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RENORMALIZED FIELD THEORY FOR THE STATIC CROSSOVER IN DIPOLAR FERROMAGNETS

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Abstract. – A field theoretical description for the static crossover in dipolar ferromagnets is presented. New non leading critical exponents for the longitudinal static susceptibility are identified and the existence and magnitude of the dip in the effective critical exponent of the transverse susceptibility found by matching techniques is scrutinized.

It was first shown by Aharony and Fisher [1] that the short range Heisenberg fixed point (FP) of the renormalization group (RG) is unstable against perturbations from the long range dipole-dipole interaction leading to a new stable dipolar FP. In subsequent papers the crossover from a critical behavior dominated by the short range exchange interaction to the asymptotic dipolar critical behavior was investigated by parquet graph [2] and matching techniques [3]. The latter follow closely the concept of phenomenological crossover theory, which leads to several difficulties: (i) the use of two temperature variables makes the analysis quite complicated and (ii) one has to assume that the dipole-dipole interaction is sufficiently weak to guarantee that the RG trajectories traverse the region close to the unstable Heisenberg FP. In ferromagnets like EuO and EuS with relatively strong dipolar interaction this assumption may not be correct. Thus the dip in the effective susceptibility exponent $\gamma_{\text{eff}}(r)$ found by these earlier studies [2] may not be universal. Here we give a field theoretical description of the crossover solely in terms of the true reduced temperature r using a generalized minimal subtraction procedure introduced by Amit and Goldschmidt [4] for bicritical points.

The Hamiltonian for a spin system with both exchange and dipolar interaction [1] is given by

$$H = -\frac{1}{2}\int_q \left[\left(r_0+q^2\right)\delta^{\alpha\beta} + g_0\frac{q^\alpha q^\beta}{q^2}\right] S_0^\alpha(\mathbf{q})\, S_0^\beta(-\mathbf{q}) - \frac{u_0}{4!}\int_{q_1}\int_{q_2}\int_{q_3} S_0^\alpha(\mathbf{q}_1)\, S_0^\alpha(\mathbf{q}_2) \times S_0^\beta(\mathbf{q}_3)\, S_0^\beta(-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3)\,. \quad (1)$$

$S_0^\alpha(\mathbf{q})$ $(\alpha = 1,2,...,n)$ are the components of the bare spin variable with n equal to the space dimensionality d. We further used the abbreviation $\int_q = \int \frac{\mathrm{d}^d q}{(2\pi)^d}$, r_0 is the bare reduced temperature and g_0 denotes the bare relative strength of the dipolar interaction.

The dipolar interaction in equation (1) breaks the symmetry of the spin fluctuations transverse and longitudinal to the wave vector $\mathbf{q}$, which is reflected in the free propagator

$$G_0^{\alpha\beta} = \frac{q^\alpha q^\beta}{q^2} G_0^L + \left(\delta^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2}\right) G_0^T, \quad (2)$$

where $G_0^i(r_0, g_0, q) = \left(r_0 + g_0\delta^{iL} + q^2\right)^{-1}$. The critical behavior is most conveniently studied by the dimensional regularization and minimal subtraction procedure of t' Hooft and Veltmann [5]. In this framework an arbitrary momentum scale μ is introduced, which allows for the definition of a dimensionless renormalized coupling constant u by $u_0 = \mu^\varepsilon Z_u S_d^{-1} u$, where the factor $S_d = 2/\left((4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)\right)$ is introduced for convenience. To remove fully the singularities it is also necessary to introduce renormalized parameters and fields according to $r_0 = Z_r r$, $g_0 = Z_g g$ and $S_0^\alpha(\mathbf{q}) = \left(Z_\Phi^L\right)^{1/2} P_{\alpha\beta}^L S^\gamma(\mathbf{q}) + \left(Z_\Phi^T\right)^{1/2} P_{\alpha\gamma}^T S^\gamma(\mathbf{q})$.

