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Abstract

Ranked gene lists are highly instable in the sense that similar measures of differential gene expression may yield very different rankings, and that a small change of the data set usually affects the obtained gene list considerably. Stability issues have long been under-considered in the literature, but they have grown to a hot topic in the last few years, perhaps as a consequence of the increasing skepticism on the reproducibility and clinical applicability of molecular research findings. In this article, we review existing approaches for the assessment of stability of ranked gene lists and the related problem of aggregation, give some practical recommendations, and warn against potential misuse of these methods. This overview is illustrated through an application to a recent leukemia data set using the freely available Bioconductor package GeneSelector.
1 Introduction

Univariate analysis is an important part of most biomedical studies investigating high-dimensional molecular data. They yield so-called rankings, for instance gene rankings in the case of microarray studies. Such rankings are displayed in almost all microarray-related biomedical publications, sometimes as the main research result, sometimes as a preliminary step to, e.g. more complex procedures like multivariate prediction of disease outcome, construction of genetic networks, or combined analyses involving different types of -omics data. A multitude of ranking criteria have been proposed in the statistical and bioinformatic literature with the aim to cope with the "small \( n \) large \( p \) problem" (where \( n \) is the number of observations and \( p \) the number of features). For example, the assessment of differential gene expression in the two-group setting has particularly attracted the attention of statisticians in recent years. Researchers typically compute a score such as the t-statistic reflecting the difference between both groups while taking the within-group variability into account. More sophisticated criteria which are particularly appropriate in the case of (very) small samples have been proposed in the literature, see recent comparison studies [1, 2, 3] for an overview of possible scores for the two-group setting.

From a practical point of view, the rank of a particular variable is often as important as the value of the statistic. Most often, the rank (not the value of the statistic) determines together with other aspects such as gene function whether the gene is selected for further analysis in future steps of the research project. For example, when there are 300 significant differentially expressed genes, the investigators will probably not test all of them extensively in further lab experiments. Conversely, they will probably examine the top-ranked genes very carefully even if no gene passes the multiple testing adjustment. Hence, providing a reliable list of top-ranking genes is probably at least as important as improving statistical power of multiple testing procedures at any price. In this spirit, Mukherjee et al [4] state that, “a realistic goal [of the statistician or bioinformatician] is to narrow the field for further analysis, to give geneticists a short-list of genes which are worth investing hard-won funds into analysing”, which Aerts et al [5] formulate as a “need for prioritization” in a slightly different context. A related problem going beyond the scope of this paper is the adjustment for multiple testing [6, 7, 8]. Here, we focus on the ordering of the genes, not on the threshold of statistical significance.

Note that the concept of top-ranking variable may refer to other situations than the usual two-group scenario in gene expression studies. For instance, the phenotype of interest may be a quantitative trait instead of a binary outcome, or even a censored survival time. Categorical phenotypes with more than two categories are
also common, and more difficult to handle than the two-group case. The concepts and ideas presented in this paper restrict neither to the comparison of two-groups nor to gene expression data, although the illustrating examples refer to such studies. Very generally, we denote as "statistic" the criterion used to rank variables, where large values of the statistic correspond to top-ranking variables.

An important aspect of univariate analyses which is in our opinion under-considered in the literature is the variability of the obtained ordered lists. By variability, we mean i) the difference between rankings obtained from the same data set based on different ranking criteria, and ii) the variability of the lists obtained with a unique method but with slightly modified versions of the data set. Ranking methods should be stable in the sense that the list remains approximately the same even after small changes in the composition of the data sets or in the data preparation. Both biomedical and statistical studies tend to ignore these two sources of variability and simply consider the obtained ranked gene list as an unequivocal and definitive result.

In the present paper, we systematically survey two important aspects of the variability of ranked lists: i) the measurement of ranking stability, which is reviewed in Section 3, and ii) the aggregation of ranked lists in the hope to obtain a more robust ranking, as introduced and discussed in Section 4. Along with a systematic literature review on stability and aggregation in the context of ranked gene lists, we summarize the state-of-the-art within a unifying framework which encompasses most of the existing procedures, we give some practical recommendations, and warn against potential misuse of methods for stability assessment and aggregation. The discussed methods are illustrated through an application to a recent leukemia microarray data set [9, 10] using the add-on Bioconductor package GeneSelector [11] in the statistical software environment R.

2 A unique gene ranking?

2.1 Notations

Although this is not always admitted in biomedical publications, the ranked lists obtained from univariate analyses should not be considered as fixed universal results. After giving a formal definition of the term "ranking" as considered in the present paper, we briefly review important sources of variations for gene rankings.

