RAND Journal of Economics
Vol. 54, No. 1, Spring 2023
pp. 104–134

Stochastic contracts and subjective evaluations

Matthias Lang*,**

Subjective evaluations are widely used, but call for different contracts from classical moral-hazard settings. Previous literature shows that contracts require payments to third parties. I show that the (implicit) assumption of deterministic contracts makes payments to third parties necessary. This article studies incentive contracts with stochastic compensation, like payments in stock options or uncertain arbitration procedures. These contracts incentivize employees without the need for payments to third parties. In addition, stochastic contracts can be more efficient and can make the principal better off compared to deterministic contracts. My results also address the puzzle about the prevalence of labor contracts with stochastic compensation.

1. Introduction

■ This article studies moral hazard if the available performance measures are nonverifiable by outsiders. The prime examples of such subjective measures of performance are subjective evaluations by supervisors, co-workers, and others. Companies and organizations widely use subjective evaluations, as verifiable or objective performance measures are often unavailable. For example, Suvorov and van de Ven (2009) report that "many firms extensively use … subjective, noncontractible performance measures," and Murphy (1993) writes: "Most often, however, performance measurement is based on subjective performance ratings." Porter et al. (2008, p.148), Dessler (2017), Rajan and Reichelstein (2006), and MacLeod and Parent (1999) confirm this extensive use of subjective performance measures. Reasons for the use of subjective performance measures are that they are more difficult for the agent to manipulate and more accurate in measuring the principal's objectives than objective measures due to, for example, multitask problems. Indeed, Gibbons (1998) concludes that "objective performance measures typically cannot … create ideal incentives."

* University of Munich (LMU), Munich, Germany; matthias.lang@econ.lmu.de.
** CESifo, Munich, Germany.
I thank Helmut Bester, Yves Breitmoser, Florian Englmaier, Vera Gruber, Amir Habibi, Martin Hellwig, Daniel Krähmer, Anja Schöttner, Sebastian Schweighofer-Kodritsch, Roland Strausz, and Stefan Terstiege for very helpful discussions and the participants of the Committee for Organizational Economics for helpful comments. Financial support from the German Research Foundation (DFG) through the CRC/TRR190 (Project number 280092119) is gratefully acknowledged.
matthias.lang@econ.lmu.de

Performance measures' subjectivity, however, requires careful contracting because payments depend on reported evaluations instead of actual performance. The contract must provide incentives to the agent to work while ensuring that the principal has no incentives to misreport the subjective evaluations. Previous literature analyzing subjective evaluations shows that contracts without third-party payments fail to incentivize employees and workers in such settings: "it is impossible to elicit subjective information under the hypothesis that the contract is budget-balancing" (MacLeod, 2003). Malcomson (1984) and Carmichael (1989) already noticed that subjective evaluations require third-party payments as budget breakers. See also Deb et al. (2016) and Bester and Münster (2016).[1] MacLeod (2003), Fuchs (2007), Chan and Zheng (2011) and MacLeod and Tan (2016) provide formal proofs for these claims.

All these models, however, implicitly assume deterministic contracts. I call a contract deterministic if payments conditional on all available performance measures are deterministic.[2] In my setting, the available performance measures are the principal's evaluation and the agent's self-assessment. In a deterministic contract, thus, payments depend only on the principal's evaluation and the agent's self-assessment and are not stochastic. The opposite of a deterministic contract is a stochastic contract, that is, a contract using stochastic compensation for some evaluations and self-assessments.

I show that focusing on deterministic contracts is restrictive. Stochastic contracts can optimally incentivize employees and workers without any payments to third parties. The intuition is as follows: Differences in risk preferences between the principal and the agent enable contracts to implement different utilities for principal and agent without relying on third parties. These differences in utilities are necessary to provide the agent with *credible* incentives to exert effort because these utility differences can incentivize the principal to report subjective evaluations truthfully.

Another advantage of stochastic contracts concerns renegotiations. Third-party payments give the principal and the agent an obvious incentive to renegotiate the contract. *Ex post*, after reporting their evaluations, both the principal and the agent want to avoid paying money to an outsider—as Hart and Moore (1988) already discussed.[3] If they agree how to split the third-party payments, both are better off *ex post*. Anticipating these renegotiations, the principal, however, has incentives to misreport the evaluation and the agent's incentives to exert effort are reduced. Stochastic contracts can ensure *ex post* budget balance. Thus, stochastic contracts can be renegotiation-proof. With stochastic contracts, either the principal or the agent is unwilling to renegotiate *ex post* once the uncertainty is realized. The party who gains in the lottery has an incentive to avoid renegotiations because renegotiations are a zero-sum game in this case. Therefore, the lottery ought to be realized as soon as the principal and the agent report their evaluations—a timing that could be included in the contract by choosing the appropriate timing of the options or stock grants.[4] To sum up, stochastic contracts allow firms and organizations to use subjective evaluations without reverting to payments to third parties.

Given the possibility of renegotiations, stochastic contracts are more profitable for the principal and, hence, more efficient than deterministic contracts. The reason is that stochastic contracts can implement more effort levels than deterministic contracts. Even if renegotiations are impossible, stochastic contracts are more efficient than deterministic contracts in some settings,

[1] "Surplus destruction is necessary in equilibrium" (Deb et al., 2016). "To ensure the principal reports truthfully, any amount that the principal deducts from the agent's compensation … must be either destroyed or diverted to a use that does not benefit the principal" (Chan and Zheng, 2011). Notice that many authors view "money-burning" as a metaphor for conflict. If there is a positive continuation probability, subjective evaluations can be implemented in an infinitely repeated game as in Levin (2003). In the infinitely repeated-game setup, conflict replaces money-burning with the amount of money that may be "burned" being constrained by the continuation value of the relationship.

[2] This definition implies that a deterministic contract need not be *ex ante* deterministic.

[3] Maskin (2002) and Maskin and Tirole (1999) use lotteries to avoid renegotiations in hold-up settings.

[4] Stochastic contracts do not solve the problem that any moral-hazard contract (including those based on objective performance measures) provides incentives to both sides to renegotiate the contract after the agent's effort choice to insure the agent from the risk included in the performance measure. Hence, the entire literature assumes commitment to the contract until performance is realized.

for example, in the model of MacLeod (2003) with imperfect correlation between the principal's evaluation and the agent's self-assessment and bounds on third-party payments. These positive effects of stochastic contract go together with other benefits of stochastic contracts, including, for example, better retention (Oyer, 2004), better screening (Oyer and Schaefer, 2005), mitigating problems of gaming (Ederer et al., 2018), and easing of financing constraints (Core and Guay, 2001).[5] My results rely only on subjective evaluations and the assumption that the agent is more risk averse than the principal. These results thus help to explain why many labor contracts use stochastic compensation, even if the random events are uninformative about agents' efforts.[6]

Next, I discuss the optimal stochastic contracts derived in this article. There are three settings to consider. First, consider an agent who is unable or unwilling to understand the principal's objectives. The optimal contract triggers stochastic compensation if the principal reports the worst evaluation. The agent's reported self-evaluation does not matter. Stochastic compensation occurs in equilibrium. An illustrative example is a contract that pays a high wage for all but the worst evaluation. For the worst evaluation, the agent receives a lower wage and with some probability a bonus as the contracting parties might be uncertain about the validity of some contractual clauses or how to interpret a poor evaluation. This uncertainty incentivizes the principal to evaluate the agent's work appropriately while maintaining the agent's incentives to work.

Second, consider an agent who is able and willing to understand the principal's objectives. Then, the optimal contract triggers stochastic compensation if the principal reports an evaluation that disagrees with the agent's self-assessment. Stochastic compensation occurs only out of equilibrium. Third and finally, consider a principal who can observe the agent's effort. Then, the optimal contract triggers stochastic compensation if the agent chooses an effort level that differs from the contractually specified one. Stochastic compensation occurs only out of equilibrium. In all three cases, the uncertainty incentivizes the principal to evaluate the agent appropriately while not interfering with the agent's incentives to work.

There are many reasons to inform and educate agents about the principal's objectives. In particular, the literature on multitasking has contributed to our understanding of the need for these instructions. My article shows that the principal wants to inform agents even in a single-task model with unidimensional effort. The reason again consists of the incentives for the principal to perform the subjective evaluation appropriately. It is easier to provide these incentives for informed agents whose self-assessment is correlated with the appropriate evaluation by the principal. In some cases, optimal contracts can avoid all losses due to the subjectivity of the principal's evaluations and attain the classical second-best as if the evaluations were an objective performance measure.

The remainder of the article is organized as follows. Section 2 discusses the related literature. In Section 3, I introduce stochastic contracts into a model of subjective evaluations. Section 4 characterizes optimal (stochastic) contracts in applied models from the literature, discusses comparative statics and efficiency. In Section 5, I prove that—in general—stochastic contracts can incentivize employees without payments to third parties and are at least as efficient as deterministic contracts. Section 6 contains the concluding remarks. The proofs are relegated to the Appendix.

2. Related literature

■ The literature on stochastic contracts neglected subjective evaluations so far. Instead, the literature shows that stochastic contracts help screening risk-averse agents. The seminal articles include Gauthier and Laroque (2014), Strausz (2006), and Arnott and Stiglitz (1988). They show that stochastic contracts can be optimal depending on the curvature of utilities. This literature considers moral hazard, for example, Bennardo and Chiappori (2003) or Lang (2017),

[5] See also the next section about the related literature.

[6] The prevalence of stochastic contracts is sometimes seen as a puzzle for standard economic theory, for example, by Kedia and Rajgopal (2009).

and adverse selection, for example, Hellwig (2007) or Stiglitz (1982). Stochastic contracts trade off losses from increasing risk for risk-averse agents against gains from mitigating incentive-compatibility constraints. These models require heterogeneous risk aversion and, in particular, a correlation between types and risk aversion to make stochastic contracts optimal. There are no gains from stochastic contracts if the agent's type and risk aversion are common knowledge as in my setting.

Alternatively, Rasmusen (1987) studies team production with objective performance measures. He shows that budget-balancing contracts require stochastic contracts to incentivize agents and mitigate free-riding problems. For a single agent, there are no free-riding problems and, hence, no reasons to use stochastic contracts. Adopting a different approach, Oyer (2004) argues that stochastic contracts increase employee retention by adjusting wages to varying outside options. Finally, Maskin (2002) and Maskin and Tirole (1999) consider hold-up settings with incomplete contracts. They show that stochastic contracts allow to avoid incomplete contracts and to deal with renegotiations. My article is the first to scrutinize stochastic contracts for subjective evaluations.

I also relate to the extensive literature on subjective performance measures. Usually that literature assumes that evaluations are observable and occur in long-term relationships. These assumptions imply relational or implicit contracts, for example, in Li and Matouschek (2013), Goldluecke and Kranz (2013), Maestri (2012), Levin (2003), Compte (1998), Kandori and Matsushima (1998), Baker et al. (1994), MacLeod and Malcomson (1989), Bull (1987), and Shapiro and Stiglitz (1984). In these dynamic settings, subjective performance measures gain some credibility due to reputation effects created by the continuation values for both contracting parties. Hence, firms can use subjective performance measures to incentivize their employees.[7] See also Malcomson (2012) and MacLeod (2007) for recent surveys.

MacLeod (2003) was the first to implement subjective performance measures in a static setting. Section 4 builds on and discusses his model in detail. MacLeod and Tan (2016) extend the model by considering malfeasance and more general timings and information structures. Deb et al. (2016) consider multiple agents working for the principal. Letina et al. (2020) add external reviewers. Lang (2019) studies optimal communication of subjective evaluations. Bester and Dahm (2018) apply subjective evaluations to markets for credence goods. Fuchs (2007) considers subjective performance measures in a finitely repeated principal-agent model. He shows that it is optimal for the principal to announce a subjective evaluation only once at the end of the interaction and not in each period similarly to ideas in Ohlendorf and Schmitz (2012). This reporting pattern ensures that the agent does not learn whether a good performance has already occurred. Then, the same final incentives can be used repeatedly. These results imply that my results are also valid in finitely repeated moral hazard settings.

3. Subjective evaluations and stochastic contracts

■ Consider a risk-neutral principal (she) employing a risk-averse agent (he). The principal proposes a contract to the agent in a take-it-or-leave-it offer. The contract specifies payments depending on reports as described later. If the agent accepts the principal's offer and signs the proposed contract, the agent exerts effort $e \in E \subseteq \mathbb{R}$. The agent's effort is unobservable by the principal. Then, the principal privately learns her subjective measure of performance $t \in \mathcal{T} \subseteq \mathbb{R}$. The agent also receives a signal $s \in \mathcal{S} \subseteq \mathbb{R}$ about his performance: his self-assessment. The two signals follow a joint distribution $F(t, s|e)$ depending on the agent's effort e. Notice that the agent's self-assessment might well be uninformative. The subjectivity of both performance measures implies that both signals are unverifiable.

After learning these subjective performance measures, principal and agent both report their evaluations: The principal evaluates the agent by reporting her evaluation $\bar{t} \in \mathcal{T}$. The agent evalu-

[7] For multiple agents, tournaments also provide credible incentives. See, for example, Carmichael (1983); Malcomson (1986); Rajan and Reichelstein (2006, 2009).

FIGURE 1

TIMING OF THE MODEL

In period 1, the agent chooses his work effort if he accepts the principal's contract offer.

In period 2, the principal observes the subjective performance t of the agent.

In period 3, the agent observes his self-assessment s.

In period 4, the principal and the agent report their evaluations.

In period 5, the principal and the agent can renegotiate their contract.

In period 6, the principal pays the agent and makes third-party payments according to the contract.

Period 5 matters only for robustness: Optimal contracts in my article do not change if renegotiations are impossible. In addition, it does not matter whether principal and agent report simultaneously in period 4 or whether sequentially the principal reports before the agent.

ates himself by reporting his self-assessment $\bar{s} \in \mathcal{S}$. The contract specifies wages $w_{\bar{t}\bar{s}}$, that depend on the reports $\bar{t}$ and $\bar{s}$, as random variables and the timing of these random variables. Thus, the agent receives a payment equal to the realization of $w_{\bar{t}\bar{s}}$. In addition, the contract can specify third-party payments $y_{\bar{t}\bar{s}} \geq 0$ by the principal. In period 5, the principal can renegotiate the contract by proposing new payments $\hat{y}$ and $\hat{w}$.[8] If the agent accepts the principal's proposal, the principal makes these new payments $\hat{y}$ and $\hat{w}$ in period 6. If the agent rejects the principal's proposal, in period 6, the principal makes the payments specified in the initial contract: $y_{\bar{t}\bar{s}}$ and the realization of $w_{\bar{t}\bar{s}}$. The principal has no commitment other than the contract. Figure 1 summarizes the timing.

Utilities are as follows: The principal's (unverifiable) profits are $B(e, t, s) - w$ if she pays a wage w. The agent's preferences are represented by utilities $U(w, e) : \mathbb{R} \times E \mapsto \mathbb{R} \cup \{-\infty\}$ if he earns a wage w and exerts effort e. The agent's utilities are increasing and concave in the wage w if $U > -\infty$. His utilities are decreasing and convex in effort e. Utilities are twice continuously differentiable if $U > -\infty$. If the agent rejects the principal's contract offer, he receives a reservation utility $\bar{u}$ and the principal earns zero profits. For technical reasons, I assume that there is a smallest effort level: $\inf E \in E$. For later reference, $\mathcal{L}$ denotes the space of simple lotteries with finitely many realizations.

For the applications in the next section and to characterize optimal contracts, I use the following assumptions which are unnecessary for my general results in Section 5.

Assumption 1. The agent exerts effort $e \in E = [0, 1)$. The principal's measure of performance is $t \in \mathcal{T} = \{1, 2, \ldots, n\}$ with $n > 2$. The agent's self-assessment is $s \in \mathcal{S} = \{0\} \cup \mathcal{T}$.

Assumption 2. The distribution $F(t, s|e)$ is such that the principal observes performance t with probability $\gamma_t(e)$ depending on the agent's effort e. The probabilities $\gamma_t(e)$ are positive for all $t \in \mathcal{T}$ and $e \in [0, 1)$ and have a derivative $\partial\gamma_t(e)/\partial e$ denoted by $\gamma_t'(e)$. The ratio $\gamma_t'(e)/\gamma_t(e)$ increases in t and the distribution is weakly convex in effort e.[9] The agent observes a self-assessment $s = t$ with probability $p \in [0, 1]$ and a self-assessment $s = 0$ with probability $1 - p$.

Assumption 3. The agent's preferences are $U(w, e) = u(w) - d(e)$. Assume the limit $\lim_{e \to 1} d(e) = \infty$ and that there is an $a \geq -\infty$ so that $\lim_{w \to a} u(w) = -\infty$.

[8] It does not matter who makes the renegotiation offer. My results remain valid if the agent can propose new payments instead. Moreover, optimal contracts remain unchanged if renegotiations are impossible.

[9] The linear specification with $\gamma_t(e) = e\gamma_t^H + (1 - e)\gamma_t^L$ and two probability measures γ_t^H and γ_t^L in MacLeod (2003) is a special case of such a distribution.

Next, I discuss my framework and compare it with the literature.

