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An analysis of net-outcome contracting with applications to equity-based compensation

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Abstract
Options, restricted stock, bonuses tied to total shareholder return, and similar equity-based compensation contracts stipulate payments that depend on stock price. Any such contract is a function of shareholder value net of the compensation payment, because stock price (1) is proportional to this net value or "net outcome" and (2) anticipates compensation-related payments and dilution. The net outcome, in turn, is reduced by the payment and so depends on the contract. Standard moral hazard analyses, wherein contractual payments are based on the gross outcome before any payment to the agent, overlook this dependency. We characterize the optimal net-outcome contract, describe its shape and pay-for-performance sensitivity, contrast it with the optimal gross-outcome contract, and discuss implications for equity-based compensation arrangements.

Keywords Dilution · Moral hazard · Team compensation · Optimal contracting · Pay-for-performance sensitivity · Stock option

JEL classification D86 · J33 · M41

1 Introduction

In most corporations, the lion's share of incentives to senior executives takes the form of equity-based compensation, such as restricted stock, performance shares, stock

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options, or bonuses tied to total shareholder return. A distinct feature of these arrangements is that the performance measure is a function of the executive's compensation payment. For example, stock options awarded to an executive are contingent claims that, if exercised, dilute the ownership interest of the shareholders and thus diminish shareholder value. Stock price is reduced because this dilution is anticipated, illustrating how the compensation arrangement (i.e., stock options) affects the performance measure (i.e., stock price). The nonlinear payoff structure implied by stock options leads to intricate interactions between compensation and the performance measure. Because shareholder value (defined as the stock price times the number of shares outstanding) is net of the value of equity-based compensation payments (i.e., a net outcome), we refer to stock options (and similar equity-based compensation) as *net contracts*. In contrast, the archetypal contract in agency theory is based on a gross outcome before any compensation paid to the agent; we call such contracts *gross contracts*. We study the optimal net contract in a standard Holmstrom (1979) moral hazard setting, characterize its slope, curvature, complexity, and pay-for-performance sensitivities (PPS), and contrast it with the optimal gross contract.

To illustrate how a gross contract and a net contract can be equivalent in substance but different in form, consider the classic sharecropping example in which the landowner is the principal and the farmhand is the agent. For simplicity, we suppress considerations of moral hazard and uncertainty and restrict attention to linear arrangements. A contract that specifies how much of the total crop the farmhand keeps is a gross contract. Such a contract might specify that the farmhand keeps one-quarter of the total crop. If the total crop is 12,000 bushels of corn, then the farmhand gets 3,000 bushels and the landowner gets the remaining 9,000 bushels. Suppose a bushel of corn sells for $1. The value of the crop will be $12,000 and the farmhand's compensation will be $3,000—which could be specified as a piece rate of 25 cents per bushel. This is also a gross contract. The value to the landowner who owns the crop, but must compensate the farmhand, is $9,000. To concretize the notion of a net contract, note that the farmhand's one-quarter share could equivalently be expressed as one-third of the net crop (i.e., one-third of 9,000 bushels of the total crop left to the landowner after the farmhand has taken his share). Further, suppose the landowner incorporates the farm and pays the farmhand a bonus equal to one-third of the value of the corporation. Then the value of the corporation is $9,000, and the bonus compensation is $3,000.

To show that stock options are akin to a bonus tied to the net value of the corporation, replace the bonus with stock options issued by the corporation to the farmhand. Suppose the landowner owns 900 shares of stock in the corporation and the corporation issues stock options with an exercise price of zero, thereby allowing the farmhand to acquire 300 shares. This is a net contract. The landowner anticipates that the farmhand will exercise the options, which will dilute the landowner's 100% interest in the corporation, implying a stock price of $10.00 per share (i.e., the value of the crop, $12,000, divided by the sum of the 900 shares issued to the landowner and the 300 new shares that will be issued to the farmhand upon option exercise). The compensation to the farmhand is stock, worth $3,000, acquired on exercise of the options. Alternatively, the corporation could issue 1,200 shares of stock in total, 900 to the landowner and 300 to the farmhand, so that, from the start, the equity stake of the farmhand is 25%, also worth $3,000. This is a gross contract. Although the net and gross contracts are economically equivalent, they have quite different slopes: 33% of the 900 shares outstanding with a net contract and 25% of the 1,200 shares outstanding with a gross contract, respectively.

While the equivalence of the gross and net contracts in the linear case above is intuitive, complications arise in practice. First, much equity-based compensation is nonlinear in stock price (in particular, the exercise price of stock options generally is not zero), so that shareholder value depends on the value of nonlinear compensation, which itself is a nonlinear function of shareholder value. Second, for an actual corporation, the outcome is the entire stream of future cash flows. Measuring these values is hard in practice because the shareholder value implied by the stream of future cash flows to the shareholders and the value of the stream of future compensation to the executives are uncertain. Third, some forms of equity-based compensation (such as stock options) are dilutive, while others (such as stock appreciation rights or a bonus based on total shareholder return) are not. Both forms, however, transfer value to employees from existing shareholders and so reduce shareholder value.

Given moral hazard and uncertainty, a direct characterization of the optimal net contract appears—to us, at least—infeasible because of the interactions between net outcome and the compensation implied by the net contract. Nevertheless, we obtain a characterization of the net contract by reference to a notional gross outcome and the associated notional gross contract. The notional gross contract allows us to characterize the optimal net contract as a well-defined function of the net outcome.

We characterize the optimal net contract's slope and its degree of convexity (or concavity). We find that the optimal net contract is convex if the agent's relative risk aversion is small, whereas it is concave if the relative risk aversion is large. This finding implies that the optimal net contract is convex with square root utility, linear with logarithmic utility, and concave with negative exponential utility. We also find that the convexity of the optimal contract varies with the level of the net outcome, first decreasing and then increasing in the net outcome. This finding implies that compensation increases at an increasing rate at higher levels of net outcome.

Our characterization of the optimal net contract has two applications. In the first, we determine the set of instruments and the weights on these instruments in the optimal net contract. An optimal contract can be thought as a mix of stock and stock options awarded to the agent. We find that the optimal net contract requires at least one additional instrument and a higher ratio of option-like instruments to stock than the optimal gross contract. The reason is that the net contract exhibits a greater degree of convexity than the gross contract. In practice, greater convexity can be achieved by (1) granting options with different maturities and exercise prices, (2) increasing the number of equity instruments, (3) tying the number of options that are granted to specific performance conditions, or (4) allowing for reload options.[1]

In the second application, we show how conventional pay-for-performance sensitivity (PPS) measures differ between economically-equivalent gross and net contracts,

[1] Larcker and Tayan (2012) document premium options in US executives' pay packages. Likewise, Canil and Rosser (2007) report that, in a representative sample of Australian corporations, 39% of option grants were granted at a discount, 29% were granted at a premium, and 32% were granted at the money.

and so do not meet the SEC's objective that disclosed pay-for-performance relations be comparable across firms with different compensation arrangements and performance measures. This finding applies whether pay is defined narrowly, as the flow of compensation in the current period, or broadly, as the change in the executive's wealth. When PPS is defined as the \$-change in compensation per \$-change in shareholder value (this is the measure used by Demsetz and Lehn 1985; Jensen and Murphy 1990; and Yermack, 1995; among others), the PPS of the net contract is always larger than the PPS of the gross contract. However, when PPS is defined as the %-change in compensation per %-change in shareholder value (as does Murphy 1986), whether the PPS of the net contract is larger or smaller than the PPS of the gross contract depends on the magnitude of the executive's fixed compensation.

In an extension, we study the case that the agent is endowed with security holdings at the contracting stage. We show that the slope and the curvature of the combined pre-existing security holdings and net contract together exhibit the same properties as in the base case. In another extension, we consider the total compensation offered to a team of employees (e.g., top management). For a self-disciplining team whose members have identical preferences and whose efforts are perfect substitutes, the slope and the curvature of the optimal net team contract exhibit the same properties as in the base case. Thus insights gained from the applications hold in these settings as well.

Our results speak to empirical studies on compensation that estimate an executive's PPS using net performance measures such as stock returns, but interpret their results in light of theoretical analyses that are based on gross measures. Because the PPS of economically-equivalent net and gross contracts differ, cross-sectional comparisons should consider whether observed differences in PPS are driven, in part, by the propensity of some corporations to offer compensation primarily in net versus gross form (loosely speaking, corporations awarding stock options versus corporations with profit-sharing plans or in which founder-CEOs are motivated primarily by the stock they already own).

Our paper relates to analytical work by Aseff and Santos (2005), Dittmann and Maug (2007), and Kadan and Swinkels (2008), who study incentive effects of stocks and options as piecewise linear payment schemes. Other research acknowledges that stock price is a net performance measure but, for the sake of simplicity, ignores the dilutive effect of compensation and studies compensation in terms of gross contracts (e.g., Hemmer, Kim, and Verrecchia 2000, Fn. 10). Bushman and Indjejikian (1993) study linear compensation contracts based on earnings and stock prices, where the stock price is the corporation's outcome less the compensation payment. Assuming linear contracts and allowing for relative performance evaluation, Black, Dikolli, Hofmann, and Pfeiffer (2021) investigate the bias in naïve OLS regression estimates of gross and net pay-for-performance sensitivities. Relative to the latter two papers, we study a broader class of utility functions, allowing for the characterization of optimal nonlinear contracts.

Section 2 presents relevant institutional arrangements. Section 3 describes the model. Section 4 studies properties of the optimal net contract. Section 5 describes implications for equity-based compensation arrangements. Section 6 extends the baseline model. Section 7 concludes. Appendix 1 presents a numerical example. Appendix 2 presents the proofs.

2 Institutional arrangements

Because shareholder value is the product of stock price times the number of shares outstanding, net contracts are those for which an executive's compensation (1) depends on the stock price and the anticipated future value transferred to the executive lowers present shareholder value or (2) has the potential to dilute the ownership interest of the existing shareholders. Stock options are a form of compensation that satisfy both criteria. Other common examples of net contracts are (i) cash bonuses based on stock price performance or similar instruments such as stock appreciation rights or phantom stock (because they satisfy the first criterion) and (ii) stock awards, grants of restricted stock units, and similar instruments, whether automatic or performance-based (because they satisfy the second criterion). Examples of gross contracts, which satisfy neither criterion, are cash bonuses based on metrics other than stock price (such as sales or nonfinancial metrics). Shares already owned by the executive are gross contracts because they entail neither a transfer of value from existing shareholders nor a dilution of shares outstanding.

The emphasis on net contracts and the dilution inherent in equity-based performance measures is pertinent to ongoing SEC rulemaking mandated by the Dodd-Frank Act. The SEC requires that corporations provide "information that shows the relationship between executive compensation actually paid and the financial performance of the issuer, taking into account any change in the value of the shares of stock and dividends of the issuer and any distributions."[2] This disclosure mandate necessitates careful consideration of the dilutive effects of compensation. To measure PPS consistently, standardization is necessary when (i) some companies award compensation in the form of a gross contract and others award compensation in the form of a net contract and (ii) the dilution due to net contracting is large.[3]

As an illustration, consider two startups that differ only in the stock and stock options they have issued. Because both are otherwise identical, the values of the companies to their respective claimants (i.e., shareholders and holders of contingent claims, namely options) are identical. Let this value be $100 million. In Company G (for gross contract), there are 10 million shares outstanding. Therefore the stock price of Company G is $10 per share. The CEO has been granted 2 million of these shares. A measure of PPS for the CEO of Company G would be the change in the CEO's wealth, expressed as a fraction of the change in shareholder value, if the value of the stock increases by one dollar. Because the CEO owns 2 million shares of stock, the CEO's

[2] The related SEC Proposed Rule Release is No. 34–7835 (dated April 29, 2015) of the Dodd-Frank Wall Street Reform and Consumer Protection Act § 953(a) (codified at 15 U.S.C. 78n(i))).