The term "data set" denotes a pair $D = (x, y)$, where the $n \times p$ matrix $x = (x_{ij})_{i=1,...,n, j=1,...,p}$ contains $n$ observations of the random vector $(X_1, \ldots, X_p)'$ (for instance the expression levels of $p$ genes), and $y = (y_1, \ldots, y_n)$ stores either the response
variable $Y$ of interest for these $n$ observations or, e.g. an experimental condition fixed by design. In this paper, we define a ranking of the variables $X_1, \ldots, X_p$ as a permutation $r = (r_j)_{j=1,\ldots,p}$ of $(1, \ldots, p)$, where $r_j$ is the rank of the variable $X_j$ with respect to its association with $Y$. A small rank indicates strong association between the considered gene and $Y$, either positive or negative. A ranking yields an ordered list $l = (l_m)_{m=1,\ldots,p}$ defined by

$$l_m = j \iff r_j = m \text{ for all } j, m = 1, \ldots, p.$$  

(1)

For instance, in the case of differential gene expression, $r_{1786} = 1$ and $l_1 = 1786$ would mean that $X_{1786}$ is identified as the most differentially expressed gene. If $l$ is an ordered list, the $k$ top genes $l_1, \ldots, l_k$ form the so-called top-$k$-list (usually $k \ll p$). For example, biomedical articles often report top-20- or top-50-lists. Note that we do not consider rankings with ties for the sake of simplicity.

### 2.2 Different ranking criteria

While methodological articles presenting new statistical ranking methods often include a comparison study for demonstrating their superiority over existing approaches, many biomedical publications show results obtained with a single statistic. However, different ranking statistics often result in very different ranked lists, even when considering top-ranking variables only. Methodological articles often highlight these differences because they can be considered as arguments in favor or against a particular method. For instance, one often reads sentences like "method A correctly identifies three well-known markers as differentially expressed, whereas these markers are not recognized as top-ranking by method B". In contrast, differences between rankings are often ignored in biomedical publications presenting a new data set, because they would make the interpretation of results more confusing and raise doubts on the reliability of the data (for instance, a referee might argue that more experiments should be performed to make the list less ambiguous).

The multiplicity of methods and the question of how to deal with the different results that they produce are very general issues in all research fields. However, they are particularly relevant in the context of different expression studies in the "$n \ll p$" setting for two reasons. Firstly, traditional approaches such as the standard t-test for two independent samples are usually considered as inappropriate in the case of very small sample sizes which are common in microarray settings, hence making the development of alternative approaches necessary. This strong need has been recognized by the research community and many teams have suggested their own
method to provide more reliable rankings of differentially expressed genes, hence indirectly giving rise to another multiplicity problem. Secondly, univariate analyses in the small \(n\) large \(p\) scenario are well-known to produce highly instable outputs, in the sense that a small change in the data or a minimal modification of the ranking criterion often results in a fully different ordering of the features.

Several tens of criteria for ranking genes according to their differential expression across two groups have been described in the literature in the last decade, making an exhaustive enumeration burdensome. The rest of this section gives a concise overview of the most widely used methods and briefly discuss their characteristics. Note that most of them can be interpreted as test statistics in the hypothesis testing framework. In this paper, they are considered as ranking criteria rather than as test statistics, since we focus on the ordering of the genes rather than on the assessment of the statistical significance. The available ranking criteria can roughly be divided into the following broad categories [3]:

- Simple approaches such as the fold change, the classical t-statistic with equal or unequal variance, Wilcoxon’s rank sum statistic.

- Ad-hoc modifications of the classical t-statistic such as the SAM statistic [12], which can also be interpreted as a Bayesian t-statistic where the denominator is stabilized by a global prior standard error, or Efron’s 90% rule derived from a mixture model [13].

- Regularization methods, for instance hierarchical Bayes methods performing regularization via prior distributions, where hyperparameters of the prior distribution are estimated from a large set of genes [14, 15, 16, 17]. A regularized t-statistic can also be obtained through a James-Stein shrinkage procedure [3].

In the rest of this paper, we illustrate the reviewed methods using the R statistical software (http://www.r-project.org) through an application to the leukemia data set available from the Bioconductor ALL package [9, 10]. The data set gives the expression levels of 12265 genes for 95 T-cells and 33 B-cells samples. The data are obtained from the ALL package with the R command lines:

```R
R> library(ALL)
R> data(ALL)
R> X<-exprs(ALL)
R> y<-phenoData(ALL)[,4]
```
The *GeneSelector* add-on package [11] is publicly available from the *Bioconductor* platform (www.bioconductor.org) and can be installed and loaded using the commands

\r\nR> install.packages("GeneSelector")
R> library(GeneSelector)
\r
A total of 15 established ranking criteria are either directly implemented in *GeneSelector* or called via wrapper functions from other packages. Given \(X\) and \(y\) as above, these functions are schematically called as RankingCriterion\((X, y, \text{parameter.1, ..., parameter.m})\). The arguments parameter.1, ..., parameter.m may optionally be used to set method parameters to a different value than the pre-defined default. Five examples including both standard methods (fold-change, ordinary t-statistic) and recently developed challengers (Limma [16], Fox-Dimmic t-statistic [17] and shrinkage t-statistic [3]) are given below.

