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ORIGINAL PAPER

The role of epistemic emotions in undergraduate students' proof construction

Sandra Schubert[1] · Reinhard Pekrun[2,3,1] · Stefan Ufer[4]

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Abstract
Proofs as epistemic tools are central to mathematical practice, as they establish and provide explanations for the validity of mathematical statements. Considering the challenge that proof construction poses to learners of all ages, prior research has investigated its cognitive determinants, but the impact of affective-motivational experiences on proof construction has been insufficiently investigated. Emotions related to knowledge acquisition (i.e., epistemic emotions) are assumed to play a key role in epistemic processes. In this study we investigated how the performance of 80 mathematics undergraduate students in a geometric proof construction task relates to the epistemic emotions experienced during proof construction. Controlling for geometry knowledge, we included control and value appraisals as antecedents in our investigation of epistemic emotions, and attention and motivation as mediators of their effects on proof construction performance. The results indicate that positive as well as negative emotions are influenced by students' appraisals, also indicating an interaction of both appraisal dimensions. Primarily enjoyment and curiosity mediate the effects of these appraisals on attention and motivation. These two markers of the proof construction process, in turn, mediate the effects of enjoyment and boredom on proof construction performance. In this study we investigated systematically the role of epistemic emotions in geometric proof construction and we offer insights that complement the existing research on the cognitive determinants of proof performance. Moreover, this study extended research on epistemic emotions into the area of proof construction, an epistemic process central to mathematics.

Keywords Proof construction · Elementary geometry · Epistemic emotions · Control-value theory · Motivation

1 Introduction

Proving statements means to find systematic explanations for their validity based on agreed assumptions (Hanna, 2018). Proof construction is one of the central epistemic processes in mathematics. According to Stylianides (2007), a proof is a mathematical argument for or against a given mathematical claim, if it employs agreed forms of reasoning (e.g., deductive reasoning), draws solely on accepted assumptions and conclusions established from these assumptions (the framing theory), and is communicated in line with agreed norms. Dealing with multi-step proofs is challenging for both students and professional mathematicians (Heinze et al., 2008).

Understanding and supporting emotional processes involved in complex mathematical knowledge acquisition has recently experienced an upsurge in mathematics education research (Cai & Leikin, 2020; Schukajlow et al., 2017) and educational psychology research (Muis et al., 2015b; Munzar et al., 2021). Scholars have made calls to investigate jointly affective and cognitive processes, in order better to understand students' mathematics learning (Barnes, 2021; Schukajlow & Rakoczy, 2016). One focal interest pertains to emotions related to knowledge acquisition (Pekrun et al., 2017). Following Stylianides (2007), we consider proof construction as an epistemic process leading to a connected sequence of arguments supporting or refuting a

✉ Stefan Ufer
ufer@math.lmu.de

1 Department of Psychology, LMU Munich, Munich, Germany

2 Department of Psychology, University of Essex, Essex, England

3 Institute for Positive Psychology and Education, Australian Catholic University, Blacktown, Australia

4 Chair of Mathematics Education, LMU Munich, Munich, Germany

mathematical claim. *Epistemic emotions*, such as surprise, confusion, and curiosity, likely arise from engagement with a proof. They are considered to be of functional importance to students' mathematical thinking (Hannula, 2012; Schukajlow et al., 2017) and the use of heuristics (Gómez-Chacón, 2017); however, their role in proof construction has not yet been considered (Sommerhoff et al., 2015). Identifying factors that shape the emergence of individuals' epistemic emotions and investigating their role in proof construction are important for understanding and supporting students' engagement with proofs.

In this study we examined the epistemic emotions that arise when students are constructing a multistep geometry proof, and how these emotions shape markers of the proof construction process, such as cognitive resources and motivation, as described in Pekrun's (2006) cognitive-motivational model of emotions and learning. While achievement emotions have been investigated frequently in educational psychology, the role of epistemic emotions in epistemic processes has been investigated only recently (Muis et al., 2015a; Pekrun et al., 2017). Results of studies in different STEM disciplines suggest that control-value appraisals are relevant to the arousal of epistemic emotions (Dowd et al., 2015; Muis et al., 2015b; Munzar et al., 2021). We propose a model that describes the complex connection between experiences of cognitive incongruities and impasses during proof construction, and antecedents and consequences of epistemic emotions.

Despite recent advances, the suggested links between epistemic emotions, their cognitive antecedents, and relevant markers of epistemic processes have not yet been considered in the context of mathematical proof construction. Specifically, the mediational mechanism linking cognitive appraisals with proof construction performance via epistemic emotions has been investigated insufficiently (Loderer et al., 2020), and existing studies have produced inconsistent findings (Muis et al., 2015b). Therefore, we investigated whether differences in university students' epistemic emotions during a geometric proof construction task are attributable to differences in their control and value appraisals, and whether epistemic emotions mediated the links among cognitive appraisals, the markers of epistemic processes (attention and motivation), and proof construction performance.

1.1 Proof construction as an epistemic process

Mathematics scholars and educational curricula designers alike recognize that understanding the role of mathematical proofs and being able to engage in proof construction is fundamental to understanding mathematics. At the same time, dealing with proofs has been recognized as "hard-to-teach and hard-to learn" at all levels of education (Stylianides & Stylianides, 2017). The reason for these difficulties is inherent in the epistemic complexity entailed in the proof concept: a proof is not only a hallmark of verifying mathematical statements within the framing theory, but also a central tool for understanding why a statement holds (Hanna, 2018). This complexity relates to the construction of new mathematical insights in scientific mathematical practice, and also to the role of proof in individual learning processes: proof construction is one way to deepen a personal mathematical belief in a conjecture to a level of confidence, that relates to the epistemic stance of "knowing" (Greene et al., 2016). Thus, the primary epistemic functions of proof are to provide (mathematical) evidence for a claim based on agreed assumptions (definitions, axioms), and explicate mathematical knowledge about why the claim holds (Hanna, 2018).

Dealing with proofs, and specifically, constructing proofs, requires a complex set of epistemic activities, such as conjecture or hypothesis generation, evidence generation, and evidence evaluation (Sommerhoff et al., 2015; Schwarz et al., 2010) described the following three main subprocesses of proof construction: *exploration* to make sense of the given claim, represent its content, and ultimately produce conjectures that serve as intermediate claims in the proof construction process (Heinze et al., 2008); *systematization* to identify possible inferences based on statements from the framing theory to generate arguments toward these claims. These two subprocesses expose a critical cognitive gap that needs to be closed to arrive at an acceptable line of argument (Schwarz et al., 2010). Finally, *inscription* concerns transforming these mental arguments into an acceptable written proof. Even though this formal quality may be conceptualized independently of the content of a proof (Ottinger et al., 2016), the line of argument may depend on, or even be changed during, the proof construction process to allow for an acceptable inscription. The specific nature of these processes may differ depending on the mathematical content of the proof. For elementary geometry proofs, for example, the role of the figure at hand, and its visualization, have been discussed in prior works (Ufer et al., 2009). In summary, proof construction may be considered an epistemic process, in which new knowledge about a given claim is generated and represented, based on knowledge from a framing theory.

