Banking Crises under a Central Bank Digital Currency (CBDC)

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Banking Crises under a Central Bank Digital Currency (CBDC)

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Abstract

One of the main concerns associated with central bank digital currencies (CBDC) is the disintermediating effect on the banking sector in general, and the risk of bank runs in times of crisis in particular. This paper examines the implications of an interest-bearing CBDC on banking crises in a dynamic bank run model with a financial accelerator. The analysis distinguishes between bank failures due to illiquidity and due to insolvency. In a numerical exercise, CBDC leads to a reduction in the net worth of banks in normal times but mitigates the risk of a bank run in times of crisis. The financial stability implications also depend on how CBDC is accounted for on the asset side of the central bank balance sheet: if CBDC issuance is offset by asset purchases, it delays the onset of both types of bank failures to larger shocks. In contrast, if CBDC issuance is offset by loans to banks, it substantially impedes failures due to illiquidity, but only marginally affects bank failures due to insolvency.

Keywords: Central Bank Digital Currency (CBDC), Financial Intermediation, Financial Stability, Bank Runs

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1 Introduction

Digital transformation has and will further change our payment and monetary system. Throughout history, money has adapted over time and its form has changed significantly – from cowrie shells and commodity money to commodity backed money and fiat money. To adjust to the developments of an increasingly digital payment system, central banks have started to explore the potential of a central bank digital currency (CBDC).[1] A retail CBDC would be inherently risk-free public money, similar to cash, but with the additional convenience of being digital and entailing the option to bear interest. These features make CBDC a closer substitute to bank deposits which could adversely affect the deposit funding of commercial banks.

One of the key issues revolves around the impact of a CBDC on the banking sector in general, but especially in times of crisis (e.g. see, ECB, 2020; Bank for International Settlements, 2018; Carney, 2018). A main concern is that CBDC may increase the risk of an aggregate bank run in any situation. Particularly when confidence in the financial system is low, the fear is that the closer substitutability to deposits may trigger bank runs under a CBDC when depositors otherwise would not have run. The reasoning is that CBDC would reduce the cost of a bank run by providing a convenient but completely safe alternative to other assets and thereby would trigger runs more easily. A further concern is that runs could unroll much faster due to (potentially unlimited) 24/7 access to CBDC.

This paper investigates the impact of a CBDC on aggregate banking crises and the possibility of systemic bank runs, for which the analysis distinguishes between banking failures due to illiquidity and insolvency.[2] To answer these questions, an interest-bearing CBDC is introduced into the dynamic bank run framework of Gertler and Kiyotaki (2015). The stylised endowment economy with financial frictions consists of households, banks, and a central bank which issues CBDC following an interst-rate rule. Serving as proxy for the firm sector, capital is modelled as a productive technology yielding a return each period that is subject to aggregate shocks. Households can place their savings in CBDC, bank deposits, or capital. While banks are most efficient in intermediating capital, they are limited by a leverage constraint. In normal times, the model features a unique equilibrium in which banks intermediate the majority of assets.

[1] According to surveys by the BIS, the share of central banks studying CBDC increased from below 70% in 2017 (Barontini and Holden, 2019) to 93 % in 2022 (Kosse and Mattei, 2023).

[2] The analysis focuses only on aggregate crises and runs. The impact on *individual* bank failures is not considered. The main view in the literature is that a CBDC is unlikely to have a major impact on *individual* bank runs (e.g. see Juks, 2018; Mancini-Griffoli et al., 2018; Meaning et al., 2018; Barrdear and Kumhof, 2022).

However, if a sufficiently large shock hits the economy, a second equilibrium emerges which is characterised by a bank run on the entire banking system. Bank failures due to illiquidity can emerge if households withdraw their deposits because they believe that other agents will run, leading to a bank run triggered by self-fulfilling beliefs. If the shocks become even larger, at a certain point only the bank run equilibrium remains and banks fail due to insolvency.

The introduction of a CBDC creates an additional liability on the central bank balance sheet that will lead to further balance sheet adjustments, contributing to the impact of a CBDC on the economy. This paper accounts for the potentially varying implications of different balance sheet adjustments by analysing CBDC issuance in the context of two policy scenarios: in the first policy scenario, the central bank offsets CBDC issuance by lending to banks *('credit policy')*; in the second policy scenario, the central bank offsets CBDC issuance by investing in the productive technology itself *('asset policy')*. Both policy scenarios would lead to an increase in the central bank balance sheet and are contrasted to the economy without a CBDC ('no CBDC').[3]

A simulation of the model offers four main insights: first, in steady state, a CBDC does not majorly affect aggregate output and prices but it does affect the composition of household savings, bank funding and capital investment, leading to a reduction in bank profits. Second, both CBDC policy scenarios tend to have a small but stabilising influence in a crisis that does not trigger a bank run. Third, these stabilising effects become more important in a bank run situation: both CBDC policy scenarios improve financial stability by postponing the emergence of bank runs to larger shocks. Fourth, the impact of a CBDC also depends on the accompanying central bank balance sheet adjustments; CBDC issuance with 'asset policy' delays the onset of both types of bank failures to larger shocks. In contrast, CBDC issuance with 'credit policy' substantially impedes failures due to illiquidity, but has little impact bank failures due to insolvency.[4] Overall, while CBDC issuance strains the banking sector in steady state by reducing deposits and net worth, it improves financial stability in times of crisis by impeding the emergence of bank runs.

What mechanism lies at the root of the result that CBDC issuance delays the emergence of bank runs – a result that runs contrary to the initially stated concerns? In the model framework, CBDC issuance makes bank runs less disruptive because the central bank channels

[3] A CBDC will increase the size of the central bank balance sheet if CBDC issuance is not fully offset by a reduction in excess reserves or cash.

[4] In fact, in the baseline calibration under CBDC with credit policy, self-fulfilling runs do not occur. However, this is not a general result and self-fulfilling runs emerge in other calibrations.

CBDC inflows back into the economy, either via credit to banks or direct capital purchases. This reduces the burden on households and increases the efficiency and price of capital. These central bank interventions that channel the CBDC inflows back into the economy slow the financial accelerator dynamics and make the occurrence of bank runs more difficult, ultimately postponing their occurrence to larger shocks.

Naturally, the central bank can take measures independent of CBDC issuance to improve financial stability and to avert systemic runs. The main insight of this model analysis is that in a bank run situation inflows into CBDC, instead of inflows into other less efficient assets, are aligned with financial stability measures and do not exacerbate bank run risks. In a bank run situation, households do not withdraw deposits because they like CBDC but because they are concerned about the safety of their deposits. By providing them with a safe and more efficient option to run into, the issuance of CBDC mitigates losses in a bank run scenario, stabilising capital prices and making runs less likely from the outset.

Context of the literature: The related literature also supports a nuanced picture of the financial stability implications of a CBDC. The issuance of CBDC is found to have ambiguous financial stability implications, depending not only on adjustments on the asset side of the central bank's balance sheet but also on information effects, the structure of the financial sector, CBDC remuneration, and other notable design parameters.

The importance of the asset side counterpart to CBDC operations is similarly highlighted by Kim and Kwon (2023) in an OLG model with a different definition of bank runs. In their model, bank runs are not self-fulfilling but set up as situations in which depositors who *need* to withdraw money cannot be fully repaid. They find that CBDC issuance that is offset by loans to banks increases financial stability, while CBDC issuance that is offset by government bond purchases deteriorates it. In the global game analysis of the two-period bank run model of Ahnert et al. (2023), CBDC remuneration has two countervailing forces that can either increase or decrease financial fragility and the probability of a bank run. They find that when banks enjoy a high degree of market power, CBDC remuneration can improve financial stability. Similarly, Skeie (2021) and Lucas (2022) underline the importance of remuneration policies for financial stability and financing conditions.

Keister and Monnet (2022) highlight that banks may adjust their maturity transformation in response to the CBDC issuance which tends to enhance financial stability. Furthermore, they argue that the issuance of CBDC gives the central bank an information advantage on financial stability conditions through which it can intervene more efficiently.[5] Consistent with the mechanism included in this paper, in the model of Keister and Monnet (2022) the anticipation of a less severe bank run scenario reduces the incentive to run from the outset. The result that CBDC issuance make banking panics less disruptive is also is in line with the findings of Williamson (2021). However, in contrast to Keister and Monnet (2022) and this paper, the lesser severity of runs tends to encourage banking panics. This opposite finding in the model of Williamson (2021) can be explained by CBDC offering an improved alternative to retail payments and thereby replacing payments that would otherwise have been made with deposits. Therefore, in their model the ability of CBDC to ensure payment efficiency during a run, reduces the cost of running and in this case makes runs more likely.

There are several papers studying the structural impact of CBDC on financial intermediation, complementing the analyses of the impact of CBDC during a financial crisis (e.g., see Keister and Sanches, 2023; Andolfatto, 2021; Chiu et al., 2023). Ahnert et al. (2022) and Chapman et al. (2023) both survey the growing CBDC literature with a specific focus on financial intermediation and stability. Several papers discuss policy measures aimed to prevent excessive CBDC holdings, such as holding limits (Bindseil et al., 2021; Assenmacher et al., 2021; Meller and Soons, 2023), tiered remuneration (Bindseil, 2019), and limited convertibility with reserves and deposits (Kumhof and Noone, 2021).

This paper contributes to the literature by analysing the impact of CBDC on banking crises and the emergence of self-fulfilling runs and insolvency in a dynamic bank run model with a financial accelerator. In this context, this paper aims to shed light on the different implications of asset side operations that offset CBDC issuance. The presented model focuses on the store of value function of money as it is most important in the context of bank runs. However, the analysis neglects the role of money as a medium of exchange even though most central banks aim to design CBDC as a means of exchange rather than an attractive store of value. Furthermore, the model poses a very stylised real setting which could be extended to an economy with a richer firm sector, nominal frictions, conventional monetary policy, and a more sophisticated government sector.

Hereinafter, Section 2 discusses central bank balance sheet adjustments to CBDC issuance in more detail. Section 3 presents the general model economy and the bank run scenarios which is subsequently simulated and analysed in Section 4. Finally, Section 5 concludes.

[5] However, similar information may already be obtainable through systems like TARGET2.

2 CBDC issuance and the central bank balance sheet

This section reviews different scenarios of how CBDC issuance can affect the central bank balance sheet. The analysis highlights the relevance of taking into account central bank balance sheet adjustments in response to CBDC issuance for equilibrium dynamics and provides motivation for the two scenarios employed in the subsequent model analysis.

General central bank balance sheet dynamics of CBDC issuance

The idea of opening the central bank balance sheet to deposits by the public is not new. Quite contrary, it was a common central bank practice until mid-20th century. As e.g. elaborated by Fernández-Villaverde et al. (2021), historically many major central banks not only allowed deposits but also granted loans to private firms and individuals. The idea of enabling deposits at the central bank for the public was prominently brought back on stage and into the electronic sphere by Tobin (1985, 1987) from which the current discussion mostly picks up.

The introduction of a CBDC would create a new type of liability that can trigger different adjustments on the central bank balance sheet, contributing in different ways to determine the equilibrium impact of a CBDC. For instance, Barker et al. (2017) argue that "although there has been a lot of discussion about how central bank digital currency could radically change payment systems – and even the financial sector as a whole – the implication for the assets on central bank balance sheets could be just as critical". However, in most CBDC model analyses, the modelling choice that offsets CBDC issuance on the central bank balance sheet remains in the background and its implications are not explicitly discussed.[6] In practice, these balance sheet adjustments will depend on monetary policy and the economic and institutional context and will be difficult to link directly to CBDC. Nonetheless, for analytical purposes, evaluating the impact of different measures in isolation can provide valuable insights.

This paper focuses on the most relevant CBDC scenarios which would lead to an increase in the central bank balance sheet. CBDC issuance will either lead to rearrangements on the liability side of the central bank balance sheet or to an increase in both, liabilities and assets. In most cases, either cash or deposits will be substituted into CBDC.[7] The substitution of cash into CBDC would simply constitute a change in the type of central bank money held by the public

[6]notable exceptions are for instance Fraschini et al. (2021) or Niepelt (2020).

[7]The argument implicitly assumes that the issuance of CBDC only leads to a redistribution but does not affect the total volume of financial assets held by households. This feature also emerges from the model analysis and is consistent with what can be expected empirically.

and would be neutral from a financial stability perspective.[8] Therefore, the following analysis abstracts from cash and focuses only on deposit substitution into CBDC. The substitution of deposits into CBDC would either lead to a reduction in excess reserves (no impact on the size of the central bank balance sheet) or require a counterposition on the asset side (increasing the size of the central bank balance sheet).[9] Situations in which deposit outflows cannot be absorbed by a reduction in excess reserves pose more serious financial stability concerns. Therefore, for simplicity the following analysis abstracts from modelling reserves and focuses on scenarios in which CBDC issuance leads to an increase in the central bank balance sheet.

Deposit substitution into CBDC via central bank credit ('credit policy')

As a first and most prominently discussed option, the central bank could increase its credit operations to banks in order to offset CBDC issuance. Such adjustments would channel lost deposit funding back to the banks in the form of central bank credit. Lending to financial institutions is already a substantial component of central bank assets in normal times but becomes particularly important in times of crisis. The role of the central bank as 'lender of last resort', dating back to Bagehot (1873), is also documented by Pattipeilohy (2016) who shows a clear shift towards private sector lending (including banks) by the Federal Reserve, Bank of England and the Eurosystem in response to the financial crisis from 2007 and onward.

Given the particular relevance of bank lending in times of crisis, the following model framework will explicitly review CBDC issuance with 'credit policy' as illustrated in Table 1.[10] This scenario also provides the basis for the 'equivalence result of public and private money' of Brunnermeier and Niepelt (2019) who show that equilibrium allocations stay unaltered under the assumption that CBDC is fully redistributed as credit to private banks (under the same conditions). Moreover, CBDC issuance with 'credit policy' is employed for instance also in the model analyses of Skeie (2021) and Kim and Kwon (2023).

Deposit substitution into CBDC via the sale of assets (asset policy)

As a second option, the central bank could accept non-bank assets as a counterposition to CBDC issuance. Such a policy scenario would imply that deposits substituted into CBDC will not be replaced with central bank funding, leading to an immediate reduction in the private bank's asset-side operations by the same amount. The assets exchanged in return for CBDC issuance

[8] In addition, focusing on situations of financial fragility, households may not want to exchange cash into CBDC but rather increase their holdings of inherently safe central bank money.

[9] See also Adalid et al. (2022) who systematically review central bank balance sheet adjustment of CBDC issuance and its impact on the balance sheet length.

[10] The analysis abstracts from explicitly taking into account collateral considerations of those credit operations.