[3] Two papers that investigate the effects of dilutive equity-based compensation on valuation are those by Core, Guay, and Kothari (2002) and Li and Wong (2005). Core et al. (2002) show how the treasury-stock method of accounting for options understates the options' dilutive effect, with the result that earnings-based valuation models yield upwardly-biased estimates of the market value. Li and Wong (2005) find that ignoring the dilutive effect of employee stock options would overstate the stock price estimate by 4% to 11.8%.

wealth increases by \$2 million. Because there are 10 million shares outstanding, the value to all shareholders increases by \$10 million. Thus PPS is 20%.

In Company N (for net contract), there are 8 million shares outstanding and additional shares are available for issue. Company N's CEO has been granted options to buy two million shares at \$1 per share and also receives a salary of \$2 million dollars.[4] The shareholders of Company N anticipate the exercise of deep-in-the money options (for which the option delta is approximately one), meaning they anticipate the dilution of their ownership due to the issuance of an additional 2 million shares of stock when the CEO exercises the options. Because Company N is identical to Company G, except for its ownership structure, the underlying value is also \$100 million. The \$100 million valuation of Company N implies a per-share value of \$10, which is the \$100 million valuation for Company N divided by 10 million shares (i.e., the 8 million now outstanding and 2 million that will be issued on exercise of the options). A one dollar increase in the stock price implies a \$2 million increase in CEO wealth (i.e., an option delta of 1 multiplied by 2 million shares under option). Because there are 8 million shares outstanding, the value to current shareholders increases by \$8 million. Thus PPS is 25%.[5]

The forgoing illustration shows how the dilution inherent in a stock option contract and appropriately anticipated by the market yields PPS values that are higher than those for a comparable gross contract. This example, like the farmhand example above, is constructed so that both CEOs' compensation contracts are nearly linear. As will be shown in later sections, nonlinear compensation contracts induce additional complexities.

In addition to mandatory disclosures of the dilutive effects of compensation, nonmandatory heuristic measures of the costs of compensation are commonplace. For example, Institutional Shareholder Services, an independent assessor of corporate governance practices, computes measures of dilution, namely, equity "burn rates." These burn rates are multi-year averages of the corporate stock and options granted to employees that are expressed as a percentage of shares outstanding. The economic importance of net contracts is illustrated by the dilution implied by an all-equity burn rate of 2% per year over 5 years, which is common in some sectors: employees receive 9.6% of shareholder value (i.e., $1 - (1 - 0.02)^5$).

Tesla and its CEO, Elon Musk, constitute a remarkable example of highly-dilutive equity-based compensation in a public corporation. In 2009, Musk was granted a package of 20,135,920 options (some performance-based), which represented 8% of

[4] The CEO of Company N receives a salary of \$2 million but must pay \$2 million to Company N to exercise the options.

[5] This calculation of PPS accords with Murphy (2013, p. 234): "Effective percentage ownership for stock options is measured by weighting each option held by the executive at the end of the fiscal year by 'Option Delta' for that option (which varies according to the exercise price and time remaining to exercise), and dividing by the total number of shares outstanding."

Tesla's common stock on the date of its initial public offering in 2010.[6] By 2013, Tesla's stock price had increased fivefold, and Musk owned 27.54% of Tesla's common stock and held options representing a further 10% of common stock.[7] In 2018, Tesla's shareholders voted to again grant an outsize option package to Musk that could dilute the interest of Tesla's shareholders by a further 12%. The eligibility conditions for the first and second tranches of these performance options were met less than 3 years later.[8] Another prominent example is Google with its CEO, Eric Schmidt.[9]

For many startups and smaller corporations, compensation to managers and founders is a significant fraction of shareholder value and results in sizable dilution. While data on compensation arrangements in startups, which are private companies, is sparse, it is nevertheless important to understand their compensation arrangements. In part, this is because some of them have grown to become market behemoths. Gornall and Strebulaev (2020) describe the dilutive effects of the several rounds of financing that so-called unicorns experience along the path to becoming public companies. In pre-IPO financing, the option pool (i.e., the number of authorized, unissued shares available to be awarded as stock options) is typically selected to be in the range of 10% to 20% of the post-money authorized shares in each round.

[6] See Tesla's S-1 registration statement filed with the Securities and Exchange Commission on January 29, 2010.

[7] Tesla's 2013 proxy statement indicates 120 million shares outstanding, of which 33,076,212 (or 27.54%) are owned by Elon Musk. In addition, Musk holds options on (i) 6,711,972 shares from a December 4, 2009 options grant, (ii) 5,274,901 shares from an August 13, 2012 options grant. The options vest only if certain performance criteria are met. These two grants represent a potential dilution of 10%.

[8] Elon Musk's compensation under this second grant of stock options was valued at up to $50 billion. The compensation consists of 12 tranches of options that vest when various performance targets are met. Each tranche represents options to buy 1% of Tesla's outstanding shares (i.e., 1.69 million shares). See Kristen Korosec (March 22, 2018), "What Elon Musk's Compensation Deal Means for Tesla Motors," http://fortune.com/2018/03/22/elon-musk-compensation-tesla/ (accessed May 28, 2018); Robert Ferris (March 21, 2018), "Elon Musk could make more than $50 billion from pay plan shareholders approved ... but he has a lot to deliver," https://www.cnbc.com/2018/03/21/tesla-shareholders-approve-elon-musks-multibilion-dollar-compensation-plan.html (accessed May 28, 2018); and Tim Higgins (July 23, 2020), "Tesla Posts Fourth-Consecutive Quarterly Profit, Defying Pandemic Shutdown," https://www.wsj.com/articles/tesla-posts-fourth-consecutive-quarterly-profit-defying-pandemic-shutdown-11595450752 (accessed July 23, 2020).

[9] When Google hired Eric Schmidt as CEO in 2001, his employment agreement provided an option to purchase 14,331,708 shares of common stock at an exercise price of $0.30, an option to buy 426,892 shares of convertible (upon an IPO) preferred stock at a purchase price of $2.3425 per share, and an annual salary of $250,000. At the time of Google's IPO in 2004, Schmidt's equity stake amounted to 6% of the corporation and was valued at over $1 billion. See p. 35 of the Google Inc. Definitive Proxy Statement for the 2005 annual meeting of shareholders (https://www.sec.gov/Archives/edgar/data/1288776/000119312505072803/ddef14a.htm, accessed June 6, 2018) and "Google IPO Aims To Change the Rules" by Kevin J. Delaney and Robin Sidel, *The Wall Street Journal* (http://www.wsj.com/articles/SB108328345314098183, accessed July 25, 2016). After a restructuring in 2012, Google became Alphabet.

3 Model

We consider the optimal compensation contract in a standard moral hazard setting with a single contracting variable, namely the net outcome of the corporation (i.e., the corporation's residual net of the agent's compensation). The model setup is chosen to parallel Holmstrom (1979). At date 0, the risk-neutral principal (shareholder) and the risk-averse agent (manager) enter into a compensation contract that provides the agent with incentives to exert unverifiable and personally costly effort at date 1. At date 2, the net outcome is realized, the agent consumes the wealth from the contract, and the principal consumes the residual. We characterize the optimal compensation contract as a function of the corporation's net outcome and term this solution the optimal *net contract.*

To elaborate, $y \in \left[\underline{y}, \bar{y}\right] \subseteq \mathbb{R}^{+}$ denotes the corporation's net outcome (e.g., the value of a limited liability corporation) with the boundary values, $\underline{y}$ and $\bar{y}$. Only the net outcome is available as a verifiable performance measure to assess the agent's effort, $a \geq 0$, implying that the agent's performance measure is inseparable from compensation. The agent's preferences are represented by the utility function,

$$U(w, a) = u(w) - k(a), \tag{1}$$

where $u(w)$ is the agent's utility for wealth, w, and the cost of effort, $k(a)$, is increasing and convex in the agent's effort, a. The agent's utility for wealth is represented by a member of the Hyperbolic Absolute Risk Aversion (HARA) class of utility functions,

$$u(w) = \frac{1-\gamma}{\gamma}\left(b \cdot \frac{w}{1-\gamma} + h\right)^{\gamma}, \tag{2}$$

with $b, h > 0$ and $bw/(1 - \gamma) + h > 0$. This class encompasses, among others, the linear, the negative exponential, the logarithmic, and the square root utility functions.

The agent's wealth equals the compensation received, w, which is based on the corporation's net outcome, y. The risk-neutral principal chooses the net contract that maximizes the expected net outcome, $\mathrm{E}[y]$, subject to the agent's individual rationality and incentive compatibility constraints, ($\mathrm{IR_N}$) and ($\mathrm{IC_N}$). The ($\mathrm{IR_N}$) constraint assures that the agent receives at least a reservation utility of $\overline{U}$. The ($\mathrm{IC_N}$) constraint ensures that the agent implements the action stipulated by the principal. The optimal net contract is the solution to the following problem:

$$\begin{aligned} (\mathrm{P_N}) : \ & \max_{w(y),a} \ \mathrm{E}[y] \\ & \text{s.t.} \\ & (\mathrm{IR_N}){:}\ \mathrm{E}[U(w(y), a)] \geq \overline{U} \\ & (\mathrm{IC_N}){:}\ a \in \operatorname{argmax}\{\mathrm{E}[U(w(y), a)]\}. \end{aligned} \tag{3}$$

Specifying (3) for a particular principal/agent relation requires a description of how the agent's effort affects net outcome. Because the net outcome is a function of the agent's compensation, which itself is a function of the net outcome, a statement about the relation between a and y requires a specification of $w(y)$, prior to solving (3). Also, the expectations in (3) are taken with respect to a density function on a and y and therefore depend on the specification of $w(y)$.

Given these complications, the optimal net contract in (3) cannot be determined directly. To overcome these complications and to describe the consequences of the agent's effort, we introduce the *notional* gross outcome before compensation, $x \in [\underline{x}, \overline{x}] \subseteq \mathbb{R}^+$. Specifically, the density $f(x,a)$ of the notional gross outcome is parameterized by the agent's effort and describes how a affects x. The density is common knowledge and satisfies usual properties such as first-order stochastic dominance. This approach allows us to specify an equivalent agency problem based on the notional gross outcome and to derive as the optimal solution the *notional gross contract*, $c(x)$. Then we characterize the optimal net contract, $w(y)$, as a function of the optimal notional gross contract, $c(x)$. Thus we exploit the property that the net outcome equals the notional gross outcome less the optimal compensation corresponding to the gross outcome; that is, $y = x - c(x)$. The relation between a and y follows in equilibrium, given the optimal net contract. While our solution technique requires consideration of the gross outcome, it is important to note that the gross outcome is a notional variable.