It is a long-standing observation that finding a line of argument for multistep proofs is a demanding epistemic process for many learners, that may lead to impasses, perceived contradictions, and cognitive dissonance (Heinze et al., 2008). Moreover, the students' epistemic goals during proof construction are based on their individual conceptions about the nature and purpose of the proof, which may deviate from the accepted norms (Chazan, 1993). This is likely to lead to insecurity and uncertainty among students about what is an acceptable proof in a specific context.

1.2 Epistemic emotions in proof construction

Epistemic emotions relate to knowledge generation and cognitive activities involved in knowledge acquisition and exploration (Vogl et al., 2019). Proof construction requires students to identify relevant task-related knowledge and integrate it into a new knowledge structure that establishes a coherent explanation of why a claim follows from agreed assumptions, and which can be inscribed as an acceptable proof. Given this direct link between proof construction, epistemic activities, and the corresponding epistemic goals, epistemic emotions should naturally arise in and influence proof construction processes.

Based on a continuous internal evaluation of expected and actual progress, epistemic emotions may arise when progress toward an epistemic goal is achieved or blocked, or, in other words, when new information and existing beliefs, knowledge structures, or recently processed information are found to be aligned or misaligned (Muis et al., 2018). Whether this is a perceived gap between individual knowledge of the framing theory and the arguments required in the proof, or failure to combine the available arguments from the framing theory into an acceptable proof, students are likely to experience impasses or uncertainty during proof construction, also labeled as *cognitive incongruities* (Fig. 1). These arise when an individual is confronted with interruptions to action plans, impasses, contradictions, anomalous events, dissonance, incongruities, unexpected feedback, or uncertainty (Berlyne, 1954). As argued above, they are likely to occur during proof construction, and they have been identified as the underlying central cause of the arousal of epistemic emotions (D'Mello & Graesser, 2012; D'Mello et al., 2014).

Furthermore, cognitive incongruities may arise from having to coordinate norms regarding the acceptable forms of reasoning (e.g., deductive reasoning), the available set of statements that may be used as reasons, and the acceptable forms of representing a proof. Students' individual evaluation of the constructed arguments and their representation may indicate that they fall short of the aspired norms (Schwarz et al., 2010). Thus, incongruities may also prevail in the form of epistemic uncertainty (Ellsworth, 2003), as it is often difficult for learners to judge the social norms of what makes a proof acceptable (Chazan, 1993; Stylianides, 2007). Therefore, students' understanding of how knowledge is justified in mathematics may be challenged during proof construction (Hofer, 2016).

All these processes may trigger different epistemic emotions, with the type of emotion depending on the type of cognitive incongruity experienced (Muis et al., 2018). Surprise is considered the initial reaction to information that is inconsistent with one´s prior knowledge. It may turn into curiosity or confusion depending on whether the incongruity resolves or persists. Similarly, epistemic enjoyment may be experienced when cognitive incongruity is resolved (Pekrun & Stephens, 2012) or when the proof construction process proceeds in line with personal epistemic beliefs (Muis et al., 2015a). However, if existing cognitive schemas or personal beliefs are strongly challenged by the insights made during proof construction, epistemic anxiety may ensue (Muis et al., 2018; Pekrun & Stephens, 2012). Frustration indicates that an epistemic goal has been repeatedly blocked, or that the resolution of incongruity seems impossible (Pekrun & Stephens, 2012). Persistent frustration has been observed to transition into boredom, marking a crucial point at which learners disengage from epistemic activities (D'Mello & Graesser, 2012). These emotional epistemic experiences can be seen as "affective markers" of cognitive (in)congruities during proof construction, indicating that cognitive discrepancies emerged or were resolved.

1.3 Control-value Perspective on Epistemic Emotions

Building on appraisal-based frameworks, Pekrun's (2006) control-value theory focuses on the following two learner variables believed to influence internal evaluations of an epistemic process and cognitive incongruity—consequently influencing the activation of epistemic emotions (Fig. 1). *Control appraisals* indicate the extent to which learners see themselves in a position to construct a proof. For example, task-related self-efficacy is one kind of control belief. *Value appraisals* describe the subjective importance of constructing a proof for the given statement.

Findings on achievement emotions support the assumptions of control-value theory: control beliefs are positively related to positive emotions (e.g., joy, hope, pride) and negatively related to negative emotions (e.g., fear, anger, hopelessness, and boredom) (Muis et al., 2015a; Pekrun et al., 2017). Also for value appraisals, positive correlations with positive emotions and negative correlations with negative emotions have been found (Forsblom et al., 2022). Studies further confirm the central postulate of control-value theory that the relationship between control appraisals and epistemic emotions should be moderated by value (Putwain et al., 2018).

While these appraisals have been tested as precursors to achievement emotions, corresponding research on epistemic emotions has been slow to emerge (Dowd et al., 2015; Muis et al., 2015b; Munzar et al., 2021; Pekrun et al., 2017). Muis et al. (2015b) examined whether perceived control for

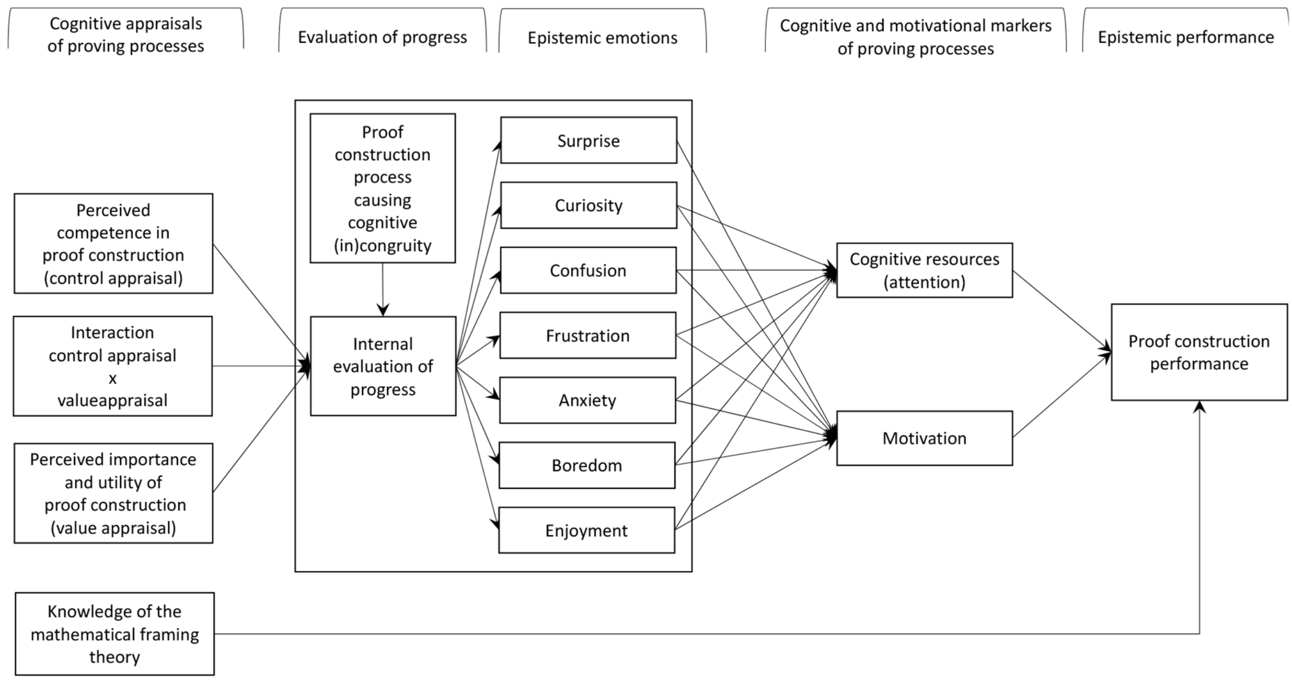


Fig. 1 Conceptual model of antecedents and effects of epistemic emotions during proof construction

