Automation and Polarization

Daron Acemoglu (MIT)
Jonas Loebbing (LMU Munich)

Discussion Paper No. 421

August 31, 2023

Collaborative Research Center Transregio 190 | www.rationality-and-competition.de
Ludwig-Maximilians-Universität München | Humboldt-Universität zu Berlin
Spokesperson: Prof. Georg Weizsäcker, Ph.D., Humboldt University Berlin, 10117 Berlin, Germany
info@rationality-and-competition.de

Automation and Polarization*

Daron Acemoglu[†] **Jonas Loebbing**[‡]

September 26, 2022

Abstract

We develop an assignment model of automation. Each of a continuum of tasks of variable complexity is assigned to either capital or one of a continuum of labor skills. We characterize conditions for interior automation, whereby tasks of intermediate complexity are assigned to capital. Interior automation arises when the most skilled workers have a comparative advantage in the most complex tasks relative to capital, and because the wages of the least skilled workers are sufficiently low relative to their productivity and the effective cost of capital in low-complexity tasks. Minimum wages and other sources of higher wages at the bottom make interior automation less likely. Starting with interior automation, a reduction in the cost of capital (or an increase in capital productivity) causes employment and wage polarization. Specifically, further automation pushes workers into tasks at the lower and upper ends of the task distribution. It also monotonically increases the skill premium above a skill threshold and reduces the skill premium below this threshold. Moreover, automation tends to reduce the real wage of workers with comparative advantage profiles close to that of capital. We show that large enough increases in capital productivity ultimately induce a transition to low-skill automation and qualitatively alter the effects of automation—thereafter inducing monotone increases in skill premia rather than wage polarization.

Keywords: assignment, automation, inequality, polarization, tasks, wages.
JEL Classification: J23, J31, O33.

*We thank Juanita Jaramillo for excellent research assistance, David Autor, Chris Busch, Moritz Drechsel-Grau and Pascual Restrepo for several useful discussions, and Hewlett Foundation and Joachim Herz Foundation for financial support.

†Massachusetts Institute of Technology, Department of Economics, E52-380, 50 Memorial Drive, Cambridge MA 02142; E-mail: daron@mit.edu.

‡LMU Munich, Center for Economic Studies, Schackstrasse 4, 80539 Munich; E-mail: jonas.loebbing@econ.lmu.de.

1 Introduction

Automation technologies, including specialized software tools, computerized production equipment, and industrial robots, have been spreading rapidly throughout the industrialized world. For example, the share of information processing equipment and software in overall investment in the US has increased from 3.5% to over 23% between 1950 and 2020 (BEA, 2021a), while the number of industrial robots per thousand workers has risen from 0.38 in 1993 to about 1.8 in 2017 (BEA, 2021b; IFR, 2018). There is now growing evidence that these technologies have not just automated a range of tasks previously performed by workers and impacted the wage structure,[1] but also have led to *polarization* of employment and wages—meaning that the negative effects have concentrated on employment and wages in the middle of the wage distribution.[2] This pattern is intimately linked to the fact that many of the tasks that have been automated used to be performed by middle-skill workers.

There is no widespread agreement on why automation has been associated with polarization, however. One explanation, suggested by Autor (2014, 2015), is related to "Polanyi's paradox", as captured by Michael Polanyi's statement that "we can know more than we can tell" (Polanyi, 1966). Put simply, many of the manual and abstract tasks embed rich tacit knowledge, making them non-routine. Because routine tasks are *technologically* easier to automate and are performed by middle-skill workers located in the middle of the wage distribution, new automation technologies have displaced labor from the middle-skill occupations and have had their most negative effects on middle-pay worker groups.

In this paper, we provide an alternative, complementary explanation: automation has focused on middle-skill tasks, because these are the most profitable ones to automate. Specifically, low-skill tasks can be performed at lower labor expenses, reducing the cost advantage of machines relative to humans.

To develop this point, we build an assignment model, combining elements from the seminal contributions by Tinbergen (1956), Sattinger (1975), and Teulings (1995, 2005), together with the model of tasks and automation in Acemoglu and Restrepo (2022). Workers are distinguished by a single-dimensional skill index, which is distributed over an interval normalized to $[0, 1]$ and tasks

[1] On the spread of automation technologies over the last 80 years, see Autor (2015), Ford (2015), Acemoglu and Restrepo (2020), Graetz and Michaels (2018) and Acemoglu and Johnson (2023).

[2] The seminal contribution on the inequality and polarization effects of automation is Autor, Levy and Murnane (2003). For a recent study of the effects of automation technologies on US wage inequality, see Acemoglu and Restrepo (2022). The polarizing effects of automation are also documented in Goos, Manning and Salomons (2009), Acemoglu and Autor (2011), and Autor and Dorn (2013).

are also distributed over the unit interval. For notational simplicity, we refer to higher-index tasks as more "complex" tasks.[3] Following the assignment literature, we assume that high-type workers have a comparative advantage in more complex tasks. Without automation, as in that literature, our model generates a monotone assignment pattern, with higher-skill workers performing more complex tasks. The distinguishing feature of our framework is that some tasks can be assigned to capital.

Under the assumption that capital does not have a comparative advantage for the most complex task and some regularity restrictions on capital productivity, we prove that the equilibrium will take one of two forms: (1) *interior automation*, where capital performs a set of intermediate tasks; or (2) *low-skill automation*, where capital takes over all tasks below a certain threshold.[4]

Interior automation is the configuration that leads to polarization, and we provide conditions under which automation is indeed interior. These conditions depend on the comparative advantage of low-skill workers relative to capital, the cost of capital, and labor supplies of different types of workers, which together determine the equilibrium wage distribution with and without automation. Intuitively, when equilibrium wages (without automation) are sufficiently low for low-skill workers, tasks in the bottom of the complexity distribution are very cheap and this reduces the profitability of performing them by capital. When this is the case, we also show that additional automation leads to both wage and employment polarization. In our model, therefore, polarization is closely linked to the fact that wages are already very low at the bottom.

In addition to establishing the existence of a unique competitive equilibrium and characterizing the conditions under which capital takes over tasks from the middle of the skill distribution, we provide a series of comparative static results for marginal (local) and large (global) changes in automation.

Our *first* result, clarifies the conditions under which automation is interior. In the baseline model, interior automation requires that wages at the bottom be sufficiently low relative to the productivity of low-skill workers and the effective cost of capital.[5] To further clarify the role of wages at the bottom, we consider a simple extension in which there is a minimum wage. In this

[3] We show that more realistic configurations, where manual tasks that require skills that machines and algorithms do not currently fully possess can be incorporated into our model and can still be mapped into our single-dimensional tasks distribution.

[4] A third possibility is no automation, which is not interesting given our focus here and will be ruled out by assuming that capital productivity is sufficiently high to make some automation profitable in equilibrium.

[5] This is the sense in which our explanation is complementary to Autor's (2014, 2015) account—when low-skill workers are relatively more productive at low-complexity tasks, this equivalently induces interior automation in our model.

case, interior automation requires that the minimum wage is not too high—otherwise, low-skill labor is too expensive and this induces low-skill automation. We also complement this result by showing that a reduction in the supply of skills at the bottom raises low-skill wages and makes a transition to low-skill automation more likely.

Our *second* result is that, as already mentioned above, a further expansion of interior automation—for example, driven by a decline in the price of capital goods—creates employment and wage polarization. Employment polarization here simply means that human workers are squeezed into smaller sets of tasks at the bottom and the top. Wage polarization takes a more specific form: relative wage changes increase as a function of the distance between the task that a skill type performs and the boundaries of the set of automated tasks. As a result, we prove that skill premia *increase* among worker types performing more complex tasks than those that are automated and *decrease* among worker types performing less complex tasks than the automated ones. Put differently, interior automation hurts (relatively) workers that are closer to the set of automated tasks. This is intuitive in view of the fact that workers closer to this set used to have a stronger comparative advantage for tasks that are now automated.

Third, we characterize the effects of automation on the *level* of real wages for different types of workers. As in Acemoglu and Restrepo (2022), whether the real wage of a given skill group declines depends on competing *displacement* and *productivity* effects, though we are able to provide more explicit conditions. One noteworthy result in this context is that the larger is the initial set of tasks that are automated, the more likely are the real wages of all skill types to increase. Moreover, we show that the productivity gains from automation are *convex* in the price of capital goods. This implies that, as capital good prices (including costs of algorithmic automation) decline further, the productivity effect strengthens, ultimately eliminating negative wage effects. These results, together, imply that the most negative consequences of displacement on workers will be at the early stages of the automation process.[6]

We present one more noteworthy result on wages, related to what we call "Wiener's conjecture", after Norbert Wiener's (1950) pioneering study of automation. Wiener claimed, "the automatic machine is the precise economic equivalent of slave labor. Any labor which competes with slave must accept the economic consequences of slave labor." This conjecture does not seem to have been

[6]In fact, because production in our model exhibits constant returns to scale and the cost of capital is constant, average wages always increase following automation, and this structure is also important for the result that when sufficiently many tasks are automated, the effects on all wages are positive. The consequences of automation on wages become more negative when the price of capital is increasing in the stock of capital, or when we depart from constant returns to scale or competitive markets. See Moll, Rachel and Restrepo (2022) and Acemoglu and Restrepo (2023).

fully borne out by economic models or the data. On the theory side, Zeira (1998) and Acemoglu and Restrepo (2018a) show that real wages will increase in the long run following automation. On the empirical side, although the real wages of low-education groups have declined over the last forty years, automation was also rapid in the 1950s and 1960s and during these decades wages for almost all demographic groups increased robustly. Our analysis suggests that Wiener's conjecture needs to be refined: different workers have different skills, and even if automated machines are like slave labor, they do not perfectly compete against all kinds of labor. Building on this intuition, we show that automation always reduces the real wages of worker types whose productivity schedule over tasks is sufficiently close to capital's productivity schedule (if such worker types exist, but they may not).

Finally, we use the model to study global—as opposed to local—effects of automation, which result when there are large declines in costs of capital goods. We show that as long as these changes keep us in the region of interior automation, their effects are qualitatively the same as those of local changes. Ultimately, however, automation expands from the interior of the set of tasks to take over all low-skill tasks. When this happens, the pattern of polarization reverses. While an expansion in interior automation hurts workers in the middle of the skill distribution the most (and lowest-skill workers are to some degree sheltered), a switch from interior to low-skill automation has its most adverse effects on lowest-skill workers. Hence, our model predicts that as automation proceeds, its inequality implications may become worse, not just quantitatively but also qualitatively.[7]

Our paper is related to several contributions in both the assignment and automation literatures. In the assignment literature, early contributions include Tinbergen (1956), Rosen (1974), Heckman and Sedlacek (1985), and Sattinger (1975, 1993). Our model more closely builds on Teulings (1995, 2005), Teulings and Gautier (2004), Costinot and Vogel (2010), and Stokey (2018). The major difference between all of these papers and our work is the presence of capital that can perform some of the tasks, thus allowing for an analysis of automation. From a technical point of view, all existing analyses within this area assume a strong form of comparative advantage (log supermodularity), which turns the problem into one of monotone assignment. Our analysis of automation relaxes overall supermodularity (though, for simplicity, we maintain log supermodularity between worker and job types).

In the automation literature, we build on earlier models where capital displaces workers in some of the tasks they used to perform. This literature and task-based models started with Zeira's (1998)

[7]On the other hand, if in this process capital productivity in already-automated tasks continues to increase, this makes negative wage level effects less likely.

seminal work and Autor, Levy and Murnane's (2003) pioneering empirical study of the polarization and inequality effects of automation. Zeira's model includes only one type of labor and does not focus on inequality implications of automation. Many subsequent works, including Acemoglu and Zilibotti (2001), Acemoglu and Restrepo (2018a,b), Berg, Buffie and Zanna (2018), Jackson and Kanik (2020), Jaimovich et al. (2021) and Hemous and Olsen (2022) allow only two types of workers, making it impossible to study wage polarization. Acemoglu and Autor (2011) study an economy with three types of workers and establish polarization when automation affects the middle group, but this structure does not allow a comprehensive analysis of the implications of different stages of automation on employment and wage patterns. Two notable exceptions are Feng and Graetz (2020) and Loebbing (2022), both of which study task-based models with a continuum of labor types, though under more restrictive assumptions. In particular, Feng and Graetz (2020) impose that automation is always interior, while Loebbing (2022) focuses on the case in which automation is always low-skill. This contrasts with our focus which is to understand *when* automation is interior and the conditions under which there is a transition to low-skill automation. Additionally, these papers do not contain our main comparative static results.

Acemoglu and Restrepo (2022) develop a general framework with multiple industries and multiple worker types to study the inequality effects of automation. In addition to providing empirical estimates of the effects of automation on US wage inequality, Acemoglu and Restrepo (2022) present a theoretical analysis of the implications of automation. Because their study lacks the specific structure imposed here (with one-dimensional heterogeneity both on the worker and job complexity side and comparative advantage between workers and tasks), it does not contain results on which tasks automation will take over. Rather, they provide equations that specify how wages of different groups will be affected as a function of the total set of tasks that are automated and "ripple effects", which capture how different skill groups compete over marginal tasks. These ripple effects cannot be explicitly characterized given their assumptions and are studied empirically. Our analysis here enables us to characterize the full equilibrium implications of changes in the prices of capital goods—including how the set of automated tasks expands and how substitution between different types of workers takes place.

The rest of the paper is organized as follows. Section 2 presents our baseline environment and defines a competitive equilibrium. Section 3 first establishes existence and uniqueness of equilibrium and some basic characterization results, and then studies the conditions under which automation is interior. Section 4 presents our main comparative static results for small changes in the cost of capital goods, thus deriving the employment and wage polarization implications of automation.

Section 5 considers global changes in automation technology and their equilibrium consequences. Section 6 concludes, while the Appendix includes several of the proofs omitted from the text.

2 Model

In this section, we introduce the basic economic environment, describe some of our assumptions and their motivation, and also define a competitive equilibrium.

2.1 Environment

We consider a static economy with a unique final good, Y, produced from a continuum of workers with skills $s \in [0,1]$ and a continuum of tasks $x \in [0,1]$. The production of the final good is given as a constant elasticity of substitution (CES) aggregate of tasks:

$$Y = \left[\int_0^1 (A_x Y_x)^{\frac{\lambda-1}{\lambda}} \, dx \right]^{\frac{\lambda}{\lambda-1}} , \tag{1}$$

where Y_x is the amount of task x, $A_x > 0$ (assumed twice continuously differentiable) is a technology parameter relevant for task x, and $\lambda > 0$ is the elasticity of substitution.

All labor types are inelastically supplied, with a density function of $l : [0,1] \to \mathbb{R}_{++}$ (which specifies the total endowment of each type of labor) and we assume that this density is continuous and has no mass points. We also assume that capital is produced out of final good with marginal cost $1/q$. We identify increases in q with greater capital productivity or equivalently lower prices of capital goods.

The task production functions are given by

$$Y_x = \int_0^1 \psi_{s,x} L_{s,x} ds + \psi_{k,x} K_x, \tag{2}$$

for all $x \in [0,1]$, where $\psi_{s,x} > 0$ and $\psi_{k,x} > 0$ denote the productivities of different factors in task x, and $L_{s,x}$ and K_x are, respectively, the amounts of labor of type s and capital allocated to the production of task x. We assume that the factor productivities $\psi_{s,x}$ and $\psi_{k,x}$ are twice continuously differentiable. Labor market clearing requires

$$\int_0^1 L_{s,x} dx = l_s \quad \text{for all } s \in [0,1] \tag{3}$$

and net output (or equivalently, consumption), which is also equivalent to total factor productivity (TFP) in this economy, is

$$C = Y - \frac{1}{q} \overline{K},$$

where $\overline{K} = \int_0^1 K_x dx$ is the aggregate capital stock.