Before we state the equivalent agency problem based on the notional gross outcome, we provide some intuition. Fig. 1 illustrates the relation between gross and net outcomes (i.e., the mapping from x to $y = x - c(x)$) for the case that the notional gross contract is strictly convex in the gross outcome. It is apparent from Fig. 1 that two complications arise in constructing the notional gross contracting problem. One is at the left end of the domain of x, and the other is at the right end. On the left, a gross contract could exist for which $c(x)$ is greater than x, which would imply $y < 0$. While it is routine in agency problems to assume that the principal has deep pockets so that gross contracts can provide payoffs to the agent that in bad states exceed the value of the project in that state, this is inconsistent with the notion that y is the value of a limited liability corporation whose value is bounded below by zero. On the right, beyond the critical value, $x_{\max}$, a convex gross contract could award the agent more than a one unit increase in compensation for a unit increase in outcome so that $c'(x) \geq 1$. At this point, $x - c(x)$ is a decreasing function of x. Thus, if the domain of gross outcome extends higher than $x_{\max}$, then there are two gross outcomes that relate to the same net outcome, so that there is not a one-to-one mapping between x and y. In summary, the notional gross contracting problem corresponding to the original net contracting problem includes bounds that are not present in the typical agency problem.

The optimal notional gross contract is the solution to the following agency problem based on the notional gross outcome; that is,

$$
\begin{aligned}
&(\mathrm{P_G}): \max_{c(x),a} \mathrm{E}[x - c(x)] \\
&\text{s.t.} \\
&(\mathrm{IR_G}): \quad \mathrm{E}[U(c(x),a)] \geq \overline{U} \\
&(\mathrm{IC_G}): \quad a \in \operatorname{argmax}\{\mathrm{E}[U(c(x),a)]\} \\
&(\mathrm{IN_G}): \quad c'(x) < 1 \\
&(\mathrm{NN_G}): \quad x - c(x) \geq 0.
\end{aligned}
\tag{4}
$$

Here the agent's contract is based on the gross outcome, $c(x)$. The risk-neutral principal chooses the contract that maximizes the expected outcome net of the agent's

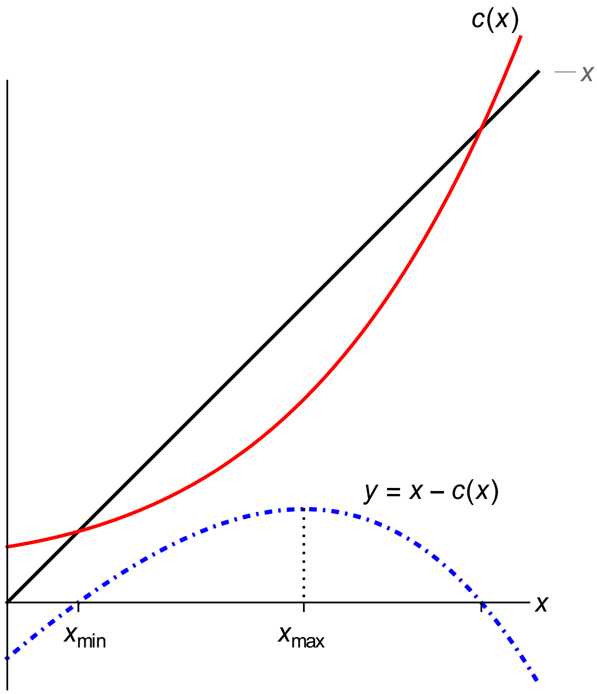

Fig. 1 Relation between notional gross outcome, compensation, and net outcome. Figure 1 depicts the notional gross contract, $c(x)$, as a function of the notional gross outcome, x, in the case of a strictly convex gross contract. The figure also depicts the associated net outcome, $y = x - c(x)$, as a function of x. Two observations emerge. (i) If $x < x_{\min}$, then the notional gross contract provides the agent with more compensation than the notional gross outcome, implying $y < 0$. (ii) If $x > x_{\max}$, then the agent's marginal compensation exceeds the marginal increase in the notional gross outcome, $c'(x) > 1$, implying that the relation between x and y is not unique. As explained in the text, the net contract is well defined if $\underline{x} \geq x_{\min}$ and $\overline{x} \leq x_{\max}$.

compensation, $E[x - c(x)]$, subject to the agent's individual rationality and incentive compatibility constraints, (IR_G) and (IC_G), and the invertibility and nonnegativity constraints, (IN_G) and (NN_G). The invertibility constraint ensures that y is an invertible function of x and implies that $c(x)$ is continuous. The nonnegativity constraint ensures that $x - c(x) = y \geq 0$.[10]

Our subsequent analysis shows that the optimal net contract from (3) follows by transforming the optimal notional gross contract from (4). For emphasis, we impose Assumptions 1 and 2 below that imply that the (IN_G) and (NN_G) constraints are satisfied. In this case, the agency problem in (4) equals the problem studied by Holmstrom (1979), and the optimal net contract follows from the familiar optimal

[10] The invertibility constraint appears in other research on optimal gross contracts. For instance, Innes (1990), Poblete and Spulber (2012), and Chaigneau, Edmans, and Gottlieb (2018) limit the contract slope to rule out situations where the principal sabotages the outcome and the agent borrows money to inflate the outcome.

gross contract. In Subsection 6.3, we relax Assumptions 1 and 2 and find that the characteristics of the optimal net contract are largely unaffected by the relaxation.

Assumption 1. The boundary value, $\underline{x}$, is sufficiently high and the boundary value, $\overline{x}$, is sufficiently low to ensure that the nonnegativity constraint is satisfied.

Assumption 2. The boundary value, $\overline{x}$, is sufficiently low such that $c'(x) < 1$ for all $x \in [\underline{x}, \overline{x}]$.

3.1 The optimal notional gross contract

We close this section by briefly presenting the solution to (4) when Assumptions 1 and 2 apply. For convenience, we assume that the costs of effort are sufficiently convex so that the first-order approach is valid.[11] We replace the (IC_G) constraint with the associated first-order condition. The optimal gross contract is characterized as follows (Holmstrom 1979):

$$\frac{1}{u'(\mathrm{c}(x))} = \lambda + \mu \cdot L(x, a), \tag{5}$$

where $\lambda > 0$ and $\mu > 0$ are the Lagrangian multipliers of the associated (IR_G) and (IC_G) constraints and $L(x,a) = f_a(x,a)/f(x,a)$ denotes the likelihood ratio. For simplicity, we assume a linear likelihood ratio, $L(x,a) = L_0(a) + L_1(a) \cdot x$, which implies $E[L(x,a)] = 0$, with $L_0(a) < 0$ and $L_1(a) > 0$ (Lambert 2001, p. 19).[12]

With HARA utility, Hemmer et al. (2000) show that the optimal contract takes the form:

$$c(x) = \frac{1-\gamma}{b} \cdot \left((b \cdot \lambda + b \cdot \mu \cdot L(x, a))^{1/(1-\gamma)} - h \right) \tag{6}$$

and is strictly increasing in the gross outcome; that is, $c'(x) > 0$.

Compensation arrangements of executives often include options that induce convexity in the payment structure. To study the convexity of contracts, Ross (2004) uses the logic of the Arrow-Pratt measure of absolute risk aversion (Pratt 1964). The curvature of the contract, M, captures the increase of the slope, relative to the slope, and equals the ratio of the second and first derivatives of the compensation function; that is,

$$M(c(x)) = \frac{c''(x)}{c'(x)}. \tag{7}$$

[11] Alternatively, we can impose standard sufficient conditions on the probability distributions to ensure the validity of the first-order approach (e.g., Jewitt 1988).

[12] Lemma 1 and Proposition 1 below do not rely on this assumption. A linear likelihood ratio holds for many distributions from the one-parameter exponential family of distributions such as exponential, normal, gamma, Poisson, and binomial (see Christensen and Feltham 2005, p. 69). As we assume a positive, bounded support for x, we consider truncated distributions. Truncating a distribution does not affect the linear likelihood ratio property.

Since $c'(x) > 0$, $M(c(x)) > 0$ indicates a convex contract, and $M(c(x)) < 0$ indicates a concave contract. Using expression (6) and following Hemmer et al. (2000), we get:

$$M(c(x)) = \frac{\gamma}{1-\gamma} \cdot \frac{\mu \cdot L_1(a)}{\lambda + \mu \cdot L(x,a)}. \tag{8}$$

Given $\lambda + \mu \cdot L(x,a) > 0$ from (5), the optimal gross contract is strictly convex if $\gamma/(1 - \gamma) > 0$ or, equivalently, if $\gamma \in (0,1)$, is linear if $\gamma \to 0$, and is strictly concave otherwise. In terms of the agent's relative risk aversion, $R(c) = -c \cdot u''(c)/u'(c)$, the optimal gross contract is strictly convex if $R(c) < bc/(bc + h)$, linear if $R(c) = bc/(bc + h)$, and strictly concave otherwise.

4 The optimal net contract

In this section, we characterize the optimal net contract and deduce its slope and curvature. To do so, we transform the optimal notional gross contract into the optimal net contract.

4.1 Characterization of the optimal net contract

To characterize the optimal net contract, we exploit the relation that the gross outcome less the compensation based on gross outcome equals the net outcome and define the function T_x: $X \to Y$ with $T_x(x) = x - c(x) = y$. Assumption 2 implies that the function is strictly increasing, $T_x'(x) = 1 - c'(x) > 0$, and so is invertible on the domain $x \in [\underline{x}, \overline{x}]$. A one-to-one mapping between x and y exists, implying that y is an equivalent statistic to x (Christensen and Feltham 2005, p. 78). We denote the inverse function $T_y : Y \to X$ with $T_y(y) = T_x^{-1}(T_x(x)) = x$. The optimal net contract is equivalent to the optimal notional gross contract state-by-state; that is,

$$w(y) = c(x) \text{ for all } x = T_y(y), \tag{9}$$

which we abbreviate as $w(y) = c(T_y(y))$. For completeness, recall that the net outcome is an equivalent statistic to the gross outcome, which implies that the probability distributions of x and y are equivalent; that is, $F(x,a) = F(T_y(y),a) = H(y,a)$ for all effort levels a. Thus the net contract in (9) induces the same effort level and ensures that agent and principal receive the same expected utility as with the optimal notional gross contract. The net contract in (9) is also optimal. Suppose to the contrary that the transformed net contract is dominated by another net contract. Such a net contract can be transformed into an equivalent gross contract, which would contradict the optimality of the gross contract in (9). The forgoing arguments prove Lemma 1.

Lemma 1. Characterization of the optimal net contract. *The optimal net contract is equivalent to the optimal notional gross contract state-by-state; that is, $w(y) = c(x)$ for all $x = T_y(y)$.*

All proofs are provided in Appendix 2.

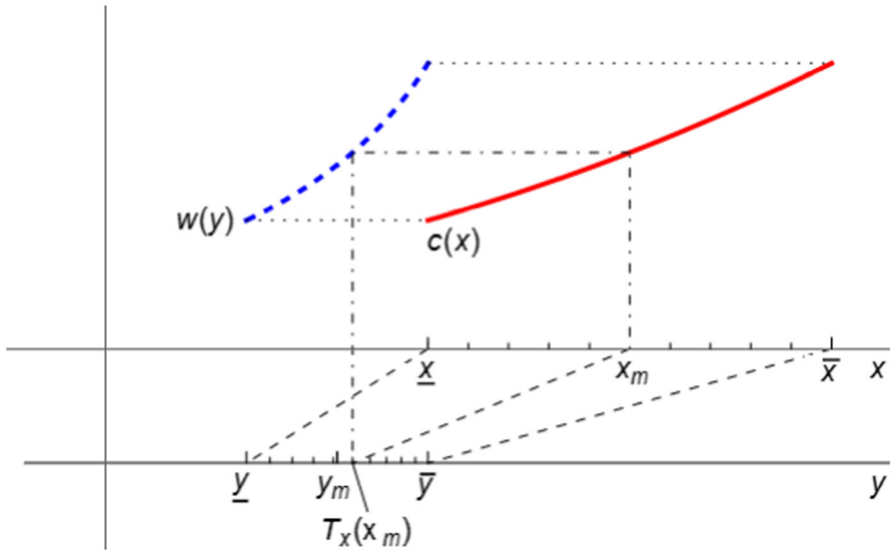

Fig. 2 Relation between the net contract and the notional gross contract. Figure 2 depicts the compensation of the notional gross contract, $c(x)$, and the compensation of the net contract, $w(y)$, for the domain of the notional gross outcomes, $[\underline{x}, \overline{x}]$, and the domain of net outcomes, $[\underline{y}, \overline{y}]$, respectively, as well as the compensation at the midpoints, $x_m = (\underline{x} + \overline{x})/2$ and $y_m = (\underline{y} + \overline{y})/2$. The related net outcome at the notional gross midpoint, x_m, is given by $T_x(x_m) = x_m - c(x_m)$.