In the conventional minimal subtraction scheme the Z factors are functions of ε $(d = 4-\varepsilon)$ and of the renormalized quantities, singular as $\varepsilon \to 0$, in order to remove the singularities in the vertex functions (VF). However in the present case this renormalization prescription is inadequate as the critical region is approached for reasons similar to the case of bicritical points [4]. Therefore we adopt the following conditions of the Z factors: (i) for finite dipolar coupling g the renormalization constants cancel the poles in ε in all VF (ii) in the limit of infinite dipolar coupling $g \to \infty$ all VF are finite order by order in u and ε.

There is no singular contribution proportional to the dipolar coupling g. This implies that the renormalization constant for g simply is given by $Z_g^{-1} = Z_\Phi^L$.

Now we turn to the RG equation for the renormalized two point VF $\Gamma_R^{(2)\alpha\beta}(r, g, \mathbf{k}, u, \mu)$, which can be decomposed into a longitudinal and transverse part in the same way as the free propagator. The corresponding RG equation is $(\alpha = L, T)$

$$0 = \left[\mu\frac{\partial}{\partial\mu} + \zeta_r\left(u, \frac{g}{\mu^2}\right) r\frac{\partial}{\partial r} + \zeta_g\left(u, \frac{g}{\mu^2}\right) g\frac{\partial}{\partial g} + \beta\left(u, \frac{g}{\mu^2}\right)\frac{\partial}{\partial u} + \zeta_\Phi^\alpha\left(u, \frac{g}{\mu^2}\right)\right]\Gamma_R^\alpha(r, g, u, \mu) \quad (3)$$

where

$$\beta\left(u, \frac{g}{\mu^2}\right) = \mu\frac{\partial}{\partial\mu} u\,|_0,\quad \zeta\left(u, \frac{g}{\mu^2}\right) = \mu\frac{\partial}{\partial\mu}\ln Z^{-1}\,|_0\,.$$

Note that due to the generalized renormalization procedure the β- and ζ-functions depend on u as well as on $\frac{g}{\mu^2}$. We find

$$\zeta_\Phi^T = -\frac{1}{12}u^2 + \frac{1}{27}u^2\frac{1}{1+\frac{\mu^2}{g}} \quad (4a)$$

$$\zeta_\Phi^L = -\frac{1}{12}u^2 - \frac{1}{144}u^2\frac{1}{1+\frac{\mu^2}{g}} \quad (4b)$$

$$\zeta_r = u - \frac{1}{4}u\frac{1}{1+\frac{\mu^2}{g}} \quad (4c)$$

$$\beta = -\varepsilon u + \left(2 - \frac{7}{12}\frac{1}{1+\mu^2/g}\right)u^2 \quad (4d)$$

The flow equation is solved by

$$\frac{1}{u(l)} = \frac{l^\varepsilon}{u} + \frac{17}{12\varepsilon}\left(1-l^\varepsilon\right) - \frac{7}{12}l^\varepsilon\int_1^l \frac{x^{-1-\varepsilon}}{1+\frac{g(x)}{\mu^2x^2}}\mathrm{d}x. \quad (5)$$

This gives four FP $(u^*,\ g^*)$: Gaussian $(0, 0)$ Heisenberg $\left(\frac{\varepsilon}{2},\ 0\right)$, Gaussian dipolar $(0, \infty)$ and dipolar $\left(\frac{12\varepsilon}{17},\ \infty\right)$, where only the dipolar FP is infrared stable.

The solution of the RG equation is

$$\Gamma_R^\alpha(r,\ g,\ k,\ u) = = \exp\left(\int_1^l \frac{\mathrm{d}\rho}{\rho}\zeta_\Phi^\alpha(\rho)\right)\Gamma_R^\alpha\left(r(l),\ g(l),\ k,\ u(l)\right) \quad (6)$$

where $\mu(l) = \mu l$, $l\frac{\mathrm{d}r(l)}{\mathrm{d}l} = r(l)\zeta_r(l)$, $l\frac{\mathrm{d}g(l)}{\mathrm{d}l} = g(l)\zeta_g(l)$ and $l\frac{\mathrm{d}u(l)}{\mathrm{d}l} = \beta(l)$ with the initial conditions $r(1) = r$, $g(1) = g$ and $u(1) = u$.

