Bachelor Thesis

Empirical comparison of Expected Utility Theory and Prospect Theory based on a self-designed survey

Department of Statistics
Ludwig-Maximilians-Universität München

Author: Franziska Laura Reichmeier
Supervisior: Dr. Christoph Jansen
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Abstract

This thesis provides an empirical comparison between the expected utility theory and the prospect theory with the aim of verifying the individual fit of each participant and the possibility of providing a model that fits as well as possible to all participants. The parameters of each model are estimated numerically using a minimum squared difference between observation(s) and an estimated model with the goal of making as few assumptions as possible in advance about the parameters that are to be estimated. Within the framework of this work, a survey with 16 binary decision situations separated into gain and loss lotteries was conducted. It was quite a simple decision context and the external circumstances are not controlled. At the end of the survey, characteristics like age gender and employments of the participants were queried.
Part of this work is investigating whether there occur violations of the expected utility and whether these violations can be explained by the prospect theory which is based on several known effects in decision-making. Additionally it was considered how the different estimated parameters of both models behave for different characteristics of participants containing the survey.

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1 Introduction

Around 20 000 decisions are made by a person every day. (Auffenberg, 2020) How many decisions have you made since waking up today? Do you remember the decision to brush your teeth or drink your coffee? Probably not. The major part of our everyday decisions is made unconsciously as part of our routines in order to relieve our brains. (Kraaz, 2021, chapter 6.2) But there are some decisions which are made consciously because they take a larger part in our lives. To give a few examples: choosing a profession, making plans for the weekend, or maybe playing a lottery at the end of the year. Due to the fact that decisions take a large part as well as time in our lives and belong to our nature as living beings, a lot of scientific fields such as psychology, philosophy, economics but also statistics are interested in them.
Games and lotteries have always played an important role in statistics. Bernoulli had his focus on exploring the mathematics behind the game. However, he also made the discovery as well as M. Allais that advising someone the lottery with the highest expectation value does not necessarily correspond with reality. (Bernoulli, 1896; Allais, 1953) Thereon John von Neumann and Oskar Morgenstern evaluated a framework of axioms for a function which is maximized by a decision maker and is meanwhile part of the expected utility theory. (von Neumann and Morgenstern, 1947) However, Daniel Kahnemann and Amos Tversky observed on their part that decision makers have preferences in multiple interdependent decision situations which may constitute a violation of the expected utility theory. This formed the motivation for the prospect theory. (Tversky and Kahnemann, 1979; Tversky and Kahnemann, 1992)
There arise two question. The first one is whether the violations of expected utility theory are really that strong outside of an experimental framework and as second whether the prospect theory is not perhaps too flexible for practice and thus an overfitting takes place. So the idea of this thesis is to compare the expected utility theory and prospect theory in terms of their applicability in practice and their generalization.
Before starting with defining both theories, there are some introductory concepts which have to be considered. Decision theory is separated into descriptive, normative and prescriptive approaches. However, in this thesis only the normative decision theory is considered. In this segment of decision theory, decision makers are advised in such a way that their well-being is maximized. (Bacci and Chiandotto, 2020, chapter 1)
Not only the goal which is wanted to be achieved by means of decision theory plays a central role, but also the external circumstances. How many decision makers are deciding in this particular situation among the different options and how much prior information about the decision situation is known. In this bachelor thesis, only decisions under risk made by a single decision maker are considered. As single decision maker counts every single human being or organization that has a common direction of interest and no conflicts among the individuals who belong to it. Whether a decision situation is classified as certain, risky or uncertain depends on the degree of prior information. When the outcome of any action is known before it is made, then the decision situation is classified as certain. When the result can not be determined in advance, but each action leads to a set of possible outcomes with associated prior probabilities, then the decision is made under risk. However, when the associated probabilities are not meaningful or even unknown, then it is a decision under uncertainty. (Luce and Raiffa, 1989, chapter 2)
As structure of this thesis it was chosen to firstly define and to explain the theoret-

ical background of the compared theories before evaluating the theories on the basis of the survey. Thus, in the second chapter before explaining both theories, the required framework of a decision situation is defined and the simplest way to make a decision according to the highest monetary outcome and its limitations in the usage are considered. This leads to the expected utility theory, followed by its boundaries and observed violations in practice. So that the second theory of this thesis - the (cumulative) prospect theory- is motivated and thereupon defined.

In the third chapter, the structure of the survey is explained. As next, a first descriptive analysis of the participants is performed and it is discussed whether and to what extent the in advance implied violations of the expected utility theory can be observed. In the fourth chapter the data is evaluated according to the expected utility theory and the prospect theory. Finally, both theories are compared with regard to their fit on the preference structure of every individual who participated the survey and both their possibilities of generalization. In the last chapter, there is a brief conclusion about the results and view of their limits in their practical application.

2 Theory

In the following chapter, a decision problem is first defined, as well as some framework which is required. Before defining the expected utility theory, the simplest tool for making a decision is regarded and especially its violations. Then, the axioms of the expected utility theory are defined as well as some problems which occur in reality. Lastly the prospect theory and its boundaries are presented which leads finally to the cumulative prospect theory.

2.1 Structure of decision problems

Goal of normative decision theory is to advise someone in making a decision, so that one can make the best possible decision in a certain situation based on a rational criterion.
Consider this example: Waking up in the morning, having a look out of the window and having to decide whether to take the umbrella or not, when leaving the house. For simplicity, there are only two possible states of weather existing on this day: rain or sunshine. So when it is raining or will start to rain on this day and the decision maker takes the umbrella with him/her, s/he will have to carry extra weight. In addition s/he has to keep the umbrella in mind, so that s/he is not leaving it in the train or somewhere else. However, when it starts raining, the big advantage is that only his/her feet will get wet. Differently, when s/he is leaving the house without the umbrella and it is raining, then s/he will get pouring wet. On the other hand, when the sun is shining throughout the whole day, and s/he is carrying the umbrella, s/he will be not only inconvenienced, but also a bit indignant for carrying the umbrella throughout the whole day. When the sun is shining and the umbrella is left at home, s/he does not have to carry extra weight and can enjoy unalloying the sun. This cleavage can be summarized in the following table 1. Thereby, the rows of table 1 are the possible actions which the decision maker can take. The columns are the external circumstances which are in the example above rain and sunshine. Consequently, the entries of the table 1 are the consequences of each action and the according state of nature. (Savage, 1951)

Action	**State of natures**	
	Rain	Sunshine
Carry umbrella	Inconvenience and wet feet	Inconvenience and minor embarrassment
Carry no umbrella	Miserable drenching	Bliss unalloyed

Table 1: Decision matrix of umbrella example

Generally spoken, for structuring a decision problem, these following three elements are needed for notation. The first one is a finite set of alternatives or actions $\mathcal{A}' = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$ among which the decision maker selects the optimal one, based on a rational criterion. So in the table 1 of the umbrella example, whether to take an umbrella or not. Furthermore, a set of states of nature $\Theta' = \{\theta_1, \ldots, \theta_j, \ldots, \theta_k\}$ is needed which represents the operating context. In the umbrella example there are the two possible states of weather: rain and sunshine. In the following, each state of nature is linked to a known probability $\pi(\theta_j)$, whereas $\sum_{j=1}^{k} \pi(\theta_j) = 1$. This corresponds to a plausibility, with which each state of nature could occur. One must add that the state of nature does not have to be related to nature per se. For simplicity in the following it will be written $\pi(\theta_j) = p_j$. In conclusion a finite set of consequences, also called results or outcomes $\mathcal{X}' = \{x_{11}, x_{12}, \ldots, x_{ij}, \ldots, x_{mk}\}$ is demanded. In this paper the consequences are usually monetary outcomes. The action a_i and true state of nature θ_j influence this set of consequences which subsequently results in:

$$x_{ij} = f(a_i, \theta_j)\ i = 1, \ldots, m;\ j = 1, \ldots, k$$

These three elements produce a so-called decision matrix (see table 2). Where the rows are the actions, the columns are the prior probabilities and the entries are the consequences. (Bacci and Chiandotto, 2020, chapter 1.3)

	Prior Probabilities					
Actions	$\pi(\theta_1)$	$\pi(\theta_2)$	$\ldots$	$\pi(\theta_j)$	$\ldots$	$\pi(\theta_k)$
a_1	x_{11}	x_{12}	$\ldots$	x_{1j}	$\ldots$	x_{1k}
a_2	x_{21}	x_{22}	$\ldots$	x_{2j}	$\ldots$	x_{2k}
$\vdots$	$\ldots$	$\ldots$	$\vdots$	$\ldots$	$\vdots$	$\ldots$
a_m	x_{m1}	x_{m2}	$\ldots$	x_{mj}	$\ldots$	x_{mk}

Table 2: Decision matrix

So with the help of a decision matrix any decision maker can analyze each action and its consequences in different states of nature. But s/he has not yet decided which action s/he takes. Regarding the decision matrix 1 of the umbrella example, at this state, no action is recommended yet.
One way to advise someone is by ordering the consequences and allocating numbers indicating how much a consequence is valued. So, the consequence is linked to a value which is called *utility*. However, there occur two problems when defining this value. The first one is, how to quantify the utility, when there exists no obvious scale of measurement. For example, how to estimate the prestige, customer goodwill and reputation of a company. The second one is: When there exists an evident scale, the true utility often does not match with the scale. As an example, it is assumed doing an unpleasant job for \$100. For many this payment of \$100 would top their income level, so they would take the task. But if they gained in advance \$1 000 000, they would probably refuse to do the assignment. Because this is a very challenging task, the aim of utility theory is to evolve and to deal with these values. (Berger, 1980, chapter 2.1)

Generally spoken, the aim of decision theory is to order the possible actions after a certain preference. Consider these two actions a_i and a_m. Hence, for ordering the actions after a certain preference structure, there is used the following notation (Luce and Raiffa, 1989, chapter 13):

i) $a_i \sim a_m$: implying the acts are equivalent by having the same utility in each state of nature

ii) $a_i \succ a_m$: implying that a_i is preferred to a_m, by strongly dominating in each state of nature

iii) $a_i \gtrsim a_m$: implying that a_i is preferred to a_m for at least one state and is preferred or indifferent to a_m for all other states by weakly dominating

A last possibility which can happen is for example: one action is not preferred or indifferent in every state of nature, but only in some states of nature. So the preference structure is determined by the event (a set of states of nature). (Savage, 1951, chapter 2.7)

In the following, the focus is on lotteries instead of actions. Therefore, the decision maker takes an action by deciding between two or multiple gambles/lotteries. In doing so, the different possible monetary outcomes x_{ij} of each lottery l_i are linked to a probability p_j. So the notation for a lottery looks like $l_i = (x_{i1}, p_1; x_{i2}, p_2; \ldots, x_{ik}, p_k)$. When there are only three possible outcomes of the lottery, the notation reduces to $l_i = (x_{i1}, p_1; x_{i2}, p_2; x_{i3})$. Thereby, it follows that $p_3 = 1 - p_1 - p_2$ because of $\sum_{j=1}^{k} p_j = 1$. This is analogous for a lottery with two possible outcomes $l_i = (x_{i1}, p_1; x_{i2})$ $(p_2 = 1 - p_1)$. If the lottery has a possible monetary outcome of zero $(l_i = (x, p_1; 0, p_2))$, the lottery simplifies to $l_i = (x, p_1)$. And when the lottery contains only one certain outcome $(p_1 = 1)$, then there is written $l_i = (x)$.[1]

The last point in this chapter is about the behavior of an individual regarding risk. There are three possible patterns. The first one is behaving *risk averse*. In this case, the expected value of the lottery is favored over the lottery itself which results as an example in $(50) \gtrsim (100, .5)$. The opposite is called *risk seeking*, this implies that the lottery is chosen over its expected value. Thus our example looks like $(50) \lesssim (100, .5)$. When the decision maker is indecisive between the lottery and its expected value $((50) \sim (100, .5))$, then s/he is behaving *risk neutral*. (Wakker, 2010, chapter 2.4)

2.2 Expected utility theory

2.2.1 Mathematical expectation criterion

One of the easiest approaches for a decision criterion one could think of is to choose the action with the highest expectation value. In literature this criterion is also known as Bayes-criterion or μ- rule. In the following, the only considered outcome is a monetary result. So it can be imagined that every action is a possible cash gamble, so that it can be decided which one a participant likes to play. In this paper the obvious scale of the result is a monetary unit (like euro or dollar). Towards Doersam, the preference value, according to which the decision is made, is given as follows (Dörsam, 2013):

$$\Phi(l_i) = \sum_{j=1}^{k} p_j * x_{ij} \tag{1}$$

Whereas the possible monetary outcomes which could occur, when choosing the lottery l_i, are $x_{i1}, x_{i2}, \ldots, x_{ik}$ with the known probabilities $p_1, p_2, \ldots, p_m$, $0 \leq p_j \leq 1$, $\sum_{j=1}^{k} p_j = 1$. So according to this, the preference structure of the lotteries have to look like

$$l_i \succ l_j \Leftrightarrow \Phi(l_i) > \Phi(l_j)$$

[1] The notation is inspired by Tversky and Kahnemann (1979) and Bacci and Chiandotto (2020)

Considering an anticipation fee for taking part at the game, then the gamble would be considered fair, when the expected value minus the anticipation fee a is zero and favorable, when this expected prize is positive ($\Phi(l_i - a) = 0$). (Jensen, 1967, chapter 3) So which of the two lottery wheels in figure 1 would be chosen this rational criterion? The notation for the example lottery wheels in figure 1 look like:

- on the left side: $l_A = (3200, .2; 3100, .25; 3000)$
- on the right side: $l_B = (7000, .3; 3000, .55)$.

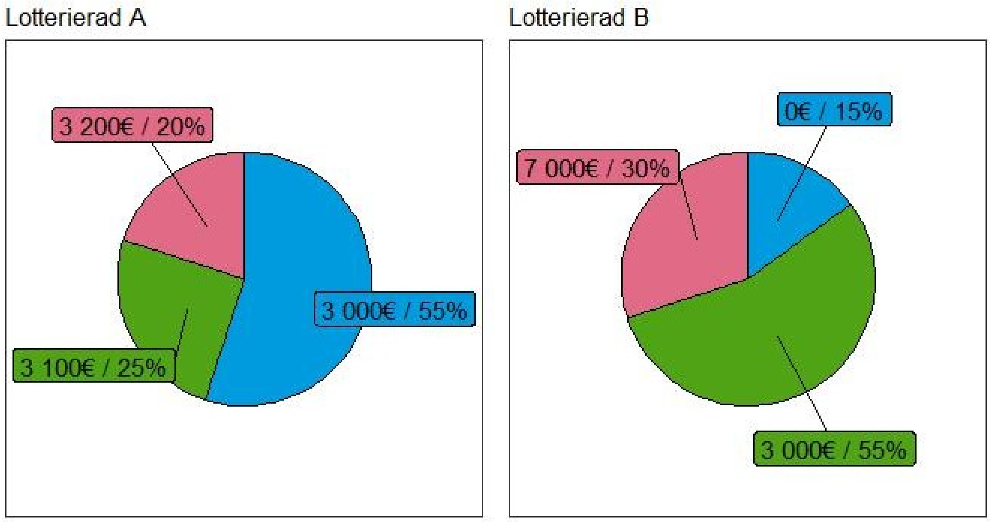

Figure 1: Decision situation with two lottery wheels. The wheel on the left side equates the lottery $l_A = (3200, .2; 3100, .25; 3000)$. On the right side the lottery is $l_B = (3000, .55; 7000, .3)$. (own figure)

Regarding the mathematical expectation criterion, lottery wheel l_B (on the right side) should be preferred to lottery wheel l_A, because:

$$\begin{aligned}\Phi(l_A) &= 0.2 * 3200 + 0.25 * 3100 + (1 - 0.2 - 0.25) * 3000 = 3065 < \\ \Phi(l_B) &= 0.3 * 7000 + 0.55 * 3000 + (1 - 0.3 - 0.55) * 0 = 3600 \\ &\Leftrightarrow l_A \prec l_B\,.\end{aligned}$$

But two problems might occur when deciding after this criterion. The first one is which lottery to advise when the expected value of all lotteries is equal? The other problem of this criterion is that people usually do not decide according to the advised lottery. So in reality most of the people would decide for the lottery wheel A in the considered example (see figure 1). This is the so-called certainty effect which will be discussed further in chapter 2.2.3. Another example, where this discrepancy between criterion and reality occurs, was introduced by Nicholas Bernoulli with the *St.Petersburg Parardox* in 1713. In this thought experiment a fair coin is thrown until head side is up. When this happens after the first toss, then the gambler is paid one coin, after the second toss two coins, after the third one four coins, after the forth one eight coins, and so on. Thus, the possible gain of this gamble results in 2^{n-1}, whereas n is the number of tosses until head comes up. So the expected value of this thought experiment is:

$$\Phi(l) = \sum_{k=1}^{\infty} \frac{1}{2^k} 2^{k-1} = \sum_{k=1}^{\infty} \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \cdots \to \infty\,.$$

As it can be seen, the expected value sums up to infinity, so that the mathematical expectation criterion recommends playing the gamble regardless of external circumstances. This could mean that the participation fee is equivalent to the total savings of the gambler, however the recommendation would be to play the gamble. Though, this does not match with reality and produces the contradiction. Already 1738, Daniel Bernoulli offers a solution for this paradox. He introduces a *moral value*, by means of which reasonable people evaluate the value of money in terms of its utility and not the obvious prize or quantity. So 100 million mean still more pleasure than 10 million, but this does not mean that 100 million are ten times more amusement than 10 million. This results in the moral expectation and the calculation looks like following:

$$\Phi(l) = \sum_{k=1}^{\infty} \frac{1}{2^k} * mv(2^k)\,.$$

The probability with which the monetary result can occur, is not multiplied with the real value anymore, but with the moral value of the quantity. In doing so, the moral value behaves as the logarithmic function of one's own wealth. (Bernoulli, 1896)
But the paradoxon is again unsolved, when the payment is changed to $exp(2^k)$. Menger showed that in fact, every unrestricted utility function can be adapted for the St. Petersburg Paradoxon. For finally solving this paradoxon, it is necessary to limit the utility function.(Menger, 1934)

2.2.2 von Neumann and Morgenstern utility theory

On the basis of this theory of utility, the goal is to define an individual and unique utility function. When a decision is made according to this expected value of utility, the subsequent action is then called rational. Therefore, conditions (axioms) are required which form the basic framework for the existence of this utility function and additionally should correctly predict human behavior.