Households		Banks		Central Bank	
A	**L**	**A**	**L**	**A**	**L**
Capital	Equity	Capital	Deposits	**Loans**	**CBDC**
Deposits	(endowment)		**Loans**		
CBDC			Equity		

Table 1: CBDC issuance with credit policy in the stylised model framework.

would depend on what asset types would be accepted by the central bank and the assets available to banks. Within the stylised model framework of this paper, the 'asset policy' scenario would translate into purchases of capital by the central bank, visualised in Table 2, as banks only hold capital as asset. In practice, this scenario would be equivalent to central bank purchases of corporate bonds. Although still an unconventional monetary policy tool, corporate bond purchases have gained importance since the financial crisis and for instance have been applied to mitigate the impact of the coronavirus pandemic (e.g. see Cavallino et al., 2020). In the literature, CBDC issuance against corporate bonds has been modelled indirectly by Fernández-Villaverde et al. (2021) and directly by Schilling et al. (2020), building on the seminal paper of Diamond and Dybvig (1983) in which the central bank invests into long-term projects (through an investment bank).

Households		Banks		Central Bank	
A	**L**	**A**	**L**	**A**	**L**
Capital	Equity	Capital	Deposits	**Capital**	**CBDC**
Deposits	(endowment)		Equity		
CBDC					

Table 2: CBDC issuance with asset purchases in the stylised model framework.

In practice, government bonds will also be a prominent assets class to be accepted for CBDC issuance under the 'asset policy' scenario. Government bonds represent the largest asset share in the balance sheet of many central banks. This feature is vividly illustrated by Pattipeilohy (2016) who developed a classification scheme of central banks by their balance sheet composition, in which 'Treasury holder' is one important type. According to their analysis, most of the major central banks fell into this category in 2015. In the literature, CBDC issuance against government bonds has been modelled by Kim and Kwon (2023), Barrdear and Kumhof (2022), and Kumhof and Noone (2021) who even argue that CBDC should only be issued against government bonds.

Both types of asset policy options, CBDC issuance against government bonds and corporate bonds would lead to a reduction in private banks' balance sheets. However, the second round and equilibrium effects will likely differ and depend on the presence of friction in the (model) economy. Within the model framework, central bank purchases of capital securities affect the valuation of capital and thus the financial accelerator. The implications of government bond purchases would depend on monetary-fiscal interactions (such as government debt dynamics), the modelling of the government sector (such as the fiscal regime) and the incentives for banks to hold government debt.[11] The model framework abstract from these considerations on monetary-fiscal interactions and abstains from explicitly modelling the government sector to remain focused on financial stability considerations.[12]

3 Model environment

This section introduces the above outlined two CBDC scenarios into the dynamic bank run framework of Gertler and Kiyotaki (2015), henceforth GK15. Following the GK15 model provides a 'no CBDC' benchmark which allows a comparison of the results under a CBDC with the results of GK15 without a CBDC. Section 3.1 outlines the stylised endowment economy with households, bankers, a central and capital as productive technology. Section 3.2 describes the emergence of banking failures due to self-fulfilling bank runs and due to insolvency.

3.1 General model framework

There are two types of goods in the economy: a non-durable consumption good C_t and a durable capital good K_t. Capital acts as a stylised proxy for the firm sector, is of fixed supply, normalised to unity, and does not depreciate. A unit of capital at time t, K_t, yields Z_{t+1} units of consumption in the next period and can be sold at the price Q_t. The productivity of capital Z_{t+1} is subject to a multiplicative aggregate shock and described by

$$Z_t = Ze^{z_t} \quad \text{with} \quad z_t = \rho z_{t-1} + \epsilon_z \tag{1}$$

where Z is aggregate productivity in steady state and ϵ_z with $E(\epsilon_z) = 0$ is a shock to productivity.

[11]For instance, the implications of CBDC issuance against government bonds would depend on whether government debt holdings would merely redistributed among agents or whether additional funding would be used to reduce (distortionary) taxes , or to expand government spending. More specifically, the implications of the latter would depend on whether government public good expenditures are modelled as a substitute, a complement, or are not considered at all in the utility function of households.

[12]Ultimately, within the stylised model framework, issuing CBDC against government bonds raises the question of what the government debt will be used for. The options ultimately lead to the question at which group of market participants the spending is targeted. Thus, the issue would just be shifted up one entity further. This can be made even more explicit by regarding the monetary and fiscal authority as a consolidated entity.

There are two types of representative agents in the economy: households and bankers, each with a unit mass of one. Households consume the nondurable consumption good and save, for which they have three options: (i) investing directly in capital; (ii) holding bank deposits; or (iii) purchasing CBDC. Banks are more efficient in managing capital but are subject to a leverage constraint. They maximise profits by investing funds from households and the central bank (under CBDC with 'credit policy') into capital. Furthermore, there is a central bank issuing CBDC to households and offsetting funds on the asset side either via 'credit policy' by granting loans to banks or via 'asset policy' by investing into capital.

3.1.1 Households

The stylised model abstracts from labour choices and households receive an endowment of $Z_t W^h$ units of the consumption good each period that is also subject to the productivity shock Z_t. Households maximise lifetime utility

$$U_t = E_t \left(\sum_{i=0}^{\infty} \beta^i ln C^h_{t+i} \right) \tag{2}$$

through the choice of consumption C_t and saving for which they have three options:
(i) Holding deposits D_t which promise to pay a (non-contingent) gross rate of return $\overline{R}^d_{t+1}$.
(ii) Directly investing in productive capital K^h_t at price Q_t, for which households face an increasing and convex management cost for each unit of capital. These efficiency costs of households reflect lacking investment expertise compared to banks. Following GK15, the management costs take the form: $f\left(K^h_t\right) = \frac{\alpha}{2}\left(K^h_t\right)^2$. In the subsequent period, households receive a payoff of $Z_{t+1}K^h_t$ on the invested capital.
(iii) Holding CBDC M_t which pays the gross rate of R^m_{t+1} set by the central bank.
Furthermore, households receive a transfer T_t from the central bank, paying out its seigniorage. If the central bank incurs losses ($T_t < 0$) the transfer turns into lump-sum taxation. The household budget constraint is thus given by:

$$C_t + D_t + Q_t K^h_t + \frac{\alpha}{2}\left(K^h_t\right)^2 + M_t = Z_t W^h + R^d_t D_{t-1} + (Z_t + Q_t)K^h_{t-1} + R^m_t M_{t-1} + T_t$$

Maximising expected utility subject to the budget constraint yields the optimality conditions specifying that the return of all three assets must be equal to the inverse of the stochastic discount factor $\Lambda_{t,t+i} \equiv \beta^i \frac{C^h_t}{C^h_{t+i}}$:

$$R_{t+1}^m = R_{t+1}^d = E_t\left(R_{t+1}^h\right) = E_t\left(\frac{C_{t+1}}{C_t}\beta^{-1}\right) = \frac{1}{\Lambda_{t,t+1}} \tag{3}$$

$$\text{where} \quad R_{t+1}^h \equiv \frac{Z_{t+1} + Q_{t+1}}{Q_t - \alpha K_t^h}$$

3.1.2 Bankers

Environment: There is a unit mass of risk-neutral bankers, running their own financial intermediary. Bankers have a finite expected lifetime and in each period, there is an i.i.d. probability of $0 < 1 - \sigma < 1$ that the banker will stop its activity and exits. Bankers enjoy consumption only upon exit by consuming their accumulated net worth. In each period, the same amount of $1 - \sigma$ new bankers enter with a starting capital of w^b, such that the number of banks remains constant[13]. While in business, bankers aim to maximise their net worth n_t by taking deposits from households d_t and loans l_t (only in the *credit policy* scenario), investing it together with their net worth into the productive technology k_t^h at price Q_t. Bankers are assumed to have a relative efficiency advantage in screening and monitoring projects and do not face the 'management costs' of capital that households must pay[14]. This setup leads to a bank's balance sheet featuring capital on the asset side and household deposits, central bank credit and net worth on the liability side of their balance sheet:

$$Q_t k_t^b = d_t + n_t + l_t. \tag{4}$$

The difference between the earnings from capital and the cost of deposits and central bank loans yields the net worth of the bank:

$$n_t = (Z_t + Q_t)k_{t-1}^b - R_t^d d_{t-1} - R_t^l l_{t-1} \tag{5}$$

The franchise value of the bank is then the sum of the discounted probability of exiting in period i and consuming the accumulated net worth ($n_t = c_t^b$ upon exit):

$$V_t = E_t\left[\sum_{i=1}^{\infty} \beta^i (1-\sigma)\sigma^{i-1} n_{t+i}\right] = E_t\left[\sum_{i=1}^{\infty} \beta^i (1-\sigma)\sigma^{i-1} c_{t+i}^b\right] \tag{6}$$

[13] The limited lifetime approach is a common modelling feature that ensures that bankers hold leverage in equilibrium and do not accumulate equity over time to evade the financial constraint.

[14] This modelling feature can also be seen in a way that bankers do face a positive management cost but the cost is normalised to zero.

The franchise value can be expressed recursively as the discounted expected probability to exit in $t+1$, plus the discounted probability weighted continuation value of the financial intermediary:

$$V_t = E_t \left[\beta(1-\sigma) n_{t+1} + \beta \sigma V_{t+1} \right] \tag{7}$$

In the absence of financial frictions, it would be optimal for all funds to be intermediated by the bank. However, bankers face an endogenous leverage constraint microfounded by a moral hazard problem: bankers can divert a certain fraction $0 < \theta < 1$ of assets for personal use. When deciding whether to 'cheat' or operate honestly, bankers compare the values of both options. With this knowledge, depositors and the central bank will only fund the bank to the extent that there is no incentive to divert funds. This leads to the constraint that the franchise value of the bank must always be greater than or equal to the gains from diverting assets which endogenously limits the debt to equity ratio of a bank.[15] However, it is assumed that the central bank is to a relatively lesser extent subject to the moral hazard risk. Due to its supervisory power and additional measures such as collateral requirements, the central bank is better at enforcing repayment from bankers, similarly modelled in Gertler and Kiyotaki (2010), Gertler and Karadi (2013) and Gertler et al. (2016b). Thus, for each unit of central bank credit supplied, a borrowing bank can only divert $\theta(1-\omega)$ with $0 < \omega < 1$. Accordingly, the moral hazard problem can be condensed into following incentive constraint:

$$V_t = E_t \left[\sum_{i=1}^{\infty} \beta^i (1-\sigma) \sigma^{i-1} n_{t+i} \right] \geq \theta \left(Q_t k_t^b - \omega l_t \right) \tag{8}$$

General optimisation: The banker chooses k_t^b, d_t, l_t each period to maximise the franchise value of the bank (7) subject to the incentive constraint (8), the evolution of net worth (5) and

[15]Naturally, this is not an accurate reflection of reality but rather a common modelling device to endogenise the amount of leverage a bank can hold. This could be done alternatively, e.g., by modelling bank regulation which would lead to a similar specification.

the balance sheet constraint (4):

$$\begin{aligned} V_t &= \max_{k_t^b, d_t, l_t} E_t\left[\beta^i(1-\sigma)n_{t+1} + \beta\sigma V_{t+1}\right] \\ \text{s.t.} \quad E_t\left[\sum_{i=1}^{\infty}\beta(1-\sigma)\sigma^{i-1}n_{t+i}^b\right] &\geq \theta\left(Q_t k_t^b - \omega l_t\right) \\ n_{t+1} &= (Z_{t+1}+Q_{t+1})k_t^b - R_{t+1}d_t - R_{t+1}^l l_t \\ Q_t k_t^b &= d_t + n_t + l_t \end{aligned} \tag{9}$$

The objective and constraints of the bank are subject to constant returns to scale; thus, they can be made independent of the total level of net worth by expressing the problem in terms of per unit of net worth (a detailed description of the reformulation and solution to the maximisation problem can be found in Appendix B). In the optimum, the bank holds as much leverage as possible so that the incentive constraint is always binding. This yields the optimality conditions:

$$(1-\omega)E_t\left[\left(R_{t+1}^b - R_{t+1}\right)\right] = E_t\left[\left(R_{t+1}^b - R_{t+1}^l\right)\right] \tag{10}$$

where $R_{t+1}^b \equiv \frac{Z_{t+1}+Q_{t+1}}{Q_t}$ is the return of a unit of capital invested by banks. Equation 10 determines the allocation of deposit funding and central bank credit. For the bank to be indifferent between the two funding sources, the expected net interest margin of using deposit financing and central bank credit must be equal, when adjusted for the relative tightness of the leverage constraint. The relatively tighter limit on deposits captured by $(1-\omega)$ implies that the central bank lending rate must be higher than the deposit rate. The lower deposit rate compensates for the tighter leverage constraint, making the banker indifferent between deposits and central bank credit. Following the argument of Gertler and Kiyotaki (2010), central bank lending can be interpreted as charging a 'penalty rate'. The rationale of this approach, dating back to Bagehot (1873), is to prevent inefficient use of central bank credit.

Optimisation under 'asset policy' : The above setting is the more general one and applies to both, the scenario with 'credit policy' and 'asset policy'. However, under the 'asset policy' scenario, the banker's problem is simplified as the central bank does not provide credit and therefore $l_t = 0$. In this case, the bank's optimisation problem reduces to the case in GK15, since deposits are the only type of leverage and the bank simply chooses deposits per unit of net worth $\frac{d_t}{n_t}$ so that the incentive constraint is binding.

3.1.3 Central Bank

The representation of the central bank focuses on issuing an interest bearing CBDC. The model framework is only expressed in real terms and does not include inflation, and therefore the model also abstracts from conventional monetary policy.[16] In addition, the model abstracts from money as medium of exchange, focusing on the role of money as store of value which is central for studying financial stability and bank run aspects.

CBDC issuance: The central bank issues CBDC following an interest rate rule, but which can be expressed equivalently in terms of a quantity rule.[17]

$$R^m_{t+1} = \delta_0 + \delta_Y \left(\frac{Y - Y_t}{Y} \right) - \delta_M M_t \tag{11}$$

Following this rule, the CBDC interest rate reacts to fluctuations in output from steady state Y and to CBDC demand. In response to a negative shock that reduces output, the CBDC interest rate increases. While this reaction may appear counterintuitive at first glance, it is important to be aware that the CBDC rate does not affect credit conditions. Instead, in this setting the CBDC interest rate is simply a saver's rate, providing households with an additional opportunity to save. If CBDC demand is high, the CBDC interest rate rule decreases and thereby dynamically stabilises the demand for CBDC by discouraging to hold more CBDC when the amount in circulation is already high.

While the issuance of CBDC on the liability side of the central bank balance sheet is the same for both analysed policy scenarios, the asset side setup differs under CBDC issuance with 'credit policy' and 'asset policy'.

Credit Policy: The rule allocating credit to the banks under CBDC with 'credit policy' is straightforward. By assumption, this policy option offsets CBDC issuance by providing credit to banks and thus all CBDC funds are channeled back to the banks:

$$L_t = M_t \tag{12}$$

[16] The combination of CBDC and conventional monetary policy is e.g. analysed in Assenmacher et al. (2023).

[17] The central bank can either determine the supply of CBDC or the interest rate. In the CBDC literature, both types of rules are analysed; for example, Gross and Schiller (2020) analyse CBDC under a price rule in which the interest rate on CBDC is set as a fixed spread to the Taylor rule. Minesso et al. (2022) analyse a Taylor-type rule (among a non-interest bearing CBDC), and Barrdear and Kumhof (2022) and Burlon et al. (2022) analyse both quantity and interest rate rules.