Next, we study the slope and the curvature of the optimal net contract. To do so, we analyze how properties of the optimal notional gross contract carry over to the optimal net contract. Figure 2 depicts a convex notional gross contract and the corresponding net contract.

Following Lemma 1, the compensation provided by the net and gross contracts is the same, state-by-state. In the state where outcome is lowest, $\underline{x} - c(\underline{x}) = \underline{y}$ and $c(\underline{x}) = w(\underline{y})$. Similarly, in the state where outcome is highest, $\overline{x} - c(\overline{x}) = \overline{y}$ and $c(\overline{x}) = w(\overline{y})$. Given a strictly increasing gross contract, differences in net outcome are more compressed than differences in the related gross outcome; that is, $\overline{y} - \underline{y} = \overline{x} - \underline{x} - (c(\overline{x}) - c(\underline{x})) < \overline{x} - \underline{x}$. Hence the domain of net outcome, $[\underline{y}, \overline{y}]$, is more compressed than the domain of gross outcome, $[\underline{x}, \overline{x}]$. The compression implies that the optimal net contract exhibits a larger slope than the optimal notional gross contract.

Further, Fig. 2 illustrates that the state indexed by the midpoint of the gross outcome, $x_m = (\overline{x} + \underline{x})/2$, does not correspond to the state indexed by the midpoint of the net outcome, $y_m = (\overline{y} + \underline{y})/2$. With a strictly convex gross contract, the net outcome at the gross midpoint exceeds the net midpoint, $T_x(x_m) = x_m - c(x_m) > y_m$. This implies that the compensation at the gross midpoint exceeds the compensation at the net midpoint, $w(T_x(x_m)) > w(y_m)$. To obtain the same level of compensation, the net contract puts more emphasis on higher values of y, making the net contract highly convex in y.

Next we confirm our intuition formally. The slope of the net contract equals the change in compensation divided by the change in outcome. For two related outcomes,

$y_0 = x_0 - c(x_0)$ and $y_1 = x_1 - c(x_1)$, where $w(y_0) = c(x_0)$ and $w(y_1) = c(x_1)$ according to (9), the slope of the net contract is given by:

$$\frac{w(y_1) - w(y_0)}{y_1 - y_0} = \frac{c(x_1) - c(x_0)}{x_1 - x_0 - (c(x_1) - c(x_0))} = \frac{c(x_1) - c(x_0)}{x_1 - x_0} \cdot \frac{1}{1 - \frac{c(x_1) - c(x_0)}{x_1 - x_0}}. \tag{10}$$

Letting x_1 converge to x_0 gives the slope, $w'(y) = c'(x) \cdot (1 - c'(x))^{-1}$ for $x = T_y(y)$. We get the same result by applying the implicit function theorem that yields the slope $w'(y) = c'(T_y(y)) \cdot T_y'(y)$, with $T_y'(y) = (1 - c'(T_y(y)))^{-1}$ from the derivative of the inverse function T_x'. That is, the slope of the optimal net contract scales the slope of the optimal notional gross contract. In a similar fashion, we obtain $w''(y) = c''(T_y(y))/(1 - c'(T_y(y)))^3$ and the curvature of the net contract, $M(w(y)) = w''(y)/w'(y)$. Proposition 1 summarizes our prior discussion.

Proposition 1. Properties of the optimal net contract. *The optimal net contract*

(i) *is strictly increasing in net outcome with a positive slope of*

$w'(y) = \frac{c'(T_y(y))}{1 - c'(T_y(y))} > 0$; *and*

(ii) *exhibits the following curvature:*

$M(w(y)) = \frac{M(c(T_y(y)))}{(1 - c'(T_y(y)))^2}$, *where the agent's preferences determine* $c(T_y(y))$ *by way of (6).*

4.2 Contract curvature

In this subsection, we show how the agent's preferences determine the curvature of the net contract. Convexity induces higher benefits for the agent if a high outcome obtains, but imposes more risk on the agent. Balancing the benefits of stronger effort incentives with the costs of a higher risk premium, we conjecture that the optimal net contract is convex (concave) if the agent's relative risk aversion is low (high). To confirm our intuition, we note that Proposition 1 (ii) implies that the net contract is convex if and only if the notional gross contract is convex. Likewise, the net contract is concave if and only if the notional gross contract is concave. Expression (8) shows how the agent's preferences relate to the convexity of the gross contract. Exploiting this relation gives Proposition 2, where $R(w) = -w \cdot u''(w)/u'(w)$ denotes the agent's relative risk aversion.

Proposition 2. Curvature of the optimal net contract and the agent's risk aversion. *The optimal net contract is*

(i) *strictly convex if the agent's relative risk aversion is sufficiently low (i.e., if* $R(w) < bw/(bw + h)$*, or, equivalently, if* $0 < \gamma < 1$*);*
(ii) *linear if the agent's relative risk aversion is intermediate (i.e., if* $R(w) = bw/(bw + h)$*, or, equivalently, if* $\gamma \to 0$*); and,*
(iii) *strictly concave if the agent's relative risk aversion is sufficiently high (i.e., if* $R(w) > bw/(bw + h)$*, or, equivalently, if* $\gamma < 0$ *or* $\gamma > 1$*).*

For example, Proposition 2 implies that the optimal net contract is convex with square root utility, linear with logarithmic utility, and concave with negative exponential utility.

Next we study how the curvature of the net contract changes with the net outcome. Of particular interest is whether the contract's convexity is greater when the net outcome is more favorable. Such a contract provides the agent with compensation that increases at an increasing rate for the most extreme favorable outcomes. In practice, such a compensation arrangement can be implemented by granting the executive tranches of premium stock options with exercise prices above the grant-date stock price. Increasing convexity in the net outcome suggests that the agent receives ever larger tranches of premium stock options.

Proposition 3 below shows that the convexity of a convex optimal net contract first decreases and then increases in the net outcome. The net contract requires ever larger compensation payments for larger values of outcome. Correspondingly, in Fig. 2, the compression associated with net outcome contracting is greater at higher values of y.

We define the variation in the curvature of the net contract as the ratio of the marginal curvature and the curvature:

$$\Delta M(w(y)) = \frac{M'(w(y))}{M(w(y))}. \tag{11}$$

The convexity of a convex contract and the concavity of a concave contract increase if $\Delta M > 0$ and decrease otherwise. The variation in the curvature of the optimal net contract is given by:

$$\Delta M(w(y)) = -\frac{1 - \frac{1+\gamma}{1-\gamma} c'\left(T_y(y)\right)}{\left(1 - c'\left(T_y(y)\right)\right)^2} \cdot \frac{\mu L_1(a)}{\lambda + \mu \cdot L\left(T_y(y), a\right)}, \tag{12}$$

as shown in the proof of Proposition 3.

Proposition 3. Variation in net contract curvature

(i) *When the agent's relative risk aversion is low, the convexity of the optimal net contract decreases in net outcome when net outcome is small and increases otherwise (i.e., if* $R(w) < bw/(bw + h)$*, then* $\Delta M \leq 0$ *for* $y \leq y_c$ *and* $\Delta M \geq 0$ *for* $y \geq y_c$*, where* y_c *is given by* $c'(T_y(y_c)) = (1 - \gamma)/(1 + \gamma)$*);*
(ii) *When the agent's relative risk aversion is high, the concavity of the optimal net contract decreases in net outcome (i.e., if* $R(w) > bw/(bw + h)$*, then* $\Delta M < 0$ *for any* y*).*

5 Comparison between net and gross contracts with applications

In the previous section, the notional gross contract was a hypothetical construct that allowed us to derive the optimal net contract. As outlined in the introduction, some corporations provide incentives primarily via stock options and restricted stock (i.e., net contracts), whereas other corporations provide incentives primarily via profit-sharing arrangements or pre-existing stock ownership (i.e., gross contracts). In this section, we compare properties of equivalent net and gross contracts. Conceptually, these contracts differ in whether the contracting variable includes or excludes the agent's compensation. Comparing equivalent net and gross contracts illustrates how the form of compensation is a determinant of the apparent relationship between pay and performance.[13] To acknowledge this change in focus, we drop the modifier "notional" in referring to the gross contract.

The results in Section 4 connect the apparent forms of optimal, equivalent net and gross contracts. Proposition 1 implies that the net contract is linear if the gross contract is linear and the net contract is convex (concave) if the gross contract is convex (concave). The net contract exhibits a larger slope and a higher degree of curvature than the gross contract. Proposition 3 shows that the convexity of a convex optimal net contract increases for higher values of net outcome. In contrast, the convexity (concavity) of a convex (concave) gross contract is decreasing in the outcome. Corollary 1 summarizes these insights and the proof provides details.

Corollary 1. Slope and curvature of net versus gross contracts *For economically equivalent optimal net and gross contracts,*

(i) *the net contract exhibits a larger slope than the gross contract;*
(ii) *the net contract is more convex (concave) than the gross contract;*
(iii) *when the agent's relative risk aversion is low and outcome is sufficiently large, the convexity of the net contract increases in net outcome, whereas the convexity of the gross contract decreases in gross outcome. When the agent's relative risk aversion is high, the concavity of the net and gross contracts decreases in the respective outcome.*

Part (iii) of Corollary 1 shows that differences in the convexity of net and gross contracts are particularly pronounced for high outcomes. As outlined in Section 2, an example of a net contract with high convexity at more favorable outcomes is Tesla's 2018 award to Musk of tranches of performance-based options. Importantly, increasing convexity with outcome is a consequence of basing pay on the net outcome. As the convexity of the benchmark gross contract always decreases in outcome, increasing convexity with outcome is inconsistent with implications from the familiar gross contract.

[13] Recall from the example in Section 2 that, although the gross and net contracts were economically equivalent, the PPS for Company G was 20% and the PPS for Company N was 25%.

5.1 Instruments

This subsection provides theoretical insights into the instruments that implement equity-based compensation contracts and compares the results with the equivalent gross contract. Our analysis shows that the net and gross contracts differ in terms of the required instruments, their respective award amounts, and their relative weights. Following Corollary 1, the optimal net contract is more convex than the optimal gross contract. Our analysis shows that higher convexity manifests in two ways: relative to the optimal gross contract, the optimal net contract uses at least one additional equity instrument and the ratio of option-like instruments to stock is higher, compared to the corresponding instruments of the optimal gross contract.

We first characterize the optimal gross contract and then derive the optimal net contract. Throughout this subsection, the agent's preferences are given by square root utility, which is in the category of low relative risk aversion in our previous results. From Propositions 1 and 2, the net and gross contracts are both convex. Given the coefficients α and β implied by (6), the optimal gross contract is quadratic,

$$c(x) = (\alpha + \beta x)^2 = f_G + v_G x + o_G x^2, \tag{13}$$

where $f_G = \alpha^2$ is the fixed payment, $v_G x = 2\alpha\beta x$ is a share of the gross outcome, and $o_G x^2 = \beta^2 x^2$ is a third instrument that is an increasing share of the gross outcome.[14] The quadratic form of (13) implies that the relative weights on the instruments satisfy $4 f_G o_G = v_G^2$ and $c'(x) < 1$ is ensured by $x < (1 - v_G)/(2o_G)$.