The theory was originally formulated by John von Neumann and Oskar Morgenstern 1947 in the chapter "The axiomatic treatment of Utility". (von Neumann and Morgenstern, 1947) But in the following, Gilboas notation is used because there the theory is displayed compactly. (Gilboa, 2009, see chapter 8.2)

First of all, a framework is required. So in this case , X is the set of alternatives. As already shown in the umbrella example (table 1), the alternatives can be anything like for example investment securities. Additionally, the set requires no structure. In this paper, X always represents a positive or negative monetary outcome. $P : X \to [0, 1]$ is the set of probability measures. First of all, the objects of choice which are the lotteries, need to be defined:

$$L = \{P : X \to [0, 1] | \#\{x|P(x) > 0\} < \infty, \sum_{x \in X} P(x) = 1\}\,.$$

Because of the finite support condition, the expression $\sum_{x \in X} P(x) = 1$ is well-defined and the lotteries are modeled by a binary relation on $\succsim \subset L \times L$. Therefore a mixing operation is defined over L for every $l_j, l_k \in L, \alpha \in [0, 1]$ and every $x \in X$ like

$$(\alpha l_j + (1 - \alpha)l_k)(x) = \alpha l_j(x) + (1 - \alpha)l_k(x)\,.$$

To simplify this notation, you can think of a two staged gamble. In the first step, it is decided whether you are allowed to play the lottery l_j with probability α or lottery l_k with probability $1-\alpha$. In the second step, the lottery is performed which then has a result.

Within this framework, Von Neumann and Morgenstern defined the following three axioms:

vNM 1. *(Weak order) For any $l_j, l_k, l_m \in L$*

is given exact one of these three following relations: $l_j \sim l_k, l_j \succ l_k, l_j \prec l_k$
and also $l_j \succ l_k, l_k \succ l_m \Rightarrow l_j \succ l_m$.

vNM 2. *(Continuity) For every $l_j, l_k, l_m \in L$, if $l_j \succ l_k \succ l_m$, there exist $\alpha, \beta \in (0,1)$ such that,*

$$\alpha l_j + (1-\alpha) l_m \succ l_k \succ \beta l_j + (1-\beta) l_m \,.$$

vNM 3. *(Independence) For every $l_j, l_k, l_m \in L$, and every $\alpha \in (0,1)$,*

$$l_j \succsim l_k \text{ iff } \alpha l_j + (1-\alpha) l_m \succsim \alpha l_k + (1-\alpha) l_m \,.$$

The first part of the weak order axiom implicate that the decision maker must be able to state preferences among L. The second one formalizes a natural transitivity among the elements of L. (Berger, 1980, chapter 2.2)
For the proof of the work, the continuity axiom is needed. It is very difficult to design an example, where the axiom is violated because it requires an infinite number of observations. For further explanation, action l_j results in \$ 1, l_k has the result \$ 0 and l_m guarantees the outcome death. So, speaking for every rational person, the preference structure is $l_j \succ l_k \succ l_m$. Because gaining \$1 is definitely preferred over gaining nothing and everyone would rather get nothing than die. Now according this axiom, there exists a probability $0 < \alpha < 1$, so that the decision maker is risking his life with probability $(1-\alpha)$ to gain \$1 because $\alpha l_j + (1-\alpha) l_m \succ l_k$. At first glance, this seems strange. However, Raiffa proposed the following counterexample. On one side of the street, a newspaper is sold for one dollar, while on the opposite side the newspaper is distributed for free. When the decision maker wants to cross the street to get the newspaper for free, then s/he is risking her/his life to get the paper for free. So s/he is considering getting hit by a car with the possible outcome death to gain \$ 1. Indeed this counterexample can be challenged again by saying that death is a constant companion of life and the decision maker might also die by not crossing the street, when s/he gets hit by a car because of a drunk driver or something similar. (Gilboa, 2009, chapter 8.2)
The independence axiom assumes that two outcomes with a certain preference structure will maintain this even when they are combined with an outcome, from which both were independent beforehand. From a normative approach, this can also be interpreted as a two-stage process. Thereby the lottery l_j is selected on the first level with probability α and on the second step the monetary outcome x is added despite of the already selected lottery l_j. (Fishburn, 1970, chapter 8.2) For a better understanding of the independence axiom, consider this example in the decision matrix 3. The decision maker has to decide if s/he wants to gamble A, where s/he surely gains \$ 1 million

	Ticket no.1 ($p_1 = \frac{1}{11}$)	Ticket no. 2-11 ($p_2 = \frac{10}{11}$)
A	\$ 1M	\$ 1M
B	\$ 0M	\$ 5M

Table 3: Decision matrix for illustrating the independence axiom

or gamble B, where s/he gains \$ 5 millions with a probability of 10/11. It does not matter now, if s/he chooses gamble A or B, but s/he has to stay consistent in her/his actions, considering now the decision matrix 4, where s/he can surely again gain \$ 1 million taking gamble A. Deciding for gamble B, s/he can gain at least \$ 1 million, with probability 0.99 and has thereby the chance of getting additional \$ 4 millions with a probability of 10 percent. So it is also important that the preference structure is consistent, so that deciding for gamble A in the first example means that the decision maker also has to decide for gamble A in the second one. Otherwise this leads to a contradiction. (Peterson, 2009, chapter 5.2)

	Ticket no.1 ($p_1 = \frac{1}{100}$)	Ticket no. 2-11 ($p_2 = \frac{10}{100}$)	Ticket no. 12-100 ($p = \frac{89}{100}$)
A	\$ 1M	\$ 1M	\$ 1M
B	\$ 0M	\$ 5M	\$ 1M

Table 4: Second decision matrix for illustrating the independence axiom

Theorem 1. *(vNM) $\succsim \subset LxL$ satisfies vNM1 - vNM3 if and only if there exists $u : x \to \mathbb{R}$ such that, for every $l_i, l_k \in L$,*

$$l_i \succsim l_k \text{ iff } \sum_{x \in X} l_i(x)u(x) \geq \sum_{x \in X} l_k(x)u(x) \tag{2}$$

Additionally, u is unique up to a positive linear transformation.

In short, the theorem[2] implies that a unique utility function exists which resembles the preferences of the decision maker. Meaning that, if a decision maker prefers a gamble over another if and only if the expected utility of the preferred game is also higher than the other one. (Gilboa, 2009, chapter 8.2)

The expected utility $EU : X \to \mathbb{R}$ is compound for the lottery $l_i = (x_{i1}, p_1; x_{i2}, p_2; \ldots; x_{ik}, p_k)$:

$$EU(l_i) = \sum_{j=1}^{k} u(x_i j) * p_j \tag{3}$$

Therefrom results the preference relation $\succ$ because according to the equation (2) in theorem vNM

$$l_i \succ l_k \text{ iff } EU(l_i) > EU(l_k)$$

Hence, the lottery with the higher expected utility value is always preferred.(Kreps, 1988, chapter 5)

[2]There are multiple approaches to prove this theorem. In chapter 8.3 Gilboa (2009) provides a good overview of the different ways of proving the theorem and their similarities and differences

Consequently, following the preference structure $\succ$ of the lottery set L, the expected utility theory will simply advise the lottery l^* with the maximal expected utility:

$$l^* \in \arg\{\max_{i=1}^{m} EU[u(x_{ij})]\} = \arg[\max_{i=1}^{m} \sum_{j=1}^{k} u(x_{ij}) p_j] \tag{4}$$

Whereas $u(x_{ij})$ is the utility of lottery l_i with the monetary outcome x_j and the prior probability p_j that x_j is gained. Accordingly, the decision matrix changes the consequences x_{ij} to the utilities $u_{ij} = u(x_{ij})$ with the accordingly prior probability p_j (see table 5). (Bacci and Chiandotto, 2020)

	Prior probabilities					
Lotteries	p_1	p_2	$\dots$	p_j	$\dots$	p_k
l_1	u_{11}	u_{12}	$\dots$	u_{1j}	$\dots$	u_{1k}
l_2	u_{21}	u_{22}	$\dots$	u_{2j}	$\dots$	u_{2k}
$\vdots$	$\dots$	$\dots$	$\vdots$	$\dots$	$\vdots$	$\dots$
l_m	u_{m1}	u_{m2}	$\dots$	u_{mj}	$\dots$	u_{mk}

Table 5: Decision matrix for deciding among lotteries with monetary outcomes and known probabilities

Now, the foundation for calculating an individual utility function is defined and the decision maker can be advised on the base of expected utility theory. But, how does the attitude towards risk impact the expected utility theory?
According to chapter 1.1, someone behaves risk averse, when the expected value of the lottery is preferred over playing the lottery (short: $E(l) \succ l$). Consider a lottery l with the two outcomes x and y. Hence, the outcome x occurs with probability p and consequently y with $1 - p$. The expected value of the lottery results in $E(l) = p * x + (1 - p) * y$ with the utility form $u[E(l)]$. The expected utility is $EU(l) = p * u(x) + (1 - p) * u(y)$. According to the definition of risk aversion, it follows that the utility of the expected value is preferred over the expected utility of lottery l, in accordance with this:

$$u[E(l)] > EU(l)\,.$$

A hypothetical utility function of a risk averse decision maker is displayed on the left side of figure 2 and leads to the conclusion that the utility function is concave. Analogous to argumentation above and by reversing the preferences and greater-than sign, it follows for a risk seeking decision maker that his/her utility function is convex. The risk-seeking utility function is displayed on the right side of figure 2. And a risk neutral utility function counts $EU(l) = u[E(l)]$, for which the identity function is mostly used. (Meyer, 2014)

2.2.3 Violation of rational behavior axioms

The expected utility theory may solve the St. Petersburg Paradox, nevertheless there occur empirical violations of the axioms which will be discussed in the following.

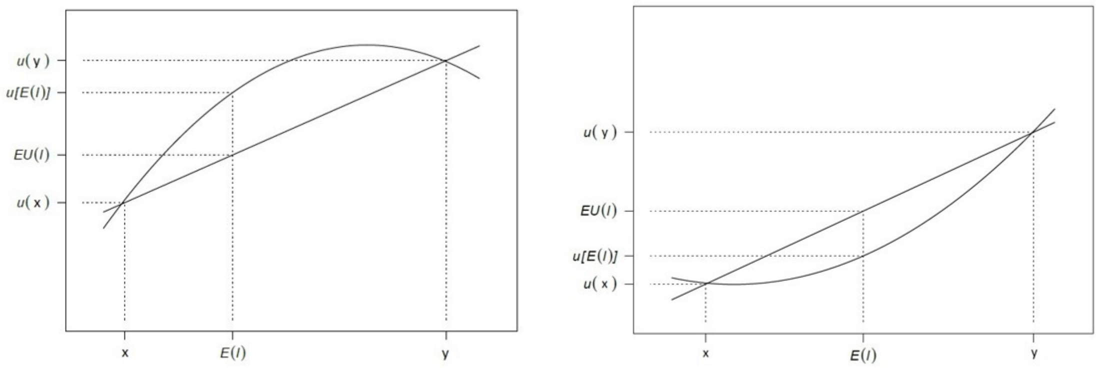

Figure 2: Two hypothetical utility functions for any decision maker who is risk averse (left), when the utility function has a concave form . In contrast a hypothetical utility function is convex for risk seeking decision maker (right).(Bacci and Chiandotto, 2020, chapter 4.2)

Starting with the violation of the weak order axiom, the second part of the axiom claims: if a lottery l_1 is preferred to lottery l_2 and lottery l_2 is preferred to l_3, then consequently l_1 is preferred over l_3. Consider the following two lotteries each with a certain monetary outcome $l_p = (35)$ and $l_q = (36)$. Obviously the lottery with the higher monetary outcome is preferred which leads to this preference structure $l_p \prec l_q$. As next, there is the lottery $l_r = (100, .5)$, where can be gained 100 monetary units with a probability of 0.5. Now, it is considered that the lottery l_r is intransitive to l_p and l_q, so the preference structure looks like $l_p \sim l_r$ and $l_q \sim l_r$. This leads to a contradiction because with the transitivity, the preference structure between l_p and l_q would change to $l_p \sim l_q$, which clearly is not the case. (Fishburn, 1970, chapter 8.3)

Another, empirical violation which is witnessed, is the so-called *certainty effect*. This breach was introduced by Maurice Allais 1953 and is also known as *Allais Paradox*. Due to the language barrier, the example of Tversky and Kahnemann (1979) is used. This variation of the Allais Paradox only differs in the smaller sizes of the monetary outcomes, while the structure remains. The example consists of the following two independent decision problems. So the situations contain:

- In the first situation the decision maker can choose between $l_1 = (2500, .33; 2400, .66)$ and $l_1' = (2400)$
- The second situation includes the lottery $l_2 = (2500, .33)$ and $l_2' = (2400, .34)$

The participants had to choose a lottery in each situation. In the first decision situation, the participants had the opportunity to choose between a lottery, where they could gain 2500 with probability 0.33, 2400 with 66 percent or nothing with probability 0.01 or they could certainly gain 2400 (with probability 100%). In the first situation, 82 % of the participants decided for the certainly lottery l_1', where they get paid 2400. So according to the expected utility theory, where $u(0) = 0$ the preference structure results in

$$\begin{aligned} l_1' \succ l_1 &\Leftrightarrow u(2400) > .33u(2500) + .66u(2400) \\ &\Leftrightarrow .34u(2400) > .33u(2500)\,. \end{aligned} \tag{5}$$

With the first decision problem in mind, the majority of the participants would have to decide again for the lottery l_2' in the second decision problem to behave according to the expected utility theory. But the majority of the participants (83%) chose the lottery l_2. Thus, the preference structure looks like this

$$l_2' \prec l_2 \Leftrightarrow .34u(2400) < .33u(2500)\,.$$

Comparing this with the formula 5, this is clearly an empirical contradiction. (Tversky and Kahnemann, 1979)
To summarize this violation: the probabilities in expected utility theory are supposed to be linear, despite the fact that the decision makers often prefer the sure outcomes over probable outcomes regardless of their value. This leads to the assumption that the probabilities of the decision makers are not linear. (Bacci and Chiandotto, 2020, chapter 3.5.2)

Individuals do not only have a biased perception, when it comes to certain outcomes, but also when it only seems that the result is certain. Usually people focus on what the differences are when comparing. For explaining the next empirical violation, the following two decision problems are considered:

- situation 1: The decision maker can choose between $l_3 = (4000, .2)$ and $l_3' = (3000, .25)$
- situation 2: This is a two-stage game. A coin is flipped which decides whether a participant of the game can enter the second stage. The coin falling correctly and the participant therefore entering the second stage occurs with a probability of .25. After reaching the second stage, the decision maker has these two possibilities $l_4 = (4000, .8)$ and $l_4' = (3000)$.

In situation 1, the majority of participants (65 %) decide for option l_3, where they can gain 4000 with a probability of 20 %. Thereby, the first situation results in:

$$l_3 \succ l_3' \Leftrightarrow .2u(4000) > .25u(3000)\,.$$

Meanwhile, 78% decided for the lottery l_4' in the second situation, so according to the certainty effect, they chose the apparently certain option. Thus, the preference structure looks like:

$$l_4 \prec l_4' \Leftrightarrow .8u(4000) < u(3000)\,.$$

And this switch of preferences from option l_3 to option l_4' is not conform with the expected utility theory. This may not be obvious at first glance, but the two situations are linked. Because of the coin flip on the first stage in the second situation, the probabilities are neither .8 nor 1, but $.8 * .25 = .2$ and $1 * .25 = .25$. This means that the quantity of the probabilities in both situations is equal, only the presentation differs.
For a better understanding, have a look at figure 3. There are displayed two decision trees, whereas the squares represent decisions made by an individual and the circles illustrate results based on random processes. On the left side is the decision tree of the first situation, where on the first stage the individual decides to play for 3000 with probability .25 or 4000 with probability .2. On the second stage a random process decides whether s/he wins the money or gets nothing. In contrast, on the right side of figure 3 is first the random process, followed by the individual decision. In this case, the decision maker faces one risky and one certain option. This structure is called

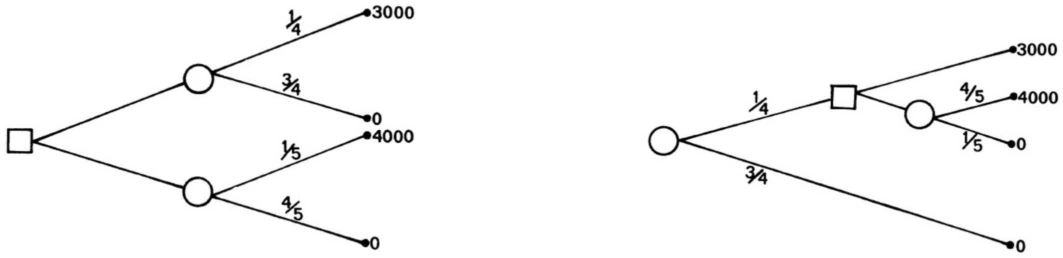

Figure 3: Two possible structures of making a decision. Thereby the left side displays the structure of choice between the lotteries l_3 and l_3' a so-called standard formulation. The right side shows the decision between l_4 and l_4' and has a so-called sequential form.(Tversky and Kahnemann, 1979)

sequential.(Tversky and Kahnemann, 1979)
Consequently, the representation of decision plays also a central role and can cause a bias and contradiction which leads to the so-called *isolation effect*.