2.2 Competitive Equilibrium

An allocation in this economy is given by a collection of density functions, $L = \{L_s\}_{s=0}^{1}$, where $L_s : [0,1] \to \mathbb{R}_+$ for each $s \in [0,1]$, and a capital allocation $K : [0,1] \to \mathbb{R}_+$. The density functions allocate labor supply of each type of labor to tasks, and the capital allocation function determines how much capital will be allocated to each task. This definition already incorporates nonnegativity constraints for all factors in all tasks. We describe an allocation with the shorthand $\{L, K\}$. Additionally, for all $s \in [0,1]$, we define the set $X_s : \{x \mid L_{s,x} > 0\}$ as the set of tasks performed by labor type s in this allocation and $X_k = \{x \mid K_x > 0\}$. We also use the terminology that tasks in the set X_k are *automated*.

We additionally designate two price functions: first, a wage function $w : [0,1] \to \mathbb{R}_+$, which determines the wage level, w_s, for each type of labor $s \in [0,1]$; and second, a task price function $p : [0,1] \to \mathbb{R}_+$, which determines the price p_x of each task $x \in [0,1]$.

A (competitive) equilibrium is defined as an allocation $\{L, K\}$ that maximizes consumption subject to labor market clearing (3), and wage and price functions, w and p, where wages are given by marginal products of the relevant labor types, i.e.,

$$\begin{aligned} w_s &= p_x \psi_{s,x} \text{ for all } x \in X_s \\ w_s &\geq p_x \psi_{s,x} \text{ for all } x \in [0,1], \end{aligned} \tag{4}$$

and task prices are given by the marginal products of tasks in final good production, i.e.,

$$p_x = \left(\frac{Y}{Y_x}\right)^{\frac{1}{\lambda}} A_x^{\frac{\lambda-1}{\lambda}} \text{ for all } x \in [0,1]. \tag{5}$$

As a consequence of consumption maximization, the cost of capital must be equal to the marginal product of capital, i.e.,

$$\begin{aligned} \frac{1}{q} &= p_x \psi_{k,x} \quad \forall x \in X_k \\ \frac{1}{q} &\geq p_x \psi_{k,x} \quad \forall x \in [0,1]. \end{aligned} \tag{6}$$

2.3 Assumptions and Motivation

We now describe some of the assumptions we will use in the next section.

Since the economy exhibits constant returns to scale and can produce output from final goods, in principle its output may be unbounded. Our first assumption ensures that the cost of capital is not so low as to generate infinite output.

Assumption 1 ***(Bounded output)***

$$\frac{1}{q} > \frac{1}{q_\infty} = \left(\int_0^1 A_x^{\lambda-1} \psi_{k,x}^{\lambda-1} \, dx \right)^{\frac{1}{\lambda-1}} .$$

Our second assumption follows the assignment literature (e.g., Teulings, 1995, 2005, Costinot and Vogel, 2010) and imposes *absolute advantage* (higher skills are more productive in all tasks) and *comparative advantage* (the productivity advantage of higher skilled workers increases more than proportionately with the task index). We impose this assumption both to simplify the analysis and also to maximize the similarity of our benchmark environment to the previous literature, which will clarify that all of the new results here are driven by the automation margin.

Assumption 2 ***(Absolute and comparative advantage)*** *For all $s > s'$, we have*

1. $\psi_{s,x} \geq \psi_{s',x}$ *for all x (*absolute advantage*).*
2. $\psi_{s,x}/\psi_{s,x'} > \psi_{s',x}/\psi_{s',x'}$ *for all $x > x'$ (*comparative advantage*).*

Comparative advantage ensures that, without capital, the equilibrium will assign higher skilled workers to higher-indexed tasks. Absolute advantage guarantees that, regardless of the levels of labor supply, higher-indexed workers will have higher wages, thus justifying our terminology of referring to higher levels of the skill index as "more skilled".

We next present a motivating example that provides a simple illustration of the comparative advantage aspect. This example has the additional advantage that it shows how multidimensional skills can be mapped into our setup with a one-dimensional skill index.

Example 1: Suppose that each task $x \in [0, 1]$ involves a combination of abstract and manual activities. Specifically, the productivity of a worker with skill level $s \in [0, 1]$ in task x will be a function of this worker's abstract and manual skills, denoted by the vector (a_s, m_s):

$$\psi_{s,x} = a_s^x m_s^{1-x}.$$

In the context of this example, a sufficient condition for comparative advantage is for workers' skill endowments (a_s, m_s) to satisfy

$$\frac{a_s}{m_s} > \frac{a_{s'}}{m_{s'}} \text{ for all } s > s'.$$

To see this, let

$$\Lambda = \log \frac{\psi(x, s)}{\psi(x, s')} = \log \frac{a_s^x m_s^{1-x}}{a_{s'}^x m_{s'}^{1-x}},$$

and by assumption, we have $\partial\Lambda/\partial x = [\log a_s - \log m_s] - [\log a_{s'} - \log m_{s'}] > 0$. A sufficient condition for absolute advantage, on the other hand, is for a_s to be strictly increasing and m_s to be non-decreasing in s.

Motivated by this pattern of comparative advantage, we will also refer to higher-index tasks as *more complex tasks*, as in Teulings (1995, 2005).

The other key dimension of our model concerns the productivity of capital relative to different labor types. Crucially, here, we do not assume overall supermodularity. However, it is convenient to put sufficient structure on the comparative advantage of capital to have a simple characterization of the set of tasks, X_k, that are assigned to capital. The next assumption achieves this.

Assumption 3 ***(Comparative advantage of capital)***

1. *For all $s \in [0,1]$, $\psi_{k,x}/\psi_{s,x}$ is quasi-concave in x.*

2. *Moreover, the most skilled workers have comparative advantage relative to capital in the most complex relative to the least complex tasks:*

$$\frac{\psi_{1,1}}{\psi_{k,1}} > \frac{\psi_{1,0}}{\psi_{k,0}}. \tag{7}$$

Intuitively, the first part of this assumption rules out situations in which the direction of comparative advantage for capital changes more than once for any given level of skill. Put differently, Assumption 3 allows some skill types to have a comparative advantage in lower-index tasks relative to capital and then again in higher-index tasks after a certain threshold. But it rules out more than one such switch. As we will see in Proposition 2, this is necessary and sufficient to ensure that the set X_k of tasks assigned to capital is convex.

The second part of the assumption imposes that there are some sufficiently complex tasks in which some type of labor still has a comparative advantage relative to capital. We next illustrate these conditions with the environment considered in Example 1.

Example 1 (continued) The first part of Assumption 3 is ensured in this example when $\psi_{k,x}$ is log concave in x. Since the productivity of each labor type is log linear in x, log concavity of capital productivity implies quasi-concavity of all relative productivity schedules $\psi_{k,x}/\psi_{s,x}$. The second part of the assumption, in turn, is equivalent to

$$\frac{a_1}{\psi_{k,1}} > \frac{m_1}{\psi_{k,0}}.$$

In other words, the highest-skill workers' abstract skills are more productive relative to capital in the most abstract tasks than their manual skills are in the most manual-intensive tasks.

3 Characterization of Equilibrium and Interior Automation

In this section, we establish existence and uniqueness of a competitive equilibrium and study the conditions under which tasks from the middle of the skill distribution will be automated.

3.1 Existence and Uniqueness

Proposition 1 ***(Existence and uniqueness)*** *Suppose Assumption 1 holds. Then, a competitive equilibrium always exists and is essentially unique in the sense that wage and price functions are uniquely determined. If in addition Assumptions 2 and 3 hold, the competitive equilibrium is unique.*

Existence follows from the fact that the competitive equilibrium maximizes net consumption in this economy, which is a continuous function of the allocation. Essential uniqueness, on the other hand, is a consequence of the fact that net consumption is a concave function of the allocation. The reason why the competitive equilibrium is essentially unique, but not fully unique under Assumption 1, is straightforward to see: some tasks may be produced at the same cost using different factors, creating indeterminacy of equilibrium allocations. Assumptions 2 and 3 rule out such indeterminacy: Assumption 2 imposes strict comparative advantage between any two types of labor and, together with Assumption 3, implies that on any subset of tasks of positive measure, there can be at most one type of labor with a productivity schedule parallel to capital's productivity schedule.[8]

3.2 Interior Automation

The next proposition confirms that, as mentioned above, the first part of Assumption 3 is enough to ensure that the set of tasks allocated to capital, X_k, is convex.

Proposition 2 ***(Convexity of assignment)*** *Suppose Assumptions 1 and 2 hold.*

1. *The allocation of labor across tasks is monotone, meaning that for any $s > s'$, if $L_{s,x} > 0$, then $L_{s',x'} = 0$ for all $x' \geq x$.*
2. *The set of automated tasks in equilibrium, X_k, is convex for all labor endowment functions and capital productivity levels if and only if Assumption 3.1 holds. Moreover, if Assumption 3.2 also holds, then the most complex tasks are performed by labor, or in other words, $1 \notin X_k$.*

[8] If a single type of labor has a productivity schedule that is parallel to capital's productivity schedule on the set of automated tasks, this does not create any indeterminacy in allocations because a single type of labor has no mass and is hence irrelevant for an allocation. Formally, an allocation is a collection of densities from an L^p space where any two densities that are equal almost everywhere are identified and represent the same allocation.

The first part of this proposition confirms that the monotonicity obtained in assignment models with log supermodularity continues to hold in our model. The second part implies that we can focus on a convex set of automated tasks. Moreover, under the second part of Assumption 3, this set will not include the most complex tasks. These observations leave three feasible configurations of automation:

1. *No automation*, where $X_k = \emptyset$ (because the cost of capital is too high).

2. *Interior automation*, where $X_k = [\underline{x}, \overline{x}]$ with $0 < \underline{x} < \overline{x} < 1$, and thus both the most complex and the least complex tasks are assigned to some labor types.[9]

3. *Low-skill automation*, where $X_k = [0, \overline{x}]$, with $0 < \overline{x} < 1$, and thus all tasks below a certain threshold of complexity are automated.

We refer to the third configuration as low-skill automation, since capital takes over tasks that used to be performed by lower-skilled workers (as in the first part of Proposition 2). The first case, no automation, is not of great interest given the focus of the current paper and we will assume below that the cost of capital is sufficiently low to ensure some automation.

We next study the conditions under which automation will be interior or low-skill. The least complex task, $x = 0$, is cheaper to produce by the least skilled worker type, $s = 0$, than by capital if

$$\frac{w_0}{\psi_{0,0}} < \frac{1/q}{\psi_{k,0}}. \tag{8}$$

When inequality (8) is satisfied, we cannot have low-skill automation, and hence from Proposition 2, we must have interior automation. This condition is intuitive. It requires that the effective wage of the least skilled workers in the least complex task (wage divided by productivity) is less than the effective cost of capital in that task (cost of capital, $1/q$, divided by the productivity of capital in that task). Whether this condition is satisfied depends on the shape of comparative advantage schedules ψ, capital productivity q and the labor supply profile l (which jointly determine the equilibrium wage for the least skilled type, w_0). For any given labor supply profile l, the following assumptions are sufficient to ensure that condition (8) is satisfied. We will study the effects of labor supply changes on these conditions in Section 5.

[9] Here the implicit assumption that the boundary tasks $\underline{x}$ and $\overline{x}$ are performed by capital is imposed for notational simplicity and is without loss of any generality.

Assumption 4 ***(Comparative advantage of least skilled workers)*** *The least skilled workers have local comparative advantage relative to capital in the least complex tasks:*

$$\frac{\partial \log \psi_{0,0}}{\partial x} < \frac{\partial \log \psi_{k,0}}{\partial x} .$$

We note that Assumption 4 imposes only *local* comparative advantage—for a comparison of the least skilled workers to capital around the least complex tasks. This comparative advantage pattern does not have to hold globally. For example, even the least skilled workers may have comparative advantage relative to capital in the most complex tasks.

Assumption 5 ***(Intermediate capital productivity)*** *Initially, $q \in (q_0, q_m)$, where q_0 is the threshold below which capital is not used in equilibrium (and thus $X_k = \emptyset$) and $q_m > q_0$ is another threshold.*[10]

Proposition 3 ***(Interior automation)*** *Suppose Assumptions 1, 2 and 3 hold. If Assumptions 4 and 5 hold as well, automation is interior, i.e., $X_k = [\underline{x}, \overline{x}]$ with $0 < \underline{x} \leq \overline{x} < 1$. If, on the other hand, Assumptions 4 and 5 do not hold, automation is low-skill for all $q > q_0$ and there is no automation for all $q \leq q_0$.*[11]

Moreover, if automation is interior, there exists a skill level $\widetilde{s} \in (0, 1)$ such that all workers with skill level $s < \widetilde{s}$ are employed in tasks $x < \underline{x}$ and all workers with skill level $s > \widetilde{s}$ are employed in tasks $x > \overline{x}$.

Proposition 3 is our first main result and provides a necessary and sufficient condition (under our other assumptions) for automation to be interior. This condition consists of the local comparative advantage condition in Assumption 4 and a restriction on the productivity of capital, as imposed in Assumption 5. The former condition is necessary and sufficient for automation to be interior initially, i.e., for a sufficiently (but not prohibitively) low productivity of capital. The latter condition ensures that the productivity of capital indeed falls into this intermediate range.

Figure 1 shows the assignment of tasks to labor and capital in the case of interior automation. Tasks in the set $X_k = [\underline{x}, \overline{x}]$ are assigned to capital, while the remaining tasks are assigned to the labor types indicated on the vertical axis.

[10] We provide a characterization of q_0 and q_m in Appendix A.3 and discuss the consequences of $q \geq q_m$ in Section 5. Note at this point that q_m is only well defined if we impose Assumption 4.

[11] If only Assumption 4 (but not 5) holds, there are two cases: if $q \leq q_0$, there is no automation; and if $q \geq q_m$, automation is low skill as we will discuss in Section 5. Note also that Assumption 5 cannot be imposed without Assumption 4 as q_m is not well defined without Assumption 4.

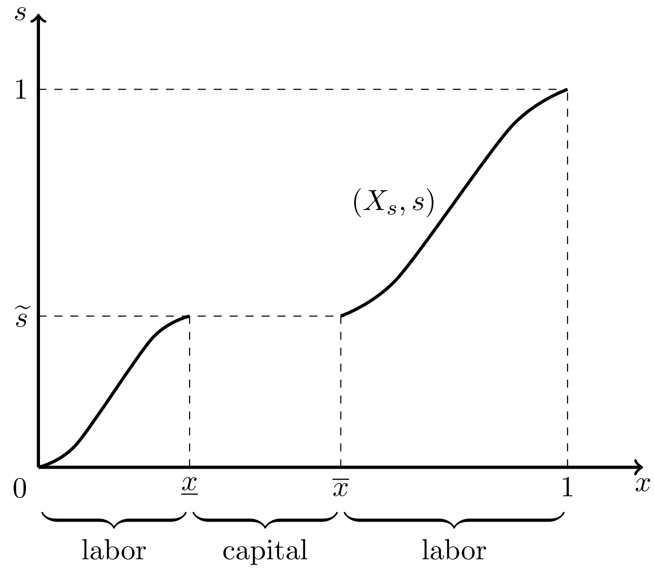

Figure 1: *Assignment of tasks to capital and labor.* Tasks in the set $X_k = [\underline{x}, \overline{x}]$ are assigned to capital, while the remaining tasks are assigned to the labor types indicated by the graph (X_s, s). In particular, tasks $x < \underline{x}$ are assigned to worker types $s < \widetilde{s}$, while tasks $x > \overline{x}$, are assigned to labor types $s > \widetilde{s}$.

The last part of Proposition 3 is a simple consequence of the comparative advantage across labor types, but also enables us to think of the distance between a skill group s and $\widetilde{s}$ as a measure of how affected this group is by automation, as we will see in the next section.

Inequality (8) provides further intuition for why automation may affect middle-skill occupations most. Fixing $\psi_{0,0}$ and treating the wage of the least skilled worker, w_0, parametrically, there are two ways in which this inequality is satisfied: either $1/(q\psi_{k,0})$ is high or w_0 is low. The first captures the economic forces proposed by Autor (2014, 2015): many of the tasks performed by lower-skill workers require a combination of tacit knowledge and manual dexterity, making them difficult to automate. The second is what we have emphasized in the Introduction: wages at the bottom are too low to make automation economically profitable.

