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Individual vs. Collective Bargaining in the Large Firm Search Model*

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Abstract

We analyze the welfare and employment effects of different wage bargaining regimes. Within the large firm search model, we show that collective bargaining affects employment via two channels. Collective bargaining exerts opposing effects on job creation and wage setting. Firms have a stronger incentive for strategic employment, while workers benefit from the threat of a strike. We find that the employment increase due to the strategic motive is dominated by the employment decrease due to the increase in workers' threat point. In aggregate equilibrium, employment is inefficiently low under collective bargaining. But it is not always true that equilibrium wages exceed those under individual bargaining. If unemployment benefits are sufficiently low, collectively bargained wages are smaller. The theory sheds new light on policies concerned with strategic employment and the relation between replacement rates and the extent of collective wage bargaining.

Keywords: search, overemployment, collective wage bargaining, wage determination.
JEL: J30, J50, J41.

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1 Introduction

An important result in the large firm search model of the labor market is that firms tend to over-employ (Cahuc et al., 2008; Cahuc and Wasmer, 2001; or Smith, 1999). The intuition for this result is due to the fundamental structure of the search friction. Once a firm and a worker are matched, both do better when striking a wage agreement that shares the rent of a match than to split up and start searching again. The firm, however, has a “first-mover-advantage” in setting the environment for the wage bargain, because the employment level is determined *before* the worker and the firm bargain over the wage.[1] If the bargained wage is a negative function of employment (a situation that e.g. occurs when the production function (or, more general the revenue function) is concave), the firm will find it advantageous to employ more workers than the wage-equals-marginal-productivity paradigm suggests.[2] Thus, in addition to the externalities prevailing in one-worker-one firm search models of the labor market (see e.g. Hosios, 1990), there will be an additional one: a wage externality.

A crucial assumption concerning the institutional organization of the wage bargain in this literature is that every individual worker bargains her individual wage with the firm. This assumption seems to be at odds with many real-world labor markets. In most western economies a dominant way of wage determination is union wage bargaining. This is true for Continental Europe where union wage coverage (i.e. union dominated wage setting) ranges between 6% and 100% depending on the sector and the country (CESifo DICE 2009) but also, to a lesser extent, for Anglo-Saxon economies. Union wage bargaining implies that the wage is bargained collectively for all employed workers in the firm. With this one might be tempted to argue that collective wage bargaining internalizes the wage externality.

This is the point of departure in this paper. Does collective wage bargaining form an institutional arrangement that fights overemployment and potentially restores efficiency in the labor market? To answer this question we extend a large firm search model to allow for collective wage bargaining. The dynamic search framework is essential for our results.[3] We analyze and compare the equilibrium outcome under collective and individual wage bargaining. With collective bargaining, the threat point of the workers will be stronger. If no agreement is reached, the firm is threatened to ‘loose’ not only the production value of one worker but that of its entire work force. Relative to individual bargaining, the increase in worker’s threat point shifts the wage setting curve outwards. However, the job creation curve (=labor demand curve) is shifted outwards, too. Intuitively, this is because workers collectively bargain a share of the average product, whereas workers individually bargain a share of marginal products. With concave revenue functions, workers collectively bargain about more concave margins, hence the wage reaction to employment changes is more severe than under

[1] In this sense, the focus is on situations in which vacancy posting is incomplete. This incompleteness is the source of the strategic employment behavior.

[2] Stole and Zwiebel (1996a, 1996b) and Wolinsky (2000) show in a static, game-theoretic setting that non-binding wage contracts and the unavailability of outside-labor during the wage bargain will also lead to overemployment. Note, however, that the setting and the source for overemployment are quite different compared to the search model. See Acemoglu and Hawkins (2006) for a search model where immediate renegotiations are taken into account.

[3] In static models with Stole-Zwiebel bargaining, collective wage bargaining removes the wage externality by hindering firms from instantaneous renegotiation (cf. Stole and Zwiebel, 1996b, Sec. III B).

individual bargaining. This results in a stronger overemployment incentive. At a given wage level the firm thus hires more workers.

As such, collective wage bargaining implies two countervailing effects. We find that at the firm-level employment falls and wages increase relative to individual bargaining. The employment decline translates to the aggregate level. Feedback effects, however, imply that equilibrium wages under collective bargaining are not necessarily larger than under individual bargaining. If the real return from not working is sufficiently low, the collectively bargained wage is smaller. Additionally, we show that the aggregate employment decline is too strong to restore efficiency. Collective bargaining generally does not internalize the wage externality.

Inter alia, the theory suggests that low replacement rates can be a reason for less pronounced collective wage bargaining in some countries. It also highlights a dual role of the strategic wage effect. On the one hand, the strategic behavior of firms implies a severe inefficiency in the market. In the collective bargaining regime, however, it prevents employment from declining too much.[4] The general point, applicable to search economies with non-linear revenue functions, is that labor market policy must take differences in bargaining regimes into account. Our model implies that moderately reducing wage flexibility (in an environment with overemployment) has beneficial effects under individual bargaining, but leads to more excessive "underemployment" under collective bargaining.

Our analysis builds on the work of Smith (1999) and Cahuc and Wasmer (2001). These papers were the first to analyze the possibility of overemployment within a Pissarides type search model.[5] Both papers showed (independently and within slightly different frameworks) that firms which operate under decreasing marginal products have a strategic incentive to employ workers up to levels in which the (technological) marginal product exceeds the marginal costs. Additionally, Cahuc and Wasmer (2001) show how the existence of an additional factor of production (they consider capital) can effectively linearize the production technology and hence remove the strategic incentive.

Understanding the role of real world bargaining regimes within the large firm search model is particularly important because the framework has become a standard device with an array of applications. Most recently, Cahuc et al. (2008) adapt the large firm setting to an economy with heterogenous labor. They show that overemployment of a particular group of workers relative to other workers depends on their relative bargaining power and their substitutability within the production process. Felbermayer and Pratt (2009) integrate the large firm search model into a model with heterogenous firms. They focus on the effects of product market regulation on unemployment. Finally, Ebell and Haefke (2009) apply the large firm model to analyze the interaction between product market deregulation and unemployment. Neither of these papers, however, analyzes the efficiency properties of institutionally different bargaining regimes observed in the cross-section of labor markets.

This paper proceeds as follows. Section 2 lays out the basic model environment and the timing of events. Section 3 characterizes the constrained efficient allocation while

[4] As such, incompleteness of vacancy posting is desirable under collective bargaining.

[5] A very early contribution is Bertola and Caballero (1994). They also consider large firms in a model with search frictions and they take the strategic/monopsonistic employment effects of firms into account. Their focus, however, is on the employment and the wage path of firms that face productivity shocks (labor hoarding). They do not analyze the effect of different bargaining institutions.

Section 4 shows how the equilibrium allocation looks like under individual and collective wage bargaining. We show that under collective bargaining, the wage externality is larger; equilibrium employment and market tightness are lower; and wages may be lower relative to individual bargaining. Section 5 analyzes the efficiency properties of both regimes. Finally, the paper concludes in Section 6 by discussing the contribution and open avenues for further research.