□ **Interpretation and discussion of the setting.** My framework captures many models in the literature as special cases, for example, MacLeod (2003), Bester and Münster (2016), as well as the one-period case of Chan and Zheng (2011) and Fuchs (2007) and the stage games in Li and Matouschek (2013), Maestri (2012), Levin (2003) and Baker et al. (1994).[10]

According to the revelation principle, it is without loss of generality to restrict attention to contracts in which payments depend only on reports $\bar{t}$ and $\bar{s}$. Stochastic compensation with random variables $w_{\bar{t}\bar{s}}$ captures stochastic contracts in a general way. The random variables might mirror court proceedings or stock options. Payments could be lotteries or the principal could discard certain messages with some probability by "turning a blind eye," as in Herweg et al. (2010). Within the class of these contracts, I often refer to the subset of deterministic contracts. A contract is *deterministic* if wages are deterministic for any combination of reports, that is, the random variables $w_{\bar{t}\bar{s}}$ are constant or, with a slight abuse of notation, $w_{\bar{t}\bar{s}} \in \mathbb{R}$ for all reports $\bar{t} \in \mathcal{T}$ and $\bar{s} \in \mathcal{S}$. Renegotiations in period 5 do not matter for optimal contracts in the next sections. The purpose of this assumption is to show the robustness of the stochastic contracts considered.

Assumption 1 specifies the domains of the agent's effort and the evaluations. Assumption 2 ensures that the distribution $\gamma_t(e)$ of the agent's performance t satisfies the monotone likelihood ratio property and that higher signals t indicate greater effort by the agent. The probability p might capture the agent's ability or willingness to understand the principal's objectives. This setting captures any degree of correlation between the principal's evaluation and the agent's self-assessment with perfect correlation for $p = 1$ and no correlation for $p = 0$. Chan and Zheng (2011), Maestri (2012), and Bester and Münster (2016) consider a different correlation structure between binary assessments. Claim A1 in Appendix A shows that optimal contracts remain qualitatively unchanged for this alternative specification.[11] Assumption 3 ensures that preferences have a tractable functional form and imposes some Inada conditions. Assumptions 1, 2, and 3 build on the moral-hazard environment with subjective performance measures introduced in MacLeod (2003). As there are many degrees of freedom in setting up more applied environments, using existing models from the literature strengthens my findings and provides better comparability.

4. Stochastic contracts: applications

■ Before characterizing optimal contracts, I write down the optimization and calculate two benchmarks: deterministic contracts without payments to third parties and optimal complete contracts for objective performance measures. The revelation principle by Myerson (1982) and the renegotiation-proofness principle by Laffont and Tirole (1990) imply that focusing on truth-telling and renegotiation-proof contracts is without loss of generality. It is helpful to define expected payments by the principal $\tilde{w}_{ts} = \mathbb{E}(w_{ts}) + y_{ts}$ and the agent's certainty equivalent $\tilde{c}_{ts}$ of the stochastic compensation w_{ts} defined by $u(\tilde{c}_{ts}) = \mathbb{E}(u(w_{ts}))$. Grossman and Hart (1983) prove that the model can be solved in two steps. First, for every level of effort e, an optimal contract and its expected costs $C(e)$ for the principal are computed. The second step determines the optimal effort level e by solving $\max_{e \in E} \mathbb{E}[B(e, t, s)|e] - C(e)$. Focusing on the first step, Program A below summarizes the principal's problem if Assumptions 1 and 2 are satisfied and the optimal values can be implemented by a renegotiation-proof contract: the principal minimizes expected wages. The participation constraint (PC) ensures that the agent accepts the principal's contract offer. The incentive compatibility (IC) guarantees that the specified effort is optimal for the agent. In addition, feasible contracts must satisfy three novel conditions. Constraints (TT_P) and (TT_A) make truth-telling optimal for the principal and the agent with respect to their subjective performance

[10] I assume a risk-averse agent. Some references assume agents to be risk neutral, in particular, to simplify the exposition in dynamic settings.

[11] Indeed, the quantitative changes are due to their binary effort choice and binary evaluations.

measures. Finally, constraint (RA) captures the fact that the agent is more risk averse than the principal.

$$C(e) = \min_{\{\tilde{w}_{ts},\tilde{c}_{ts}\}_{t,s\in\mathcal{T}}} \sum_{t\in\mathcal{T}} (p\tilde{w}_{tt} + (1-p)\tilde{w}_{t0})\gamma_t(e) \tag{A}$$

$$\text{subject to} \sum_{t\in\mathcal{T}} (pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e))\gamma_t(e) \geq \bar{u}, \tag{PC}$$

$$e \in \arg\max_{\tilde{e}} \sum_{t\in\mathcal{T}} (pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e))\gamma_t(\tilde{e}) \tag{IC}$$

$$p\tilde{w}_{tt} + (1-p)\tilde{w}_{t0} \leq p\tilde{w}_{\bar{t}t} + (1-p)\tilde{w}_{\bar{t}0} \qquad \forall t, \bar{t} \in \mathcal{T}, \tag{TT$_P$}$$

$$\sum_{t\in\mathcal{T}} U(\tilde{c}_{t0}, e)\gamma_t(e) \geq \sum_{t\in\mathcal{T}} U(\tilde{c}_{t\bar{s}}, e)\gamma_t(e) \text{ and } \tilde{c}_{tt} \geq \tilde{c}_{t\bar{s}} \quad \forall t \in \mathcal{T}, \forall \bar{s} \in \mathcal{S}, \tag{TT$_A$}$$

$$\tilde{w}_{ts} \geq \tilde{c}_{ts} \qquad \forall t \in \mathcal{T}, \forall s \in \mathcal{S}. \tag{RA}$$

I begin by confirming the observation in the literature that deterministic contracts require payments to third parties to incentivize the agent to exert any meaningful effort.[12]

Lemma 1. Deterministic contracts without payments to third parties, that is, $y_{\bar{t}\bar{s}} = 0$ and $w_{\bar{t}\bar{s}} = \tilde{w}_{\bar{t}\bar{s}} = \tilde{c}_{\bar{t}\bar{s}}$ for all $\bar{t} \in \mathcal{T}$ and $\bar{s} \in \mathcal{S}$, cannot implement meaningful effort, $e > \min E$.

Intuitively, contracts for subjective performance measures need three kinds of incentives. First, the contract must incentivize the agent to exert effort. Second, the contract must incentivize the principal to evaluate the agent appropriately. Third, the contract must incentivize the agent to monitor the principal's evaluation if possible. Deterministic contracts without payments to third parties cannot provide these three incentives simultaneously and the agent exerts no effort. In particular, the principal has an incentive to evaluate the agent negatively to save wage payments. The only disciplining effect is the possibility that the agent could flag this misreporting and impose a contractual penalty on the principal if the contract allows for this. This penalty could incentivize the principal to evaluate the agent appropriately. Without third parties, the agent receives these payments. Hence, these payments make it optimal for the agent to pretend that even appropriate evaluations by the principal are biased. Therefore, it is impossible to incentivize the principal to evaluate the agent appropriately whenever wages vary in the principal's reports. Appropriate evaluations are only possible for a fixed-wage contract $w_{ts} = w \in \mathbb{R}$ because evaluations have no consequences in such a contract. Such a contract, however, cannot incentivize the agent. Thus, the agent exerts no effort, $e = \min E$.

Formally, consider the subgame once performance is realized. In this subgame, principal and agent report their evaluations. This subgame is a constant-sum game. Therefore, it is impossible for the agent's wages to vary in the principal's message. Consequently, Bester and Münster (2016) conclude that "money burning is required" to establish incentives for subjective performance measures. Thus, the literature uses payments to third parties to incentivize agents with subjective performance measures. Yet, these payments are unappealing for many reasons, and there are some claims in the literature that it is unclear how common payments to third parties are in reality.

As a *benchmark*, consider a classical moral-hazard setting, in which performance measures are verifiable and contractible. Then, constraints (TT$_P$) and (TT$_A$) are irrelevant. Optimal contracts are deterministic with no payments to third parties. Thus, the contract is renegotiation

[12] See the references in the introduction and, in particular, MacLeod (2003), Fuchs (2007), Chan and Zheng (2011), and MacLeod and Tan (2016).

proof and constraint (RA) binds. Optimal contracts minimize the principal's expected payments subject to the agent's PC and IC. The textbook solution to this problem is the optimal complete contract.

Lemma 2. Given Assumptions 1–3 and implementable effort $e > 0$, the optimal complete contract is $w_{ts} = c_t^* \in \mathbb{R}$ and $y_{ts} = 0$ for all $t \in \mathcal{T}$ and $s \in \mathcal{S}$. The values c_t^* are defined by

$$\frac{1}{u'(c_t^*)} = \mu_0 + \mu_1 \frac{\gamma_t'(e)}{\gamma_t(e)} \qquad \forall t \in \mathcal{T} \tag{1}$$

with the Lagrange multiplier μ_0 of the PC and the Lagrange multiplier μ_1 of the IC. Better performances yield higher wages, that is, wages c_t^* strictly increase in performance t.

The monotone likelihood ratio property implies increasing wages. If $\lim_{w\to\infty} u(w) = \infty$, any effort $e \in [0, 1)$ is implementable. For later reference, I denote the principal's expected costs of such a complete contract as $C^c(e)$. I follow the convention that $C^c(e) = \infty$ for nonimplementable effort e.

This article, however, focuses on *subjective* performance measures that are nonverifiable. Therefore, the additional constraints (TT_P) and (TT_A) do matter. To understand their relevance, suppose that the principal and the agent were to agree to the optimal complete contract determined by Eq. (1). Analyzing the problem backwards, the agent's message does not matter as $w_{ts} = c_t^*$ does not depend on his message s. The principal wants to minimize her wage payments. Therefore, she always reports the worst evaluation $t = 1$ because c_t^* increases in the evaluation t and, thus, $c_1^* = \min_t c_t^*$. Hence, the agent anticipates a wage of c_1^* independently of her performance. Thus, she optimally chooses effort $e = \min E$. Consequently, in the optimum these additional constraints are binding.

Optimal contracts depend on whether the principal can observe the agent's effort and on whether the agent is able and willing to understand the principal's objectives, that is, whether the agent's self-assessment is informative. After characterizing optimal contracts, I discuss efficiency and comparative statics of optimal contracts in risk aversion and in correlation p.

□ **Uninformative self-assessments.** If the agent is unable or unwilling to understand the principal's objectives, his self-assessments are uninformative, $p = 0$. This setting corresponds to Chan and Zheng (2011, one-period case),[13] Fuchs (2007, one-period case),[14] MacLeod (2003, No correlation), and Section IV in Levin (2003, stage game).[15] Then, nobody, particularly not the agent, can cross-check the principal's subjective evaluation of the agent's work. Hence, the principal's expected payments must be independent of her evaluation. Nonetheless, the principal can incentivize the agent even in that case as I show next. To incentivize the agent to exert effort, his expected utilities must depend on the evaluation by the principal. Therefore, stochastic payments must be used in equilibrium to avoid payments to third parties.

Proposition 1. If the agent's self-assessment is uninformative ($p = 0$) and effort $e > 0$ is implementable, the following contract is optimal given Assumptions 1–3: $y_{ts} = 0$ and

$$w_{ts} = \begin{cases} \omega^* & \text{if } t > 1 \\ \Delta & \text{if } t = 1 \end{cases}$$

[13] They assume $\mathcal{T} = \{1, 2\}$, no renegotiations, binary effort, and a correlated self-assessment for the agent which the contract does not directly use. Optimal contracts do not change qualitatively though. In their Section 5, they study contracts that directly use the correlated self-assessment. Even then, optimal contracts do not change qualitatively. See Claim A1 in Appendix A.

[14] He considers $\mathcal{T} = \{1, 2\}$, no renegotiations, and binary effort. Optimal contracts do not change qualitatively. Indeed the quantitative changes are due to their binary effort choice and binary evaluations.

[15] He studies continuous evaluations t and no renegotiations. Optimal contracts do not change qualitatively.

with a lottery $\Delta \in \mathcal{L}$, a wage $\omega^* \in \mathbb{R}$, and a certainty equivalent $b < \omega^*$ defined by

$$u(\omega^*) = \bar{u} + d(e) - \frac{\gamma_1(e)}{\gamma_1'(e)} d'(e) \quad \text{and} \quad u(b) = \bar{u} + d(e) + \frac{1-\gamma_1(e)}{\gamma_1'(e)} d'(e).$$

The lottery Δ has a mean ω^*, a certainty equivalent b, and is realized at the end of period 4.

A simple implementation of such a contract is: If the agent receives any evaluation except the worst one, the principal pays the agent a wage ω^*. If the agent receives the worst evaluation, the principal pays the agent a lower wage and with some probability a bonus. The principal's expected payments are ω^*, whereas the agent's certainty equivalent equals b.

The optimal contract only considers whether the agent receives the worst possible evaluation by the principal. Otherwise, evaluations do not matter. In particular, the agent's self-assessment and her reports are irrelevant. This contract reminds of solutions in moral hazard settings with risk-neutral agents and limited liability. Notice that the agent is risk averse here. Hence, it is the subjectivity of evaluations that drives this result. If the agent receives the worst evaluation, the principal's payments are stochastic like in an arbitration or court procedure. The agent values the stochastic payments for the worst evaluation $t = 1$ as if he receives a risk-free wage b. Nonetheless, the principal expects payments of $\omega^* > b$. Optimal contracts use lotteries in equilibrium to provide the agent with incentives to exert effort and to ensure that truth-telling is optimal for the principal and the agent. Notice that the subjectivity of her evaluation hurts the principal because the wage costs in this contract are above $C^c(e)$. Optimal stochastic contracts, however, imply higher efficiency than optimal deterministic contracts, as we discuss below at the end of Section 4.

In some cases, the principal can implement the optimal contract by an asset transfer. Consider a project with random payoffs $\rho(t)$. The principal offers the agent to work on this project or provides financing for this project with the following contract: After evaluating the project, the principal has the option to buy the project from the agent at a price $\pi = \omega^*$.[16] If the principal does not buy the project, the agent is left with the project. For such a contract to fit with my setting, I set the principal's benefits $B(e, 1, s) = 0$ and $B(e, t, s) = \rho(t)$ for $t > 1$. For the contract to provide correct incentives, the agent's certainty equivalent of $\rho(1)$ has to equal b and

$$\mathbb{E}[\rho(1)] \leq \omega^* \leq \mathbb{E}[\rho(t)] \quad \text{for all } t > 1.$$

This asset transfer provides the correct incentives analogously to the contract in Proposition 1 above. In particular, if $t > 1$, the principal buys the project as the project's value is above the contractually fixed price: $\pi \leq \mathbb{E}[\rho(t)]$ for all $t > 1$. If $t = 1$, the principal does not buy the project as the project's value is below the price: $\pi \geq \mathbb{E}[\rho(1)]$. Hence, the agent anticipates receiving $\pi = \omega^*$ for subjective evaluations $t > 1$ and a certainty equivalent of b for $t = 1$. Therefore, Proposition 1 ensures that this contract in combination with the asset transfer incentivizes the agent—dealing in an elegant way with the issue of renegotiations. Next, turn to informative self-assessments by the agent.

□ **Informative self-assessments.** If the agent is able and willing to understand the principal's objectives, his self-assessments are informative, $p > 0$. Then, the setting corresponds, for example, to MacLeod (2003, Perfect/Imperfect Correlation) and the stage game in Section II.C of Baker et al. (1994).[17]

[16] For the use of such option contracts, see also Bester and Krähmer (2017) and Bester and Krähmer (2012).

[17] They consider $p = 1$, no renegotiations, and $\mathcal{T} = \{1, 2\}$. Optimal contracts do not change qualitatively, though. Indeed the quantitative changes are due to their binary evaluations.

Proposition 2. If the agent's self-assessment is informative ($p > 0$) and effort $e > 0$ is implementable, the following contract is optimal given Assumptions 1–3: $y_{ts} = 0$ and

$$w_{ts} = \begin{cases} c_t^* & \text{if } s = 0 \text{ or } t = s \\ \Delta_t & \text{otherwise} \end{cases}$$

with the optimal complete contract c_t^* defined in equation (1) and lotteries $\Delta_t \in \mathcal{L}$ that have a mean c_n^*/p, a certainty equivalent c_t^*, and are realized at the end of period 4.

Optimal contracts use the agent's self-assessment to cross-check the principal's evaluation of his work. Therefore, the principal evaluates the agent correctly. In equilibrium, the agent reports either $\bar{s} = 0$ or $\bar{s} = t$. Hence, the equilibrium wages in the optimal contract resemble the wages in the optimal complete contract discussed above. The agent's informative self-assessment ensures that there are no losses due to the subjectivity of the evaluations. This self-assessment allows the contract to employ the agent to monitor the principal in evaluating him appropriately. As the principal designs the contract, the principal monitors herself in this way. This monitoring serves as a commitment device for the principal to evaluate the agent appropriately and thus to make the incentives, that were specified in the contract, credible. The contract in Proposition 2 makes truth-telling optimal for the agent by ensuring that his expected utilities do not depend on his self-assessment. The lotteries guarantee that the expected wages, however, depend on the agent's self-assessment. In particular, conflicts between the principal's evaluation and the agent's self-assessment increase the expected payments by the principal. Therefore, it is optimal for the principal to report her subjective evaluation of the agent truthfully.

Notice that optimal contracts include stochastic compensation for all evaluations—no matter whether the principal's evaluation is good or bad. Stochastic payments occur whenever the principal's evaluation conflicts with the agent's self-assessment. These conflicts occur only out of equilibrium and might explain why arbitration or court procedures exist but are infrequently used following good evaluations. A simple way to implement optimal contracts is: A contract specifies n bonus levels $(c_t^*)_{t=1}^n$ calculated in Proposition 2. The principal discretionarily proposes one of these bonus levels. If the agent agrees, the principal pays the proposed bonus. If the agent disagrees, there is an arbitration process with a random outcome.