Proposition 3 clarifies that these two explanations are linked, because the wage is endogenous. However, they are also distinct, and one way of illustrating this is to consider variations in the wage at the bottom of the distribution, holding the other parameters of the model constant.

The simplest way of doing this is by imposing a minimum wage in the model (the alternative, via changes in the labor supply function, is explored in Section 5). Here we discuss briefly the implications of a minimum wage, $\underline{w}$. In the presence of a binding minimum wage, the equilibrium will involve rationing—some worker types may not be hired. This requires an obvious change in the definition of equilibrium, which we omit to save space. It is also straightforward to see that the set of rationed workers will always be of the form $[0, \underline{s}]$ (see Teulings, 2000). Except for rationing, the same equilibrium conditions as in our analysis so far apply. Then we have:

Proposition 4 ***(Minimum wages and automation)*** *Suppose Assumptions 1, 2, 3, 4 and 5 hold, so that in the competitive equilibrium without the minimum wage, we have interior automation. Now consider a minimum wage of $\underline{w} > 0$, which leads to the rationing of workers with skills in $[0, \underline{s}]$. If in addition we have*

$$\frac{\partial \log \psi_{\underline{s},0}}{\partial x} \geq \frac{\partial \log \psi_{k,0}}{\partial x}, \tag{9}$$

then inequality (8) is violated and we transition to low-skill automation.

Intuitively, without the minimum wage, labor performing low-skill tasks is cheap, and this makes automating these tasks unprofitable, ensuring interior automation. When the minimum wage is imposed, this leads to the rationing (unemployment) of some low-skill workers, and more generally makes skills at the bottom more expensive, making the automation of the tasks previously performed by low-skill workers profitable. It is also useful to observe the role of condition (9): without this condition, some of the workers with skill above $\underline{s}$ may find it profitable to take the lowest-complexity tasks. Notice also that this condition is compatible with Assumption 4, since the comparison is for different skill levels. In fact, the juxtaposition of these two conditions highlights that in our model conditions for the comparative advantage of capital and labor are endogenous, since which skill type's productivity is compared to capital's productivity is determined in equilibrium.

3.3 Characterization of Equilibrium

We next provide a characterization of equilibrium when Assumptions 1 to 3 hold and $q \geq q_0$ (so that there is automation in equilibrium). Under these assumptions, the set of automated tasks takes the form $X_k = [\underline{x}, \overline{x}]$ with $\underline{x} \leq \overline{x} < 1$. This leaves the sets $[0, \underline{x})$ and $(\overline{x}, 1]$ for labor, with skills above a threshold $\widetilde{s} \in [0, 1)$ employed in $(\overline{x}, 1]$ and those below $\widetilde{s}$ employed in $[0, \underline{x})$.

By standard arguments from the assignment literature, the allocation of skills to tasks in either of the two sets $[0, \underline{x})$ and $(\overline{x}, 1]$ can be described by an assignment function $X : [0, \widetilde{s}) \cup (\widetilde{s}, 1] \to [0, \underline{x}) \cup (\overline{x}, 1]$ mapping skills to tasks.[12] The assignment function is differentiable (except at the

[12] Recall that X_s was defined as the set of tasks performed by skill s, and thus in general X should be a correspondence. However, under Assumptions 1 to 3, X_s is a singleton and henceforth we treat X as a function.

threshold $\widetilde{s}$), strictly increasing and onto (see Costinot and Vogel, 2010). These properties are illustrated in Figure 1.

Condition (4) implies that every worker type is assigned to the task in which its marginal value product is maximized. Thus, we have

$$\log w_s = \log p_{X_s} + \log \psi_{s,X_s} = \max_x \{\log p_x + \log \psi_{s,x}\} \ .$$

An envelope argument then yields the differential equation

$$(\log w_s)' = \frac{\partial \log \psi_{s,X_s}}{\partial s} \quad \forall s \neq \widetilde{s}, \tag{10}$$

which we can think of as determining wages given assignment and a boundary condition (where $(\log w_s)'$ denotes the derivative of the function $\log w_s$ with respect to s). When Assumptions 4 and 5 hold and automation is interior, the boundary condition is provided by the requirement that in both tasks $\underline{x}$ and $\overline{x}$, production must be equally costly with capital and skill $\widetilde{s}$:

$$\frac{w_{\widetilde{s}}}{\psi_{\widetilde{s},\underline{x}}} = \frac{1/q}{\psi_{k,\underline{x}}} \quad \text{and} \quad \frac{w_{\widetilde{s}}}{\psi_{\widetilde{s},\overline{x}}} = \frac{1/q}{\psi_{k,\overline{x}}}.$$

When automation is low-skill, we have $\underline{x} = 0$ and $\widetilde{s} = 0$ and the first equality becomes an inequality—it must be weakly more costly to produce task $\underline{x} = 0$ with skill $\widetilde{s} = 0$ than with capital—while the second equality provides the relevant boundary condition. So, taking logs, we have

$$\log w_{\widetilde{s}} = \log \psi_{\widetilde{s},\overline{x}} - \log \psi_{k,\overline{x}} - \log q \geq \log \psi_{\widetilde{s},\underline{x}} - \log \psi_{k,\underline{x}} - \log q, \tag{11}$$

with equality when automation is interior and with $\widetilde{s} = \underline{x} = 0$ when automation is low-skill. Note also that, when automation is interior, the wage function characterized by (10) and (11) has a kink point at $\widetilde{s}$, where the assignment function jumps upwards.

Intuitively, equation (10) ensures that all workers find it optimal to sort into the tasks assigned to them. This requires that the marginal return to skill at any level s is given by the marginal productivity gain in the task assigned to s, X_s.

Next, we can combine the equilibrium conditions for wages (4), task prices (5), and task production (2) to obtain an expression for inverse labor demand,

$$w_s = Y^{\frac{1}{\lambda}} A_{X_s}^{\frac{\lambda-1}{\lambda}} \psi_{s,X_s}^{\frac{\lambda-1}{\lambda}} L_{X_s}^{-\frac{1}{\lambda}},$$

where L_{X_s} is the marginal density of labor over tasks. A change of variable allows us to express this density in terms of the density of labor over skills, $X'_s L_{X_s} = l_s$, for $s \neq \widetilde{s}$. Using this and rearranging, we obtain the labor demand curve

$$\frac{l_s}{X'_s} = \frac{Y A_{X_s}^{\lambda-1} \psi_{s,X_s}^{\lambda-1}}{w_s^{\lambda}} \quad \forall s \neq \widetilde{s}. \tag{12}$$

The labor demand curve here takes the form of a differential equation for the assignment function, given wages. If automation is interior, the assignment function has two branches, one on $[0, \widetilde{s})$ and one on $(\widetilde{s}, 1]$. If automation is low-skill instead, only the upper branch exists. For the lower branch, the boundary condition is

$$\lim_{s \nearrow \widetilde{s}} X_s = \underline{x} \tag{13}$$

whereas for the upper branch, the boundary condition is given by

$$\lim_{s \searrow \widetilde{s}} X_s = \overline{x}\ . \tag{14}$$

Intuitively, if labor demand in task X_s is high (e.g., because aggregate output is high or the wage of skill s low), equation (12) requires that the density of labor supplied to task X_s is high as well. This is achieved if the slope of the assignment function X'_s is small, which means that more workers are squeezed into a few tasks in the neighborhood of X_s.

Overall, we have a two-dimensional system of differential equations (and boundary conditions) for wages and assignment. This system fully characterizes equilibrium together with the production function (1), the capital allocation rule

$$K = \text{argmax} \left\{ Y - \frac{\overline{K}}{q} \right\}\ ,$$

and the requirement that any task is assigned to some production factor. If automation is interior, this requires $X_0 = 0$ and $X_1 = 1$. If automation is low-skill, only $X_1 = 1$ is required.

Our characterization displays the two channels via which automation, as induced by a decline in the price of capital, affects wages and assignment. The first is a *displacement effect* as in Acemoglu and Restrepo (2022): a decline in the price of capital (an increase in q) reduces the boundary condition for wages (11) and hence, given assignment, the wage of worker type $\widetilde{s}$. This implies, in particular, that workers directly competing with capital must either relocate to other tasks or accept a wage decline in proportion to the reduction of the price of capital. The second is a *productivity effect*, driven by the fact that a lower price of capital raises aggregate output Y. From equation (12), the productivity effect raises labor demand in all tasks proportionately and, for a given assignment, wages for all skill levels rise proportionately as well.

Finally, we can derive a simple expression for the share of capital in national income, which will be useful when discussing the TFP effects of automation. Combining the task production function

(2), task prices (5), and equation (6) for the marginal product of capital, we obtain

$$\frac{1}{q} = Y^{\frac{1}{\lambda}} A_x^{\frac{\lambda-1}{\lambda}} \psi_{k,x}^{\frac{\lambda-1}{\lambda}} K_x^{-\frac{1}{\lambda}} .$$

Then, solving for capital, integrating over $[\underline{x}, \overline{x}]$ and dividing by qY yields the share of capital in gross output as

$$\alpha_k = \frac{\overline{K}/q}{Y} = \Gamma_k q^{\lambda-1} , \tag{15}$$

where

$$\Gamma_k = \int_{\underline{x}}^{\overline{x}} A_x^{\lambda-1} \psi_{k,x}^{\lambda-1} \, dx$$

is the task share of capital, a productivity-weighted measure for the set of automated tasks. Equation (15) shows that a decline in the price of capital has two distinct effects on the capital share: a capital deepening effect (as captured by $q^{\lambda-1}$) the sign of which depends on whether tasks are complements or substitutes; and the effect of the expansion of the task share of capital, the counterpart of the displacement effect on wages discussed above.

4 Local Effects of Automation

In this section, we assume that Assumptions 1, 2, 3, 4 and 5 hold so that automation is interior, and study the implications of a small decline in the price of capital goods (an increase in q), which will expand the set of automated tasks. Our main results characterize the polarization and inequality consequences of automation.

4.1 Employment Polarization

Proposition 5 ***(Automation and employment polarization)*** *Suppose Assumptions 1, 2, 3, 4 and 5 hold and consider a small increase in the productivity of capital* $d \log q > 0$. *Then,*

$$d\underline{x} < 0 \quad \textit{and} \quad d\overline{x} > 0$$

(automation expands in both directions) and

$$dx_s < 0 \textit{ for all } s \in (0, \widetilde{s}) \quad \textit{and} \quad dx_s > 0 \textit{ for all } s \in (\widetilde{s}, 1)$$

(the assignment function shifts down below the set of automated tasks and shifts up above the set).

Moreover, if $\lambda \geq 1$, *the labor share always decreases. If* $\lambda < 1$, *there exists a threshold for capital productivity* $\hat{q} > q_0$ *such that the labor share decreases if* $q \in (q_0, \hat{q})$.

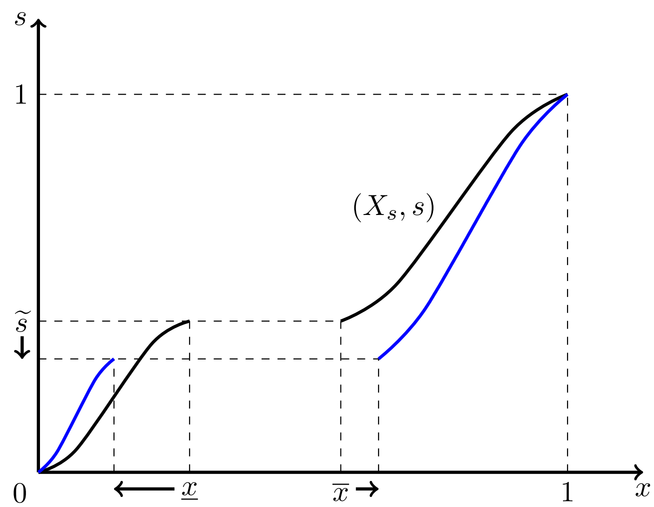

Figure 2: *Employment Polarization.* In response to a local increase in capital productivity, the set of automated tasks expands in both directions and workers move towards the extremes of the task distribution, here illustrated for the case with $d\widetilde{s} < 0$.

The first part of this proposition establishes that a decline in the price of capital goods (or an increase in the productivity of capital) always expands the set of automated tasks on both sides, and relatedly, it shifts the assignment of workers further towards the two extremes of the task distribution, as shown in Figure 2. This result thus implies that the employment polarization pattern documented in Autor, Levy and Murnane (2003), Goos, Manning and Salomons (2009) and Acemoglu and Autor (2011) always applies so long as we consider a small increase in the set of automated tasks, starting from interior automation.[13]

The second part provides conditions under which the labor share declines. There are two channels via which automation affects the labor share (see also our discussion of equation (15) for the capital share). First, the expansion of the set of automated tasks, established in the first part of the proposition, always decreases the labor share. Second, the productivity gain in tasks that are already automated decreases the labor share when tasks are substitutes ($\lambda \geq 1$) but raises it when tasks are complements ($\lambda < 1$). Hence, when tasks are complements, the total effect is ambiguous. Yet, even in this case, the proposition shows that the expansion of the set of automated tasks dominates and the labor share declines in the initial stages of automation (when the productivity of capital is small enough).

[13]It is also straightforward to show that if there were technological constraints on what tasks could be automated (as in Acemoglu and Restrepo, 2018a) and these dictated that only tasks in the set $[\underline{x}, \overline{x}]$ could be automated, and we consider an expansion of the set with $d\underline{x} < 0$ and $d\overline{x} > 0$, then the same employment polarization result would hold.

4.2 Wage Polarization

The next proposition gives one of our most important results:

Proposition 6 ***(Automation and wage polarization)*** *Suppose Assumptions 1, 2, 3, 4 and 5 hold and consider a small increase in the productivity of capital $d \log q > 0$. Then, there is wage polarization in the sense that skill premia increase above the threshold task $\widetilde{s}$ and decrease below this threshold. Or equivalently,*

$$d \log w_s > d \log w_{s'} \text{ for all } s < s' \in (0, \widetilde{s}] \quad \text{and} \quad d \log w_s < d \log w_{s'} \text{ for all } s' > s \in [\widetilde{s}, 1).$$

The wage polarization result contained in Proposition 6 is similar to the finding in Acemoglu and Autor (2011), but as briefly discussed in the Introduction, in that paper, this result was a direct consequence of the fact that there were three types of workers, and automation was assumed to affect the middle type. Here, we see that wage polarization reflects much more general forces and applies throughout the distribution, and regardless of exactly where automation is taking place (provided that we start from interior automation). We are not aware of other results of this sort in the literature.

The economics of this result is again related to the competing displacement and productivity effects. The former directly harms the earnings of workers who used to perform the previously-automated tasks, while the latter benefits all workers symmetrically. Notably, the displacement effect does not just impact directly-affected workers (whose previous tasks are taken over by capital), but *all* workers, because of the general pattern of substitutability between worker types. These "ripple effects" are also present in Acemoglu and Restrepo (2022), but in our setting, they can be shown to depend only on the distance of a skill group to the threshold type $\widetilde{s}$. This, combined with the symmetric productivity effects, immediately yields the result in Proposition 6.

Proposition 6 shows how skill premia change, generating a pattern of wage polarization. Other important questions are whether the real wage *level* of some worker types will decline following the expansion in automation and whether the top or the bottom of the wage distribution will be more affected. The next proposition answers these questions.