2 Model

2.1 Environment

Consider a search and bargaining economy in continuous time.[6] There is an infinite countable number of large firms indexed by i and a mass L of homogenous workers who each supply 1 unit of labor inelastically. All agents are risk-neutral, infinitely-lived, anonymous, and discount future income at constant rate r. Firms are endowed with a homogenous production technology $F(N_i)$, $F' > 0 \geq F''$, $F(0) = 0$, and produce a unique consumption good which is the numéraire.[7] At every point in time, $N_i(t)$ (≥ 0) is predetermined and cannot be increased instantaneously due to the search friction. To recruit a worker, a firm must open a vacancy at cost $c > 0$. The aggregate number of matches between workers and firms per unit time is given by $M(U,V)$, where M is the labor market matching function. M is an increasing function of the aggregate number of vacancies V and the pool of unemployed workers $U = L - N$, concave, and homogenous of degree 1. Matches are random so that, in each short time interval dt, a vacant position is filled with probability $\frac{M}{V}dt = M\left(\frac{U}{V},1\right)dt \equiv \lambda_m(\theta)\,dt$, where $\lambda_m' < 0$, $\eta(\theta) \equiv \lambda_m' \frac{\theta}{\lambda_m} \in (-1,0)$, and $\lim_{\theta \downarrow 0} \lambda_m = +\infty$. An increase in the ratio of open positions to searching workers, i.e. the tightness $\theta \equiv \frac{V}{U}$, lowers the rate at which hiring firms match with a worker. An unemployed worker will match with a firm with probability $p(\theta) \equiv \frac{M}{U} = \theta\lambda_m$, $p' > 0$. Occupied jobs are exogenously destroyed by job-specific i.i.d. shocks at Poisson rate λ_s. While workers are employed with a firm, their wages are determined either by individual (I) or collective (C) wage bargaining. $\zeta \in \{I, C\}$ indicates the respective wage setting regime. In both cases, we focus on negotiations about the contemporaneous wage. Under individual wage bargaining, the firm negotiates simultaneously with each employee (more details below). Under collective wage bargaining, all employees delegate the wage negotiation to a representative worker and decide jointly whether to work. Note that our notion of collective wage bargaining, or collective bargaining for short, is similarly narrow as Stole and Zwiebel (1996b)'s. We abstract from a wide range of important differences between individual and collective bargaining to concentrate sharply on the fact that, with diminishing returns to the product of labor, collective bargaining requires employees to jointly bargain about more concave margins than individuals.[8]

[6] The time argument is suppressed unless this might cause confusion.

[7] We treat a firm's labor force N_i as continuous variable. Alternatively, $N_i \in Z$ and sufficiently large firms yield qualitatively similar results. As in Cahuc et al. (2008) $N_i^j F^{(j)}(N_i)$ is assumed to be continuous in zero ($F^{(j)}$ denotes the jth derivative of F) to ensure convergence of integrals under individual bargaining.

[8] In practice, a "firm-level union" or "collective" bargaining commonly implies a variety of important features suppressed in our notion of collective wage bargaining which may themselves have important consequences for the bargaining process. These features include bargaining about both wages and employment,

2.2 Timing

At every point in time, each firm announces a number of vacancies and bargains the wage with its current employees. We assume that vacancy posting (arguably the more costly transaction) occurs logically before the wage bargain.[9] By the very nature of the search friction, the vacancy choice that determines employment at $t+dt$, $V_i(t)$, precedes the wage negotiation in which agents bargain about the wage $w_i(t+dt)$. For a large class of production functions and demand structures, the wage bargaining outcome is a function of the firm's employment level at the time.[10] In the following applications, this function is continuous and differentiable. Importantly, it is rationally anticipated by the firm when vacancies are posted. As a result, the firm's optimal vacancy choice at time t takes into account that hiring an additional worker changes the wage cost per worker (a wage externality exists).

3 Pareto Efficient Allocation: Social Planner

Before turning to the decentral equilibrium in the economy, we first derive the efficient allocation. Consider a utilitarian planner who faces the environment with search frictions described above. Aggregate employment increases by matches of workers and hiring firms and decreases by destroyed jobs:

$$dN = \lambda_m(\theta)\,V dt - \lambda_s N dt. \tag{1}$$

The planner chooses a sequence of vacancies that, given (1), maximizes social welfare, i.e., the present value of income minus benefit payments and vacancy costs,

$$SW(t) = \int_t^\infty e^{-r(\tau-t)}\left[F(N) + [L-N]\,b - cV\right] d\tau. \tag{2}$$

By standard arguments, the planner's problem can be written recursively, with N as the only state.[11] The value function solves the Bellman equation

$$rSW(N) = \max_V \left\{F(N) + (L-N)b - cV + \frac{dSW(N)}{dt}\right\}. \tag{3}$$

Making use of (1), we can write

$$\frac{dSW(N)}{dt} = SW'(N)\left[\lambda_m(\theta)V - \lambda_s N\right]. \tag{4}$$

With this, optimality requires the expected search costs to equal the increase in welfare due to newly hired workers:

$$SW(N)' = \frac{c}{\lambda_m(\theta) + \lambda_m'(\theta)\theta}. \tag{5}$$

working conditions, retirement packages, and so on. Cf. Bauer and Lingens (2009b) on "efficient" bargaining in search models. Moreover, appointing a representative may also alter the threat points and open up another source of strategic interactions.

[9] This implies that a firm still has to pay a (vacancy posting) cost if, off the equilibrium path, it decides to walk away from a wage negotiation.

[10] In this paper, we model firms as price-takers and impose concavity on the production function. Alternative assumptions that imply curvature on the product of labor yield qualitatively similar results.

[11] The derivation of the Bellman equation can e.g. be found in Turnovsky (2000).

The planner takes into account that posting an additional vacancy increases the search cost through the congestion externality. Solving the planner's problem, we get

Proposition 1 (Planner Solution, Smith 1999) *In steady state, the planner solution to the optimal vacancy posting problem is given by*

$$\frac{c}{\lambda_m(\theta)} \frac{r + \lambda_s - \lambda_m \theta \eta(\theta)}{1 + \eta(\theta)} = F'(N) - b, \tag{6}$$

where $\eta(\theta) \equiv \frac{\lambda_m'(\theta)}{\lambda_m(\theta)} \theta$.

Proof. Differentiating the maximized Bellman equation with respect to N gives a differential equation for the evolution of the co-state variable SW',

$$\begin{aligned} rSW'(N) &= F'(N) - b - c\frac{dV}{dN} + SW''(N)\left[\lambda_m(\theta)V - \lambda_s N\right] \\ &\quad + SW'\left\{\left[\lambda_m'(\theta)\theta + \lambda_m(\theta)\right]\frac{dV}{dN} - \lambda'(\theta)\theta^2\frac{\partial U}{\partial N} - \lambda_s\right\}. \end{aligned}$$

In steady state, $\frac{dN}{dt} = 0$. Substituting with (5) and noting that $\frac{\partial U}{\partial N} = -1$ by definition gives the solution. ■

Proposition 1 derives the policy function (which is basically the same as the one in Pissarides, 2000 ch. 8), i.e. the optimal level of labor market tightness given some employment level. Together with the Beverdige curve (combinations of θ and N for which $\frac{dN}{dt} = 0$, implicitly given by (1)), (6) provides a benchmark allocation for the following analysis. Note that obviously the planner does not take wage effects into account. This is due to the fact that the wage (in general) is only a distribution device. In the decentral equilibrium, however, the firm can strategically affect the wage by its employment choice. Thus, the wage bargain affects the optimal allocation. Hence, the standard Hosios (1990) condition does not restore efficiency in the large firm case (cf. Smith 1999).

We next focus on the decentral situation and characterize the equilibrium under both bargaining regimes. In the individual bargaining case, we naturally draw on Smith (1999) and Cahuc and Wasmer (2001).

4 Equilibrium

We assume that both firms and workers are anonymous and focus on symmetric Markov equilibria where strategies depend exclusively on payoff relevant state variables. We further limit attention to symmetric steady state search equilibria, where employment and wages are the same in all firms and aggregate employment and the market tightness are constant. By search equilibrium we mean a vector (N, θ, w) that simultaneously solves the forward-looking job creation conditions and wages that support the bargaining outcomes.