\r
R> fc <- RankingFC\((X, y)\)
R> tstat <- RankingTstat\((X, y)\)
R> limma <- RankingLimma\((X, y, \text{proportion = 0.01})\)
R> foxdimmic <- RankingFoxDimmic\((X, y, m = 100)\)
R> shrinkt <- RankingShrinkageT\((X, y)\)
\r
The top-lists yielded by the ranking procedures can be easily displayed using the function toplist:

\r
R> methodlist <- list(fc, tstat, limma, foxdimmic, shrinkt)
R> topten <- matrix\(\text{nrow = 10, ncol = length(methodlist)}\)
R> for \((i \text{ in seq(along = methodlist)})\) topten[, \(i\)] <- toplist(methodlist[[\(i\)]], top = 10, show = FALSE)$index
R> colnames(topten) <- unlist(lapply(methodlist, slot, "method"))
R> print(topten)
\r
\r
<table>
<thead>
<tr>
<th>Foldchange</th>
<th>ordinaryT</th>
<th>Limma</th>
<th>FoxDimmicT</th>
<th>ShrinkageT</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1,]</td>
<td>8399</td>
<td>8399</td>
<td>8399</td>
<td>8399</td>
</tr>
<tr>
<td>[2,]</td>
<td>8173</td>
<td>8225</td>
<td>8225</td>
<td>8225</td>
</tr>
<tr>
<td>[3,]</td>
<td>9478</td>
<td>3268</td>
<td>3268</td>
<td>8173</td>
</tr>
<tr>
<td>[4,]</td>
<td>8172</td>
<td>5064</td>
<td>5064</td>
<td>9478</td>
</tr>
<tr>
<td>[5,]</td>
<td>11834</td>
<td>1174</td>
<td>1174</td>
<td>11834</td>
</tr>
<tr>
<td>[6,]</td>
<td>6702</td>
<td>7106</td>
<td>7106</td>
<td>2673</td>
</tr>
<tr>
<td>[7,]</td>
<td>2673</td>
<td>8172</td>
<td>8172</td>
<td>7414</td>
</tr>
</tbody>
</table>
While gene 8399 is unambiguously at the very top position, there are already three different candidates for the second position. Furthermore, except for gene 8399, there is no gene present in all top-ten lists, as output by the method `GeneSelector`:

```r
R> genesel <- GeneSelector(methodlist, maxrank = 10)
R> show(genesel)
```

GeneSelector run with gene rankings from the following statistics:
- Foldchange
- ordinaryT
- Limma
- FoxDimmicT
- ShrinkageT

Number of genes below threshold rank 10 in all statistics: 1

```r
R> which(slot(genesel, "selected") == 1)
[1] 8399
```

The multiplicity of available methods may lead to a substantial publication bias in the sense that some researchers might choose their ranking statistic on the basis of its results. For example, if the "favorite gene" is not identified as top-ranking by a particular criterion, researchers are likely to try out another statistic or even to develop a new assessment procedure that better corresponds to the expected or observed data structure. By favorite feature, we mean for instance a feature that is expected to be relevant based on biological knowledge, the feature that is expected to rank best based on a previous study, or conversely a feature which has not yet been identified as relevant in this context and may thus yield an innovative marker. The strategy consisting of choosing the ranking statistic a posteriori based on the obtained results should be banned, since it may yield a substantial optimistic bias and lead to "wrong research findings" [18], as quantitatively assessed in the case of high-dimensional class prediction [19].

### 2.3 Different perturbed versions of the data set

When addressing the variability of gene lists, one can also consider the variations resulting from small changes in the data set. One may obtain a completely different
ranked list, not only by using a different ranking statistic but also by considering subsets (or other modified versions) of the data set. Ideally, one would expect that the ranked list remains approximately the same even if, e.g. a few observations are removed from the data set. This kind of stability is often investigated through resampling techniques, which are briefly reviewed in the rest of this section together with two additional methods.

Very generally, resampling can be defined as “randomly drawing a pseudo data set out of the available data set”. There are many different resampling variants whose theoretical properties are relatively well-known, see for instance the textbooks on bootstrap [20] and on resampling [21]. These two most popular approaches are briefly reviewed in the rest of this section.

There are several commonly used methods to generate a set of perturbed datasets \( \{\tilde{D}_b, \ b = 1, \ldots, B\} \) from a given original dataset \( D \). In the Jackknife procedure (also denoted as "subsampling"), a number \( d \) (\( d < n \)) of observations are randomly selected and removed from the data set. When the number of distinct subsets of size \( d \) is moderate, they are often all considered successively, i.e. \( B = n!/(d!(n-d)!). \) Otherwise, the subsets of observations to be removed are generated randomly, with \( B \) usually depending on the induced computational expense. The special case "leave-one-out" is obtained when one removes only \( d = 1 \) observation at a time.

An alternative to the Jackknife is the so-called "bootstrap" method. One draws \( n \) observations out of \( n \) with replacement, which means that some observations appear more than once in the perturbed data set, whereas other (in average \( (1 - 1/n)^n \approx 36.8\% \)) are excluded. Similarly to the Jackknife, bootstrap sampling is repeated \( B \) times with \( B \) depending on the induced computational expense and on the maximal number of distinct bootstrap samples, which can be computed by combinatorial considerations.