solving a complex mathematics problem and value for that problem predicted the epistemic emotions experienced by 5th grade students. Path analyses revealed that both control and value were predictors of epistemic emotions. Specifically, the more the students felt that they were in control of their problem-solving, the less likely they were to experience confusion and anxiety. Further, the more the students valued mathematics, the more curiosity and enjoyment they experienced in solving the problem. Conversely, value was negatively related to the experience of confusion, frustration, anxiety, and boredom. Munzar et al., (2021) confirmed these relationships between control-value appraisals and epistemic emotions in a similar context. Neither study found interaction effects of control and value in predicting epistemic emotions.

The subsequent effects of emotions on cognitive performance are detailed in a cognitive-motivational sub-model of the control-value theory (Pekrun, 2006). In this model it is assumed that it is insufficient to explain the effects of emotions by solely considering cognitive mechanisms. The motivational consequences of emotions are considered equally important in specifying their effects on learning and problem-solving. Prominent *markers* of these processes comprise the *cognitive resources* invested in the task (*attention*) and experience of intrinsic *motivation* (Fig. 1).

As epistemic emotions are by definition task-related, it can be assumed that these emotions should be directed toward the object that causes the emotion (i.e., a proof construction task), thus allocating memory resources to construct the proof (Foster & Keane, 2015). With surprise mainly arising before either curiosity or confusion (Vogl et al., 2019), motivational consequences of this epistemic emotion have not yet been systematically investigated in educational contexts. Curiosity has been found to mobilize the exploration of new knowledge (Berlyne, 1954; Litman et al., 2005). Similarly, enjoyment was found to intrinsically motivate learners to engage with a task (Pekrun et al., 2002). These cognitive and motivational mechanisms, stimulated by surprise, curiosity, and enjoyment, may ultimately result in enhanced performance (Chevrier et al., 2019) (Fig. 1).

Confusion, despite being a negative activating emotion, has also been found to motivate learners to engage actively with learning materials (D'Mello & Graesser, 2012; D'Mello et al., 2014). However, this requires that confusion ultimately be resolved, as it might otherwise impair performance (Muis et al., 2015b). Therefore, mixed effects of confusion on attention and motivation, and on the resulting performance, may be expected (D'Mello & Graesser, 2012; D'Mello et al., 2014). Similarly, existing research on other negative activating emotions, such as frustration and anxiety, shows that their effects on cognitive and motivational processes are not consistently negative (Baker et al., 2010; Muis et al., 2015b). Frustration can promote exploration and learning if positive expectancies to solve the problem are maintained (Baker et al., 2010; Pekrun & Stephens, 2012). Anxiety can motivate knowledge-generating behavior that aims to reduce uncertainty and fulfill the needs for epistemic control (Miceli & Castelfranchi, 2005). While high anxiety can be detrimental to learning, it can also induce strong extrinsic motivation to invest effort and avoid failure (Pekrun et al., 2002). Overall, existing research has suggested detrimental effects of frustration and anxiety on knowledge generation (D'Mello & Graesser, 2012; Pekrun & Stephens, 2012). Lastly, boredom is related to low investment of cognitive resources in the task (i.e., low attention), low motivation, and a desire to escape the situation (Götz et al., 2019), and detrimental effects of reduced attention towards the task (Pekrun et al., 2010; Tze et al., 2016) on performance have been reported.

The effects of epistemic emotions on performance are defined by the complex interplay among individual learner characteristics, task requirements, and the motivational and cognitive subprocesses. Although compared to other academic emotions, epistemic emotions are relatively understudied, the above considerations suggest that positive epistemic emotions (such as curiosity or enjoyment) should have positive effects and boredom should have an overall negative effect on performance (Chevrier et al., 2019). It is difficult to derive clear predictions for negative activating emotions (frustration and anxiety). However, the findings of previous studies on emotions and academic performance point to expecting predominantly negative effects of frustration or anxiety on academic performance (Götz & Hall, 2013).

1.4 Goals and Questions

In the present study, we investigated the conceptual model displayed in Fig. 1, using a multistep elementary geometry proof construction task with university students. Elementary geometry was considered a field in which all participants had sufficient knowledge concerning the framing theory.

We investigated whether interindividual differences in students' experiences of epistemic emotions during proof construction were attributable to control and value appraisals. We focused on seven emotions related to epistemic processes, namely, surprise, curiosity, confusion, joy, frustration, anxiety, and boredom. Additionally, we examined the joint impact of epistemic emotions on markers of the proof construction processes, that is, the motivation invested in and attention (i.e., cognitive resources) allocated to the proof construction task. These, in turn, were expected to predict proof construction performance. Based on our model (Fig. 1), we approached the following five sets of questions:

Q1. To which extent do epistemic emotions, including curiosity, joy, surprise, anxiety, confusion, frustration, and boredom, arise during proof construction?

Q2. We were interested in the question of the role of the hypothesized antecedents of epistemic emotions: do these influence the intensity of epistemic emotions? We expected that control and value appraisals would predict the experience of epistemic emotions (H2a). We expected stronger relations between control appraisals and epistemic emotions for participants with higher value appraisals (H2b).

Q3. What is the role of epistemic emotions for proof construction? We expected that epistemic emotions, including attention and motivation, would predict markers of proof construction processes. We expected positive relations for students' experiences of curiosity and joy during proof construction with attention, motivation, and proof construction performance—and negative relations for anxiety, boredom, and frustration (H3).

Q4. To which extent do markers of the proving process predict proof construction performance? We expected that both markers—attention and motivation—would positively predict performance beyond knowledge of the framing theory (H4).

Q5. Do epistemic emotions (Q5a) and markers of the proving process (Q5b) act as mediators as specified in the conceptual model (Fig. 1)? We expected that epistemic emotions mediate links between cognitive appraisals, attention, and motivation (H5a), and that attention and motivation mediate the links between epistemic emotions and performance (H5b).