Next we study the equivalent net contract. Using $T_x(x) = x - (f_G + v_G x + o_G x^2) = y$ yields $x = [1 - v_G - \sqrt{1 - 2v_G - 4o_G y}]/[2o_G] = T_y(y)$, where invertibility requires $v_G < 1/2$ and that we consider the negative root. The gross contract in (13) can be restated in terms of net outcome, yielding the equivalent net contract, $c(T_y(y)) = f_G + v_G T_y(y) + o_G T_y(y)^2 = w(y)$. Using a Taylor series approximation allows us to display the net contract as a polynomial[15]:

$$w(y) = f_N + v_N y + o_N y^2 + h_N y^3 + O\left(y^4\right), \tag{14}$$

with coefficients
$f_N = \frac{2\left(1 - v_G - \sqrt{1 - 2v_G}\right)}{v_G^2} f_G$, $v_N = \frac{1 - \sqrt{1 - 2v_G}}{\sqrt{1 - 2v_G}}$, $o_N = \frac{o_G}{(1 - 2v_G)^{3/2}}$, and $h_N = \frac{2o_G^2}{(1 - 2v_G)^{5/2}}$.

The net contract can be interpreted as comprising a fixed payment, f_N, and variable compensation from stock, $v_N y$, and additional option-like instruments, $o_N y^2$ and $h_N y^3$, where v_N, o_N, and h_N represent the quantities of the corresponding financial instruments. In practical arrangements, the necessary convexity can be achieved with a basket of options having different maturities and exercise prices.

[14] In more detail, the square root utility is characterized by $\gamma = 1/2$, $b = 1/2$, and $h = 0$ in (2). The optimal gross contract defined in (6) implies $\alpha = (\lambda + \mu L_0(a))/2$ and $\beta = \mu L_1(a)/2$. As the quadratic form of (13) implies a convex relation between x and $c(x)$, the literature often interprets $o_G x^2$ as compensation from option-like instruments.

[15] While higher-order elements exist, terminating the Taylor series after the cubic element is sufficient to illustrate the difference from the gross contract.

In comparing the contracts, note that one additional instrument is necessary in the net contract and the ratio of option-like instruments to stocks (interpreted as the higher-order terms in the polynomial) is higher (i.e., $h_N > 0$ and $o_N/v_N > o_G/v_G$). Intuitively, the net contract is more convex than the gross contract and the higher convexity is introduced in this manner. Proposition 4 summarizes our findings.

Proposition 4. Instruments needed to implement net contracts *For an agent with square root utility, the approximation of the optimal net contract in (14) requires at least one additional option-like instrument and contains a higher ratio of option-like instruments to stock compared to the optimal gross contract in (13).*

From expression (13), the gross contract is a quadratic function, implying that the expected gross compensation, $E[c(x)]$, is a function of the mean and the variance (i.e., the first and second central moments of the distribution of the gross measure), but not of the skewness of the gross measure (i.e., the third central moment). In contrast, due to the additional instrument, the net contract is a higher-order polynomial, implying that the expected net compensation, $E[w(y)]$, is a function of the mean, the variance, and the skewness of the net measure.

5.2 Pay-for-performance sensitivity

This subsection provides theoretical insights into the pay-for-performance sensitivities that describe the relation between changes in executive compensation and changes in shareholder value. In accordance with SEC regulations, corporations must provide information about the relation between actual compensation and performance.[16] As in the empirical literature on executive compensation, PPS measures are estimated based on relatives of net outcome, such as stock price or total shareholder return. In contrast, the theoretical literature typically characterizes the optimal PPS measures based on gross outcome. In the following, we study properties of PPS for equivalent net and gross contracts.

We return to the general case of HARA utility functions, including optimal convex and concave contracts. Two of the most common measures of PPS are[17]:

$$PPS^I = \frac{\Delta \text{ compensation}}{\Delta \text{ shareholder value}} \quad \text{and} \quad PPS^{II} = \frac{\Delta \ \ln(\text{compensation})}{\Delta \ \ln(\text{shareholder value})}. \qquad (15)$$

PPS^I measures the dollar change in compensation per dollar change in shareholder value. PPS^{II} measures the percentage change in compensation for a percentage change in shareholder value and is termed the pay-for-performance elasticity.[18]

[16] In its draft rules to amend Item 402 of Regulation S-K (Release No.: 34–7835), the SEC proposes "that the compensation covered by the disclosure be 'executive compensation actually paid.'"

[17] Edmans, Gabaix, and Landier (2009) provide an overview of PPS measures. A third measure proposed by Holmstrom (1992) is PPS^{III} = Δcompensation/Δln(shareholder value). The properties of PPS^{III} are largely consistent with the properties of PPS^{II}.

[18] To capture an executive's incentives, empirical proxies for executive compensation in PPS measures are often based on the total change in executives' wealth, including the change in the value of pre-existing security holdings. As outlined in Subsection 6.1, our insights are largely unaffected by the presence of these holdings.

To provide a proper reference point for the empirical literature, we characterize the PPS measures of the optimal net contract. Corresponding to (15), PPS_N^I is the slope and PPS_N^{II} is the elasticity of the optimal net contract:

$$PPS_N^I = w^{'}(y) \text{ and } PPS_N^{II} = \frac{w^{'}(y)}{w(y)/y}. \tag{16}$$

Both PPS measures are functions of the realized net outcome and generally provide a local, not global, description of the relation between compensation and net outcome (i.e., shareholder value).

The PPS measures of the optimal gross contract are straightforward:

$$PPS_G^I = c^{'}(x) \text{ and } PPS_G^{II} = \frac{c^{'}(x)}{c(x)/x}. \tag{17}$$

A comparison of the PPS measures of the optimal net and gross contracts in (16) and (17) is misleading because it overlooks the distinction between net and gross outcomes. To compare the PPS measures, we restate (17) in terms of the net outcome, y:

$$PPS_G^I = \frac{w^{'}(y)}{1+w^{'}(y)} \text{ and } PPS_G^{II} = \frac{w^{'}(y)}{w(y)/y} \cdot \frac{1+w(y)/y}{1+w^{'}(y)} \text{ for } y = T_x(x) \cdot \tag{18}$$

To derive (18), note that gross outcome equals net outcome plus net compensation, $x = y + w(y)$. Further, $c(x) = w(y)$ and $c'(x) = w'(y)/(1 + w'(y))$ at $y = T_x(x)$. The proof of Proposition 5 below provides details.

The slope of the optimal net contract exceeds the slope of the optimal gross contract, implying that PPS_N^I in (16) always exceeds PPS_G^I in (18). The relation between PPS_N^{II} and PPS_G^{II} is more subtle. Because the net outcome is gross outcome less compensation, PPS_N^{II} is smaller than PPS_G^{II} if the compensation is sufficiently large (and thus net outcome is sufficiently small). Comparing (16) and (18) yields $PPS_N^{II} < PPS_G^{II}$ if $w(y)/y > w'(y)$. Proposition 5 summarizes.

Proposition 5. Pay-for-performance sensitivities *For* $y = T_x(x)$, *a comparison of the PPS measures in (16) and (18) yields:*

(a) $PPS_N^I > PPS_G^I$ *for all* y*;*

(b) $PPS_N^{II} < PPS_G^{II}$ *if and only if* $w(y)/y - w'(y) > 0$.

We use two examples to illustrate Proposition 5. First, with logarithmic utility, optimal net and gross contracts are linear and $PPS_N^{II} < PPS_G^{II}$ when the fixed payment of the net contract is positive.[19] That is, $w(y)/y = (f_N + v_N y)/y > w'(y) = v_N$ if $f_N > 0$.

[19] The example in Section 2 assumes a gross outcome of \$100 million and the CEO of Company G receives stock-based compensation of \$20 million. The resulting net outcome is \$80 million, and the CEO of Company N receives economically-equivalent option-based compensation of \$20 million. As outlined, $PPS_N^I = 0.25 > PPS_G^I = 0.20$, whereas $PPS_N^{II} = 0.25 \cdot (80/20) = 1$ and $PPS_G^{II} = 0.20 \cdot (100/20) = 1$ because the fixed payment is zero.

Second, with square-root utility, optimal contracts are convex. Panels A and B in Fig. 3 depict the optimal gross and net contracts.[20] Panel C plots the PPS measures (16) and (18) for the optimal net and gross contract as a function of net outcome. Consistent with Proposition 5, we get $PPS_N^I > PPS_G^I$ and $PPS_N^{II} < PPS_G^{II}$ for low values of net outcome.

The following implications emerge. (1) Because, in practice, PPS is largely driven by equity-based compensation, the empirical estimates of PPS in archival studies correspond to the measures described in (16). (2) Because PPS_N^{II} generally attains a wider range of values than PPS_G^{II}, in cases where CEO incentives exhibit significant convexity (e.g., when a large portion of the contract's value is due to near-the-money options or options with contingent performance vesting requirements), PPS_N^{II} is likely a naïve proxy of ex ante incentives. (3) Because the slope and the convexity of the optimal net contract both increase with net outcome, an (incorrect) naïve inference that low PPS causes low outcome would be more strongly supported by archival evidence of pay and performance realizations drawn from net contracts with substantial convexity. (4) Cross-sectional comparisons ought therefore to consider whether observed differences in PPS are driven, in part, by divergence in the propensities of corporations to provide compensation in net or gross form.

6 Extensions of the baseline model

In this section, we show how the analysis adapts to the cases of an agent with pre-existing security holdings and a team of agents. We also show that the characteristics of the optimal net contract are largely unaffected when Assumptions 1 and 2 are relaxed.

6.1 Pre-existing security holdings

In practice, incentives derive from both equity-based compensation awarded in the current period and from stock and option holdings granted in prior periods. Although we analyze a static model, our analysis offers insight into actual settings, which are dynamic. To connect the static model to actual practice, we observe that the agent is motivated partly by the change in value of pre-existing security holdings and partly by the flow of compensation granted in the current period, where the latter is regarded as the increment necessary to optimally motivate the agent.[21] Combined, these two parts provide the total effort incentives and the agent's incentive compatibility and individual rationality constraints are adjusted accordingly. Essentially, the principal rebalances the agent's total incentives and chooses the flow compensation to maximize the net outcome subject to the modified incentive compatibility and individual rationality constraints.[22] Our analysis shows that the curvature of the combined change in value of pre-existing

[20] The example in Figure 3 shows that the additional instrument in the net contract is important and the ratio of option-like instruments to stock is higher than the ratio of the corresponding gross instruments.

[21] New incentives may be necessary because the executive has exercised options and sold stock, or because previously-granted options have fallen deep out of the money.

[22] Rhodes (2016) is an example of an archival/empirical analysis that regards newly-awarded compensation as an increment that rebalances the agent's incentives to maximize shareholder value.