The central bank redirects lost deposit funding exchanged in CBC back into the banking sector on a 1:1 basis and grants loans L_t to banks equivalent to the amount of CBDC. The interest rate on central bank credit R_t^l is endogenously determined.
The flow of central bank funds is then given by

$$M_t + R_t^l L_{t-1} = R_t^m M_{t-1} + L_t + T_t \tag{13}$$

where in each period the central bank issues CBDC M_t and receives the loan repayments plus interest from the previous period. It uses the funds to repay the claims and interest on CBDC from the previous period and additionally grants new loans. The remainder is distributed as transfers T_t to the households (or as tax in the case of a negative residue). In the absence of a bank run, the interest rate on loans is higher than the interest rate on CBDC; therefore, the transfers are positive despite being small.

Asset Policy: In contrast, under 'asset policy', the central bank does not accommodate CBDC with credit but buys capital itself. When investing into credit, it is reasonable to assume that the central bank is less efficient than the bank but more efficient than private households. Therefore, when the central bank purchases capital it faces – similar to households but at a lower level – an increasing and convex cost of $f\left(K_t^{cb}\right) = \frac{\alpha_{cb}}{2}\left(K_t^{cb}\right)^2$ with $\alpha_{cb} < \alpha$.
The central bank fully offsets the issuance of CBDC by investments in capital and therefore buys as much capital, such that the gross investment equals the amount of CBDC supplied to households, leading to:

$$Q_t K_t^{cb} + \frac{\alpha_{cb}}{2}(K_t^{cb})^2 = M_t \tag{14}$$

The flow of funds of the central bank evolves similarly to Equation 13: the funds from CBDC and last period's capital returns are used to repay the claims of the CBDC from the previous period and for investments into new capital, plus management costs:

$$M_t + (Z_t + Q_t)K_{t-1}^{cb} = R_t^M M_{t-1} + Q_t K_t^{cb} + \frac{\alpha_{cb}}{2}(K_t^{cb})^2 + T_t \tag{15}$$

3.1.4 Aggregation, Timing and Equilibrium

Summarising the individual bank measures (denoted in lowercase letters) leads to the banking sector aggregate (denoted in capital letters). As there are a constant number of symmetric banks normalised to one and an equal amount of banks exit enter each period, aggregate net worth evolves according to:

$$N_t = \sigma \left[(Z_t + Q_t) K_{t-1}^b - R_t^d D_{t-1} - R_t^l L_{t-1} \right] + W^b \quad \text{with} \quad (1-\sigma) w^b = W^b \tag{16}$$

Capital is of fixed supply and normalised to unity, therefore capital holding of households K_t^h , banks K_t^b and the central bank K_t^{cb} add up to:

$$K_t^h + K_t^b + K_t^{cb} = 1 \tag{17}$$

Under CBDC with 'asset policy' $K_t^{cb} \geq 0$ and under CBDC with 'credit policy' $K_t^{cb} = 0$ as the central bank does not invest into the productive technology itself.

Gross output amounts to:

$$C_t^h + C_t^b + \frac{\alpha}{2} \left(K_t^h \right)^2 + \frac{\alpha_{cb}}{2} \left(K_t^{cb} \right)^2 = Z_t + Z_t W^h + W^b \tag{18}$$

However, output is defined without management costs i.e.

$$Y_t = C_t^h + C_t^b \tag{19}$$

The sequence of events is as follows: at the beginning of period t, the productivity of capital Z_t is realised. The new allocation of deposits, CBDC, loans, and capital investments are then determined. Deposits and loans are issued in a way that the moral hazard constraint is satisfied and the banker does not have the incentive to divert assets. The next period begins with the realisation of return on the capital invested the previous period.

Under CBDC with 'credit policy', the equilibrium is given by the vector of real prices Q_t, R_t^d, R_t^m, R_t^l and a vector of quantities $Z_t, K_t^b, K_t^h, D_t, M_t, L_t, N_t, \frac{V_t}{N_t}, C_t^h, T_t$. Under CBDC with 'asset policy' the equilibrium is determined by the slightly different vector of real prices Q_t, R_t, R_t^M and a vector of quantities $Z_t, K_t^b, K_t^h, K_t^{cb} D_t, M_t, N_t, \frac{V_t}{N_t}, C_t^h, T_t$. A compilation of the equilibrium equation under both policies can be found in Appendix C.

3.2 Bank failures in the model environment

The equilibrium outlined above is based on the mutual beliefs of households about the deposit decisions of other households, in particular that they will not participate in a bank run. Under certain conditions, a second equilibrium among the above outlined 'banking equilibrium' exists, in which the belief of a bank run triggers a bank run in equilibrium ('bank run equilibrium'). The following section examines the conditions under which a bank run equilibrium exists and how it unfolds under the different CBDC policy scenarios. The analysis considers only systemic bank runs, where banks are identical and subject only to aggregate shocks. In addition, the productivity shock to the economy that opens up the possibility of a bank run is unanticipated and therefore bank runs are also unanticipated.[18]

3.2.1 Types of bank failures

Bank failures due to illiquidity

In each period, households decide whether to roll over their deposits to the next period. The individual decision to roll over deposits critically depends on the expectations of other households. If a household expects that other households will not roll over deposits and if this would leave the bank with zero assets, then it is individually rational not to roll over deposits as well. In such cases of a systemic bank, all households withdraw their deposits and banks are forced to liquidate their assets at the fire-sale price Q_t^*. If the funds from the sold assets are less than the claims on the deposits, depositors must accept an equal haircut x_t with

$$x_t = \frac{(Q_t^* + Z_t)K_{t-1}^b}{R_t^d D_{t-1}} \tag{20}$$

on their claims $R_t^d D_{t-1}$.[19]

A bank run equilibrium exists if the haircut value becomes less than 1 ($x_t < 1$). Conversely, if $x_t \geq 1$, all funds can be paid out in full and there is no reason to run, even if all other households run. Thus, the *existence* of a bank run equilibrium is determined by economic fundamentals, with the fire-sale value of the assets determining the haircut rate x_t. Whether a bank run is *triggered* ultimately depends on the beliefs of households, with the bank run being induced in the model via a sunspot shock. If a bank run results in a default, the strategic complementarity

[18] Gertler and Kiyotaki (2015) and Gertler et al. (2016a) further study anticipated banking panics, allowing for a risk premium on deposits that is based on bank run probabilities.

[19] In contrast to Diamond and Dybvig (1983), there is no sequential service constraint assumption and depositors share the remaining funds equally across all depositors.

in behaviour makes the belief of a bank run a self-fulfilling prediction. Self-fulfilling bank runs can be considered as *bank failures due to illiquidity* and are possible if:

$$(Q_t^* + Z_t)K_{t-1}^b < R_t^d D_{t-1} \tag{21}$$

Bank failures due to insolvency

If a sufficiently large shock hits the economy and the bank's liabilities exceed the value of its capital even at the non-fire-sale price Q_t, it is rational for households to withdraw their deposits – regardless of the beliefs of other households. These bank runs driven by economic fundamentals can be considered as bank failures due to insolvency and are possible if:

$$\begin{aligned} &(Q_t + Z_t)K_{t-1}^b < R_t^d D_{t-1} \quad \text{(in the CBDC asset policy \& no CBDC case)} \\ &(Q_t + Z_t)K_{t-1}^b < R_t^d D_{t-1} + R_t^l L_{t-1} \quad \text{(in the CBDC credit policy case)} \end{aligned} \tag{22}$$

The sole difference between the illiquidity and insolvency conditions in the 'no CBDC' scenario and the 'asset policy' CBDC scenario is the pricing of capital. There is a further difference in the 'credit policy' CBDC scenario: In the case of bank failures due to illiquidity, it is assumed that only households but not the central bank participate in the bank run. Instead, the central bank continues to lend to banks (see a more detailed description below). In contrast, in the case of bank failures due to insolvency, the central bank also withdraws its funding support and banks fail when the asset valuation cannot cover the full liabilities (i.e. deposits and central bank loans). Hence, the condition for a bank failure due to insolvency is met when the fundamental valuation of assets is less than than the total liabilities (deposits and credit). In contrast, the condition for a bank failure due to illiquidity is met when the fire-sale valuation of assets is less than the value of deposits, i.e. only a subset of the total liabilities.

Equilibrium ranges

The above scenarios lead to three possible equilibrium regions: in the first region, there is a unique equilibrium with functioning financial intermediation ('banking equilibrium'). In the second region, the banking equilibrium and the bank run equilibrium coexist. Lastly, in the third region, only the bank run equilibrium remains. Based on the above conditions for the two types of bank failures, Table 3 shows the thresholds for the different equilibrium ranges, conditional on the evolution of capital productivity Z_t.

Equilibria	Type of bank failures	CBDC asset policy & No CBDC	CBDC credit policy
Banking	None	$Z_t \geq \frac{R^d_t D_{t-1}}{K^b_{t-1}} - Q^*_t$	
Banking + Bank run	Illiquidity	$\frac{R^d_t D_{t-1}}{K^b_{t-1}} - Q^*_t > Z_t \geq \frac{R^d_t D_{t-1}}{K^b_{t-1}} - Q_t$	$\frac{R^d_t D_{t-1}}{K^b_{t-1}} - Q^*_t > Z_t \geq \frac{R^d_t D_{t-1} + R^l_t L_{t-1}}{K^b_{t-1}} - Q_t$
Bank run	Insolvency	$Z_t > \frac{R^d_t D_{t-1}}{K^b_{t-1}} - Q_t$	$Z_t > \frac{R^d_t D_{t-1} + R^l_t L_{t-1}}{K^b_{t-1}} - Q_t$

Table 3: Equilibrium ranges based on capital productivity Z_t.

3.2.2 Bank runs in the different policy scenarios

How does a bank run unfold?[20] As a reference point, it is useful to first review the baseline scenario without a CBDC: a large shock is realised, triggering a bank run and households have no other other option but to invest in capital themselves. However, households are only willing to purchase capital at a lower price due to their increasing marginal costs of managing capital: the higher costs of managing capital lead a fire-sale price of capital that is much lower than the non-bank-run price of capital. Although socially detrimental, individually it is still optimal to run and receive a haircut on deposit claims rather than losing the full deposit value.

In the 'asset policy' CBDC scenario, bank runs unfold in a similar way: households withdraw all deposits from the bank, the bank fails and closes down. The main difference here is that households can hold CBDC as a second asset. The relative return to households between holding capital and CBDC, determines the share that flows into CBDC and into capital. In this case, households but also the central bank invest into capital in the period of the bank run.

In the 'credit policy' CBDC scenario, bank runs remain similar for households but unfold differently for banks. As long as the bank is merely illiquid but not insolvent (see the conditions outlined above), only households participate in the run and withdraw their deposits while the central bank continues to grant credit. Households withdraw their deposits with a haircut, leaving the bank with no assets but outstanding debt to the central bank. In the period of the run, the bank defaults on its claims on the central bank. Without an intervention, the bank would go bankrupt. Instead, the central bank, in its spirit as 'lender of last resort', continues to lend to the bank in the amount of its CBDC inflows which enables the bank to continue to operate.[21] In the period of the bank run, the bank invests into capital only with central bank

[20]The set of the condensed equilibrium conditions at the time of the bank run can be found in the Appendix C.
[21]Note that this approach may raise moral hazard concerns, which could be mitigated by close supervision.

credit as it has zero net worth and zero deposits. Due to the outstanding claims of banks to the central bank, the capital return in the period of the run is fully paid out to the central bank. Ultimately, these measures prevent bank failures but also leave banks with zero net worth in the period after the run.

The central bank usually makes profit as the interest rate on credit is larger than the rate on CBDC. However, in the period of the bank run, the central bank absorbs the loss on the defaulted loans and the losses are distributed to households via lump-sum taxes (the transfers become negative). Nevertheless, in the period after the bank run, the central bank recovers some losses as it collects the full profit on invested capital by the bank and redistributes it to households[22]. In the period after the bank run, the banking sector starts to recover. Thus, in the 'credit policy' scenario, banks do not fail and their ability to intermediate capital efficiently can be maintained in the period of the bank run. At the same time, banks still start the period after the bank run with zero net worth and therefore are in the same situation as in the scenarios with 'no CBDC' and CBDC with 'asset policy': with zero net worth it is as if these banks had failed and the sector must recover via newly entering banks.

4 Model analysis

This section examines the dynamics of the economy under three different scenarios: CBDC with 'credit policy', CBDC with 'asset policy', and 'no CBDC'. After explaining the calibration of the numerical exercise in Section 4.1, Section 4.2 analyses the implications of these three model scenarios in steady state. Subsequently, Section 4.3 examines the response of the economy to capital productivity shocks that do not lead to bank failures, whereas Section 4.4 focuses on bank failures and analyses the impact of the different CBDC policy scenarios. Specifically, the latter section investigates the required shock sizes after which self-fulfilling bank runs become possible (Section 4.4.1) and the shock sizes that lead to insolvency (Section 4.4.2). The aim of the analysis in this stylised endowment economy setup is not to aim for precisely matching empirical properties but rather to investigate the mechanisms influencing financial stability through the issuance of CBDC in the differing policy scenarios outlined above.

4.1 Calibration and simulation

Table 4 lists all the parameter choices. Most of the parameters present in the 'no CBDC' scenario are taken from GK15, with the exception of the share of divertable assets $\theta = 0.22$, which is

[22] The set of the equilibrium conditions at the time of the bank run can be found in Appendix C.

updated to the value of Gertler et al. (2019). The introduction of a CBDC introduces additional parameters. Under CBDC issuance with 'credit policy', central bank credit to banks is, similar to deposit funding, also subject to a moral hazard constraint but only to half the extent and thus ω is set to 0.5 (e.g. as in Gertler and Karadi, 2013). Similarly, under CBDC issuance with 'asset policy', the management cost of capital purchases α_{cb} that the central bank faces is set to half the efficiency costs that households face.

There is no empirical counterpart to guide the choice of values of the CBDC interest rate rule. Both, δ_0 and δ_M affect the steady state share of CBDC, set to equal 10% of total household saving. As most central banks aim to design CBDC as a means of exchange rather than an attractive store of value (Group of Central Banks, 2020), this is a relatively high and conservative value and in line with the cash-to-GDP ratio for 2021 for the US (9.2%) and euro area (12.8%) (Bank for International Settlements, 2020). The output coefficient of the CBDC rule δ_Y set to 0.26, following the calibration of Minesso et al. (2022) who analyse the international implications of CBDC with a Taylor-type rule. With this calibration, the output coefficient is somewhat larger than the output coefficient in conventional Taylor-type rules, however the CBDC rule does not reflect conventional monetary policy and is therefore not comparable to the calibration of standard Taylor rules. Furthermore, the CBDC rule demand coefficient δ_M is set such that an increase in the demand for CBDC has a distinct but moderate effect on the CBDC interest rate. Finally, periods are calibrated to be equal to one quarter.