If the scale of the measurements is metric, perturbed datasets can also be generated by adding noise to the data matrix, for instance independent normally distributed terms with zero mean and "small" (feature-specific) variance.

If the \( y_i \) take values in a finite set of labels, and there is uncertainty about the correctness of label assignments, the consequences of erroneous labeling might be investigated by generating perturbed datasets by changing the label of a number \( d \) of randomly selected observations. Note that changing the labels usually affects the results more than simply removing the observations. In practice, \( d \) should thus be small.

In the context of univariate analyses, one can then repeat the variable ranking procedure for each modified version \( D_b \) of the original data set successively and
compare the resulting rankings by measuring their similarities. Such strategies have often been used to study the stability of gene rankings [22, 23, 24, 25, 26, 27, 28, 29]. These methods are applied in medical research for empirically checking the stability of the identified genes [30, 31]. Bootstrap sampling and Jackknife are by far the most widely used procedures.

As an illustration, three of the perturbation schemes outlined above (Jackknife, bootstrap and label swap) are applied together with the Limma ranking procedure [16] using the GeneSelector package. As a preparatory step, the methods GenerateFoldMatrix and GenerateBootMatrix are called in order to specify how the perturbed datasets have to be generated. Note that the function set.seed is used here for the sake of reproducibility.

```r
R> set.seed(1)
R> leave10out <- GenerateFoldMatrix(y = y, replicates = 50, k = floor(length(y) * 0.1), minclassize = 20)
R> set.seed(2)
R> boot <- GenerateBootMatrix(y = y, replicates = 50, maxties = 3, minclassize = 20)
R> set.seed(3)
```

The first function call randomly selects about 10% percent of all observations for 50 iterations, while the second one produces 50 bootstrap data sets. Note that the two methods GenerateFoldMatrix and GenerateBootMatrix allow resampling with constraints (as specified by the argument minclassize corresponding to the minimum number of observations from each class), which is often necessary in the case of very small samples. The chosen ranking procedure can then be applied to the perturbed data sets using the method RepeatRanking, for example:

```r
R> limma_leave10out <- RepeatRanking(limma, leave10out, scheme = "subsampling")
R> limma_change10 <- RepeatRanking(limma, leave10out, scheme = "labelexchange")
R> limma_boot <- RepeatRanking(limma, boot)
```

The Limma ranking is computed based on perturbed data sets obtained by removing 10% of the observations using the first command or by changing the label of these observations using the second command, while the third command computes the rankings based on bootstrap samples. Again, the method toplist allows to visualize the results: among others, it outputs the number of times each gene is placed at a particular top-position.

In the literature, such approaches have been used for two main purposes: i) the assessment of the stability of variable rankings, and ii) the derivation of aggregated
rankings which are believed to be more accurate or more stable than rankings obtained from the original data set. Both approaches are reviewed in Sections 3 and 4, respectively.

3 Stability of gene rankings

3.1 Framework

In this article, the term stability refers to the similarity between rankings obtained with different (similar) criteria (see Section 2.2) or based on different perturbed version of the data set (see Section 2.3). This section is devoted to the measurement of stability in terms of similarities between lists in this context. Note that one may also investigate the stability of ranked lists across different platforms or across different studies within a meta-analysis. It is well-known that results from different studies often show poor overlap, and that the combination of several studies in form of a meta-analysis may yield more reliable results. Many groups compare and combine results from, e.g. different platforms or different labs [32, 33, 34]. While some approaches are rank-based [33], most meta-analysis methods are based on the statistic itself and thus do not fit into the scope of the present article. A recent overview can be found in [35].

In the statistics and bioinformatics literature, many quantitative criteria have been proposed for comparing two or more gene lists, which can roughly be divided into set-based (Section 3.2) and distance-based methods (Section 3.3). In our framework, we consider two rankings $r$ and $r'$ associated to two lists $l$ and $l'$ through Eq. (1). A stability measure is a function $s$ of $r$, $r'$ and possibly additional parameters that measures the similarity between the two lists. Reasonable stability measures often focus on the top-ranking variables, because similarities between the ranks of relevant variables are more interesting than similarities between irrelevant variables. For instance, usual approaches may concentrate on top-$k$-lists (for some $k \ll p$) or give more importance to the top of the list through appropriate weighting. Note that in practice the stability measures reviewed below often have to be accommodated to take more than two rankings into account. There are essentially two approaches to do that. Some criteria can be generalized in a way that they consider all rankings simultaneously, especially the criteria based on the concept of overlap [36]. The second approach consists of summarizing pairwise stability measures, for instance through averaging [29]. This can be done by considering all pairs of rankings successively [29], or by comparing a "reference list" (for instance the list derived from the original data set) to all the other [23].
3.2 Set-based methods for stability assessment

Approaches of the first type compare sets consisting of a fixed number $k$ of top-genes, e.g., $k = 100$. Common criteria for measuring the similarity between two gene lists are the size of the intersection of the two top-$k$-lists

$$s^{(1)}(r, r', k) = \sum_{j=1}^{p} I(r_j \leq k \land r'_j \leq k),$$

where $I$ denotes the indicator function ($I(A) = 1$ if $A$ is true, $I(A) = 0$ otherwise), or the proportion $s^{(1)}(r, r', k)/k$ of genes in the top-$k$-list from $l$ that are also in the top-$k$-list from $l'$, also denoted as percentage of overlap (POG) [37]. Such stability measures are used by numerous researchers [28, 29, 38, 39].