Panel A. Gross contract

$$c(x) = 0.3113 + 0.3698x + 0.1098x^2 \quad \text{for} \quad x \in [1,2].$$

Panel B. Net contract

$$w(y) = 0.5459 + 0.9598y + 0.8268y^2 + 0.6977y^3 + O(y^4) \quad \text{for} \quad y \in [0.2091, 0.5097].$$

Panel C. PPS measures for varying values of net outcome

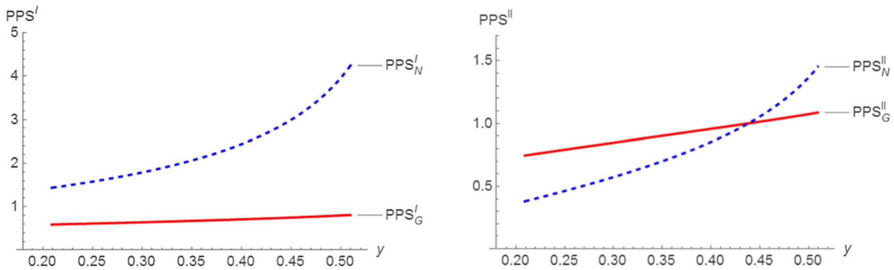

Fig. 3 Numerical example: PPS measures for convex contracts. Figure 3 presents the optimal net and gross contracts and the related pay-for-performance sensitivities (PPS) for a particular moral hazard problem, where the agent's preferences are characterized by square-root utility and uncertainty is represented by a truncated gamma distribution. Details are in Appendix A. Panels A and B show the optimal gross and net contracts as functions of the gross and net outcomes, *x* and *y*, respectively. Panel C depicts PPS as functions of the net outcome. For each graph in Panel C, we omit the gross outcome axis and present PPS_G as a function of the net outcome. In Fig. 2, the gross and net outcomes are represented by separate axes so that the mapping from the gross outcome to the related net outcome is explicit; here the mapping is implicit

security holdings and flow compensation has the same properties as described in Proposition 1.

The agent's incentives, in the absence of any contract, were assumed to be zero in the main analysis (i.e., endowed incentives are normalized to zero). If instead the agent already owns corporate stock or other equity compensation prior to date 0 that vest at date 2, the endowed incentives are nonzero. We denote the agent's date 2 wealth from pre-existing security holdings, which includes the change in value of the securities from date 1 to date 2, as $q(y)$. Corresponding to (3), the principal's problem is characterized by:

$$\begin{aligned}
&(P_N): \ \max_{w(y),a} \ \mathrm{E}[y] \\
&s.t. \\
&(IR_N): \ \ \mathrm{E}[U(w(y)+q(y),a)] \geq \widetilde{U} \\
&(IC_N): \ \ a \in \operatorname{argmax}\{\mathrm{E}[U(w(y)+q(y),a)]\}.
\end{aligned} \tag{19}$$

The agent's reservation utility, $\widetilde{U}$, is the utility from the liquidated value of the pre-existing security holdings along with the value to the agent of the best outside employment alternative. Technically, $\widetilde{U}$ is a constant in the principal's choice problem.

What matters in the contracting problem is that $\widetilde{w}(y) = w(y) + q(y)$ motivates the agent to implement the optimal second-best action. To achieve this, the compensation awarded incremental to $q(y)$ must result in total compensation of $\widetilde{w}(y)$. Since $q(y)$ is a constant for a given y, pointwise optimizing (19) with respect to $w(y)$ is equivalent to pointwise optimizing with respect to $\widetilde{w}(y)$. Consequently the solution to (19) corresponds to the solution of (3).

To determine $\widetilde{w}(y)$, consider the equivalent notional gross contracting problem. Let $p(x)$ denote the agent's date 2 wealth from pre-existing security holdings as a function of gross outcome. Thus $p(x)$ is equivalent to $q(y)$ and, as before, $c(x)$ is equivalent to $w(y)$. Then the agent's utility is given by $u(c(x) + p(x)) - k(a)$. The principal is the residual claimant and chooses the optimal notional gross contract as the solution to the following agency problem; that is,

$$\begin{array}{ll} (P_G): & \max_{c(x),a} \; \mathrm{E}[x - c(x) - p(x)] \\ s.t. & \\ (IR_G): & \mathrm{E}[U(c(x) + p(x), a)] \leq U \\ (IC_G): & a \in \operatorname{argmax} \{\mathrm{E}[U(c(x) + p(x), a)]\} \\ (IN_G): & c'(x) + p'(x) < 1 \\ (NN_G): & x - c(x) - p(x) \geq 0. \end{array} \tag{20}$$

To derive the optimal notional gross contract, sum the flow compensation and the date 2 value of pre-existing security holdings and define $\widetilde{c}(x) = c(x) + p(x)$, which is analogous to (6). The curvature of the optimal notional gross contract and the pre-existing security holdings are $M(c(x)) = c''(x)/c'(x)$ and $M(p(x)) = p''(x)/p'(x)$.

We obtain the optimal combined net contract $\widetilde{w}(y)$ by noting that net outcome equals gross outcome less the payments from flow compensation and pre-existing security holdings; that is, $\widetilde{T}_x(x) = x - c(x) - p(x) = y$ with the inverse $\widetilde{T}_y(y) = \widetilde{T}_x^{-1}\left(\widetilde{T}_x(x)\right)$ $= x$. In line with Proposition 1, the slope of the optimal net contract scales the slope of the notional gross contract and equals

$$w'(y) = \frac{c'(x)}{1 - c'(x) - p'(x)} > 0 \;\text{ for all } x = \widetilde{T}_y(y). \tag{21}$$

Combined, the compensation and the change in value from pre-existing security holdings have a curvature equal to $M(w(y) + q(y)) = M(c(x) + p(x))/(1 - c'(x) - p'(x))^2$ for all $x = \widetilde{T}_y(y)$. The optimal net contract's curvature is adjusted accordingly:

$$M(w(y)) = \frac{M(c(x) + p(x))}{(1 - c'(x) - p'(x))^2} - \frac{p'(x) \cdot (M(p(x)) - M(c(x) + p(x)))}{c'(x) \cdot (1 - c'(x) - p'(x))} \;\text{ for all } x = \widetilde{T}_y(y). \tag{22}$$

We close this section by studying the effect of pre-existing security holdings on the PPS measures. Like Edmans et al. (2009), we replace "compensation" in (15) with "change in wealth" to obtain wealth-performance sensitivity measures, where the change in wealth includes both the change in value of the pre-existing security holdings and the compensation. Corollary 2 summarizes our findings.

Corollary 2. Properties of the optimal net contract with pre-existing security holdings *With pre-existing security holdings, the optimal net contract is* $w(y) = c(x)$

for all $x = \widetilde{T}_y(y)$ *with the slope and curvature outlined in (21) and (22). For any* $y = \widetilde{T}_x(x)$, *a comparison of the wealth-adjusted PPS measures yields* $PPS_N^I > PPS_G^I$; *further,* $PPS_N^{II} < PPS_G^{II}$ *if and only if* $w(y)/y - w'(y) > q'(y) - q(y)/y$.

The rankings of wealth-performance sensitivity measures in Corollary 2 are the same as the rankings of pay-performance sensitivity measures in Proposition 5. If the agent has pre-existing stock and option-like holdings (i.e., $q(y) = v_H y + o_H y^2$, where v_H and o_H denote the quantities of stock and option-like holdings, respectively), then the condition $PPS_N^{II} < PPS_G^{II}$ reduces to $w(y)/y - w'(y) > o_H y$. While this inequality is unaffected by the agent's stock holdings, greater option-like holdings will cause this inequality to be violated, implying $PPS_N^{II} > PPS_G^{II}$.

6.2 Team contracts

In many cases, net incentive contracts are also relevant for corporations that provide equity-based compensation to a group of employees, such as the top management team. When these incentive contracts cover many employees, dilution is substantial. This subsection describes how our single-agent analysis applies to a team of agents capable of monitoring each other. We show that the slope and curvature of the optimal net team contract exhibit the same characteristics as in Proposition 1. Consequently, the implications of the single agent analysis also apply to empirical settings where corporations provide compensation to a group of employees. The analysis formally connects the practitioner notion of burn rate (i.e., the dilution caused by total pay to the team) and shareholder value.

We start by establishing the notional team problem based on the corporation's gross outcome that depends on the efforts of N agents that work in a team and observe each other's effort. To preserve tractability, we consider identical agents with HARA utility preferences described by (1) and (2) and that the agents' individual efforts are perfect substitutes. The density of the notional gross outcome equals $f(x, a_1 + \ldots + a_N)$ with a linear likelihood ratio of $L_0(a_1 + \ldots + a_N) + L_1(a_1 + \ldots + a_N) \cdot x$. Identical agents imply identical contracts and identical induced level of efforts, i.e., $c_i(x) = c(x)$ and $a_i = a$ for all $i = 1, \ldots, N$. As in the single-agent problem (4), the optimal notional gross contracts for the team of agents solve the following agency problem:

$$\begin{aligned}
&(P_{GT}): \ \max_{c(x),a} \ \mathrm{E}[x - N \cdot c(x)] \\
&s.t. \\
&(IR_{Gi}): \ \mathrm{E}[u(c(x), Na)] - k(a) \geq \overline{U} \\
&\text{for all } i = 1, \ldots, N \\
&(IC_{Gi}): \ a \in \operatorname{argmax} \{\mathrm{E}[U(c(x), Na)] - k(a)\} \\
&\text{for all } i = 1, \ldots, N \\
&(IN_G): N \cdot c'(x) < 1 \\
&(NN_G): \ x - N \cdot c(x) \geq 0.
\end{aligned} \tag{23}$$

The agents' efforts parameterize the density of notional gross outcome and can be restated as $f(x, Na) = g(x, a)$, where a and g capture the effect of team effort on gross outcome. Using the fact that any utility function is well-defined subject to a positive-affine transformation, we restate the utility function from (1) such that $N^{\gamma} \cdot (u(c(x)) - k(a)) = u(N \cdot c(x)) - N^{\gamma}k(a)$. With individual rationality and incentive compatibility constraints being identical across agents, we recast the agency problem (23) such that the principal chooses team compensation, $C(x) = N \cdot c(x)$, and team effort to maximize the expected gross outcome less expected team compensation; that is,

$$\begin{aligned}
&\left(\widetilde{\mathrm{P}}_{\mathrm{GT}}\right): \ \max_{C(x),a} \ \mathrm{E}_g[x - C(x)] \\
&s.t. \\
&(IR_G): \ \mathrm{E}_g[u(C(x),a)] - N^{\gamma}k(a) \geq N^{\gamma}\overline{U} \\
&(IC_G): \ a \in \operatorname{argmax}\left\{\mathrm{E}_g[u(C(x),a)] - N^{\gamma}k(a)\right\} \\
&(IN_G): \ C'(x) < 1 \\
&(NN_G): \ x - C(x) \geq 0,
\end{aligned} \tag{24}$$

where $\mathrm{E}_g[\cdot]$ denotes expected value based on g. The optimal notional gross team contract is solved in the same way as program (4).

We obtain the optimal net team contract, $W(y)$, by noting that net outcome equals gross outcome less team compensation; that is, $\overline{T}_x(x) = x - C(x) = y$ with the inverse $\overline{T}_y(y) = x$. The slope and curvature of the optimal net team contract is obtained as in Proposition 1. Corollary 3 summarizes our finding.

Corollary 3. Properties of the optimal net team contract *The optimal net team contract is* $W(y) = C(x)$ *for all* $x = \overline{T}_y(y)$ *and has the following slope and curvature:* $W'(y) = \frac{C'\left(\overline{T}_y(y)\right)}{1 - C'\left(\overline{T}_y(y)\right)} > 0$ *and* $M(W(y)) = \frac{M\left(C\left(\overline{T}_y(y)\right)\right)}{\left(1 - C'\left(\overline{T}_y(y)\right)\right)^2}$.

Thus the insights for the PPS measures in Proposition 5 carry over to the optimal net team contract.