Parameter	**Value**	**Source**	**Description**
α	0.008	GK15	Marginal management costs households
α_{cb}	0.004	new	Marginal management costs central bank (CAP)
β	0.99	GK15	Discount rate
σ	0.95	GK15	Bankers survival probability
θ	0.22	GK15	Share of divertable assets
ω	0.5	GK13	Advantage in seizure rate for central bank credit (LOB)
ρ	0.95	GK15	Serial Correlation of productivity shock
W^h	0.045	GK15	Household endowment
W^b	0.0011	GK15	Bankers endowment
δ_0	1.0194	new	CBDC base parameter
δ_Y	0.26	new	CBDC rule output coefficient

Continued on next page

δ_M	0.1	new	CBDC rule demand coefficient

Table 4: Parameter Calibration.
GK15=Gertler and Kiyotaki (2015), GK13=Gertler and Karadi (2013)

The simulation largely follows the computational procedure of Gertler and Kiyotaki (2015), outlined in more detail in the online appendix. The response to a shock without a bank run is computed as a shock to the nonlinear perfect foresight model. To simulate a shock followed by a bank run, the equilibrium path is split into three components and calculated starting backwards. First, the path of the economy after the bank run is calculated, starting from one period after the bank run. This is possible because the terminal values (i.e. steady state values) and the initial values of the backward-looking variables are known. Based on the post-bank run paths of the variables, the forward-looking bank run equilibrium can be calculated. Finally, until the period in which the bank run occurs, the equilibrium path is the same as under the shock without a bank run. While in principle a bank run can occur in any period, the period of the shock is the period most susceptible. Therefore, the following analysis focuses on bank runs emerging in the period of the shock and and only last one period. Furthermore, the analysis restricts deposits, bank net worth, and bank consumption to be non-negative.

4.2 Steady state implications of a CBDC

In steady state, the issuance of a CBDC does not affect aggregate quantities, only allocations. Table 5 compares the steady state values of the model under 'credit policy', 'asset policy', and the baseline scenario with 'no CBDC'.[23] Steady state output and most prices are the same across all three scenarios. Accordingly, steady state welfare, defined as aggregate consumption, is the same across all scenarios.[24]

[23]Values are rounded up to the fourth decimal. Differences beyond the fourth decimal are so small that values will be considered as the same.

[24]Welfare falls short of the first best scenario without financial frictions in which only banks invest into capital.

Variable	CBDC credit policy	CBDC asset policy	No CBDC
Output Y_t	0.0586	0.0586	0.0586
Capital			
Bank capital K_t^b	0.6468	0.5552	0.6462
Household capital K_t^h	0.3532	0.3520	0.3538
Central bank capital K_t^{cb}	/	0.0928	/
Consumption			
Household consumption C_t^h	0.0551	0.0552	0.0549
Banker consumption C_t^b	0.0035	0.0034	0.0037
Financial assets			
Deposits D_t	0.4904	0.4954	0.5803
CBDC M_t	0.0939	0.0939	/
Loans to banks L_t	0.0941	/	/
Banking sector			
Bank net worth N_t	0.0686	0.0655	0.0717
Bank leverage ϕ_t	9.4337	8.4940	9.0085
Interest rates & prices			
Price of capital Q_t	1.0005	1.014	1
Capital return R_t^b	1.0518	1.0518	1.0518
Deposit rate R_t^d	1.0404	1.0404	1.0404
CBDC rate R_t^M	1.0404	1.0404	/
Loan rate R_t^L	1.0461	/	/

Table 5: Comparison of steady state values.
Note: returns are expressed in annualised values

While total welfare is unchanged by CBDC, there is redistribution from banks to households. Household consumption slightly benefits from CBDC issuance and the banking sector – especially banks' net worth and consumption – is strained by CBDC in steady state. However, the increase in household consumption by less than 1% is marginal and originates from the lump-sum transfers of the central bank to households. While the portfolio allocation of household savings changes more substantially (Figure 1), this does not affect household wealth as equilibrium returns are equal across assets and prices remain unchanged. Per calibration target, CBDC holdings account for 10% of total household savings. The share of capital investment remains relatively constant. Therefore, flows into CBDC originate entirely from deposits and reduce the percentage of deposit holdings by about 10%-points.

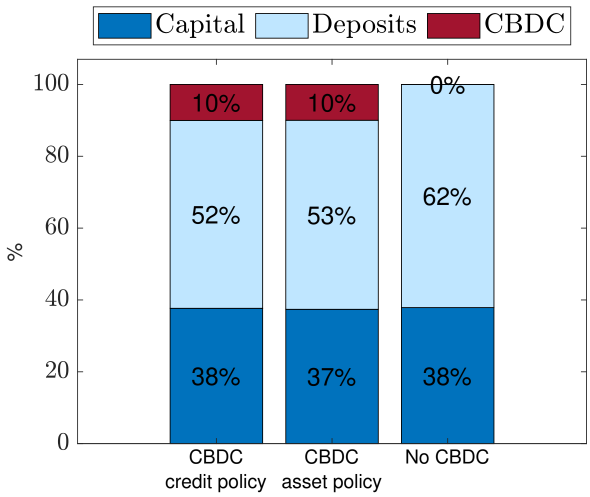

Figure 1: Households savings in steady state.

The consumption of bankers is lower due to lesser net worth and profitability of banks. Under CBDC with 'asset policy', the issuance of CBDC leads to a reduction in the size of the balance sheet, while under CBDC with 'credit policy' the lost deposit funding is replaced by central bank credit, leading to a balance sheet size that remains similar (Figure 2). Both policy scenarios strain profitability: while the issuance of CBDC under 'credit policy' reduces profitability through higher funding costs of central bank credit, the issuance of CBDC under 'asset policy' reduces net worth through a smaller balance sheet size. This smaller balance sheet, and hence the capital holdings of banks under CBDC with 'asset policy' is compensated by central bank capital holdings and the central bank takes over some of the capital investments which were previously held by banks (Figure 3).

To summarise, both CBDC policy options do not change steady state aggregate output and welfare but only affect the distributions between households and banks.

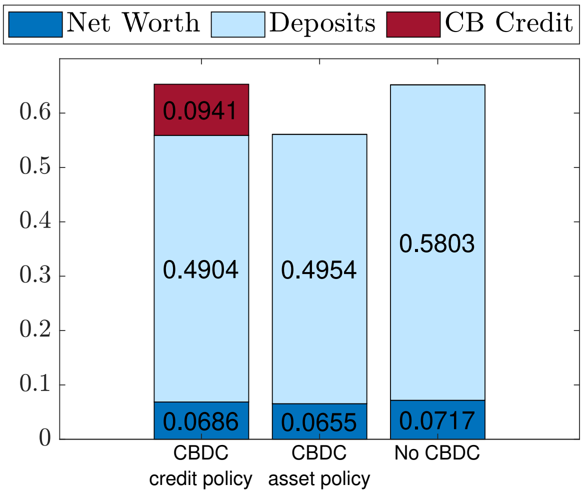

Figure 2: Bank funding in steady state.

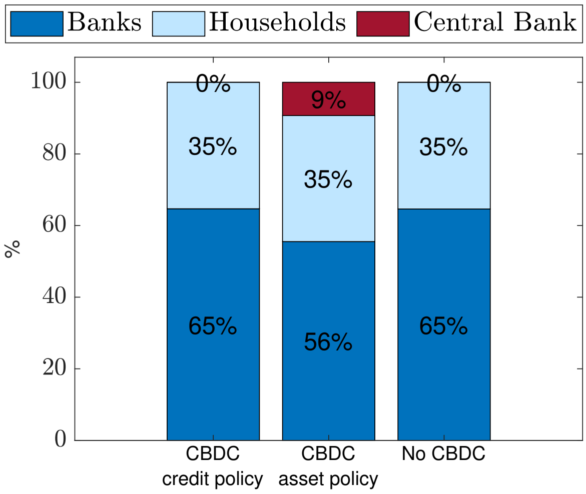

Figure 3: Capital allocation in steady state.

4.3 Financial shocks under a CBDC

The following section investigates the impact of a CBDC in response to an unanticipated 5% shock to capital productivity that does not trigger a bank run. Figure 4 shows the response of key variables to the shock. Overall, the issuance of CBDC tends to reduce output losses, albeit only marginally. The slightly dampened impact on output can be explained by the stabilising effect of CBDC issuance on capital prices under both policy scenarios. Under CBDC with 'credit policy', capital prices are stabilised by refinancing bank funding with increased loans from the central bank. Under CBDC with asset policy capital prices are stabilised through increased capital holdings by the central bank that substitute for household capital holdings.

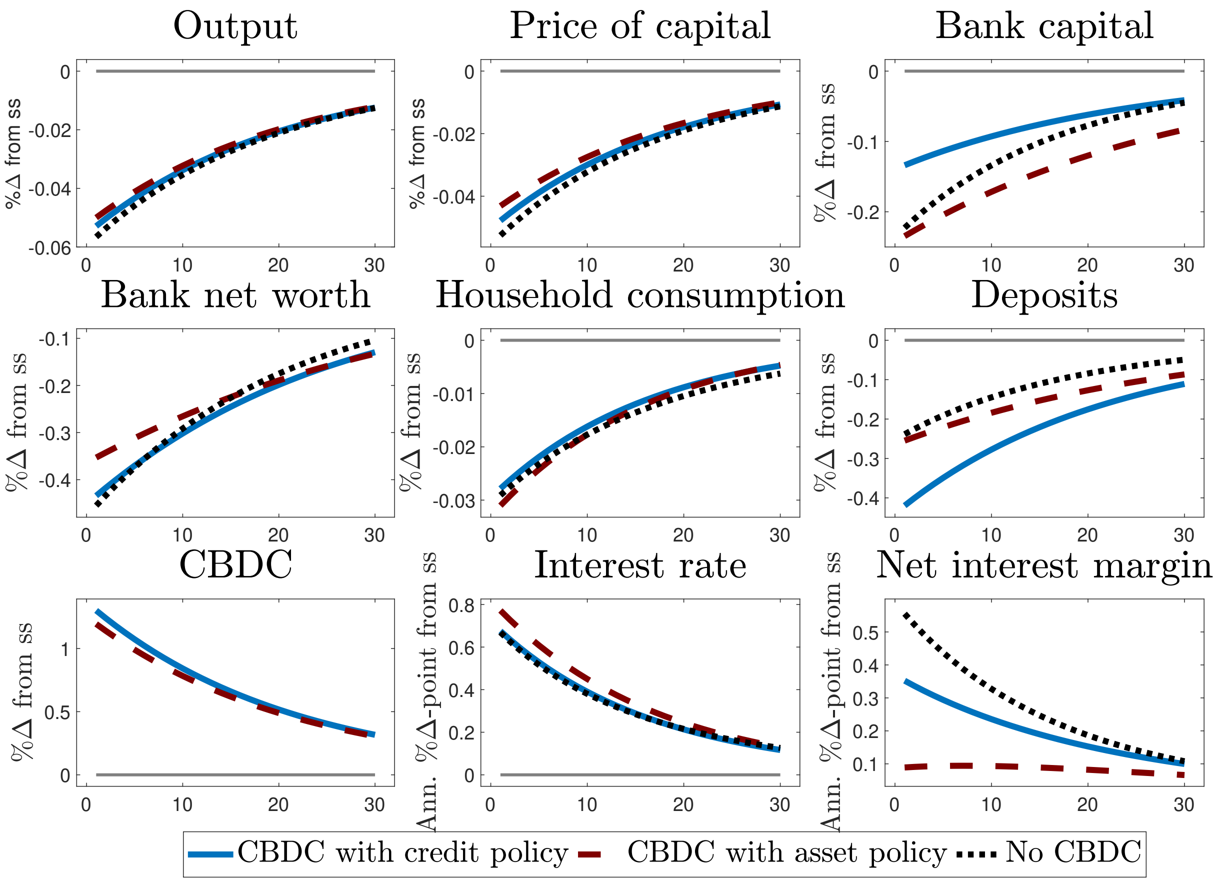

Figure 4: No-bank-run response to a 5% shock to capital productivity.

The simulated shock to capital quality tightens the bank's leverage constraint, forcing a reduction in bank capital. Under CBDC issuance with 'asset policy', bank capital and deposits fall similarly strong but more persistently than in the 'no CBDC' scenario. In contrast to the 'no CBDC' scenario, households do not have to fully absorb the reduction in bank capital but can also hold CBDC which increases in response to the shock. To offset the inflow in CBDC holdings on the balance sheet, the central bank increases its investments in bank capital. Those activities alleviate household managements costs and stabilise the price of capital which leads to a smaller drop in bank net worth than in the alternative scenarios.

Under CBDC issuance with 'credit policy', the decline in bank capital is smaller than under the alternative scenarios. The decrease in deposits that is replaced by CBDC is channelled back to the bank in the form of central bank credit, mitigating the drop in bank capital and stabilising the price of capital. However, central bank credit is provided at higher funding costs and leads to a similar decline in bank net worth as in the 'no CBDC' scenario. Nevertheless, the optimal funding structure of banks still shifts towards central bank credit because the increase in the bank's interest margin is higher for loans than for deposits. This also explains why the decline in deposits is steeper than in the alternative scenarios.

In all scenarios, the decline in household consumption is similar, being least severe under 'credit policy', while the decline in bankers' consumption is least severe under 'asset policy'. Considering household and bank consumption jointly, the aggregated and discounted welfare across periods in response to the shock is higher under the CBDC scenarios ('asset policy' : 4.9566, 'credit policy': 4.9573) than in the 'no CBDC' scenario (4.9546). Taking stock, CBDC issuance tends to mitigate the response to capital quality shocks by stabilising asset prices, rather than exacerbating them but which comes at the cost of larger deposit outflows.

Insights from alternative model calibrations

Most parameter values for the simulation are adopted from Gertler and Kiyotaki (2015). However, there are a few parameters that have been newly introduced and are less established in the literature. The following paragraphs briefly summarise the main insights from: (i) varying the dynamic coefficients δ_Y and δ_M in the CBDC interest rate rule; (ii) varying the capital management costs of the central bank α_{cb}; and (iii) varying the leverage constraint for central bank credit to banks ω. These alternative calibrations are also presented more detaileldy in Appendix A.

If the CBDC interest rate rule is more responsive to output fluctuations from steady state, this leads to a stronger stabilisation of output, consumption, capital prices, and bank net worth. Such stabilisation comes at the cost of higher deposit disintermediation and a larger increase in CBDC, also resulting in a larger central bank balance sheet. In contrast, a stronger response to CBDC demand makes the CBDC interest rate less attractive and leads to stronger automatic stabilisation of CBDC issuance, which in turn comes at the cost of lower stabilisation in output and capital prices.

The same mechanisms explain the optimal values of δ_Y and δ_M: the optimal δ_Y that would maximise welfare (aggregate discounted total consumption) in response to a capital quality shock would be as high as possible, whereas the optimal δ_M would be as low as possible. However, these welfare considerations do not take into account the negative implications of excessive central bank intervention in response to a shock, such as potential moral hazard side effects.