In some cases, the principal can implement the optimal contract by an asset transfer. Consider a project with random payoffs $\rho(t)$ as before. The principal offers the agent to work on this project or provides financing for this project with the following contract: After evaluating the project, the principal proposes a price from the set of contractually fixed prices $(\pi_t)_{t=1}^n = (c_t^*)_{t=1}^n$ calculated in Proposition 2. If the agent agrees to the proposed price, the principal buys the project from the agent at that price.[18] If the agent disagrees with the proposed price, the agent is left with the project. For such a contract to fit with my setting, I set the principal's benefits to $B(e, t, s) = \rho(t)$. For the contract to provide correct incentives, the agent's certainty equivalent of $\rho(t)$ has to equal c_t^* and

$$\mathbb{E}[\rho(t)] \geq (c_t^* - (1 - p)c_1^*)/p \quad \text{for all } t \in \mathcal{T}. \tag{2}$$

This asset transfer provides the correct incentives analogously to the contract in Proposition 2 above. In equilibrium, the principal buys the project at price π_t because any other price proposal $\pi_{t'}$ yields lower profits as with probability p the agent's self-assessment reveals the principal's deviation and the agent rejects the principal's offer:

$$\mathbb{E}[\rho(t)] - \pi_t \geq p \cdot 0 + (1 - p)(\mathbb{E}[\rho(t)] - \pi_{t'}) \quad \text{for all } t' < t \text{ by Inequality (2).}$$

The agent agrees to a proposed price π_t if her self-assessment is $0 \neq s \leq \bar{t}$ or $s = 0$ as he is indifferent between the price π_t and the certainty equivalent of c_t^* of the project. Hence, the

[18] See Footnote 16 for such option contracts in the literature.

agent anticipates receiving π_t for subjective evaluation t. Therefore, Proposition 2 ensures that this contract in combination with the asset transfer optimally incentivizes the agent to exert the specified effort. Finally, I consider observable effort.

□ **Observable effort.** Some articles, for example, Letina et al. (2020, without observers) and the stage games in Li and Matouschek (2013), MacLeod and Malcomson (1989), and Bull (1987), study observable effort. To accommodate observable effort, I adjust Assumptions 1 and 2 so that the principal observes the agent's (nonverifiable) effort, that is, $t = e$.

Assumption 1a. The agent exerts effort $e \in E = [0, 1)$. The principal's measure of performance is $t \in \mathcal{T} = E$.

Assumption 2a. The distribution $F(t, s|e)$ is such that the principal observes performance t=e with probability 1.

Proposition 3. If $\sup_w u(w) > \bar{u} + d(\hat{e})$, the following contract implements effort $\hat{e}$ at first-best costs given Assumptions 1a, 2a, and 3: $y_{ts} = 0$ and

$$w_{ts} = \begin{cases} u^{-1}(\bar{u} + d(\hat{e})) & \text{if } t = \hat{e} \\ \Delta & \text{otherwise.} \end{cases}$$

The lottery $\Delta \in \mathcal{L}$ has a mean $u^{-1}(\bar{u} + d(\hat{e}))$, satisfies $\mathbb{E}u(\Delta) \leq \bar{u} + d(0)$, and is realized at the end of period 4.

It is well known that verifiable effort allows the principal to attain the first best. As effort is naturally observable for the agent, he can perfectly check the principal's evaluation. Therefore, there are no losses due to the subjectivity of evaluations in contrast to Proposition 1. The agent can monitor the principal to evaluate him appropriately. Even if the agent were to forget his effort choice and receive no or a fully uninformative self-assessment, however, it would be possible to attain the first best here. The lotteries allow the principal to be indifferent between her reports so that truthful reporting is optimal for her. Notice that optimal contracts include stochastic compensation only out of equilibrium but potentially for any evaluation—regardless of whether the principal's evaluation is good or bad. Stochastic payments occur whenever the principal's evaluation indicates a conflict, that is, the principal reports the agent choosing not the effort level specified by the contract. These conflicts occur only out of equilibrium.

In some cases, the principal can implement the optimal contract by an asset transfer. Consider a project with random payoffs $\rho(t)$ as before. The principal offers the agent to work on this project or provides financing for this project with the following contract: After evaluating the project, the principal has the option to buy the project from the agent at a price $\pi = u^{-1}(\bar{u} + d(\hat{e}))$. If the principal does not buy the project, the agent is left with the project. For such a contract to fit with my setting, I set the principal's benefits $B(e, t, s) = \rho(t)$. For the contract to provide correct incentives, $\mathbb{E}[\rho(\hat{e})] = u^{-1}(\bar{u} + d(\hat{e}))$, $\mathbb{E}[\rho(t)]$ increases in t and $\mathbb{E}u(\rho(t)) \leq \bar{u} + d(0)$ for $t \leq \hat{e}$. The asset transfer provides the correct incentives analogously to the contract in Proposition 3 above. In particular, if $e \geq \hat{e}$, the principal buys the project as the project's value is above the contractually fixed price: $\mathbb{E}[\rho(e)] \geq \pi$. If $e < \hat{e}$, the principal does not buy the project as the project's value is below the price: $\mathbb{E}[\rho(e)] < \pi$. Hence, the agent anticipates receiving the price π for effort $e \geq \hat{e}$ and retaining the project with expected utility below $\bar{u} + d(0)$ for effort $e < \hat{e}$. Therefore, this contract in combination with the asset transfer optimally incentives the agent to exert effort $\hat{e}$.

In summary, Propositions 1, 2, and 3 characterize optimal contracts for subjective evaluations. These optimal contracts provide agents with economically meaningful incentives without payments to third parties. Thus, it is possible and optimal to avoid third-party payments by using stochastic contracts. The next section studies the efficiency of stochastic contracts in detail.

□ **Efficiency of stochastic contracts.** Before comparing the efficiency of stochastic and deterministic contracts, let me clarify the usual notion of efficiency in moral-hazard settings. *Ex ante* efficiency considers the sum of the principal's and the agent's *ex ante* expected utilities for a given effort e. By standard arguments, optimal contracts satisfy the agent's PC with equality. Therefore, the agent's *ex ante* expected utilities equal his reservation utility u. Thus, a contract increases *ex ante* efficiency if and only if the contract increases the principal's *ex ante* expected profits, that is, the *ex ante* expected costs $C(e)$ decrease. Remember that I call a contract *deterministic* if $w_{\bar{t}\bar{s}} = \tilde{c}_{\bar{t}\bar{s}}$ for all reported evaluations $\bar{t} \in \mathcal{T}$ and $\bar{s} \in \mathcal{S}$. I show later in Section 5 that—in general—optimal stochastic contracts are at least as efficient as deterministic contracts. Here, I begin by comparing stochastic contracts to deterministic contracts without third-party payments. In all three settings considered above, stochastic contracts increase efficiency.

Lemma 3. Assume effort $e > 0$, Assumptions 1–3 or 1a, 2a, and 3, and that third-party payments are impossible, that is, $y_{\bar{t}\bar{s}} = 0$ for all reports $\bar{t} \in \mathcal{T}$ and $\bar{s} \in \mathcal{S}$. Stochastic contracts increase efficiency compared to deterministic contracts.

The reason is that without third-party payments deterministic contracts cannot incentivize agents whereas stochastic contracts can incentivize agents as discussed above. The next benchmark is the most interesting one. I compare stochastic contracts without third-party payments to deterministic contracts with third-party payments. In all three settings considered above, stochastic contracts increase efficiency.

Proposition 4. Suppose effort $e > 0$ and Assumptions 1–3 or 1a, 2a, and 3. Stochastic contracts increase efficiency compared to deterministic contracts with third-party payments.

The reason is that any third-party payments imply successful renegotiations.[19] Therefore, third-party payments cannot incentivize truthful reporting of the evaluations. Stochastic contracts can incentivize truthful reporting and effort provision by the agent as these contracts are renegotiation proof according to Propositions 1, 2, and 3. Given the importance of renegotiations for this argument, I finally turn to a comparison without renegotiations.

Thus, suppose renegotiations are impossible. I compare stochastic contracts to deterministic contracts with bounded third-party payments. This upper bound might be due to credibility or renegotiation issues. In repeated games, this upper bound could correspond to the continuation value of the relationship. See also Proposition 8 in MacLeod (2003) for a static model with bounded third-party payments. If third-party payments are sufficiently bounded, it is intuitive that stochastic contracts increase efficiency in all three settings considered above. Yet what happens for higher bounds? It turns out that stochastic contracts can increase efficiency for any bound if we focus on self-assessments with small $p > 0$.

Proposition 5. Assume that renegotiations are impossible, third-party payments are bounded, effort $e > 0$, and Assumptions 1–3. Stochastic contracts increase efficiency compared to deterministic contracts for any bounds on third-party payments and sufficiently small $p > 0$.

Intuitively, for p sufficiently small, most of the time the agent cannot cross-check the principal's evaluation. Hence, the principal anticipates any deviations from truth-telling to be detected with small probabilities. Therefore, only very large third-party payments can incentivize the principal to report her evaluation correctly. Any bound on third-party payments makes this

[19] Renegotiations allow the contracting parties to avoid paying money to an outsider.

impossible, and thus, the principal has to adjust the deterministic contract making it less efficient than the optimal stochastic contract. Before turning to the general model in Section 5, I discuss comparative statics of optimal contracts in risk aversion and in correlation, in the next two sections.

□ **Comparative statics in risk aversion.** To study comparative statics in risk aversion, I consider utilities where the disutility of effort is measured in monetary terms. This alternative assumption regarding utilities is as follows:

Assumption 3a. The agent's preferences are $U(w, e) = -\exp(-k(w - d(e)))$ with a $k > 0$. Assume that the limit $\lim_{e\to 1} d(e) = \infty$. The agent's reservation utility is $\bar{u} = -\exp(-k\bar{w})$ with some $\bar{w} \in \mathbb{R}$. Finally, assume that the distribution $\gamma_t(e)$ is log-convex.

Under Assumption 3, any change in risk preferences would imply a change in the disutility of effort. Therefore, it is impossible to derive meaningful comparative statics in risk aversion under Assumption 3. Assumption 3a solves this problem because it allows to change the agent's risk aversion without changing the agent's disutility of effort. Appendix B contains optimal contracts for Assumption 3a replacing Assumption 3.

I begin with the optimal complete contract for verifiable performance measures as a benchmark. Optimal contracts are deterministic with no payments to third parties:

$$w_{ts} = c_t^{*\mathrm{CARA}} = d(e) + \frac{1}{k}\ln\left(\exp(k\bar{w}) + \mu_1 k\left(\frac{\gamma_t'(e)}{\gamma_t(e)} + kd'(e)\right)\right) \qquad \forall t \in \mathcal{T}, s \in \mathcal{S}$$

with the Lagrange multiplier μ_1 of the incentive compatibility.[20] For vanishing risk aversion, $k \to 0$, any frictions disappear and the first best is attainable. For very high risk aversion, $k \to \infty$, only trivial effort $e = 0$ is implementable. Beyond that, comparative statics of optimal contracts in risk aversion are ambiguous in this benchmark. Grossman and Hart (1983) summarized: "Very little can be said."

Turning to subjective evaluations, I begin with uninformative self-assessments, $p = 0$. Proposition B1 in Appendix B confirms that, similar to Proposition 1, the following contract is optimal:

$$w_{ts} = \begin{cases} \omega^{*\mathrm{CARA}} & \text{if } t > 1 \\ \Delta & \text{if } t = 1. \end{cases}$$

The lottery $\Delta \in \mathcal{L}$ has mean $\omega^{*\mathrm{CARA}}$ and a certainty equivalent of b^{CARA}. Thus, the optimal contract offers a deterministic wage whenever the principal does not report the worst evaluation. For the worst evaluation, payments are stochastic ensuring that the principal does not gain by reporting this worst evaluation, although the agent's certainty equivalent is lower than his wage for better evaluations. In equilibrium, the principal reports all evaluations truthfully.

Proposition 6. Suppose $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$, Assumptions 1, 2, and 3a, and that the agent's self-assessment is uninformative, that is, $p = 0$. The wage

$$\omega^{*\mathrm{CARA}} = \bar{w} + d(e) - \frac{1}{k}\ln\left(1 + \frac{\gamma_1(e)}{\gamma_1'(e)}kd'(e)\right) \underset{k\to 0}{\longrightarrow} \bar{w} + d(e) - \frac{\gamma_1(e)}{\gamma_1'(e)}d'(e)$$

increases in the agent's risk aversion k. Similarly, the certainty equivalent

$$b^{\mathrm{CARA}} = \bar{w} + d(e) - \frac{1}{k}\ln\left(1 - \frac{1 - \gamma_1(e)}{\gamma_1'(e)}kd'(e)\right) \underset{k\to 0}{\longrightarrow} \bar{w} + d(e) + \frac{1 - \gamma_1(e)}{\gamma_1'(e)}d'(e)$$

[20] Lemma B1 in Appendix B provides the formal result.

increases in risk aversion k. For vanishing risk aversion, $k \to 0$, the difference $\omega^{*\mathrm{CARA}} - b^{\mathrm{CARA}}$ converges to $-d'(e)/\gamma_1'(e)$.

The optimal contract does not pin down a unique lottery Δ, but there are infinitely many lotteries that satisfy the requirements on the mean and the certainty equivalent. Therefore, general comparative statics of the lottery Δ are impossible. For vanishing risk aversion, $k \to 0$, however, the riskiness of the lottery must diverge to infinity to generate a positive risk premium for an (almost) risk-neutral agent. In addition, I study two common and simple classes of lotteries below: the normal and the binary distribution. I begin with normally distributed lotteries. Then

$$\Delta^1 \sim \mathcal{N}\left(\omega^{*\mathrm{CARA}}, 2\frac{\omega^{*\mathrm{CARA}} - b^{\mathrm{CARA}}}{k}\right)$$

captures the stochastic payments in the contract.[21] Next, I turn to binary lotteries. Consider Δ^2 as a lottery that pays $\omega^{*\mathrm{CARA}} + z$ and $\omega^{*\mathrm{CARA}} - z$ with a probability of half each. Then,

$$z = \frac{1}{k}\ln\left(\exp(k(\omega^{*\mathrm{CARA}} - b^{\mathrm{CARA}})) + \sqrt{\exp(2k(\omega^{*\mathrm{CARA}} - b^{\mathrm{CARA}})) - 1}\right)$$

or, equivalently, $\cosh(kz) = \exp(k(\omega^{*\mathrm{CARA}} - b^{\mathrm{CARA}}))$ yields the required certainty equivalent for the agent. Both examples show that the comparative statics of the variance in the agent's risk aversion are ambiguous.

Next, I study informative self-assessments, $p > 0$. Proposition B2 in Appendix B confirms that, similar to Proposition 2, the following contract is optimal:

$$w_{ts} = \begin{cases} c_t^{*\mathrm{CARA}} & \text{if } s = 0 \text{ or } t = s \\ \Delta_t & \text{otherwise} \end{cases}$$

with the optimal complete contract $c_t^{*\mathrm{CARA}}$ defined above. The lotteries Δ_t have a mean of $c_n^{*\mathrm{CARA}}/p$ and a certainty equivalent of $c_t^{*\mathrm{CARA}}$. Thus, the optimal contract guarantees deterministic equilibrium wages because the agent truthfully reports her self-assessment $s \in \{t, 0\}$ in equilibrium. These equilibrium wages coincide with the benchmark wages. Therefore, subjective evaluations do not imply any losses for the principal in the case of informative self-assessments, $p > 0$. Out of equilibrium, the principal or the agent could misreport. Such reports imply stochastic payments out of equilibrium. These lotteries ensure that the principal does not gain by misreporting the subjective evaluation although the agent's certainty equivalent remains unchanged.

Proposition 7. Suppose $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$, Assumptions 1, 2, and 3a, and that the agent's self-assessment is (partially) informative, that is, $p > 0$. For vanishing risk aversion, $k \to 0$, equilibrium wages converge to

$$w_{ts} \underset{k\to 0}{\longrightarrow} \bar{w} + d(e) + \frac{\gamma_t'(e)}{\gamma_t(e)} \frac{d'(e)}{\sum_{\hat{t}=1}^{n} \frac{(\gamma_{\hat{t}}'(e))^2}{\gamma_{\hat{t}}(e)}}$$

for $s \in \{0, t\}$ and $t \in \mathcal{T}$, and expected out-of-equilibrium payments converge to

$$\mathbb{E}(w_{ts}) \underset{k\to 0}{\longrightarrow} \frac{1}{p}(\bar{w} + d(e)) + \frac{\gamma_n'(e)}{\gamma_n(e)} \frac{d'(e)}{p\sum_{\hat{t}=1}^{n} \frac{(\gamma_{\hat{t}}'(e))^2}{\gamma_{\hat{t}}(e)}}$$

for $s \neq 0, t$ and $t \in \mathcal{T}$.

[21] The variance of this lottery usually decreases in risk aversion, in particular, for low-risk aversion: $\lim_{k\to 0} \frac{\partial \mathrm{Var}(\Delta^1)}{\partial k} < 0$. It is easy, however, to construct examples in which the variance of this lottery Δ^1 increases in risk aversion because the difference $\omega^{*\mathrm{CARA}} - b^{\mathrm{CARA}}$ also depends on the agent's risk aversion. Consider, for example, $d'(e) = k = 1$, $\gamma_1(e) = 1/2$, and $\gamma_1'(e) = -0.55$.

To study the comparative statics of the stochastic out-of-equilibrium wages again I turn to the two classes of lotteries introduced above. For normally distributed lotteries,

$$\Delta_t^1 \sim \mathcal{N}\left(c_n^{*\text{CARA}}/p, 2\frac{c_n^{*\text{CARA}}/p - c_t^{*\text{CARA}}}{k}\right)$$

captures the stochastic payments out of equilibrium. The variance of this lottery decreases in risk aversion for sufficiently small risk aversion:

$$\lim_{k \to 0} \frac{\partial \text{Var}(\Delta_t^1)}{\partial k} < 0$$

for all t. Returning to binary lotteries with outcomes $c_n^{*\text{CARA}}/p - z_t$ and $c_n^{*\text{CARA}}/p + z_t$, the values z_t defined by

$$\cosh(kz_t) = \exp(k(c_n^{*\text{CARA}}/p - c_t^{*\text{CARA}}))$$

yield the required certainty equivalent for the agent. This value z decreases in the agent's risk aversion k for sufficiently small risk aversion.