6.3 Optimal net contract with unbounded support

In this subsection, we show that the characteristics of the optimal net contract are largely unaffected when relaxing Assumptions 1 and 2. Assume that the gross outcome is nonnegative and may be unboundedly large, $x \in \mathbb{R}^+$. As before, we first determine the optimal notional gross contract in (4) and then transform it into the optimal net contract. For convenience, we restate the invertibility constraints in (4), ($\mathrm{IN_G}$): $c'(x) \leq z = 1 - \varepsilon$ for all $x \in \mathbb{R}^+$, where ε is an arbitrarily small number. The notional gross contract equals:

$$c(0) \leq 0 \ \text{ for } x = 0; \tag{25}$$

$$\frac{1}{u'(c(x))} = \lambda + \mu \cdot L(x,a) \ \text{ for all } x > 0 \ \text{ and } c'(x) < z; \tag{26}$$

$$c(x) = f_G + z \cdot x \text{ for all } x > 0 \text{ and } c'(x) = z. \tag{27}$$

When gross outcome equals the lower bound of zero, the nonnegativity constraint, (NN_G): $0 - c(0) \geq 0$, implies that the agent's compensation is not positive. When gross outcome is positive and the invertibility constraint does not bind, the gross contract has the same structure as in (5). When gross outcome is sufficiently large, the invertibility constraint binds and the slope of the gross contract equals $c'(x) = z$, implying that the gross contract is linear in gross outcome.

Since invertibility is preserved everywhere on the domain, we can transform the piecewise gross contract in (25)–(27) into an equivalent net contract over a right unbounded interval, $y \geq \underline{y} = 0 - c(0) \geq 0$. The payments provided by the net and the gross contract are the same, state-by-state. For the lower boundary value, (25) implies $w(\underline{y}) = c(0)$. Moving rightward, eq. (26) implies that the net contract has the same structure as the optimal net contract from Proposition 1 until the invertibility constraint binds. When the invertibility constraint binds, the gross contract in (27), $c(x) = f_G + z \cdot x$, implies that the net outcome is given by $y = x - c(x) = (1 - z)x - f_G$ and the net contract is linear in net outcome, $w(y) = [(1 + z)f_G + z \cdot y]/(1 - z)$. Finally, $y = (1 - z)x - f_G$ and unbounded positive support for x together imply unbounded positive support for y.

The transformed net contract is optimal. Suppose to the contrary that the transformed net contract is dominated by another net contract. Such a net contract can be transformed into an equivalent gross contract, which would contradict the optimality of the notional gross contract in (25)–(27).

Proposition 6 summarizes our finding.

Proposition 6. Optimal unbounded net contracts *For unbounded positive support* $y \geq \underline{y}$, *the optimal net contract is given by:*

$$w(y) = \begin{cases} c(0) & \text{for } \underline{y} \\ c(T_y(y)) & \text{for all } y > \underline{y} \text{ such that } c'(T_y(y)) < z \\ \dfrac{1+z}{1-z} \cdot f_G + \dfrac{z}{1-z} \cdot y & \text{for all } y > \underline{y} \text{ such that } c'(T_y(y)) = z. \end{cases}$$

We close this subsection by studying how the optimal net contract performs, relative to the optimal gross contract. With unlimited positive support $x \in \mathbb{R}^+$, the optimal gross contract is given by eq. (6) and is the solution to a program without the invertibility and nonnegativity constraints. In contrast, the optimal net contract in Proposition 6 is the solution to a program with both constraints. When the invertibility constraint binds, the optimal net contract is strictly dominated by the optimal gross contract. The invertibility constraint binds when the agent's relative risk aversion is low because the optimal gross contract is strictly convex and its slope exceeds one for large values of x. Proposition 7 summarizes.

Proposition 7. Optimal unbounded net versus gross contracts *When the agent's relative risk aversion is low, the optimal gross contract strictly dominates the optimal net contract for unbounded positive support of gross outcome.*

Proposition 7 implies that the best net contracting solution is no better than the best gross contracting solution. While it is sometimes more efficient (and never less efficient) to write contracts based on the gross outcome, note that gross outcome is often unavailable for contracting. Recall that shareholder value (and hence the stock price) is the value shareholders expect to receive, net of all costs, including compensation costs. Thus shareholder value is a net outcome whenever total compensation includes dilutive equity-based compensation, such as stock options. For public companies, this value is the market price determined by competitive stock traders. Therefore the net outcome is readily available for contracting. The gross outcome is not priced by competitive traders and so would have to be calculated (somehow) for it to be available for contracting. Adding back compensation to a corporation's market value to derive a notional gross outcome is hard, because the shareholder value implied by the stream of future cash flows and the value of the stream of future compensation are both uncertain. One reason for the ubiquity of net contracts is the ready availability of a market-determined, objective stock price. This stands in sharp contrast to the difficulty and subjectivity of the adding back calculation that would be required to implement a gross contract.

7 Conclusion

We provide insights into key characteristics of an important class of compensation arrangements, namely, net contracts. Net contracts are those for which an executive's compensation (1) depends on the stock price and the anticipated future value transferred to the executive lowers the present shareholder value or (2) has the potential to dilute the ownership interest of the existing shareholders. Examples of net contracts include stock options and restricted stock. To analyze the link between equity-based compensation and shareholder value, we construct and solve a novel agency program. We use this solution to characterize the net contract's shape and the variety of equity instruments needed to implement this shape. We extend the analysis to address executives' pre-existing security holdings and team-based compensation considerations. Our findings provide a potential explanation for highly convex contracts along with a varying number of equity instruments in startups and smaller corporations where executive compensation is a significant fraction of shareholder value. To provide insights into the executives' incentives, we also relate the contract shape to various empirical measures of pay-for-performance sensitivity.

Our analysis identifies challenges regulators face in writing rules mandating disclosure of the relation between equity-based compensation and performance, as well as challenges investors face in interpreting such disclosures. Computing pay-for-performance consistently across gross and net contractual forms—even when the contracts are economically-equivalent—is problematic.[23] In practice, this difficulty is compounded by the facts that (i) compensation arrangements for a single employee may include both gross and net elements; (ii) some of these elements may be nonlinear; (iii) companies have multiple employees, each with their own compensation arrangements; and (iv) pay-for-performance sensitivity is a local measure.

Our framework suggests avenues for future analytical research: while we consider pure net contracts, practical arrangements of executive compensation often entail a mixture of equity-based compensation and annual bonus plans using financial and nonfinancial performance measures, suggesting a mixture of net and gross contracting elements. Practical compensation arrangements also often cover multiple years. For example, given a stock option program with different maturities, granting and exercising stock options implies time-varying dilutive effects on executives' incentives, suggesting that measures of PPS vary over time. Our analysis of employee teams raises the interesting empirical question of how anticipated dilution (what practitioners estimate as burn rates) relate to corporate performance and measures of compensation cost prescribed by GAAP and the SEC.

Appendix 1: Numerical example

Considering a truncated gamma distribution, Fig. 3 illustrates our results in Subsection 5.2 for the case of square root utility. From an empirical perspective, the truncated gamma distribution is similar to the lognormal distribution and is often used in option pricing. Specifically,

$$f(x,a) = \frac{1}{a}\exp[-(x/a)]\frac{(x/a)^{n-1}}{(n-1)!}s(a) \text{ for } x \in [\underline{x}, \overline{x}], \tag{28}$$

where $a \geq 0$ is the agent's action, n is an integer greater than 1 that characterizes the skewness of the distribution, and $s(a)$ ensures $\int_{\underline{x}}^{\overline{x}} f(x,a)dx = 1$.

In Fig. 3, $n = 2, \underline{x} = 1, \overline{x} = 2$, $k(a) = a/20$, and $\overline{U} = 1$. The optimal action is $a^* = 0.7289$. We verified numerically that the optimal action is a global maximum.

[23] Harmonization of PPS measures across gross and net contracts is straightforward when the contracts are linear. One way to accomplish this is to convert gross pay-for-performance sensitivity to a net basis by removing from the denominator the manager-owned component of shareholder value. In the case of Company G in Section 2, this has the effect of changing the denominator from \$100 million to \$80 million. As a result, the PPS for companies G and N are both 25%.

Appendix 2: Proofs

Proof of Lemma 1. Follows as outlined in the text.

Proof of Proposition 1. With $y = x - c(x) = T_x(x)$ and the inverse function, $x = T_x^{-1}(y) = T_y(y)$, we get $w'(y) = c'(T_y(y)) \cdot T_y'(y)$ and $w''(y) = c''(T_y(y)) \cdot T_y'(y)^2 + c'(T_y(y)) \cdot T_y''(y)$, with $T_y'(y) = (1 - c'(T_y(y)))^{-1}$ and $T_y''(y) = c''(T_y(y))/(1 - c'(T_y(y)))^3$. Hence we get:

$$w'(y) = \frac{c'(T_y(y))}{(1-c'(T_y(y)))},$$
$$w''(y) = \frac{c''(T_y(y))}{(1-c'(T_y(y)))^2} + \frac{c'(T_y(y)) \cdot c''(T_y(y))}{(1-c'(T_y(y)))^3} = \frac{c''(T_y(y))}{(1-c'(T_y(y)))^3}, \text{ and} \tag{29}$$

$$M(w(y)) = \frac{w''(y)}{w'(\mathrm{y})} = \frac{c''(T_y(y))}{c'(T_y(y))} \frac{1}{(1 - c'(T_y(y)))^2} = \frac{M(c(T_y(y)))}{(1 - c'(T_y(y)))^2}.$$

Proof of Proposition 2. Since $c'(T_y(y)) < 1$, we get $\mathrm{sgn}[M(w(y))] = \mathrm{sgn}[M(c(x))]$. From the discussion around (8), we get $\mathrm{sgn}[M(c(x))] = \mathrm{sgn}[\gamma/(1 - \gamma)]$. Noting that

$$u'(w) = b\left(b \cdot \frac{w}{1-\gamma} + h\right)^{\gamma-1}, u''(w) = -b^2\left(b \cdot \frac{w}{1-\gamma} + h\right)^{\gamma-2}, \text{and } R(w)$$
$$= \left(\frac{1}{1-\gamma} + \frac{h}{bw}\right)^{-1} \tag{30}$$

provides the result.

Proof of Proposition 3. Note that the variation of the curvature is given by:

$$M(w(y)) = \frac{w''(y)}{w'(y)},\ M'(w(y)) = \frac{w'''(y)}{w'(y)} - \left(\frac{w''(y)}{w'(y)}\right)^2, \text{and } \Delta M(w(y))$$
$$= \frac{w'''(y)}{w''(y)} - \frac{w''(y)}{w'(y)}. \tag{31}$$

For net contracts, we get from (29):

$$w'''(y) = c'''(T_y(y)) \cdot T_y'(y)^3 + 3T_y'(y)T_y''(y)c''(T_y(y)) + c'(T_y(y)) \cdot T_y'''(y)$$
$$. \tag{32}$$

Using $T_y'''(y) = [c'''(T_y(y))(1 - c'(T_y(y))) + 3c''(T_y(y))]/(1 - c'(T_y(y)))^5$, we get:

$$w'''(y) = \frac{c'''\left(T_y(y)\right)\left(1 - c'\left(T_y(y)\right)\right) + 3c''\left(T_y(y)\right)^2}{\left(1 - c'\left(T_y(y)\right)\right)^5}. \tag{33}$$

For the gross contract, we obtain from (6):

$$\begin{aligned} c'(x) &= \mu L_1(a) \cdot (b \cdot (\lambda + \mu \cdot L(x,a)))^{\gamma/(1-\gamma)}, \\ c''(x) &= \mu^2 L_1^2(a) \cdot \tfrac{b\gamma}{1-\gamma} \cdot (b \cdot (\lambda + \mu \cdot L(x,a)))^{(2\gamma-1)/(1-\gamma)}, \text{ and} \\ c'''(x) &= \mu^3 L_1^3(a) \cdot \tfrac{b^2\gamma(2\gamma-1)}{(1-\gamma)^2} \cdot (b \cdot (\lambda + \mu \cdot L(x,a)))^{(3\gamma-2)/(1-\gamma)}. \end{aligned} \tag{34}$$

Using (29), (33), and (34) gives:

$$\begin{aligned} \Delta M(w(y)) &= \frac{w'''(y)}{w''(y)} - \frac{w''(y)}{w'(y)} \\ &= -\frac{1 - \frac{1+\gamma}{1-\gamma}c'(T_y(y))}{(1 - c'(T_y(y)))^2} \frac{\mu L_1(a)}{\lambda + \mu \cdot L\left(T_y(y), a\right)}, \end{aligned} \tag{35}$$

where $\mu L_1(a) > 0$ and $\lambda + \mu \cdot L(T_y(y),a) > 0$ by (5).