Relaxing the assumptions about the central bank's capital management costs in case of

CBDC with 'asset policy', and the leverage constraint for central bank credit to banks in the case of CBDC with 'credit policy', tends to lead to a higher effectiveness of central bank measures (i.e. a higher stabilisation and smaller increases in CBDC and the central bank balance sheet). Yet, one may question how reasonable it would be to fully relax those assumptions. The opposite picture emerges when these assumptions are further restricted. Although, overall the impact of variations is relatively small, especially for variations in capital management costs under CBDC with 'asset policy'.

4.4 Bank failures

Following on from Section 3.2 which distinguished between the different types of bank failures, this section investigates bank failures and the different equilibrium regions in the numerical example. Specifically, it addresses the questions: how large does a shock to the steady state have to be (i) for a self-fulfilling bank run equilibrium to emerge and enabling a bank failure due to illiquidity, and (ii) to cause a bank failure due to insolvency?

Figure 5 provides the answer to these questions by showing the equilibrium regions for the different policies in the context of the numerical example.[25] In the baseline case without CBDC, self-fulfilling bank runs become possible when capital productivity shocks exceed 2.5%. When capital shocks exceed 8.04%, all deposits are withdrawn regardless of the beliefs of other households and banks become insolvent. Including CBDC issuance in the model framework improves financial stability by postponing both types of bank failure to larger shocks. Under CBDC issuance with 'asset policy', self-fulfilling runs become possible when capital productivity shocks exceed 8.57% and banks become insolvent at shock sizes above 11.3%. Under CBDC issuance with 'credit policy', the lender of last resort measures prevent self-fulfilling runs in the baseline calibration, but not in general. At the same time, under CBDC issuance with 'credit policy', banks already become insolvent at shock sizes above 8.26%, a region similar to the insolvency threshold of no CBDC issuance. The following subsections will further dissect the mechanisms at play for the equilibrium regions in the different policy scenarios.

[25] Note that the higher the simulated shock, the more unstable the simulations become.

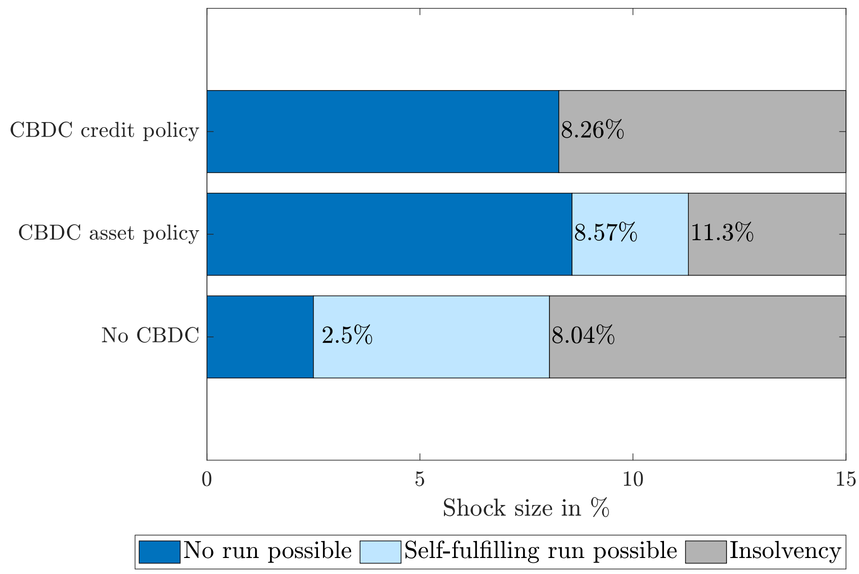

Figure 5: Shock sizes to steady state levels that are needed i) for the emergence of a self-fulfilling run, leading to default due to illiquidity and ii) for triggering a fundamental bank run, leading to insolvency in the period of the shock.

4.4.1 Bank failures due to illiquidity

Bank runs without a CBDC

Figure 6 compares the bank run and no bank run scenarios after a 5% shock to capital productivity in the case without CBDC issuance. As long as households believe that other households will not lose faith and run on the bank, the banking equilibrium prevails and the shock has the dynamics as displayed in Figure 4 (black solid lines in Figure 6 and black dotted lines in Figure 4). However, if households believe that the shock prompts others to withdraw their deposits, it triggers a bank run in the period of the shock (grey dashed lines in Figure 6).

The bank run leads to a complete withdrawal of deposits, forcing the banks to liquidate all their capital. All banks fail as their net worth is wiped out during the run. Only households are left to purchase capital and as they are less efficient than banks, they face increasing marginal costs for managing capital. Therefore, capital is sold at a fire-sale price, a price much lower than it would have been in the absence of a run. Overall, the bank run leads to a sharp drop in output and consumption. In the period after the bank run, the banking sector starts to recapitalise. Due to the reduced capital productivity and price, banks are initially able to earn high profit margins, leading to high interest rates and a rapid recovery.

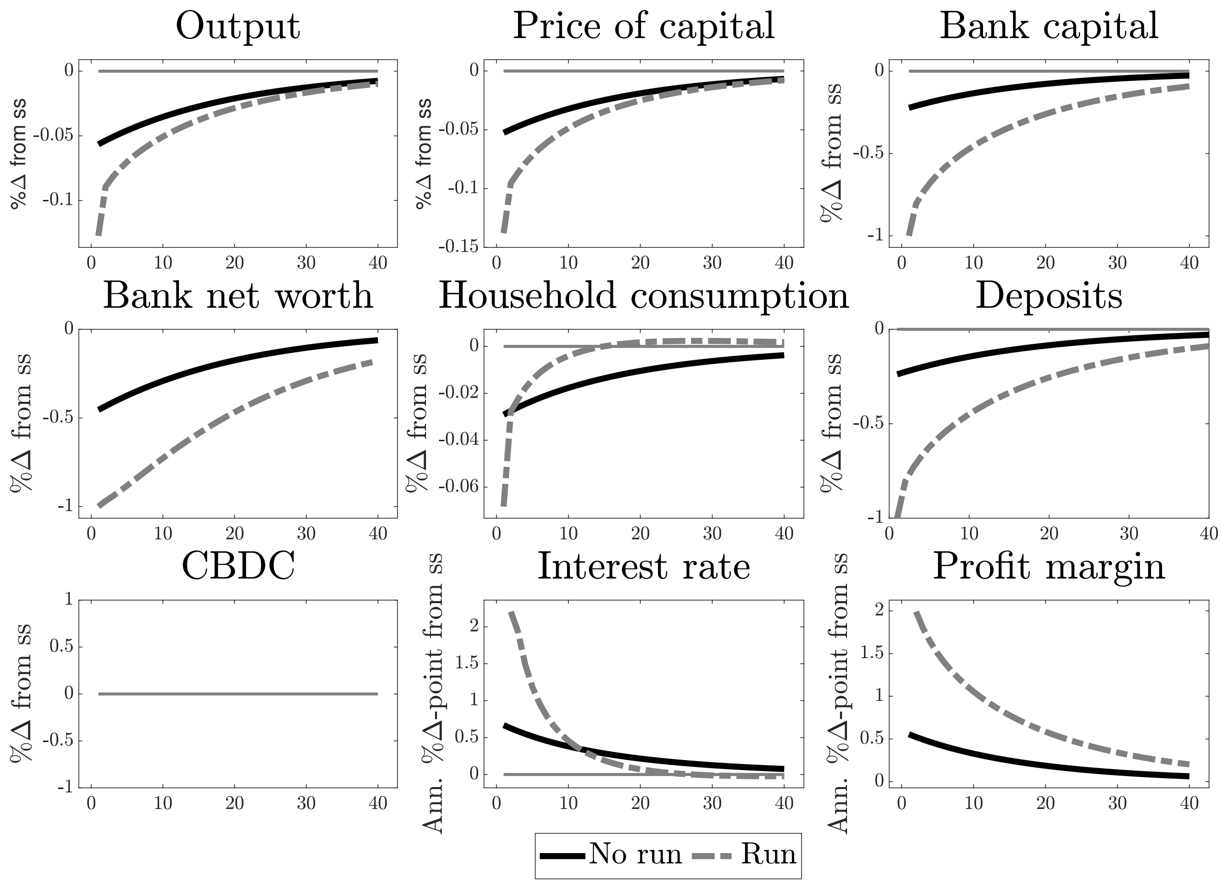

Figure 6: 'No CBDC scenario' – 5% Shock to productivity with and without a bank run in the period of the shock.

Bank runs under CBDC with asset policy

CBDC issuance with 'asset policy' requires a much larger shock to enable the possibility of self-fulfilling runs. Figure 7 compares the bank run (light red dashed lines) and no bank run (red solid lines) scenarios after a 10% shock to capital productivity. In response to the bank run, the price of capital falls sharply, although in relative terms less than in the 'no CBDC' scenario. The dampened decline in capital prices translates into relatively less severe output losses and consumption losses compared to the 'no CBDC' case, but still more severe than the no bank run scenario.

The central bank's asset purchases stabilise the fire-sale price of assets, thereby mitigating output losses in the bank run scenario. When withdrawing deposits from the bank, households can now hold both capital and CBDC. The additional option to hold CBDC makes the run less costly as households do not have to hold all their funds in the form of capital. Instead, some of the capital is also held by the central bank. Jointly holding the capital stock during the bank run stabilises the fire-sale price for two reasons: first, although the central bank is less efficient in managing capital than the bank, it is more efficient and faces lower capital management costs than households. Second, capital is held jointly by households and the central bank and, as capital management costs are convex, this further mitigates the overall efficiency losses of capital.

The stabilising influence of central bank asset purchases on the fire-sale price of capital is sufficiently large to fully cover deposit claims even in response to larger shocks and thereby postpones the occurrence of self-fulfilling bank runs. While a bank run under CBDC issuance with asset policy still leads to a welfare deterioration, the additional welfare loss is smaller which leads to a more resilient economy towards self-fulfilling runs.

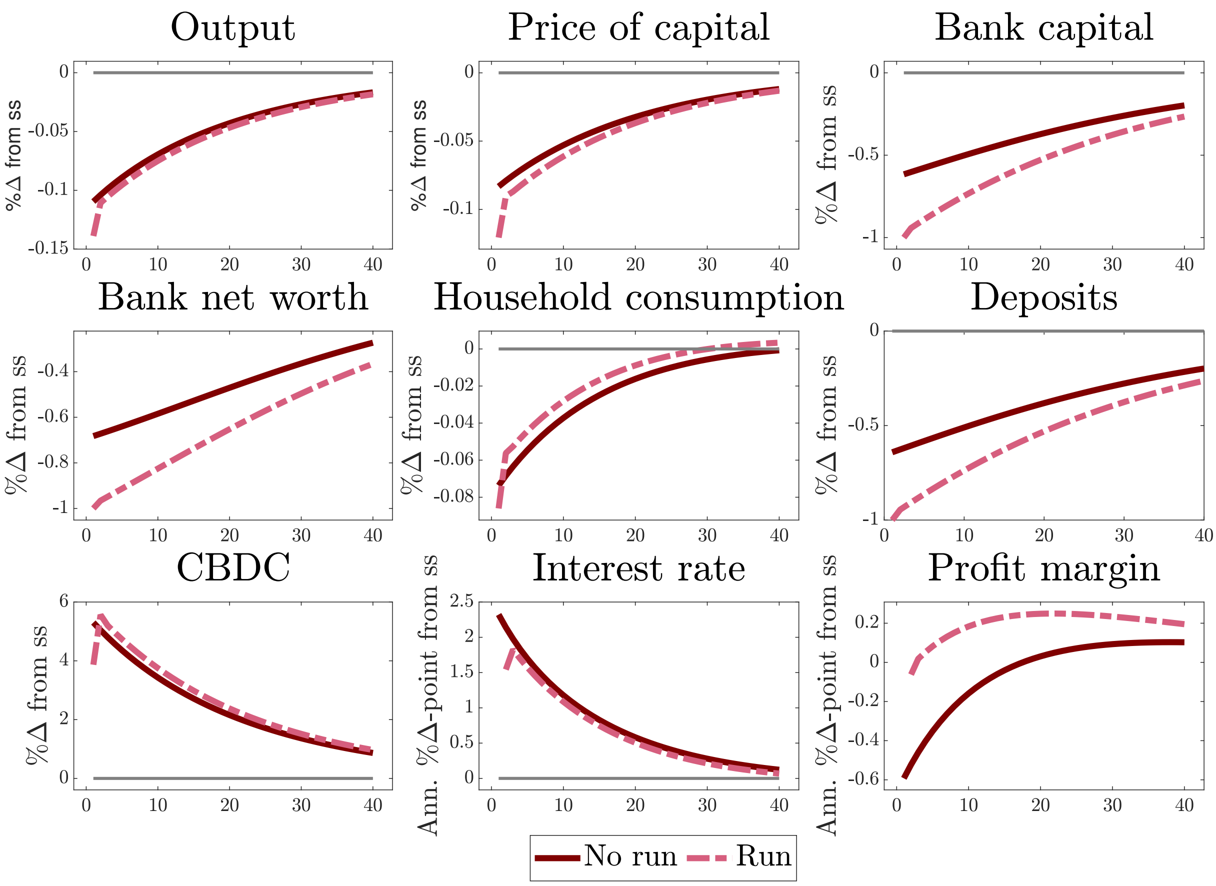

Figure 7: CBDC with asset policy scenario – 10% Shock to productivity with and without a bank run in the period of the shock.

Bank runs under CBDC with credit policy

The issuance of CBDC with ‘credit policy’ increases the threshold for self-fulfilling runs further.

In the baseline calibration, there is no region in which the banking and the bank run equilibrium coexist. This result rests on the assumption that the central bank does not participate in self-fulfilling bank runs but only withdraws funding once the bank is insolvent.[26]

There are two forces that jointly impede the occurrence of bank runs under CBDC with 'credit policy'. First, in the event of a bank run, only a fraction of the bank's liabilities are withdrawn, as both deposits and central bank credit are on the bank's balance sheet. Therefore, the fire-sale price of assets needs to cover only the share of liabilities that are deposits as the central bank does not engage in the bank run. In this way, central bank credit provides the bank with a stable funding source which creates an additional buffer on the bank's balance sheet. This additional central bank credit buffer results in a lower share of assets that would be withdrawn in a bank run scenario and makes the funding outflow easier to absorb. The larger the pre-run central bank credit holdings are, the higher the hurdle for the emergence of a self-fulfilling bank run. Second, instead of withdrawing funds, the central bank simultaneously provides the bank with additional liquidity in the amount of inflows into CBDC. The continued credit provision enables the bank to invest in capital even in case of a bank run. With the bank being more efficient in managing capital, this greatly stabilises the capital price. Acting as lender of last resort, the central bank provision of credit further impedes the emergence of bank runs. Those two forces break the 'doom-loop' of self-fulfilling runs in the baseline calibration.