Finally, I scrutinize observable effort. Proposition B3 in Appendix B confirms that, similar to Proposition 3, the following contract is optimal:

$$w_{ts} = \begin{cases} \bar{w} + d(e) & \text{if } t = e \\ \Delta & \text{otherwise.} \end{cases}$$

The lottery Δ has a mean $\bar{w} + d(e)$ and a risk premium of at least $d(e) - d(0)$. Thus, the optimal contract offers a deterministic wage that is paid whenever the principal reports the contractually specified effort. In equilibrium, the agent always chooses this effort, and the principal reports truthfully so that equilibrium wages are deterministic.

Proposition 8. Suppose Assumptions 1a, 2a, and 3a are satisfied. Equilibrium wages w_{es} do not depend on the agent's risk aversion. The off-equilibrium payments Δ depend on the agent's risk aversion. In particular, the riskiness of the lottery decreases in the agent's risk aversion k as the risk premium is constant.

To study the comparative statics of the stochastic payments out of equilibrium, again turn to the two classes of lotteries introduced above. For normally distributed lotteries, the lotteries

$$\Delta^1 \sim \mathcal{N}(\bar{w} + d(e), \sigma^2)$$

with $\sigma^2 \geq 2(d(e) - d(0))/k$ capture the stochastic payments in the contract. The lower bound on the variance of this lottery is decreasing and convex in the agent's risk aversion.

Returning to binary lotteries with outcomes $\bar{w} + d(e) - z$ and $\bar{w} + d(e) + z$, the following equation defines a lower bound on the value z

$$\cosh(kz) = \exp(k(d(e) - d(0))).$$

That bound decreases in k and has a limit of $\lim_{k \to 0} \partial z/\partial k = -\infty$ for small risk aversion.

□ **Comparative statics in correlation.** Before turning to a more general setting in the next section, I study comparative statics of optimal contracts in correlation between the principal's and the agent's evaluation. For observable effort and in the benchmark for verifiable performance, the agent's self-assessment does not matter. Therefore, optimal contracts in these two cases do not depend on the amount of correlation between the principal's and the agent's evaluation. Thus, I focus on subjective evaluations with unobservable effort here. I scrutinize what happens if the agent's self-assessment becomes more informative about the principal's evaluation.

Proposition 9. Given Assumptions 1–3, the more informative the agent's self-assessment is,

- the lower the principal's expected costs of the contract are.
- the more wage levels optimal contracts distinguish.

Beginning with uninformative self-assessments, the optimal contract specifies just two utility levels for the agent according to Proposition 1. This is less efficient than the objective benchmark which is fully differentiated. For higher correlation, optimal contracts are as efficient as the second-best benchmark according to Proposition 2. Therefore, the principal's expected costs of the contract decrease in correlation. Positive correlation between the principal's and the agent's evaluation allows the agent to cross-check the principal's reported evaluation. Therefore, optimal contracts can avoid any frictions arising from the subjectivity of the evaluations. Thus, incentivizing the agent to exert effort becomes cheaper for the principal. Remember that one interpretation of the correlation p is how much the agent is able and willing to understand the principal's objectives. It seems plausible that the principal can sometimes increase the informativeness of the agent's self-assessment by informing and educating agents about her objectives. Thus, we can interpret this result as an incentive for the principal to inform and educate agents about her objectives.

Next, going from a contract ($p = 0$) with two different utility levels for the agent to a contract ($p > 0$) that is fully differentiated and has a different wage for each evaluation means that optimal contracts have more wage levels the higher the correlation. With uninformative self-assessments, the agent cannot cross-check the principal's reported evaluation, and thus, the expected wage payments for the principal have to be constant for all evaluations. Optimal contracts pay a lower wage in terms of the agent's certainty equivalent only for the worst reported evaluation. With informative self-assessments, equilibrium wages are the same for the principal and the agent. Therefore, the basic trade-off of moral hazard between insurance and incentives ensures that optimally each evaluation has its own wage level. Thus, the number of different wage levels increases from two to n (full differentiation).

Focusing on informative self-assessments, Proposition 2 immediately shows that expected out-of-equilibrium wages and the riskiness of out-of-equilibrium wages are decreasing in the correlation between the principal's and the agent's evaluation. The higher the correlation, the more likely it is that the agent observed the principal's evaluation and hence, the more likely that any misreporting by the principal triggers out-of-equilibrium payments. Thus, it is possible to reduce expected out-of-equilibrium payments without affecting the incentives of the principal to report the evaluation correctly. Equilibrium wages do not depend on the correlation for informative self-assessments. Thus, observational data can identify the change from no correlation to some correlation but not a further increase in correlation.

A natural follow-up question concerns the generality of my results about stochastic contracts. The next section studies the abstract setting without the additional assumptions and shows that stochastic contracts in general can provide appropriate incentives.

5. Stochastic contracts: the general case

■ This section considers the general moral-hazard setting with subjective performance evaluations *without* Assumptions 1–3. This general setting captures many models in the literature as special cases, as discussed above. As shown above in Lemma 1, deterministic contracts require payments to third parties to incentivize the agent to exert any meaningful effort. In contrast, stochastic contracts do not need payments to third parties. Such contracts can incentivize agents at least as well as deterministic contracts with third-party payments.

Theorem 1. Suppose that a deterministic contract $\mathcal{W}'$ incentivizes the agent to exert effort e. Then, there is a (stochastic) contract $\mathcal{W}$ (without payments to third parties) that incentivizes the agent to exert effort e. The statement remains valid if renegotiations are impossible.

Intuitively, Theorem 1 constructs a contract $\mathcal{W}$ that uses lotteries as stochastic compensation. The principal's utilities of such a lottery are $\mathbb{E}(w_{\tilde{t}\tilde{s}})$, whereas the agent's utilities are $\mathbb{E}u(w_{\tilde{t}\tilde{s}})$. The principal's and the agent's certainty equivalences of a given lottery $w_{\tilde{t}\tilde{s}}$ differ because the agent is more risk averse than the principal. This difference in risk preferences guarantees that the principal's certainty equivalent of these wages is higher than the agent's certainty equivalent. Therefore, it is *possible* to impose a penalty on the principal and reduce her utilities while not increasing the agent's utilities. This possibility allows the contract to provide all three incentives that I discuss following Lemma 1: the agent to exert effort, the principal to evaluate the agent appropriately, and the agent to monitor the principal's evaluation.

Formally, the subgame (beginning in period 4) once performance is realized, in which the principal and the agent report their evaluations, is no longer a constant-sum game and the agent's utilities can vary with the principal's reported evaluation. Hence, it is possible to provide all three incentives simultaneously. Consequently, stochastic contracts can provide agents with economically meaningful incentives without payments to third parties. The next corollary establishes that stochastic contracts outperform deterministic contracts with third-party payments because they are more profitable for the principal.

Corollary 1. Optimal stochastic contracts are at least as profitable for the principal as deterministic contracts. Sometimes stochastic contracts strictly increase the principal's profits. The statement remains valid if renegotiations are impossible.

6. Conclusion

■ This article studies incentive contracts based on subjective evaluations. Many firms use subjective evaluations. Frequently, subjective evaluations are more difficult for the agent to manipulate and more accurate in measuring the principal's objectives than objective performance measures. Classical contracts fail to incentivize employees based on subjective evaluations because they neglect credibility issues. In addition to incentivizing employees to exert effort, contracts must guarantee that supervisors report their evaluations truthfully. Therefore, subjective evaluations require novel contracts.

The literature on these novel contracts has implicitly assumed deterministic contracts. This restriction to deterministic contracts implies that contracts require payments to third parties. Otherwise, it is impossible to provide incentives to the agent. I show that stochastic contracts can optimally incentivize employees without the need for third-party payments. Hence, contracts are budget balanced. Stochastic compensation, like stock options or shares, ensures that contracts provide incentives both for truthful reporting of evaluations and—at the same time—for the agent to exert effort. This stochastic compensation is costly for the risk-neutral principal but provides less utility for the risk-averse agent. Stochastic contracts can even increase the principal's profits by making deviations from truth-telling unprofitable and, thus, ensuring the credibility of the agent's incentives. In these cases, the principal strictly prefers to use stochastic contracts. An additional major benefit of stochastic contracts is that they are *ex post* efficient and, thus, renegotiation proof.

To follow the literature (see the references above), I assume a risk-neutral principal. Nonetheless, my optimal contracts do not require this assumption. All that is required is that the principal is less risk averse than the agent. This assumption is very common. In addition, there is a strong economic intuition for the principal being less risk averse because she can usually diversify her risks. This is not the case for the agent in most labor contracts because employees find it difficult to diversify or insure their labor income. Thus, the assumption of risk-neutral agents is mainly made in dynamic settings to abstract from the implications of consumption smoothing for optimal contracts.

Optimal contracts use stochastic compensation. One might wonder how flexible firms are in designing stochastic compensation. Firms are very flexible in designing stock option plans by using, for example, different exercise prices, vesting periods, and conditions. Therefore, companies can construct the required lotteries from stock options. Common financial products, such as exchange-traded options, are available for this purpose, and no specialized intermediaries are necessary. Alternatively, firms can use restricted stock units. As firms can issue restricted stock units themselves, this might be a reason why restricted stock units as a form of compensation recently became popular in the United States.

Other examples of stochastic payments are uncertain arbitration procedures and legal uncertainty about which contractual clauses are valid. The contracting parties might be unsure how a disagreement is to be interpreted and what wages are appropriate. Finally, stochastic payments could involve conflicts, like working to rule, strikes, or walkouts. Compensation in stock options, restricted stock units, or shares is well documented, whereas other forms of stochastic payments have received less attention in the literature but appear no less important: "Stock option grants to nonexecutive employees have become an important component of compensation policy in recent decades," as Hochberg and Lindsey (2010) summarize the empirical evidence on stock options as a form of stochastic compensation.[22] The emphasis here is on equity-based compensation for rank-and-file employees because there might be additional reasons to use such a compensation for executives. Rank-and-file employees individually have little, if any, effect on stock prices, so that equity-based compensation offers no informational benefit. In addition, firms grant relatively small amounts of options or stocks to rank-and-file employees compared with the total number of a firm's stocks. Therefore, the employee gains only a very small share of any value added. Hence, equity-based compensation does not seem optimal for incentivizing rank-and-file employees. Nevertheless, the use of equity-based compensation for rank-and-file employees is widespread and growing: "The use of equity-based compensation for employees below the executive rank has been growing rapidly during the last decade" (Bergman and Jenter, 2007). In frameworks with objective evaluations, this usage is hard to explain. Stochastic compensation seems to contradict basic intuitions derived from moral-hazard models: "the prevalence of broad based option plans remains a puzzle for standard economic theory" (Kedia and Rajgopal, 2009). My article provides a simple rationale for the use of stochastic contracts.

Appendix A: Proofs

Section 4 assumes a particular structure of correlation between the principal's evaluation and the agent's self-assessment which is common in the literature. Some articles consider a more general correlation but for binary signals only, for example Section 5 in Chan and Zheng (2011, one-period case), Bester and Münster (2016), and the stage game in Maestri (2012).[23] The next claim shows that even with this more general correlation, optimal contracts remain qualitatively unchanged from Proposition 1.

Claim A1. Given Assumptions 1–3, in the setting of Proposition 5 of Chan and Zheng (2011, one-period case), optimal contracts remain qualitatively unchanged compared with my Proposition 1.

Proof. In the setting of Section 5 with T=1 Chan and Zheng (2011), evaluations and self-assessments are $\mathcal{T} = \mathcal{S} = \{1, 2\}$. They consider a joint distribution allowing for correlation between both assessments. Effort is binary, $e \in \{0, 1\}$, so that Chan and Zheng (2011) focus on incentivizing effort $e = 1$. They assume $c(0) = 0$, $c(1) = c > 0$, as well as $\gamma_2(1) > \gamma_2(0)$ and $\text{Prob}(t = 2|e = 1, s = 2) > \max\{\text{Prob}(t = 2|e = 1, s = 1), \text{Prob}(t = 2|e = 0, s = 2), \text{Prob}(t = 2|e = 0, s = 1)\}$. For the one-period case considered here, without loss of generality assume a discount factor of one. Finally, their Proposition 5 assumes $\text{Prob}(t = 2|e = 1, s = 1) \geq \gamma_2(0)$.

[22] Core and Guay (2001) confirm these claims: "The corporate use of stock option plans for nonexecutive employees is widespread." See also Babenko and Sen (2016), who report that in their sample of 663 large US firms in the years 1996 to 2011, an average rank-and-file employee was granted about 780 stock options each year with a Black-Scholes fair value of $6981. Call et al. (2016), Kim and Ouimet (2014), Kedia and Rajgopal (2009), and Oyer and Schaefer (2005) perform additional empirical analyses of broad-based compensation with stocks and stock options.

[23] Bester and Münster (2016) and Maestri (2012) consider continuous effort, however.

Proposition 5 in Chan and Zheng (2011) shows for T=1 that an optimal contract implies utilities of

$$u(\tilde{c}_{22}) = u(\tilde{c}_{21}) = \bar{u} + c + \frac{\gamma_1(1)}{\gamma_1(0) - \gamma_1(1)}c \quad \text{and} \quad u(\tilde{c}_{12}) = u(\tilde{c}_{11}) = \bar{u} + c - \frac{1 - \gamma_1(1)}{\gamma_1(0) - \gamma_1(1)}c.$$

As $\gamma_2(1) > \gamma_2(0)$ implies $\gamma_1(0) > \gamma_1(1)$, it is easy to see that $\tilde{c}_{22} > \tilde{c}_{11}$. Adjusting the values ω^* and b in the contract of Proposition 1 accordingly yields the optimal stochastic contract:

$$w_{ts} = \begin{cases} \tilde{c}_{22} & \text{if } t > 1 \\ \Delta & \text{if } t = 1 \end{cases}$$

The lottery Δ has a mean $\tilde{c}_{22}$, a certainty equivalent $\tilde{c}_{11}$, and is realized at the end of period 4. Therefore, the alternative correlation does not change optimal contracts qualitatively. For the same reasons as in Proposition 1, the contract is renegotiation proof. □

Proof of Lemma 1. The setting of MacLeod (2003) is a special case of my general setting. Nevertheless, the proof of MacLeod (2003) carries over and remains valid. Suppose third-party payments are impossible and $y_{ts} = 0$ for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. "After making their subjective evaluations, the principal and agent play a constant-sum game when making their reports. From the min-max theorem such a game has a unique value and hence the agent's compensation cannot depend upon [the principal's evaluation] t" (MacLeod, 2003). □

Proof of Lemma 2. Regarding the agent's incentive compatibility (IC), the first-order approach is valid here because the distribution induced by $\gamma_t(e)$ is convex. According to Grossman and Hart (1983), Rogerson (1985), and Kirkegaard (2017), the convexity of the distribution function condition together with the convexity of $d(\cdot)$ and the monotone likelihood ratio property (MLRP) guarantees that the first-order approach is valid. Hence, I can rewrite the agent's incentive compatibility (IC) as

$$\sum_{t\in\mathcal{T}} \gamma_t'(e)(pu(\tilde{c}_{tt}) + (1-p)u(\tilde{c}_{t0})) = d'(e).$$

Consider Program A without the constraints (TT_P) and (TT_A) for truth-telling. It is straightforward that the solution to this relaxed problem is $\tilde{w}_{ts} = \tilde{c}_{ts} = \tilde{c}_{t0} = c_t^*$ with a certainty equivalent c_t^* for all $t, s \in \mathcal{T}$ if a solution exists. Neglecting the agent's incentive compatibility (IC), the solution to this relaxed problem is $c_t^* = u^{-1}(\bar{u} + d(e))$ for all $t \in \mathcal{T}$ if effort e is implementable. Therefore, the agent's incentive compatibility is binding because the solution to the relaxed problem violates the agent's incentive compatibility. Consequently, the Lagrange multiplier μ_1 of the agent's incentive compatibility is positive in equation (1).

Optimization with respect to c_t^* with the Lagrange multipliers of the participation constraint μ_0 and the incentive compatibility μ_1 determines the optimal complete contract as

$$\gamma_t(e) - \mu_0 u'(c_t^*)\gamma_t(e) - \mu_1 u'(c_t^*)\gamma_t'(e) = 0,$$

$$\frac{1}{u'(c_t^*)} = \mu_0 + \mu_1 \frac{\gamma_t'(e)}{\gamma_t(e).} \tag{A1}$$

Again, we see that the Lagrange multiplier μ_1 is positive: If $\mu_1 = 0$, then equation (A1) implies that wages c_t^* are constant in t, violating the incentive compatibility (IC). Hence, $\mu_1 > 0$. The right-hand side of equation (A1) increases in $t \in \mathcal{T}$ due to the monotone likelihood ratio property. Therefore, the strict concavity of $u(\cdot)$ implies that the solution c_t^* is unique and that c_t^* increases in $t \in \mathcal{T}$. Given that effort e is implementable, this concludes the proof.