What happens, when the payments are no longer gains but losses? In the following, losses are displayed with $-x$ and gains stay x. After the expected utility theory, the preferences have to stay consistent regardless of the range of outcomes. So consider these two choices:

- situation 1: $l_5 = (4000, .8)$ and $l_5' = (3000)$
- situation 2: $l_6 = (-4000, .8)$ and $l_6' = (-3000)$

So in the first one, the decision maker can choose whether to behave risky and get \$ 4000 with probability 0.8 or to be risk averse and to take the certain \$ 3000. In the second decision situation, the gain situation is simply mirrored, so that the decision maker has to decide if s/he prefers to lose \$ 4000 with probability 0.8 or to lose for sure \$ 3000. In the first place, the majority decided for the certain option ($l_5 \prec l_5'$), however most decision makers preferred the risk seeking option ($l_6 \succ l_6'$) in the loss situation. Thus, it comes again to an empiric violation. Generally spoken, when the *framing effect* arises, people prefer taking a risk in loss situations. But when it comes to gains, they often prefer the risk less option. (Tversky and Kahnemann, 1979)
But how does this match with insurances? When people like to take a risk, when it is about losses, how can insurances still exist and further why does everyone recommend taking insurance? As asserted in the experiment, the perception of insurances compared to losses is different. Insurances are seen as an investment in safety, as well as they trigger social norms. Nevertheless the presentation of lotteries and insurances is different. Consider choosing between $l_7 = (-5000, .001)$ and $l_7' = (-5)$. When you are gambling, paying \$5 does not relieve you of having to play the gamble a second and maybe a third time. Insurance behaves differently, it is clear that someone has to pay once only \$5 and be safe from losing money afterwards again. So choosing the safety option in a gamble does not automatically mean that someone is safe from playing again and eventually losing money. (Fischhoff et al., 1988)

2.3 Prospect theory

As seen in the chapter above, there exists some violations of the expected utility theory. In the following chapter, there is presented a theory which tries to involve the sychological effects of individuals in making a decision. This theory is called *prospect theory* and was developed by Daniel Kahnemann and Amos Tversky 1979 and advanced in 1992. The importance of this theory displays in winning the nobel prize in economic sciences 2002. (Nobel-Prize, 2002)

The most fundamental matter needed for discussing this theory is prospects. Above it were already defined prospects, but lotteries called. A recap: Lotteries or rather in this theory prospects $(x_1, p_1, \ldots, x_n, p_n)$ connect an outcome x_i with a probability p_i. Tough, it is essential that $\sum_{j=1}^{m} p_j = 1$. To simplify the notation, there is written (x) for a (riskless) prospect, thus the outcome x is certain. And, when the prospect looks like $(x, p; 0, 1-p)$, we simplify it to (x, p). In the following the term prospect will be used for lotteries, when talking about the prospect theory. [3]

The theory is divided into two stages of development. In the first phase, called editing, the representation of the prospects is simplified. In the phase of evaluation, the second one, a formal model is evolved.
In the following paragraph, the editing phase and its major tools to reformulate the options are introduced. The first four presented operations make changes in the prospects independently from each other.
As discussed in chapter 2.2.3, individuals usually do not perceive different monetary outcomes in the same way. Thus, they measure outcomes as gains and losses. Thus, a reference point is needed which separates the outcomes into gains and losses. This reference point, especially its location, can be influenced by the expectations of the decision maker and the representation of the offered prospects. Because the prospects are consequently coded as gains or losses, this operation is called *Coding*. Defining a reference point is one of the major differences between prospect theory to the expected utility theory.
In the editing phase, prospects with identical outcomes and probabilities are simplified through combining, thus the following operation is called *Combination*. For example, the prospect $(200, .25; 200, .25)$ can be shortened to $(200, .5)$. This reduced form will be then evaluated later on.
Another aspect of the editing phase is the *simplification* of probabilities and outcomes. In this case, the components of the prospects are rounded and are changed for example from $(101, .49)$ to $(100, .5)$. This involves that very implausible outcomes are eliminated.
Also the next tool reduces the prospects. Consider this example: The prospect is changed from $(300, .8; 200, .2)$ to $(100, .8)$. This means that with the help of the so-called *segregation*, the risk less component of the prospect is removed. In the example the 200 monetary units are certain, only the difference between 300 and 200 is uncertain and thus relevant.
The following operation corresponds to the isolation effect which was discussed in the previous chapter. The *cancellation* is used for a set of two or more prospects. Just like the isolation effect also this tool ignores the common components of two or more prospects. For example the decision between $(200, .2; 100, .5; -50, .3)$ and (200, .2; 150, .5; -100, .3) is reduced to $(100, .5; -50, .3)$ and $(150, .5; -100, .3)$ by deleting the same outcomes.
For the last tool, a further explanation is needed. In chapter 2.1, the preference structure for e.g. $l_i \succ l_m$ says that l_i strongly dominates l_m in each state. This implies that

[3] Notation again inspired by Bacci and Chiandotto (2020) and Tversky and Kahnemann (1979)

each outcome of l_i is better than l_m. This last operation in the editing phase removes each prospect which is dominated by another one and is therefore unnecessary. For example the prospect $(500, .2; 100, .5)$ dominates the prospect $(500, .15; 100, .5)$. Thus, the latter one can be removed and will not be evaluated in the next step. This form of reduction is called *Dominance*. (Tversky and Kahnemann, 1979)

After converting the prospects into a simpler form and eliminating redundant prospects, the evaluation can be started.

Compared to the expected utility theory, the two main differences are

- the monetary outcomes are not final asset, but separated into gains and losses
- the values (utilities) of the outcomes are multiplied by a decision weight and not the probability

So for the evaluation are required two functions: the value function and the probability weighting function. (Tversky and Kahnemann, 1979)

To formulate the new idea, Tversky and Kahnemann built a mathematical frame. S is a finite set of states of nature, whereby the so called events are a subset of S. Again, X is a set of consequences/outcomes, whereby it contains a neutral outcome. In the case of this thesis with monetary outcomes, this reference point is 0. Therefore, all outcomes above zero are determined as gains, and all below as losses (denoted by $-x$). For the uncertain prospect function $f : S \to X$ is applied for every $s \in S$ to $f(s) = x$, whereby $x \in X$. (Tversky and Kahnemann, 1992)

The first component is the value function $v(x)$ which can be interpreted as the subjective individual outcome x of a prospect. Because of the rational violations of the expected utility theory the value function satisfies the following points. The value function:

(i) is defined on the changes in relation to the reference point

(ii) has concavity for gains and convexity for losses

(iii) is steeper for losses than for gains

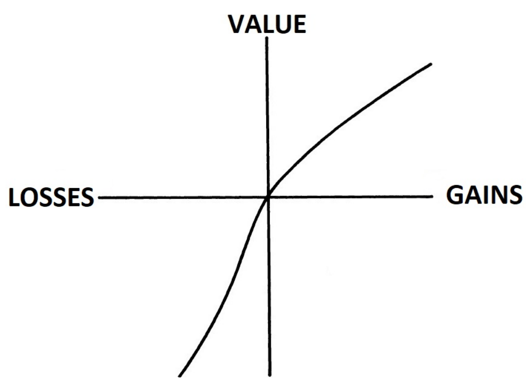

Figure 4: A possible value function of the prospect theory (Tversky and Kahnemann, 1986)

This is summed up in figure 4. So the first point is important because humans rather recognize changes or differences than a final asset. Also this separation of the outcomes allows us to evaluate slightly different functions for gains and losses which correspond to the framing effect. The human perception of change differs by the size of the asset. So recognizing a change between three degrees and six degrees is easier for people, than a change from 13 degrees to 16 degrees. This can be applied to monetary changes. The change in the gain from 100 to 200 seems greater than a change from 1100 to 1200. Considering this argumentation, it follows that the value function has concave form for the monetary outcomes above the reference point, where: $v''(x) < 0$ for $x > 0$. Simultaneous , this can be adapted for the losses. Therefore the value function is assumed to be convex, this means: $v''(x) > 0$ for $x < 0$. Lastly, the value function is considered to be steeper for losses than for gains. The reason is the following consideration. There are two symmetrical fair bets, whereas $x > y \geq 0$. Therefore the lotteries are $l_1 = (x, .5; -x)$ and $l_2 = (y, .5; -y)$. The aversion to symmetrical bets is increasing with the size of the outcomes, thus the smaller magnitude is preferred:

$$v(y) + v(-y) > v(x) + v(-x) \Leftrightarrow v(-y) - v(-x) > v(x) - v(y)\,.$$

Consequently the value function is steeper for losses because when putting $y = 0$ the value function results in $v(x) < -v(-x)$. (Tversky and Kahnemann, 1979)

The second required component for the evaluation is the probability weighting function. The probability weighting function yields decision weights on the probabilities. In doing so, these decision weights do not satisfy any probability axioms. The decision weight $w(p)$ can be seen as an impact of p on the overall value of the prospect. Thus, the probability weighting function is defined as $w : [0, 1] \to [0, 1]$ and is naturally growing. If the expectation principle matches the impact of the probability, then the probability weighting function reduces to: $w(p) = p$. The marginal points of the probability range are defined as $w(0) = 0$ and $w(1) = 1$, thus impossible events with a probability of zero are disregarded.
The probability weighting function is assumed with some characteristics. The first one is the *subadditivity*. Therefore, it is supposed that $w(rp) > rw(p)$ for $0 < r < 1$. This feature only has to hold for small probabilities. The next one is the *over-weighting* of small p, therefore $w(p) > p$. This issue is reflected in the preference for l_7 over l'_7 which was discussed in the previous chapter. Important to mention here is that there is a difference between the overweighting of p and the overestimation in the evaluation of the probabilities of rare events. Lastly the *subcertainty*which directly follows the overweighting of small probabilities. This means that for all $0 < p < 1$: $w(p)+w(1-p) < 1$. This refers to l_1 and l'_1 in the previous chapter as follows:

$$\begin{aligned}
&l_1 : v(2400) > w(.33)v(2500) + w(0.66)v(2400) \Leftrightarrow (1 - w(.66))v(2400) > w(0.33)v(2400)\\
&l'_1 : w(.33)v(2500) > w(.34)v(2400)\\
&\Rightarrow 1 - w(.66) > w(.34) \text{ or } w(.66) + w(.34) < 1\,.
\end{aligned}$$

Based on these previous assumptions, the probability function could have the form of the convex function in figure 5. When $w(p) = p$, the probability weighting function results in the bisecting line of figure 5. (Tversky and Kahnemann, 1979)

So the calculation of the prospect $f = (x, p; y, q)$ looks like

$$PT(x, p; x, q) = w(p)v(x) + w(q)v(y) \tag{6}$$

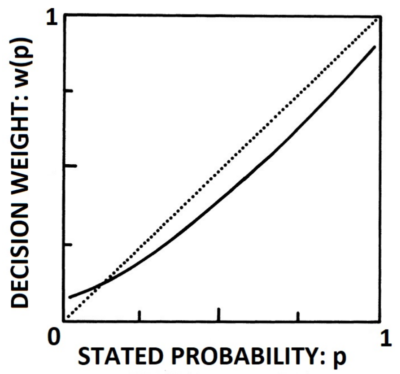

Figure 5: A possible probability weighting function according to the prospect theory (solid line). The dashed line is the identity function. (Tversky and Kahnemann, 1986)

Whereas v(0) = 0, w(0) = 0, w(1) =1. Hence, for the certain prospect (x) it follows that the prospect model is identical with the value function $(PT(x, 1) = w(1)v(x) = v(x))$. When the probabilities p, q of the prospect $(x, p; y, q)$ add up to 1 and the prospects are not mixed, meaning there are either gains or losses ($x > y > 0$ or $x < y < 0$), then the equation (6) reduces to

$$PT(x, p; y, q) = v(y) + w(p)[v(x) - v(y)] \tag{7}$$

As one can see in equation (7), the prospect is separated in the riskless component, in this case y, so the minimum of gain or loss, which is certain and achievable, when choosing this game. The other part is the risky component, so the additional gain or loss, which is uncertain. This second component of decision weight is operated on the difference of value of the outcomes x and y. This segregation is already taken in the editing phase of strictly positive and negative prospects. (Tversky and Kahnemann, 1979)

For each prospect f, a number $PT(f)$ is applied, so that the preference structure of the (cumulative) prospect theory corresponds to (Tversky and Kahnemann, 1992)

$$f \succsim g \text{ iff } PT(f) \geq PT(g)$$

Though the prospect theory was criticized for violating stochastic dominance, Tversky and Kahnemann even admitted that direct violations of dominance are prevented by the editing phase through recapping and eliminating unnecessary prospects, but indirect violations are not prevented. (Tversky and Kahnemann, 1979)
As example consider these three prospects:

- $l_1 = (-100000, .5)$
- $l_2 = (-45000)$
- here the representation differs because the gamblers have to pay a participation fee of 45 000 and can then gain 45 000 or lose 50 000 each with a probability of .5, thus $l_3 = (-95000, .5)$.

In this case clearly l_3 dominates l_1, but many gamblers prefer l_1 over l_2 and l_2 over l_3. (Raiffa, 1970) The structure of prospect theory can not prevent this kind of violation

regarding stochastic dominance.
Machina criticized that violating stochastic dominance is unavoidable because the prospect theory model can not keep up the monotonicity. Consequently any individual will choose some prospects over others which are stochastically dominating them. (Machina, 1982, chapter 4.8)
This has also been observed in an experiment. Consider these two prospects

- $l_2 = (45, .06; 30, .01; -15, .03)$
- $l_2' = (45, .07; -10, 0.01; -15, .02)$

It is not obvious at first glance, but l_2' dominates l_2. Nevertheless, the majority of the participants chose l_2 and thus violated the dominance. But Tversky and Kahnemann did not see this as a problem of their theory by arguing that the theory predicts this violation in some observations. (Tversky and Kahnemann, 1986)

However, Tversky and Kahnemann evolved the *cumulative prospect theory* (short: CPT) by combining the main ideas of the prospect theory and the rank-dependent model of Quiggin. Before presenting the CPT, the main idea of the rank-dependent model of Quiggin shall briefly be discussed.

One of the central aspects of the rank-dependent theory is the weighting function formulated as transformation on cumulative probabilities which is conditioned on the ordered outcomes. So, the probabilities are attached to a probability weighting function which looks like

$$\pi_i(p) = w(\sum_{j=1}^{i} p_j) - w(\sum_{j=1}^{i-1} p_j)\,. \tag{8}$$

As displayed in equation (8), the decision weight is influenced by all probabilities (not only p_i) and the position in the preference ranking. Therefore $f(\sum_{j=1}^{i} p_j)$ is assigned to get the outcome x_i or worse. So the decision weight of outcome x_i is compound of the difference between having the outcome x_i or worse and having a result worse than the outcome x_i. The shape of f is influenced by the decision makers approach towards risk.
Another central point of the rank-dependent theory is the weaker formulation of the axioms than in vNM expected utility theory. For example the independence axiom is rephrased so that, it only depends on the probabilities and their decision weights and independent of the outcomes. (Quiggin, 1982)

In 1992, Tversky and Kahnemann adapted this idea for the prospect theory. Rather than transforming every probability on its own, the CPT changes the entire cumulative distribution function. Thus, one of the advantages of CPT in comparison to prospect theory, is the different cumulative probability weighting functions for gains and losses. (Tversky and Kahnemann, 1992)

So the prospects $f = (x_{-k}, p_{-k}; \ldots; x_m, p_m)$ are ordered like

$$x_{-k} \leq \cdots \leq x_{-1} \leq 0 \leq x_1 \leq \cdots \leq x_m$$

and consists of an outcome x_j which can occur based on the familiar probability p_j (Wakker, 2010, chapter 9). Thus, the probability weighting function $w^a : [0,1] \rightarrow [0,1]$ is strictly increasing and fulfills the two conditions $w^a(0) = 0$ and $w^a(1) = 1$, whereas $a = \{-,+\}$. The value function $v : X \rightarrow Re$ is either strictly increasing and once again fulfills the condition $v(x_0) = v(0) = 0$. Consequently, this results in:

$$V(f) = V(f^+) + V(f^-)$$
$$V(f^+) = \sum_{j=0}^{m} \pi_j^+ v(x_j), \quad V(f^-) = \sum_{j=-k}^{0} \pi_i^- v(x_j) \tag{9}$$

As you can see in equation 9, the model is divided into a model for gains and for losses. Thereby the decision weights for gains $\pi^+(f^+) = (\pi_0^+, \pi_1^+, \ldots, \pi_m^+)$ are specified by:

$$\begin{aligned} \pi_m^+ &= w^+(p_m) \\ \pi_j^+ &= w^+(p_j + \cdots + p_m) - w^+(p_{j+1} + \cdots + p_m), \quad 0 \leq j \leq m-1 \end{aligned}$$

Accordingly, the decision weights of the gains refer to the difference between the outcome is at least as good as x_j and the outcome is strictly better than x_j. Analogously, the decision weights for losses $\pi^-(f^-) = (\pi_{-c}^-, \pi_{-c+1}^- \ldots, \pi_0^-)$ are defined by:

$$\begin{aligned} \pi_{-k}^- &= w^-(p_{-k}) \\ \pi_j^- &= w^-(p_{-k} + \cdots + p_j) - w^-(p_{-k} + \cdots + p_{j-1}), \quad 1-k \leq j \leq 0 \end{aligned}$$

The interpretation of this definition is then analogously to the decision weights of the gains: the difference between the outcome is at least as bad as x_j and the outcome is strictly worse than x_j. The CPT model implies the relation $w^-(p) = 1 - w^+(1-p)$ between the cumulative weighting function for gains and losses, in contrast to the prospect theory which implies $w^+(p) = w^-(p)$.
When $\pi_j = \pi_j^+$, for $j \geq 0$ and $\pi_j = \pi_j^-$, for $j < 0$, the equation (9) simplifies to

$$V(f) = \sum_{j=-k}^{m} \pi_j v(x_j) \tag{10}$$

The cumulative weighting function for gains w^+ is assumed to be concave near the origin and becoming convex (see figure 6). According to this inverted S-shape, the cumulative weighting function is steeper at the boundary points and smoother in the middle. As in figure 5 displays the cumulative weighting function of the gains w^+ and losses w^- are in comparison alike, but not equal. (Tversky and Kahnemann, 1992)

To draw a first comparison between the two theories: The (cumulative) prospect theory seems far more complex and involves a lot of known effects regarding making a decision under risk. In order for this model to be flexible, several parameters are required. The quantity of estimated parameters can become too large, when the number is not restricted. (Tversky and Kahnemann, 1992) This flexibility can result in overfitting. Then the model has a good fit on the sample, but lacks generalization. The model takes up too much noise, so that the general prediction gets worse.(Wakker et al., 2014) In contrast to the expected utility theory impresses with its simplicity and little required parameters.

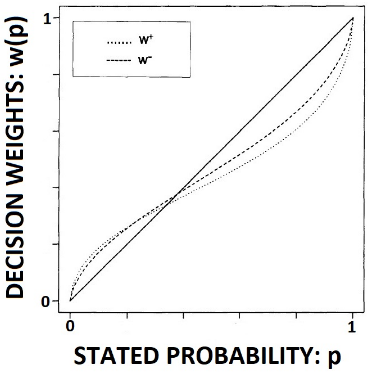

Figure 6: Two possible probability weighting functions of CPT for gains and losses. The dashed line is the proabability weighting function for gains and the dotted line for losses. As illustrated the probability weighting function for gains is slightly more curved than for losses, implying that the risk aversion for gains is higher(Tversky and Kahnemann, 1992)

3 Survey

In the following chapter there is given an overview over the lotteries which were part of the survey. Additionally, a first descriptive analysis is performed.