4.1 Job Creation

Each firm chooses a path for vacancies to maximize the present value of its profit flows, $\Pi_i(t) = \int_t^\infty e^{-r[\tau - t]} \pi_i(\tau)\, d\tau$, taking as given the tightness of the labor market and the evolution of its workforce,

$$dN_i = \lambda_m(\theta) V_i dt - \lambda_s N_i dt, \quad N_i \geq 0. \tag{7}$$

The firm's vacancy choice determines future employment and thereby potentially alters the firm's bargaining position. In a sense, the search friction allows the firm to act as monopsonist in the wage bargain. When calculating the instantaneous profit (revenues minus wage and vacancy costs), the firm rationally anticipates this relationship, allowing the wage to depend on employment:

$$\pi_i = F(N_i) - w_i(N_i) N_i - cV_i. \tag{8}$$

Since the firm's problem is stationary, we solve it recursively. Π_i must satisfy the Bellman equation

$$r\Pi_i(N_i) = \max_{V_i} \left\{ F(N_i) - w_i(N_i) N_i - cV_i + \frac{d\Pi_i(N_i)}{dt} \right\}. \tag{9}$$

The right-hand side describes the firms optimal vacancy choice.[12] Making use of (7), the continuation value can be written as $\frac{d\Pi_i}{dt} = \Pi_i' \left[\lambda_m(\theta) V_i - \lambda_s N_i\right]$. This gives the first order maximization condition derived from (9):

$$\Pi_i'(N_i) = \frac{c}{\lambda_m(\theta)}. \tag{10}$$

Vacancies are chosen to set the search cost equal to the marginal value of employment. Differentiating the maximized Bellman equation with respect to N_i, using the envelope condition, and the law of motion of N_i yields the value of the marginal job in steady state,

$$\Pi_i'(N_i) = \frac{F'(N_i) - w_i(N_i) - w_i'(N_i) N_i}{r + \lambda^s}. \tag{11}$$

An additional worker raises output by F' and adds w_i to the wage bill as long as the job is not destroyed. Crucially, every employee (potentially) affects the wage bargain (i.e., $w_i' \neq 0$). Thus, when considering the marginal return of a worker, the firm takes the production effect (F') and the strategic effect in the wage bargain (w') into account. If e.g. a marginal worker decreases the bargained wage (as will be the case below), the marginal value of a worker increases (and will be larger than her purely "technological" production value).[13] Thus, the firm can exert monopsonistic power since it 'chooses' (by means of vacancy posting) employment before the wage bargain takes place.

Combining (10) and (11) yields a first expression for the firm's job creation curve:

$$F'(N_i) - w_i(N_i) - w_i'(N_i) N_i = \frac{r + \lambda_s}{\lambda_m(\theta)} c. \tag{12}$$

If $w' = 0$, (12) boils down to the well-known labor demand curve in search economies with costly vacancy posting, $w_i = F' - \frac{r+\lambda^s}{\lambda_m(\theta)} c$. Here, $w' \neq 0$ shifts the labor demand curve.

[12] We assume that the firm faces a deterministic evolution of its workforce. We could also assume that this was random without changing the results.

[13] Note that in standard monopsony models (see Manning, 2003) it is usually assumed that $w' < 0$ due to a positively sloped labor supply curve. This will not hold in the search framework we consider.

4.2 Wage Determination

Following Bertola and Caballero (1994) and the tradition of the seminal search literature (cf. Diamond 1981, 1982; Mortensen 1978, 1982; Pissarides 1985, 2000), we assume that wages are determined by bilateral generalized axiomatic Nash bargaining.[14] The "threat point" in the individual bargaining case is equal to the option of looking for an alternative partner to match with. For the worker(s), this is the value of unemployment; for the firm, this is the value of producing without the marginal worker. In the collective bargaining case the threat point is the value of a complete hold-up (i.e. a strike). Workers 'receive' the (flow) value of being unemployed. The firm cannot produce for that instant of time. Consequently the value in this case is $r\Pi_i(0)$.[15]

To begin with, consider the net returns from a worker-firm match. The worker's (expected) lifetime utility is $W^z = E_t \int_t^\infty e^{-r[\tau - t]} z d\tau$ where $z \in \{w, b\}$ denotes her instantaneous income (workers cannot save) and the corresponding employment state.[16] Expectations are formed with respect to future income (in a symmetric steady state, z "jumps" stochastically back and forth between a uniform constant w and b). When working for a firm at a constant wage, W_i^w supports the Bellman equation

$$rW_i^w = w_i + \lambda^s \left[W^b - W_i^w \right]. \tag{13}$$

The worker earns w_i and suffers a capital loss $\tilde{W}_i \equiv W_i^w - W^b$ in case she looses her job. As both parties take the worker's outside option W^b as given, (13) can be solved for the worker's net (flow) return:

$$r\tilde{W}_i = r \frac{w_i - rW^b}{r + \lambda^s}. \tag{14}$$

Under collective wage bargaining, employees delegate the bargain to a representative worker who carries the negotiation out for them. Her net return is the sum of all employees' net returns,

$$r\tilde{W}_{i,c}(N_i) \equiv N_i r\tilde{W}_i^w. \tag{15}$$

Consider next the net return for the firm. Under individual bargaining, the fact that wages are bargained simultaneously allows us to consider a representative firm-worker

[14] While bilateral bargaining is a natural choice due to the quasi-rents, simultaneous bilateral bargaining may be perceived as too simple an assumption that may be replaced by multilateral bargaining using the Shapley value. Note, however, that the sharing rule derived under bilateral Nash bargaining implies a lower bound on the wage effect, which is the main focus of our paper. Further, applying the standard Nash sharing rule replicates the wage schedule obtained under Stole-Zwiebel bargaining in the static model, which is a useful benchmark. Finally, simultaneous bilateral bargaining allows us to apply the envelope condition to derive the firm's net return from a match (Π' is in fact the added value of any worker in each bargain). Since we do not consider immediate renegotiations, Acemoglu and Hawkin's (2006, footnote 3) criticism of Cahuc et al. (2008)'s approach, which is to assume immediate renegotiations and use the envelope condition throughout, does not apply.

[15] Due to the continuous time framework, there will be continuous wage bargaining. Hence, it seems natural to assume that the bargaining parties split the 'instantaneous' pie and consider their flow values.

[16] Although employment at the firm level follows a deterministic process, the labor market state of one individual evolves stochastically (the firm knows that it will be separated from some workers, but not from whom.)

pair. The net (flow) return from employing a worker is equal to the marginal value $r\Pi_i'$. Under collective bargaining this return is equal to the ex-post (after vacancies are posted) flow value of production with the given number of employees

$$r\tilde{\Pi}_i(N_i) = F(N_i) - w_i(N_i)N_i. \tag{16}$$

Given the net-returns in both configurations, we next determine the corresponding bargaining outcomes. Under individual bargaining, the wage is given by

$$w_i = \arg\max\left\{\left(r\tilde{W}_i\right)^{\beta}\left(r\Pi_i'\right)^{1-\beta}\right\}, \tag{17}$$

where $\beta \in (0,1)$ denotes the worker's bargaining weight. Using (11) and (14), the first-order maximization condition derived from (17) satisfies the linear first-order differential equation

$$w_i = [1-\beta]rW^b + \beta\left[F'(N_i) - w_i'(N_i)N\right]. \tag{18}$$

Following Cahuc and Wasmer (2001) and Cahuc et al. (2008), we assume the boundary condition $\lim_{N_i\to 0} Nw(N) = 0$.[17] This particular solution satisfies

$$w_i^{WS,I} = [1-\beta]rW^b + N_i^{-\frac{1}{\beta}}\int_0^{N_i} x^{\frac{1-\beta}{\beta}}F'(x)\,dx = [1-\beta]rW^b + \int_0^1 x^{\frac{1-\beta}{\beta}}F'(N_i x)\,dx. \tag{19}$$

The wage is set above the worker's outside option. The "mark-up" is a weighted average of marginal productivity.