Next we study the behavior of the transverse and longitudinal two point VF in the asymptotic region $(l \to 0)$. (i) For $T = T_c$ and by choosing the flow parameter l according to $\frac{k}{\mu(l)} = 1$ we find

$$\Gamma_R^\alpha(0,\ g,\ k,\ u) \propto \begin{cases} g + ck^{2+\zeta_\Phi^{L*}} & \alpha = L \\ k^2 + \zeta_\Phi^{T*} & \alpha = T \end{cases} \quad (7)$$

where c is a constant, $u^* = \lim_{l\to 0} u(l) = \frac{12}{17}\varepsilon$ and $\zeta_\Phi^{\alpha *} = \lim_{l\to 0}\zeta_\Phi^\alpha(l)$. From equation (7) we may identify the leading exponent $\eta^T = -\zeta_\Phi^{T*} = \frac{20}{867}\varepsilon^2$ and a new non leading critical exponent $\eta^L = -\zeta_\Phi^{L*} = \frac{13}{289}\varepsilon^2$. (ii) For $\mathbf{k} = 0$ and by choosing $\frac{r(l)}{\mu^2 l^2} = 1$ one gets

$$\Gamma_R^\alpha(r,\ g,\ 0,\ u) \propto \begin{cases} g + \tilde{c}r^{(2+\zeta_\Phi^{L*})/(2-\zeta_r^*)} & \alpha = L \\ r^{(2+\zeta_\Phi^{T*})/(2-\zeta_r^*)} & \alpha = T \end{cases} \quad (8)$$

where $\tilde{c}$ is a constant. Therefrom we find the leading critical exponents

$$2\nu = 2\left(2-\zeta_r^*\right)^{-1} = 1 + \frac{9}{34}\varepsilon + \frac{7013}{58956}\varepsilon^2$$

and $\gamma_T = \nu(2-\eta_T)$ for the transverse and the non leading exponent $\gamma_L = \nu(2-\eta_L)$ for the longitudinal susceptibility.

Now we study the crossover of the transverse susceptibility at zero wave vector $\mathbf{k} = 0$. We find for the effective critical exponent defined by $\gamma_{\text{eff}} = \frac{\partial \ln \chi^{-1}}{\partial \ln r}$

$$\gamma_{\text{eff}}(r,\ g,\ u) = 1 + u(l)\left[\frac{1}{2} - \frac{1}{8}\frac{g}{r}\ln\left(1+\frac{r}{g}\right)\right] \quad (9)$$

with $l = \sqrt{r}/\mu$ and $u(l)$ is given by equation (5).

Figure 1 shows $\gamma_{\text{eff}}(r,\ g)$ *versus* $\frac{r}{g}$ at fixed dipolar couplings for a series of initial values u. For weak dipolar systems $\left(g = 10^{-6}\right)$ all curves join and the minimum of the effective susceptibility exponent is a universal property of the system, whereas for stronger dipolar systems $\left(g = 10^{-2}\right)$ it depends on u whether there is a minimum or not. If $u \geq u_H^*$ there is a minimum at the same position as for $u = u_H^*$, but for $u \leq u_H^*$ the dip in γ_{eff} diminishes with decreasing u.

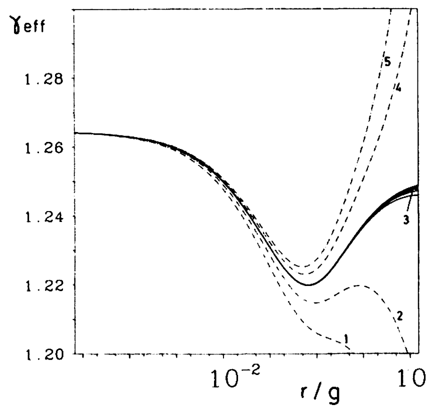

Fig. 1. – Effective susceptibility exponent γ_{eff} for $g = 10^{-6}$ (solid), $g = 10^{-2}$ (point-dashed), $\varepsilon = 1$ and a series of initial values $u = \frac{2+k}{10}$ with $k = 1, ..., 5$ indicated in the graph.

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