I conclude the proof by some remarks on implementability. To implement no effort, $e = 0$, set $c_t^* = u^{-1}(\bar{u} + d(0))$ for all $t \in \mathcal{T}$. This wage c_t^* is well defined if and only if effort $e = 0$ is implementable. The limit $\lim_{w\to\infty} u(w) = \infty$ or $\bar{u}$ sufficiently low ensures that the wage c_t^* is well defined and hence, that effort $e = 0$ is implementable. Positive effort $e > 0$ is implementable for $\lim_{w\to\infty} u(w) = \infty$ if the constraint set is nonempty (Page, 1987). To implement positive effort, $e > 0$, consider the contract

$$c_t = \begin{cases} c_1 & \text{if } t \in \mathcal{T}_g \\ c_2 & \text{otherwise} \end{cases}$$

with the set $\mathcal{T}_g = \{t \in \mathcal{T} | \gamma_t'(e) \geq 0\}$ and c_1 as well as c_2 determined below. The contract satisfies the agent's incentive compatibility (IC) if

$$d'(e) = \sum_{t\in\mathcal{T}_g} u(c_1)\gamma_t'(e) + \sum_{t\in(\mathcal{T}\setminus\mathcal{T}_g)} u(c_2)\gamma_t'(e) = (u(c_1) - u(c_2)) \sum_{t\in\mathcal{T}_g} \gamma_t'(e) \tag{A2}$$

because $\sum_{t\in\mathcal{T}} \gamma_t(e) = 1 \Rightarrow \sum_{t\in\mathcal{T}} \gamma_t'(e) = 0$ and, hence,

$$\sum_{t\in\mathcal{T}_g} \gamma_t'(e) = -\sum_{t\in(\mathcal{T}\setminus\mathcal{T}_g)} \gamma_t'(e).$$

The assumption $\lim_{w\to a} u(w) = -\infty$ implies that it is always possible to satisfy the incentive compatibility by setting c_2 sufficiently close to a as defined in Assumption 3. According to the definition of the set $\mathcal{T}_g$ and the monotone likelihood ratio property, the sum $\sum_{t\in\mathcal{T}} \gamma_t'(e)$ in equation (A2) is positive. Therefore, equation (A2) uniquely determines $u(c_1) - u(c_2)$. Moreover, $d' > 0$ implies $c_1 > c_2$. The contract satisfies the PC if

$$d(e) + \bar{u} = u(c_1)\sum_{t\in\mathcal{T}_g}\gamma_t(e) + u(c_2)\sum_{t\in(\mathcal{T}\setminus\mathcal{T}_g)}\gamma_t(e) = u(c_2) + (u(c_1) - u(c_2))\sum_{t\in\mathcal{T}_g}\gamma_t(e)$$

because $\sum_{t\in\mathcal{T}}\gamma_t(e) = 1$ and, hence,

$$\sum_{t\in\mathcal{T}_g}\gamma_t(e) = 1 - \sum_{t\in(\mathcal{T}\setminus\mathcal{T}_g)}\gamma_t(e).$$

Plugging in the above solution for $u(c_1) - u(c_2)$ uniquely determines $u(c_2)$. The values for $u(c_1)$ and $u(c_2)$ are feasible if $\lim_{w\to\infty} u(w) = \infty$. Therefore, the constraint set is nonempty. The above contract also proves that the costs of an optimal contract are lower than $\max\{c_1, c_2\} < \infty$ for implementable effort e.

By the standard arguments, it is not optimal to use third-party payments or stochastic compensation. Hence, the contract is renegotiation proof. □

Proof of Proposition 1. With uninformative self-assessments, the principal's truth-telling constraint (TT_P) implies $\tilde{w}_{t0} = \tilde{w}_{\bar{t}0}$ for all $t, \bar{t} \in \mathcal{T}$. Expected wages must be constant in the principal's message because the contract cannot detect any deviations from truth-telling by the principal. Thus, define $\omega = \tilde{w}_{t0}$ for a $t \in \mathcal{T}$. In addition, the agent's truth-telling constraint (TT_A) implies

$$\sum_{t\in\mathcal{T}} u(\tilde{c}_{t0})\gamma_t(e) \geq \sum_{t\in\mathcal{T}} u(\tilde{c}_{t\bar{s}})\gamma_t(e)$$

for all $\bar{s} \in \mathcal{T}$. As the values $\tilde{c}_{ts}$ for all $t, s \in \mathcal{T}$ matter only off the equilibrium path, without loss of generality, we can set $\tilde{c}_{ts} = \tilde{c}_{t0}$ for all $t, s \in \mathcal{T}$ to satisfy the agent's truth-telling constraint (TT_A). Adjusting Program A accordingly yields Program B′:

$$\min_{\omega, \tilde{c}_{t0}} \omega \tag{B′}$$

$$\text{subject to} \sum_{t\in\mathcal{T}} u(\tilde{c}_{t0})\gamma_t(e) - d(e) \geq \bar{u}, \tag{PC}$$

$$\sum_{t\in\mathcal{T}} \gamma_t'(e)u(\tilde{c}_{t0}) = d'(e) \tag{IC}$$

$$\omega \geq \tilde{c}_{t0} \qquad\qquad \forall t \in \mathcal{T}. \tag{A3}$$

The next steps calculate the optimal ω and $\tilde{c}_{t0}$. Define ν_0, ν_1 and β_t to be the Lagrange multipliers of the PC, IC, and constraint (A3) in Program B′, respectively. If $\tilde{c}_{t0} = \omega$ for all $t \in \mathcal{T}$, the contract violates the agent's IC. Therefore, there is an evaluation $t^* \in \mathcal{T}$ with stochastic payments, that is, $\omega > \tilde{c}_{t^*0}$. Then, the complementary slackness condition yields $\beta_{t^*} = 0$. Optimization of the Lagrangian with respect to $\tilde{c}_{t^*0}$ results in

$$-\nu_0 u'(\tilde{c}_{t^*0})\gamma_{t^*}(e) - \nu_1 u'(\tilde{c}_{t^*0})\gamma_{t^*}'(e) = 0.$$

Hence,

$$\nu_0 + \nu_1 \frac{\gamma_{t^*}'(e)}{\gamma_{t^*}(e)} = 0. \tag{A4}$$

The monotone likelihood ratio property ensures that $\frac{\gamma_t'(e)}{\gamma_t(e)}$ strictly increases in $t \in \mathcal{T}$. In addition, ν_1 must be positive because the solution to Program B without the agent's incentive compatibility (IC) is $\omega = \tilde{c}_{t0} = u^{-1}(\bar{u} + d(e))$ for all $t \in \mathcal{T}$, and this solution violates constraint (IC). Therefore, equation (A4) can hold for at most one $t^* \in \mathcal{T}$. Hence, $\tilde{c}_{t0} = \omega$ and $\beta_t \geq 0$ for all $t \in \mathcal{T} \setminus \{t^*\}$.

Assume to the contrary $t^* \neq 1$. This assumption implies $t^* > 1$. Optimization of the Lagrangian with respect to $\tilde{c}_{10}$ results in $-\nu_0 u'(\tilde{c}_{10})\gamma_1(e) - \nu_1 u'(\tilde{c}_{10})\gamma_1'(e) + \beta_1 = 0$. Hence,

$$\nu_0 + \nu_1 \frac{\gamma_1'(e)}{\gamma_1(e)} = \frac{1}{u'(\tilde{c}_{10})}\frac{\beta_1}{\gamma_1(e)}.$$

Equation (A4), $\nu_1 > 0$, $t^* > 1$, and the monotone likelihood ratio property imply that the left-hand side of the last equation is negative. The right-hand side is nonnegative because constraint (A3) is binding for $t = 1$ and $\beta_1 \geq 0$. This contradiction proves that $t^* = 1$.

Plugging these results into the PC and the IC yields:

$$u(\omega^*)(1-\gamma_1(e)) + u(\tilde{c}_{10}^*)\gamma_1(e) - d(e) = \bar{u},$$

$$(u(\tilde{c}_{10}^*) - u(\omega^*))\gamma_1'(e) = d'(e),$$

because $1 = \sum_{t\in\mathcal{T}} \gamma_t(e) \Rightarrow 0 = \sum_{t\in\mathcal{T}} \gamma_t'(e) = \gamma_1'(e) + \sum_{t\in\mathcal{T}\setminus\{1\}} \gamma_t'(e)$. Solving the first equation for $u(\tilde{c}_{10}^*)$ gives $u(\tilde{c}_{10}^*) = [\bar{u} + d(e) - u(\omega^*)(1-\gamma_1(e))]/\gamma_1(e)$. Inserting this value for $u(\tilde{c}_{10}^*)$ into the second equation yields $(\bar{u} + d(e) - u(\omega^*))\gamma_1'(e) = \gamma_1(e)d'(e)$ and finally results in

$$u(\omega^*) = \bar{u} + d(e) - \frac{\gamma_1(e)}{\gamma_1'(e)}d'(e) \quad \text{and} \quad u(\tilde{c}_{10}^*) = \bar{u} + d(e) + \frac{1-\gamma_1(e)}{\gamma_1'(e)}d'(e).$$

These values of ω^* and $\tilde{c}_{10}^*$ allow to characterize the optimal contract. For this purpose, define $b = \tilde{c}_{10}^*$ and consider a lottery $\Delta \in \mathcal{L}$ with a certainty equivalent b, that is, $\mathbb{E}u(\Delta) = u(b)$, and $\mathbb{E}(\Delta) = \omega^*$ that is realized at the end of period 4. Strict concavity of u and the assumption $\lim_{w\to a} u(w) = -\infty$ ensure existence of such a lottery Δ. Now consider the contract stated in Proposition 1. The contract implements $\tilde{c}_{ts} = \tilde{w}_{ts} = \omega^*$ for all $t > 1$ and all $s \in \mathcal{S}$ as well as $\tilde{c}_{1s} = b = \tilde{c}_{10}^*$ for all $s \in \mathcal{S}$. Hence, the contract in Proposition 1 implies $\omega = \omega^*$ and

$$u(\tilde{c}_{t0}) = \begin{cases} u(\omega^*) & \text{if } t > 1 \\ u(b) = u(\tilde{c}_{10}^*) & \text{if } t = 1. \end{cases}$$

The definition of $\tilde{c}_{t0}^*$ ensures that the contract satisfies the agent's PC and IC. The agent's truth-telling constraint (TT_A) is satisfied because his utilities are independent of his message. The principal's truth-telling constraint (TT_P) is also satisfied because expected wages are independent of her message. The contract also satisfies constraint (RA). Consequently, the contract in Proposition 1 is feasible. The contract is also optimal because ω^* and $\tilde{c}_1^*$ are optimal in Program B.

It remains to show that the contract is renegotiation proof and that the contract uses no third-party payments. Any third-party payments $y_{ts} > 0$ are not renegotiation proof, as the principal could propose to skip the third-party payment ($\hat{y}_{ts} = 0$) and pay half of it to the agent ($\hat{w}_{ts} = w_{ts} + y_{ts}/2$). The agent would be happy to accept any such proposal which makes the principal better off. Therefore, third-party payments are not renegotiation proof.

The lottery Δ in Proposition 1 is realized at the end of period 4. Denote this realization by $\mathcal{O}(\Delta)$. Hence, when renegotiations occur in period 5, renegotiations are a zero-sum game. Any proposal that the agent accepts (i.e., any wage above $\mathcal{O}(\Delta)$) makes the principal worse off. The agent rejects any proposal that makes the principal better off (i.e., any wage below $\mathcal{O}(\Delta)$). Hence, the contract is renegotiation proof. □

Proof of Proposition 2. The proof proceeds in two steps. The first step constructs lotteries that satisfy the conditions in the proposition. The second step proves optimality of the contract stated in the proposition.

First, I show how to construct the lotteries Δ_t. Consider, for example, a lottery $\Delta_t \in \mathcal{L}$ that pays $c_n^*/p + z_t$ and $c_n^*/p - z_t$ with equal probabilities and that is realized at the end of period 4. This lottery yields an expectation of c_n^*/p. Choosing $z_t \in \mathbb{R}$ appropriately ensures $\mathbb{E}u(\Delta_t) = u(c_t^*)$ because $c_n^*/p \geq c_n^* \geq c_t^*$ for all $t \in \mathcal{T}$. Strict concavity of u and the assumption $\lim_{w\to a} u(w) = -\infty$ guarantee that there is a unique and bounded z_t for all $t \in \mathcal{T}$. This construction establishes existence of the lotteries Δ_t.

Second, consider optimality. On the equilibrium path, the contract in Proposition 2 makes the principal pay a wage of c_t^* to the agent. Lemma 2 shows that the optimal complete contract c_t^* satisfies the agent's PC and IC. Hence, the agent accepts the contract in Proposition 2 and exerts effort e. Independently of the agent's message s, his utility is $u(c_t^*) = \mathbb{E}u(\Delta_t)$. Therefore, truth-telling is optimal for the agent and his truth-telling constraint (TT_A) is satisfied. Suppose the agent reports his self-assessment s truthfully. If the principal evaluates the agent correctly, she pays c_t^*. If the principal deviates to an evaluation $\bar{t} \neq t$, she expects to pay

$$p\tilde{w}_{\bar{t}t} + (1-p)\tilde{w}_{\bar{t}0} = pc_n^*/p + (1-p)c_{\bar{t}}^* = c_n^* + (1-p)c_{\bar{t}}^* > c_n^* = \max_{r\in\mathcal{T}} c_r^* \geq c_t^*.$$

Therefore, truth-telling is optimal for the principal and her truth-telling constraint (TT_P) is satisfied. Constraint (RA) is trivially satisfied.

In summary, the contract implements equilibrium payments of c_t^*. Remember that the optimal complete contract c_t^* is a solution to a relaxed problem without the truth-telling constraints (TT_P) and (TT_A). Consequently, the contract in Proposition 2 optimally incentivizes the agent based on subjective evaluations.

It remains to show that the contract is renegotiation proof and that the contract uses no third-party payments. Any third-party payments $y_{ts} > 0$ are not renegotiation proof, as the principal could propose to skip the third-party payment ($\hat{y}_{ts} = 0$) and pay half of it to the agent ($\hat{w}_{ts} = w_{ts} + y_{ts}/2$). The agent would be happy to accept any such proposal which makes the principal better off. Therefore, third-party payments are not renegotiation proof.

The lotteries Δ_t in Proposition 2 are realized at the end of period 4. Denote this realization by $\mathcal{O}(\Delta_t)$. Hence, when renegotiations occur in period 5, renegotiations are a zero-sum game. Any proposal that the agent accepts (i.e., any wage above $\mathcal{O}(\Delta_t)$) makes the principal worse off. The agent rejects any proposal that makes the principal better off (i.e., any wage below $\mathcal{O}(\Delta_t)$). Hence, the contract is renegotiation proof. □

Proof of Proposition 3. First, construct a lottery $\Delta \in \mathcal{L}$ with the specified properties. The lottery pays $u^{-1}(\bar{u}+d(e))+z$ and $u^{-1}(\bar{u}+d(e))-z$ with equal probabilities and is realized at the end of period 4. This lottery yields an expectation of $u^{-1}(\bar{u}+d(e))$. Choosing $z \in \mathbb{R}$ appropriately ensures $\mathbb{E}u(\Delta)=\bar{u}+d(0)$. Strict concavity of u and the assumption $\lim_{w\to a} u(w)=-\infty$ guarantee that there is a unique and bounded z. This construction establishes existence of the lottery Δ.

Second, consider incentives. In period 4, the principal's expected payoffs do not depend on her report. Therefore, reporting $t=e$ is optimal for the principal. If the agent's chooses effort e, her utilities in the contract are $\bar{u}+d(e)-d(e)=\bar{u}$. Hence, she accepts the contract and her participation constraint (PC) is satisfied. If the agent chooses any other report or effort, her expected utilities are at most $\bar{u}+d(0)-d(0)\le\bar{u}$. Therefore, the agent optimally chooses effort e and reports $s=e$.

The contract implements any effort $e\in[0,1)$ at first-best costs. It remains to show that the contract is renegotiation proof and that the contract uses no third-party payments. Any third-party payments $y_{ts}>0$ are not renegotiation proof, as the principal could propose to skip the third-party payment ($\hat{y}_{ts}=0$) and pay half of it to the agent ($\hat{w}_{ts}=w_{ts}+y_{ts}/2$). The agent would be happy to accept any such proposal which makes the principal better off. Therefore, third-party payments are not renegotiation proof.

The lottery Δ in Proposition 3 is realized at the end of period 4. Denote this realization by $\mathcal{O}(\Delta)$. Hence, when renegotiations occur in period 5, renegotiations are a zero-sum game. Any proposal that the agent accepts (i.e., any wage above $\mathcal{O}(\Delta)$) makes the principal worse off. The agent rejects any proposal that makes the principal better off (i.e., any wage below $\mathcal{O}(\Delta)$). Hence, the contract is renegotiation proof. □

Proof of Lemma 3. Lemma 1 proves that deterministic contracts without third-party payments cannot incentivize agents. This remains true if renegotiations are impossible. Propositions 1, 2, and 3 establish that stochastic contracts can incentivize agents if effort e is implementable. Hence, if effort e is not implementable with the optimal stochastic contract, stochastic contracts weakly increase efficiency. If effort e is implementable with the optimal stochastic contract, stochastic contracts strictly increase efficiency. Notice that Lemma 3 is valid no matter whether renegotiations are possible or not.

Considering the second step of Grossman and Hart (1983), the sum of utilities increases if $\int_{\mathcal{T},\mathcal{S}} B(e,t,s)\mathrm{d}F(t,s|e) > \int_{\mathcal{T},\mathcal{S}} B(0,t,s)\mathrm{d}F(t,s|0)$ for some implementable e. □

Proof of Proposition 4. Third-party payments are not renegotiation proof. Given any positive third-party payment $y_{ts}>0$, the principal could propose to skip the third-party payment ($\hat{y}_{ts}=0$) and pay half of it to the agent ($\hat{w}_{ts}=w_{ts}+y_{ts}/2$). The agent would be happy to accept any such proposal which makes the principal better off. The renegotiation-proofness principle by Laffont and Tirole (1990) implies that it is without loss of generality to focus on renegotiation-proof contracts, that is, deterministic contracts without third-party payments. Hence, deterministic contracts cannot incentivize agents according to Lemma 1. Propositions 1, 2, and 3 establish that stochastic contracts can incentivize agents if effort e is implementable. Hence, if effort e is not implementable with the optimal stochastic contract, stochastic contracts weakly increase efficiency. If effort e is implementable with the optimal stochastic contract, stochastic contracts strictly increase efficiency.

