voting under EV cannot be ruled out, this cannot lead to a severe distortion of the sincere preferences. Both the most liked and the most disliked alternatives are safe from manipulation. Possibly, some indifferent candidates will be elevated, but this is an much milder distortion than those under PV, or BR. Thus, UV comes out on top under strategic as well as under sincere voting.

7. THE CONDORCET CRITERION

In the preceding sections I argued in favor of UV from various points of view. The voting theorist may still not be convinced, since I have not discussed the relationship of UV to the Condorcet criterion (CON). A CON alternative is one that defeats every other alternative in binary contests by MV. The CON is the most frequently invoked criterion for evaluating alternative voting rules. Since a CON alternative may not exist, the test consists in counting how often different rules fail to select a CON alternative when it does exist. UV may not select the CON alternative, as is illustrated by Table 1.

Table 1 about here

Alternative *b* is either preferred or indifferent for all voters and wins the UV vote. In binary votes, *a* scores over *b* by 65/35 and over *c* by 30/0, assuming that voters who are indifferent between two alternatives abstain.

The relationship between CON and UV is best understood by considering first the case of AV. Brams and Fishburn (1983, Ch. 3), base their advocacy of AV in large measure on the following argument. Suppose that voter preferences are *dichotomous*, in the sense that for any

voter, the alternatives can be assigned to two sets, *A* and *B*, such that any element of *A* is strictly preferred to any element of *B* and any two elements of the same set are indifferent. To allow for tied votes, define a CON winner as an alternative that is not defeated by any other alternative in a binary choice. Under the assumptions made, there must be one or more CON winners and they coincide with the AV winners.

Brams and Fishburn stress that this result holds only for AV and that other voting rules may conflict with CON. While the statement is true, it depends crucially on the fact that both AV and CON employ the same 2-valued scale. A similar result holds for an arbitrary scale. Assume that preferences are *k*-chotomous in the following sense: Let A_j, $j = k, k-1, ..., 1$ be a collection of sets such that for any $a \in A_i, b \in A_j, i > j \Rightarrow a \succ b$ and $a, b \in A_j \Rightarrow a \approx b, \forall j$. Consider an UV with an S_k voting scale. Then an alternative in A_j would receive the score *j*. By assumption, the score of any alternative depends on that alternative alone. An UV winner has, by definition, a higher aggregate score than a UV looser and would therefore defeat a UV looser also in a binary contest *if the scale remains unchanged and the vote continues to be sincere.* Under these assumptions, any UV winner is also a CON winner and vice versa.

In a real world situation, the italicized condition will not be met for $k>2$. The reason is that in a binary choice, the strategic incentive to exploit the full range of the scale so as to exert a maximum of voting power is too great. Consequently a binary scale is used universally for binary choices. The fact that a $k>2$ scale is necessarily shrunk in going from a multi-alternative to a binary choice explains why a UV-*k* winner may not be a conventional CON winner. The interpretation must be that the UV winner is the better choice, because she was selected by means of a finer, more accurate scale.

Interestingly, Arrow (1951, p. 32) makes an essentially identical argument, but draws an entirely different conclusion. He considers the addition of utilities first for three alternatives with a 3-valued scale and, after dropping one alternative, a choice between the remaining two on a 2-valued scale. He notices an apparent preference reversal and thus violation of his condition on independence of irrelevant alternatives. He concludes that the addition of utilities makes no sense.

8. EMPIRICAL EVALUATION OF VOTING RULES

Most empirical work on voting has investigated how voter preferences are determined.[1] This research does not deal with the central issue of voting theory, determining the best voting rule for aggregating given preferences. The time is ripe for moving the evaluation of voting rules from the theoretical to the empirical plane. I believe that this can be done relatively easily by questioning voters on the way to, or from the polls. This is the moment when they are best informed concerning issues and candidates and also know their final choice. Strangely, this optimal resource appears to have been exploited only by the media seeking to forecast the result a few hours, or days ahead, not by academic researchers seeking to resolve more fundamental issues.

Following is a sketch of how a questionnaire for this purpose might be constructed:

Outline of a voter questionnaire to be used just before or after a vote

Part I: The voter should be asked to state her sincere preferences

This should be done in two ways:

a. As a weak ordering by placing one or more names in boxes labeled 'first choice', 'second choice', etc.. A final box should be labeled 'don't know', or 'cannot rank'.

b. As cardinal rankings on both 2-valued AV and 3-valued EV scales. A 'don't know' box should be included here also.

Part II: The voter is asked how she would actually, i.e. strategically, vote under PV, AV and EV.

By comparing the results of Parts I and II, it would be possible to test a basic implication of the preceding section: Preference distortions in going from sincere to strategic voting will be severe under RV, mild or non existent under UV.

Part III: Voting on voting rules

Democracy means deciding important societal issues by vote. The voting rule to be used is itself such an issue. It therefore seems natural to ascertain how voters would vote on the different voting rules. Several aspects of such a vote would have to be considered to make it meaningful. This includes explanations of basic properties of different rules, particularly the defects of PV that are inherited by multistage procedures that use PV at each step.

The single stage methods that I would put to a vote are: PV, AV and EV. The multistage methods are: PV with runoff; instant runoff voting (IRV), which is currently being promoted in the US with some success; AV and EV with a runoff between the two most highly rated

[1] For a recent example and survey of the literature see Merrill and Grofman (1999).

candidates. The reason for this last entry is that voters would be better able to inform themselves about just two remaining candidates than about many.

For the vote on the voting rules I would use the single stage methods. These may yield conflicting results, but I would expects considerable uniformity.

Research along the indicated lines would produce valuable knowledge about how voters would vote under alternative voting rules as well as regarding their preference among the rules.

9. CONCLUSIONS

Political voting theory and practice are the only fields that consistently restrict the freedom of respondents to express their preferences, or evaluations, without restriction on a given scale. In all other fields where the aggregation of evaluations takes place, in survey research, or in performance evaluations in schools and businesses, restricting the evaluations reported on a given scale was never an issue. Similarly, abstract collective choice theory never restricts the expression of preferences.

This paper has concentrated on the theoretical arguments for general UV, with an unspecified number of scale divisions. On pragmatic grounds I favor evaluative voting with the (1,0,-1) scale. I believe that it would be highly motivating for voters to be able to vote against as well as for candidates, or issues. Since the general electorate tends to be very poorly informed regarding both candidates and issues, a finer division of the voting scale appears to be both unnecessary and possibly confusing to the voters. These pragmatic aspects are discussed at some length in my previous paper.

Regarding progress in the adoption of UV, realism forces me to be skeptical. Altering the RV procedures in common use is difficult. Politicians elected under a given voting rule are loath to change. Equally significant is the failure of voting theorists to establish a minimal consensus on what should be done and to project that consensus onto public awareness.[1]

[1] The difficulties that stand in the way of going from theory to practice in the context of approval voting are discussed by Brams and Fishburn (2003).

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ABREVIATIONS

AV	approval voting
BC	Borda count
EV	evaluative voting
IRV	instant runoff voting
MR	majority rule
PV	plurality voting
RV	restricted voting
SWF	social welfare function
UV	utilitarian voting

Table 1

UV May Differ from CON

	a	*b*	*c*
35	1	0	-1
35	-1	1	1
30	1	0	0
	30	35	0