Consider next the collective bargaining case. Here, the wage outcome is given by

$$w_i = \arg\max\left\{\left(r\tilde{W}_{i,c}\right)^{\beta}\left(r\tilde{\Pi}_i\right)^{1-\beta}\right\}. \tag{20}$$

Combining the first order maximization condition derived from (20) with (14) and (16), we derive the wage equation under collective bargaining:

$$w_i^{WS,C} = [1-\beta]rW^b + \beta\frac{F(N_i)}{N_i}. \tag{21}$$

The wage outcome is a weighted average of the workers' outside option and the average product, with the weight given by the workers' bargaining "strength" β. As under individual bargaining, the labor market tightness only affects the bargaining outcome through the workers' outside option.

Having derived the wage schedules, we are now in the position to determine the wage externality under both regimes. Both (19) and (21) imply that a marginal increase of employment allows the firm to bargain all it's wages downwards. Let $\delta^{\zeta} \equiv dw_i^{WS,\zeta}/dN_i$. Under individual bargaining, the wage externality derived from (19) is equal to[18]

$$\delta^I = \left[-\frac{1}{\beta}N_i^{-\frac{1}{\beta}}\int_0^{N_i} x^{\frac{1-\beta}{\beta}}F'(x)\,dx + F'(N)\right]N_i^{-1} = \int_0^1 x^{\frac{1}{\beta}}F''(N_i x)\,dx \quad (<0). \tag{22}$$

[17] I.e., $w_i \to \infty$ for $N_i \to 0$ is imposed. This particular solution yields a wage equation that coincides with the Stole-Zwiebel wage schedule.

[18] The second equality uses integration by parts,

$$\frac{1}{\beta}N_i^{-\frac{1}{\beta}}\int_0^{N_i} x^{\frac{1-\beta}{\beta}}F'(x)\,dx = F'(N_i) - N_i\int_0^1 x^{\frac{1}{\beta}}F''(N_i x)\,dx.$$

Analogously, under collective bargaining, (21) implies positive wage savings for the firm:

$$\delta^C = \frac{\beta}{N_i}\left[F'(N_i) - \frac{F(N_i)}{N_i}\right] \quad (<0). \tag{23}$$

In both regimes, concavity of the labor revenue function results in a negative relationship between the wage outcome and the firm's employment level.

The marginal and the average product are related by the production elasticity $\alpha(N) \equiv F'\frac{N}{F}$. Comparing (22) and (23) under the assumption that α is a constant (e.g. in the "Cobb-Douglas" case), we conclude that hiring an additional worker under collective bargaining allows the firm to save more on existing wages than under individual bargaining.

Proposition 2 (Size of wage externality) *Suppose $\alpha' = 0$. At any given employment level, collective wage bargaining induces a higher wage externality than individual bargaining.*

Proof. Given N_i, the relative (absolute) difference due to hiring an additional worker is

$$\delta \equiv \left|\delta^I\right| - \left|\delta^C\right| = \frac{\beta}{N_i}\left[F'(N_i) - \frac{F(N_i)}{N_i} - \frac{N_i}{\beta}\int_0^1 x^{\frac{1}{\beta}} F''(N_i x)\, dx\right]. \tag{24}$$

If $\delta < 0$, collective bargaining implies larger wage reductions at the given employment level. With $\alpha' = 0$ and $F'' < 0$, it holds that $F = \kappa N_i^{\alpha}$ where $\kappa > 0$ and $0 < \alpha < 1$. (24) becomes

$$\delta = -\beta N_i^{\alpha-2}\left[1-\alpha\right]^2 \frac{1-\beta}{1-\left[1-\alpha\right]\beta} \quad (<0). \tag{25}$$

Hence, $\left|\delta^I\right| < \left|\delta^C\right|$. ■

Given the derivation of the wage equation, the intuition for Proposition 2 is straightforward. In general, the wage declines in employment if the labor revenue function is subject to diminishing returns. Concavity implies that any inframarginal product of labor falls by more than the marginal product if employment increases marginally. As collective bargaining shifts negotiations into more concave regions of the labor revenue function, savings on existing wages increase. This increases the incentive for using vacancy posting strategically. We finally note that neither wage externality depends on the worker's outside option. As such, the strategic component is not subject to changes in the labor market tightness.

4.3 Equilibrium at the firm level

Firms and workers take aggregate variables as given. The firm-level equilibrium pair $\left(N_i^{\zeta}, w_i^{\zeta}\right)$ simultaneously satisfies the job creation condition and the wage equation as a function of aggregate employment and the labor market tightness.

Making use of footnote (18) to combine (22) and (12), we derive an intuitive expression for the job creation curve under individual bargaining:

$$w_i^{JC,I} = F'(N_i) - N_i \int_0^1 x^{\frac{1}{\beta}} F''(N_i x)\, dx - \frac{r+\lambda^s}{\lambda_m(\theta)} c. \tag{26}$$

Employment is chosen to balance the returns from hiring a worker, i.e., revenues from production, savings on existing wages, and savings on vacancy costs, with the associated wage cost.

Combining (23) and (12) provides us with the job creation curve under collective bargaining:

$$w_i^{JC,C} = F'(N_i) + \beta \left[\frac{F(N_i)}{N_i} - F'(N_i) \right] - \frac{r + \lambda^s}{\lambda_m(\theta)} c. \tag{27}$$

This job creation curve is remarkably similar to the one under individual bargaining. Again, the firm chooses employment optimally by balancing marginal costs and marginal returns. The marginal gain in form of savings on wages is different in both cases. With a concave production function of the form $F = \kappa N_i^{\alpha}$ the strategic incentive in the collective case is larger than the individual one. Thus, $w_i^{JC,C} > w_i^{JC,I}$ (see Proposition 2 for a similar reasoning). The job creation under collective bargaining will be to the north of that under individual bargaining.

Summarizing, we observe two countervailing effects of the collective bargaining regime. Firms will increase their labor demand since the marginal gain of employment increases due to the strategic effect. Workers who bargain collectively, however, are more powerful and hence can acquire a larger piece of the pie. In the following we focus on the aggregate effect of the bargaining institution.

Collecting terms allows an alternative interpretation of the job creation curve (27):

$$w_i^{JC,C} = (1 - \beta) F'(N_i) + \beta \frac{F(N_i)}{N_i} - \frac{r + \lambda^s}{\lambda_m(\theta)} c. \tag{28}$$

Depending on the distribution of the bargaining "strength", the main determinant of employment is the marginal product (if the firm has lots of bargaining power, i.e. β is close to zero) or the larger average product (if the workers have most of the bargaining power, i.e. β is close to 1).

Under individual bargaining, combining the wage curve (19) and the job creation condition (26) yields an implicit expression for the partial equilibrium employment, N_i^I:

$$F'\left(N_i^I\right) - rW^b = N_i^I \int_0^1 x^{\frac{1}{\beta}} F''\left(N_i^I x\right) dx + \frac{r + \lambda^s}{1 - \beta} \frac{c}{\lambda_m(\theta)}. \tag{29}$$

The corresponding wage, obtained from (26) and (29), satisfies

$$w_i^I = rW^b + \frac{\beta}{1 - \beta} \frac{r + \lambda^s}{\lambda_m(\theta)} c. \tag{30}$$

Workers receive their reservation wage rW^b and, like in standard models, a fraction $\frac{\beta}{1-\beta}$ of the appropriately discounted savings on hiring costs due to the match.

Combining (21) and (27), we find a firm's employment in the collective bargaining case:

$$F'\left(N_i^C\right) - rW^b = \frac{r + \lambda^s}{1 - \beta} \frac{c}{\lambda^m(\theta)}. \tag{31}$$

We can readily compare this expression to (29) for a given identical outside option and market tightness.