How can banks under CBDC with 'credit policy' become insolvent without opening the possibility for self-fulfilling runs? As described above, the price threshold $\overline{Q}_t^*$ that triggers self-fulfilling banks is only conditional on the fraction of deposits $\overline{Q}_t^* = \frac{R_t^d D_{t-1}}{K_{t-1}^b} - Z_t$. In contrast, the price threshold $\overline{Q}_t$ that triggers insolvency is easier to be reached because the central bank also stops funding in the event of insolvency: $\overline{Q}_t = \frac{R_t^d D_{t-1}}{K_{t-1}^b} + \frac{R_t^l L_{t-1}}{K_{t-1}^b} - Z_t$. While Q_t and Q_t^* are the two equilibrium prices in the banking and bank run equilibrium, $\frac{R_t^d D_{t-1}}{K_{t-1}^b} - Z_t$ are the same in both scenarios. For this reason, under the baseline calibration, even on the brink of insolvency, the fire-sale value of assets would be sufficient to cover deposit claims and would prevent a self-fulfilling run.[27] At the same time, the non-fire-sale value of assets will not be sufficient to

[26]If the central bank were also to participate in self-fulfilling runs, this would largely remove the stabilising effect of CBDC issuance and enable bank runs also in the baseline calibration.

[27]Why are bank runs not possible under the baseline calibration, even on the verge of insolvency? After all, switching from deposit funding to credit funding comes at a higher interest rate, which should trigger insolvency? The answer is no, in this case the non-fire-sale asset price Q_t wound remain marginally above the insolvency threshold and therefore the central bank will continue to provide credit as the bank is not yet insolvent. Bearing in mind, the insolvency threshold is conditional on households not expecting a bank run. Therefore, if the beliefs of households' would change and they would run on the bank, this will only affect the fire-sale price but not the non-fire sale price. Because under no-run beliefs the bank would remain solvent, the central bank will continue

cover *all* liabilities of the bank (i.e., deposits *and* central bank credit). Therefore, under the baseline calibration, the bank can fail due to insolvency without opening up the possibility of a self-fulfilling run.

Overall, by interceding as lender of last resort with the funds from CBDC, the central bank altogether eliminates the possibility of a run in the context of the numerical example. However, this does not preclude self-fulfilling bank runs under credit policy in the general setting. Self-fulfilling bank runs under CBDC with credit policy are possible under alternative calibrations.

Which of the two channels preventing the bank run under the baseline calibration is stronger and what are the implications of reducing both forces, individually and jointly? The first force works via the share of credit on the bank's liabilities in the period before the bank run. Reducing δ_0 in the central bank's CBDC rule to $\delta_0 = 1.01015$ diminishes central bank credit in steady state to almost zero, with $L_{st.st.} = 0.0005$ (instead of $L_{st.st.} = 0.0941$ under the baseline calibration with $\delta_0 = 1.0194$). Muting the first channel is not sufficient to enable self-fulfilling bank runs before the insolvency threshold. The second channel of providing central bank credit via CBDC inflows is sufficiently strong to still prevent the bank run from the outset. The second channel can be relaxed by increasing the reaction to CBDC demand in the CBDC rule to δ_M to disincentivise CBDC uptake. Setting $\delta_M = 0.5$ (instead of $\delta_M = 0.1$ under the baseline calibration), self-fulfilling bank runs become possible from shock sizes above 6.3%. Relaxing both forces jointly, with $\delta_0 = 1.01015$ and $\delta_M = 0.5$, enables self-fulfilling bank runs already from shock sizes above 2.9%.

The role of friction for the emergence of self-fulfilling runs

Both types of CBDC issuance – with 'asset policy' and 'credit policy' – delay the emergence of the bank run equilibrium to larger shocks by reducing frictions during a bank run. This stabilises the fire-sale price of assets, mitigates welfare losses and makes banks runs more difficult to emerge. The contributions of the different model frictions to the emergence of bank runs are examined in more detail below.

The main friction that makes bank runs costly is the capital management cost of households α. Reducing the household capital management costs to almost zero with $\alpha = 0.001$ ($\alpha = 0.008$ in the baseline calibration) delays the emergence of self-fulfilling runs in the 'no CBDC' scenario to the brink of insolvency at shocks above 21.9% (vs. 2.5% under the baseline calibration).

to provide credit in the event of a bank run, thereby preventing the bank run even at the brink of insolvency. In short, a self-fulfilling run cannot trigger insolvency because they are distinct equilibrium concepts that are conditional on household beliefs.

Conversely, increasing capital management costs to $\alpha = 0.015$ in the scenario without CBDC enables a bank run equilibrium already in steady state in the absence of any shocks.

While additional frictions in the CBDC scenarios also play a role for the emergence of bank runs, they are not as central as household capital management costs α and the parameters of the CBDC reaction rule. In the scenario of CBDC issuance with 'asset policy', capital management costs of the central bank are introduced as additional friction. Reducing the central bank's capital management costs to $\alpha_{cb} = 0.001$ (instead of $\alpha_{cb} = 0.004$ in the baseline calibration) delays the emergence of self-fulfilling runs from shocks above 8.6% under the baseline calibration to shocks above 9.7%. Conversely, setting central bank capital management costs to the capital management costs of the households with $\alpha_{cb} = 0.008$ reduces the shock threshold for the emergence of bank runs to shocks above 7.7%. In the CBDC scenario with 'credit policy', the leverage constraint for central bank credit ω is introduced as an additional friction. Setting $\omega = 1$ removes the leverage constraint but increases the costs of central bank credit which leads to the possibility of self-fulfilling bank runs at shocks above 2.1%. Conversely, setting $\omega = 0$ leads to the same leverage constraint but also to the same funding costs as on deposits. Similarly to the baseline calibration, this does not open up the possibility of self-fulfilling runs before the insolvency threshold.

Moreover, bank runs are not anticipated. How would the results change if bank runs were anticipated, for example as in Gertler and Kiyotaki (2015) or Gertler et al. (2016a)? Accounting for the possibility of bank failures would lead to a risk premium on deposits and increase bank funding costs. As the introduction of CBDC in the model framework makes bank runs less likely by delaying their occurrence to larger shocks, the risk premium on deposits would decrease in the presence of CBDC. Therefore, introducing anticipation of bank runs into the analysis would not materially change the insights of this analysis and results would be strengthened further. In addition, allowing for anticipation of bank failures, CBDC would be more favourable for the profitability of banks as the introduction of CBDC would reduce the risk premium and thus the deposit funding costs and therefore further mitigate financial fragility.

4.4.2 Bank failures due to insolvency

Figure 8 displays the evolution of deposits to increasing shock sizes under the different policy scenarios. Banks become insolvent in response to a shock when the non-fire-sale value of assets becomes smaller than the liabilities. Thus, the conditions for a default due to illiquidity and default due to insolvency only differ in the price of capital for the 'no CBDC' and CBDC with ‘asset policy’ scenarios. For CBDC with ‘credit policy’, the conditions for bank failure due to illiquidity and insolvency also differ in the assets that are taken into account. While the central bank does not engage in self-fulfilling runs and the fire-sale value of assets only needs to cover deposits, the central bank credit claims are taken into account in the calculation of the insolvency conditions.

The size of shocks that thrust banks into insolvency under CBDC with ‘credit policy’ is similar but a bit later than the scenario without CBDC. While in both scenarios the overall balance sheet size is similar, the stabilisation of capital prices under CBDC with ‘credit policy’ leads to a deferral of insolvency to slightly larger shock. In contrast, the shock size that triggers insolvency under CBDC with ‘asset policy’ is substantially larger. The higher insolvency threshold can be explained by the smaller bank balance sheet under CBDC with ‘asset policy’ and also by the stabilisation of the capital price.

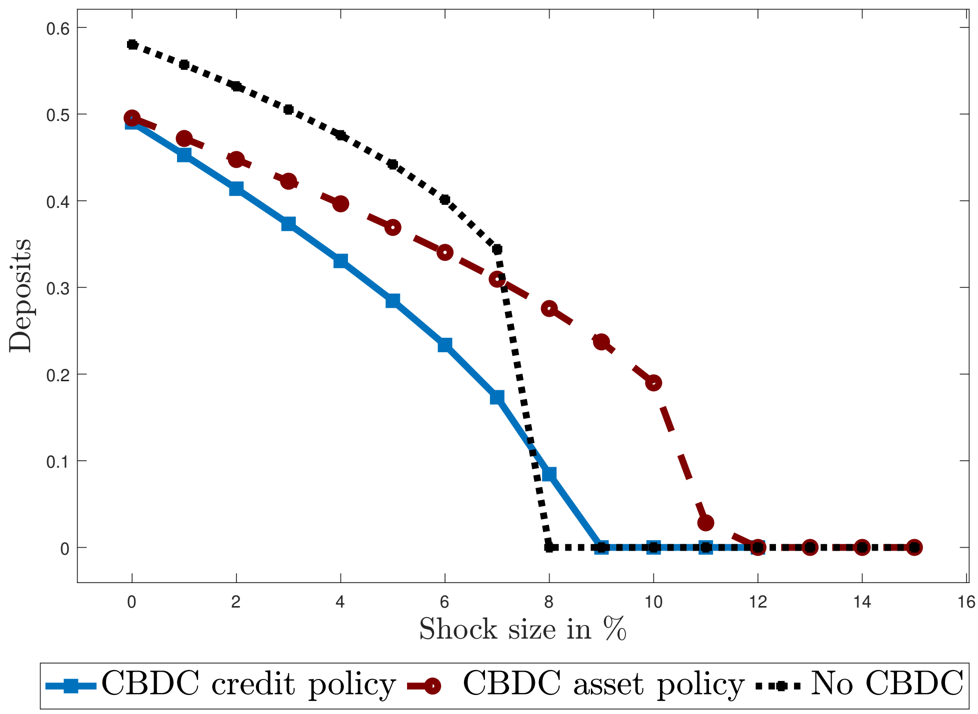

Figure 8: Evolution of deposits in the period of the shock with increasing capital shock sizes under the different policy scenarios and without emergence of self-fulfilling runs.

To summarise, under both policy scenarios CBDC affects the equilibrium regions in a way that supports financial stability. Under CBDC with ‘asset policy’, the emergence of both bank run types is postponed to substantially larger shock. Under CBDC with ‘credit policy’, the possibility of self-fulfilling runs does not arise in the baseline calibration, while insolvency is triggered at a similar shock size as in the 'no CBDC' scenario.

5 Conclusion

One of the main concerns about a CBDC is its disintermediating effect on the banking sector and especially the increased risk of a bank run in times of crisis. This paper analyses the impact of CBDC on financial stability as an extension of the dynamic bank run model with a financial accelerator by Gertler and Kiyotaki (2015).

CBDC issuance creates an additional type of liability on the central bank balance sheet which will lead to further balance sheet adjustments. In the model analysis, I account for the potentially different impact of these balance sheet adjustments by analysing CBDC issuance in the context of two different asset side policies: by granting loans to banks (CBDC with 'credit policy') and by purchasing capital (CBDC with 'asset policy').

The stylised model analysis offers several insights: in the steady state, a CBDC does not affect output and welfare. Instead, it affects the composition of household savings, bank funding and capital investment, ultimately reducing bank profits. In response to shocks that do not trigger a bank run, the issuance of CBDC does not exacerbate, but rather tends to mitigate, output and welfare losses by stabilising asset prices. At the same time, the presence of CBDC also leads to larger deposit outflows. Most importantly, the stabilisation of asset prices improves financial stability by deferring the emergence of bank failures due to illiquidity (caused by self-fulfilling runs) and bank failures due to insolvency to larger shocks. Overall, I find that a CBDC strains the banking sector in normal times by reducing deposits and net worth. Yet, contrary to prevailing concerns, CBDC improves financial stability in times of crisis by stabilising capital prices through asset-side adjustments that follow CBDC issuance.

This analysis is carried out in a stylised setting that abstracts from several features that could be considered in future research. For instance by: (i) bringing the analysis from a real to a nominal setting that allows for inflation dynamics and conventional monetary policy, (ii) introducing a richer financial sector and embedding a more complex structure of the central bank balance sheet that includes reserves, cash, government bonds and a collateral framework, (iii) incorporating the function of money as a medium of exchange into the analysis (iv) allowing for richer dynamics and frictions at firm level.

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Appendix

A Comparison to alternative model specifications

A.1 Varying CBDC rule parameters

The following section investigates how the response to a capital productivity shock changes, varying the two coefficients δ_Y and δ_M in the CBDC interest rate rule $R^m_{t+1} = \delta_0 + \delta_Y \left(\frac{Y - Y_t}{Y}\right) - \delta_M M_t$ under both 'credit policy' and 'asset policy'.

The coefficient δ_Y captures the reaction strength of the CBDC interest rate rule to fluctuations in output from its steady state value. Figures 9 and 10 display variations in δ_Y in the 'credit policy' and 'asset policy' scenario. Compared to the baseline calibration (solid lines, following the calibration of Minesso et al. 2022), δ_Y is doubled to 0.52 (higher output stabilisation, dashed lines) and halved to 0.13 (lower output stabilisation, dotted lines). Variations in δ_Y have the same qualitative implications for CBDC issuance with 'credit policy' and 'asset policy': a higher response to output fluctuations increases the stabilisation of output, consumption, the price of capital, and bank net worth. Yet, this comes at the cost of a higher deposit disintermediation and a greater increase in CBDC issuance. The effects are reversed in the case of a lower responsiveness to output fluctuations i.e. lower stabilisation but lower disintermediation.

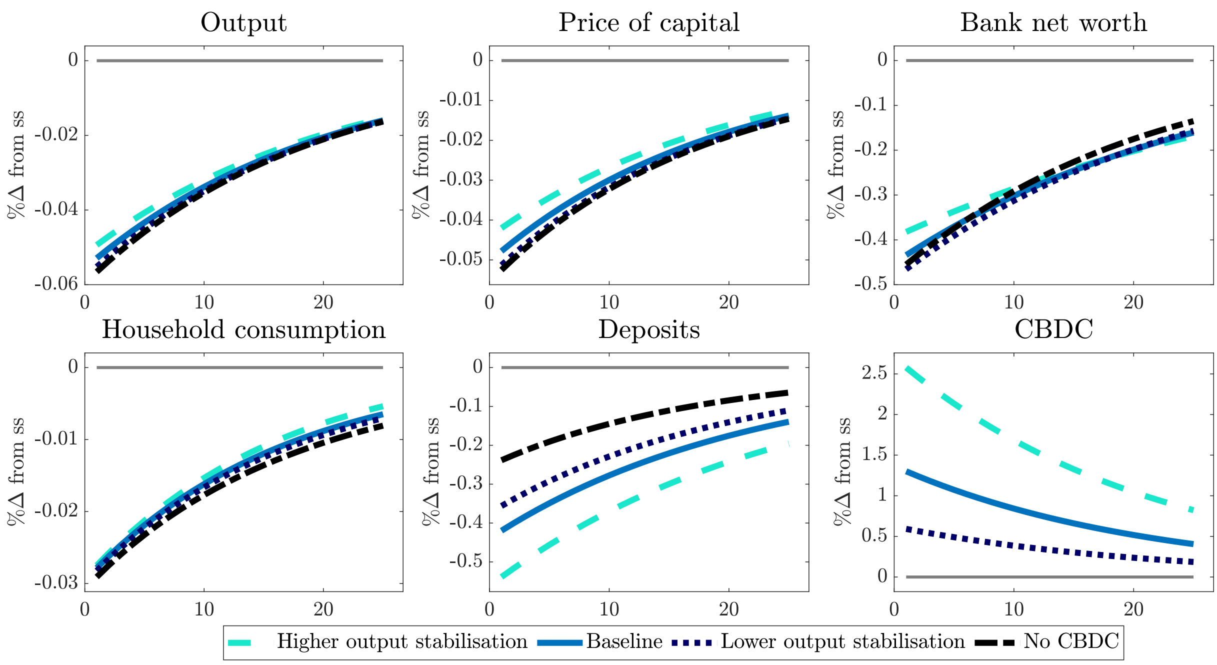

Figure 9: Response to a 5% shock to productivity under CBDC with 'credit policy' for high and low CBDC rule output coefficient δ_Y.