Proposition 7 ***(Automation and wage levels)*** *Suppose Assumptions 1, 2, 3, 4 and 5 hold and consider a small increase in the productivity of capital $d \log q > 0$.*

1. *The average wage in the economy always increases.*

2. *There exists a threshold for capital productivity $\hat{q} > q_0$ such that if $q \in (q_0, \hat{q})$, then for some $\delta_1, \delta_2 > 0$, we have $d \log w_s < 0$ for all $s \in (\widetilde{s} - \delta_1, \widetilde{s} + \delta_2)$.*

3. *Suppose that there exists s' such that $\psi_{s',x}/\psi_{k,x}$ is constant in x. Then for some $\delta_2, \delta_2 > 0$, we have $d \log w_s < 0$ for all $s \in (s' - \delta_1, s' + \delta_2)$.*

4. *Suppose that $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$. Then, there exists a threshold for capital productivity $\widetilde{q} < q_m$ (where q_m is the upper bound imposed by Assumption 5) such that if $q \in (\widetilde{q}, q_m)$, the inequality between the top and the bottom of the skill space increases. That is,*

$$d \log w_0 < d \log w_1 \ .$$

The first part follows immediately from Euler's theorem given constant returns to scale, the constant price of capital (in terms of the final good), and competitive factor markets. In particular, we always have that

$$\int_0^1 \frac{\alpha_s}{1-\alpha_k} d \log w_s \, ds = d \log c = \frac{\alpha_k}{1-\alpha_k} > 0,$$

where α_s and α_k are the income shares of skill s and capital, respectively, and so the left-most term is the change in the average wage in the economy. Hence, the average wage always increases following an expansion in automation.[14]

The second part is also intuitive. When the initial level of capital productivity is low, the set of automated tasks is small. This implies that a marginal increase in q generates only a small productivity effect, and the most affected worker type, $\widetilde{s}$, necessarily experiences a real wage decline (due to the displacement effect). In fact, the decline in the real wage extends to a set of workers around $\widetilde{s}$, because the wage level effects are continuous in skill. This result highlights the central role of the magnitude of the productivity effect, which we characterize in the next subsection.

The third part provides a refinement of Norbert Wiener's conjecture discussed in the Introduction. Namely, if a worker type has a productivity profile very similar to that of capital, then Wiener's intuition that production using capital will cause the impoverishment of this worker type is correct.[15] However, in contrast to Wiener's general statement, this is not true for all workers,

[14] As discussed in footnote 6, this result is itself a consequence of some of the special assumptions that are typically imposed in these types of models, including ours, and can also be relaxed. Since this is not our main focus, we do not explore this issue further in this paper.

[15] Strictly speaking, Proposition 7 requires the worker type to have exactly the same productivity profile as capital. In Proposition 9, we extend this to worker types whose productivity profile is sufficiently similar—but not exactly equal—to that of capital. This extension is easier to formalize when studying global changes in capital productivity, so we defer it to Proposition 9.

but only for specific worker types. Indeed, we know from the first part that average wages (and hence the wages of some skill levels) have to increase.

Finally, the fourth part shows that automation increases the inequality between high- and low-skill workers, at least if the productivity of capital is high enough. The intuition for this result is as follows. If $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$, then automation will proceed in an unbalanced way, approaching the bottom of the task space as q grows large (see next section). As automation tilts towards the bottom, so do its displacement effects on wages, reducing wages at the bottom relative to the top of the skill space.

4.3 Productivity

The next proposition provides a characterization for the productivity effects of automation, using a second-order Taylor expansion.

Proposition 8 ***(Productivity effects)*** *Suppose Assumptions 1, 2, 3, 4 and 5 hold and consider a small increase in the productivity of capital* $\Delta \log q > 0$. *Then we have*

$$\Delta \log TFP \approx \frac{\alpha_k}{1-\alpha_k} \Delta \log q + \frac{1}{1-\alpha_k} \left[\lambda - 1 + \frac{\partial \log \Gamma_k}{\partial \overline{x}} \frac{d\overline{x}}{d \log q} - \frac{\partial \log \Gamma_k}{\partial \underline{x}} \frac{d\underline{x}}{d \log q} \right] (\Delta \log q)^2 \ .$$

The first term in the approximation is an immediate consequence of Hulten's theorem. The first-order effect of an increase in capital productivity is equal to capital's share in net output. It shows clearly that the TFP gain will be smaller when α_k is small, thus confirming the result in Proposition 7.2: when the productivity of capital is low to start with, or equivalently only a few tasks are initially automated, then the productivity effect is small (which is the reason why negative wage effects are more likely in this case).

The second term captures two distinct but related effects. First, the expansion of the task share of capital Γ_k tends to make the TFP effects of an increase in capital productivity convex: an increase in capital productivity expands the set of automated tasks and thus generates a bigger base on which additional productivity gains can be obtained. Second, when holding the set of automated tasks fixed, an increase in capital productivity increases or decreases the share of capital in national income, depending on whether tasks are complements ($\lambda < 1$) or substitutes ($\lambda > 1$). If tasks are complements, an increase in capital productivity leads to an increase in the labor share and a decrease in the capital share. This counters the effect from the expansion of the set of automated tasks and the implied convexity of TFP effects. In contrast, if tasks are substitutes, an increase in capital productivity leads to a decrease in the labor share and an increase in the capital share, amplifying convexity.

5 Global Effects of Automation

In this section, we consider non-infinitesimal (potentially large) changes in the productivity of capital. We distinguish two cases. In the first, studied in the next subsection, after this change, automation still remains interior. In the second, studied in the subsequent subsection, we transition from interior to low-skill automation. Finally, we also discuss additional comparative statics with respect to labor supply changes.

5.1 Non-Local Changes with Interior Automation

Proposition 9 ***(Polarization with large changes in automation)*** *Suppose Assumptions 1, 2, 3, 4 and 5 hold and consider a potentially large increase in the productivity of capital $\Delta \log q > 0$, which still satisfies Assumption 5. Let $\widetilde{s}' = \widetilde{s} + \Delta \widetilde{s} \in (0, 1)$ be the new threshold skill level.*

1. *Then, automation expands in both directions and causes employment polarization. That is,*

$$\Delta \underline{x} < 0 \quad \textit{and} \quad \Delta \overline{x} > 0,$$

and

$$\Delta x_s < 0 \textit{ for all } s \in (0, \widetilde{s}') \quad \textit{and} \quad \Delta x_s > 0 \textit{ for all } s \in (\widetilde{s}', 1).$$

2. *There is wage polarization in the sense that skill premia increase above the threshold skill $\widetilde{s}'$ and decrease below this threshold. Or equivalently,*

$$\Delta \log w_s > \Delta \log w_{s'} \textit{ for all } s < s' \in (0, \widetilde{s}'] \quad \textit{and} \quad \Delta \log w_s < \Delta \log w_{s'} \textit{ for all } s' > s \in [\widetilde{s}', 1).$$

3. *The average wage always increases, and moreover, there exists a threshold for capital productivity $\hat{q} > q_0$ such that if $q + \Delta q \in (q_0, \hat{q})$, then for some $\delta_1, \delta_2 > 0$, we have $d \log w_s < 0$ for all $s \in [\widetilde{s} - \delta_1, \widetilde{s} + \delta_2]$.*

4. *Let $\gamma_{s,k}^{max} = \max_x \{\log \psi_{s,x} - \log \psi_{k,x}\} - \min_x \{\log \psi_{s,x} - \log \psi_{k,x}\}$ for some s. Then,*

$$\Delta \log w_s \leq \gamma_{s,k}^{max} - \Delta \log q.$$

In particular, if $\gamma_{s,k}^{max} < \epsilon$, then every $\Delta \log q > \epsilon$ will reduce the wage of workers with skill level s.

5. *Suppose that $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$. Then, if $q + \Delta q$ is sufficiently close to q_m, the inequality between the top and the bottom of the skill space increases, i.e.,*

$$\Delta \log w_0 < \Delta \log w_1.$$

In summary, this proposition establishes that our main employment and wage polarization results do not depend on whether we consider small or large changes in capital productivity—provided that automation starts out and remains interior. Moreover, as before, when we initially have relatively few tasks automated (or the productivity of capital is still relatively low), an expansion in automation hurts workers around the skill threshold $\widetilde{s}$. Our refinement of Wiener's conjecture also extends to this case: the wages of worker types with productivity profiles sufficiently similar to capital's will decline (but, as before, wages cannot decline for all worker types). Finally, under the same conditions as in the local analysis, the impact of automation on the wage distribution is asymmetric: inequality increases between high-skill and low-skill workers.

We will next see that, in contrast to this case, when automation ceases to be interior, we obtain very different comparative statics.

5.2 Transition to Low-Skill Automation

We now consider a non-local change in capital productivity that induces a transition from interior to low-skill automation.

Proposition 10 ***(Transition to low-skill automation)*** *Suppose Assumptions 1, 2, 3 and 4 hold. Suppose also that*

$$\frac{\psi_{0,0}}{\psi_{k,0}} < \frac{\psi_{0,1}}{\psi_{k,1}} \tag{16}$$

and Assumption 5 holds initially. Now consider a potentially large increase in the productivity of capital $\Delta \log q > 0$ that violates Assumption 5. Then:

1. *Automation transitions from interior to low-skill, so all low-complexity tasks are taken over by capital. That is, $\Delta \underline{x} = -\underline{x}$.*

2. *This transition does not induce employment polarization. Instead, the assignment function shifts up everywhere, $\Delta X_s > 0$ for all $s < 1$.*

3. *It does not induce wage polarization either. Instead, skill premia increase over the entire skill space. That is,*

$$\Delta \log w_s < \Delta \log w_{s'} \quad \textit{for all } s < s'.$$

Next, suppose Assumptions 1, 2, 3, 4 and condition (16) *hold but we start from $q \geq q_m$ already, so that Assumption 5 is violated. Then, a further increase in the productivity of capital shifts up the assignment function everywhere and skill premia increase over the entire skill space (i.e., there is no longer any employment or wage polarization).*

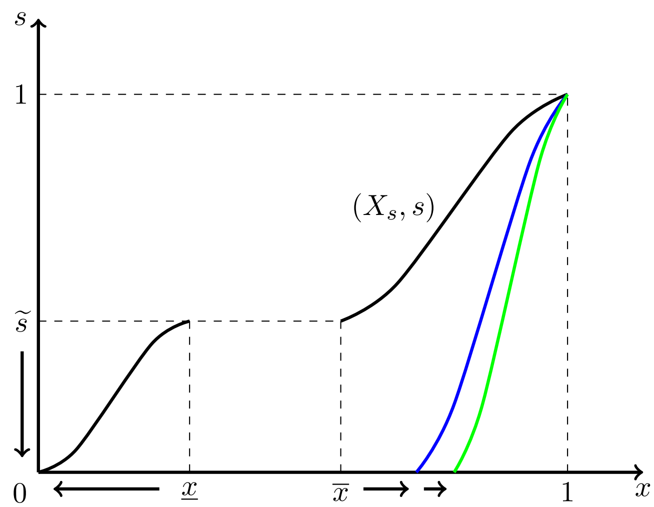

Figure 3: *Transition to low-skill automation.* If capital productivity grows large enough, automation becomes low-skill. A further increase in capital productivity then pushes all workers towards the upper end of the task distribution.

The most important result in this proposition is that for a sufficiently large capital productivity, automation transitions from interior to low-skill and this transition changes the wage effects of automation qualitatively. In contrast to the wage polarization pattern we have seen so far, once automation becomes low-skill it induces monotone increases in wage inequality—whereby automation impacts the lowest-skill workers most negatively. In this case, the employment polarization effects of automation vanish as well: further automation now pushes all workers towards more complex tasks. Figure 3 diagrammatically illustrates this transition.

Note finally that Proposition 10 is stated under condition (16), which ensures that the ratio of capital productivity to the productivity of the least skilled workers is greater at the lowest complexity tasks than at the highest complexity tasks. To understand the importance of this assumption, note that as q increases towards q_∞, all tasks will asymptotically become automated. This pushes all workers towards those tasks in which they have global comparative advantage relative to capital, i.e., in which their productivity relative to capital is maximized. If condition (16) holds, even the least skilled workers have global comparative advantage relative to capital in the most complex tasks, so they are ultimately forced to relocate towards these tasks.[16] This induces a transition from interior to low-skill automation and increases all skill premia.[17]

[16]If this process pushed low-skill workers' wages below their reservation wages, then rather than relocating towards more complex tasks, they would leave the labor force, further speeding up the transition to low-skill automation.

[17]If condition (16) is not satisfied, automation remains interior indefinitely. To see this, note that we have already imposed in Assumption 3.2 that the most skilled workers have global comparative advantage relative to capital in the most complex tasks, so they will never be displaced from these tasks. In contrast, if both condition (16) and

5.3 Implications of Labor Supply Changes

Finally, we consider the implications of changes in the labor supply profile. Although this could have been studied for local changes, it is more convenient to discuss these comparative statics in the case of global changes. The main comparative static is given in the next proposition.

Proposition 11 ***(Labor Supply Changes and Automation)*** *Suppose Assumptions 1, 2, 3, 4 and condition (16) hold and Assumption 5 holds under the initial labor supply l. Now consider a change in labor supply such that $\Delta \log l_s < \Delta \log l_{s'}$ for all $s < s'$ (an increase in the relative supply of more skilled workers). Then, the threshold capital productivity q_m at which automation transitions from interior to low-skill declines, $\Delta q_m < 0$.*

Moreover, suppose that $q_m + \Delta q_m < q$ so that Assumption 5 is violated after the labor supply change and automation transitions from interior to low-skill. Then, skill premia increase in the bottom part of the wage distribution and decrease in the upper part. Specifically, there exists $\hat{s} \in (\widetilde{s}, 1)$ (where $\widetilde{s}$ is the threshold skill before the labor supply change) such that

$$\Delta \log w_s < \Delta \log w_{s'} \text{ for all } s < s' \in (0, \hat{s}] \quad \text{and} \quad \Delta \log w_s \geq \Delta \log w_{s'} \text{ for all } s' > s \in [\hat{s}, 1).$$

There are two important results contained in this proposition. First, relative scarcity of low-skill workers accelerates the transition to low-skill automation. The intuition is provided by inequality (8) and is related to our results about the effects of a minimum wage in Proposition 4: a lower relative supply of low-skill workers raises their wage and makes it more profitable to automate the tasks previously performed by the low-skilled. Put differently, automation is initially interior in Proposition 11 because low-skill workers have local comparative advantage relative to capital in less complex tasks (Assumption 4), *but also* because they are relatively abundant and hence relatively cheap. Lower wages make automating low-complexity tasks less profitable and lead to interior automation.

A second important result in Proposition 11 is a type of upward-sloping relative demand for skills. Given a fixed assignment of workers and capital to tasks, the monotone increase in the supply of skills would have reduced skill premia. However, the response of equilibrium assignment qualitatively changes this pattern. Low-skill automation becomes more likely and this reduces the relative wages of low-skill workers and raises skill premia at the bottom. Other instances of greater relative supply of skills leading to higher skill premia are present in models of directed

Assumption 3.2 were violated, automation would transition from interior to high-skill at some point (whereby the most complex tasks would be taken over by capital).

technological change (because greater abundance of skilled workers encourages more skill-biased technological change, as in Acemoglu, 1998, 2007), and in models of search and matching (because with more skilled workers around, more employers make investments complementary to skilled workers and search for them, as in Acemoglu, 1999). In the model here, a similar outcome arises, even though the equilibrium is competitive and there is no endogenous innovation. Rather, this result is driven by the response of the equilibrium assignment of tasks between capital and labor.

6 Conclusion

There has been rapid automation of a range of tasks across the industrialized world over the last four decades. There is growing evidence that this automation has fueled both inequality and polarization—whereby middle-skilled workers have been displaced from their jobs and have experienced relative wage declines.

To develop a deeper understanding of the causes of polarization, this paper has built an assignment model of automation. In our model, each of a continuum of tasks of variable complexity is assigned to either capital or one of a continuum of labor skills. Our model generalizes existing assignment models, which typically impose global supermodularity conditions that ensure monotone matching between workers and tasks. In contrast, in our model with capital there is no global supermodularity.

We prove existence and essential uniqueness of competitive equilibria and characterize conditions under which automation is interior, meaning that it is tasks of intermediate complexity that are assigned to capital. Put simply, interior automation arises when the most skilled workers have a comparative advantage in the most complex tasks relative to capital and other labor, and because the wages of the least skilled workers are sufficiently low relative to their productivity and the effective cost of capital in low-complexity tasks. Highlighting the role of wages at the low-end of the wage distribution, we demonstrate that minimum wages and other sources of higher wages at the bottom make interior automation less likely.