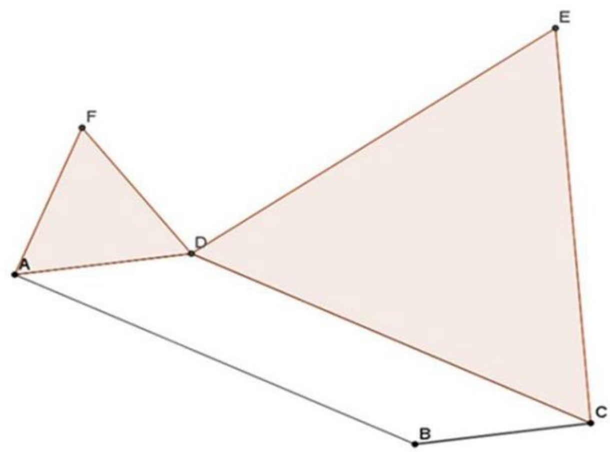
2 Method

2.1 Sample

Students from mathematics-intensive programs ($N=80$; age $M=22.9$ years, $SD=4.5$ years; enrolled in programs for mathematics teacher education 50.1% mathematics BA 16.3%; mathematics MA 12.5%; other subjects, e.g., physics 21.1%; 33 females, 2 missing) at a large research-intensive German university were invited to participate in the study via university mailing lists and postings on notice boards. Participation in the study was voluntary, with the possibility of receiving course credit depending on the provision in the study program. On average, the participants were in their fourth semester ($M_{\text{Semester}} = 4.28$, $SD_{\text{Semester}} = 3.07$).

2.2 Procedure

Small groups of students (not more than 6 at a time) participated and worked individually in a laboratory room at



Consider an arbitrary parallelogram ABCD in the plane with equilateral triangles constructed over the sides [CD] and [DA], as for example in the diagram above.

Prove that ∢FEB = ∢BFE.

Fig. 2 Proof construction task

the university. The first author conducted the individual sessions. First, students' knowledge of the mathematical framing theory relating to elementary geometry was assessed. Then they studied an example of a proof construction task, together with an acceptable proof, and a list of statements that formed the framing theory. Control and value appraisals were assessed in relation to this task. Subsequently, the students were asked to solve the proof construction task displayed in Fig. 2 (Alqassab et al., 2018). They were allowed to use the list of statements from the framing theory (Appendix 4). Participants took an average of $M = 23.67$ ($SD = 11.00$) minutes to construct the proof. Directly after task completion, the students reported the intensity of epistemic emotions experienced, as well as their task-specific attention and motivation. Epistemic emotions, motivation and attention were surveyed retrospectively to avoid interrupting or disturbing the proof construction process (Pekrun et al., 2017).

2.3 Measures

Control appraisals. The students' subjective competencies of proof construction were assessed with a 12-item scale based on the Self-Description Questionnaire (SDQ; Marsh 1990) (e.g., "I have always done well in such geometric problems.") with answer options from 1 (strongly disagree) to 5 (strongly agree).

Value appraisals. Perceived values for geometric proof construction were measured using six items from Pekrun et al.'s (2017) task value scale, covering intrinsic task value (e.g., "I like solving such geometric problems"), utility value, and attainment value. The participants responded on a scale ranging from 1 (do not agree at all) to 5 (completely agree).

Epistemic emotions. The students' subjective experiences of epistemic emotions were measured using the short version of the Epistemic Emotions Scale (EES; Pekrun et al., 2017). Each of the seven items addressed one emotion (i.e., curious, surprised, confused, anxious, frustrated, excited, bored). Intensity ratings were assessed on a scale ranging from 1 (not at all) to 5 (very strong).

Motivation. Motivation was measured using four subscales from the Situational Motivation Scale (SIMS; Guay et al., 2000). To represent a continuum from high to low levels of self-determined regulation, the students were asked "Why [they were] engaged in this task?". Subscales with four items each addressed their intrinsically motivated behaviors ("…because I thought that this task was interesting"), their identified regulation, external regulation, and amotivation ("…there may be good reasons to do this task, but personally I don´t see any"). The students answered on a scale ranging from 1 (not at all) to 5 (completely). The students' ratings were combined into an overall index of motivation ranging from negative values for strong extrinsic motivation to positive values for strong intrinsic motivation (cf. Appendix 1).

Attention. The 6-item scale (Wild & Schiefele, 1994) to measure students' attention during the proof construction task assessed the students' lack of concentration, distractibility, and task-irrelevant thinking during task completion (e.g., "It was hard for me to stay focused during the task") on a scale ranging from 1 (not at all) to 5 (completely). Values were reversed so that higher scores represented more attention allocated to the task.

Framing theory knowledge. The students' knowledge of the mathematical framing theory was assessed using a series of 35 true/false questions on elementary geometry theorems (Alqassab et al., 2018). One point was awarded for each correct answer (Cronbach's $\alpha = 0.76$).

Proof construction performance. A typical proof was used as a reference to code the participants' proof attempts. All students employed the same approach to the task. We coded which of the possible intermediate claims (proof steps), that were necessary for a complete proof, were visible in the proof attempts, and whether they were supported by reference to appropriate geometric definitions or theorems. All data were coded by two independent raters, indicating very good overall reliability (ICC = 0.83, Cohen's $\kappa = 0.74$–0.96, $M = 0.83$, $SD = 0.11$). Three measures were computed as follows: (1) the number of correct intermediate claims in the proof attempt; (2) the number of conclusions for which a mathematical justification from the framing theory was provided; and (3) the quality of those arguments

Table 1 Descriptive statistics for all measures

	No. items	*M*	*SD*	*α*	*Mean* $r_{i(t-i)}$	*Possible range*	*Observed range*	*Skewness*	*Kurtosis*	*N*
Control	12	4.12	0.53	0.85	0.55	1–5	2.5-5	−0.55	0.08	80
Value	6	3.13	0.81	0.87	0.69	1–5	1.3-5	0.01	−0.32	80
Curiosity	1	3.29	1.16	-	-	1–5	1–5	−0.49	−0.53	80
Enjoyment	1	2.53	1.12	-	-	1–5	1–5	0.43	−0.39	80
Surprise	1	2.40	1.21	-	-	1–5	1–5	0.37	-1.00	80
Anxiety	1	1.44	0.76	-	-	1–5	1–4	1.72	2.25	80
Boredom	1	1.68	0.99	-	-	1–5	1–5	1.50	1.50	80
Confusion	1	2.61	1.13	-	-	1–5	1–5	0.39	−0.59	80
Frustration	1	2.89	1.27	-	-	1–5	1–5	0.14	−0.95	80
Attention	6	3.68	1.06	0.96	0.88	1–5	1.2-5	−0.49	−0.78	80
Motivation (overall index)	16	1.45	4.64	-	-	-	-2.5-2.88	−0.90	−0.49	80
Intrinsic regulation	4	3.02	0.99	0.81	0.64	1–5	1–5	0.05	−0.40	80
Identified regulation	4	2.95	1.03	0.85	0.70	1–5	1–5	−0.09	−0.86	80
External regulation	4	3.10	1.03	0.85	0.69	1–5	1–5	−0.34	−0.63	80
Amotivation	4	2.22	0.92	0.83	0.65	1–5	1-4.75	0.61	−0.26	80
Proof Performance (z-Scores)	3	0.00	0.91	0.88	0.87	-	-1.3-2.2	0.60	−0.78	80
Framing theory knowledge	35	30.4	3.72	-	-	0–35	15–35	-1.42	2.55	80

Note. $^*p<.05$, $^{**}p<.01$, $^{***}p<.001$, $^{\circ}p<.10$.