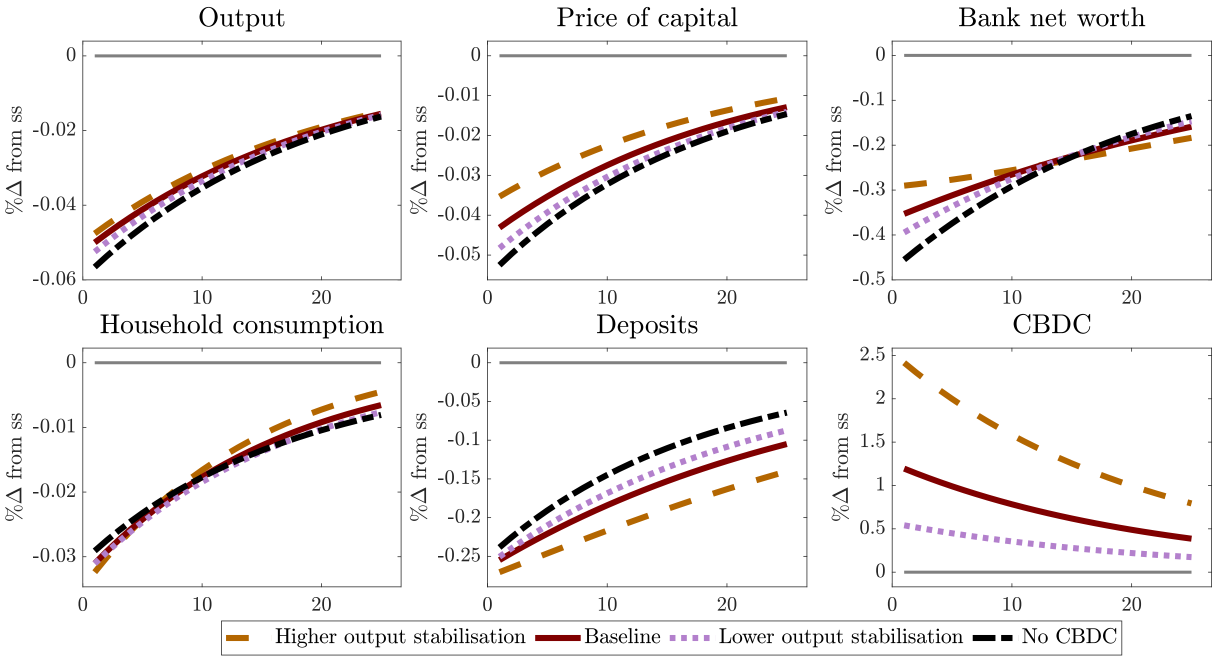

Figure 10: Response to a 5% shock to productivity under CBDC with 'asset policy' for high and low CBDC rule output coefficient δ_Y.

The coefficient δ_M captures the responsiveness of the CBDC interest rate rule to CBDC demand and thereby stabilising fluctuations in CBDC. Figures 11 and 12 show variations in δ_M in the 'credit policy' and 'asset policy' scenario. Compared to the baseline calibration (solid lines) of $\delta_M = 0.1$, δ_M is increased to 0.15 (higher CBDC demand response, dashed lines) and reduced to 0.075 (lower CBDC demand response, dotted lines). Variations in δ_M lead to similar observations as the variations in δ_Y and have again have the same qualitative implications for CBDC issuance with 'credit policy' and 'asset policy'. Making the CBDC interest rate rule less attractive in response to increased CBDC demand (i.e. a higher CBDC demand response) leads to a less stabilisation, but also to a smaller increase in CBDC supply (and vice versa). However, the difference in the variations is relatively small, except for the implications on the deposit response where the differences are most pronounced.

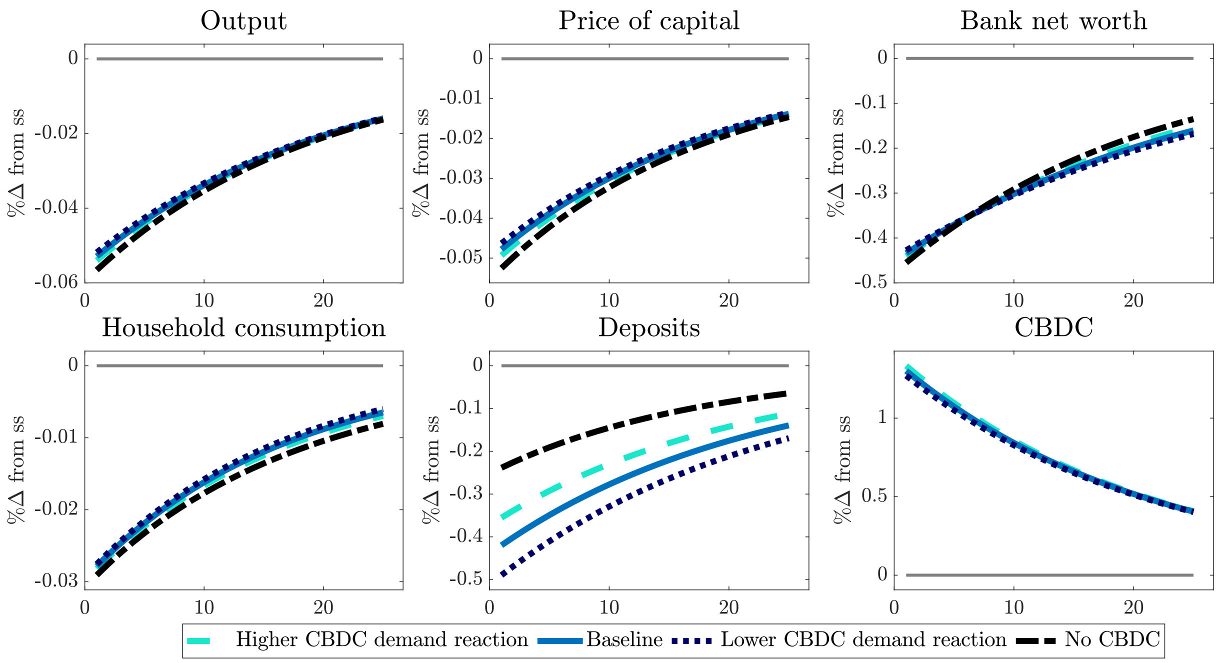

Figure 11: Response to a 5% shock to productivity under CBDC with 'credit policy' for high and low CBDC rule CBDC demand coefficient δ_M

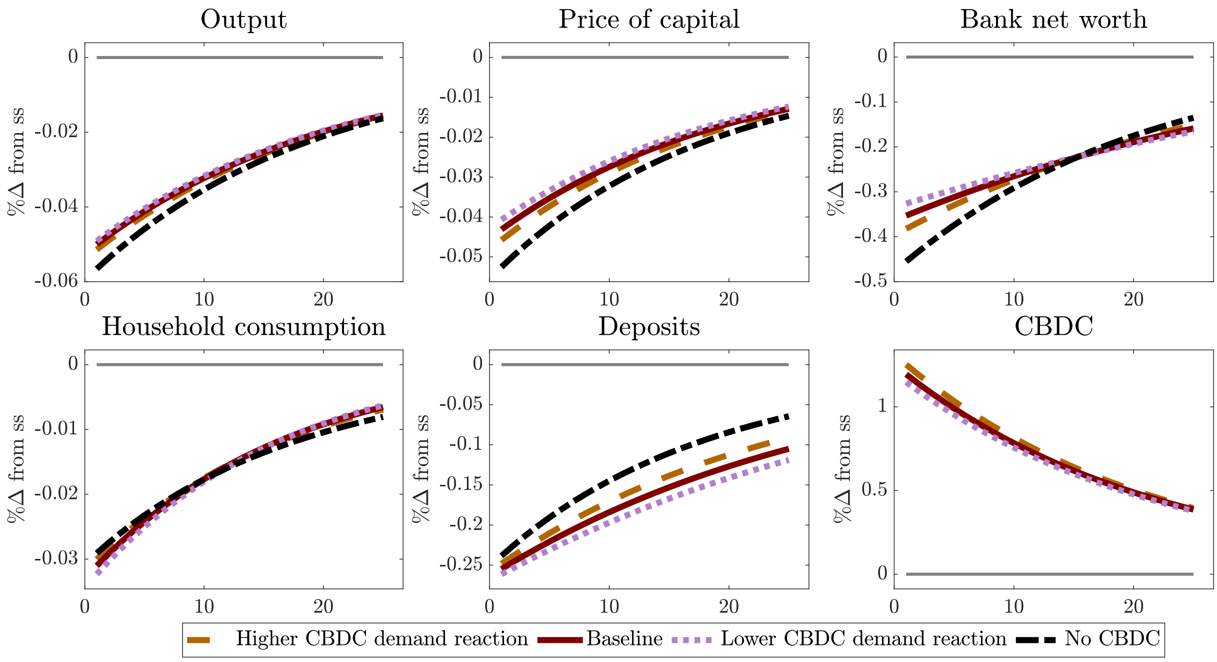

Figure 12: Response to a 5% shock to productivity under CBDC with 'asset policy' for high and low CBDC rule CBDC demand coefficient δ_M.

A.2 Varying CBDC credit and asset policy assumptions

The following section investigates how the response to a capital productivity shock changes, varying the two additional assumptions underlying CBDC issuance with 'credit policy' and 'asset policy'.

For CBDC issuance with 'credit policy', it is assumed that the leverage constraint of central bank credit to banks is less binding than for deposits, due to superior supervisory powers and collateral requirements. Figure 13 displays variations in the relative strength of the leverage constraint $V_t \geq \theta\left(Q_t k_t^b - \omega l_t\right)$ for central bank credit, as captured by ω which is set to 0.5 in the baseline calibration (blue solid lines). Assuming no leverage constraint on central bank credit (dashed turquoise lines) leads to lower deposit disintermediation and slightly higher stabilisation of output and consumption, while requiring the smallest increase in CBDC. Contrary, applying the same leverage constraint strength as deposits (blue dotted lines) leads to the same output and consumption response as in the case without CBDC, the highest deposit disintermediation and the highest increase in CBDC.

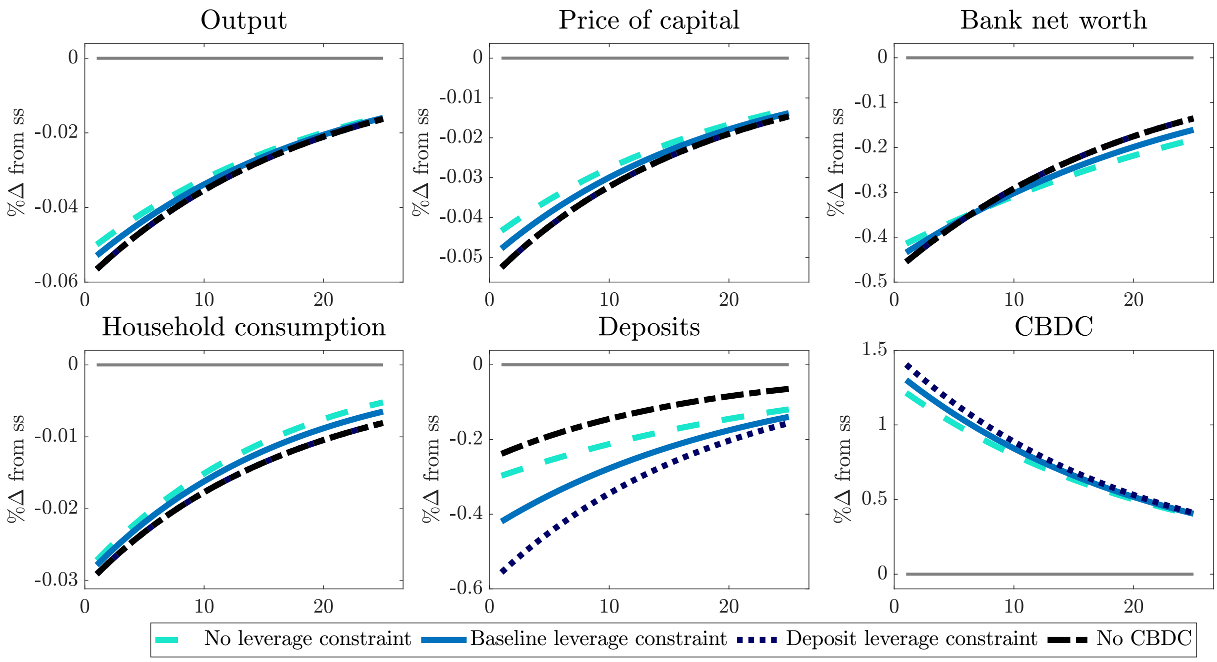

Figure 13: Response to a 5% shock to productivity under CBDC with credit policy and varying leverage constraint for central bank credit.

Similarly, for CBDC with 'asset policy', the central bank is assumed to face lower capital management costs than households, but which are of the same functional form $f\left(K_t^{cb}\right) =$

$\frac{\alpha_{cb}}{2}\left(K_t^{cb}\right)^2$. Figure 14 displays variations in the capital management costs of the central bank α_{cb} which is set to half of the capital management costs of households in the baseline calibration (red solid lines). Assuming no capital management costs for central bank capital purchases (dashed orange lines) and the same capital management costs as households (purple dotted lines) leads to very similar responses to the baseline case. Under the functional form of increasing marginal household management costs, varying α_{cb} has relatively little effect. Instead, more relevant for the results is that the capital management costs are born from two different agents in the economy rather than only one.