We provide a series of local and global comparative statics, showing how further automation impacts wages and assignment patterns. Most importantly, when automation starts and remains interior, a lower cost of capital (or greater capital productivity) causes employment polarization: middle-skill workers are displaced from middle-complexity tasks and are pushed towards higher or lower parts of the complexity distribution. This type of automation also causes wage polarization: the skill premium monotonically increases above a skill threshold and monotonically declines below the same threshold. Moreover, automation tends to reduce the real wage of workers with comparative advantage profiles close to that of capital.

Our global comparative static results additionally establish that large enough increases in capital productivity ultimately induce a transition to low-skill automation, whereby the pattern of comparative statics changes qualitatively. In particular, after this transition to low-skill automation, further declines in the cost of capital no longer cause employment or wage polarization. Rather, they have a monotone effect on the skill premium.

Despite its richness, our framework is fairly tractable and opens the way to further analysis of the changing assignment patterns in modern labor markets. Fruitful areas of future inquiry include the following. First, automation has been going on together with a changing structure of tasks and an evolving distribution of skills over at least the last 250 years. This can be introduced into our framework by simultaneously expanding the range of tasks and skills, and would be an important area for future work. Second, the productivity of capital in various tasks should in principle change endogenously, responding to which tasks are being assigned to capital—or are likely to be assigned to capital in the future. This issue can be investigated in an extended version of our framework in which the direction of technological change and capital productivity across tasks are endogenized. Third, in practice multiple tasks may be assigned to a worker because either there are economies of scope or other types of task complementarities, and extending this class of models to "one-to-many" matching is another important area for further inquiry. Last but not least, the framework here can be used to further refine the empirical investigation of the relationship between automation and inequality, for example, by adding more structure and predictions to studies such as Acemoglu and Restrepo (2022).

A Proofs for Section 3: Characterization of Equilibrium and Interior Automation

A.1 Proof of Proposition 1: Existence and Uniqueness

Existence An equilibrium allocation maximizes net output subject to labor market clearing, given by

$$\int_0^1 L_{s,x} dx \leq l_s \quad \text{for all } s. \tag{17}$$

To prove existence, it is useful to split the problem of net output maximization into two steps. First, we fix the aggregate capital stock $\overline{K}$ and maximize gross output for given $\overline{K}$ and subject to (17). This is a problem of maximizing a continuous function over a compact set, such that a maximizer is guaranteed to exist. Let $F(\overline{K}, l)$ denote the maximal gross output for given $\overline{K}$ and labor supply l.

In the second step, we choose $\overline{K}$ to maximize net output $F(\overline{K}, l) - \overline{K}/q$. This is again a continuous problem, but $\overline{K}$ can be any positive real number, so we have to establish boundedness. For this, note that

$$\lim_{\overline{K} \to \infty} \frac{\partial F(\overline{K}, l)}{\partial \overline{K}} = \left(\int_0^1 A_x^{\lambda-1} \psi_{k,x}^{\lambda-1} \, dx \right)^{\frac{1}{\lambda-1}}.$$

Thus, Assumption 1 ensures that

$$\lim_{\overline{K} \to \infty} \frac{\partial F(\overline{K}, l)}{\partial \overline{K}} < \frac{1}{q}$$

such that net output is bounded and attains its maximum for finite $\overline{K}$.

Essential Uniqueness For essential uniqueness of equilibrium, note that net output is concave in the allocation and the set of feasible allocations is convex. This implies that, while the equilibrium allocation itself may not be unique, the Frechet derivative of net output is constant across all equilibrium allocations.[18] Hence, equilibrium wages are unique. The same argument applies to task prices when writing the maximization of net output as a maximization over task inputs, including task production functions as constraints, i.e.,

$$\max_{\{Y_x\}_{x=0}^1, L, K} \left[\int_0^1 (A_x Y_x)^{\frac{\lambda-1}{\lambda}} \, dx \right]^{\frac{\lambda}{\lambda-1}} - \frac{1}{q} \int_0^1 K_x \, dx$$

subject to task production (2) and labor market clearing (3).

[18] One way to see this is as follows. The set of maximizers of a concave function on a convex set is a face of the hypograph of the function. Thus, there exists a supporting hyperplane of the hypograph that contains the entire set of maximizers. Together with differentiability, this immediately implies that the derivative of the function is constant on the set of maximizers.

Uniqueness Our proof of Proposition 2 below shows that under Assumptions 1, 2 and 3, the labor allocation L and the set of tasks performed by capital X_k are uniquely determined given wages. Moreover, given prices, X_k and the labor allocation, choosing the output-maximizing capital allocation K is a strictly concave problem with a unique solution.

Thus, given wages and task prices, the equilibrium allocation is determined uniquely. Since equilibrium wages and task prices are unique, the equilibrium allocation is unique as well.

A.2 Proof of Proposition 2: Convexity of Assignment

Monotonicity Monotonicity of the labor allocation under comparative advantage assumptions is a standard result. One way to prove it, which is useful for our argument in the next step below, is presented here. Let

$$S_x^{min} = \underset{s}{\operatorname{argmin}} \{\log w_s - \log \psi_{s,x}\}$$

be the set of skills that produce task x at minimal cost. By Assumption 2, $\log w_s - \log \psi_{s,x}$ is strictly submodular, so Topkis' monotonicity theorem (Topkis 1998) implies that if $s \in S_x^{min}$, $s' \in S_{x'}^{min}$ and $x > x'$, then $s \geq s'$. Moreover, if for some x there exist $s, s' \in S_x^{min}$ with $s > s'$, then all skill levels in (s', s) can only be assigned to x. This creates a mass point in the density of labor over tasks, such that $p_x = 0$, contradicting condition (4). Hence, S_x^{min} is a singleton for all x. Inverting this correspondence, we obtain $X_s \subseteq \{x | s \in S_x^{min}\}$. Finally, from the properties of S_x^{min} it follows immediately that if $x \in X_s$, $x' \in X_{s'}$ and $s > s'$, then $x > x'$.

Convexity We start with the following lemma which will be useful to establish properties of X_k throughout the paper.

Lemma 1 *Suppose Assumptions 1, 2 and 3.1 hold and let*

$$\omega_x = \min_s \left\{ \frac{w_s \psi_{k,x}}{\psi_{s,x}} \right\}$$

be the minimal effective unit cost of producing the amount $\psi_{k,x}$ of task x with labor. Then, the set of automated tasks is equal to the upper level set of ω at level $1/q$,

$$X_k = \{x | \omega_x \geq 1/q\}.$$

Proof. The unit cost of producing the amount $\psi_{k,x}$ of task x with capital is $1/q$. Hence on X_k, we must have $1/q \leq \omega_x$. Moreover, if $1/q < \omega_x$ at some x, then x must be in X_k. Hence,

$$\{x | \omega_x > 1/q\} \subseteq X_k \subseteq \{x | \omega_x \geq 1/q\}.$$

Now suppose that X_k and $\{x|\omega_x \geq 1/q\}$ differ by a set of strictly positive measure. Then, since the labor endowment has no mass points, a strictly positive measure of skills must be assigned to a subset of $\{x|\omega_x = 1/q\}$. In particular, there must exist skill levels $s_1 < s_2 < s_3$ assigned to tasks $x_1 < x_2 < x_3$ in $\{x|\omega_x = 1/q\}$. Moreover, since the cost-minimizing skill S_x^{min} is unique for every task (see first step of the proof), we must have

$$\frac{w_{s_2}\psi_{k,x_1}}{\psi_{s_2,x_1}} > \frac{w_{s_1}\psi_{k,x_1}}{\psi_{s_1,x_1}} = \frac{w_{s_2}\psi_{k,x_2}}{\psi_{s_2,x_2}} = \frac{w_{s_3}\psi_{k,x_3}}{\psi_{s_3,x_3}} < \frac{w_{s_2}\psi_{k,x_3}}{\psi_{s_2,x_3}}.$$

But this string of relations contradicts the quasi-concavity of $\psi_{k,x}/\psi_{s_2,x}$. Hence, the difference between X_k and $\{x|\omega_x \geq 1/q\}$ must be of measure zero and we can set $X_k = \{x|\omega_x \geq 1/q\}$ without loss of generality. ■

The "if" part of Proposition 2.2 follows from Lemma 1. By Assumption 3.1, $w_s\psi_{k,x}/\psi_{s,x}$ is quasi-concave in x for all s. Thus, ω_x is the lower envelope of quasi-concave functions and as such it is quasi-concave itself. Hence, its upper level sets are convex and so is X_k.

Next, consider the "only if" part. We will prove that if $\psi_{k,x}/\psi_{s,x}$ is not quasi-concave in x for some s, then there exists a labor endowment l and capital productivity q such that X_k is not convex. For this it turns out useful to rewrite labor market clearing as

$$\int_0^s \int_0^1 L_{s',x}\, dx\, ds' = H_s \quad \text{for all } s,$$

where H_s is the cumulative distribution function of labor endowments. This specification allows to embed mass points as jumps in H_s.

Suppose now that $\psi_{k,x}/\psi_{s',x}$ is not quasi-concave in x for s' and consider the case where only skill s' is supplied,

$$H_s = \mathbb{I}_{s>s'} = \begin{cases} 0 & \text{if } s < s' \\ 1 & \text{if } s \geq s'. \end{cases}$$

Since $\psi_{k,x}/\psi_{s,x}$ is not quasi-concave, there exist $x_1 < x_2 < x_3$ such that

$$\frac{w_{s'}\psi_{k,x_1}}{\psi_{s',x_1}} > \frac{w_{s'}\psi_{k,x_2}}{\psi_{s',x_2}} < \frac{w_{s'}\psi_{k,x_3}}{\psi_{s',x_3}}.$$

By Euler's theorem, $w_{s'}$ equals net output. Next note that net output is continuous in the allocation and in q, and hence Berge's maximum theorem applies and implies that equilibrium net output is continuous in q (and equilibrium allocations are upper hemicontinuous in q). Moreover, net output is also increasing in q. Thus, the wage $w_{s'}$ is continuously increasing in q and there exists a value for q such that

$$\frac{w_{s'}\psi_{k,x_1}}{\psi_{s',x_1}}, \frac{w_{s'}\psi_{k,x_3}}{\psi_{s',x_3}} > \frac{1}{q} > \frac{w_{s'}\psi_{k,x_2}}{\psi_{s',x_2}},$$

which implies that X_k cannot be convex.

It remains to extend the result to labor endowments without mass points, which is a simple continuity argument. Net output is continuous in allocations while the set of feasible allocations is continuous in the endowment cumulative density function H. Thus by the maximum theorem, the set of equilibrium allocations is upper hemicontinuous in H. Since there is no equilibrium allocation generating a convex X_k under the endowment function $\mathbb{I}_{s>s'}$ considered above, we can construct a sequence of differentiable endowment functions with strictly positive derivative, $\{H^{(n)}\}_{n\in\mathbb{N}}$, that converges to $\mathbb{I}_{s>s'}$; for sufficiently large n, the set of automated tasks cannot be convex.

No High-Skill Automation Suppose $1 \in X_k$ (to derive a contradiction). Then, it must be cheaper to produce task $x = 1$ with capital than with labor, $w_s\psi_{k,1}/\psi_{s,1} \geq 1/q$ for all s. Moreover, the first step of the proof ("monotonicity") implies that among all labor types, task $x = 1$ can be produced at the lowest cost using skill $s = 1$, $S_1^{min} = \{1\}$. Thus, $\frac{w_s\psi_{k,1}}{\psi_{s,1}} > \frac{1}{q}$ for all $s < 1$. Together with Assumption 3.2, this implies that there exists an $\epsilon > 0$ such that $w_s\psi_{k,0}/\psi_{s,0} > 1/q$ for all $s \in (1-\epsilon, 1]$, i.e., it is cheaper to produce the least complex task with capital than with the most skilled workers.

Hence, for every $s \in (1-\epsilon, 1)$, we have established that s is strictly more expensive than capital in both the most and the least complex task. Quasi-concavity of $\psi_{k,x}/\psi_{s,x}$ (Assumption 3.1) requires that this extends to all tasks:

$$\frac{w_s\psi_{k,x}}{\psi_{s,x}} > \frac{1}{q} \quad \text{for all } s \in (1-\epsilon, 1).$$

But this implies that skill levels $s \in (1-\epsilon, 1)$ cannot be assigned to any task in equilibrium, which is clearly incompatible with an equilibrium allocation maximizing (finite) net output. Hence, we must have $1 \notin X_k$: the most complex task is not automated.

A.3 Characterization of Capital Productivity Thresholds

Here we provide a characterization of the two productivity thresholds q_0 and q_m.

No Automation Threshold For q_0, suppose that we restrict capital usage to zero in all tasks and consider the output-maximizing allocation of labor. Let w^0 be the resulting wage vector such that $\omega_x^0 = \min_s\{w_s^0\psi_{k,x}/\psi_{s,x}\}$ is the corresponding minimum labor cost function. Then, we can define

$$\frac{1}{q_0} = \max_x \omega_x^0,$$

as the prohibitive cost of capital. Capital will be used in equilibrium if and only if $q \geq q_0$.

Interior Automation Threshold To characterize q_m suppose that Assumption 4 holds, such that $\psi_{k,x}/\psi_{0,x}$ is strictly increasing in x on a neighborhood of $x = 0$. Then, we define a threshold task $\overline{x}_m$ as the smallest $x \in (0, 1)$ such that

$$\frac{\psi_{k,0}}{\psi_{0,0}} = \frac{\psi_{k,x}}{\psi_{0,x}}.$$

That is, the productivity ratio between capital and the least skilled workers is the same at task $\overline{x}_m$ and task 0. Such an $\overline{x}_m$ exists if and only if

$$\frac{\psi_{0,0}}{\psi_{k,0}} < \frac{\psi_{0,1}}{\psi_{k,1}} ,$$

which is condition (16) imposed in our Proposition 10. If condition (16) does not hold, set q_m equal to the upper bound q_∞.

Now, suppose that condition (16) holds and we restrict capital to tasks below $\overline{x}_m$ and labor to tasks above $\overline{x}_m$. Then we choose the allocation that maximizes net output subject to these restrictions. Let w_s^m be the resulting wage function. Note that w_s^m is strictly increasing in q, allowing us to define q_m as the unique value of q that solves

$$\frac{1}{q_m} = \frac{w_0^m(q_m)\psi_{k,\overline{x}_m}}{\psi_{0,\overline{x}_m}},$$

where we wrote $w_0^m(q)$ to emphasize the dependence of w_0^m on q. Intuitively, this condition equates the costs of producing task $\overline{x}_m$ with capital and with the least skilled workers.

Finally, note that if $q = q_m$, the restriction of capital to tasks below $\overline{x}_m$ and labor to tasks above $\overline{x}_m$ is not binding, and in this case we have $X_k = [0, \overline{x}_m]$.

A.4 Proof of Proposition 3: Interior Automation

Interior Automation We start by showing that under Assumptions 1-5, automation is interior. We already know from Proposition 2 and our definition of q_0 in Appendix A.3 that under Assumptions 1-3 and if $q > q_0$, automation is interior or low-skill. In particular, the set of automated tasks takes the form $[\underline{x}, \overline{x}]$ with $\underline{x} < \overline{x} < 1$. It remains to show that $\underline{x} > 0$ if $q < q_m$.

In Appendix A.3 we have shown that Assumption 4 allows us to define q_m such that $X_k = [0, \overline{x}_m]$ if $q = q_m$. Moreover, we show in Propositions 5 and 10 that the set of automated tasks is strictly increasing in q on $q \geq q_0$, i.e., $X_k(q) \supset X_k(q')$ for all $q > q' \geq q_0$.[19] Now suppose that $X_k = [0, \overline{x}]$ for some $q \in (q_0, q_m)$. Since X_k is strictly increasing in q, we must have $\overline{x} < \overline{x}_m$ in this case. Hence,

$$\frac{1}{q} \leq \frac{w_0\psi_{k,0}}{\psi_{0,0}} < \frac{w_0\psi_{k,x}}{\psi_{0,x}} \quad \text{for all } x \in (0, \overline{x}],$$

[19]Proposition 5 shows this when starting from q such that automation is interior while Proposition 10 considers the case where the initial q is such that automation is low skill.

where the second inequality comes from the definition of $\overline{x}_m$ and the fact that $\overline{x} < \overline{x}_m$. This implies that skill 0 cannot be allocated to tasks $(0, \overline{x} + \epsilon)$ for some $\epsilon > 0$, and the same holds for all skills in some neighborhood of 0 by continuity of wages and productivities. But then, since the labor allocation is monotone (Proposition 2.1), no skill type can be allocated to $(\overline{x}, \overline{x} + \epsilon)$. Since these tasks are also not performed by capital, they would not be performed at all, which is incompatible with equilibrium. Hence, we must have $\underline{x} > 0$ if $q \in (q_0, q_m)$.