(i.e., clear reference to the objects in the figure). All three indices were z-standardized and averaged to create a measure of proof construction performance.

2.4 Statistical analyses

There were no missing values in the dataset. We specified a manifest path model using the R package lavaan (Rosseel et al., 2021) based on standardized data (z-scores). The model coefficients were estimated using the Maximum Likelihood estimator. To account for small sample sizes and potential non-normality, we employed bootstrapping when computing standard errors (10,000 samples; Hayes & Preacher 2013) as this method provides more precise estimates and yields more statistical power. Model fit was assessed using the following cutoffs: RMSEA ≤ 0.06, SRMR ≤ 0.06, and CFI/TLI ≥ 0.97 (Hu & Bentler, 1999).

An interaction term of control and value appraisals was modelled as the product of the standardized appraisal scores (Aiken et al., 1991). Slopes for the relation between control appraisals and emotions at different levels of value were calculated as defined parameters. The following correlations were included: correlations between the appraisals, their interaction term, and the framing theory knowledge; correlations between all epistemic emotions; and correlations between attention and motivation. To improve model fit, a path from value to attention was added. The results of the models with and without this path were largely the same (cf. Appendix 3), but we decided to report data from the model that fitted the data according to usual cutoff criteria. The students' knowledge of the mathematical framing theory was included as a covariate of proof construction performance.

3 Results

3.1 Preliminary analyses

Distribution. Each measure was examined for reliability, skewness, kurtosis, and outliers (Table 1). Mean item-total correlations (part-whole corrected) were above 0.54 for all items, and reliabilities were good (α = 0.81– 0.96).

Correlations among epistemic emotions. A positive correlation was observed between curiosity and enjoyment (Table 2); these emotions were negatively correlated with confusion, frustration, and boredom. Confusion was substantially linked to frustration, while boredom was unrelated to confusion and frustration. Likely due to small variance, anxiety was not significantly related to other epistemic emotions or other variables. Similarly, surprise was unrelated to all variables except confusion and curiosity.

Since surprise and anxiety did not show significant correlations with appraisals, markers of the proof construction process or proof construction performance (rs < 0.18), they were excluded from subsequent analyses.

Proposed path model. To identify potential multicollinearity effects, we calculated Variance Inflation Factors (VIF) for each predictor group. VIFs were below a conservative cut-off value of 2.5 for all measures. The proposed path model showed a good model fit ($\chi^2(20) = 23.77$, p = .25;

Table 2 Pearson correlations between all measures

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
Control	-												
Value	−0.03	-											
Curiosity	0.19	0.55**	-										
Enjoyment	0.32*	0.41**	0.61**	-									
Surprise	−0.07	0.17	0.26*	0.12	-								
Anxiety	−0.03	0.11	−0.02	0.07	0.15	-							
Boredom	−0.05	−0.21°	−0.48**	−0.40**	−0.12	−0.09	-						
Confusion	−0.23*	−0.24*	−0.25*	−0.26*	0.29*	0.20°	0.11	-					
Frustration	−0.16	−0.10	−0.12	−0.25*	0.21°	0.21°	−0.02	0.48**	-				
Attention	0.35**	0.00	0.34*	0.53**	−0.04	0.00	−0.37**	−0.34**	−0.41**	-			
Motivation	0.20°	0.59**	0.75**	0.68**	0.18	0.16	−0.54**	−0.32*	−0.20°	0.39**	-		
Performance	0.24*	0.03	0.18	0.31*	0.10	−0.04	−0.24*	−0.20°	−0.25*	0.43**	0.38**	-	
Framing theory knowledge	0.16	0.15	0.17	0.20°	0.18	0.05	−0.04	−0.10	−0.04	0.09	0.17	0.31**	-

Note. *$p < .05$, **$p < .01$, ***$p < .001$, °$p < .10$.

CFI = 0.99; TLI = 0.96; RMSEA = 0.049; SRMR = 0.039). For a post-hoc power analyses, see Appendix 2.

3.2 Main analyses

Appraisals and experiences during proof construction (Q1). Mean levels of cognitive appraisals and attention were moderate to high; mean levels of control appraisals were particularly high (Table 1). The students' task motivation averaged from moderate to high levels, with amotivation being the least intensively reported. Standard deviations indicated variability in appraisal endorsement, attention, and motivation across participants. Students reported a broad range of positive and negative epistemic emotions—curiosity, enjoyment, confusion, and frustration were the most intensively reported emotions. Anxiety and boredom were the least frequently reported factors. Surprise was reported with moderate intensity, with large variation.

Antecedents of epistemic emotions (Q2). As expected, higher control and value appraisals were related to more intense experiences of curiosity and enjoyment (Ha2), further qualified by a significant value by control interaction for curiosity, which reached significance only in tendency[1] for enjoyment (H2b, Table 3).

Control showed a positive relationship with curiosity at higher levels of value ($\beta_{M+2SD} = 0.61$, $p < .01$, Fig. 3) that became weaker at the mean value ($\beta_M = 0.21$, $p < .05$), and was negative, albeit non-significant at lower levels of value ($\beta_{M-2SD} = -0.19$, $p = .37$). Similarly, control showed a stronger positive relationship with enjoyment at higher levels of value ($\beta_{M+2SD} = 0.71$, $p < .01$) that became weaker at the mean value ($\beta_M = 0.35$, $p < .001$), and was non-significant at lower levels of value ($\beta_{M-2\,SD} = -0.02$, $p = .92$).

In tendency, higher control and higher value negatively predicted confusion; no significant interaction emerged. While frustration was unrelated to appraisals, boredom was negatively predicted by subjective value attributed to the task, and the interaction term between control and value appraisals approached significance. Follow-up analyses showed that the relationship between control and boredom decreased from a descriptively positive relationship at lower levels of value ($\beta_{M-2SD} = 0.37$, $p = .35$) to a negative relationship at higher levels of value ($\beta_{M+2SD} = -0.54$, $p = .05$). Thus, H2b is corroborated only for curiosity.

Epistemic emotions, attention, and motivation (Q3a). Corroborating H3, positive epistemic emotions were positively related to motivation (curiosity and enjoyment) and attention (enjoyment). Negative epistemic emotions were negatively related to attention (frustration and, in tendency but yet insignificant, boredom) and motivation (boredom,

1 We speak of a *tendency* for non-significant statistics with $p < .10$.

Table 3 Path coefficients for control and value as antecedents of epistemic emotions

	Curiosity		Enjoyment		Boredom		Confusion		Frustration	
	β	*SE*	*β*	*SE*	*β*	*SE*	*β*	*SE*	*β*	*SE*
Value	0.54***	0.09	0.40***	0.09	−0.22*	0.10	−0.23°	0.12	−0.10	0.11
Control	0.21*	0.09	0.35***	0.10	−0.09	0.12	−0.23°	0.12	−0.17	0.12
Value x Control	0.20*	0.10	0.18°	0.11	−0.23°	0.12	−0.04	0.13	−0.05	0.12
R^2	0.36		0.30		0.10		0.11		0.04	

Note. All outcome and predictor variables were standardized (z scored). The product term for the interaction effect is the product of the individual standardized variables. The product term was not re-standardized.