Considering the second step of Grossman and Hart (1983), the sum of utilities increases if $\int_{\mathcal{T},\mathcal{S}} B(e,t,s)\mathrm{d}F(t,s|e) > \int_{\mathcal{T},\mathcal{S}} B(0,t,s)\mathrm{d}F(t,s|0)$ for some implementable e. □

Proof of Proposition 5. Propositions 1, 2, and 3 are also valid if renegotiations are impossible. These results prove that stochastic contracts can incentivize agents in an optimal way. Proposition 2 shows that stochastic contracts can achieve expected wage costs of $C^c(e)$ (as defined following Lemma 2) for any degree of correlation and $p>0$. Now, suppose that contracts are deterministic. To describe optimal contracts in this case, I denote the bound on the third-party payments as S. If $S<\omega^*-b$, with the values ω^* and b defined in Proposition 1, deterministic contracts cannot implement effort e. If $(\omega^*-b)/p > S \ge \omega^*-b$, the optimal deterministic contract is

$$\tilde{w}_{ts}=\omega^* \quad \text{and} \quad \tilde{c}_{ts}=\begin{cases} b & \text{if } t=1 \\ \omega^* & \text{otherwise} \end{cases} \tag{A5}$$

To show optimality, I re-interpret the value $\tilde{w}_{ts}$ as the deterministic wage payment by the principal. Then, the agent earns a wage $\tilde{c}_{ts}$, whereas the amount $\tilde{w}_{ts}-\tilde{c}_{ts}$ is paid to third parties. Thus, Proposition 1 implies that contract (A5) is optimal.

If $S\ge(\omega^*-b)/p$, the optimal contract is

$$\tilde{w}_{ts}\begin{cases} =\tilde{c}_{tt} & \text{if } t=s \\ \in[\tilde{c}_{tt},\tilde{c}_{tt}+S] & \text{if } s=0 \\ =\tilde{c}_{tt}+S & \text{otherwise.} \end{cases}$$

In addition, there is a $\bar{t}\in\mathcal{T}$ such that

$$\tilde{c}_{ts}\begin{cases} =\tilde{c}_{ns'} & \text{if } t>\bar{t} \\ <\tilde{c}_{(t+1)s'} & \text{otherwise} \end{cases}$$

for all $t \in \mathcal{T}$ and all $s, s' \in \mathcal{S}$ according to Proposition 8 in MacLeod (2003). Moreover, $\bar{t} < n-1$ if $S < (c_n^* - c_1^*)/p$. If $S \geq (c_n^* - c_1^*)/p$, the optimal deterministic contract has $\bar{t} = n-1$, $\tilde{w}_{t0} = \tilde{c}_{tt}$ and in equilibrium implements wage payments c_t^* for the agent at the same costs $C^c(e)$ as the optimal complete contract. Therefore, all deterministic contracts are more expensive than $C^c(e)$ if $S < (c_n^* - c_1^*)/p$. If the correlation p is sufficiently small, $S < (c_n^* - c_1^*)/p$ for any finite bound S. Consequently, stochastic contracts are strictly more efficient and more profitable for the principal if the alignment p between the agent's self-assessment and the principal's evaluation is sufficiently small and effort e is implementable with the optimal stochastic contract. Hence, if effort e is not implementable with the optimal stochastic contract, deterministic contracts cannot implement effort e and stochastic contracts weakly increase efficiency. If effort e is implementable with the optimal stochastic contract, stochastic contracts strictly increase efficiency. □

Proof of Proposition 6. The condition $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$ ensures that effort e is implementable according to Proposition B1 in Appendix B. Higher risk aversion k increases the certainty equivalent b in the contract in Proposition B1:

$$\frac{\partial b}{\partial k} = \frac{1}{k^2}\ln\left(1 - \frac{1-\gamma_1(e)}{\gamma_1'(e)}kd'(e)\right) - \frac{1}{k}\frac{1}{1 - \frac{1-\gamma_1(e)}{\gamma_1'(e)}kd'(e)}\frac{1-\gamma_1(e)}{\gamma_1'(e)}d'(e)$$

$$= \frac{1}{k^2}\ln\left(1 - \frac{1-\gamma_1(e)}{\gamma_1'(e)}kd'(e)\right) - \frac{1}{k}\frac{1}{\frac{\gamma_1'(e)}{1-\gamma_1(e)} - kd'(e)}d'(e) > 0$$

because the MLRP ensures $\gamma_1'(e) < 0$ so that $\frac{\gamma_1'(e)}{1-\gamma_1(e)} < 0$ and $\frac{1-\gamma_1(e)}{\gamma_1'(e)} < 0$ and the logarithm is positive. In addition, the last fraction being negative guarantees that the second term is also positive.

Higher risk aversion k increases the wage $\omega^{*\text{CARA}}$:

$$\frac{\partial \omega^{*\text{CARA}}}{\partial k} = \frac{1}{k^2}\ln\left(1 + \frac{\gamma_1(e)}{\gamma_1'(e)}kd'(e)\right) - \frac{1}{k}\frac{1}{1 + \frac{\gamma_1(e)}{\gamma_1'(e)}kd'(e)}\frac{\gamma_1(e)}{\gamma_1'(e)}d'(e) > 0$$

because the negative $\gamma_1'(e)$ ensures that the logarithm $\ln(1 + \frac{\gamma_1(e)}{\gamma_1'(e)}kd'(e))$ is decreasing and concave in k and equals zero for $k = 0$. Thus, Taylor's theorem yields $\ln(1 + \frac{\gamma_1(e)}{\gamma_1'(e)}kd'(e)) > k\frac{\partial \ln(1+\frac{\gamma_1(e)}{\gamma_1'(e)}kd'(e))}{\partial k}$. Therefore, the derivative is positive.

Next, consider the difference $\omega^{*\text{CARA}} - b$. There is a $\bar{\gamma} \in (0, 1/2)$ so that higher risk aversion k decreases this difference for $\gamma_1(e) < \bar{\gamma}$ and increases this difference for $\gamma_1(e) > \bar{\gamma}$. The threshold $\bar{\gamma}$ for $\gamma_1(e)$ is determined by $(2\bar{\gamma} - 1)\gamma_1'(e) = 2\bar{\gamma}kd'(e)(1-\bar{\gamma})$ and depends on the risk aversion k. The reason is the following:

$$\frac{\partial \omega^{*\text{CARA}} - b}{\partial k}$$

$$= -\frac{1}{k^2}\ln\left(1 - \frac{kd'(e)}{\gamma_1'(e) + \gamma_1(e)kd'(e)}\right) + \frac{1}{1 - \frac{kd'(e)}{\gamma_1'(e)+\gamma_1(e)kd'(e)}}\frac{-\gamma_1'(e)d'(e)}{k(\gamma_1'(e) + \gamma_1(e)kd'(e))^2}$$

$$= -\frac{1}{k^2}\ln\left(1 - \frac{kd'(e)}{\gamma_1'(e) + \gamma_1(e)kd'(e)}\right) + \frac{1}{k}\frac{1}{\gamma_1'(e) - (1-\gamma_1(e))kd'(e)}\frac{-\gamma_1'(e)d'(e)}{\gamma_1'(e) + \gamma_1(e)kd'(e)}$$

because Taylor's theorem yields

$$-\ln\left(1 - \frac{kd'(e)}{\gamma_1'(e) + \gamma_1(e)kd'(e)}\right) + k\frac{\partial \ln\left(1 - \frac{kd'(e)}{\gamma_1'(e)+\gamma_1(e)kd'(e)}\right)}{\partial k}\begin{cases} < 0 & \text{for } \gamma_1(e) < \bar{\gamma}, \\ >0 & \text{for } \gamma_1(e) > \bar{\gamma} \end{cases}$$

as the logarithm $\ln(1 - \frac{kd'(e)}{\gamma_1'(e)+\gamma_1(e)kd'(e)})$ equals zero for $k = 0$, $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$ and

$$\frac{\partial \ln\left(1 - \frac{kd'(e)}{\gamma_1'(e)+\gamma_1(e)kd'(e)}\right)}{\partial k} = \frac{-1}{\gamma_1'(e) - (1-\gamma_1(e))kd'(e)}\frac{\gamma_1'(e)d'(e)}{\gamma_1'(e) + \gamma_1(e)kd'(e)} > 0,$$

$$\frac{\partial^2 \ln\left(1 - \frac{kd'(e)}{\gamma_1'(e)+\gamma_1(e)kd'(e)}\right)}{\partial k^2} = \frac{(2\gamma_1(e) - 1)\gamma_1'(e) + 2\gamma_1(e)kd'(e)(\gamma_1(e) - 1)}{(\gamma_1'(e) - (1-\gamma_1(e))kd'(e))^2}\frac{d'(e)^2\gamma_1'(e)}{(\gamma_1'(e) + \gamma_1(e)kd'(e))^2},$$

so that the logarithm is increasing and concave in k for $\gamma_1(e) < \bar{\gamma}$ and the logarithm is increasing and convex in k for $\gamma_1(e) > \bar{\gamma}$.

Finally, consider the limit to risk neutrality, $k \to 0$. L'Hospital's rule yields $b \to \bar{w} + d(e) + \frac{1-\gamma_1(e)}{\gamma_1'(e)}d'(e)$, $\omega^{*\text{CARA}} - b \to -d'(e)/\gamma_1'(e)$, and $\omega^{*\text{CARA}} \to \bar{w} + d(e) - \frac{\gamma_1(e)}{\gamma_1'(e)}d'(e)$.

For CARA utilities and a normal distribution, the risk premium equals the variance times $k/2$. Rearranging yields the necessary variance of the lottery. The CARA utilities of a binary lottery with probabilities half each are equal to $\cosh(\cdot)$. Plugging in the definition of the cosh yields the term for the value z. □

Proof of Proposition 7. The condition $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$ ensures that effort e is implementable according to Proposition B1 in Appendix B for uninformative self-assessments. With informative self-assessments, the principal has more flexibility and can implement more effort levels. Therefore, the condition $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$ ensures that effort e is implementable also for informative self-assessments.

Proposition B2 in Appendix B shows that the equilibrium wages equal the optimal complete contract $c_t^{*\text{CARA}}$ as calculated in Lemma B1 in Appendix B:

$$c_t^{*\text{CARA}} = d(e) + \frac{1}{k}\ln\left(\exp(k\bar{w}) + \mu_1 k\left(\frac{\gamma_t'(e)}{\gamma_t(e)} + kd'(e)\right)\right).$$

Taking the derivative with respect to the agent's risk aversion k yields

$$\frac{\frac{\partial\mu_1}{\partial k}k(\frac{\gamma_t'(e)}{\gamma_t(e)} + kd'(e)) + \mu_1(\frac{\gamma_t'(e)}{\gamma_t(e)} + 2kd'(e)) + \bar{w}\exp(k\bar{w})}{k\left(\exp(k\bar{w}) + \mu_1 k\left(\frac{\gamma_t'(e)}{\gamma_t(e)} + kd'(e)\right)\right)} - \frac{1}{k^2}\ln\left(\exp(k\bar{w}) + \mu_1 k\left(\frac{\gamma_t'(e)}{\gamma_t(e)} + kd'(e)\right)\right).$$

For the limit to risk neutrality, $k \to 0$, apply L'Hospital's rule. Hence,

$$w_{ts} \underset{k\to 0}{\longrightarrow} d(e) + \partial \ln\left(\exp(k\bar{w}) + \mu_1 k\left(\frac{\gamma_t'(e)}{\gamma_t(e)} + kd'(e)\right)\right)/\partial k\Big|_{k=0}$$

for $s \in \{0, t\}$ and all $t \in \mathcal{T}$. For bounded μ_1, this derivative equals $\bar{w} + \mu_1\frac{\gamma_t'(e)}{\gamma_t(e)}$. Calculating the agent's expected utility, it is straightforward to see that the agent's PC is satisfied for any μ_1 in the limit as $\sum_t \gamma_t'(e) = 0$. Turn to the agent's incentive compatibility in the limit. Plugging in the limit wages yields:

$$\sum_t \gamma_t'(e)(\bar{w} + \mu_1\frac{\gamma_t'(e)}{\gamma_t(e)}) = d'(e).$$

Solving for the limit μ_1, we get $\mu_1 \to d'(e)/(\sum_t \frac{(\gamma_t'(e))^2}{\gamma_t(e)}))$ confirming the assumption of bounded μ_1. Therefore,

$$w_{ts} \underset{k\to 0}{\longrightarrow} \bar{w} + d(e) + \frac{\gamma_t'(e)}{\gamma_t(e)}\frac{d'(e)}{\sum_{\hat{t}=1}^n \frac{(\gamma_{\hat{t}}'(e))^2}{\gamma_{\hat{t}}(e)}}$$

for $s \in \{0, t\}$ and all $t \in \mathcal{T}$. Similarly, out-of-equilibrium expected wages converge to

$$\mathbb{E}(w_{ts}) \underset{k\to 0}{\longrightarrow} \frac{1}{p}(\bar{w} + d(e)) + \frac{\gamma_n'(e)}{\gamma_n(e)}\frac{d'(e)}{p\sum_{\hat{t}=1}^n \frac{(\gamma_{\hat{t}}'(e))^2}{\gamma_{\hat{t}}(e)}}$$

for all $s \neq 0, t$ and all $t \in \mathcal{T}$ because $\mathbb{E}(\Delta_t) = c_n^{*\text{CARA}}/p$ for all t.

For CARA utilities and a normal distribution, the risk premium equals the variance times $k/2$. Rearranging yields the necessary variance of the lottery. As $\lim_{k\to 0} c_n^{*\text{CARA}}/p - c_t^{*\text{CARA}} > 0$, the risk premium is positive in the limit and the variance of the lottery has to decrease in the risk aversion for all evaluations t for sufficiently small risk aversion.

The CARA utilities of a binary lottery with probabilities half each are equal to $\cosh(\cdot)$. Again the value z decreases in the risk aversion for all evaluations t for sufficiently small risk aversion. □

Proof of Proposition 8. Begin with the equilibrium wages $\bar{w} + d(e)$ calculated in Proposition B3 in Appendix B. These equilibrium wages do not depend on the agent's risk aversion and are deterministic. The mean and the risk premium of the off-equilibrium payments Δ are constant in the agent's risk aversion. A constant risk premium translates in a riskiness that decreases in the agent's risk aversion.

For CARA utilities and a normal distribution, the risk premium equals the variance times $k/2$. Rearranging yields a variance of at least $2(d(e) - d(0))/k$. That lower bound on the variance is decreasing and convex in the agent's risk aversion with the limit $\lim_{k\to 0} \partial \text{Var}(\Delta^1)/\partial k = -\infty$.

The CARA utilities of a binary lottery with probabilities half each are equal to $\cosh(\cdot)$. Again, the value z decreases in the risk aversion with the limit $\lim_{k\to 0} \partial z/\partial k = -\infty$. □

Proof of Proposition 9. For uninformative self-assessments, the optimal contract is less efficient than the objective benchmark according to Proposition 1. For higher correlation, optimal contracts are as efficient as the second-best benchmark according to Proposition 2. Therefore, the principal's expected costs of the contract decrease in correlation.

For uninformative self-assessments, the optimal contract has two wage levels according to Proposition 1. For higher correlation, optimal contracts have n wage levels according to Proposition 2.