Low-Skill Automation Next, we show that automation is low-skill if Assumptions 4 and 5 do not hold (while still imposing Assumptions 1-3). If Assumption 4 does not hold, we have

$$\frac{\partial \log \psi_{k,0}}{\partial x} - \frac{\partial \log \psi_{0,0}}{\partial x} \leq 0.$$

By log supermodularity of labor productivity (Assumption 2), it follows that

$$\frac{\partial \log \psi_{k,0}}{\partial x} - \frac{\partial \log \psi_{s,0}}{\partial x} < 0 \quad \text{for all } s > 0.$$

Together with quasi-concavity of $\psi_{k,x}/\psi_{s,x}$ (Assumption 3), this implies that $w_s \psi_{k,x}/\psi_{s,x}$ is decreasing in x for all $s > 0$. By continuity of productivity schedules, the same must then hold for $s = 0$.

So, ω_x is the lower envelope of a family of decreasing functions and must therefore be decreasing itself. This implies that $\{x|\omega_x \geq 1/q\}$ is either empty (if $q < q_0$) or contains zero (if $q \geq q_0$). Using Lemma 1, the same holds for the set of automated tasks X_k.

Labor Allocation Finally, the result that there exists $\widetilde{s} \in (0, 1)$ such that skills below $\widetilde{s}$ are allocated to tasks below X_k and skills above $\widetilde{s}$ are allocated to tasks above X_k is an immediate consequence of monotone labor allocations (Proposition 2.1) and the fact that no positive mass of skills can be allocated on X_k. The latter is implied by our proof of Lemma 1.

A.5 Proof of Proposition 4: Minimum Wages and Automation

The first part of the proof follows closely the proof of low-skill automation for the case where Assumptions 4 and 5 do not hold in Proposition 3. In particular, condition (9) together with comparative advantage across labor types (Assumption 2) implies that $\psi_{k,x}/\psi_{s,x}$ is decreasing in x for all $s > \underline{s}$ and, by continuity, also for $s = \underline{s}$ (where $\underline{s}$ is such that all skills below $\underline{s}$ are non-employed due to the minimum wage). Thus, the minimum labor cost function ω_x is decreasing and the set of automated tasks, which is equal to $\{x|\omega_x \geq 1/q\}$, is either empty or contains zero.

The second part is to show that, if $q \in (q_0, q_m)$ as in Assumption 5, then the set of automated tasks remains non-empty after the introduction of the minimum wage. For this, we compare the assignment problems without capital, with and without the minimum wage.

Without capital, our setting has been studied extensively in the literature (e.g., Costinot and Vogel 2010). The introduction of the minimum wage is equivalent to a shift in the lower bound of the skill space from 0 to $\underline{s}$. Without capital, it leads to a decline in skill premia along the entire skill space, with the wage of the least skilled remaining worker type $\underline{s}$ increasing and the wage of the most skilled worker type ($s = 1$) decreasing (Teulings, 2000; Costinot and Vogel, 2010). Let w_s^0 be the wage function without capital and without minimum wage (and ω_x^0 the associated minimum labor cost function) and $w_s^{0,min}$ the wage function without capital but with minimum wage. Then,

$$\frac{1}{q} < \max_x \omega_x^0 \leq \max_x \frac{w_{\underline{s}}^0 \psi_{k,x}}{\psi_{\underline{s},x}} \leq \frac{w_{\underline{s}}^{0,min} \psi_{k,0}}{\psi_{\underline{s},0}},$$

where the first inequality uses that $q > q_0$ by Assumption 5, the second follows from the definition of ω_x^0 as the lower envelope of all workers' effective cost, and the last inequality is implied by $w_{\underline{s}}^0 \leq w_{\underline{s}}^{0,min}$ and $\psi_{k,x}/\psi_{\underline{s},x}$ being decreasing in x. The inequalities imply that if X_k were empty, we had $0 \in X_k$ by Lemma 1, a contradiction. Hence, X_k remains non-empty after introduction of the minimum wage.

B Proofs for Section 4: Local Effects of Automation

We use our characterization of the wage and the assignment function in terms of the differential equation system (10)-(14) to conduct comparative statics with respect to capital productivity. Implicitly, this imposes Assumptions 1 to 3 and $q \geq q_0$.

We consider a small change in capital productivity $d \log q$ (if $q = q_0$ we impose $d \log q > 0$ such that our equilibrium characterization continues to hold) and study its first-order effects on wages and assignment. From equations (10) and (12), we obtain the variational equations

$$(d \log w_s)' = \frac{\partial^2 \log \psi_{s,X_s}}{\partial s \partial x} dX_s \tag{18}$$

$$\begin{aligned}(dX_s)' =& \lambda \frac{l_s w_s^\lambda}{Y A_{X_s}^{\lambda-1} \psi_{s,X_s}^{\lambda-1}} d \log w_s - \frac{l_s w_s^\lambda}{Y A_{X_s}^{\lambda-1} \psi_{s,X_s}^{\lambda-1}} d \log Y \\ &- (\lambda - 1) \frac{l_s w_s^\lambda}{Y A_{X_s}^{\lambda-1} \psi_{s,X_s}^{\lambda-1}} \left(\frac{\partial \log A_{X_s}}{\partial x} + \frac{\partial \log \psi_{s,X_s}}{\partial x} \right) dX_s,\end{aligned} \tag{19}$$

which hold for all $s \neq \widetilde{s}$. The boundary conditions for the upper branch of these variations, i.e., the branch on $(\widetilde{s}, 1]$, are given by

$$\begin{aligned} d\log w_{\widetilde{s}}^{+} &= \left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x}\right) d\overline{x} + \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s} d\widetilde{s} - d\log q - (\log w_{\widetilde{s}})'^{+} d\widetilde{s} \\ &= \left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x}\right) d\overline{x} - d\log q \qquad (20) \\ dX_{\widetilde{s}}^{+} &= d\overline{x} - (X_{\widetilde{s}})'^{+} d\widetilde{s} \\ &= d\overline{x} - \frac{l_{\widetilde{s}} w_{\widetilde{s}}^{\lambda}}{Y A_{\overline{x}}^{\lambda-1} \psi_{\widetilde{s},\overline{x}}^{\lambda-1}} d\widetilde{s}, \qquad (21) \end{aligned}$$

where the superscript '+' denotes the right-side limit of the respective function. The boundary conditions for the lower branch (which exists only if automation is interior) are

$$\begin{aligned} d\log w_{\widetilde{s}}^{-} &= \left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x}\right) d\underline{x} + \frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} d\widetilde{s} - d\log q - (\log w_{\widetilde{s}})'^{-} d\widetilde{s} \\ &= \left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x}\right) d\underline{x} - d\log q \qquad (22) \\ dX_{\widetilde{s}}^{-} &= d\underline{x} - (X_{\widetilde{s}})'^{-} d\widetilde{s} \\ &= d\underline{x} - \frac{l_{\widetilde{s}} w_{\widetilde{s}}^{\lambda}}{Y A_{\underline{x}}^{\lambda-1} \psi_{\widetilde{s},\underline{x}}^{\lambda-1}} d\widetilde{s}, \qquad (23) \end{aligned}$$

with the superscript '−' denoting left-side limits.

From the upper branch of the system, we can obtain the change in assignment of the most skilled workers, $dX_1(d\overline{x}, d\widetilde{s})$, as a function of $d\overline{x}$ and $d\widetilde{s}$. Analogously, if automation is interior, the lower branch yields $dX_0(d\underline{x}, d\widetilde{s})$, the change in assignment of the least skilled workers as a function of $d\underline{x}$ and $d\widetilde{s}$. Both of these changes must be zero in equilibrium, which defines functions $d\overline{x}(d\widetilde{s})$ and $d\underline{x}(d\widetilde{s})$. The following lemma establishes some properties of $d\overline{x}(d\widetilde{s})$ and $d\underline{x}(d\widetilde{s})$.

Lemma 2 *Suppose Assumptions 1, 2 and 3 hold, $q \geq q_0$ and $d\log q > 0$. Then, the function $d\overline{x}(d\widetilde{s})$ is strictly increasing, satisfies $d\overline{x}(0) > 0$ and*

$$\left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x}\right) d\overline{x}(0) < d\log q + \frac{1}{\lambda} d\log Y.$$

Moreover, if $\underline{x} > 0$, $d\underline{x}(d\widetilde{s})$ is strictly increasing, satisfies $d\underline{x}(0) < 0$ and

$$\left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x}\right) d\underline{x}(0) < d\log q + \frac{1}{\lambda} d\log Y.$$

Proof. We focus on the results for $d\overline{x}(d\widetilde{s})$. The proof for $d\underline{x}(d\widetilde{s})$ proceeds analogously. Define

$$d\log \widetilde{w}_s = d\log w_s - \frac{1}{\lambda} d\log Y$$

as the wage change net of the productivity effect, i.e., the pure displacement effect of automation.

With this, we can write the variational equations (18) and (19) more compactly as

$$(d \log \widetilde{w}_s)' = \gamma(s) dX_s$$

$$(dX_s)' = \alpha(s) dX_s + \beta(s) d \log \widetilde{w}_s$$

where $\gamma(s), \beta(s) > 0$ for all s and the initial values are

$$d \log \widetilde{w}_{\widetilde{s}}^+ = \left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x} \right) d\overline{x} - d \log q - \frac{1}{\lambda} d \log Y$$

$$dX_{\widetilde{s}}^+ = d\overline{x} - \frac{l_{\widetilde{s}} w_{\widetilde{s}}^{\lambda}}{Y A_{\overline{x}}^{\lambda-1} \psi_{\widetilde{s},\overline{x}}^{\lambda-1}} d\widetilde{s}.$$

We next show that the implied change dX_1 is strictly increasing in both initial values. First consider $dX_{\widetilde{s}}^{+(1)} < dX_{\widetilde{s}}^{+(2)}$ with $d \log \widetilde{w}_{\widetilde{s}}^+$ equal in both cases. The implied path $dX_s^{(2)}$ starts above $dX_s^{(1)}$. Suppose now $dX_s^{(2)}$ crosses $dX_s^{(1)}$ for the first time at a skill level s_1. Clearly, this crossing must be from above, that is, we need $(dX_{s_1}^{(2)})' \leq (dX_{s_1}^{(1)})'$. But from the differential equations, we obtain

$$(dX_{s_1}^{(1)})' - (dX_{s_1}^{(2)})' = \beta(s_1)(d \log \widetilde{w}_{s_1}^{(1)} - d \log \widetilde{w}_{s_1}^{(2)}) < 0,$$

where the last inequality follows from the fact that $d \log \widetilde{w}_s^{(2)}$ starts from the same value as $d \log \widetilde{w}_s^{(1)}$ but increases at a faster rate until s_1 because $dX_s^{(2)} > dX_s^{(1)}$ for $s < s_1$. It follows that the two paths cannot cross and we have $dX_1^{(2)} > dX_1^{(1)}$.

The reasoning for the second initial value, $d \log \widetilde{w}_{\widetilde{s}}^+$, follows a similar line. If we increase $d \log \widetilde{w}_{\widetilde{s}}^+$, the path dX_s will have a larger slope initially and hence move upwards for s slightly above $\widetilde{s}$. Then, the argument for why it can't cross its original path again, is the same as above.

We have thus shown that the change dX_1 is strictly increasing in both initial values. Both initial values, in turn, are increasing in $d\overline{x}$. Moreover, $dX_{\widetilde{s}}^+$ is strictly decreasing in $d\widetilde{s}$ while $d \log \widetilde{w}_{\widetilde{s}}^+$ is unaffected by $d\widetilde{s}$. Thus, dX_1 is strictly increasing in $d\overline{x}$ and strictly decreasing in $d\widetilde{s}$. Then, setting $dX_1 = 0$ yields the first claim of the lemma: $d\overline{x}$ is strictly increasing in $d\widetilde{s}$.

Next, we show that if $d\widetilde{s} = 0$, then $dX_1 = 0$ requires to set $d\overline{x}$ such that the initial value $dX_{\widetilde{s}}^+$ is positive while $d \log \widetilde{w}_{\widetilde{s}}^+$ is negative (recall from above that $d \log \widetilde{w}_{\widetilde{s}}^+$ is negative at $d\overline{x} = 0$). To see why this must be the case, suppose we were to start with both initial values negative. Then, dX_s could never attain zero for any $s \geq \widetilde{s}$ by reasoning analogous to that used above: If dX_s approaches zero from below, its derivative will turn negative because $\beta(s) d \log \widetilde{w}_s$ is negative ($d \log \widetilde{w}_s$ starts from a negative initial value and declines from there because dX_s is negative initially). Similarly, if we start with both initial values positive (and at least one strictly positive), dX_s can never attain zero either, because as it approaches zero, its derivative will become positive by reversing the arguments from the case with both initial values negative.

Hence, when setting $d\widetilde{s}$ to zero, then $dX_1 = 0$ requires $d\overline{x}$ to be strictly positive but sufficiently small for $\log \widetilde{w}_{\widetilde{s}}^{+}$ to be strictly negative, which are the second and third claims of the lemma. ■

B.1 Proof of Proposition 5: Automation and Employment Polarization

Expansion of Automation Suppose now that Assumptions 4 and 5 hold such that automation is interior. In this case, condition (11) requires that

$$\underbrace{\left(\frac{\partial \psi_{\widetilde{s},\overline{x}}}{\partial x} + \frac{\partial \log \psi_{k,\overline{x}}}{\partial x}\right)}_{=\gamma_{\widetilde{s},\overline{x}}} d\overline{x}(d\widetilde{s}) + \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s} d\widetilde{s} = \underbrace{\left(\frac{\partial \psi_{\widetilde{s},\underline{x}}}{\partial x} + \frac{\partial \log \psi_{k,\underline{x}}}{\partial x}\right)}_{=\gamma_{\widetilde{s},\underline{x}}} d\underline{x}(d\widetilde{s}) + \frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} d\widetilde{s}, \quad (24)$$

where we have already inserted the functions $d\overline{x}(d\widetilde{s})$ and $d\underline{x}(d\widetilde{s})$ derived from equations (18) to (23) above. Rearranging and signing terms, we obtain:

$$\underbrace{\gamma_{\widetilde{s},\overline{x}}}_{\geq 0} d\overline{x}(d\widetilde{s}) - \underbrace{\gamma_{\widetilde{s},\underline{x}}}_{\leq 0} d\underline{x}(d\widetilde{s}) = \underbrace{\left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} - \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s}\right)}_{<0} d\widetilde{s}.$$

By Lemma 2, the left-hand side of this equation is increasing while the right-hand side is strictly decreasing in $d\widetilde{s}$. Thus, the equation determines a unique equilibrium change $d\widetilde{s}^{*}$.

If $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)$ is strictly positive, then $d\widetilde{s}^{*}$ must be strictly negative, which implies that $d\underline{x}(d\widetilde{s}^{*}) < 0$ (by Lemma 2) and

$$d\overline{x}(d\widetilde{s}^{*}) = \frac{\gamma_{\widetilde{s},\underline{x}}}{\gamma_{\widetilde{s},\overline{x}}} d\underline{x}(d\widetilde{s}^{*}) + \frac{1}{\gamma_{\widetilde{s},\overline{x}}} \left(\frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} - \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s}\right) d\widetilde{s}^{*} > 0.$$

Note here that $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0) > 0$ can only hold if $\gamma_{\widetilde{s},\overline{x}} > 0$.