β the standardized coefficient. **p* < .05 ***p* < .01, ****p* < .001, °*p* < .10.

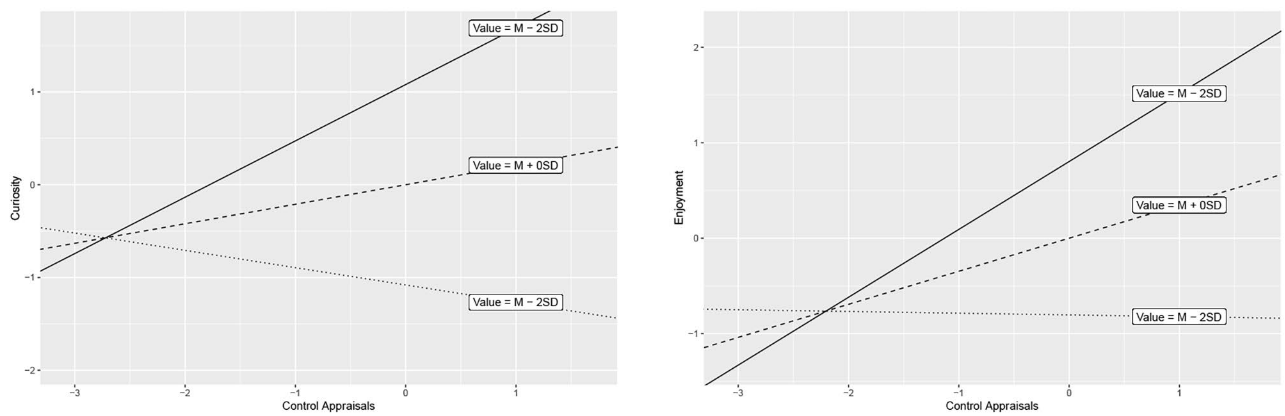


Fig. 3 Relations between control and curiosity (left) and enjoyment (right) for different levels of value

Table 4). Confusion did not significantly predict attention or motivation.

Table 4 Path coefficients predicting motivation and attention by epistemic emotions

	Attention		Motivation	
	β	$SE_β$	*β*	$SE_β$
Curiosity	0.09	0.14	0.45***	0.08
Enjoyment	0.43***	0.12	0.30***	0.09
Boredom	−0.22°	0.12	−0.19**	0.07
Confusion	−0.14	0.11	−0.10	0.07
Frustration	−0.26*	0.11	−0.02	0.08
R^2	0.49		0.69	

Note. β standardized regression coefficient. The standardized path coefficient for the effect of value on attention was *β* = −0.34, *SE* = 0.14, *p* < .01. **p* < .05, ***p* < .01, ****p* < .001, °*p* < .10.

Table 5 Path coefficients predicting performance by motivation, attention, and the framing theory knowledge

	Performance	
	β	$SE_β$
Attention	0.32**	0.11
Motivation	0.21*	0.11
Framing Theory Knowledge	0.24*	0.11
R^2	0.28	

Note. β standardized regression coefficient.

**p* < .05, ***p* < .01, ****p* < .001, °*p* < .10.

Associations between attention, motivation, and proof construction performance (Q4). The model explained more than one-fourth (28%) of the variance in proof construction performance (Table 5). Knowledge of the mathematical framing theory significantly predicted the quality of proof constructions. Corroborating H4, attention and motivation devoted to constructing the proof both substantially contributed to higher-quality proof constructions.

Indirect effects of appraisals on attention and motivation via epistemic emotions (Q5a). Under all direct effects of appraisals on emotions, only the effect of value on attention was retained in the final model (*β* = −*0.34, SE* = *0.13, p* < *.01)*. This direct negative effect was unexpected.

Beyond this, emotions mediated the effects of control on motivation and attention (Table 6). Enjoyment and curiosity mediated the effects of control and value on motivation, but only enjoyment mediated relationships between participants' control and value appraisals and their attention. This result indicates that more value attributed to the task spurs curiosity, which, in turn, motivates students to perform more intense work. Moreover, the more the control and value are attributed to the task, the more students enjoy constructing a proof, which, in turn, enhances their attention and motivation. Negative emotions (confusion, frustration, and boredom) did not function as significant mediators. Thus, H5a was corroborated for positive epistemic emotions.

Table 6 Coefficients and confidence intervals for mediation analyses between cognitive appraisals and attention and motivation via epistemic emotions

	Estimates		95% CI	
	β	*SE*	*LL*	*UL*
Control → Curiosity → Motivation (+)	0.09*	0.04	0.01	0.20
Control → Enjoyment → Motivation (+)	0.10**	0.04	0.03	0.19
Control → Boredom → Motivation (+)	0.02	0.03	– 0.02	0.08
Control → Confusion → Motivation (+)	0.02	0.02	– 0.01	0.07
Control → Frustration → Motivation (+)	0.00	0.02	– 0.03	0.04
Total Indirect Effect: Control → Motivation (+)	*0.24***	*0.08*	*0.09*	*0.40*
Value → Curiosity → Motivation (+)	0.25***	0.06	0.12	0.37
Value → Enjoyment → Motivation (+)	0.12*	0.05	0.03	0.24
Value → Boredom → Motivation (+)	0.04	0.03	– 0.00	0.11
Value → Confusion → Motivation (+)	0.02	0.02	– 0.01	0.08
Value → Frustration → Motivation (+)	0.00	0.01	– 0.01	0.04
Total Indirect Effect: Value → Motivation (+)	*0.43****	*0.08*	*0.28*	*0.58*
Control → Curiosity → Attention (+)	0.02	0.03	– 0.04	0.10
Control → Enjoyment → Attention (+)	0.15*	0.06	0.04	0.27
Control → Boredom → Attention (+)	0.02	0.03	– 0.03	0.09
Control → Confusion → Attention (+)	0.03	0.04	– 0.01	0.12
Control → Frustration → Attention (+)	0.04	0.04	– 0.01	0.14
Total Indirect Effect: Control → Attention (+)	*0.26***	*0.08*	*0.11*	*0.43*
Value → Curiosity → Attention (+)	0.05	0.07	– 0.09	0.20
Value → Enjoyment → Attention (+)	0.17**	0.07	0.06	0.31
Value → Boredom → Attention (+)	0.05	0.04	0.00	0.13
Value → Confusion → Attention (+)	0.03	0.03	– 0.02	0.11
Value → Frustration → Attention (+)	0.02	0.03	– 0.04	0.09
Total Indirect Effect: Value → Attention (+)	*0.33***	*0.10*	*0.13*	*0.54*

Note. + positive expected effect. LL lower limit, UL upper limit. *$p<.05$, **$p<.01$, ***$p<.001$, °$p<.10$.

Indirect effects of epistemic emotions on proof construction performance via markers of the proof construction process (Q5b). The direct effects of epistemic emotions on proof construction performance were not included in the final model, since attention and motivation mediated the effects of epistemic emotions on performance (H5b, Table 7). Contrary to the prominent role of curiosity as a mediator between appraisals and the students' attention and motivation, the indirect effect of curiosity on proof construction performance failed to reach statistical significance. Enjoyment showed a significant indirect effect on performance, that primarily stemmed from an indirect effect via students' attention. A significant total indirect (negative) effect of boredom on performance was observed. Here, mediation via attention was slightly stronger than via motivation, but both specific indirect effects failed to reach significance. The indirect effects of frustration and confusion on proof construction performance were not significant.