In the same vein, Irizarry et al [40] suggest a visualization technique called ”CAT plot” (standing for ”Correspondence At the Top”) that represents the proportion of overlap of the top-$k$-lists versus $k$. A variant can be obtained by dividing the size of the intersection by the size of the union, yielding

$$s^{(2)}(r, r', k) = \frac{s^{(1)}(r, r', k)}{2k - s^{(1)}(r, r', k)}.$$

The stability measure $s^{(2)}(r, r', k)$ is often used in practice [36, 39]. It can be seen as a special of Jaccard’s index obtained when the two compared sets are of the same size $k$. For all these methods, there is inevitably some arbitrariness in the choice of the cut-off parameter $k$. In practice, one might consider several different arbitrarily chosen numbers of genes (e.g. $k = 50$, $k = 100$, $k = 200$) [2]. An other pitfall of these methods is that they follow the nothing-or-all principle. All genes in the subset are given the same weight independently of their rank, whereas the other are completely ignored. The major advantage of such approaches is their simple interpretation in terms of intersection.

The so-called overlap score [41] implemented in the package 'OrderedList' [42] is also related to this class of stability measures, except that it considers similarities between the genes from both the top and the bottom of the lists, corresponding to e.g. up- and down-regulation. A pitfall of this approach is that the list is considered as symmetric with the top-genes and bottom-genes being implicitly equally relevant. If, as assumed in the present article, only the top-genes are important (i.e. if both up- and down-regulated appear at the top of the list in the two-group setting), the
score can be reformulated as

\[ s^{(3)}(r, r', \alpha) = \sum_{k=1}^{p} w^{(k)}_{\alpha} \cdot s^{(1)}(r, r', k), \]

where \( w^{(k)}_{\alpha} \) \((k = 1, \ldots, p)\) are weights decreasing with increasing \( k \). In this representation, the score is a weighted combination of the sizes of the intersections of the two top-\( k \)-lists for different \( k \) values. It avoids the difficult choice of \( k \) but involves an additional parameter \( \alpha \) determining the weights \( w^{(k)}_{\alpha} \). Yang et al [41] suggest to choose exponential weights of the form \( w^{(k)}_{\alpha} = e^{-\alpha k} \) in order to put more weight on the top of the list. The parameter \( \alpha \) determines the depth of the top-list that is considered as relevant for the comparison of the two lists. They suggest a procedure for choosing the optimal value of \( \alpha \): the score is computed for different values of \( \alpha \), both for subsamples as well as well for subsamples with permuted class labels. The optimal value of \( \alpha \) is then the value that best separates permuted subsamples from subsamples with true class labels in ROC analysis. To our knowledge, this approach is the only one that determines the "depth of the comparison" based on an objective criterion.

Other criteria which can also be formalized in a similar manner are the overlapping probability of top ranking gene lists derived based on the hypergeometric distribution [43], or Dice-Sorensen’s index and Ochiai’s index [39]. Note that it is also usual to consider top-lists of different sizes, for instance when the results of different studies on the same disease are compared in the context of meta-analysis. As an example, the so-called recovery score [23] considers lists of genes with p-values falling below a given threshold rather than top-\( k \)-lists. This method is thus not purely rank-based, although it can be used in a similar way as the methods described above.

Set-based stability measures can be computed using the method `GetStabilityOverlap` from the `GeneSelector` package. For example, the stability of the Limma ranking procedure when 10% of the observations are removed from the data can be assessed with the `GetStabilityOverlap` through

```r
R> limma_leave10out_setbased <- GetStabilityOverlap(limma_leave10out, 
  scheme = "original", decay = "exponential", alpha = 0.16) 
R> summary(limma_leave10out_setbased, measure = "intersection", position = 10) 
```

summary of intersection counts (with respect to reference data set):

<table>
<thead>
<tr>
<th>Min.</th>
<th>1st Qu.</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qu.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.800</td>
<td>0.900</td>
<td>0.900</td>
<td>0.938</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Figure 1: **Left:** Rank in the perturbed data sets (10% left out) against rank in the original data set. **Right:** Average percentage of overlap and average overlap score against the list position $k$.

expected score in the case of no-information: 0.0007920792

R> summary(limma_leave10out_setbased, measure = "overlap", position = 10)

summary of overlap scores (with respect to reference data set):