Proposition 3 (Employment at the firm level) *Suppose $F'' < 0$ and take as given some θ and rW^b. Then $N_i^C < N_i^I$.*

Proof. As $N_i^I \int_0^1 x^{\frac{1}{\beta}} F''(N_i x)\, dx < 0$, $F'\left(N_i^I\right) < F'\left(N_i^C\right)$. Concavity completes the proof. ■

To determine the partial equilibrium wage with collective bargaining, we make use of the definition of α in the wage curve (21):

$$w_i^{WS,C} = (1-\beta)\, rW^b + \frac{\beta}{\alpha(N_i)} F'(N_i). \tag{32}$$

Combining (32) and (31), the wage satisfies

$$w_i^C = (1-\beta)\, rW^b + \frac{\beta}{\alpha\left(N_i^C\right)} \left[rW^b + \frac{r+\lambda^s}{1-\beta} \frac{c}{\lambda^m(\theta)} \right], \tag{33}$$

where N_i^C is (implicitly) determined by (31). In the special case in which $\alpha' = 0$, the wage is directly given in (33). Wages are increasing in the curvature of the production technology as inversely measured by α. Note that the wage in the individual case does not depend on the curvature of F. Here, $\alpha < 1$ (concavity) and $\beta > 0$ allow the workers to bid up the wage. This is a similar result as obtained from static neoclassical models with collective bargaining (see e.g. Layard et al., 2005). Note, however, that the economic intuition behind these results is different. In standard static models the curvature of the production function is a measure for the trade-off between wages and employment that a worker collective (i.e. a union) faces. If this trade-off is in favor of the union (curvature of F is large) the bargained wage will be high. In the search model, the bargained wage will be larger, too. The reason is, however, that the strategic effect of employment for the *firm* is large. The union only participates in this large strategic effect.

Proposition 4 (Relative wages at the firm level) *Take some θ and rW^b as given. If F is concave, $w_i^I < w_i^C$.*

Proof. Rearranging (33) yields

$$w_i^C = rW^b + \frac{\beta}{1-\beta} \frac{r+\lambda^s}{\lambda^m(\theta)} c + \beta \left[\frac{1}{\alpha\left(N_i^C\right)} - 1 \right] \left[rW^b + \frac{r+\lambda^s}{\lambda^m(\theta)} \frac{c}{1-\beta} \right]. \tag{34}$$

Concavity implies $\alpha\left(N_i^C\right) < 1$. Comparing this expression with (30) completes the proof. ■

4.4 Labor market equilibrium

In steady-state, wages are constant and equal in all firms. The equilibrium system is characterized by four variables: aggregate employment $N(t)$, the total number of vacancies $V(t)$, the wage, and the worker's reservation wage $rW^b(t)$. The four equations that determine these four variables are the wage-setting function, the job creation condition, the law of motion for N, and the equilibrium evaluation of rW^b. By standard arguments, the last object solves

$$rW^b = b + p(\theta)\left[W_i^w - W^b\right] \tag{35}$$

where b (≥ 0) is a real return, measured in terms of the consumption good, that accrues to the worker during search. Symmetric wages imply $W_i^w = W^w$. Combining (13) and (35) we derive the equilibrium value of working:

$$W^w - W^b = \frac{w-b}{r+\lambda^s+p(\theta)}. \tag{36}$$

Plugging (36) back in (35) yields a familiar expression for the outside option:

$$rW^b = b + p(\theta)\frac{w-b}{r+\lambda^s+p(\theta)} = \frac{r+\lambda^s}{r+\lambda^s+p(\theta)}b + \frac{p(\theta)}{r+\lambda^s+p(\theta)}w. \tag{37}$$

As usual, the reservation wage is increasing in w and θ.

Finally, in steady-state, the law of motion for N yields a "Beverdige curve": $VM\left(\frac{\bar{L}-N}{V},1\right) = N\lambda^s$. We normalize $\bar{L} \equiv 1$ and divide by $1-N$:

$$(1-N)\,\theta\lambda^m(\theta) - N\lambda^s = 0. \tag{38}$$

Definition 1 (Equilibrium) *A symmetric steady state equilibrium is a path of constant w, N, V, and ρW^b that satisfies (19) and (26) under individual bargaining and (21) and (27) under collective bargaining, respectively, as well as (37) and (38).*

The two wage setting regimes differ with respect to the equilibrium "policy function" $V = V(N)$. Making use of the matching technology and the law of motion for employment, $V(N)$ can equivalently be written as $N(\theta)$. In what follows, we solve for $N(\theta)$ under the two regimes.

4.4.1 The Aggregate Policy Function

Recall that equilibrium under individual bargaining is described by the Beveridge curve (38), the wage setting curve (19), the job creation curve (26), and the reservation wage equation (37). Combining the reservation wage equation (37) with the partial equilibrium wage under individual bargaining (30) gives the reservation wage as a function of θ:

$$rW^b = b + p(\theta)\frac{\beta}{1-\beta}\frac{c}{\lambda^m(\theta)}. \tag{39}$$

Combining the partial equilibrium equations with the reservation wage equation and dropping the firm subscript, we derive an implicit expression for the policy function:

$$\Gamma_I(N,\theta) \equiv \int_0^1 x^{\frac{1}{\beta}-1}F'(Nx)\,dx - \beta b - \frac{\beta}{1-\beta}\frac{c}{\lambda^m(\theta)}\left[r+\lambda^s+\beta p(\theta)\right] = 0. \tag{40}$$

Γ_I defines a strictly downward sloping policy function in θ -N-space. For $\theta \downarrow 0$, $N \downarrow N_0 > 0$ where N_0 is determined by $\int_0^1 x^{\frac{1}{\beta}-1}F'(N_0x)\,dx = \beta b$. It follows that a unique intersection with the Beveridge curve (which starts at the origin and is strictly increasing) exists for some $\theta, N > 0$.

If the production function is of the form $F = N^{\alpha}$, i.e. the elasticity of production is a constant (and $\kappa = 1$), the policy function solves[19]

$$N(\theta) = \left\{ \frac{\frac{\alpha}{1-[1-\alpha]\beta}}{b + \left[\theta + \frac{r+\lambda^s}{\beta\lambda^m(\theta)}\right] \frac{\beta}{1-\beta} c} \right\}^{\frac{1}{1-\alpha}}. \tag{41}$$

Differentiating (41) shows that an increase in the production elasticity α raises employment. Note that the wage externality shifts the policy function outwards (see Appendix 7.8). Employment increases with the strategic vacancy posting effect.

To complete the characterization of equilibrium, in principle we need to solve for equilibrium market tightness θ^I and equilibrium employment N^I using the Beveridge Curve and (41). While an explicit solution requires a specification for λ^m, such a solution is not necessary to characterize the differences between the two bargaining regimes as the Beveridge curve is not affected by changes in the bargaining structure.[20]

Finally, with knowledge of the equilibrium market tightness θ^I, the equilibrium wage is readily obtained from (30) and (37):

$$w^I = b + \frac{\beta}{1-\beta} \frac{c}{\lambda^m(\theta^I)} \left[r + \lambda^s + p\left(\theta^I\right)\right]. \tag{42}$$

Consider next the equilibrium under collective bargaining. We again derive the aggregate policy function using the reservation wage equation and the partial equilibrium conditions:

$$F'(N) \left\{1 - \frac{\beta p(\theta)}{r+\lambda^s} \left[\frac{1}{\alpha(N)} - 1\right]\right\} = b + \frac{r+\lambda^s+\beta p(\theta)}{1-\beta} \frac{c}{\lambda^m(\theta)}. \tag{43}$$

Unlike in the individual bargaining case, the production elasticity now affects the slope of the reservation wage equation. For example, if α is constant, a 1% increase in the reservation wage no longer raises the equilibrium wage by 1%, but by $1 + \beta\left(\frac{1}{\alpha} - 1\right)\%$ $(> 1\%)$. To ensure existence of a steady state solution with positive employment (i.e., a solution to (43)), we have to impose

$$\alpha(N) = F'(N) \frac{N}{F(N)} > \frac{\beta p(\theta)}{r+\lambda^s+\beta p(\theta)}. \tag{44}$$

Intuitively, for the firm to employ positive amounts of labor, the marginal product must not be "too small" relative to the average product. The intuition for this requirement is that the feedback effect from the aggregate equilibrium to the firm-level wage setting behavior must not be too large. If this was the case the wage would 'accelerate', leading to a break-down of the economy. In this situation employment would be zero.