Table 7 Coefficients and confidence intervals for mediation analyses between epistemic emotions and performance via attention and motivation

	Estimates		95% CI	
	β	*SE*	*LL*	*UL*
Curiosity → Attention → Performance (+)	0.03	0.05	– 0.06	0.13
Curiosity → Motivation → Performance (+)	0.10°	0.05	0.00	0.21
Total Indirect Effect: Curiosity → Performance (+)	*0.12°*	*0.07*	*– 0.01*	*0.25*
Enjoyment → Attention → Performance (+)	0.14*	0.06	0.03	0.29
Enjoyment → Motivation → Performance (+)	0.06°	0.04	0.00	0.14
Total Indirect Effect: Enjoyment → Performance (+)	*0.20****	*0.06*	*0.09*	*0.33*
Boredom → Attention → Performance (–)	– 0.07	0.05	– 0.10	0.00
Boredom → Motivation → Performance (–)	– 0.04	0.03	– 0.10	0.00
Total Indirect Effect: Boredom → Performance (–)	*– 0.11**	*0.05*	*– 0.23*	*0.03*
Confusion → Attention → Performance (–)	– 0.05	0.04	– 0.14	0.02
Confusion → Motivation → Performance (–)	– 0.02	0.02	– 0.07	0.01
Total Indirect Effect: Confusion → Performance (–)	*– 0.07*	*0.04*	*– 0.16*	*0.01*
Frustration → Attention → Performance (–)	– 0.08°	0.05	– 0.18	0.01
Frustration → Motivation → Performance (–)	– 0.01	0.02	– 0.05	0.03
Total Indirect Effect: Frustration → Performance (–)	*– 0.09°*	*0.05*	*– 0.19*	*0.00*

Note. + positive expected effect, – negative expected effect. LL lower limit, UL upper limit. *$p<.05$, **$p<.01$, ***$p<.001$, °$p<.10$.

4 Discussion

Although research on emotions has become prominent in the mathematics education literature, little attention has been paid to epistemic emotions. Moreover, affective characteristics have been rarely considered when studying proof construction. To our knowledge, few studies in mathematics education have investigated cognitive, affective, and motivational processes during complex mathematical tasks in the past. In this study we examined the relationship between university students' subjective appraisals of a proof construction task, their emotional epistemic experiences, attention allocated to and motivation invested in the task, and proof construction performance, controlling for knowledge of the framing theory. We explored whether epistemic emotions function as mediators linking subjective appraisals with attention and motivation, and whether students' attention and motivation mediated the relationships between epistemic emotions and proof construction performance. The findings of the study provide support for the following claims: (1) Students experience a range of epistemic emotions during proof construction, with substantial inter-individual differences. However, some emotions are experienced at a low intensity, on average. (2) Primarily positive epistemic emotions depend on students' control and value appraisals of the task. (3) Epistemic emotions function as mediators between these appraisals and markers of the proof construction process (attention and motivation). (4) We observed partial support for the assumption that value moderates the relationship between control appraisals and emotional experiences. (5) Positive and negative emotions function as predictors of proof construction performance indirectly, via the markers of the proof construction process.

4.1 Epistemic emotions during proof construction (Q1)

The qualitatively distinct positive and negative epistemic emotions reported by the participants reflect that proof construction is a knowledge-generating activity that triggers epistemic emotions. Curiosity and enjoyment were the most intensely reported emotions. This is encouraging in terms of the historically exclusive emphasis on fear and anxiety (Ma, 1999), and mirrors a recent finding in the context of an international mathematics competition showing that the dominant emotional experiences of participants are indeed positive (Greensfeld & Deutsch, 2020). Our results shed light on the emotional experiences of university students enrolled in undergraduate mathematics (teacher education) programs, who are the main addressees of proof-related activities at the university level.

In line with our assumption that proof construction processes are likely to induce cognitive incongruity (DeBellis & Goldin, 2006), frustration and confusion were also reported by the students during proof construction. Encouragingly, the proof construction task in this study did not provoke strong anxiety or boredom. The first reflects the definition of epistemic anxiety, which is assumed to be triggered only by strong uncertainties about beliefs in a proposition or in the framing theory (Hookway, 2008; Muis et al., 2015a). The low reports of boredom indicated that the proof construction task rendered an appropriate level of challenge for our sample, as boredom can stem from either too low task demands coupled with high abilities or from too high task demands coupled with low abilities (Pekrun et al., 2010).

Only a small intensity of surprise was reported. Recent research indicates that surprise is an initial reaction to cognitive incongruity, but rapidly turns into either curiosity or confusion (Vogl et al., 2019). Therefore, later experiences of curiosity or confusion may have overwritten memories of feeling surprised, by retrospective reporting after the task. In summary, proof construction was accompanied by a rich palette of positive as well as negative epistemic emotional experiences, showing that proof construction is an emotionally charged endeavor.

4.2 Antecedents of epistemic emotions (Q2)

We assumed that students' appraisals would affect their evaluation of cognitive incongruencies during the proof construction process and, in turn, their emotional experiences. In line with prior research, the relationship between students' appraisals and their reported positive epistemic emotions ranged from small to moderate magnitude (Bieg et al., 2013). While the interaction assumption between control and value proposed by Pekrun (2006) has not yet been tested for epistemic emotions (Muis et al., 2015a; Munzar et al., 2021), the relationships between subjective control and curiosity, and between subjective control and joy were amplified by high levels of reported value. The relationships disappeared at low levels of reported value, suggesting that high levels of control cannot compensate for low levels of value. This result is critical for intervention measures designed to stimulate positive epistemic emotions.

Conversely, in line with previous research on the antecedents of negative epistemic emotions (Muis et al., 2015a; Munzar et al., 2021), the more the students valued the proof construction activity, the less likely they were to experience boredom and (in tendency) confusion. The perceived importance of proof construction seemed to act as a protective shield from experiencing negative epistemic emotions (Pekrun et al., 2010). No significant interaction of both appraisals emerged for negative epistemic emotions, although higher value appraisals tended to dampen the negative relationship between control and boredom. This result partially resonates with recent findings, where the highest level of boredom was experienced for high control and low value (Bieg et al., 2013; Putwain et al., 2018). Our finding corroborates that boredom is primarily induced by a lack of value appraisals, possibly combined with very high control. This does not contradict the suggested curvilinear relationship between control and boredom, meaning that boredom might also be experienced when facing a too high level of challenge (Pekrun et al., 2010).

Primarily positive epistemic emotions seemed to depend on the appraisals of the task, for the sample of university students in this study. Lower values of explained variance for negative emotions indicated that negative emotions might be more strongly affected by evaluation processes during proof construction than by students' a priori appraisals.