<table>
<thead>
<tr>
<th></th>
<th>Min.</th>
<th>1st Qu.</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qu.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>overlap</td>
<td>0.863</td>
<td>0.916</td>
<td>0.940</td>
<td>0.935</td>
<td>0.959</td>
<td>1.000</td>
</tr>
</tbody>
</table>

expected score in the case of no-information: 0.0003353366

The first summary command displays the mean percentage of overlap with $k = 10$, while the second one shows the corresponding mean overlap score. As specified through the argument `scheme="original"`, similarity is measured between the ranking from the original data set and the rankings from perturbed data sets (and not between all pairs of perturbed data sets). The graphics depicted in Figure 1 can be obtained by applying the `plot` method to the objects output by `RepeatRanking` and `GetStabilityOverlap`, respectively. We conclude that the removal of only 10% of the observations modifies the top-10-list moderately, with a percentage of overlap of 85% or more. Obviously, the stability is lower when more observations are removed, or when perturbed data sets are generated by bootstrap. Note that the stability also depends on the ranking method.
3.3 Distance-based for stability assessment

Approaches of the second type are based on the distance between the two considered rankings $r$ and $r'$, where the definition of the term “distance” depends on the particular method. Such methods may be more difficult to interpret for applied scientists than approaches of the first type. An example is the Canberra distance $\sum_{j=1}^{p} |r_j - r'_j|/(r_j + r'_j)$ and the Canberra distance with location parameter $k+1$ which is defined as $\sum_{j=1}^{p} |\min(r_j, k+1) - \min(r'_j, k+1)|/\{\min\{r_j, k+1\} + \min\{r'_j, k+1\}\}$ and focuses on the top-$k$-list [29]. These measures give more weight to the top of the list through the denominator $r_j + r'_j$. Other well-known examples are Spearman’s rank correlation and Kendall’s tau. However, in their standard form they do not give more weight to the top of the list. Hence, it makes sense to consider modified versions such as the top-$k$-list Kendall distance [33] or Spearman’s footrule [44] that focus on a given number $k$ of top-genes.

3.4 Remarks on stability

It seems that, in practice, two gene lists may show apparently poor overlap although each list comprises mostly truly differentially expressed genes [45]. Based on this idea, an alternative stability criterion accounting for correlation between genes was recently proposed [37]. Roughly, a variant of the percentage of overlap is computed by counting both the genes shared by the two top-lists and the genes from one of the top-list that are significantly correlated with at least one gene from the other top-list. Based on this measure, the authors conclude that gene lists are often not as different as they might seem at the first glance. Note, however, that this method is not purely rank-based since it uses additional information on correlations.

Beside the critical assessment of the results’ reliability, there are other motivations for assessing the stability of univariate rankings. One of these motivations is the optimized choice of a ranking criterion or of the parameters involved in a given ranking criterion. Several variants of this approach are proposed by different groups of authors [46, 47]. They suggest to “learn” the ranking criterion based on the reproducibility of the ranked lists obtained from bootstrap samples, i.e. to select the parameter value or the ranking criterion yielding the highest stability. However, we claim that, although stability should be an important criterion for choosing a particular statistic, the choice should not be based solely on stability, because a stable ranking procedure does not necessarily rank the genes properly. This becomes obvious if we consider a ranking criterion that always assigns each gene to the same rank independently of the sample. Such a ranking would obviously be stable, but fully
inappropriate. In this vein, the so-called fold-change criterion (which considers the mean difference between the two groups after log-transformation of gene expression data) is often found to yield higher stability than criteria involving some variance estimation, but it is also well-known that it may not identify the right genes. In a nutshell, an ideal ranking criterion has both a low variance (i.e. a high stability) and a low bias.

The second motivation for assessing the stability of ranking procedures is that stability provides an objective criterion for computing the minimal sample size required in an experiment. In this spirit, one can determine whether the sample size is sufficient based on the stability of the obtained ranking [23]. Using this method, these authors found that a stability plateau is reached for sample sizes between \( n = 8 \) and \( n = 15 \) with the real data sets that they investigated. They argue that stability-based determination of the minimal sample size is more realistic than model-based methods which rely on the quality of the data generating model - the power being unknown in real data studies. The "probability of selection" method [22] is based on a similar idea. In this framework, the power is defined as the probability for a gene with true rank \( \leq k_1 \) to be ranked among the first \( k_0 \) genes, where \( k_0 \) and \( k_1 \) are adequately chosen integers. The power is assessed for different sample sizes in data driven simulations.