Rewriting (43) using the definition of α yields an implicit expression for the policy function in analogy to the individual case:

$$\Gamma_C(N,\theta) \equiv \frac{\beta p(\theta)}{r+\lambda^s} \left[\frac{F(N)}{N} - F'(N)\right] - F'(N) + b + \frac{c}{\lambda^m(\theta)} \frac{r+\lambda^s+\beta p(\theta)}{1-\beta} = 0. \tag{45}$$

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[19] $N(0) = \left\{\frac{1}{1-[1-\alpha]\beta} \frac{\alpha}{b}\right\}^{\frac{1}{1-\alpha}}$ (> 0), $\lim_{\theta\to\infty} N(\theta) = 0$, $N' < 0$.

[20] In θ-N-space, the Beverdige curve is $N = \frac{\theta\lambda^m(\theta)}{\theta\lambda^m(\theta)+\lambda^s} = \frac{1}{1+\frac{\lambda^s}{p(\theta)}}$ (≤ 1).

Proposition 5 (Existence of unique steady states) *Suppose (44) holds. Then, a unique steady with positive employment and positive market tightness exists.*

Proof. We are left to prove existence in the collective bargaining case. We verify that Γ_C defines a strictly downward sloping function if and only if (44) holds. By the implicit function theorem, $N(\theta)$ is decreasing in θ if $\frac{\partial \Gamma_C(N,\theta)}{\partial N} > 0$ since $\frac{\partial \Gamma_C(N,\theta)}{\partial \theta} > 0$. Observe that differentiating $\Gamma_C(N, \theta)$ with respect to N yields the same sign as

$$-\frac{\beta p(\theta)}{N}\left[\frac{F(N)}{N} - F'(N)\right] - [r + \lambda^s + \beta p(\theta)] F''(N). \tag{46}$$

Hence, $\frac{\partial \Gamma_C(N,\theta)}{\partial N} > 0$ if

$$\frac{N F''(N)}{F'(N) - \frac{F(N)}{N}} > \frac{\beta p(\theta)}{r + \lambda^s + \beta p(\theta)}. \tag{47}$$

Differentiating the identity in (44) gives $NF'' > \frac{\beta p(\theta)}{r+\lambda^s+\beta p(\theta)}\left[F'(N) - \frac{F(N)}{N}\right]$, i.e., upon rearranging, (47). We conclude that (44) ensures $N' < 0$. As $\Gamma_C(N_0, 0) = 0$ also yields $F'(N_0) = b$, and hence $N_0 > 0$, a unique (intersection with the Beveridge curve and hence a unique) steady state exists. ■

Beveridge curve and the policy function determine (as before) equilibrium employment and market tightness, θ^I and N^I, respectively. With θ^C, we could readily solve for the equilibrium wage by combining the reservation wage equation and the partial equilibrium wage, (33) and (37):

$$w^C = \frac{1 + \beta\chi\left(N^C\right)}{1 - \beta\chi\left(N^C\right)\frac{p(\theta^C)}{r+\lambda^s}} b + \frac{1}{\alpha(N)}\frac{\beta}{1-\beta}\frac{c}{\lambda^m(\theta)}\frac{r + \lambda^s + p\left(\theta^C\right)}{1 - \beta\chi\left(N^C\right)\frac{p(\theta^C)}{r+\lambda^s}}, \tag{48}$$

where $\chi(N) \equiv \frac{1}{\alpha(N)} - 1$.

We are now in the position to compare the equilibrium allocations under both bargaining regimes. Combining Γ_I and Γ_C for a given θ provides us with the distance between the policy functions under both bargaining regimes at any given θ :

$$\frac{1}{\beta}\int_0^1 x^{\frac{1}{\beta}-1} F'\left(N^I x\right) dx = F'\left(N^C\right) - \frac{\beta p(\theta)}{r + \lambda^s}\left[\frac{F\left(N^C\right)}{N^C} - F'\left(N^C\right)\right]. \tag{49}$$

Note that, for some technologies, collective bargaining implies $N^C = 0$ while $N^I > 0$ under individual bargaining.[21] We are thus left to compare employment levels under the assumption that (44) holds.

Proposition 6 (Equilibrium allocation under both regimes) *Concavity of F yields $N^C < N^I$ and $\theta^C < \theta^I$.*

[21] This is the case if (44) fails, which here shows up as requirement for the right-hand side of (49) to be positive:

$$\frac{F'(N_C)}{\frac{F(N_C)}{N_C} - F'(N_C)} = \frac{F'(N_C)}{\frac{1}{\alpha(N)} - 1} > \frac{\beta p(\theta)}{r + \lambda^s}. \tag{50}$$

Proof. Starting with (49), $\beta p(\theta) > 0$ implies

$$F'\left(N^C\right) - \frac{1}{\beta}\int_0^1 x^{\frac{1}{\beta}-1} F'\left(N^I x\right) dx = \frac{\beta p(\theta)}{r+\lambda^s}\left[\frac{F\left(N^C\right)}{N^C} - F'\left(N^C\right)\right] > 0$$

Hence,

$$\begin{aligned} F'\left(N^C\right) &> \frac{1}{\beta}\int_0^1 x^{\frac{1}{\beta}-1} F'\left(N^I x\right) dx = F'\left(N^I\right) - N^I \int_0^1 x^{\frac{1}{\beta}} F''\left(N^I x\right) dx \\ &\Leftrightarrow \quad F'\left(N^C\right) - F'\left(N^I\right) > -N^I \int_0^1 x^{\frac{1}{\beta}} F''\left(N^I x\right) dx > 0. \end{aligned}$$

Concavity implies that $N^C < N^I$. The positively sloped Beveridge curve completes the proof. ■

Thus, although the strategic incentive for overemployment in the collective bargaining regime exceeds that of the individual regime, employment will be larger under the latter one. The reason is that collectively bargaining workers can appropriate a larger piece of the pie. This decreases the employment incentive for firms and overcompensates the incentive for strategic overemployment. This is the same result as at the firm level. The robustness of partial equilibrium results, however, does not hold for equilibrium wages, which we consider next.

We showed that $w^I(\theta) < w^C(\theta)$ in partial equilibrium. Moreover, Proposition 6 showed that $\theta^C < \theta^I$, leading to $p\left(\theta^C\right) < p\left(\theta^I\right)$. The collectively bargained wage is, hence, larger given labor market tightness. However, it could well be that the decrease in the (equilibrium) probability of finding a job (and the associated decrease in the outside option) leads to an overall wage decrease in the collective regime.