Analogously, if $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)$ is strictly negative, then $d\widetilde{s}^{*}$ must be strictly positive, which implies that $d\overline{x}(d\widetilde{s}^{*}) > 0$ and $d\underline{x}(d\widetilde{s}^{*}) < 0$.

Finally, if $\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0) - \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)$ equals zero, then $d\widetilde{s}^{*}$ must be zero as well, such that $d\overline{x}(\widetilde{s}^{*}) > 0$ and $d\underline{x}(d\widetilde{s}^{*}) < 0$ follow immediately from Lemma 2.

At this point, note also that

$$\max\{\gamma_{\widetilde{s},\overline{x}} d\overline{x}(d\widetilde{s}^{*}), \gamma_{\widetilde{s},\underline{x}} d\underline{x}(d\widetilde{s}^{*})\} \leq \max\{\gamma_{\widetilde{s},\overline{x}} d\overline{x}(0), \gamma_{\widetilde{s},\underline{x}} d\underline{x}(0)\} < d\log q + \frac{1}{\lambda} d\log Y,$$

where the second inequality follows from Lemma 2. This implies that the initial values $d\log \widetilde{w}_{\widetilde{s}}^{+}$ and $d\log \widetilde{w}_{\widetilde{s}}^{-}$ must both be negative at $d\widetilde{s}^{*}$. This result will be useful in the next step of the proof.

Employment Polarization We have just shown above that $d\log \widetilde{w}_{\widetilde{s}}^{+}$ is strictly negative in equilibrium. This implies that the initial value $dX_{\widetilde{s}}^{+}$ in the dynamic system for dX_s and $d\log \widetilde{w}_s$ must be strictly positive:

$$dX_{\widetilde{s}}^{+} = d\overline{x} - \frac{l_{\widetilde{s}} w_{\widetilde{s}}^{\lambda}}{Y A_{\overline{x}}^{\lambda-1} \psi_{\widetilde{s},\overline{x}}^{\lambda-1}} d\widetilde{s} > 0.$$

If it were negative, we could never attain $dX_1 = 0$ by the reasoning in the proof of Lemma 2.

Suppose now that at some skill $s_1 > \widetilde{s}$, dX_s turns negative, that is, it crosses zero from above: $dX_{s_1} = 0$ and $(dX_{s_1})' \leq 0$. To obtain $dX_1 = 0$, dX_s must at some point $s_2 > s_1$ attain zero again, this time from below: $dX_{s_2} = 0$ and $(dX_{s_2})' \geq 0$. The differential equation for $(dX_s)'$, however, implies that

$$(dX_s)' = \beta(s) d \log \widetilde{w}_s \quad \text{for } s = s_1, s_2.$$

We must therefore have $d \log \widetilde{w}_{s_1} \leq 0$. Since dX_s is negative between s_1 and s_2, we will also have $d \log \widetilde{w}_{s_2} < 0$ by the equation for $(d \log \widetilde{w}_s)'$. This in turn implies $(dX_{s_2})' < 0$, a contradiction.

As a result, dX_s cannot cross zero but stays positive until dX_1.[20] Analogous reasoning yields $dX_s < 0$ for $0 < s < \widetilde{s}$.

Labor Share Instead of proving the results for the labor share directly, we prove that the inverse of these results holds for the capital share. From equation (15), we obtain the response of the capital share to the increase in capital productivity as

$$\begin{aligned} d\alpha_k &= \alpha_k(\lambda - 1) d \log q + q^{\lambda-1} \frac{\partial \Gamma_k}{\partial \underline{x}} d\underline{x} + q^{\lambda-1} \frac{\partial \Gamma_k}{\partial \overline{x}} d\overline{x} \\ &= \alpha_k(\lambda - 1) d \log q - q^{\lambda-1} \left(A_{\underline{x}}^{\lambda-1} \psi_{k,\underline{x}}^{\lambda-1} \right) d\underline{x} + q^{\lambda-1} \left(A_{\overline{x}}^{\lambda-1} \psi_{k,\overline{x}}^{\lambda-1} \right) d\overline{x}. \end{aligned} \tag{25}$$

The last two terms are strictly positive by our employment polarization result, such that the capital share increases if $\lambda \geq 1$.

If $\lambda < 1$, the total effect on the capital share depends on the relative strength of the capital deepening effect (first term) and the expansion of the set of automated tasks (second and third term). If $q = q_0$, we have $\alpha_k = 0$, such that the capital deepening effect vanishes. Moreover, Lemma 2 implies that

$$\max\{|d\overline{x}|, |d\underline{x}|\} \geq \min\{|d\overline{x}(0)|, |d\underline{x}(0)|\} > 0,$$

which means that the expansion of the set of automated tasks does not vanish. So, we must have $d\alpha_k(q_0) > 0$. Finally, note that $d\alpha_k$ (considering the perturbation $d \log q > 0$) is a right-hand derivative and as such it is continuous from the right, i.e.,

$$\lim_{q \searrow q_0} d\alpha_k = d\alpha_k(q_0) > 0.$$

[20] We can exclude the case where dX_s has a critical zero (a point where dX_s is tangent to zero but does not cross it). This is because a critical zero would imply $(dX_s)' = 0$ and $dX_s = 0$. But then, the entire upper branch of dX_s would be identical zero, which is incompatible with the initial value $dX_{\widetilde{s}}^{+}$ being strictly positive.

This proves that $d\alpha_k > 0$ in some right neighborhood of q_0.

B.2 Proof of Proposition 6: Automation and Wage Polarization

By (18), we have

$$d \log w_s - d \log w_{s'} = \int_{s'}^{s} \frac{\partial^2 \log \psi_{t,X_t}}{\partial s \partial x} dX_t \, dt.$$

By Assumption 2.2 and our employment polarization result in Proposition 5, this expression is strictly positive for all $s > s' \geq \widetilde{s}$ and also for all $s < s' \leq \widetilde{s}$.

B.3 Proof of Proposition 7: Automation and Wage Levels

We have already proved part 1 of the proposition in the main text. Here we prove parts 2 to 4.

Part 2 Condition (11) implies that

$$d \log w_{\widetilde{s}} = \begin{cases} \frac{\partial \log \psi_{\widetilde{s},\underline{x}}}{\partial s} d\widetilde{s} + \gamma_{\widetilde{s},\underline{x}} d\underline{x} - d \log q - (d \log w_{\widetilde{s}})'^{-} = \gamma_{\widetilde{s},\underline{x}} d\underline{x} - d \log q & \text{if } d\widetilde{s} \geq 0 \\ \frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial s} d\widetilde{s} + \gamma_{\widetilde{s},\overline{x}} d\overline{x} - d \log q - (d \log w_{\widetilde{s}})'^{+} = \gamma_{\widetilde{s},\overline{x}} d\overline{x} - d \log q & \text{if } d\widetilde{s} \leq 0 \end{cases} \tag{26}$$

where $\gamma_{\widetilde{s},\underline{x}}$ and $\gamma_{\widetilde{s},\overline{x}}$ are defined as in equation (24). Now suppose at first that $q = q_0$. Then, $\overline{x} = \underline{x}$ and $\gamma_{\widetilde{s},\underline{x}} = \gamma_{\widetilde{s},\overline{x}} = 0$ because $\underline{x} = \overline{x}$ is a maximizer of the effective labor cost function ω_x. So, we obtain $d \log w_{\widetilde{s}} = -d \log q < 0$.

Next, for $d \log q > 0$, $d \log w_{\widetilde{s}}$ is a right-hand derivative and thus must be continuous from the right. So, $d \log w_{\widetilde{s}} < 0$ in a right neighborhood of q_0. Finally, this extends to skills in some neighborhood around $\widetilde{s}$ because the wage change $d \log w_s$ is continuous in s.

Part 3 If $\psi_{s',x}/\psi_{k,x}$ is constant, we must have

$$\frac{w_{s'}}{\psi_{s',x}} = \frac{1/q}{\psi_{k,x}}$$

for all x.[21] Differentiating this, we obtain

$$d \log w_{s'} = -d \log q < 0.$$

Since the change $d \log w_s$ is continuous in s, we obtain $d \log w_s < 0$ for all s in some neighborhood of s'.

[21] Otherwise, either the set of automated tasks were empty (if the equation held with $<$ instead of $=$) or skill s' could not be assigned to any task (if the equation held with $>$ instead of $=$).

Part 4 Suppose at first that $q = q_m$ and consider $d\log q < 0$. It is easy to check that for $d\log q < 0$, the reasoning of Lemma 2 can be adjusted to imply that $d\overline{x}(d\widetilde{s})$ is still strictly increasing but now $d\overline{x}(0) < 0$ and

$$\left(\frac{\partial \log \psi_{\widetilde{s},\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x}\right) d\overline{x}(0) > d\log q + \frac{1}{\lambda} d\log Y.$$

Since at $q = q_m$ we have $\underline{x} = 0$ and $\widetilde{s} = 0$, the analogous results for $d\underline{x}$ do not apply. Instead, we have $dX_{\widetilde{s}}^{-} = 0$ and hence by equation (23),

$$d\underline{x}(d\widetilde{s}) = \frac{l_0 w_0^{\lambda}}{Y A_{\underline{x}}^{\lambda-1} \psi_{\widetilde{s},\underline{x}}^{\lambda-1}} d\widetilde{s}.$$

So $d\underline{x}(\widetilde{s})$ is strictly increasing and $d\underline{x}(0) = 0$.

As we consider $d\log q < 0$ starting from q_m, equation (11) holds and so does its variational counterpart (24). Then, by reasoning analogous to that in the first part of the proof of Proposition 5, we can show that $d\overline{x} < 0$ and $d\underline{x} \geq 0$. Next, by the same reasoning as in the second part of the proof of Proposition 5, we obtain that $dX_s < 0$ for all $s \in (0, 1)$. By the argument of the proof of Proposition 6, this implies that $d\log w_1 < d\log w_0$.

Finally, note that for $d\log q < 0$, the changes $d\log w_1$ and $d\log w_0$ are left-hand derivatives and as such they are continuous from the left. So, we have that $d\log w_1 < d\log w_0$ in response to $d\log q < 0$ for all q in some left neighborhood of q_m. But for $q \in (q_0, q_m)$, wages are differentiable in q and we obtain the reverse for $d\log q > 0$, i.e., $d\log w_1 > d\log w_0$ in response to $d\log q > 0$ for all q in some left neighborhood of q_m.

B.4 Proof of Proposition 8: Productivity Effects

TFP, or net output, satisfies

$$c = \max_{\overline{K}} F(\overline{K}, l) - \frac{\overline{K}}{q}$$

where F is maximal output subject to aggregate factor supplies $\overline{K}$ and l (see proof of Proposition 1). So by the envelope theorem, we obtain $dc/dq = \overline{K}/q^2$ and hence:

$$\frac{d\log c}{d\log q} = \frac{\overline{K}/q}{c} = \frac{\overline{K}/q}{Y - \overline{K}/q} = \frac{\alpha_k}{1-\alpha_k}.$$

The second-order term is then given by

$$\frac{d\log(\alpha_k/(1-\alpha_k))}{d\log q} = \frac{1}{1-\alpha} \frac{d\log \alpha_k}{d\log q},$$

which by our previous result (25) can be written as

$$\frac{1}{1-\alpha_k}\left(\lambda - 1 + \frac{\partial \log \Gamma_k}{\partial \underline{x}} \frac{d\underline{x}}{d\log q} + \frac{\partial \log \Gamma_k}{\partial \overline{x}} \frac{d\overline{x}}{d\log q}\right).$$

Combining these first- and second-order terms yields our second-order Taylor approximation of $\Delta \log c$.

C Proofs for Section 5: Global Effects of Automation

C.1 Proof of Proposition 9: Polarization with Large Changes in Automation

Part 1 The first result, the expansion of the set of automated tasks, is a direct consequence of its local counterpart in Proposition 5 and the fundamental theorem of calculus. In particular, we can obtain $\Delta \underline{x}$ by integrating a series of local changes $d\underline{x}$ from q to $q + \Delta q$. Since $[q, q + \Delta q] \subset (q_0, q_m)$, all of these local changes are strictly negative by Proposition 5 and, hence, we have $\Delta \underline{x} < 0$. Analogously, we obtain that $\Delta \overline{x} > 0$.

For the corresponding changes in labor assignment, we split the skill space into three intervals. First on $(0, \min\{\widetilde{s}', \widetilde{s}\})$, all of the local changes dX_s when moving from q to $q + \Delta q$ are strictly negative (by Proposition 5), such that we obtain $\Delta X_s < 0$ for all $s \in (0, \min\{\widetilde{s}', \widetilde{s}\})$. Analogously, all local changes on $(\max\{\widetilde{s}', \widetilde{s}\}, 1)$ are strictly positive by Proposition 5 such that we have $\Delta X_s > 0$ for all $s \in (\max\{\widetilde{s}', \widetilde{s}\}, 1)$.

Finally the skills on $(\min\{\widetilde{s}', \widetilde{s}\}, \max\{\widetilde{s}', \widetilde{s}\})$ switch from one side of the set of automated tasks to the other when capital productivity increases from q to $q + \Delta q$. Suppose at first that $\widetilde{s}' < \widetilde{s}$. In this case, $X_s < \underline{x} < \overline{x}^{new} < X_s^{new}$ for all $s \in (\widetilde{s}', \widetilde{s})$. So, $\Delta X_s > 0$ for all $s \in (\widetilde{s}', \widetilde{s})$. Analogously, if $\widetilde{s} < \widetilde{s}'$, we have $X_s > \overline{x} > \underline{x}^{new} > X_s^{new}$ and therefore $\Delta X_s < 0$ for all $s \in (\widetilde{s}, \widetilde{s}')$.

Part 2 By equation (10) and Assumption 2.2 (supermodularity of $\log \psi_{s,x}$), the local skill premium $(\log w_s)'$ is strictly increasing in the task assigned to s. With the employment polarization result from part 1, this immediately implies wage polarization as stated in part 2 of the proposition.

Part 3 These two results follow immediately from the fundamental theorem of calculus and their local counterparts in Proposition 7, along the lines of part 1 above.

Part 4 Consider first the equilibrium under $q \geq q_0$ and any skill s. Since there must be some task in which capital is less costly than s, we have

$$w_s \geq \min_x \left\{ \frac{\psi_{s,x}}{\psi_{k,x} q} \right\}.$$

Under the new capital productivity $q^{new} = q + \Delta q$, there must be some task performed by skill s in equilibrium, so we must have

$$w_s^{new} \leq \max_x \left\{ \frac{\psi_{s,x}}{\psi_{k,x} q^{new}} \right\}.$$

Combining the previous two inequalities, we obtain

$$\frac{w_s^{new}}{w_s} \leq \frac{\max_x \{\psi_{s,x}/\psi_{k,x}\}}{\min_x \{\psi_{s,x}/\psi_{k,x}\}} \frac{q}{q^{new}},$$

or, in logs,

$$\Delta \log w_s \leq \max_x \{\log \psi_{s,,x} - \log \psi_{k,x}\} - \min_x \{\log \psi_{s,x} - \log \psi_{k,x}\} - \Delta \log q.$$

Part 5 We have shown in Appendix A.3 that $q_m < q_\infty$ if $\psi_{0,0}/\psi_{k,0} < \psi_{0,1}/\psi_{k,1}$. So at first suppose that the new capital productivity $q^{new} = q + \Delta q$ is equal to q_m. Since the equilibrium allocation is continuous in q, the same reasoning as in part 1 of the proof implies that $X_s^{new} > X_s$ for all $s \in (\widetilde{s}', 1)$.[22] Moreover, by definition of q_m we have $\widetilde{s}' = 0$ (see Appendix A.3). Thus, $X_s^{new} > X_s$ for all $s \in (0, 1)$ and, by the same reasoning as in part 2, $\Delta \log w_1 > \Delta \log w_0$.

Next, since wages are continuous in q, $\Delta \log w_1 > \Delta \log w_0$ must hold for all q^{new} in some lower neighborhood of q_m, which is claim 5 of the proposition.