4 Deriving more accurate ranking criteria through aggregation

4.1 Framework

An issue related to the stability and similarity of gene rankings is the derivation of aggregated rankings in the hope that they will be more reliable than the original ranking obtained using the original data set. The term "reliability" is of course difficult to define in the context of gene rankings, since the truth is not known. In practice, reliability is often assessed based on simulated data, which we consider as suboptimal since this approach usually oversimplifies the data structure, or by comparing the obtained top-genes to previous biological knowledge. Alternatively, some authors assess their lists of genes within a supervised learning framework, i.e. by employing them for prediction and computing the resulting error rate using an appropriate evaluation design [48]. In the slightly different context of variable selection for regression, the advantages of repeating the procedure on a large number of subsamples is well-documented [49], with nice consistency properties.
Formally, an aggregated ranking or an aggregated list are obtained by "summarizing" a collection of rankings \( \{r(b), \ b = 1, \ldots, B\} \) or a collection of lists \( \{l(b), \ b = 1, \ldots, B\} \), respectively. Note that this framework is very general. A collection of rankings/lists can be obtained both from a collection of perturbed data sets (for a fixed ranking procedure) or, conversely, from applying different ranking procedures to a fixed data set. Meta-analysis approaches that combine results from different studies using ranked-based procedures also fit in this framework.

4.2 Examples of aggregation procedures

A straightforward method to combine several rankings consists of computing a univariate summary statistic for each gene \( j \), e.g. the mean \( \frac{1}{B} \sum_{b=1}^{B} r_{jb} \) or a quantile of \( r_{jb}, \ b = 1, \ldots, B \). The genes are then re-ranked according to their summary statistic. In particular, high quantiles (for instance 0.75, 0.9, 0.95) may be relevant because they favor genes which are never badly ranked, hence potentially eliminating genes whose apparent good rank is due to a single outlying observation.

Another approach focuses on the frequency of selection of the considered gene \( j \) within the top-\( k \)-list over different rankings \( r^{(1)}, \ldots, r^{(B)} \), also denoted as the “selection probability function” [22]. The idea of selecting genes based on their frequency of selection among a large number of perturbed data sets is also proposed in several similar variants by numerous independent researcher groups [28, 29, 50]. A major pitfall of all these approaches is that they heavily depend on the arbitrarily fixed threshold \( k \). By trying several thresholds successively, one would obtain substantially different rankings. Another problem is that they ignore the rank of the selected genes within the considered subset. As outlined in Section 3 in the context of stability assessment, they follow the all-or-nothing principle. However, their simplicity (they are easy to interpret) and their versatility (they can also be used to measure stability) are major advantages.

A more complex approach is based on Markov chain models and inspired from webpage ranking methodology [33]. Given a collection of rankings \( r^{(1)}, \ldots, r^{(B)} \), one extracts all genes \( j \) with rank not greater than a threshold \( k \) in at least one of the rankings, i.e. one considers the set \( \mathcal{E}_k = \{j : \exists b \in \{1, \ldots, B\} : r_{jb} \leq k\} \). Each gene \( j \in \mathcal{E}_k \) represents one state of a Markov chain, with \( \mathcal{E}_k \) being the state space. The event \( r_{jb} < r_{j'b} \) is interpreted as a transition from state \( j' \) to state \( j \). The corresponding transition matrix of the Markov chain is estimated based on the frequencies of the events \( \{r_{jb} < r_{j'b}\} \) (for \( j, j' \in \mathcal{E}_k \)) in the collection of rankings \( r^{(1)}, \ldots, r^{(B)} \). The obtained estimate is modified to achieve ergodicity, which guarantees the existence of a unique stationary distribution of the Markov
An aggregated ranking is then obtained by ordering the genes decreasingly according to their stationary probabilities.

Another recently proposed procedure that uses the sets $E_k$ is based on the cross-entropy Monte-Carlo optimization technique [44]. It searches iteratively for the optimal list in terms of the minimization of the sum of weighted distances between the candidate aggregated list and each of the input lists. Two different distance measures are considered: Kendall’s tau and Spearman's footrule. An advantage of this method is that it can put different weights to each input list depending on its importance/reliability. Note that the two approaches based on $E_k$ yield aggregated rankings for the top-genes only.

Aggregation can be simply performed using the GeneSelector package:

\begin{verbatim}
R> methodRR <- MergeMethods(methodlist)
R> agg_mean <- AggregateSimple(methodRR, measure = "mean")
R> agg_q90 <- AggregateSimple(methodRR, measure = "quantile", q = 0.9)
R> agg_MC <- AggregateMC(methodRR, type = "MCT", maxrank = 100)
\end{verbatim}

While the first command merges the five considered rankings (fold-change, ordinary t, Limma, Fox-Dimmic t and shrinkage t), the three next lines yield aggregated rankings using the averaging procedure, the 90%-quantile and the Markov chain model, respectively. Each function call produces a further ranking, which can again be analyzed with the toplist function.

\begin{verbatim}
R> toptenagg <- cbind(mean = toplist(agg_mean, show = F)$index,
                     q90 = toplist(agg_q90, show = F)$index,
                     MC = toplist(agg_MC, show = F)$index)
R> print(toptenagg)
\end{verbatim}

\begin{verbatim}
mean  q90  MC
[1,] 8399  8399  8399
[2,]  8225  8225  8225
[3,]  3268  8172  3268
[4,]  8172  3268  8172
[5,]  9478  8321  9478
[6,]  7106 11834  8173
[7,]  8173  7106  7106
[8,]  8321  5064  8321
[9,]  5064  9478  5064
[10,] 11834  8173 11834
\end{verbatim}
As becomes obvious from this simple example, aggregation does not solve the problem of the multiplicity of possible rankings, since different aggregation schemes produce different rankings. Hence, aggregation is rather a means of averaging out the peculiarities of each single ranking than a solution to the confusing multiplicity of ranking methods. It can also be seen from the aggregation outputs that the sophisticated Markov chain method and the simple averaging procedure produce very similar results. Note that many other aggregation methods are conceivable to "summarize" a collection of rankings. In particular, this topic is related to dimension reduction: to compress several rankings into a single one, one could also use techniques like principal components analysis.