According to Proposition 2, expected out-of-equilibrium wages equal c_n^*/p and, thus, decrease in the correlation p. In addition, the risk premium of out-of-equilibrium wages equal $c_n^*/p - c_t^*$ for an evaluation t. This risk premium decreases in the correlation p. □

Proof of Theorem 1. Stochastic contracts allow to replicate the outcomes of any deterministic contract with payments to third parties. The revelation principle ensures that it is without loss of generality to focus on direct mechanisms. In the space of deterministic contracts, hence, it is without loss of generality to consider contracts with w'_{ts} paid by the principal to the agent and y'_{ts} paid by the principal to third parties. We can replicate any such contract using a stochastic contract $\mathcal{W}$ without payments to third parties. For this purpose, I study three cases. First, consider w'_{ts} such that $U(w'_{ts}, e) > -\infty$. The new contract has

$$y_{ts} = 0 \qquad \text{and} \qquad w_{ts} = \begin{cases} w'_{ts} & \text{if } y'_{ts} = 0 \\ \Delta_{ts} & \text{if } y'_{ts} > 0 \end{cases}$$

with lotteries $\Delta_{ts} \in \mathcal{L}$ for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. The lotteries have a mean $w'_{ts} + y'_{ts}$, a risk premium y'_{ts} for the agent, and are realized at the end of period 4. Thus, $\mathbb{E}U(w_{ts}, e) = U(w'_{ts}, e)$. Consider, for example, a lottery $\Delta_{ts} \in \mathcal{L}$ that pays $w'_{ts} + y'_{ts} + z_{ts}$ and $w'_{ts} + y'_{ts} - z_{ts}$ with equal probabilities. This lottery yields an expectation of $w'_{ts} + y'_{ts}$. Choosing $z_{ts} \geq 0$ appropriately ensures

$$\mathbb{E}U(w_{ts}, e) = \mathbb{E}U(\Delta_{ts}, e) = U(w'_{ts}, e)$$

for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. Strict concavity of U in w guarantees that there is a unique and bounded z_{ts} for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$: for fixed t and s, I define the function

$$\mathcal{Z}(z) = U(w'_{ts} + y'_{ts} + z, e)/2 + U(w'_{ts} + y'_{ts} - z, e)/2 - U(w'_{ts}, e).$$

Obviously, $\mathcal{Z}(0) = U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e) > 0$ as U increases in w. Next, I construct $\bar{z} > 0$ such that $\mathcal{Z}(\bar{z}) < 0$. I denote $U_w = \partial U/\partial w$. Notice that for $z > y'_{ts}$ we have $\mathcal{Z}(z) =$

$$\begin{aligned}
& \frac{1}{2}U(w'_{ts} + y'_{ts} + z, e) + \frac{1}{2}U(w'_{ts} + y'_{ts} - z, e) - U(w'_{ts}, e) \\
\overset{\text{concavity of } U \text{ in } w}{<} \quad & U(w'_{ts} + y'_{ts}, e) + \frac{1}{2}zU_w(w'_{ts} + y'_{ts}, e) \\
& + \frac{1}{2}(U(w'_{ts}, e) - U(w'_{ts} + y'_{ts}, e) + U(w'_{ts} + y'_{ts} - z, e) - U(w'_{ts}, e)) - U(w'_{ts}, e) \\
\overset{\text{concavity of } U \text{ in } w}{<} \quad & U(w'_{ts} + y'_{ts}, e) + \frac{1}{2}zU_w(w'_{ts} + y'_{ts}, e) \\
& - \frac{1}{2}y'_{ts}U_w(w'_{ts} + y'_{ts}, e) + \frac{1}{2}(y'_{ts} - z)U_w(w'_{ts}, e) - U(w'_{ts}, e) \\
= \quad & U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e) - \frac{1}{2}(y'_{ts} - z)(U_w(w'_{ts} + y'_{ts}, e) - U_w(w'_{ts}, e)) \\
= \quad & U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e) + \frac{1}{2}(y'_{ts} - z)(U_w(w'_{ts}, e) - U_w(w'_{ts} + y'_{ts}, e)).
\end{aligned}$$

The last line equals zero for

$$z = \bar{z} = y'_{ts} + 2\frac{U(w'_{ts} + y'_{ts}, e) - U(w'_{ts}, e)}{U_w(w'_{ts}, e) - U_w(w'_{ts} + y'_{ts}, e)}.$$

The value $\bar{z} > y'_{ts} > 0$ is positive because $U(w'_{ts} + y'_{ts}, e) > U(w'_{ts}, e)$ as well as $U_w(w'_{ts}, e) > U_w(w'_{ts} + y'_{ts}, e)$. Therefore, $\mathcal{Z}(\bar{z}) < 0$. Together, $\mathcal{Z}(0) > 0$, $\mathcal{Z}(\bar{z}) < 0$, and the intermediate value theorem guarantee that there is a $\tilde{z} \in (0, \bar{z})$ such that $\mathcal{Z}(\tilde{z}) = 0$. Hence, an appropriate $z_{ts} \in \mathbb{R}_0^+$ exists for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. This construction establishes existence of lotteries Δ_{ts}. Unsurprisingly, the lotteries Δ_{ts} are not uniquely determined. There are many lotteries $\Delta_{ts} \in \mathcal{L}$ with the required mean and risk premium.

The second case considers w'_{ts} and y'_{ts} such that $U(w'_{ts} + y'_{ts}, e) = -\infty$. Then, the new contract has $y_{ts} = 0$ and $w_{ts} = w'_{ts} + y'_{ts}$. The PC ensures that t and s occur only out of equilibrium. Finally, the third case considers w'_{ts} and y'_{ts} such that $U(w'_{ts}, e) = -\infty$ and $U(w'_{ts} + y'_{ts}, e) > -\infty$. Then, the new contract has $y_{ts} = 0$ and $w_{ts} = \Delta_{ts}$ with a lottery $\Delta_{ts} \in \mathcal{L}$ that has mean $w'_{ts} + y'_{ts}$ and some probability for w'_{ts} occurring. Again, the PC ensures that t and s occur only out of equilibrium. The second and third cases have to be considered for technical reasons, but seem irrelevant in applications.

To sum up, the new contract $\mathcal{W}$ provides the same expected utilities for every combination of reports for the agent as the deterministic contract $\mathcal{W}'$ that uses payments to third parties and similarly for the principal. The principal's expected costs in the new contract are

$$\mathbb{E}(w_{ts}) = \mathbb{E}(w'_{ts} + y'_{ts} + \Delta_{ts}) = w'_{ts} + y'_{ts}$$

which are the same as in the old contract for all $t \in \mathcal{T}$ and all $s \in \mathcal{S}$. Therefore, the reporting strategies of contract $\mathcal{W}'$ also form an equilibrium in the reporting subgame in contract $\mathcal{W}$. Hence, the agent is willing to participate and receives the same incentives as in the deterministic contract $\mathcal{W}'$. Importantly, there are no payments to third-parties in the new contract $\mathcal{W}$. For the same reasons as in Proposition 1, the contract $\mathcal{W}$ is renegotiation proof. Thus, the contract $\mathcal{W}$ can be implemented if renegotiations are possible as well as if renegotiations are impossible. Consequently, the contract $\mathcal{W}$ incentivizes the agent to exert effort e without payments to third parties. □

Proof of Corollary 1. Following the construction in Theorem 1, for every contract $\mathcal{W}'$ with payments to third parties there is a (stochastic) contract $\mathcal{W}$ without such payments that yields the same utilities for the principal and the agent for every combination of reports. Then, it is an equilibrium for the principal to follow her reporting strategy in the previous contract $\mathcal{W}'$ also in the new contract $\mathcal{W}$ and for the agent to follow the same reporting strategy and to choose the same level of effort as in the previous contract $\mathcal{W}'$. The expected costs and benefits for the principal are the same in contract $\mathcal{W}$ as in the previous contract $\mathcal{W}'$. Consequently, optimal stochastic contracts without third-party payments are at least as profitable for the principal as deterministic contracts with third-party payments.

In addition, Proposition 5 shows that stochastic contracts can increase the principal's profits. □

Appendix B: Monetary costs of effort

Lemma B1. Given Assumptions 1, 2, and 3a and implementable effort $e > 0$, the optimal complete contract is

$$w_{ts} = c_t^{*\text{CARA}} = d(e) + \frac{1}{k}\ln\left(\exp(k\bar{w}) + \mu_1 k\left(\frac{\gamma_t'(e)}{\gamma_t(e)} + kd'(e)\right)\right) \qquad \forall t \in \mathcal{T}, s \in \mathcal{S} \tag{B1}$$

with the Lagrange multiplier μ_1 of the incentive compatibility. Better performances yield higher wages, that is, wages $c_t^{*\text{CARA}}$ strictly increase in performance t.

Proof of Lemma B1. Regarding the agent's IC, the first-order approach is valid here because the distribution induced by $\gamma_t(e)$ is log-convex. Fagart and Fluet (2013) show that for Assumption 3a the first-order approach is valid if the distribution is log-convex, disutilities $d(\cdot)$ are convex, and the MLRP is satisfied. Hence, I can rewrite the agent's incentive compatibility as

$$\sum_{t\in\mathcal{T}} \gamma_t'(e)(pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e)) + \sum_{t\in\mathcal{T}} k\gamma_t(e)(pU(\tilde{c}_{tt}, e) + (1-p)U(\tilde{c}_{t0}, e))d'(e) = 0$$

because $\partial U(c, e)/\partial e = -kd'(e)\exp(-k(c - d(e)) = kU(c, e)d'(e)$.

Consider Program A without the constraints (TT_P) and (TT_A) for truth-telling. It is straightforward that the solution to this problem is $\tilde{w}_{ts} = \tilde{c}_{ts} = \tilde{c}_{t0} = c_t^{*\text{CARA}}$ with a certainty equivalent $c_t^{*\text{CARA}}$ for all $t, s \in \mathcal{T}$ if a solution exists. Neglecting the agent's incentive compatibility, the solution to this relaxed problem is $c_t^{*\text{CARA}} = \bar{w} + d(e)$ for all $t \in \mathcal{T}$. Therefore, the agent's incentive compatibility is binding because the solution to the relaxed problem violates the agent's incentive compatibility. Consequently, the Lagrange multiplier μ_1 of the agent's incentive compatibility is positive in equation (B1).

Optimization with respect to $c_t^{*\text{CARA}}$ with the Lagrange multipliers of the participation constraint μ_0 and the incentive compatibility μ_1 determines the optimal complete contract:

$$\gamma_t(e) - \mu_0\frac{\partial U(c_t^{*\text{CARA}}, e)}{\partial c_t^{*\text{CARA}}}\gamma_t(e) - \mu_1\frac{\partial U(c_t^{*\text{CARA}}, e)}{\partial c_t^{*\text{CARA}}}\gamma_t'(e) - \mu_1 k\gamma_t(e)\frac{\partial U(c_t^{*\text{CARA}}, e)}{\partial c_t^{*\text{CARA}}}d'(e) = 0,$$

$$\frac{1}{\frac{\partial U(c_t^{*\text{CARA}}, e)}{\partial c_t^{*\text{CARA}}}} = \frac{1}{k}\exp(k(w - d(e)) = \mu_0 + \mu_1\frac{\gamma_t'(e)}{\gamma_t(e)} + \mu_1 kd'(e) \tag{B2}$$

The Lagrange multiplier μ_1 is positive. The right-hand side of equation (B2) increases in $t \in \mathcal{T}$ due to the monotone likelihood ratio property. Therefore, positive monotonicity of the exponential function implies that $c_t^{*\text{CARA}}$ increases in $t \in \mathcal{T}$. Summing up the first line of above equations over t yields:

$$1 + \mu_0 k\sum_{t\in\mathcal{T}} U(c_t^{*\text{CARA}}, e)\gamma_t(e) + \mu_1 k\left(\sum_{t\in\mathcal{T}} U(c_t^{*\text{CARA}}, e)\gamma_t'(e) + \sum_{t\in\mathcal{T}} k\gamma_t(e)U(c_t^{*\text{CARA}}, e)d'(e)\right) = 0$$

because $\partial U(c, e)/\partial c = k\exp(-k(c - d(e)) = -kU(c, e)$. The complementary slackness condition ensures that the term in brackets equals zero as it is equal to the agent's incentive compatibility (IC). By Proposition 2 in Grossman and Hart (1983) and Claim 3 in Chade and De Serio (2002), we know that the agent's PC is binding. Thus, the agent's expected utilities equal $\bar{u}$. Therefore, $\mu_0 = -1/k\bar{u} = \exp(k\bar{w})/k$. Plugging this value into equation (B2) and rearranging, yields equation (B1). Finally, notice that effort $e = 0$ is always implementable by a wage $c_t^{*\text{CARA}} = \bar{w} + d(0)$ for all $t \in \mathcal{T}$.

By standard arguments, it is not optimal to use third-party payments or stochastic compensation. Hence, the contract is renegotiation proof. □

Proposition B1. If $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$ and the agent's self-assessment is uninformative, the following contract is optimal for implementing effort $e > 0$ given Assumptions 1, 2, and 3a:

$$w_{ts} = \begin{cases} \omega^{*\text{CARA}} & \text{if } t > 1 \\ \Delta & \text{if } t = 1 \end{cases}$$

with a lottery $\Delta \in \mathcal{L}$, that is realized at the end of period 4, has a mean $\omega^{*\text{CARA}}$ as well as a certainty equivalent $b^{\text{CARA}} = \bar{w} + d(e) - \frac{1}{k}\ln(1 - \frac{1-\gamma_1(e)}{\gamma_1'(e)}kd'(e))$, and a wage $\omega^{*\text{CARA}} = b^{\text{CARA}} + \frac{1}{k}\ln(1 - \frac{kd'(e)}{\gamma_1'(e) + \gamma_1(e)kd'(e)}) > b^{\text{CARA}}$.

Proof of Proposition B1. With uninformative self-assessments, the principal's truth-telling constraint (TT_P) implies $\tilde{w}_{t0} = \tilde{w}_{\bar{t}0}$ for all $t, \bar{t} \in \mathcal{T}$. Expected wages must be constant in the principal's message because the contract cannot detect any deviations from truth-telling by the principal. Thus, define $\omega = \tilde{w}_{t0}$ for a $t \in \mathcal{T}$. In addition, the agent's truth-telling constraint (TT_A) implies

$$\sum_{t\in\mathcal{T}} U(\tilde{c}_{t0}, e)\gamma_t(e) \geq \sum_{t\in\mathcal{T}} U(\tilde{c}_{t\bar{s}}, e)\gamma_t(e)$$

for all $\bar{s} \in \mathcal{T}$. As the values $\tilde{c}_{ts}$ for all $t, s \in \mathcal{T}$ matter only out of equilibrium, without loss of generality, we can set $\tilde{c}_{ts} = \tilde{c}_{t0}$ for all $t, s \in \mathcal{T}$ to satisfy the agent's truth-telling constraint (TT_A). Adjusting Program A accordingly yields Program B':

$$\min_{\omega, \tilde{c}_{t0}} \omega \tag{B'}$$

$$\text{subject to} \sum_{t\in\mathcal{T}} U(\tilde{c}_{t0}, e)\gamma_t(e) \geq \bar{u}, \tag{PC}$$

$$\sum_{t\in\mathcal{T}} \gamma_t'(e)U(\tilde{c}_{t0}, e) + k\gamma_t(e)U(\tilde{c}_{t0}, e)d'(e) = 0 \tag{IC}$$

$$\omega \geq \tilde{c}_{t0} \qquad \forall t \in \mathcal{T}. \tag{B3}$$

The next steps calculate the optimal ω and $\tilde{c}_{t0}$. Define ν_0, ν_1 and β_t to be the Lagrange multipliers of the PC, IC, and constraint (B3) in Program B', respectively. If $\tilde{c}_{t0} = \omega$ for all $t \in \mathcal{T}$, the contract violates the agent's incentive compatibility (IC). Therefore there is an evaluation $t^* \in \mathcal{T}$ with stochastic payments, that is, $\omega > \tilde{c}_{t^*0}$. Then, the complementary slackness condition yields $\beta_{t^*} = 0$. Optimization of the Lagrangian with respect to $\tilde{c}_{t^*0}$ results in

$$-\nu_0 \frac{\partial U(\tilde{c}_{t^*0}, e)}{\partial \tilde{c}_{t^*0}}\gamma_{t^*}(e) - \nu_1 \frac{\partial U(\tilde{c}_{t^*0}, e)}{\partial \tilde{c}_{t^*0}}\gamma_{t^*}'(e) - \nu_1 k\gamma_{t^*}(e)\frac{\partial U(\tilde{c}_{t^*0}, e)}{\partial \tilde{c}_{t^*0}}d'(e) = 0.$$

Hence,

$$\nu_0 + \nu_1 \frac{\gamma_{t^*}'(e)}{\gamma_{t^*}(e)} + \nu_1 kd'(e) = 0. \tag{B4}$$

The monotone likelihood ratio property ensures that $\frac{\gamma_t'(e)}{\gamma_t(e)}$ strictly increases in $t \in \mathcal{T}$. In addition, ν_1 must be positive because the solution to Program B' without the agent's IC is $\omega = \tilde{c}_{t0} = \bar{w} + d(e)$ for all $t \in \mathcal{T}$ and this solution violates constraint (IC). Therefore, equation (B4) can hold for at most one $t^* \in \mathcal{T}$. Hence, $\tilde{c}_{t0} = \omega$ and $\beta_t \geq 0$ for all $t \in \mathcal{T} \setminus \{t^*\}$.

Assume to the contrary $t^* \neq 1$. This assumption implies $t^* > 1$. Optimization of the Lagrangian with respect to $\tilde{c}_{10}$ results in

$$-\nu_0 \frac{\partial U(\tilde{c}_{10}, e)}{\partial c_{10}}\gamma_1(e) - \nu_1 \frac{\partial U(\tilde{c}_{10}, e)}{\partial \tilde{c}_{10}}\gamma_1'(e) - \nu_1 k\gamma_1(e)\frac{\partial U(\tilde{c}_{10}, e)}{\partial \tilde{c}_{10}}d'(e) + \beta_1 = 0.$$

Hence,

$$\nu_0 + \nu_1 \frac{\gamma_1'(e)}{\gamma_1(e)} + \nu_1 kd'(e) = \frac{1}{\frac{\partial U(\tilde{c}_{10}, e)}{\partial \tilde{c}_{10}}} \frac{\beta_1}{\gamma_1(e)}.$$

Equation (B4), $\nu_1 > 0$, $t^* > 1$ and the monotone likelihood ratio property imply that the left-hand side of the last equation is negative. The right-hand side is nonnegative because constraint (B3) is binding for $t = 1$ and $\beta_1 \geq 0$. This contradiction proves that $t^* = 1$.