Proposition 7 (Equilibrium wages and size of the benefits) *Suppose $\alpha' = 0$. Then, $w^I \lesseqgtr w^C$ is possible. Remarkably, $w^C < w^I$ if b is sufficiently small.*

Proof. We rewrite the collective wage in analogy to the individual wage, $w^I = b + \frac{\beta}{1-\beta}\frac{c}{\lambda^m(\theta^I)}\left[r + \lambda^s + p\left(\theta^I\right)\right]$:

$$w^C = \tag{51}$$

$$b + \frac{\beta}{1-\beta}\frac{c}{\lambda^m\left(\theta^C\right)}\left[r+\lambda^s+p\left(\theta^C\right)\right] \tag{52}$$

$$+\frac{\beta\left(\frac{1}{\alpha}-1\right)\left(1+\frac{p\left(\theta^C\right)}{r+\lambda^s}\right)}{1-\beta\left(\frac{1}{\alpha}-1\right)\frac{p\left(\theta^C\right)}{r+\lambda^s}} b \tag{53}$$

$$+\frac{\beta}{1-\beta}\frac{c}{\lambda^m\left(\theta^C\right)}\left[r+\lambda^s+p\left(\theta^C\right)\right]\left[\frac{1}{\alpha-\beta(1-\alpha)\frac{p\left(\theta^C\right)}{r+\lambda^s}} - 1\right]. \tag{54}$$

The first part, (52), is less than w^I because $\theta_C^* < \theta_I^*$ and $p' > 0$, $\lambda^{m\prime} < 0$. The second part, (53), is positive under (44); the third part, (54), is again negative under (44). The proposition follows as $b \to 0$ makes (53) negligibly small. ■

The 'feedback' effect between the firm level and the aggregate level may be such that the decrease of the outside option dominates the initial wage hike due to collective bargaining. This is obviously more likely if the value of having no job, i.e. the unemployment benefit/value of leisure is small. The interesting consequence of this result is that union formation is not necessarily beneficial for workers. Thus, (endogenous) union formation is more likely in an environment where the unemployment benefits are quite generous. This may be an explanation for the dominance of collective bargaining in Continental Europe compared to Anglo-Saxon economies.

Up to this point, we have compared wages and employment under the two different bargaining institutions. An important question is wether collective bargaining is some sort of second best setting that internalizes the wage externality. The next section focuses on this question and compares the outcome under the different bargaining regimes with the planner solution.

5 Efficiency Properties and Policy

Since equilibrium employment and labor market tightness were characterized implicitly, we apply an indirect approach to analyze the efficiency properties of the two bargaining regimes. To this end we compare the policy functions under individual and collective bargaining, repeated here in (56) and (57) for convenience, to the one chosen by a social planner, repeated in (55):

$$F'(N) - b = \frac{c}{\lambda_m(\theta)} \frac{r + \lambda_s - p(\theta)\eta(\theta)}{1 + \eta(\theta)} \tag{55}$$

$$F'(N) - b - F'(N) + \frac{1}{\beta} \int_0^1 x^{\frac{1}{\beta}-1} F'(Nx)\, dx = \frac{c}{\lambda^m(\theta)} \frac{r + \lambda^s + \beta p(\theta)}{1 - \beta} \tag{56}$$

$$F'(N) - b - \frac{\beta p(\theta)}{r + \lambda^s} \left[\frac{F(N)}{N} - F'(N) \right] = \frac{c}{\lambda^m(\theta)} \frac{r + \lambda^s + \beta p(\theta)}{1 - \beta} \tag{57}$$

Let us impose the well-known Hosios (1990) condition, i.e. $-\eta(\theta) = \beta$. With this restriction, the congestion externality of vacancy posting is internalized. In this case, the right-hand sides of (55)–(57) are identical. Both decentral allocations, however, are also characterized by inefficiencies arising from the bargaining process and strategic employment incentives. It is obvious from inspecting the policy functions, assuming the Hosios condition to hold, that in general neither individual bargaining (cf. Smith 1999) nor collective bargaining yields efficiency. Comparison of the three policy functions leads us to the following result.

Proposition 8 (Efficiency) *Consider a concave production/revenue function, i.e.* $F'' < 0$, *and suppose the Hosios condition holds. Let* N^* *and* θ^* *denote the efficient levels of employment and the labor market tightness, respectively. Then,* $N^C < N^* < N^I$. *Moreover,* $\theta^I > \theta^* > \theta^C$. *If production is linear,* $F'' = 0$, *the equilibrium allocation* (N, θ) *under both bargaining regimes coincides and is equal to the efficient allocation.*

Proof. Fix some arbitrary $\bar{N}$. At this level of employment the left-hand side of (55) is smaller than that of (56). This follows from the fact that with concave production $-F'(N) + \frac{1}{\beta} \int_0^1 x^{\frac{1}{\beta}-1} F'(Nx)\, dx = -\int_0^1 x^{\frac{1}{\beta}} F''(Nx)\, dx$ (integration by parts). Finally, the left-hand side of (55) is larger than that of (57). This is a direct consequence of the assumption of a concave production function. In θ-N-space, the policy function of the planner will be strictly below the policy function derived under individual bargaining and strictly above the policy function derived under collective bargaining (the right-hand side of the policy functions is strictly increasing in θ). A positively sloped Beveridge curve completes the proof. With linear production $F'' = 0$ and $\frac{F}{N} - F' = 0$. The policy functions are then identical and as such are employment and the labor market tightness. ■

As suggested earlier, the individual bargaining regime results in inefficient overemployment. Controlling for congestion, firms post too many vacancies because the induced increase in employment decreases the wage for all employed workers. Collective bargaining potentially acts as a counterforce to this employee hold-up. The collective decision whether to work provides a channel for wages to increase and employment to decrease, thus acting against the firm's overemployment incentive. As we have seen, however, the larger threat point allows workers to bargain too high a wage. Thus, employment will 'undershoot' and decrease beyond the socially optimal level. Collective bargaining is generally not an appropriate institutional setting to internalize the adverse effects of the wage externality.

If the production function was linear in N, the wage externality vanishes under both bargaining regimes. In this situation the firm has no incentive to use vacancy posting strategically. Moreover, however, there will be no difference between collective and individual bargaining either. In general, the bargained wage (under both regimes) is a function of the worker's production "value". This value is the average product under collective bargaining and the marginal product under individual bargaining. Both coincide with linear production. Assuming a linear production/revenue function not only removes the wage externality but also equates the bargaining outcomes, and hence equilibrium allocations, under both bargaining regimes. Thus, linear production ensures efficiency if the Hosios condition is imposed.

Note, however, that if we were able to remove only the strategic wage effect ($w' \equiv 0$) without having to impose a linear production/revenue function, the result would be different. In this (hypothetical) situation, the equilibrium outcome in the economy with individual bargaining is efficient if the Hosios condition holds. The economy with collective bargaining, however, is again characterized by "underemployment" because the workers collectively can bargain for higher wages due to the threat of going on strike. Policies with the (side) effect of "switching off" (or dampening) the wage externality lead to (an increase of) efficiency under individual bargaining whereas the same policies reduce welfare under collective bargaining. We discuss some implications of these findings after a short classification.

6 Discussion and Conclusion

Collective wage bargaining is usually associated with an inefficient allocation in the labor market. The strategic advantage of bargaining collectively and deciding jointly whether to work (i.e. forming a union) allows employees to bargain higher wages (by means of rationing labor), see McDonald and Solow (1981). This view is supported by standard neoclassical models in which the bargaining setting is the only friction in the labor market. More recently, it has become widely acknowledged that the labor market is characterized by a variety of important frictions. First and foremost, the fact that heterogeneities (which come in many forms) prevent instantaneous matching has become a key element in most prominent modeling frameworks (see, e.g. Pissarides, 2000, or Rogerson et al., 2005). In this paper, we analyzed the nature of the interaction between the search friction and the collective bargaining friction. Building on the work of Smith (1999), Cahuc and Wasmer (2001) and Cahuc et al. (2008) we allowed for collective bargaining in a search and bargaining economy with concave production. Collective bargaining affects this economy via two channels at the firm level. First, workers who bargain collectively strive for higher wages. Since they have the possibility to go on strike and shut down production, the firm is more eager to strike an agreement. Second, the firm anticipates this hold-up and will over-employ strategically in order to decrease the wage. Comparing the adverse effects on wage setting and the wage externality, we derived a number of interesting results. In our simple framework, aggregate employment under collective bargaining always falls short of that under individual bargaining. The wage hike effect will always dominate the overemployment effect. The analogue, however, is not necessarily true for the wage. At the aggregate level the collectively bargained wage is smaller than the equilibrium wage under individual bargaining if the replacement rate is sufficiently low. This effect is due to the fact that the endogenous outside option, which is a function of prevailing wages *and* the reemployment probability, decreases under collective bargaining. Moreover, we show that *both* bargaining regimes deliver inefficient allocations.