4.3 Remarks on aggregation

This article focuses on the aggregation of ranked lists. Of course, it is also possible to aggregate the ranking criterion itself and then rank the genes based on this aggregated criterion. For example, a weighted sum of the t-statistics obtained from subsamples can be derived [51]. A related approach ranks genes by averaging the p-values obtained in a large number of bootstrap samples [4]. Note that simply averaging p-values gives virtually more weight to the perturbed data sets for which the considered gene is not among the top-genes. Moreover, we point out that one should be cautious while interpreting p-values obtained from bootstrap samples, since they are known to be in average smaller than those from the original sample [52, 53]. Hence, averaged bootstrapped p-values should be interpreted as ranking criterion only rather than as p-values within a significance testing framework.

Furthermore, we note that simply averaging p-values, statistics or ranks obtained from different lists may miss interesting information contained in the distribution of the statistic of a particular variable across the different lists. For instance, in the case of lists obtained from $B$ subsamples, not only the average statistic or rank reveal interesting patterns, but also its tails, i.e. its extreme values obtained for particular subsamples/bootstrap samples. Extreme values may indicate the presence of outliers whose removal from the data set yield completely different ranks or statistics. In this spirit, Pepe et al [22] point out in the context of their "probability of selection" framework that the whole survivor function gives a a more full description of sampling variability in the ranking.

Lastly, one should be careful while interpreting aggregated ranked lists if the top-genes are highly correlated. In subsamples or bootstrap samples, variables that are highly correlated are likely to have similar ranks. If the top-genes are highly correlated, they have to "share" the first places and may appear at the top of the
5 Conclusion

The stability of gene rankings should be routinely investigated as an important part of univariate analyses, since gene lists usually show considerable variability. In particular, it makes sense to examine the consistency of the results across different ranking criteria or different slightly modified versions of the available data set. Many procedures have been proposed in the literature for this purpose. The choice of the stability measure may depend on the sought objective. Sophisticated approaches like the overlap score \[41, 42\] should probably be preferred to ad-hoc set-based criteria if the goal is to compare the stability of two ranking criteria or the stability with respect to two different response variables. A further advantage of this approach is that it provides a solution to the difficult question of the depth of the comparison. However, simple measures such as the percentage of overlap might be more helpful to get an easily interpretable insight into the stability of the results. Aggregation, in particular aggregation of results obtained from slightly perturbed versions of the data set, is expected to yield more accurate rankings. Moreover, it is useful in practice to synthesize the results of different methods for interpretation purposes. Note, however, that there are again multiple possible aggregation schemes and thus potentially multiple aggregated rankings, as observed with the cross-entropy method using Kendall’s tau and Spearman’s footrule as distance measures \[44\]. Hence, aggregation does not intrinsically solve the problem that one does not know "which ranking is the right one".

Although the present article focuses on rankings, similar problems occur in the context of multivariate analyses and variable selection \[38, 39, 54, 55, 56\]. Before applying a "gene signature" in clinical practice, the stability of this signature may be assessed based on resampling techniques \[57\]. Similarly, one can also assess the stability of, e.g. inferred network structures or clustering outputs. We actually believe that stability issues will gain much attention in future biomedical research, because they are tightly connected to the problem of "fishing for significance" and "false research findings" \[18\] in several respects. Firstly, if the found gene list is instable with respect to the data set (i.e. if slightly modified versions of the data set yield fully different rankings), it is intuitively expected to validate poorly on a new independent data set. In this sense, the assessment of stability can be considered as a kind of preliminary (non-expensive) pseudo-validation step. Secondly, instability with respect to the method of analysis (i.e. when several procedures are applied...
successively on the same data set) increases the temptation to “fish for significance” by trying numerous methods until one of them returns satisfying results. This approach obviously yields an optimistic bias and potentially leads to false research findings. Hence, we believe that variability across methods should be adequately acknowledged and reported in publications.
Keypoints:

- A ranked gene list should not be considered as a unique definitive result. It makes sense to study the stability of a list by considering alternative ranking criteria and/or slightly modified versions of the data set.

- Many stability measures and aggregation methods have been proposed to handle multiple ranked lists. Methods based on the concept of overlap and frequencies of selection are the easiest to interpret.

- Many important methods for stability assessment and aggregation of gene lists are implemented in the Bioconductor package **GeneSelector**.
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