3.1 Structure of questionnaire

The survey obtained 16 decision-making situations and was divided into two parts which was a gain and a loss section. Each decision-making situation consisted of two lottery wheels so that each situation had a binary decision structure. In the first eight questions, the participants exclusively could hypothetically win money or in the worst case gain nothing. The lotteries/prospects looked like:

no.	Lottery wheel A	$E(l_A)$	Lottery wheel B	$E(l_B)$
1	(12 000, .35; 3000, .25)	4950	(6000, .7; 3000, .25)	4950
2	(12 000, .001; 3000, .25)	762	(6000, .002; 3000, .25)	762
3	(7000, .3; 3000, .55)	3750	(3200, .2; 3100,. 25; 3000)	3065
4	(7000; .3)	2100	(3200, .2; 1500, .25)	1015
5	In the beginning of the next lottery, a fair coin is thrown (p=0.5) which decides whether the participant is allowed to turn the lotterywheel or not, when s/he enters the second step, the wheels look like:			
	(7000; .6)	2100	(3200, .4; 1500, .5)	1015
6	(5000, .35; 3000, .35)	2800	(2500, .65; 4000, .15)	2225
7	(4500, .25; 3500, .35)	2350	(3000, .1; 1500; .8)	1500
8	(30 000, .05; 4000, .3)	2700	(2000, .2; 1500, .7)	1450

Table 6: First eight decision situations and the associated lotteries/prospects of the survey which exclusively consist of gains (and no possible loss)

In the last eight questions, the participants could only lose hypothetical money. The questions in the loss section were identical to the gains, despite the sign having been changed from positive to negative. The corresponding table 16 can be found in the appendix. The order of the questions within the gains and losses was random and so was the side of the depiction of the lottery wheels (as A or B respectively left or right side). In table 6 and table 16, the depiction of the wheels are structured in such a way that the risk averse decision is on the right side and the risk seeking decision is on the left side.

The both lotteries in each question were displayed as lottery wheels so that for example the first decision situation looked like in figure 1 in chapter 2.2.1. The idea to display the lotteries/prospects as lottery wheels was inspired by Abdellaoui.(Abdellaoui et al., 2008) To clearly mark the difference between the gain and loss situation, both had different color palettes so that for example the counterpart of the first gain decision situation (accordingly the loss situation) looked like in figure 7. Each box around the wheel contains the possible monetary outcome with the respective probability. Losses are marked through the different color scheme, the minus mark before the theoretical possible outcome and a text above this section in the survey. All possible amounts of money as well as nothing (accordingly no gain and no loss) were specified. The size of the circular segments of the lottery wheels correspond to the corresponding probabilities of each outcome. The participants expressed their preference by choosing the particular wheel. The corresponding questionnaire can be found in the appendix at survey form.

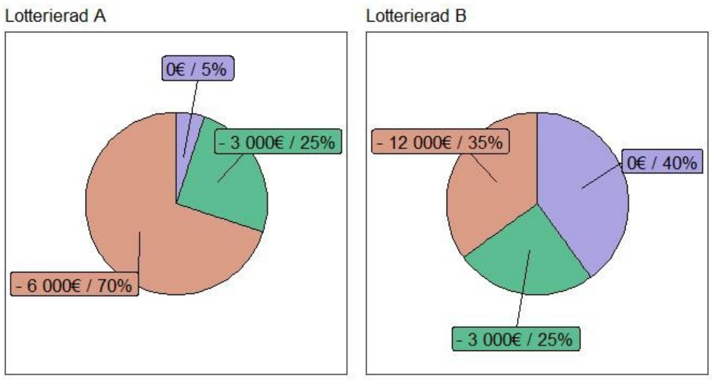

Figure 7: Lotteries of the ninth decision situation which is the counterpart to the first decision situation. This decision situation consists of exclusively losses and in the best case a monetary outcome of zero

According to Wakker, when the monetary outcomes are chosen too low, the utility function is nearly linear.(Wakker and Deneffe, 1996) Consequently, the size of the monetary outcomes were chosen relatively high. In addition, it was noted that the expected value of the respective lottery wheels is not obvious so that the participants do not choose by maximizing the expected value. Therefore, most of the lottery wheels have three possible outcomes.
In the beginning, the participants were told that there exists neither a right nor a wrong decision. The only thing that matters is their preference in this survey. The participation was on a voluntary basis and there did not exist any monetary advantage. The participants were asked to answer all questions and only give one answer at a time but were not forced to do so. As result, a few gave multiple answers or none in single cases. When both lottery wheels were chosen, the preference structure is defined as indifferent ($l_{iA} \sim l_{iB}$). While the cases where no answer was given are interpreted as incomparable. The participant did not make a decision, consequently his/her preference structure is not complete. Because a complete preference structure is a condition of the expected utility theory as well as of the prospect theory, these four observation which contain incomparable preference structures are banned from the following evaluation. At the end of the survey, the characteristics age, gender and employment of the participants were asked.

The survey was structured in such a way that it could cause violations of the axioms of the expected utility theory which already have been discussed in chapter 2.2.3.
The first direct violation of the rational axioms of the expected utility theory which can happen, is between decision situation one and two. According to the expected utility theory the preference structure have to be consistent. Exactly this means, if lottery wheel B is preferred over the lottery A in the first decision situation, then also the lottery wheel B has to be preferred in the second situation because:

$$l_{1A} \prec l_{1B} \Leftrightarrow .35 * u(12000) + .25 * u(3000) < .7 * u(6000) + .25 * u(3000)$$
$$\Leftrightarrow .35 * u(12000) < .7 * u(6000) \Leftrightarrow .001 * u(12000) < .002 * u(6000) \Leftrightarrow l_{2A} \prec l_{2B}$$

But when the preference in the second situation changes to A, then this inconsistency can be explained with the certainty effect. In this case the participants would prefer a nearly certain effect over a possible regardless of each quantity.
A change in the preference structure the other way round (from A to B) implies also a violation of the expected utility theory but can not be explained with the certainty effect.
In the expected utility theory, the same relevance has to be assigned to the probabilities of the events despite their value. (Bacci and Chiandotto, 2020, chapter 3.5.2) Since, in addition, the losses are the mirrored gains in this survey, the certainty effect can therefore be observed also between decision situation nine and ten. If and how many participants behaved accordingly the certainty effect will be discussed in the following chapter.

The next effect which was considered in this survey, arises in subgames like the decision situation 5. On the first step is thrown a fair coin which decides whether a gambler is allowed to enter the second step or not with a probability of 0.5. Then on the second step, the lottery wheel is turned and the decision maker can theoretically win money. This means for the preference structure of situation 5 for example:

$$\begin{aligned} l_{5A} \succ l_{5B} &\Leftrightarrow 0.5 * (0.6 * u(7000)) > 0.5 * (.4 * u(3200) + .5 * u(1500)) \\ &\Leftrightarrow .3 * u(7000) > .2 * u(3200) + .25 * u(1500) \Leftrightarrow l_{4A} \succ l_{4B} \end{aligned}$$

So according to the expected utility theory, the preference structure of situation 5 and 4 have to be identical. An inconsistency in the preference structure can be explained with the isolation effect when the preference structure looks like: The risky option A is chosen in situation four and then switched to the risk averse wheel B in situation five. According to the argument above, the isolation effect could also occur in the loss section between decision situations twelve and thirteen.

The last effect which was considered in this survey is the framing effect. In short: individuals behave risk averse when it is about gain situations and risk seeking when it is about losses. Since the losses are simply the mirrored gains, the preference structure has to be identical for gains and the belonging losses according to the expected utility theory. As an example consider the first and ninth decision situation as well as the preference structure $l_{1A} \prec l_{1B}$ so that:

$$\begin{aligned} l_{1A} \prec l_{1B} \Leftrightarrow 0.35 * u(12000) + 0.25 * u(3000) < 0.7 * u(6000) + 0.25 * u(3000) \\ \Leftrightarrow 0.35 * u(-12000) + 0.25 * u(-3000) < 0.7 * u(-6000) + 0.25 * u(-3000) \Leftrightarrow l_{9A} \prec l_{9B} \end{aligned}$$

Consequently, lottery wheel B has to be preferred over lottery wheel A in the ninth decision situation according to the expected utility theory. In contrast, the choice of lottery wheel A in situation nine (and still lottery B in the first situation) would imply the framing effect. A change in the preference structure the other way round also violates the axioms of the expected utility theory but are not explainable with the framing effect.
This preference structure between gains and losses follows analogous for all the remaining decision situation.

3.2 Descriptive analysis

It was possible to participate in the survey from 01.03.2023 until 04.04.2023. As mentioned, in addition to the preference structure of the lottery wheels, the three characteristics: age, gender and employment were collected. Every participant answered this section of characteristics so that there are no missing values.
In total, 123 individuals participated in the survey, of which 119 observations can be evaluated. Thereof 68 women and 51 men are included, whereby one person answered the gender question with non-binary and female, so she was assigned female because of the lack of the non-binary group.

The stacked barplot (see figure 8) displays the age distribution grouped by gender. Therefore the age is displayed on the x-axis and the corresponding number of participants on the y-axis as well as the bars are separated into gender by color. The age of the participants ranges from 14 to 68. As it can be clearly seen in the figure 8, the majority of participants are under 30 years old so that this age group makes up nearly 75% of the sample. Furthermore, the majority in this age group are female. In contrast, the 30-39 age group are all men and consist of three observations. The age group of 40 to 49 contains ten observations, while four are female and six are male. The 50 to 59 year old contains fourteen observations, of which exactly half is female. Five participants were 60 years old or older. What is striking is that the age distribution is univariate so that nearly 20 observations are 23-year-olds and consequently make up the greatest age character. And the other thing to admit is that not every age between 14 and 68 is present in this survey. Especially, from the age of 29 onwards, there are gaps in the age distribution.

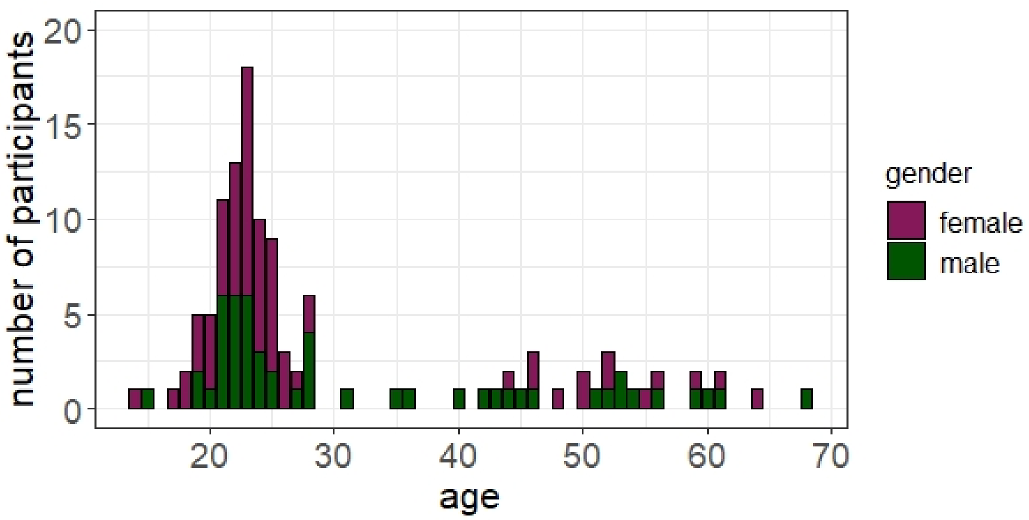

Figure 8: Barplot of the age of the participants separated by age

The last characteristic should capture the salary. In order to achieve the highest possible response rate, it was not asked directly about the salary, instead the participants had to choose between four options of employment, with which one can hopefully draw conclusions about the salary. The first and biggest group, with about half of the sample is, the trainee group (see table 18). This includes all persons who are still in training, meaning pupils, students and apprentices. Some of the trainees allocated that they are also employees. Lastly, they were grouped only as trainees because the salary of a side job is not comparable to the salary of a fulltime job. As it is displayed in figure 9, this group of trainees only consists of people under 30 years. The next largest group are the employees with 43 percent of the sample. The youngest of this group is 19 years old and the oldest is 64 years old. Thus, this group covers almost the entire age distribution. A few said that they are employed and self-employed. These are included only as freelancers. This next category of self-employed consists of 6 individuals, whereby 4 of them are under 30 and the last two are 50 years old. Lastly, there was the option of others which can include anything else like unemployment, gap year, pension et cetera. The one person who falls under this category, is probably a pensioner because of the additional age statement of 68.

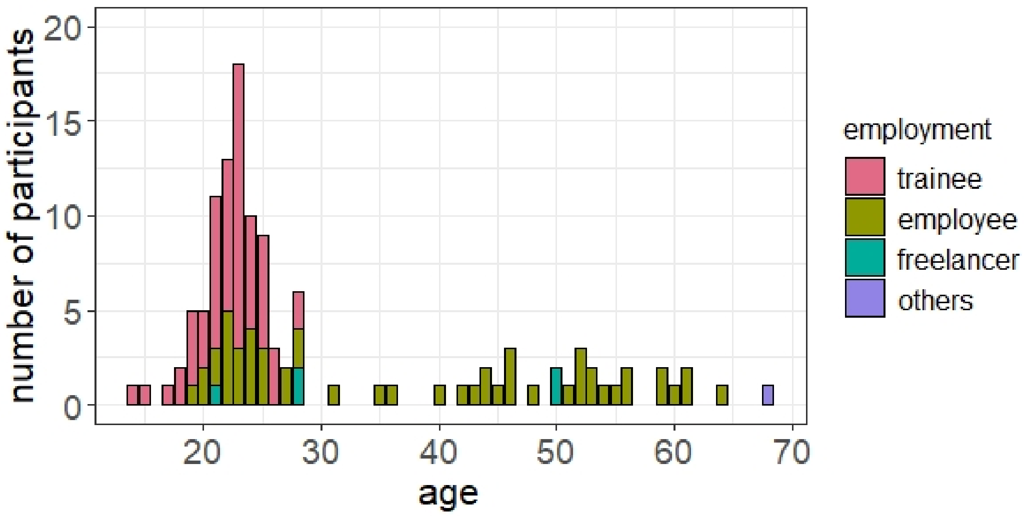

Figure 9: Barplot of the age of the participants grouped by their employment

After the descriptive analysis of the characteristics of the participants, it is time to have a look at the answering of the questions. As already mentioned the complete response rate was quite high and in a few cases except the few duple and incomplete answers. More information about it is in the appendix in table 19, where the numbers of the choices in each decision situation are written.

But maybe there exists a preference structure over all decision situations which multiple participants have chosen? This question will now be considered in more detail in the following and in the corresponding table 7.
The maximum number of participants sharing the same preference structure over all questions is three. Thereby, the pattern which they have chosen is standing out. In the decision situations, where they could hypothetical win money, they preferred always the more riskless lottery wheel B. Except in the second situation, where they preferred A. And in the loss situation, they always preferred the riskier lottery wheel A except again in the decision situation 10 which is the mirrored loss situation of the decision situation two. This means that the certainty effect and framing effect can be observed in this preference structure, however, more on that later. Additionally, there exists three other patterns which each were used by two participants. Consequently, the overall preference structure seems quite unique for every individual.

overall preference structure	number of participants	total number of patterns
BABBBBBBABAAAAAA	3	114
BBBBBBBBABBAAAAB	2	
BABBBBBBABAAABAB	2	
preference structure for gains	**number of participants**	**total number of patterns**
BABBBBBB	16	54
BBBBBBBB	11	
BABBBABB	6	
preference structure for losses	**number of participants**	**total number of patterns**
ABAAAAAA	6	75
BBBBBBBB	5	
ABABBAAA	5	

Table 7: Three most frequent matching patterns of preference structure over the 16, first eight and last eight decision situations

When separating the decision situations into gain and losses and searching for patterns within both parts, then there exists more overlaps.
The maximum which is reached in the gains section, is 16. Thus, 16 participants gave for the gains section the same preference structure which was choosing the more riskless lottery wheel B except for decision situation two, where they choose the riskier wheel A. Also here, this means the certainty effect occurs between situation one and two. These 16 participants are more than ten percent of the sample. Another nearly ten percent of the sample (11 participants) have either the same preference structure which only differs in the preference of situation two to the gain structure above. Hence, they chose over all the gain situations the riskless lottery wheel B. Thus, these eleven participants decided accordingly the expected utility theory in the gain section. Consequently, in the gain section are more common preference structures.
Lastly, there is searched for a preference structure among the loss situation. In this case the unified preference structure is weaker. The maximum number of the equal preference structure is six. Accordingly six participants preferred the riskier lottery wheel A, except again in situation 10, where they chose wheel B.
Summarizing: The total quantity of different patterns, looking only at the gain section is the lowest with 54. This implies that in the gain section the most overlaps between the preference structure of the participants exist. This is followed by the loss section, where 75 different patterns exist so that in this section the preference structure of the individuals seems more unique than in the gain section. Lastly, in the overall preference structure exist the most different patterns with 114 so that the overall preference structure is quite unique. Consequently, it might make sense to create a decision theoretical model with different risk parameters for gains and losses in order to create a preference structure which fittest the most perfectly. But it has to be admitted that the number of all possible patterns, when only profits or losses are considered, is significantly lower with 256 (2^8) than when looking at the overall preference structure ($2^{16} = 65536$). Conversely, this means that the probability of overlaps is higher by the preference structure of gains and losses (compared to the overall preference structure), assuming that someone randomly selected a pattern.

As displayed above a few effects that motivate the prospect theory were included in the survey. The next step is to check whether and to what extent these occur. For this purpose, the preference structure between two linked decision situations is considered. In all the effects considered, a pattern with three possibilities emerges. The first one is that participants were consistent according to expected utility in their responses and chose the risk seeking lottery A or risk averse lottey B in both situations. In the second and third possibility they have violated the expected utility theory but only one of the two violations can be explained with the effect.

As explained in the chapter above, the preference pattern in decision situations one and two have to be equal according to the expected utility theory. In both situations the expected value of each lottery wheel is the same but the probabilities and the possible monetary outcomes differ. So that one can say that lottery wheel B is more certain because that someone has a positive monetary outcome is in total more probable despite of the size. This means, when someone decides for lottery wheel A in situation one, then s/he either has to take lottery wheel A in situation two. The mosaic plot in figure 10 displays on the x-axis the first decision situation and separated in the rate of answer A and B. A few decided for the lottery wheel A in the first situation and thus the bulk preferred the more certain option B. In contrast, the majority preferred in situation two the option A which has the higher involved monetary outcome. (The second decision situation is displayed on the y-axis of figure 10.) As a consequence about 70% participants behaved according to the certainty effect which equates to the top right field in figure 10. About a quarter of participants behaved according to the expected utility theory by choosing in both situations either option A or option B (ratio of top left field plus ratio of bottom right field). Consequently almost all participants behaved according to either the expected utility theory or the certainty effect in this case.

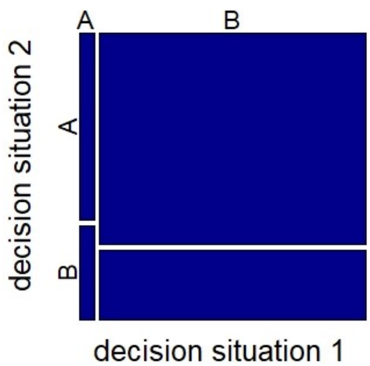

Figure 10: Mosaic plot of the response behavior between decision situation two $l_{2A} = (12000, .001; 3000, .25)$ vs. $l_{2B} = (6000, .002; 3000, .25)$ (on the y-axis) and decision situation one with the lotteries $l_{1A} = (12000, .35; 3000, .25)$ and $l_{1B} = (6000, .7; 3000, .25)$ (on the x-axis). Thereby the top right field equates the proportion of participants who behaved according to the certainty effect.