Note that $R(w) < bw/(bw + h)$ is equivalent to $0 < \gamma < 1$ and $R(w) > bw/(bw + h)$ is equivalent to $\gamma < 0$ or $\gamma > 1$. For $0 < \gamma < 1$, (i) follows from $\text{sgn}[\Delta M(w(y))] = \text{sgn}[(1 + \gamma)c'(T_y(y)) - (1 - \gamma)]$ and $0 < c'(T_y(y)) < 1$. For $\gamma < 0$ or $\gamma > 1$, (ii) follows from $1 - \frac{1+\gamma}{1-\gamma}c'\left(T_y(y)\right) > 0$.

Proof of Corollary 1. (i) and (ii) follow from Proposition 1. (iii) follows from Proposition 3 and, using (34), the variation of the curvature for the gross contract equals:

$$\Delta M(c(x)) = \frac{c'''(x)}{c''(x)} - \frac{c''(x)}{c'(x)} = -\frac{\mu L_1(a)}{\lambda + \mu \cdot L(x,a)} < 0, \tag{36}$$

since $\mu L_1(a) > 0$ and $\lambda + \mu \cdot L(x, a) > 0$ by (5).

Proof of Proposition 4. Follows as outlined in the text, with

$$\frac{v_N}{o_N} = \frac{\left(1 - \sqrt{1 - 2v_G}\right)(1 - 2v_G)}{o_G} \leq \frac{v_G}{o_G}. \tag{37}$$

The relation holds only with equality for $v_G = 0$.

Proof of Proposition 5. We define PPS like Edmans et al. (2009, p. 4891). For net contracts, with the shareholders' net return given by $(1 + r_y)\text{E}[y] = y$,

$$PPS_N^I = \frac{\Delta \text{ compensation}}{\Delta \text{ shareholder value}} = \frac{dw(y)}{dr_y} \Big/ \frac{dy}{dr_y} = \frac{\partial w(y)}{\partial y}\frac{\partial y}{\partial r_y} \Big/ \frac{\partial y}{\partial r_y} = w^{'}(y) \text{ and}$$
$$PPS_N^{II} = \frac{\Delta \text{ ln(compensation)}}{\Delta \text{ ln (shareholder value)}} = \frac{d\ln(w(y))}{dr_y} \Big/ \frac{d\ln(y)}{dr_y} = \frac{1}{w(y)}\frac{\partial w(y)}{\partial y}\frac{\partial y}{\partial r_y} \Big/ \left(\frac{1}{y}\frac{\partial y}{\partial r_y}\right) = \frac{w^{'}(y)}{w(y)/y}. \tag{38}$$

For gross contracts, with the shareholders' gross return $(1 + r_x)\text{E}[x] = x$, we get:

$$PPS_G^I = \frac{\Delta \text{ compensation}}{\Delta \text{ shareholder value}} = \frac{dc(x)}{dr_x} \Big/ \frac{dx}{dr_x} = \frac{\partial c(x)}{\partial x}\frac{\partial x}{\partial r_x} \Big/ \frac{\partial x}{\partial r_x} = c^{'}(x) \text{ and}$$
$$PPS_G^{II} = \frac{\Delta \text{ ln(compensation)}}{\Delta \text{ ln(shareholder value)}} = \frac{d\ln(c(x))}{dr_x} \Big/ \frac{d\ln(x)}{dr_x} = \frac{1}{c(x)}\frac{\partial c(x)}{\partial x}\frac{\partial x}{\partial r_x} \Big/ \left(\frac{1}{x}\frac{\partial x}{\partial r_x}\right) = \frac{c^{'}(x)}{c(x)/x}. \tag{39}$$

To restate the PPS measures of the optimal gross contract in terms of the net outcome, we note that the gross outcome equals the net outcome plus net compensation, $T_y(y) = y + w(y) = x$. Since $T_y^{'}(y) = 1 + w^{'}(y) > 0$, the inverse function exists, $T_x(x) = T_y^{-1}(T_y(y))$. The gross compensation is $c(x) = c(T_x(x)) = w(y)$, and the marginal gross compensation follows as $c^{'}(T_x(x)) \cdot T_x^{'}(x)$, with $c'(T_x(x)) = w'(y)$ and $T_x^{'}(x) = (1 + w^{'}(y))^{-1}$ from the derivative of the inverse function, $T_y^{'}$. Using these transformations in (17) gives (18).

Proof of Corollary 2. With $y = x - c(x) - p(x) = \widetilde{T}_x(x)$ and the inverse function, $x = \widetilde{T}_y(y)$, we get $w^{'}(y) = c^{'}\left(\widetilde{T}_y(y)\right) \cdot \widetilde{T}_y^{'}(y)$ and $w^{''}(y) = c^{''}\left(\widetilde{T}(y)\right) \cdot \widetilde{T}_y^{'}(y)^2 + c^{'}\left(\widetilde{T}_y(y)\right) \cdot \widetilde{T}_y^{''}(y)$, with

$$\widetilde{T}_y^{'}(y) = \left(1 - c^{'}\left(\widetilde{T}_y(y)\right) - p^{'}\left(\widetilde{T}_y(y)\right)\right)^{-1} \tag{40}$$

and

$$\widetilde{T}^{''}(y) = \left[c^{''}\left(\widetilde{T}_y(y)\right) + p^{''}\left(\widetilde{T}_y(y)\right)\right] / \left[1 - c^{'}\left(\widetilde{T}_y(y)\right) - p^{'}\left(\widetilde{T}_y(y)\right)\right]^3. \tag{41}$$

For sake of simplicity, we suppress the notion and get:

$$\begin{aligned} w^{'} &= \frac{c^{'}}{1-c^{'}-p^{'}}, \\ w^{''} &= \frac{c^{''}}{\left(1-c^{'}-p^{'}\right)^2} + \frac{c^{'}\cdot\left(c^{''}+p^{''}\right)}{\left(1-c^{'}-p^{'}\right)^3}, \text{ and} \\ \frac{w^{''}}{w^{'}} &= \frac{M(c+p)}{\left(1-c^{'}-p^{'}\right)^2} - \frac{p^{'}\cdot(M(p)-M(c+p))}{c^{'}\cdot\left(1-c^{'}-p^{'}\right)} \end{aligned} \tag{42}$$

with curvatures $M(c+p) = \frac{c^{''}+p^{''}}{c^{'}+p^{'}} = \frac{c^{'}\cdot M(c)+p^{'}\cdot M(p)}{c^{'}+p^{'}}$, $M(p) = \frac{p^{''}}{p^{'}}$, and $M(c) = \frac{c^{''}}{c^{'}}$.

Similar to (38) and (39), the wealth-performance sensitivity measures are given by:

$$\begin{aligned} PPS_N^I &= \frac{\Delta \text{ agent}'\text{s wealth}}{\Delta \text{ shareholder value}} = w'(y) + q'(y), \\ PPS_N^{II} &= \frac{\Delta \ \ln\left(\text{agent}'\text{s wealth}\right)}{\Delta \ \ln(\text{shareholder value})} = \frac{w'(y)+q'(y)}{[w(y)+q(y)]/y}, \\ PPS_G^I &= c'(x) + p'(x), \text{ and} \\ PPS_G^{II} &= \frac{c'(x)+p'(x)}{[c(x)+p(x)]/x}. \end{aligned} \tag{43}$$

As in the proof of Proposition 5, we use the inverse function $\widetilde{T}_y(y) = y + w(y) + q(y)$ $= x$ with $\widetilde{T}_y'(y) = 1 + w'(y) + q'(y)$ and $\widetilde{T}_x'(x) = \left(1 + w'(y) + q'(y)\right)^{-1}$, to restate the PPS measures of the optimal gross contract in terms of the net contract:

$$\begin{aligned} PPS_G^I &= \frac{w'(y)+q'(y)}{1+w'(y)+q'(y)} \text{ and} \\ PPS_G^{II} &= \frac{w'(y)+q'(y)}{1+w'(y)+q'(y)} \frac{y+w(y)+q(y)}{w(y)+q(y)}. \end{aligned} \tag{44}$$

A comparison of (43) and (44) provides the result.

Proof of Corollary 3. Follows from the arguments in the text.

Proof of Proposition 6. Note that the invertibility constraint, $c'(x) \leq z$, implies that $c(x)$ is continuous in $x \in \mathbb{R}^+$. As outlined, for $x = 0$, the nonnegativity constraint implies (25); that is, $c(0) \leq 0$. Note, $c'(x) < 1$ and $c(0) \leq 0$ ensure nonnegativity; that is, $x - c(x) > 0$ for all x, because

$$c(x) = \int_0^x c'(u)du + c(0) < \int_0^x 1 \cdot du + c(0) = x + c(0) \leq x. \tag{45}$$

Hence we can replace the (NN_G) constraint by the constraint $c(0) \leq 0$ and consider the Lagrange function

$$\begin{aligned} L = & \int_0^\infty (x - c(x))f(x,a)dx + \lambda\left(\int_0^\infty u(c(x))f(x,a)dx - k(a) - \overline{U}\right) \\ & + \mu\left(\int_0^\infty u(c(x))f_a(x,a)dx - k'(a)\right) + \varsigma(x)\left(z - c'(x)\right) - \eta c(0), \end{aligned} \tag{46}$$

with the Lagrange multipliers $\varsigma(x)$ for (IN_G) and η for $c(0) \leq 0$.

If (IN_G) is not binding, then $\varsigma(x) = 0$ and pointwise optimization yields for $x > 0$:

$$-f(x,a) + \lambda \cdot u'(c(x))f(x,a) + \mu \cdot u'(c(x))f_a(x,a) = 0, \tag{47}$$

implying (26). If (IN_G) is binding; that is, $c'(x) = z$, integration yields (27).

Proof of Proposition 7. For a binding (IN_G) constraint, the result follows immediately from Proposition 6. The (IN_G) constraint binds if $\lim_{x\to\infty} c'(x) > 1$. For low risk aversion $\gamma \in (0, 1)$, the gross contract is strictly convex and $c'(x)$ is increasing. Solving the equation $c'(x_c) = 1$, where $c(x)$ is given by (6), yields

$$x_c = \frac{(\mu \cdot L_1(a))^{-(1-\gamma)/\gamma} - b \cdot (\lambda + \mu \cdot L_0(a))}{b \cdot \mu \cdot L_1(a)} \tag{48}$$

and $c'(x) > 1$ if $x > x_c$.

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