Plugging these results into the PC and the IC yields:

$$U(\omega^{*\text{CARA}}, e)(1 - \gamma_1(e)) + U(\tilde{c}_{10}^{*\text{CARA}}, e)\gamma_1(e) = \bar{u},$$

$$(U(\tilde{c}_{10}^{*\text{CARA}}, e) - U(\omega^{*\text{CARA}}, e))\gamma_1'(e) + kd'(e)\bar{u} = 0,$$

because $1 = \sum_{t\in\mathcal{T}} \gamma_t(e) \Rightarrow 0 = \sum_{t\in\mathcal{T}} \gamma_t'(e) = \gamma_1'(e) + \sum_{t\in\mathcal{T}\setminus\{1\}} \gamma_t'(e)$. Solving the first equation for $U(\tilde{c}_{10}^{*\text{CARA}}, e)$ gives $U(\tilde{c}_{10}^{*\text{CARA}}, e) = [\bar{u} - U(\omega^{*\text{CARA}}, e)(1-\gamma_1(e))]/\gamma_1(e)$. Inserting this value for $U(\tilde{c}_{10}^{*\text{CARA}}, e)$ into the second equation yields $(\bar{u} - U(\omega^{*\text{CARA}}, e))\gamma_1'(e) = -\gamma_1(e)kd'(e)\bar{u}$ and finally results in $U(\tilde{c}_{10}^{*\text{CARA}}, e) = \bar{u}(1 - \frac{1-\gamma_1(e)}{\gamma_1'(e)}kd'(e))$,

$$\omega^{*\text{CARA}} = \bar{w} + d(e) - \frac{1}{k}\ln(1 + \frac{\gamma_1(e)}{\gamma_1'(e)}kd'(e)) \text{ and } \tilde{c}_{10}^{*\text{CARA}} = \bar{w} + d(e) - \frac{1}{k}\ln(1 - \frac{1-\gamma_1(e)}{\gamma_1'(e)}kd'(e)).$$

These values of $\omega^{*\text{CARA}}$ and $\tilde{c}_{10}^{*\text{CARA}}$ are well-defined for $kd'(e) < -\gamma_1'(e)/\gamma_1(e)$ and allow to characterize the optimal contract. Therefore, the condition $kd'(e) < |\gamma_1'(e)/\gamma_1(e)|$ ensures that effort e is implementable. For this purpose, define $b^{\text{CARA}} = \tilde{c}_{10}^{*\text{CARA}}$ and consider a lottery $\Delta \in \mathcal{L}$ with $\mathbb{E}\exp(k(b^{\text{CARA}} - \Delta)) = 1$ and $\mathbb{E}(\Delta) = \omega^{*\text{CARA}}$ that is realized at the end of period 4. Now consider the contract stated in Proposition B1. The contract implements $\tilde{c}_{ts} = \tilde{w}_{ts} = \tilde{w}_{1s} = \omega^{*\text{CARA}}$ for all $t > 1$ and all $s \in \mathcal{S}$ as well as $\tilde{c}_{1s} = b^{\text{CARA}}$ for all $s \in \mathcal{S}$. Hence, the contract implies $\omega = \omega^{*\text{CARA}}$ and

$$u(\tilde{c}_{t0}) = \begin{cases} u(\omega^{*\text{CARA}}) & \text{if } t > 1 \\ u(b^{\text{CARA}}) = u(\tilde{c}_{10}^{*\text{CARA}}) & \text{if } t = 1. \end{cases}$$

The definition of $\tilde{c}_{t0}^{*\text{CARA}}$ ensures that the contract satisfies the agent's PC and his IC. The agent's truth-telling constraint (TT_A) is satisfied because his utilities are independent of his message. The principal's truth-telling constraint (TT_P) is also satisfied because expected wages are independent of her message. The contract also satisfies constraint (RA). Consequently, the contract in Proposition B1 is feasible. The contract is also optimal because $\omega^{*\text{CARA}}$ and $\tilde{c}_1^{*\text{CARA}}$ are optimal in Program B'. For $kd'(e) \geq -\gamma_1'(e)/\gamma_1(e)$, that is, large risk aversion or high disutility of effort, effort e is not implementable. For the same reasons as in Proposition 1, the contract is renegotiation proof. □

Proposition B2. If the agent's self-assessment is informative, that is, $p > 0$, and effort $e > 0$ is implementable, the following contract is optimal given Assumptions 1, 2, and 3a:

$$w_{ts} = \begin{cases} c_t^{*\text{CARA}} & \text{if } s = 0 \text{ or } t = s \\ \Delta_t & \text{otherwise} \end{cases}$$

with the optimal complete contract $c_t^{*\text{CARA}}$ defined in Lemma B1 in Appendix B and lotteries $\Delta_t \in \mathcal{L}$. The lotteries Δ_t have a certainty equivalent $c_t^{*\text{CARA}}$ and a mean $c_n^{*\text{CARA}}/p$ and are realized at the end of period 4.

Proof of Proposition B2. On the equilibrium path, above contract makes the principal pay a wage of $c_t^{*\text{CARA}}$ to the agent. Lemma B1 shows that the optimal complete contract $c_t^{*\text{CARA}}$ satisfies the agent's PC and IC. Hence, the agent accepts the contract and exerts effort e. Independently of the agent's message s, his utility is $U(c_t^{*\text{CARA}}, e) = \mathbb{E}U(\Delta_t, e)$. Therefore, truth-telling is optimal for the agent and his truth-telling constraint (TT_A) is satisfied. Suppose the agent reports his self-assessment s truthfully. If the principal evaluates the agent correctly, she pays $c_t^{*\text{CARA}}$. If the principal deviates to an evaluation $\bar{t} \neq t$, she expects to pay

$$p\tilde{w}_{\bar{t}t} + (1-p)\tilde{w}_{\bar{t}0} = pc_n^{*\text{CARA}}/p + (1-p)c_{\bar{t}}^{*\text{CARA}} = c_n^{*\text{CARA}} + (1-p)c_{\bar{t}}^{*\text{CARA}} > c_n^{*\text{CARA}} = \max_{r\in\mathcal{T}} c_r^{*\text{CARA}}.$$

Therefore, truth-telling is optimal for the principal and her truth-telling constraint (TT_P) is satisfied. Constraint (RA) is trivially satisfied.

In summary, the contract implements equilibrium payments of $c_t^{*\text{CARA}}$. Remember that the optimal complete contract $c_t^{*\text{CARA}}$ is a solution to a relaxed problem without the truth-telling constraints (TT_P) and (TT_A). Consequently, the contract in Proposition B2 optimally incentivizes the agent based on subjective evaluations. For the same reasons as in Proposition 2, the contract is renegotiation proof. □

Proposition B3. The following contract attains effort e at first-best costs given Assumptions 1a, 2a, and 3a:

$$w_{ts} = \begin{cases} \bar{w} + d(e) & \text{if } t = e \\ \Delta & \text{otherwise.} \end{cases}$$

with a lottery $\Delta \in \mathcal{L}$ that has a mean $\bar{w} + d(e)$ and a certainty equivalent below $\bar{w} + d(0)$ and that is realized at the end of period 4.

Proof of Proposition B3. In period 4, the principal's expected payoffs do not depend on her report. Therefore, reporting $t = e$ is optimal for the principal. If the agent's chooses effort e, her utilities in the contract are $-\exp(-k(\bar{w} + d(e) - d(e))) = \bar{u}$. Hence, she accepts the contract and her PC is satisfied. If the agent chooses any other effort, her utilities are at most $\mathbb{E}(-\exp(-k(\Delta - d(0)))) = \exp(kd(0))\mathbb{E}(-\exp(-k\Delta)) \leq -\exp(-k\bar{w}) = \bar{u}$. Therefore, the agent optimally chooses effort e.

The contract implements any effort $e \in [0, 1)$ at first-best costs. For the same reasons as in Proposition 3, the contract is renegotiation proof. □

Acknowledgements

■ Open access funding enabled and organized by Projekt DEAL.

References

Arnott, R. and Stiglitz, J.E. "Randomization with Asymmetric Information." *RAND Journal of Economics*, Vol. 19 (1988), pp. 344–362.

Babenko, I. and Sen, R. "Do Nonexecutive Employees Have Valuable Information? Evidence from Employee Stock Purchase Plans." *Management Science*, Vol. 62 (2016), pp. 1878–1898.

Baker, G., Gibbons, R., and Murphy, K.J. "Subjective Performance Measures in Optimal Incentive Contracts." *Quarterly Journal of Economics*, Vol. 109 (1994), pp. 1125–1156.

Bennardo, A. and Chiappori, P.-A. "Bertrand and Walras Equilibria under Moral Hazard." *Journal of Political Economy*, Vol. 111 (2003), pp. 785–817.

Bergman, N.K. and Jenter, D. "Employee Sentiment and Stock Option Compensation." *Journal of Financial Economics*, Vol. 84 (2007), pp. 667–712.

Bester, H. and Dahm, M. "Credence Goods, Costly Diagnosis and Subjective Evaluation." *Economic Journal*, Vol. 128 (2018), pp. 1367–1394.

Bester, H. and Krähmer, D. "Exit Options in Incomplete Contracts with Asymmetric Information." *Journal of Economic Theory*, Vol. 147 (2012), pp. 1947–1968.

Bester, H. and Krähmer, D. "The Optimal Allocation of Decision and Exit Rights in Organizations." *RAND Journal of Economics*, Vol. 48 (2017), pp. 309–334.

Bester, H. and Münster, J. "Subjective Evaluation versus Public Information." *Economic Theory*, Vol. 61 (2016), pp. 723–753.

Bull, C. "The Existence of Self-Enforcing Implicit Contracts." *Quarterly Journal of Economics*, Vol. 102 (1987), pp. 147–159.

Call, A.C., Kedia, S., and Rajgopal, S. "Rank and File Employees and the Discovery of Misreporting: The Role of Stock Options." *Journal of Accounting and Economics*, Vol. 62 (2016), pp. 277–300.

Carmichael, H.L. "The Agent-Agents Problem: Payment by Relative Output." *Journal of Labor Economics*, Vol. 1 (1983), pp. 50–65.

Carmichael, H.L. "Self-Enforcing Contracts, Shirking, and Life Cycle Incentives." *Journal of Economic Perspectives*, Vol. 3 (1989), pp. 65–83.

Chade, H. and De Serio, V. N.V. "Risk Aversion, Moral Hazard, and the Principal's Loss." *Economic Theory*, Vol. 20 (2002), pp. 637–644.

Chan, J. and Zheng, B. "Rewarding Improvements: Optimal Dynamic Contracts with Subjective Evaluation." *RAND Journal of Economics*, Vol. 42 (2011), pp. 758–775.

Compte, O. "Communication in Repeated Games with Imperfect Private Monitoring." *Econometrica*, Vol. 66 (1998), pp. 597–626.

Core, J.E. and Guay, W.R. "Stock Option Plans for Non-Executive Employees." *Journal of Financial Economics*, Vol. 61 (2001), pp. 253–287.

Deb, J., Li, J., and Mukherjee, A. "Relational Contracts with Subjective Peer Evaluations." *RAND Journal of Economics*, Vol. 47 (2016), pp. 3–28.

Dessler, G. *Human Resource Management*, 15th ed. Saddle River, NJ: Pearson Prentice Hall, 2017.

Ederer, F., Holden, R., and Meyer, M. "Gaming and Strategic Opacity in Incentive Provision." *The RAND Journal of Economics*, Vol. 49 (2018), pp. 819–854.

Fagart, M.-C. and Fluet, C. "The First-Order Approach When the Cost of Effort Is Money." *Journal of Mathematical Economics*, Vol. 49 (2013), pp. 7–16.

Fuchs, W. "Contracting with Repeated Moral Hazard and Private Evaluations." *American Economic Review*, Vol. 97 (2007), pp. 1432–1448.

Gauthier, S. and Laroque, G. "On the Value of Randomization." *Journal of Economic Theory*, Vol. 151 (2014), pp. 493–507.

Gibbons, R. "Incentives in Organizations." *Journal of Economic Perspectives*, Vol. 12 (1998), pp. 115–132.

Goldluecke, S. and Kranz, S. "Renegotiation-Proof Relational Contracts." *Games and Economic Behavior*, Vol. 80 (2013), pp. 157–178.

Grossman, S.J. and Hart, O.D. "An Analysis of the Principal-Agent Problem." *Econometrica*, Vol. 51 (1983), pp. 7–45.

Hart, O. and Moore, J. "Incomplete Contracts and Renegotiation." *Econometrica*, Vol. 56 (1988), pp. 755–785.

Hellwig, M.F. "The undesirability of randomized income taxation under decreasing risk aversion." *Journal of Public Economics*, Vol. 91 (2007), pp. 791–816.

Herweg, F., Müller, D., and Weinschenk, P. "Binary Payment Schemes: Moral Hazard and Loss Aversion." *American Economic Review*, Vol. 100 (2010), pp. 2451–2477.

Hochberg, Y.V. and Lindsey, L. "Incentives, Targeting, and Firm Performance: An Analysis of Non-executive Stock Options." *Review of Financial Studies*, Vol. 23 (2010), pp. 4148–4186.

Kandori, M. and Matsushima, H. "Private Observation, Communication and Collusion." *Econometrica*, Vol. 66 (1998), pp. 627–652.

Kedia, S. and Rajgopal, S. "Neighborhood Matters: The Impact of Location on Broad Based Stock Option Plans." *Journal of Financial Economics*, Vol. 92 (2009), pp. 109–127.

Kim, E.H. and Ouimet, P. "Broad-Based Employee Stock Ownership: Motives and Outcomes." *The Journal of Finance*, Vol. 69 (2014), pp. 1273–1319.

Kirkegaard, R. "A Unifying Approach to Incentive Compatibility in Moral Hazard Problems." *Theoretical Economics*, Vol. 12 (2017), pp. 25–51.

Laffont, J.-J. and Tirole, J. "Adverse Selection and Renegotiation in Procurement." *The Review of Economic Studies*, Vol. 57 (1990), pp. 597–625.

Lang, M. "Legal Uncertainty as a Welfare Enhancing Screen." *European Economic Review*, Vol. 91 (2017), pp. 274–289.

Lang, M. "Communicating Subjective Evaluations." *Journal of Economic Theory*, Vol. 179 (2019), pp. 163–199.

Letina, I., Liu, S., and Netzer, N. "Delegating Performance Evaluation." *Theoretical Economics*, Vol. 15 (2020), pp. 477–509.

Levin, J. "Relational Incentive Contracts." *American Economic Review*, Vol. 93 (2003), pp. 835–857.

Li, J. and Matouschek, N. "Managing Conflicts in Relational Contracts." *American Economic Review*, Vol. 103 (2013), pp. 2328–2351.

MacLeod, W.B. "Optimal Contracting with Subjective Evaluation." *American Economic Review*, Vol. 93 (2003), pp. 216–240.

MacLeod, W.B. "Reputations, Relationships, and Contract Enforcement." *Journal of Economic Literature*, Vol. 45 (2007), pp. 595–628.

MacLeod, W.B. and Malcomson, J.M. "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment." *Econometrica*, Vol. 57 (1989), pp. 447–480.

MacLeod, W.B. and Parent, D. "Job Characteristics and the form of Compensation." *Research in Labor Economics*, Vol. 18 (1999), pp. 177–242.

MacLeod, W.B. and Tan, T.Y. "Opportunism in Principal-Agent Relationships with Subjective Evaluation." *NBER Working Paper* 22156, 2016.

Maestri, L. "Bonus Payments versus Efficiency Wages in the Repeated Principal-Agent Model with Subjective Evaluations." *American Economic Journal: Microeconomics*, Vol. 4 (2012), pp. 34–56.

Malcomson, J.M. "Work Incentives, Hierarchy, and Internal Labor Markets." *Journal of Political Economy*, Vol. 92 (1984), pp. 486–507.

Malcomson, J.M. "Rank-Order Contracts for a Principal with Many Agents." *The Review of Economic Studies*, Vol. 53 (1986), pp. 807–817.

Malcomson, J.M. "Relational Incentive Contracts." In R. Gibbons and J. Roberts, eds., *Handbook of Organizational Economics*, ch. 25. Princeton, NJ: Princeton University Press, 2012.

Maskin, E. "On Indescribable Contingencies and Incomplete Contracts." *European Economic Review*, Vol. 46 (2002), pp. 725–733.

Maskin, E. and Tirole, J. "Unforeseen Contingencies and Incomplete Contracts." *Review of Economic Studies*, Vol. 66 (1999), pp. 83–114.

Murphy, K.J. "Performance Measurement and Appraisal: Merck Tries to Motivate Managers to Do It Right." *Employment Relations Today*, Vol. 20 (1993), pp. 47–62.

Myerson, R.B. "Optimal Coordination Mechanisms in Generalized Principal-Agent Problems." *Journal of Mathematical Economics*, Vol. 10 (1982), pp. 67–81.

Ohlendorf, S. and Schmitz, P.W. "Repeated Moral Hazard and Contracts with Memory: The Case of Risk-Neutrality." *International Economic Review*, Vol. 53 (2012), pp. 433–452.

Oyer, P. "Why do Firms Use Incentives That Have no Incentive Effects?" *Journal of Finance*, Vol. 59 (2004), pp. 1619–1650.

Oyer, P. and Schaefer, S. "Why Do Some Firms Give Stock Options to All Employees?: An Empirical Examination of Alternative Theories." *Journal of Financial Economics*, Vol. 76 (2005), pp. 99–133.

Page, F.H. "The Existence of Optimal Contracts in the Principal–Agent Model." *Journal of Mathematical Economics*, Vol. 16 (1987), pp. 157–167.

Porter, C., Bingham, C., and Simmonds, D. *Exploring Human Resource Management*. London: McGraw-Hill Education, 2008.

Rajan, M.V. and Reichelstein, S. "Subjective Performance Indicators and Discretionary Bonus Pools." *Journal of Accounting Research*, Vol. 44 (2006), pp. 585–618.

Rajan, M.V. and Reichelstein, S. "Objective versus Subjective Indicators of Managerial Performance." *The Accounting Review*, Vol. 84 (2009), pp. 209–237.

Rasmusen, E. "Moral Hazard in Risk-Averse Teams." *RAND Journal of Economics*, Vol. 18 (1987), pp. 428–435.

Rogerson, W.P. "The First-Order Approach to Principal-Agent Problems." *Econometrica*, Vol. 53 (1985), pp. 1357–1367.

Shapiro, C. and Stiglitz, J.E. "Equilibrium Unemployment as a Worker Discipline Device." *The American Economic Review*, Vol. 74 (1984), pp. 433–444.

Stiglitz, J.E. "Self-Selection and Pareto Efficient Taxation." *Journal of Public Economics*, Vol. 17 (1982), pp. 213–240.
Strausz, R. "Deterministic versus Stochastic Mechanisms in Principal-Agent Models." *Journal of Economic Theory*, Vol. 128 (2006), pp. 306–314.
Suvorov, A. and van de Ven, J. "Discretionary Rewards as a Feedback Mechanism." *Games and Economic Behavior*, Vol. 67 (2009), pp. 665–681.