In comparison to above, the certainty effect can be observed in fewer cases, when considering losses instead of gains (see figure 21 in the appendix). For decision situation nine and ten, the certainty effect makes the smallest share of all the possible decision patterns due to that the participant satisfied the guidance. This means that the share of violation of the expected utility theory which can not be explained with the certainty effect is higher. As a result the certainty effect is not a as common violation of the expected utility theory when considering losses like in the respective gain situation.

Next, there is a comparison of the answering structure between the decision situation four and five in figure 11. On the x axis, the lotteries of decision situation four is displayed, whereby the respective shares are almost the same size. On the y-axis are the lottery wheels of the decision situation five. When comparing the size of the fields, it is noticeable that both possible decision patterns according to the expected utility theory make up the biggest share because the bottom right field is the largest followed by the top left one which both correspond to the preference structure according to the expected utility theory. As a result, in total 30% of the participants did not behave consistent with the expected utility theory, with over 20% deciding according to the isolation effect. The bottom left field in figure 11 equates to the share of participants who behaved according to the isolation effect because a decision maker decides in the fourth decision situation for the risk seeking option A and switches then in a two-stage situation for the risk averse option B.

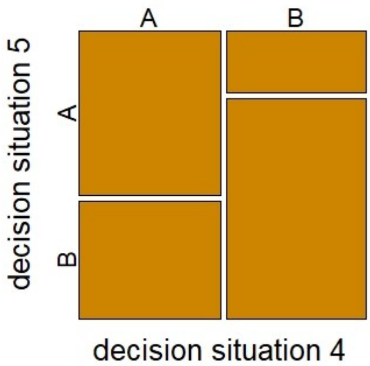

Figure 11: Mosaicplot of preference structure between decision situation four on the x-axis ($l_{4A} = (7000; .3) l_{4B} = (3200, .2; 1500, .25)$) and the two-stage lotteries $l_{5A} = (7000, .5{*}.6)$ and $l_{5B} = (3200, .5{*} .4; 1500, .5{*}.5)$ on the y-axis. The bottom left field corresponds to the share of the sample who decided according to the isolation effect

As it is the case with the certainty effect, the isolation effect is also observable in the losses but again makes the smallest share of all possible decision patterns (see figure 22 in the appendix). So that it is not an ordinary violation of the expected utility theory as compared to the gains.

The last implied violation is the framing effec which can appear between all the gain situations and their mirrored loss situations. In figure 12 on the x-axis are the two lottery wheels of decision situation one displayed, while on the y-axis is the mirrored loss situation which is decision situation nine. The biggest field of the mosaic plot 12 is the top right one. To land in this field, someone decides in the gain situation for the risk averse option B and switches his/her preference to the risk seeking option A in the belonging loss situation. This results in about 60% of the participants who decided according to the framing effect. One-third of the sample decided consistent with the expected utility theory. The other associated mosaic plots can be found in the appendix.
Summarizing the framing effect, mosaicplot 12 shows the case of the framing effect, where it is observable the most. In comparison to the mosaicplot 23 in the appendix

shows the weakest case of the framing effect. In this case the preference structure changes the other way round from risk seeking in the gain situation versus risk averse in the loss situation and in addition, the violation of the expected utility theory has the biggest share here. The participants hold their preference structure most strongly in the mosaic plot 29 in the appendix.

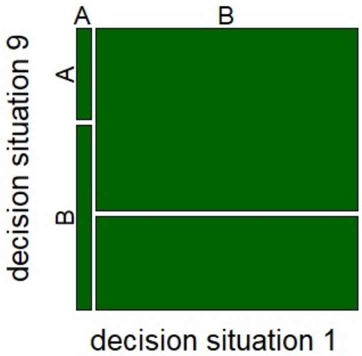

Figure 12: Mosaicplot of the response pattern between decision situation one with the lotteries $l_{1A} = (12000, .35; 3000, .25)$ and $l_{1B} = (6000, .7; 3000, .25)$ (x-axis) as well as situation nine $l_{9A} = (-12000, .35; -3000, .25)$ vs. $l_{9B} = (-6000, .7; -3000, .25)$ (y-axis). The top right field implies the portion of the sample who behaved accordingly the framing effect.

The effects which have been the motivation for evolving the prospect theory can be observed in this survey. However, in some cases they seem to occur more often than in other ones. Though, in each implied violation there exists participants who stayed consistent in their choices and thus their preference structure can be reflected by the expected utility theory. Not only the violations in form of effects are observed and dominate but the other possibility of violating the expected utility theory predominate in single cases.

4 Results

In this following chapter the estimation of the parameters is explained before the estimated parameters are discussed also with regard to the collected characteristics of the participants. Finally, the two theories are compared with respect to their goodness of fit and generalizability.

4.1 Estimation

The expected utility theory has an influence only on the monetary outcomes and thus the utility function. In this thesis it was was assumed that one parameter influences the utility function in terms of behavior regarding risk. This results in the following utility function:

$$u(x) = \begin{cases} x^r & \text{if } x \geq 0\,, \\ -(-x)^r & \text{if } x < 0\ . \end{cases}$$

Thereby, x is a possible lottery payoff and r determines the curvature of the utility function. Thus, when $r < 1$, the utility function is concave (for $x \geq 0$) and implies risk aversion, whereby $r > 0$. In contrast, there is a willingness to take a risk, when $r > 1$ and hence the utility function is convex (for $x \geq 0$). Neutrality regarding risk appears, when the risk parameter is one and hence the utility function is added up to the identity function. (Wakker, 2008) In this thesis, the risk parameter r influences similar the losses, but the form is different so that in case of $x < 0$ the utility form is convex for $r < 1$, but still implies risk aversion. The reason for this is the mirrored monetary outcomes in the loss section.

In the survey, the participants were asked to choose between lottery $l_A = (x_{A,1}, p_1; x_{A,2}, p_2; \ldots; x_{A,k}, p_k)$ and lottery $l_B = (x_{B,1}, p_1; x_{B,2}, p_2; \ldots; x_{B,k}, p_k)$ over a series of g questions. The expected utility for each lottery wheel A with the unknown risk parameter r is calculated accordingly for each decision situation:

$$EU_g^A(l_A) = \sum_{j=1}^{k} p_j * u(x_{A,j}) = \begin{cases} \displaystyle\sum_{j=1}^{k} p_j * x_{A,j}^r & \text{if } x_{A,j} \geq 0\ , \\ \displaystyle\sum_{j=1}^{k} p_j * (-(-x_{A,j})^r) & \text{if } x_{A,j} < 0\ . \end{cases} \tag{11}$$

Thereby $x_{A,j}$ is one of the possible payoffs with the corresponding probability p_j of any lottery wheel A over the g questions.
The calculation for each lottery wheel B is analogous to equation 11, consequently $EU_g^B(l_B) = \sum_{j=1}^{k} p_j * u(x_{B,j})$. (Bocqueho et al., 2013)

In order to decide upon a lottery wheel in one out of the g decision situations, it is at first required to compare the expected utilities of lottery wheel A and B. This leads to Δ which is the difference between the expected utility of lottery A and B in each decision situation g (Bocqueho et al., 2013):

$$\Delta_g^{EU} = |EU_g^A| - |EU_g^B|\,.$$

In order to choose a lottery, the wheel with the higher expected utility is preferred which results in δ_g^{EU} for every decision situation g with the unknown risk parameter r:

$$\delta_g^{EU} = \begin{cases} 1 & \text{if } \Delta_g^{EU} \geq 0 \ , \\ 0 & \text{if } \Delta_g^{EU} = 0 \quad , \\ -1 & \text{if } \Delta_g^{EU} < 0 \ . \end{cases}$$

In contrast to the expected utility theory, the prospect theory has an influence on the monetary outcomes via the value function. Within the following framework the form of the value function $v(x)$ is dependent on two parameters meaning one for positive and one for negative outcomes:

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \ , \\ -(-x)^\beta & \text{if } x < 0 \ , \end{cases}$$

whereby $\alpha, \beta > 0$. Again, x are the possible lottery payoffs. In this case, the value function is separated into a gain and a loss section so that for each section different parameters regarding risk can be evaluated. The curvature of the value function for gains is determined by α and the one for the losses β. Consequently, the value function is concave (convex) when $\alpha < 1$ ($\beta < 1$) and implies risk aversion. In contrast to $\alpha > 1$, ($\beta > 1$) implies risk seeking because of the convex (concave) value function. Again, $\alpha, \beta = 1$ leads to a linear value function and assumes risk neutrality. (Abdellaoui et al., 2008)
In the prospect theory it is assumed, that the value function is steeper for losses than for positive monetary outcomes. In this survey this assumption plays no central role because no mixed prospects (lotteries) existed, in which case this additional parameter would have had an impact on the decision situation. More detailed information can be found in Appendix.

The calculation for the prospect (respectively lottery) A $(x_{A,1}, p_1; \ldots; x_{A,k}, p_k)$ and B $(x_{B,1}, p_1; \ldots; x_{B,k}, p_k)$ in the gth decision situation looks like

$$PT_g^A(l_A) = \sum_{j=1}^{k} w(p_j) * v(x_{A,j}) \ , \ PT_g^B(l_B) = \sum_{j=1}^{k} w(p_j) * v(x_{B,j}) \ , \tag{12}$$

whereas $v(x_{A,j})$ is the value function of the possible monetary outcome $x_{A,j}$ which occurs with the corresponding probability p_j.(Bocqueho et al., 2013) This probability does not have to be regarded as linear so that in equation 12, the weighting probability function $w(\cdot)$ is calculated as:

$$w(p) = \exp(-(-\ln p)^\gamma) \, .$$

The curvature of the probability weighting function is assigned by γ, whereby $\gamma > 0$. When $\gamma < 1$, the probability weighting function has an inverse S-shape. This implies that with decreasing γ, the distinction for different sizes of probability disappears and all probabilities are perceived as being equal. In less extreme cases $\gamma < 1$ leads to overweighting small probabilities and underweighting larger probabilities. In contrast to $\gamma < 1$, when $\gamma > 1$ the probability weighting function follows the s-shape so that in the most extreme case, probabilities are only recognized as 0 or 1. When $\gamma = 1$, the probability function is simply $w(p) = p$. (Prelec, 1998)

Also when applying prospect theory, it is necessary to compare the size of the expected utilities of the prospects with each other, so that index Δ_g corresponds to the difference between the expected utility of the prospect A and B in each decision situation g (Bocqueho et al., 2013):

$$\Delta_g^{PT} = |PT_g^A| - |PT_g^B| \, .$$

In order to choose a lottery, the wheel with the higher expected utility is preferred which results in δ_g^{PT} for every decision situation g with the unknown set of parameters (α, β, γ):

$$\delta_g^{PT} = \begin{cases} 1 & \text{if } \Delta_g^{PT} > 0 \ , \\ 0 & \text{if } \Delta_g^{PT} = 0 \ , \\ -1 & \text{if } \Delta_g^{PT} < 0 \ . \end{cases}$$

When it comes to estimating th atitude towards risk, there are two approaches. The first one aims to find the ideal parameters of the expected utility theory as well the prospect theory for every single participant. It is searchead for the (set of) parameters which minimizes the sum of the quadratic difference between the chosen lottery and the expected lottery over all the sixteen decision situations for each participant i. Consequently, the estimation looks for the expected utility theory like:

$$\sum_{g=1}^{16} (y_{ig} - \delta_g^{EU})^2 \rightarrow \text{ Min} \tag{13}$$

Thereby y_{ig} is the observation of participant i in decision situation g. According to the above $y_{ig} = 1$ when the observed preferred lottery is the riskier choice A. If lottery wheel B was chosen and thus $y_{ig} = -1$. When the participant was indifferent between the two options$y_{ig} = 0$, . For the prospect theory δ_g^{EU} changes to δ_g^{PT}.
This means that there is estimated a (set of) parameter(s) which has the best possible goodness of fit on the answered individual preference structure overall sixteen decision situations for each single participant according to the expected utility theory or the prospect theory.
Afterwards, a statement based on the median participant is made so that in terms of the expected utility theory $\hat{r}_{median}$ results as follows (Fahrmeir et al., 2016, chapter 2.2.1):

$$\hat{r}_{median} = \begin{cases} \hat{r}_{\frac{n+1}{2}} & \text{if n uneven,} \\ \frac{1}{2}(\hat{r}_{\frac{n}{2}} + \hat{r}_{\frac{n}{2}+1}) & \text{if n even,} \end{cases}$$

whereby n is the number of observations. The median is calculated analogous for the prospect theory and the associated set of parameters $(\hat{\alpha}_{median}, \hat{\beta}_{median}, \hat{\gamma}_{median})$.

As a second approach, a global attempt is used. So that the (set of) parameter(s) that minimizes the quadratic difference for every individual participant is not chosen but instead the quadratic difference is minimized on the whole sample. In the case of the expected utility theory $\hat{r}$ equates the parameter which minimizes:

$$\sum_{i=1}^{n} \sum_{g=1}^{16} (y_{ig} - \delta_g^{EU})^2 \rightarrow \text{ Min} \tag{14}$$

Once again y_{ig} is the observation of participant i in decision situation g so that $y_{ig} = 1$ applies when the observed preferred lottery is the riskier choice A and in contrast to $y_{ig} = -1$ when lottery B was chosen. Also when a participant was indifferent between the two lotteries $y_{ig} = 0$. Analogous δ_g^{EU} changes to δ_g^{PT} for the prospect theory.

Because the preference structure over the sixteen decision wheels does not change anymore, when $r, \alpha, \beta > 1$, it was assumed that $\alpha, \beta, r \in (0, 1.5)$.
So in order to find the risk parameter which fits the best with the individual/global preference structure, the easiest way was calculating the quadratic difference for every $r \in \{0.1, 0.4, 0.5, 0.6, 0.7, 1, 1.1\}$ and in the following choosing the $\hat{r_i}$ or $\hat{r}$, that minimizes the quadratic difference. Such loose grid for estimating $\hat{r_i}/\hat{r}$ was chosen because for example every $r \in (0, .34]$ estimates an equal preference structure over all 16 decision situations so that a finer grid is simply not required with this structure of decisions situations and this parameter cannot even be estimated more precisely.

The minimizing of the quadratic difference under the prospect theory was a bit more complicated than for the expected utility theory. Here a grid of 0.1 steps was not sufficient and a finer grid would be very computationally expensive so that the Simulated Annealing Algorithms was used to optimize the quadratic difference regarding the individual and global approach. One reason for the use of the Simulated Annealing Algorithms is that this algorithm has the capacity to minimize on quite a rough surface which was the case here. One additional advantage is that this algorithm uses exclusive function values. (Datacamp, n.d.)

4.2 Estimated parameters

Before estimating the parameter of both models, the participants are grouped by their behavior regarding risk. When someone preferred lottery A at least six times (out of eight), s/he is assigned as risk seeking. The other way round, when s/he answered lottery B at least six times, s/he is assigned as risk averse. When neither of the two cases fit, then the person is classified as mixed.

		Losses			
		Risk averse	Mixed	Risk seeking	Total
	Risk averse	10	33	12	55
Gains	Mixed	12	31	12	55
	Risk seeking	5	3	1	9
	Total	27	67	25	

Table 8: Classification of participants in term of their risk attitude

It stands out that in the gain situations, the bulk of participants is classified as behaving risk averse or mixed. Nine participants are defined as behaving risk seeking.

In contrast to the gain lotteries, the majority of participants are allocated as neither behaving risk averse nor risk seeking in a loss situation. The quantity of participants behaving risk averse and participants answering risk seeking is nearly equal. In total, 42 participants are classified consistent in the gain and loss section as risk averse, mixed or risk seeking. The remaining 65% are classified various behavior patterns in the risk and loss section.

4.2.1 Expected utility theory

As mentioned above the parameter of the individual behavior regarding risk is estimated through iteration so that $\hat{r}_i$ and in conclusion $\hat{r}_{median}$ can only take values from the interval $\{0.1, 0.4, 0.5, 0.6, 0.7, 1, 1.1\}$. This has to be kept in mind, when regarding the figure 13, where on the x axis one finds the possible values of $\hat{r}_i$ and on the y-axis the accordingly number of participants, whose specified preference structure best matches with the $\hat{r}_i$. The parameter which fits the most participants is $\hat{r}_i = 0.1$ so that this value is estimated for over 60% of the sample and is consequently also the median, consequently $\hat{r}_{median} = 0.1$. This value calculates a preference structure which advises in all 16 decision situations the risk averse lottery B. The value which has the second most frequent best fit to the preference structure of the participants is $\hat{r}_i = 1$. This implies that the participants with this estimated parameter rather behaves risk neutral in front of their preference structure. In total, about 80% of the sample have an estimated risk parameter, which implies risk aversion because $\hat{r}_i < 1$. In contrast, one participant was estimated to be risk seeking.

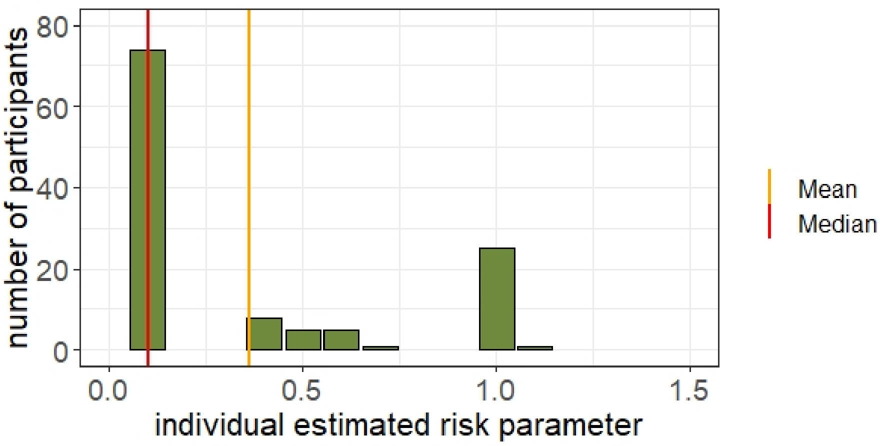

Figure 13: Barplot of the counts of the individual estimated risk parameter $\hat{r}_i$. Also drawn in is $\hat{r}_{median}$ (red line) and the average of $\hat{r}_i$

As already mentioned above and displayed in figure 14, $\hat{r}_{median} = 0.1$. This means that the typical behavior regarding risk in the sample was very risk averse in terms of the expected utility theory. The utility function which is displayed in figure 14 is calculated with $r = 0.1$. This equates not only $\hat{r}_{median}$ but also the global estimated parameter $\hat{r}$. As it can be seen the utility function is convex in the gain and concave in the loss section. The strength of the risk aversion is reflected on the y-axis which has a small range.