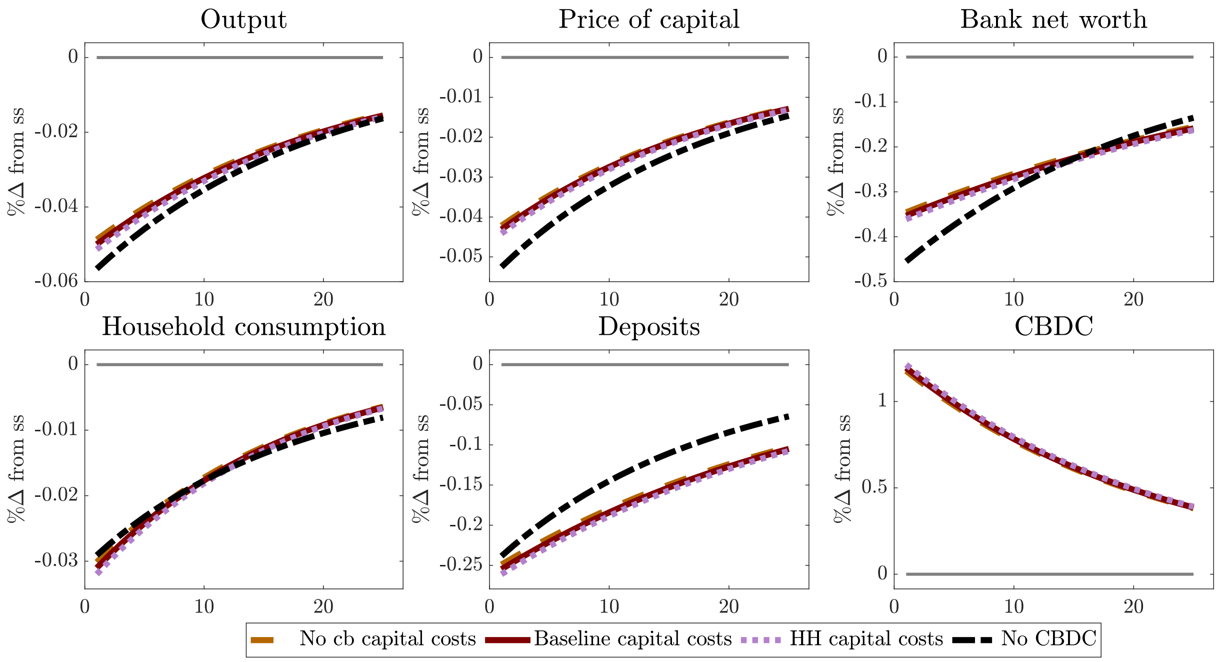

Figure 14: Response to a 5% shock to productivity under CBDC with 'asset policy' and varying central bank capital management costs.

B Reformulating and solution to the optimisation problem of the banker in terms of per unit of net worth

To make the maximisation problem independent of its initial conditions, equation 9 needs to be expressed in terms of per unit of net worth. For this, the evolution of net worth (5) is combined with the flow of funds constraint (4) and reformulated such that

$$
\begin{aligned}
n_{t+1} &= R^b_{t+1} Q_t k^b_t - R_{t+1} d_t - R^L_{t+1} l_t \quad \text{with} \quad R^b_{t+1} \equiv \frac{Z_{t+1}+Q_{t+1}}{Q_t} \\
\Rightarrow n_{t+1} &= R^b_{t+1} n_t + (R^b_{t+1} - R_{t+1}) d_t + (R^b_{t+1} - R^L_{t+1}) l_t \\
\Rightarrow \frac{n_{t+1}}{n_t} &= R^b_{t+1} + (R^b_{t+1} - R_{t+1}) \tilde{d}_t + (R^b_{t+1} - R^L_{t+1}) \tilde{l}_t \\
& \text{with} \quad \tilde{d}_t \equiv \frac{d_t}{n_t} \quad \text{and} \quad \tilde{l}_t \equiv \frac{l_t}{n_t}
\end{aligned}
$$

where R^b_{t+1} is the return of a unit of capital invested by banks, $\tilde{d}_t$ are deposits per unit of net worth, $\tilde{l}_t$ is central bank credit per unit of net worth and $\frac{n_{t+1}}{n_t}$ is the growth rate of net worth.

Likewise, we can then express the value function as:

$$
\begin{aligned}
V_t &= E_t \left[\beta(1-\sigma) n_{t+1} + \beta \sigma V_{t+1} \right] = E_t \left[\Omega_{t+1} n_{t+1} \right] \quad \text{with} \quad \Omega_{t+1} \equiv \beta \left(1 - \sigma + \sigma \frac{V_{t+1}}{n_{t+1}} \right) \\
\Rightarrow \Psi_t \equiv \frac{V_t}{n_t} &= E_t \left[\Omega_{t+1} \frac{n_{t+1}}{n_t} \right] \\
&= E_t \left[\Omega_{t+1} \left(R^b_{t+1} + (R^b_{t+1} - R_{t+1}) \tilde{d}_t + (R^b_{t+1} - R^l_{t+1}) \tilde{l}_t \right) \right] \\
&= \nu_t + \mu^d_t \tilde{d}_t + \mu^l_t \tilde{l}_t \\
\text{with:} \quad & \nu_t \equiv \Omega_{t+1} R^b_{t+1}, \quad \mu^d_t \equiv \Omega_{t+1} \left(R^b_{t+1} - R_{t+1} \right), \quad \mu^l_t \equiv \Omega_{t+1} \left(R^b_{t+1} - R^l_{t+1} \right)
\end{aligned}
\tag{23}
$$

where Ω_{t+1} is the weighted average of the discounted marginal value of net worth to exiting and to remaining bankers. Ψ is the franchise value of the bank per unit of asset and can thus be interpreted as Tobin's q ratio. Similarly, the flow of funds constraint (4) can be inserted into the incentive constraint (8) and expressed in terms of per unit of net worth:

$$
\begin{aligned}
\frac{V_t}{n_t} &\geq \theta \left(d_t + n_t + l_t - \omega l_t \right) / n_t \\
\Rightarrow \Psi_t &\geq \theta \left(1 + \tilde{d}_t + (1-\omega) \tilde{l}_t \right)
\end{aligned}
\tag{24}
$$

Finally, the reformulated optimisation problem is choosing leverage (i.e. deposits and credit per unit of net worth ($\tilde{d}_t$), $\tilde{l}_t$) each period to maximise the franchise value per unit of net worth which can also be regarded as Tobin's q ratio:

$$
\Psi_t = \nu_t + \mu^d_t \tilde{d}_t + \mu^l_t \tilde{l}_t \quad \text{s.t.} \quad \Psi_t \geq \theta \left(1 + \tilde{d}_t + (1-\omega) \tilde{l}_t \right) \tag{25}
$$

This leads to the Lagrangian function

$$L_t = \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t + \lambda_t \left[\nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t - \theta \left(1 + \tilde{d}_t + (1-\omega)\tilde{l}_t \right) \right] \tag{26}$$

with the Kuhn Tucker conditions

$$\frac{\partial L}{\partial \tilde{d}_t} = \mu_t^d + \lambda_t \mu_t^d - \lambda_t \theta \leq 0 \quad \tilde{d}_t \geq 0 \quad \text{and} \quad \tilde{d}_t \frac{\partial L}{\partial \tilde{d}_t} = 0 \tag{27}$$

$$\frac{\partial L}{\partial \tilde{l}_t} = \mu_t^l + \lambda_t \mu_t^l - \lambda_t \theta (1-\omega) \leq 0 \quad \tilde{l}_t \geq 0 \quad \text{and} \quad \tilde{l}_t \frac{\partial L}{\partial \tilde{l}_t} = 0 \tag{28}$$

$$\frac{\partial L}{\partial \lambda_t} = \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t - \theta \left(1 + \tilde{d}_t + (1-\omega)\tilde{l}_t \right) \geq 0 \quad \lambda_t \geq 0 \quad \text{and} \quad \lambda_t \frac{\partial L}{\partial \lambda_t} = 0 \tag{29}$$

To show that the incentive constraint must be binding, assume conversely that the constraint is slack and thus $\lambda_t = 0$: this would imply for equations 27 $\mu_t^d \leq 0$ and equation 28 $\mu_t^l \leq 0$. These are the expected discounted excess returns on capital financed by deposits (μ_t^d) and central bank credit (μ_t^l). However, to be economically meaningful, the excess returns must be greater than zero. Therefore, in any relevant economic scenario $\mu_t^d > 0$ and $\mu_t^l > 0$ and thus $\lambda_t \neq 0$ and the incentive constraint is binding with $\lambda_t > 0$. This result can be interpreted intuitively by looking at the objective function which is strictly increasing in $\tilde{d}_t$ and $\tilde{l}_t$. Thus, it would be optimal for the bank to set these variables as high as possible. However, the incentive constraint limits the size of the leverage, being linear in $\tilde{d}_t$ and $\tilde{l}_t$. Therefore, in the optimum, the bank sets its leverage as high as possible such that the incentive constraint is always binding. The Kuhn-Tucker conditions collapse to

$$\frac{\partial L}{\partial \tilde{d}_t} = \mu_t^d + \lambda_t \mu_t^d - \lambda_t \theta = 0 \tag{30}$$

$$\frac{\partial L}{\partial \tilde{l}_t} = \mu_t^l + \lambda_t \mu_t^l - \lambda_t \theta (1-\omega) = 0 \tag{31}$$

$$\frac{\partial L}{\partial \lambda_t} = \nu_t + \mu_t^d \tilde{d}_t + \mu_t^l \tilde{l}_t - \theta \left(1 + \tilde{d}_t + (1-\omega)\tilde{l}_t \right) = 0 \tag{32}$$

yielding the following optimality conditions:

$$\frac{\theta}{\mu_t^d} = \frac{\theta(1-\omega)}{\mu_t^l} \Leftrightarrow (1-\omega)\mu_t^d = \mu_t^l$$

$$\Rightarrow (1-\omega)E_t\left[\left(R_{t+1}^b - R_{t+1}\right)\right] = E_t\left[\left(R_{t+1}^b - R_{t+1}^L\right)\right] \tag{33}$$

C Overview of equations in the different models

Description	**Equation**	**CBDC credit policy**	**CBDC asset policy**	No CBDC
Households				
Deposit Euler	$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^d = 1$	✓	✓	✓
CBDC Euler	$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^m = 1$	✓	✓	
Capital Euler	$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^h = \beta \frac{C_t^h}{C_{t+1}^h} \left(\frac{Z_{t+1}+Q_{t+1}}{Q_t+\alpha K_t^h} \right) = 1$	✓	✓	✓
Banking Sector				
Balance Sheet	$Q_t K_t^b = D_t + N_t + L_t$	✓		
	$Q_t K_t^b = D_t + N_t$		✓	✓
Net Worth	$N_t = \sigma \left[(Z_t + Q_t) K_{t-1}^b - R_t^d D_{t-1} - R_t^l L_{t-1} \right] + W^b$	✓		
	$N_t = \sigma \left[(Z_t + Q_t) K_{t-1}^b - R_t^d D_{t-1} \right] + W^b$		✓	✓
Max Problem	$\Psi_t = E_t \{ \beta (1 - \sigma + \sigma \Psi_{t+1}) \left[R_{t+1}^b + (R_{t+1}^b - R_{t+1}^d) \frac{D_t}{N_t} + (R_{t+1}^b - R_{t+1}^l) \frac{L_t}{N_t} \right] \}$	✓		
	$\Psi_t = E_t \left\{ \beta (1 - \sigma + \sigma \Psi_{t+1}) \left[R_{t+1}^b + (R_{t+1}^b - R_{t+1}^d) \frac{D_t}{N_t} \right] \right\}$		✓	✓
	with $R_{t+1}^b = \frac{Z_{t+1}+Q_{t+1}}{Q_t}$			
Leverage Constraint	$\Psi_t \geq \theta \left(1 + \frac{D_t}{N_t} + (1-\omega) \frac{L_t}{N_t} \right)$	✓		
	$\Psi_t \geq \theta \left(1 + \frac{D_t}{N_t} \right)$		✓	✓
Leverage Combination	$\left(R_{t+1}^b - R_{t+1}^l \right) = (1-\omega) \left(R_{t+1}^b - R_{t+1}^d \right)$	✓		
Central Bank				
CB Balance	$M_t + R_t^l L_{t-1} = R_t^m M_{t-1} + L_t + T_t$	✓		
	$M_t + (Z_t + Q_t) K_{t-1}^{cb} = R_t^m M_{t-1} + Q_t K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 + T_t$		✓	
CBDC interest rate	$R_{t+1}^m = \delta_0 + \delta_Y \left(\frac{Y - Y_t}{Y} \right) - \delta_M M_t$	✓	✓	
CB Lending	$L_t = M_t$	✓		
CB Capital	$Q_t K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = M_t$		✓	
Aggregation and Market Clearing				
Total Capital	$K_t^b + K_t^h = 1$	✓		✓
	$K_t^b + K_t^h + K_t^{cb} = 1$		✓	
Total Output	$C_t^h + \frac{\alpha}{2} \left(K_t^h \right)^2 + \frac{1-\sigma}{\sigma} \left(N_t - W^b \right) = Z_t + z_t W^h + W^b$	✓		✓
	$C_t^h + \frac{\alpha}{2} \left(K_t^h \right)^2 + \frac{1-\sigma}{\sigma} \left(N_t - W^b \right) + \frac{\alpha_{cb}}{2} \left(K_t^{cb} \right)^2 = Z_t + z_t W^h + W^b$		✓	

Table 6: Overview of equations in the different policy scenarios.

Description	**Equation**	**CBDC credit policy**	**CBDC asset policy**	No **CBDC**
Households				
CBDC Euler	$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^m = 1$	✓	✓	
Capital Euler	$\beta \frac{C_t^h}{C_{t+1}^h} R_{t+1}^h = \beta \frac{C_t^h}{C_{t+1}^h} \left(\frac{Z_{t+1}+Q_{t+1}^*}{Q_t^*+\alpha K_t^h} \right) = 1$	✓	✓	✓
Banking Sector				
Balance Sheet	$Q_t^* K_t^b = L_t$	✓		
Central Bank				
CB Balance	$M_t + R_t^l L_{t-1} = R_t^m M_{t-1} + L_t + T_t$	✓		
	$M_t + (Z_t + Q_t^*) K_{t-1}^{cb} = R_t^m M_{t-1} + Q_t^* K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 + T_t$		✓	
CBDC interest rate	$R_{t+1}^m = \delta_0 + \delta_Y \left(\frac{Y - Y_t}{Y} \right) - \delta_M M_t$	✓	✓	
CB Lending	$L_t = M_t$	✓		
CB Capital	$Q_t^* K_t^{cb} + \frac{\alpha_{cb}}{2} (K_t^{cb})^2 = M_t$		✓	
Aggregation and Market Clearing				
Total Capital	$K_t^b + K_t^h = 1$	✓		
	$K_t^h + K_t^{cb} = 1$		✓	
	$K_t^h = 1$			✓
Total Output	$C_t^h + \frac{\alpha}{2} \left(K_t^h \right)^2 = Z_t + z_t W^h$	✓		✓
	$C_t^h + \frac{\alpha}{2} \left(K_t^h \right)^2 + \frac{\alpha_{cb}}{2} \left(K_t^{cb} \right)^2 = Z_t + z_t W^h$		✓	

Table 7: Overview of bank run equilibrium equations in the different model variants