C.2 Proof of Proposition 10: Transition to Low-Skill Automation

Low-Skill Automation We start by showing that if automation is low-skill, it remains low-skill when capital productivity increases further. Moreover, further increases in capital productivity shift all skills towards more complex tasks and raise all skill premia, which is the last part of the proposition.

If automation is low-skill, only the upper branch of the differential system (10) to (14) applies while we have $\widetilde{s} = 0$ and $\underline{x} = 0$. We now conjecture that a marginal increase in capital productivity $d \log q > 0$ leaves the threshold skill and the lower bound of the set of automated tasks unchanged, $d\widetilde{s} = 0$ and $d\underline{x} = 0$, i.e., automation remains interior. Then, from Lemma 2 we obtain that $d\overline{x} > 0$, while the first part of equation (11) implies that

$$d \log w_0 = \left(\frac{\partial \log \psi_{0,\overline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\overline{x}}}{\partial x} \right) d\overline{x} - d \log q > -d \log q.$$

[22] Continuity of the equilibrium allocation in q follows from Berge's maximum theorem (see the proof of Proposition 2 above) and uniqueness of the allocation (by Proposition 1), which allows to strengthen the conclusion of the maximum theorem from upper hemicontinuity to continuity.

To verify our initial conjecture that automation remains interior, we can now check that the second part of equation (11) remains satisfied, i.e.,

$$d\log w_0 \geq \left(\frac{\partial \log \psi_{0,\underline{x}}}{\partial x} - \frac{\partial \log \psi_{k,\underline{x}}}{\partial x}\right) d\overline{x} - d\log q = d\log q,$$

which we have shown above. Thus, we have shown that if automation starts low-skill, then it remains low-skill when capital productivity increases. So, automation is low-skill for all $q \geq q_m$.

What are the employment and wage effects of an increase in capital productivity when automation is low-skill? We have already shown that $d\widetilde{s} = 0$ and $d\overline{x} > 0$. Then, the same reasoning as in part 2 ("Employment Polarization") of the proof of Proposition 5 implies that the assignment function shifts up everywhere, i.e., $dX_s > 0$ for all $s \in (0,1)$. By the same reasoning as in the proof of Proposition 6, this in turn implies that all skill premia increase, i.e., $d\log w_s$ is strictly increasing in s.

By the fundamental theorem of calculus, these local effects of low-skill automation extend to increases in capital productivity of any size when starting from $q \geq q_m$.

Transition We have shown above that automation is low-skill for all $q \geq q_m$. This implies that if we start from $q < q_m$ and consider a change Δq such that $q + \Delta q \geq q_m$, then $\Delta \underline{x} = -\underline{x}$ (i.e., automation transitions from interior to low-skill), which is claim 1 of the proposition.

Task Upgrading We have shown in the first part above ("Low-Skill Automation") that $dX_s > 0$ for all $s \in (0,1)$ in response to a marginal increase $d\log q > 0$ starting from $q \geq q_m$. Moreover, in Proposition 5, we have shown that $dX_s > 0$ for all $s \in (\widetilde{s}, 1)$ in response to $d\log q > 0$ when starting from $q \in (q_0, q_m)$, where $\widetilde{s}$ is the threshold skill level at q. Integrating these marginal changes between $q \in (q_0, q_m)$ and $q^{new} \geq q_m$, we obtain that $\Delta X_s > 0$ for all $s \in (\widetilde{s}, 1)$.

For skills $s \leq \widetilde{s}$, which switch to the other side of the set of automated tasks when capital productivity increases to q^{new}, we have

$$X_s < \underline{x} < \overline{x}^{new} < X_s^{new},$$

and hence $\Delta X_s > 0$ as well.

Rise in Skill Premia The task upgrading result in the previous part immediately implies that all skill premia increase, i.e., $\Delta \log w_s$ is strictly increasing in s, by the reasoning of part 2 of Proposition 9.

C.3 Proof of Proposition 11: Labor Supply Changes and Automation

We start with a useful lemma on the wage effects of labor supply changes which holds for all settings where production is concave and linear homogeneous in labor and wages equal marginal products.

Lemma 3 *Consider any two labor endowments $l > 0$ and $l^{new} > 0$ with corresponding wage functions w and w^{new}. Then, if $w_s \leq w_s^{new}$ for all s, we must have $w = w^{new}$.*

Proof. By Euler's homogeneous function theorem, we have

$$\begin{aligned} c^{new} - c &= \int_0^1 w_s^{new} l_s^{new}\, ds - \int_0^1 w_s l_s\, ds \\ &= \int_0^1 (w_s^{new} - w_s) l_s^{new}\, ds + \int_0^1 w_s (l_s^{new} - l_s)\, ds \\ \Rightarrow \qquad \int_0^1 (w_s^{new} - w_s) l_s^{new}\, ds &= c^{new} - c - \int_0^1 w_s (l_s^{new} - l_s)\, ds \leq 0, \end{aligned}$$

where the inequality in the last line follows from concavity of net output in labor inputs. The last line shows that it is impossible to have $w_s \leq w_s^{new}$ for all s and with strict inequality on a subset of skills of strictly positive measure. Hence, if $w_s \leq w_s^{new}$ for all s, then $w = w^{new}$, i.e., the two wage functions are equal almost everywhere. ■

The important implication of Lemma 3 is that a labor supply change alone can never cause all wages to increase or all wages to decrease. Instead there will always be some wages that increase and some that decrease, except in the case where the wage function is completely unchanged.

Low-Skill Automation Threshold We can now prove that the threshold where automation transitions from interior to low-skill, q_m, is strictly decreasing in relative skill supply. First, recall from Appendix A.3 that q_m is defined as the unique solution to

$$\frac{1}{q_m} = \frac{w_0^m(q_m)\psi_{k,\overline{x}_m}}{\psi_{0,\overline{x}_m}}, \tag{27}$$

where w^m is the wage function obtained under the restriction that capital can only be allocated to tasks below $\overline{x}_m$ and labor only to tasks above $\overline{x}_m$. This equation has a unique solution because the left-hand side is strictly decreasing and the right-hand side strictly increasing in q_m.

Now consider an increase in relative skill supply, $\Delta \log l$ with $\Delta \log l_s$ strictly increasing in s. We know from prior work that such a change in labor supply lowers all skill premia in the pure labor assignment model, i.e., if no capital were used (Costinot and Vogel, 2010). With the restriction that capital must be assigned below and labor above $\overline{x}_m$, the labor allocation is determined as in a pure labor assignment model. Hence, the result from prior work applies and the wage change

$\Delta \log w_s^m$ must be strictly decreasing in s. By Lemma 3, the wage change cannot be negative for all skill levels and we must have $\Delta \log w_0^m > 0$.[23] Thus, the right-hand side of equation (27) increases, such that q_m must decrease to solve equation (27), $\Delta q_m < 0$.

Wage Effects Consider now an increase in relative skill supply $\Delta \log l$ such that $q \in (q_0, q_m)$ initially but $q > q_m^{new}$ after the change.

We know that before the change, automation is interior and $\overline{x} < \overline{x}_m$, where $\overline{x}_m$ is the value defined in the characterization of q_m in Appendix A.3.[24] After the change, by construction of q_m^{new}, we must have $\overline{x}^{new} > \overline{x}_m$. So, for all skills $s \in [0, \widetilde{s}]$, we have

$$X_s < \overline{x} < \overline{x}^{new} \leq X_s^{new}$$

such that the assignment function increases strictly on $[0, \widetilde{s}]$.

Suppose now that the assignment function shifts up everywhere. Then, by the reasoning of part 2 of the proof of Proposition 9, $\Delta \log w_s$ is increasing in s. However, since task $x = 0$ is assigned to skill $s = 0$ before the change and to capital afterwards (while the productivities of capital and labor are unchanged), the wage w_0 must increase strictly. This requires that all wages increase, $\Delta \log w_s > 0$ for all s, in contradiction to Lemma 3. So, there must exist a skill level $\hat{s} \in (\widetilde{s}, 1)$ such that the new assignment function X^{new} crosses the old function X from above at $\hat{s}$.

Next, suppose that X^{new} crosses X again at some skill $s_1 \in (\hat{s}, 1)$, this time from below. Then, we have $X_{s_1}^{new} = X_{s_1}$ and $X_{s_1}'^{new} \geq X_{s_1}'$. Moreover, there is another skill $s_2 > s_1$ (potentially but not necessarily equal to 1) such that $X_{s_2}^{new} = X_{s_2}$ and $X_{s_2}'^{new} \leq X_{s_2}'$ because the two assignment functions must intersect at $s = 1$. Now, since we also have $l_{s_2}^{new}/l_{s_1}^{new} > l_{s_2}/l_{s_1}$, the ratio of labor supply in X_{s_2} over X_{s_1} is strictly greater under l^{new} than under l. By equation (12), this implies

$$\frac{w_{s_2}^{new}}{w_{s_1}^{new}} < \frac{w_{s_2}}{w_{s_1}}.$$

But since X^{new} crosses X from below at s_1, we have $X_s^{new} \geq X_s$ for $s \in [s_1, s_2]$ such that equation (10) implies that

$$\frac{w_{s_2}^{new}}{w_{s_1}^{new}} \geq \frac{w_{s_2}}{w_{s_1}},$$

which yields a contradiction.

[23] Note that the proof of Lemma 3 only uses linear homogeneity and concavity of net output in labor, so it also applies to the situation where capital is restricted to tasks below and labor to tasks above $\overline{x}_m$.

[24] If we had $\overline{x} \geq \overline{x}_m$ before the change, then no skill could be assigned to any task below $\overline{x}_m$ by construction of $\overline{x}_m$ and hence we would have $\underline{x} = 0$, which contradicts interior automation.

We have therefore established that there exists $\hat{s} \in (\widetilde{s}, 1)$ such that $\Delta X_s > 0$ for all $s \in (0, \hat{s})$ and $\Delta X_s \leq 0$ for all $s \in [\hat{s}, 1]$. With the reasoning from part 2 of the proof of Proposition 9, this implies that $\Delta \log w_s$ is strictly increasing on $[0, \hat{s}]$ and decreasing on $[\hat{s}, 1]$.

References

Acemoglu, D. (1998): "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality," *Quarterly Journal of Economics*, **113** (4), 1055–1089.

——— (1999): "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," *American Economic Review*, **89** (5), 1259–78.

——— (2007): "Equilibrium Bias of Technology," *Econometrica*, **75** (5), 1371–1409.

Acemoglu, D. and D. H. Autor (2011): "Skills, Tasks and Technologies: Implications for Employment and Earnings," in *Handbook of Labor Economics*, **4** Elsevier, 1043–1171.

Acemoglu, D. and S. Johnson (2023): *Progress: The Thousand-Year Struggle Over Technology and Shared Prosperity*, Public Affairs.

Acemoglu, D. and P. Restrepo (2018a): "The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment," *American Economic Review*, **108** (6), 1488–1542.

——— (2018b): "Modeling Automation," *AEA Papers and Proceedings*, **108**, 48–53.

——— (2020): "Robots and Jobs: Evidence from US Labor Markets," *Journal of Political Economy*, **128** (6), 2188–2244.

——— (2022): "Tasks, Automation, and the Rise in US Wage Inequality," *Econometrica*, forthcoming.

——— (2023): "Distorted Automation: Tasks, Labor Market Rents and Unemployment in Search-Matching Model."

Acemoglu, D. and F. Zilibotti (2001): "Productivity Differences," *Quarterly Journal of Economics*, **116** (2), 563–606.

Autor, D. H. (2014): "Polanyi's Paradox and the Shape of Employment Growth," Working Paper 20485, NBER.

——— (2015): "Why Are There Still So Many Jobs? The History and Future of Workplace Automation," *Journal of Economic Perspectives*, **29** (3), 3–30.

Autor, D. H. and D. Dorn (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the U.S. Labor Market," *American Economic Review*, **103** (5), 1553–1597.

Autor, D. H., F. Levy, and R. J. Murnane (2003): "The Skill Content of Recent Technological Change: An Empirical Exploration," *Quarterly Journal of Economics*, **118** (4), 1279–1333.

BEA (2021a): "Fixed Assets Accounts Tables, Table 2.7: Investment in Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type," US Bureau of Economic

Analysis.

——— (2021b): "National Income and Product Accounts, Table 6.8: Persons Engaged in Production by Industry," US Bureau of Economic Analysis.

Berg, A., E. F. Buffie, and L.-F. Zanna (2018): "Should we fear the robot revolution? (The correct answer is yes)," *Journal of Monetary Economics*, **97**, 117–148.

Costinot, A. and J. Vogel (2010): "Matching and Inequality in the World Economy," *Journal of Political Economy*, **118** (4), 747–786.

Feng, A. and G. Graetz (2020): "Training Requirements, Automation, and Job Polarisation," *Economic Journal*, **130** (631), 2249–2271.

Ford, M. (2015): *Rise of the Robots: Technology and the Threat of a Jobless Future*, Basic Books.

Goos, M., A. Manning, and A. Salomons (2009): "Job Polarization in Europe," *American Economic Review*, **99** (2), 58–63.

Graetz, G. and G. Michaels (2018): "Robots at Work," *Review of Economics and Statistics*, **100** (5), 753–768.

Heckman, J. J. and G. Sedlacek (1985): "Heterogeneity, Aggregation, and Market Wage Functions: An Empirical Model of Self-Selection in the Labor Market," *Journal of Political Economy*, **93** (6), 1077–1125.

Hémous, D. and M. Olsen (2022): "The Rise of the Machines: Automation, Horizontal Innovation, and Income Inequality," *American Economic Journal: Macroeconomics*, **14** (1), 179–223.

IFR (2018): "World Robotics 2018," International Federation of Robotics.

Jackson, M. O. and Z. Kanik (2020): "How Automation that Substitutes for Labor Affects Production Networks, Growth, and Income Inequality," Working Paper.

Jaimovich, N., I. Saporta-Eksten, H. Siu, and Y. Yedid-Levi (2021): "The macroeconomics of automation: Data, theory, and policy analysis," *Journal of Monetary Economics*, **122**, 1–16.

Loebbing, J. (2022): "An Elementary Theory of Directed Technical Change and Wage Inequality," *Review of Economic Studies*, **89** (1), 411–451.

Moll, B., L. Rachel, and P. Restrepo (2022): "Uneven Growth: Automation's Impact on Income and Wealth Inequality," *Econometrica*, forthcoming.

Polanyi, M. (1966): *The Tacit Dimension*, Doubleday.

Rosen, S. (1974): "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, **82** (1), 34–55.

Sattinger, M. (1975): "Comparative Advantage and the Distributions of Earnings and Abilities," *Econometrica*, **43** (3), 455–468.

——— (1993): "Assignment Models of the Distribution of Earnings," *Journal of Economic Literature*, **31** (2), 831–880.

Stokey, N. L. (2018): "Technology, Skill, and the Wage Structure," *Journal of Human Capital*, **12** (2), 343–384.

Teulings, C. N. (1995): "The Wage Distribution in a Model of the Assignment of Skills to Jobs," *Journal of Political Economy*, **103** (2), 280–315.

——— (2000): "Aggregation Bias in Elasticities of Substitution and the Minimum Wage Paradox," *International Economic Review*, **41** (2), 359–398.

——— (2005): "Comparative Advantage, Relative Wages, and the Accumulation of Human Capital," *Journal of Political Economy*, **113** (2), 425–461.

Teulings, C. N. and P. A. Gautier (2004): "The Right Man for the Job," *Review of Economic Studies*, **71** (2), 553–580.

Tinbergen, J. (1956): "On the Theory of Income Distribution," *Weltwirtschaftliches Archiv*, **77** (2), 156–173.

Topkis, D. M. (1998): *Supermodularity and Complementarity*, Princeton University Press.

Wiener, N. (1950): *The Human Use of Human Beings: Cybernetics and Society*, Houghton Mifflin.

Zeira, J. (1998): "Workers, Machines, and Economic Growth," *Quarterly Journal of Economics*, **113** (4), 1091–1117.