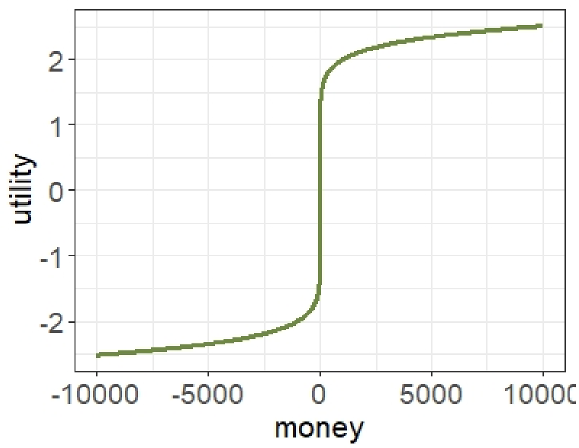

Figure 14: The utility function based on median of the individual estimated risk parameter r which equates also equates the global estimated parameter $\hat{r}$ of the whole sample

As next, it is discussed, if the median risk parameter differs for various groups of participants in order to make a statement about the typical behavior regarding the different categories.
First, the sample is grouped by gender and then the asocciated risk parameter $\hat{r}_{median}$ is calculated whose results are shown in table 9. When the median risk parameter is considered, there is no identifiable difference regarding the typical risk attitude between woman and man in the sample. Only when the mean value is considered, it becomes apparent that on average the female participants answered slightly more risk averse than the male participants.

	female	male
$\hat{r}_{median}$	0.1	0.1
$\frac{1}{n}\sum_{i=1}^{n}\hat{r}_i$	0.301	0.439

Table 9: Risk parameter $\hat{r}_{median}$ and average $\hat{r}_i$ separated after gender

In the next distinction after the employment in table 10, there is also no difference regarding the typical behavior regarding risk observable. Only the category others differs, but because this group contains of one observation, it is excluded of the following comparison. However, differences between the groups only become visible again when the average individual risk parameter is considered. The mean risk parameter in the trainee and employee category is identical so that on average both categories behave more risk seeking than the freelancers because of the higher mean individual risk parameter $\hat{r}_i$. The other way around the freelancers answered on average more risk averse than the rest of the sample.

	trainees	employees	freelancers
$\hat{r}_{median}$	0.1	0.1	0.1
$\frac{1}{n}\sum_{i=1}^{n}\hat{r}_i$	0.360	0.360	0.240

Table 10: Risk parameter $\hat{r}_{median}$ and average $\hat{r}_i$ separated after employment

The last captured characteristic was the age. For the evaluation regarding the risk behavior, the participants were divided into age groups, as shown in table 11. When considering the median of the individual risk parameter separated into groups, again there are not many differences. For usual all groups are estimated with a risk averse behavior, except for the 30- to 39 year-olds and the over 60 year-olds who have a risk parameter which implies that they are typically more risk seeking in comparison to the other age groups. In order to perceive also more sophisticated differences between the four other categories of age, a look is thrown onto the mean of the individual risk parameter. Both groups of the 60- to 70 year- olds and the 30- to 39 year-olds remain the most seeking. In contrast, the under 20 years-olds are considered the most risk averse on average followed by the 50 to 59 years-old.

	> 20	20-29	30-39	40-49	50-59	$60-69$
$\hat{r}_{median}$	0.1	0.1	0.6	0.1	0.1	0.6
$\frac{1}{n}\sum_{i=1}^{n}\hat{r}_i$	0.220	0.379	0.567	0.300	0.286	0.560

Table 11: Risk parameter $\hat{r}_{median}$ and average $\hat{r}_i$ separated after age

4.2.2 Prospect theory

For the estimation of the prospect theory three parameters are required. The two parameters which influence the value function are β and α. As already explained above in more detail, α as well as β are restricted so that $\alpha, \beta \in (0, 1.5)$. This was necessary to guarantee that individual as well as global the best possible set of parameters is estimated.

The distribution of the individual risk aversion parameter in the gain section $\hat{\alpha}_i$ results in figure 15. This distribution is right skewed. The individual estimated parameter $\hat{\alpha}_i$ is less than one for nearly all observations. What is also recognizable in figure 15 is that the majority of the participants have an individual estimated parameter of less than 0.5. This implies that a lot of participants answered the first section of the survey risk averse. This observation reflects in $\hat{\alpha}_{median}$ which is estimated to be 0.283 so that the typical behavior in the gain lotteries is risk aversion for the whole sample.

In contrast to the individual risk parameter regarding positive results, the distribution of the individual risk behavior parameter of the losses in figure 16 is bivariate. More participants have a preference structure which seems to be risk seeking because there are estimated more $\hat{\beta}_i > 1$. However, still over half of the sample is estimated to behave risk averse. Because the distribution of the individual estimated $\hat{\beta}_i$ in figure 16 is bivariate the highest density of parameters are at the respective boundary points of the selected interval of β.
Summarizing, the participants seem to be more diverse in the loss section regarding their own preference structure and also their behavior regarding risk seems more to differ than in the gain section. This has already been observed in chapter 3.2. In total, the participants have answered the loss section less risk averse than the gain section which is mirrored in $\hat{\beta}_{median} = 0.506$ because it is higher to the compared typical gain risk parameter $\hat{\alpha}_{median}$.

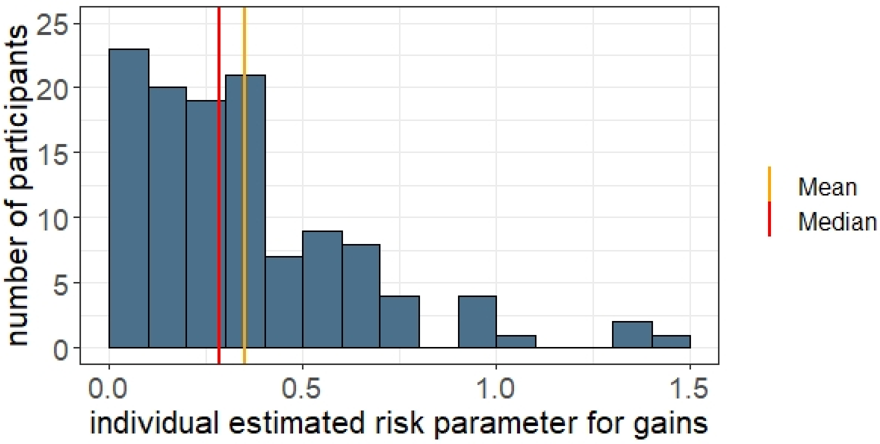

Figure 15: Distribution of the individual risk parameter of the value function in the gain section. Thereby $\hat{\alpha}_{median} = 0.283$ (red line) and the average of $\hat{\alpha}$ is 0.349 (orange line)

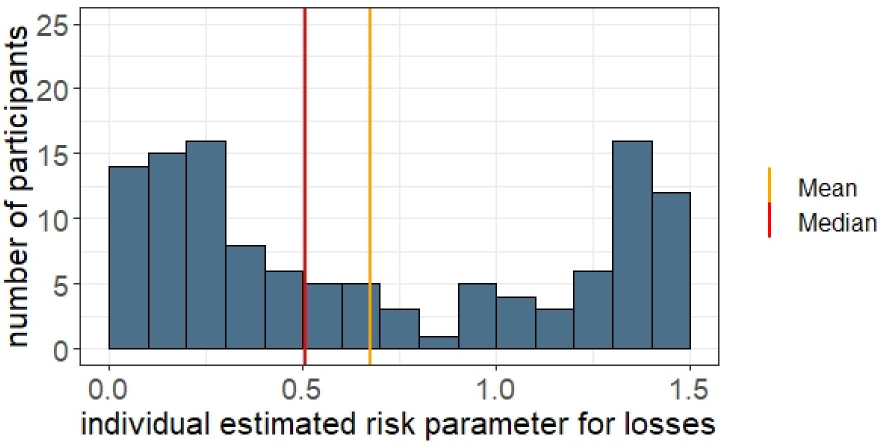

Figure 16: Distribution of the individual risk parameter of the value function in the loss section. Thereby $\hat{\alpha}_{median} = 0.506$ (red line) and the average of $\hat{\beta}$ is 0.676 (orange line)

Next, the value function of the median estimated parameters is displayed in figure 17 on the left side. In order to calcualte the value function, it have been used for positive monetary section $\hat{\alpha}_{median} = 0.283$ and for negative monetary section $\hat{\beta}_{median} = 0.506$. Thus, the participants tend to typically answer risk averse despite of the positive or negative monetary outcome. However, they typically decided in the gain section more risk averse than the lottery wheels containing losses. This is reflected in figure 17 through the steeper form of the negative monetary outcomes compared to the gains.
In contrast to the expected utility theory, especially the median risk parameter regarding the negative monetary outcomes has been estimated significantly higher. In addition, the median risk parameters regarding risk and losses differ clearly. In comparison, the value function of the global parameters of the whole sample is in figure on 17 is on the right side. These are quite interesting because the set of parameters which describes all participant at same time best, estimates a very risk averse parameter for the gains with $\hat{\alpha} = 0.085$ and a risk seeking parameter for the losses $\hat{\beta} = 1.332$ so that the global parameter for the gains agrees with the implied behavior of global estimation of the expected utility theory for the over all monetary results.

The last estimated parameter γ has an impact on the probability weighting function.

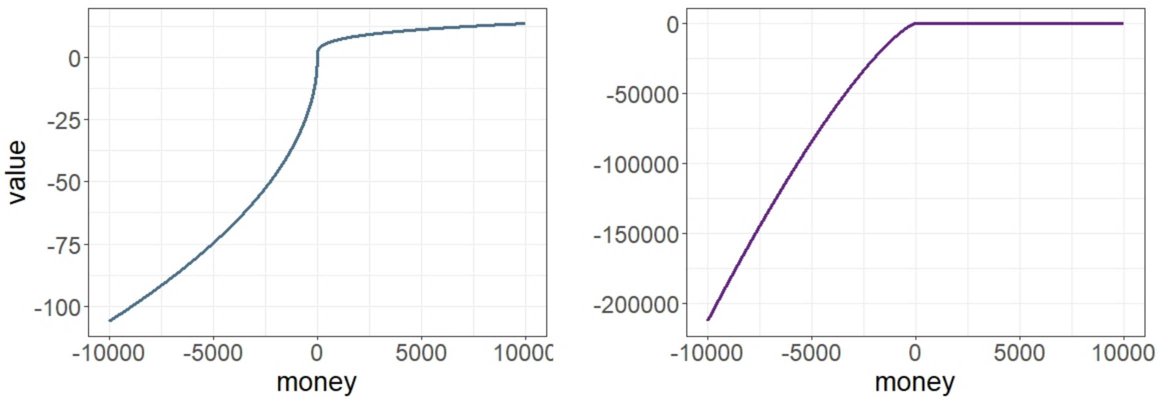

Figure 17: The value function based on the median data $(\hat{\alpha}_{median}, \hat{\beta}_{median}) = (.283, .506)$ on the left side. The value function of the global estimated risk parameters $(\hat{\alpha}, \hat{\beta}) = (.085, 1.332)$ is displayed on the right side

When $\gamma < 1$, it implies the certainty effect (see chapter 2.2.3) which indicates that small probabilities are overweighted and big probabilities are underweighted. The probability weighting function is then inverse S shaped. When having a look at the distribution of the individual estimated parameter for each participant at figure 18, first of all it is striking that the estimated parameter has a range from zero to 16. Thereby, the majority of observations have an estimated $\hat{\gamma}_i \in (0, 2)$. Only a few observations are outliers with $\hat{\gamma}_i > 2$. It can be observed that more than 40% of the sample have an estimated $\gamma < 1$ which implies the certainty effect. In short, this also implies that about 60% of the participants have a $\hat{\gamma}_i$ bigger than one which does not imply the certainty effect. This implied behavior which is not conform with the certainty effect for more than half of the sample is reflected in the $\hat{\gamma}_{median} = 1.360$ as well as the average of $\hat{\gamma}_i$ with 2.027.

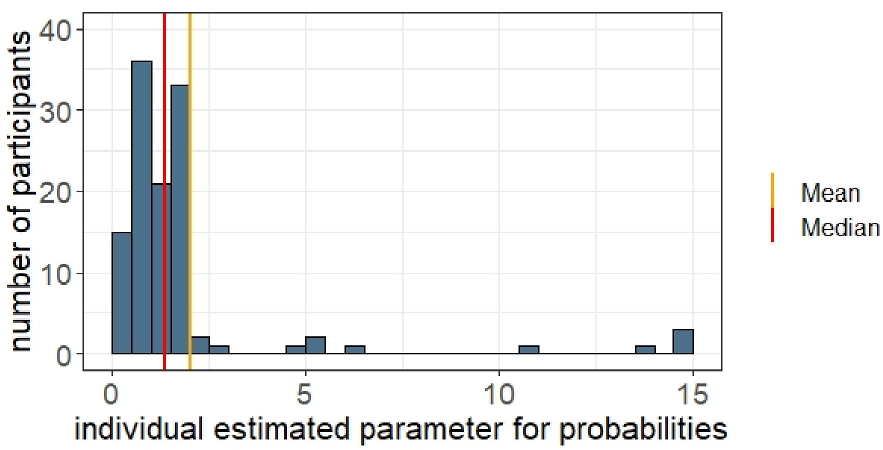

Figure 18: Distribution of the parameter of the probability weighting function. Thereby, the median of γ has a value of 1.360 and the mean is 2.027

As already mentioned, more than the half of the participants have do not a parameter value $\hat{\gamma}_i$ which implies the certainty effect so that this is reflected in the median of this estimated parameter. In figure 19, the median of this parameter is displayed as calculated probability weighting function in blue. By comparing it to the identity function in grey, it becomes clear that the probability weighting function is not linear, although the $\hat{\gamma}_{median}$ is not very far from one. The global estimator $\hat{\gamma}$ is even bigger with 1.743 so that the probability weighting function is more curved. In general, $\hat{\gamma}_i$ and $\hat{\gamma}$ seems to contain information, which cannot be displayed through the respectively risk parameter of gains and losses.

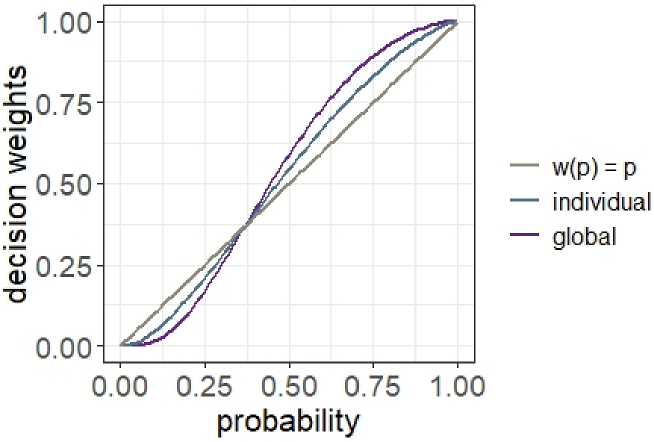

Figure 19: The probability weighting function based on the median data ($\hat{\gamma} = 1.360$) in blue and on the global estimator ($\hat{\gamma} = 1.743$) in purple. The identity function $w(p) = p$ is displayed in grey

Next, the median of each estimated parameter grouped by the three collected characteristics is compared .
First of all, the parameters grouped by age are presented in table 12. The parameter regarding typical risk behavior $\hat{\alpha}_{median}$ and $\hat{\beta}_{median}$ grouped by gender is less than one for both categories, but smaller for woman than for man which implies that the female participants typically answered the survey more risk averse than the male participants in gain and loss lotteries. However, $\hat{\gamma}_{median}$ is greater than one for both groups so that usually small probabilities tend to be underestimated and in contrast to big probabilities over estimated. Because $\hat{\gamma}_{median}$ of woman is bigger than for man, this effect is normally slightly more noticeable for women. Because the $\hat{\gamma}_{median}$ of male participants is close to one, the probability weighting function is usually nearly linear.

	female	male
$\hat{\alpha}_{median}$	0.228	0.375
$\hat{\beta}_{median}$	0.483	0.506
$\hat{\gamma}_{median}$	1.387	1.178

Table 12: $(\hat{\alpha}_{median}, \hat{\beta}_{median}, \hat{\gamma}_{median})$ separated after gender

Again the employment category others is excluded of the following evaluation in table 13 because of the single observation. In the gain section all three remaining employment categories are estimated as typically behaving risk averse despite positive or negative monetary results. The most usually risk averse in gain situations are the trainees, closely followed by the employees and lastly with a bigger difference the freelancers. Also for the negative monetary outcomes, $\hat{\beta}_{median}$ implies risk aversion for all three work categories. Compared to the risk behavior towards gains, the structure is the other way round here, the freelancers have the lowest $\hat{\beta}_{median}$ followed by the employees and finally the trainees. The parameter $\hat{\gamma}_{median}$ is bigger than 1 for all three employment categories. Thereby, the parameter of $\hat{\gamma}_{median}$ of the freelancers is the smallest of all three groups and close to 1 so that their probability weighting function is usually very conform to the identity of the probabilities. The $\hat{\gamma}_{median}$ of the employees and freelancers implies that they normally tend to underweight small probabilities and overweight big probabilities. This effect is estimated to be normally bigger for the trainees than for the employees.

	trainees	employees	freelancers
$\hat{\alpha}_{median}$	0.242	0.284	0.509
$\hat{\beta}_{median}$	0.623	0.408	0.381
$\hat{\gamma}_{median}$	1.405	1.360	1.018

Table 13: $(\hat{\alpha}_{median}, \hat{\beta}_{median}, \hat{\gamma}_{median})$ grouped after employment

When considering the usual risk parameter for gains ($\hat{\alpha}_{median}$) in table 14, it is striking that with increasing age, the participants in the survey are normally less risk averse (excluded the 30- to 39-year-olds who are in total only three male observations in this survey). This structure of behaving less risk averse with growing age is also observable in the loss section from the age of 40. The youngest participants who are under 20-years-old are normally the most risk averse in the loss section of all age categories (except the 30-to 39-year-olds). What stands out is that the 20-to 29-year-olds are -after the over 60-years-old- the most typical risk seeking of all age categories in terms of negative results. The parameter of the probability weighting function indicates for the age up to 49 years that these participants typically follow the pattern of overweighting small probabilities and underweighting big probabilities. This effect is the opposite for participants over 50 years. In the last two age categories the certainty effect is usually observable because $\gamma < 1$ so that small probabilities are overweight and big probabilities underweight. It must be admitted that the probability weighting function for the 50- to 59-year-olds is close to one and thus nearly linear so that a tendency exists in this group for the certainty effect, but not as strong compared to the over 60- year-olds.