4.3 Indirect effects of appraisals on cognitive-motivational markers (Q4)

The mediating effects of epistemic emotions emerged for curiosity, and enjoyment, in explaining the relationships between appraisals and the cognitive-motivational markers of the proof construction process. Significant effects were found for enjoyment, with higher control and value appraisals relating positively to joy during proof construction, and joy relating positively to attention and motivation. Similarly, stronger control and value appraisals were found to be positively related to curiosity, which was positively related to motivation (but not to attention). The remaining epistemic emotions did not play a significant role in linking appraisals with attention and motivation.

4.4 Effects of epistemic emotions (Q3)

We considered two mechanisms that mediate the relationship between epistemic emotions and performance. Enjoyment positively predicted attention devoted to the task, indicating the propensity of enjoyment to stimulate motivation and the use of cognitive resources (Buff et al., 2011; Pekrun et al., 2002). Frustration and boredom, on the other hand, predicted a lack of concentration, distractibility, and task-irrelevant thinking. This aligns with theories that frustration constrains self-regulation and academic performance by occupying valuable cognitive resources (Munzar et al., 2021), and that bored students are prone to be distracted by task-irrelevant information (Pardos et al., 2014). Both mechanisms reduce students' cognitive capacity to enact cognitively and metacognitively demanding strategies (Pekrun et al., 2010).

In this study, motivation was observed to be closely linked to curiosity and enjoyment. This finding shows the importance of positive epistemic emotions in engaging students, and corroborates their adaptive functions for knowledge acquisition (Murayama et al., 2019). Conversely, boredom was observed to be strongly negatively related to the motivation invested in proof construction, in line with boredom's label as a "motivational barrier" (Baker et al., 2010; Pekrun et al., 2010; Tze et al., 2016; Vogel-Walcutt et al., 2012, p. 90).

From a theoretical perspective, confusion can be productive or unproductive (D'Mello et al., 2014). This might explain the null findings on its effects for the sample-level analyses in this study; while some studies reported that confusion can be problematic when it is not resolved (Muis et al., 2015b), others indicated that students may benefit from the confusion resolution process (Munzar et al., 2021).

4.5 Indirect effects of epistemic emotions on performance (Q5)

The significant indirect effect of enjoyment on proof construction performance was mediated by attention and, in tendency, motivation. Conversely, more intense boredom was accompanied by weaker proof construction performance; however, both specific indirect effects did not reach statistical significance. Additionally, all other indirect effects failed to reach statistical significance. Overall, epistemic emotions and knowledge of the framing theory explained approximately one-fourth of the variance in proof construction performance. Prior research shows substantial variance explanation by the framing theory knowledge together with other cognitive learner characteristics, such as problem-solving skills (Ufer et al., 2008). This suggests that future research should consider emotions in combination with these additional explanatory variables. Moreover, further mediating markers of the proof construction process, such as effort and task engagement (Wang et al., 2016) may be relevant, as suggested in Pekrun's (2006) framework.

4.6 Limitations and Outlook

Several limitations of this study need to be considered. First, the study is based on specific assumptions about potential mediation and interaction effects. Even though these effects are parts of established theories and have been supported by numerous studies in other contexts, they may be challenged by more complex theories in the future. A more fine-grained temporal resolution of activities and measurements during the experiment would be necessary to investigate these relations. Our design precludes causal inferences. While the causal hypotheses in this study were theoretically grounded and all variables were measured in line with suggested temporal dynamics, we decided to measure epistemic emotions, attention, and motivation retrospectively directly after task completion. Studies involving experimental priming of cognitive incongruities and specific manipulations of control or value appraisals are needed (Vogl et al., 2019). Further research could investigate whether state transition analyses of dynamic changes in epistemic emotions can address the questions of whether and how perceptions of control may adjust throughout proof construction (Chevrier et al., 2019; Munzar et al., 2021).

Second, we focused on a specific proof task in elementary geometry. Further theoretical work and empirical studies may investigate if and how the content area of the proof, e.g., in terms of the necessity of relying on symbolic representations, may influence motivational processes.

Third, this study relied primarily on the participating students' self-reports. These could have been coupled with other methods to trace students' emotions allowing fluent assessment of emotional experiences. A more qualitative analysis of students' emotional experiences alongside their cognitive processes would be of interest, but also methodologically demanding. Facial recognition, skin conductance or think-aloud-protocols may not only account for the in situ dynamics of emotions described above, but also for the fact that some experiences simply elude from memory.

Fourth, the between-subject design employed in this study is not suited to examining intraindividual mechanisms of epistemic emotions. Future studies should rectify this shortcoming.

Finally, given the relatively small size of the current sample, we could not use latent analyses to test our hypotheses. Moreover, the power to detect small direct and indirect effects was limited. Therefore, future studies should include larger sample sizes.

Our study focused on the role of epistemic emotions on motivational and attentional processes during proof construction. Students' control and value beliefs functioned as antecedents of these variables in our study. A different line of research investigates emotional experiences, motivation, or attention to the development of affective dispositions towards mathematics (Hill et al., 2021; Kosiol et al., 2019).

4.7 Practical Implications

The findings of this study suggest that students' epistemic emotions and cognitive as well as motivational factors impact proof construction performance. Thus, interventions and professional development for (university) teaching staff should target not only cognitive aspects but also motivational and affective processes. While poor performance in proof construction has long been attributed to cognitive factors, such as knowledge or problem-solving skills (Ufer et al., 2008), recent frameworks urge researchers to attend to affective factors of mathematical cognition (Beltrán-Pellicer & Godino, 2020). This study's findings provide not only important insight into the functional mechanisms of epistemic emotions, but also indicate that they can act as affective markers that can point to cognitive difficulties students encounter during proof construction (e.g., cognitive incongruities). Further, providing training to students and teachers to handle negative emotional experiences adaptively might be helpful to sustain their positive emotional experiences. Here, meta-analytic findings show that emotion interventions targeting cognitive change through appraisals (in light of this study's results, conveying value) can alleviate negative emotions (Webb et al., 2012), and prior studies offer a variety of techniques to regulate students' emotions effectively (Harley et al., 2019).

5 Conclusions

To the best of our knowledge, this study is the first to investigate the functions of epistemic emotions during proof construction. It shows that proof construction is an affectively charged experience, as the participating students reported a range of positive and negative epistemic emotions. We modeled the emergence of epistemic emotions by considering control and value appraisals regarding proof construction. As hypothesized, enjoyment and curiosity were related to appraisal interactions. In considering the effects of epistemic emotions, the findings suggest that curiosity and enjoyment enhance students' attention and motivation invested in proof construction, while boredom hinders their concentration and motivation. Confusion and frustration did not negatively impact students' attention, motivation, or proof construction performance. Future studies investigating within-person dynamics may help to explain these null effects. This study's findings also suggest that students' appraisals are linked to their attention and motivation indirectly through positive epistemic emotions, and that positive as well as negative epistemic emotions are linked with the students' proof construction performance through the emotions' effects on attention and motivation. Therefore, evoking curiosity and fostering enjoyment regarding proof construction is a means of promoting adaptive epistemic processes among students that will ultimately benefit their proof construction performance.

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on previous versions of the manuscript. All authors read and approved the final manuscript.

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Declarations

Conflict of interest The authors have no conflicts of interest to declare.

Informed consents Participation in the study required informed consent.

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