	> 20	20-29	30-39	40-49	50-59	60-69
$\hat{\alpha}_{median}$	0.222	0.240	0.937	0.321	0.352	0.4151
$\hat{\beta}_{median}$	0.268	0.633	0.177	0.308	0.331	0.705
$\hat{\gamma}_{median}$	1.117	1.400	1.509	1.531	0.962	0.731

Table 14: $(\hat{\alpha}_{median}, \hat{\beta}_{median}, \hat{\gamma}_{median})$ separated after age

What must be added at the end from the end of the evaluation of the prospect theory is, that the parameters $(\hat{\alpha}_{median}, \hat{\beta}_{median}, \hat{\gamma}_{median})$ for the typical behavior regarding risk and probabilities do not always fit with the observations which motivated this theory. It matches with the observations that the value function is typically risk averse for positive monetary outcomes. However, the fact that people are usually risk seeking in the loss section is only true in the sense that they are more risk seeking compared to the gain section. But the estimated $\hat{\beta}_{median}$ does not correspond to a usual risk seeking form in contrast to the global estimator $\hat{\beta}$, which forms the value function slightly concave for negative monetary outcomes. Also the parameter of the probability weighting function $\hat{\gamma}_{median}$ speaks against the motivating observations because its form does not imply the certainty effect.

4.3 Fit of theories

Finally, there is a comparison of the fit of both theories on the data. First of all, the individual goodness of fit is considered for both theories. Lastly, the quality of the generalization of both models is collated.

Starting with the comparison of the individual fit of both theories on each participant. Therefore, the distribution of the minimum of the estimated (set of) parameters of each participant is compared. In the case of the expected utility theory, these values result in the sum of quadratic differences between the preference structure of every single participant over the sixteen decision situations and the preference structure of his/her estimated parameter $\hat{r}_i$ which provides the best possible individual fit. When the individual and the estimated preference structure are matching over the total sixteen decision situations, then this value is zero and grows with every case where there is no match by four (or two in the case of indifferent). Due to the quantity of 16 decision situations, this criterion can take a maximum value of 64. This is analogous for the model of the prospect theory, but instead of the individual risk parameter $\hat{r}_i$, the individual estimated set of parameters $(\hat{\alpha}_i, \hat{\beta}_i, \hat{\gamma}_i)$ is used.
In figure 20 the distribution of the individual differences is displayed, on the left side for the expected utility theory and on the right side for the prospect theory. Starting with the expected utility theory (left boxplot), where these values of differences range from eight to 28. This means that the estimated parameter can display the individual preference structure up to fourteen places in the best case. Consequently, two decision situations are reflected wrongly. Accordingly, in the worst case seven preference spots are recorded incorrectly. The median lies by 20 so that for one half of the sample at least eleven decision situations are given back correctly. In contrast, the other half of the sample has five up to seven decision situations given back wrong.
The boxplot on the right side of figure 20 displays the distribution of the values of differences for the prospect theory. Here the range is from zero to 21 so that some observations exist, where the prospect theory can reflect the preference structure over the sixteen decision situations perfectly and in the worst cases, the preference structure is describedcorrectly in ten out of sixteen decision situations. The median of the right boxplot in figure 20 lies by eight so that for one half of the sample their preference structure can be expressed in at least fourteen decision situations correctly, when using the model of the prospect theory. One can say that the prospect theory can reproduce the individual preference structure over the sixteen decision situations better in comparison to the expected utility theory because the prospect theory produces the preference structure in the worst case, as good as the expected utility theory for one half of the sample.

For the comparison of the quality of the generalization of both models, the sample is randomly divided into training and test data. Subsequent the two models are esti-

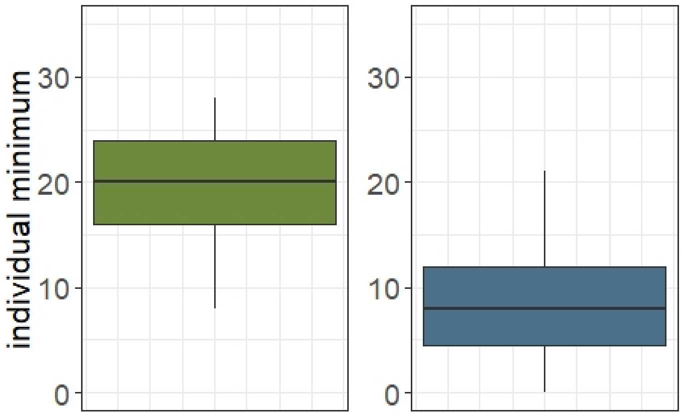

Figure 20: Boxplot of the individual global minimum of expected utility theory (left) and prospect theory (right)

mated on 80% of the sample and the goodness of fit is calculated on the remaining 20% of the data. For estimating the unknown (set) of parameter(s) the global approach in equation 14 is used.

Afterwards the goodness of fit is checked with quadratic difference of equation 13, but summed over all observations and then scaled afterwards, so that it looks like:

$$\frac{1}{n}(\sum_{i=1}^{n}(\sum_{g=1}^{16}(y_{ig} - \delta_g)^2)\,. \tag{15}$$

For the expected utility theory is thus used $\hat{r}$. This is analogous for the parameters $(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$ of the prospect theory. This criterion works in a way that a smaller value also means a better average fit of the model with the global estimated parameters on the test data which implies a better generalization of the theory in comparison to the other one.

When comparing the fit of the test data to the estimated preference structure of each theory in table 15, it is striking that the prospect theory has a smaller value than the expected utility theory so that the global estimated preference structure of the prospect theory matches better on the observations of the test data. This difference is not big, but existing.

Expected utility theory	Prospect theory
26.917	23.583

Table 15: Values of equation 15 which implies the goodness of fit of both theories on the test data

If one now keeps in mind that the prospect theory has three times as many parameters as the expected utility theory in the context of this thesis and is therefore much more complicated to estimate, one could well argue to prefer the expected utility. Although, table 15 implies that in this framework of decision situations happened no overfitting - but how would the value in table 15 change when the model would be tested on a completely different setting of decision situations.

In summary on can say: The mapping of individual preference structures works better with prospect theory than it does with the expected utility theory. The prospect theory can represent the decisions of individuals more accurately and even manages to perfectly reflect the preference structure for a few participants. Also in the case of ability of generalization the prospect theory has a better fit than the expected utility theory. Nevertheless the expected utility theory represents the global preference structure slightly worse but nevertheless similarly well which is why the expected utility theory is still attractive due to the fewer parameters required and thus less in danger of overfitting. If one still keeps in mind that the survey was designed in such a way that violations of the expected utility theory can take place, one can definitely argue to favor the expected utility theory especially if the decision situations are simple and do not contain implied violations. Finally, it can be said which theory to use depends on the purpose one wants to achieve. If the individual risk behavior wants to be observed then the prospect theory is better. In contrast, if a model is required which advises people on making a decision the expected utility theory might be equally as good, if not better because of its simplicity.

5 Conclusion

Both the expected utilty theory and the prospect theory have their advantages and disadvantages when it comes to making decisions under risk.
The prospect theory is a more complex model that considers several known effects in decision-making and thus requires more parameters, which can result in overfitting and poor generalization. On the other hand, the expected utility theory is a simpler model that requires fewer parameters, making it normally more attractive for generalizations. Nonetheless, it was observed within the framework of this work that the prospect theory is able to present the decision-making behavior of individuals much more accurately than the expected utility theory. Concrete it enables a better mapping of individual preference structures and can perfectly reflect the preference structure of a few participants. Also regarding to generalization abilities the prospect theory outperforms the expected utility theory.
In considering which theory to use, it depends on the purpose. If the goal is to observe and to reflect individual risk behavior and maybe to draw conclusions about them, the prospect theory is the more suitable choice. In contrast, if a model is required to advise people in making decisions, the expected utility theory might had achieved a slightly poorer generalization score compared to the prospect theory but through consideration and argumentation it can still be the tool of choice especially when the decision situations are rather simple and do not contain implied violation as it was the case here.

One last interesting point which can be discussed further in the future is how to involve also observations with incomplete preference structures in both theories. Even if one tries to avoid an incomplete preference structure, it cannot be prevented and will occur.
This survey had decision situations that do not necessarily occur on a daily basis especially with the lotteries consiting only losses, so it would be interesting to find more close to reality situations, where both theories can be adapted and compared how they perform in a more difficult setting.

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A Further information

A.1 Survey form

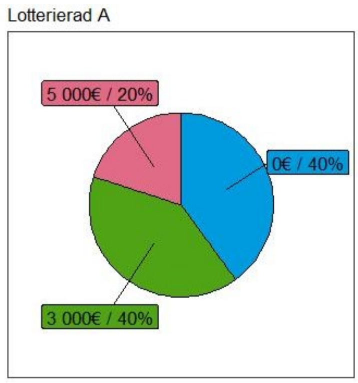
Umfrage zur Entscheidungstheorie

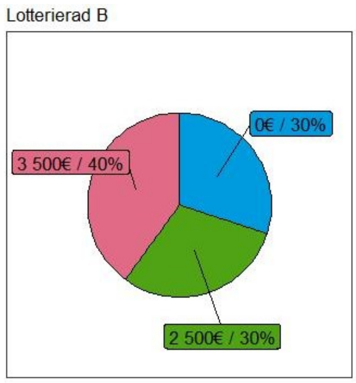
Vielen Dank, dass Sie an der Umfrage für meine Bachelorarbeit zum Thema Entscheidungstheorie teilnehmen.

Hier eine kurze Anleitung: Im Folgenden werden immer zwei Lotterieräder gezeigt. Stellen Sie sich vor, Sie müssen entscheiden, an welcher Lotterie Sie lieber teilnehmen möchten, indem Sie das jeweilige Kästchen anklicken. Es gibt dabei weder eine richtige noch eine falsche Antwort. Es geht ausschließlich um Ihre persönliche Vorliebe beim Treffen von Entscheidungen. Es gibt kein echtes Geld zu gewinnen. Damit Ihr Fragebogen ausgewertet werden kann, ist es zwingend notwendig, dass Sie bei allen Fragen eine Entscheidung treffen. Noch ein kleiner Tipp: Falls sie den Fragebogen mit Ihren Handy beantworten, sieht man die Lotterieräder besser, wenn man den Bildschirm dreht, so dass dieser quer ist.

Beispiel

Im Folgenden ein kurzes Beispiel. Sie sehen zwei Lotterieräder: Beim Lotterierad A (links) haben Sie die Möglichkeit, 5000€ mit einer Wahrscheinlichkeit von 20% zu gewinnen, 3000€ mit einer Wahrscheinlichkeit von 40% oder keinen Gewinn zu erzielen mit einer Wahrscheinlichkeit von 40% . Beim Lotterierad B (rechts) haben Sie die Möglichkeit 3500€ mit einer Wahrscheinlichkeit von 40% zu gewinnen, 2500€ mit einer Wahrscheinlichkeit von 30% und keinen Gewinn zu erzielen mit einer Wahrscheinlichkeit von 30%. Wählen Sie nun das Lotterierad aus, dass Sie lieber wollen:





○ Lotterierad A
○ Lotterierad B

Datenschutzerklärung

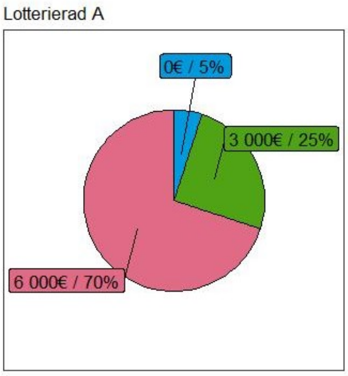
Einwilligungserklärung gemäß Datenschutz für eine Umfrage zum Thema Entscheidungstheorie

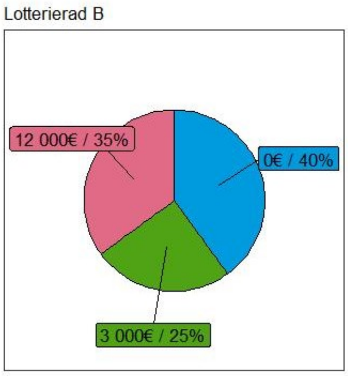
Auf folgenden Seiten werden ein paar Fragen zum Thema Entscheidungstheorie gestellt. Ziel der Umfrage ist es, den Nutzen von unterschiedlichen Entscheidungstheorien zu bewerten.Im Abschluss der Umfrage werden zudem nähere Informationen zu Ihrer Person (Alter, Geschlecht, etc.) abgefragt, um auch soziale Faktoren in die Auswertung mit einbeziehen zu können und somit die Bewertung zu verbessern.
Die Teilnahme an dieser Umfrage ist ohne die Nennung ihres Namens, sowie ohne eine Registrierung möglich.

○ Ich bin einverstanden

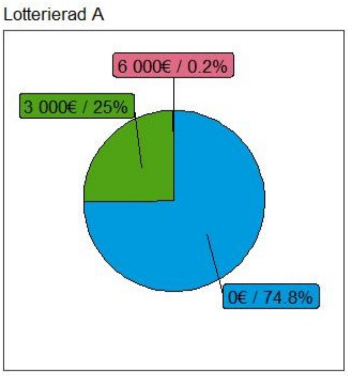
Gewinnlotterien

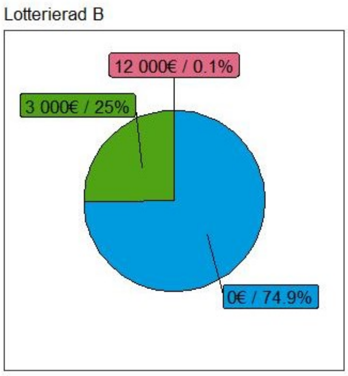
Im Folgenden haben Sie immer die Wahl zwischen zwei Lotterierädern, bei denen Sie potenziell einen Gewinn erzielen können. Die Chance, Geld zu verlieren, ist ausgeschlossen.



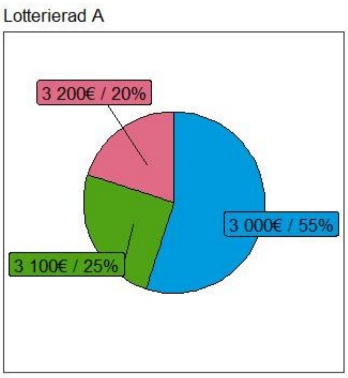


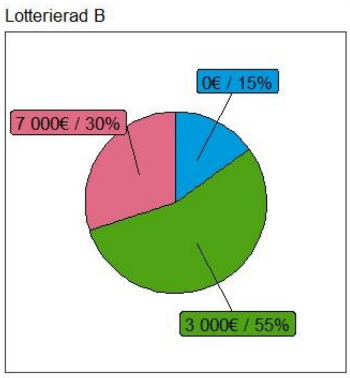
○ Lotterierad A
○ Lotterierad B



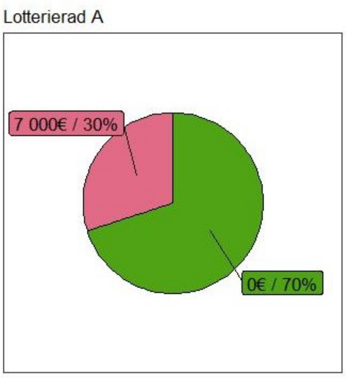


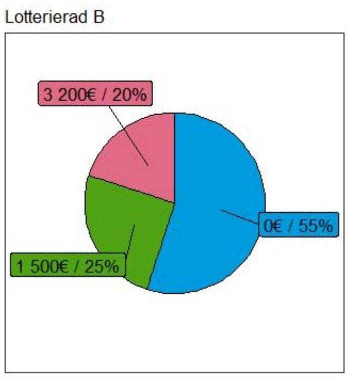
○ Lotterierad A
○ Lotterierad B





○ Lotterierad A
○ Lotterierad B

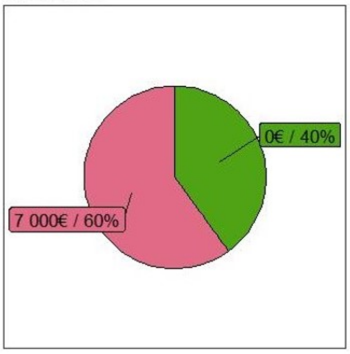




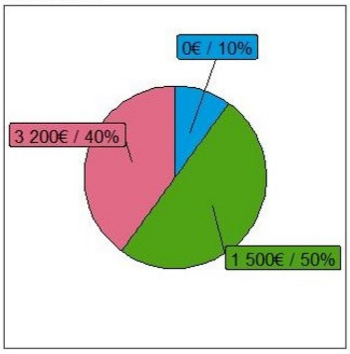
○ Lotterierad A
○ Lotterierad B

Bei folgender Lotterie wird vorab eine faire Münze geworfen, die über die Teilnahme an der Lotterie entscheidet. Wenn Kopf oben liegt, dann dürfen Sie ein Lotterierad auswählen. Wenn Zahl oben liegt, dann dürfen Sie nicht teilnehmen und haben somit auch keine Chance Geld zu gewinnen.
Da Kopf bei Ihnen oben liegt, haben Sie nun die Wahl zwischen den zwei folgenden Lotterieräder:

Lotterierad A

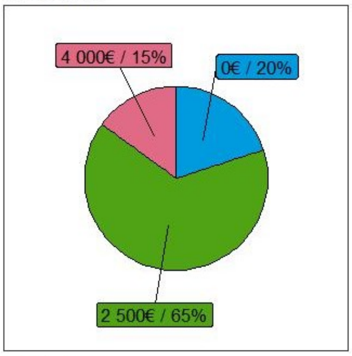


Lotterierad B

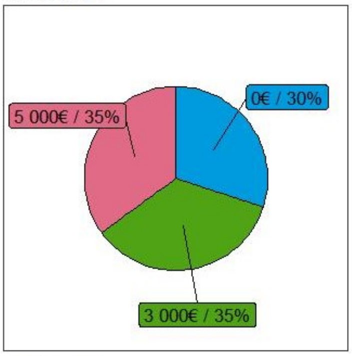


○ Lotterierad A
○ Lotterierad B

Lotterierad A

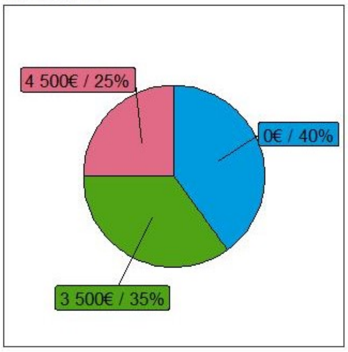


Lotterierad B

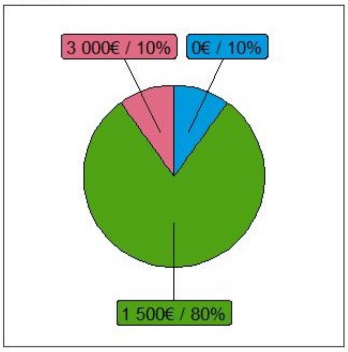


○ Lotterierad A
○ Lotterierad B

Lotterierad A

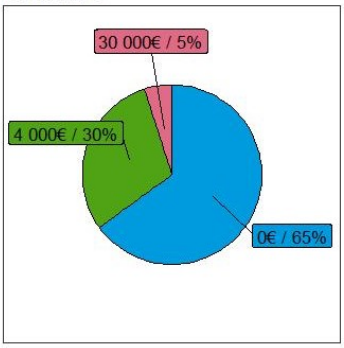


Lotterierad B

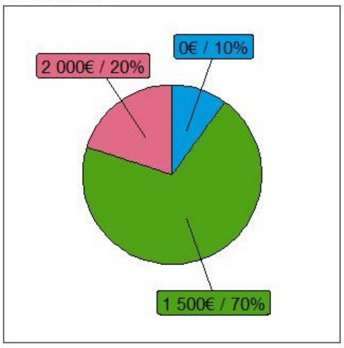


○ Lotterierad A
○ Lotterierad B

Lotterierad A



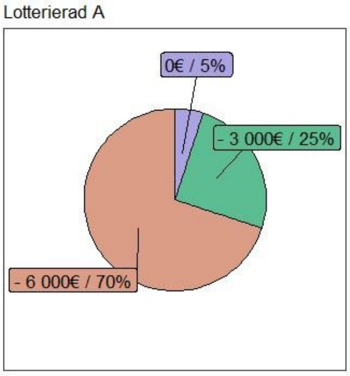
Lotterierad B

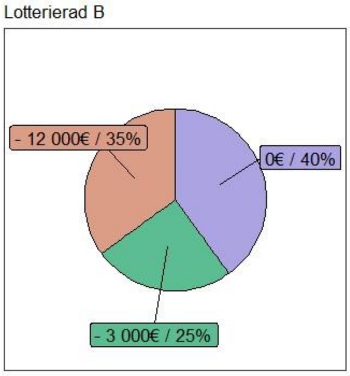


○ Lotterierad A
○ Lotterierad B

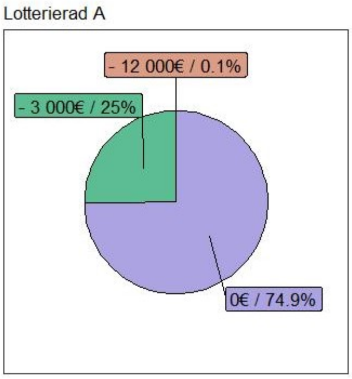
Verlustlotterien

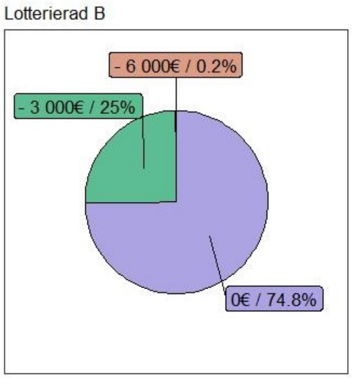
Im Folgenden haben Sie immer die Wahl zwischen zwei Lotterierädern, bei denen Sie potenziell Geld verlieren können. Die Chance, Geld zu gewinnen, ist ausgeschlossen.



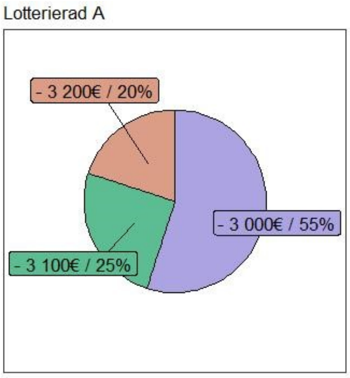


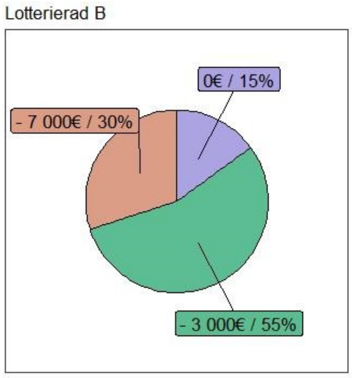
○ Lotterierad A
○ Lotterierad B



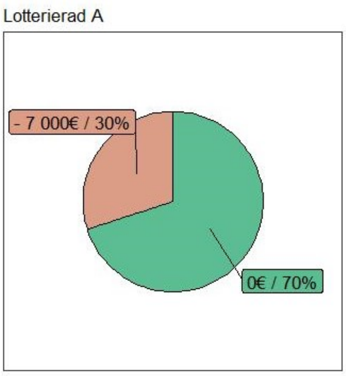


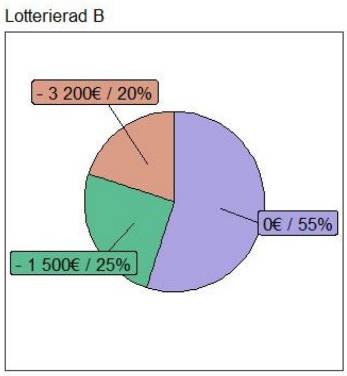
○ Lotterierad A
○ Lotterierad B





○ Lotterierad A
○ Lotterierad B

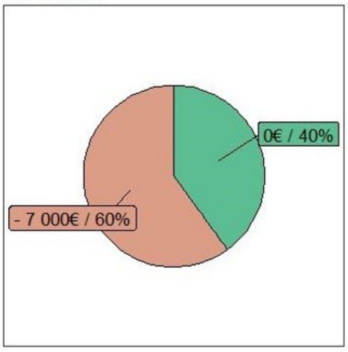




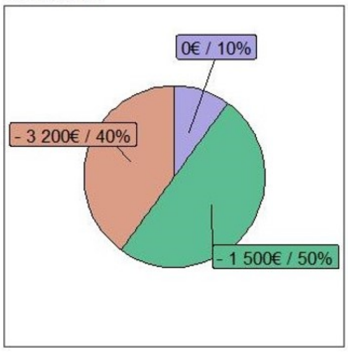
○ Lotterierad A
○ Lotterierad B

Bei folgender Lotterie wird vorab eine faire Münze geworfen, die über die Teilnahme an der Lotterie entscheidet. Wenn Kopf oben liegt, dann müssen Sie ein Lotterierad auswählen. Wenn Zahl oben liegt, dann dürfen Sie nicht teilnehmen und verlieren somit auch kein Geld.
Da Kopf bei Ihnen oben liegt, müssen Sie eine Entscheidung zwischen den zwei folgenden Lotterierädern treffen:

Lotterierad A

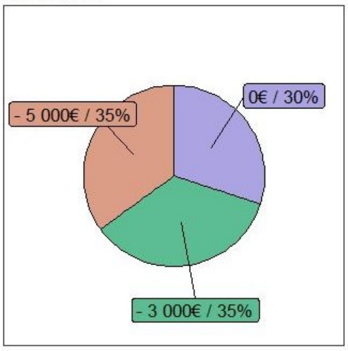


Lotterierad B

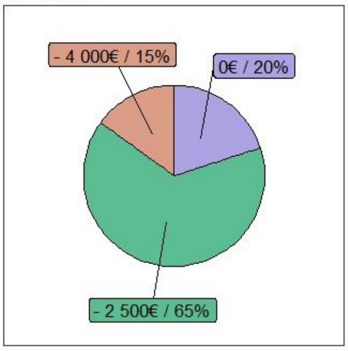


○ Lotterierad A
○ Lotterierad B

Lotterierad A

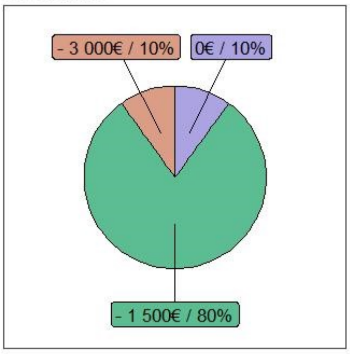


Lotterierad B

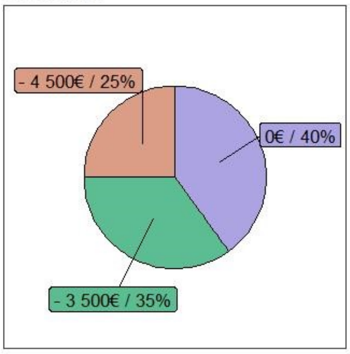


○ Lotterierad A
○ Lotterierad B

Lotterierad A

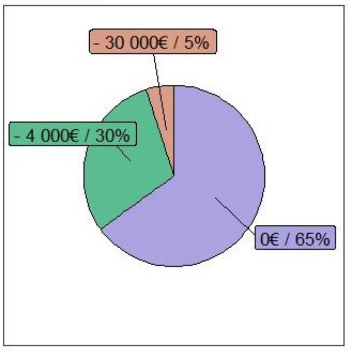


Lotterierad B

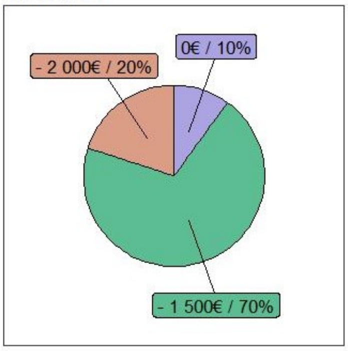


○ Lotterierad A
○ Lotterierad B

Lotterierad A



Lotterierad B



○ Lotterierad A
○ Lotterierad B

Charakteristika

Im Folgenden werden ein paar nähere Informationen zu Ihrer Person abgefragt.

Geben Sie bitte Ihr Alter an: ______

Geben Sie bitte Ihr Geschlecht an:

- ○ Weiblich
- ○ Männlich
- ○ Nicht-Binär
- ○ Keine Angabe

Geben Sie bitte Ihr Angestelltenverhältnis an:

- ○ Schüler/in, Student/in, Auszubildende/r
- ○ Angestellte/r
- ○ Selbstständige/r
- ○ Sonstiges

Ende

Bitte senden Sie Ihr Formular noch ab.

Vielen Dank für Ihre Zeit und Teilnahme.

A.2 Graphics and tables

no.	Lottery wheel A	$E(l_A)$	Lottery wheel B	$E(l_B)$
9	(-12 000, .35; -3000, .25)	-4950	(-6000, .7; -3000, .25)	-4950
10	(-12 000, .001; -3000, .25)	-762	(-6000, .002; -3000, .25)	-762
11	(-7000, .3; -3000, .55)	-3750	(-3200, .2; -3100,. 25; -3000)	-3065
12	(-7000; .3)	-2100	(-3200, .2; -1500, .25)	-1015
13	In the beginning of the next lottery, a fair coin is thrown (p=0.5), which decides whether the participant have to turn the lotterywheel or not, when s/he enters the second step, the wheels look like:			
	(-7000; .6)	-2100	(-3200, .4; -1500, .5)	-1015
14	(-5000, .35; -3000, .35)	-2800	(-2500, .65; -4000, .15)	-2225
15	(-4500, .25; -3500, .35)	-2350	(-3000, .1; -1500; .8)	-1500
16	(-30 000, .05; -4000, .3)	-2700	(-2000, .2; -1500, .7)	-1450

Table 16: Lotteries/prospects of the lottery wheels in the survey, which exclusively consists of losses (and no possible gains)

female	male
68	51

Table 17: Gender of the participants

trainee	employee	freelancer	others
60	53	5	1

Table 18: Employment of participants

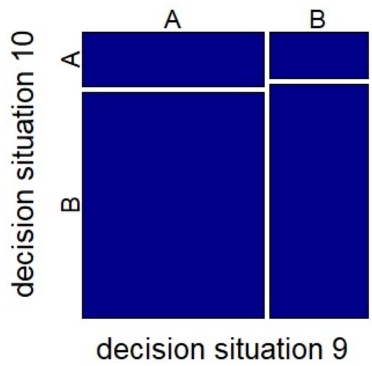


Figure 21: Displaying the share in preference structures between decision situation nine and ten

Decision situation	lottery wheel A	lottery wheel B	indifferent
1	6	113	
2	89	30	
3	34	85	
4	60	59	
5	48	71	
6	47	72	
7	30	89	
8	28	91	
9	77	42	
10	22	97	
11	73	46	
12	47	71	1
13	54	65	
14	54	65	
15	74	44	1
16	51	66	2

Table 19: Number of the responses in each decision situation

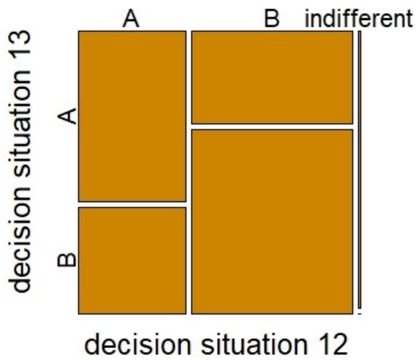


Figure 22: Displaying the share in preference structures between decision situation twelve and thirteen

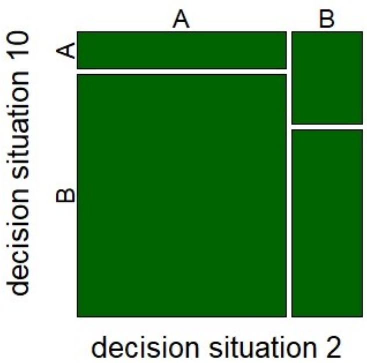


Figure 23: Displaying the share in preference structures between decision situation two and ten

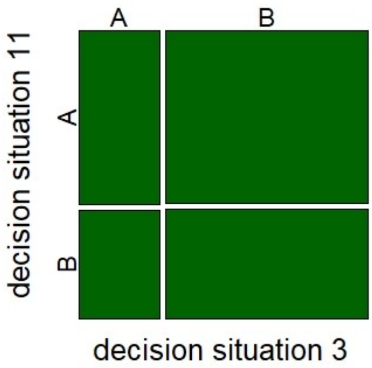


Figure 24: Displaying the share in preference structures between decision situation three and eleven

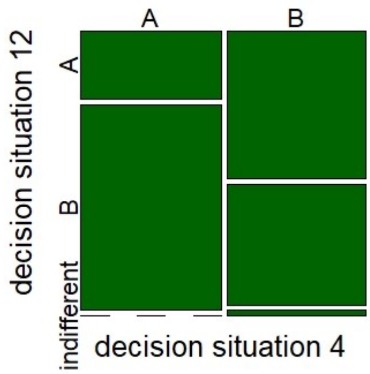


Figure 25: Displaying the share in preference structures between decision situation four and twelve

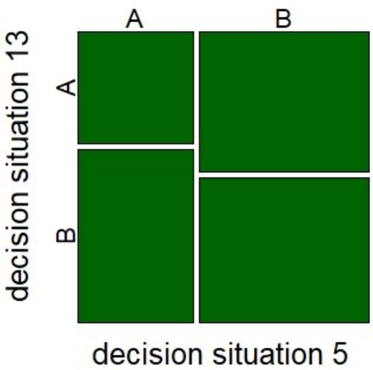


Figure 26: Displaying the share in preference structures between decision situation five and thirteen

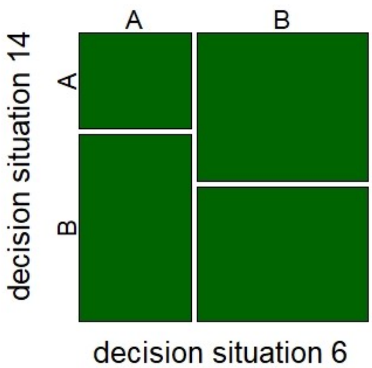


Figure 27: Displaying the share in preference structures between decision situation six and fourteen

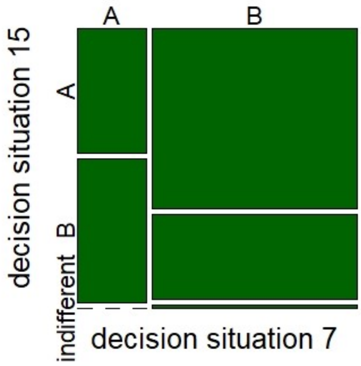


Figure 28: Displaying the share in preference structures between decision situation seven and fifteen

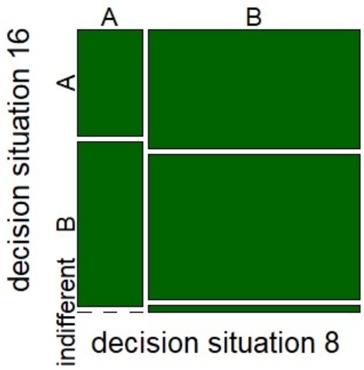


Figure 29: Displaying the share in preference structures between decision situation eight and sixteen

A.3 Possible additional parameter for value function

In the following is a brief explanation of why the additional parameter for negative results does not come into effect in terms of the construction of the survey.

It is assumed, that there exists a preference structure between the two prospects $l_{gA} \ominus l_{gB}$ and all possible monetary outcomes of both prospects are $x_{ig} \leq 0$. Additionally the value function looks like $v(x_{ig}) = -\lambda(-x_{ig})^{\beta}$, so that:

$$\begin{aligned}
l_{gA} \ominus l_{gB} &\Leftrightarrow \sum_{j=1}^{m} w(p_j) * v(x_{gj}) \ominus \sum_{j=1}^{m} w(p_j) * v(x_{gj}) \\
&\Leftrightarrow \sum_{j=1}^{m} w(p_j) * (-\lambda * (-x_{ig})^{\beta}) \ominus \sum_{j=1}^{m} w(p_j) * (-\lambda * (-x_{ig})^{\beta}) \\
&\Leftrightarrow \lambda \sum_{j=1}^{m} w(p_j) * (-(-x_{ig})^{\beta}) \ominus \lambda \sum_{j=1}^{m} w(p_j) * (-(-x_{ig})^{\beta}) \\
&\Leftrightarrow \sum_{j=1}^{m} w(p_j) * (-(-x_{ig})^{\beta}) \ominus \sum_{j=1}^{m} w(p_j) * (-(-x_{ig})^{\beta})
\end{aligned}$$

A.4 Files in the electronic attachment

- datapreperation.R: File for preparation of the data
- daten.csv: processed data
- descriptiv.R: creation of the graphics and tables in the descriptive analysis
- evaluation.R: (programming of the) estimation of the parameters
- graphics_evaluation.R: creation of the graphics in the results chapter
- lotteries.R: To generate the lottery wheels of the decision situations included in the survey
- UmfragezurEntscheidungstheorie.csv: collected data

Declaration of Authorship

I hereby declare that I have written this thesis independently and have not used any other than the indicated aids. The passages of the work, which are taken or the sense of other works (including internet sources) are taken, have been marked with the source.

Munich, 09.05.2023, Franziska Laura Reichmeier