These insights have important policy relevance. First, labor markets with search frictions and convex production technologies/revenue functions are characterized by strategic overemployment if workers bargain their wages individually. With collective bargaining one could have hoped for 'internalizing' this effect. Our results suggests that this assertion is not warranted. Collective bargaining results in 'underemployment' from an efficiency standpoint and thus cannot act as a second-best institutional setting. Second, policies that aim at changing wage flexibility (minimum wages, regulation of the length of contracts) are found to have countervailing effects depending on the wage setting regime. With individual bargaining, less wage flexibility reduces the strategic wage effect and is likely to increase welfare in situations where firms over-employ. Under collective bargaining the opposite is true. Less wage flexibility decreases the overemployment incentive and hence makes the 'underemployment' effect of collective bargaining worse. The general message is that labor market reforms have to take the bargaining regime into account.

Our framework offers a number of avenues for fruitful further research. First, analyzing the issue of optimal wage flexibility (or the optimal contract length) in a model that allows for different institutional settings of the wage bargain is an important task. With this we can get a deeper insight into the effects and interactions of various frictions in the labor market. Moreover, it is interesting to see how the dynamic behavior of wages and employment (over the business cycle) differs under different bargaining institutions. This is especially important to reconcile the different labor market experience of Continental Europe vis-à-vis the U.S. (see e.g. Hall, 2003 and Shimer, 2004 for an analysis of the effects of wage rigidity under individual bargaining). Second, the issue of different bargaining set-ups within the collective bargaining scheme deserves further attention. In static neoclassical models it is well known that bargaining over wages and employment increases efficiency (see e.g. McDonald and Solow, 1981; see, however, Oswald, 1993). Is this also true in a dynamic search model with large firms? Depending on whether or not bargaining over wages and vacancies offers a way to achieve efficiency gains under certain conditions, we can shed new light on how to organize labor markets and help ranking different bargaining institutions according to their welfare implications. Third, our paper is a step towards a better understanding of the emergence of collective vs. individual bargaining as called for by Cahuc et al. (2008, 961). In their heterogenous worker search and bargaining economy, the wage bargains of different groups of workers are interdependent in a, in retrospect, straightforward way (the degree of overemployment of a group of workers is a function of the technological substitutability between these groups and their relative bargaining weights). An integrated model of heterogenous labor and individual vs. collective wage bargaining will help improving our knowledge of the relative performance and existing patterns of wage setting institutions predominant in the real world.

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7 Referee's Appendix

7.1 Derivation of equation (11)

Differentiating the maximized Bellman equation with respect to N_i, gives a differential equation for the costate variable Π_i' :

$$\begin{aligned} r\Pi_i'(N_i) &= F'(N_i) - w_i'(N_i)N_i - w_i(N_i) - cV_i'(N_i) \\ &\quad + \Pi_i''(N_i)\frac{dN_i}{dt} - \left[\lambda^s - \lambda^m(\theta)V_i'(N_i)\right]\Pi_i'(N_i). \end{aligned} \tag{58}$$

From the f.o.c., $V'[-c + \lambda^m \Pi_i'] = 0$. In steady state, $dN_i/dt = 0$. Rearranging gives (11).

7.2 Derivation of equations (18) and (19)

The first-order maximization condition derived from the individual wage bargain satisfies

$$\beta\frac{\frac{\partial\tilde{W}_i}{\partial w_i}}{\tilde{W}_i} + (1-\beta)\frac{\frac{\partial\Pi_i'}{\partial w_i}}{\Pi_i'} = 0. \tag{59}$$

We earlier showed $\Pi_i' = \frac{F'(N_i) - w_i(N_i) - w_i'(N_i)N_i}{r+\lambda^s}$ and $\tilde{W}_i = \frac{w_i - rW^b}{r+\lambda^s}$. When bargaining the wage, N_i is held fixed and w_i is taken parametric. Hence, paying the worker an additional \$ changes the return for the firm by $-\frac{1}{r+\lambda^s}$. Adding a \$ to the wage changes the worker's utility by $\frac{\partial\tilde{W}_i}{\partial w_i} = \frac{1}{r+\lambda^s}$. Taken together, the f.o.c. in (59) becomes the usual sharing rule $\beta\Pi_i' = (1-\beta)\tilde{W}_i$. Substituting with Π_i' and $\tilde{W}_i$, the bargained wage satisfies the linear differential equation

$$w_i = \beta w_i'(N_i)N_i + \beta F'(N_i) + (1-\beta)rW^b. \tag{60}$$

This is equation (18) in the main text. In what follows, we suppress the firm index for the remainder of this section and replicate the solution obtained by Cahuc et al. (2008). Dropping the constant $(1-\beta)rW^b$ and rearranging, the wage solves

$$\frac{dw}{dN} + \frac{w}{\beta N} = \frac{F'(N)}{N}. \tag{61}$$

The associated homogenous equation (the LHS of (61) set equal to zero) has the solution

$$w = C(N)N^{-\frac{1}{\beta}}, \tag{62}$$

whereby

$$\frac{dw}{dN} = -\frac{1}{\beta}CN^{-\frac{1}{\beta}-1} + C'(N)N^{-\frac{1}{\beta}}. \tag{63}$$

Using (62) and (63) in (61) gives $C' = F'(N)N^{\frac{1}{\beta}-1}$ or, after integration,

$$C(N) = \int_0^N F'(y)y^{\frac{1}{\beta}-1}dy + D, \tag{64}$$

where D is a constant of integration. The general solution to (61) is found by combining (64) and (62):

$$w = N^{-\frac{1}{\beta}} \int_0^N F'(y)\, y^{\frac{1}{\beta}-1} dy + DN^{-\frac{1}{\beta}}. \tag{65}$$

Making the substitution $y = zN$, the first term becomes $N^{-\frac{1}{\beta}} \int_0^N F'(y)\, y^{\frac{1}{\beta}-1} dy = N^{-\frac{1}{\beta}} \int_0^1 F'(zN)\,(zN)^{\frac{1}{\beta}-1} N dz$, so that

$$w = \int_0^1 F'(zN)\, z^{\frac{1}{\beta}-1} dz + DN^{-\frac{1}{\beta}}. \tag{66}$$

To pin down a particular solution, we follow CMW and impose $\lim_{N\to 0} w(N)\,N = 0$, whereby $D = 0$. Adding the constant $(1-\beta)\,rW^b$ and noting (65), we get the solution in (19).

7.3 Derivation of equation (21)

The first-order maximization condition derived from (20) satisfies

$$\beta \frac{\frac{\partial \tilde{W}_{i,c}}{\partial w_i}}{\tilde{W}_i, c} + (1-\beta) \frac{\frac{\partial \tilde{\Pi}_i}{\partial w_i}}{\tilde{\Pi}_i} = 0. \tag{67}$$

Making use of (14) and (16), we get $\beta \frac{F}{N} - \beta w = (1-\beta)\, w_i - (1-\beta)\, rW^b$. Solving for w_i gives (21).

7.4 Derivation of equation (25)

Integrating the definition $dF/dN - (\alpha/N)F(N) = 0$, $\int_{F(\bar{\kappa})}^{F(N)} F^{-1} dF = \alpha \int_{\bar{\kappa}}^{N} N^{-1} dN$ for some $\bar{\kappa} > 0$ shows $\ln F(N) = \alpha \ln N + \ln F(x) - \alpha \ln x$, or equivalently $F(N) = \kappa N^{\alpha}$, $\kappa \equiv F(\bar{\kappa})/\bar{\kappa}^{\alpha}$ (> 0). Accordingly,

$$\begin{aligned} \delta &= \frac{\beta}{N_i}\left[F'(N) - \frac{F(N_i)}{N_i} - \frac{N_i}{\beta}\int_0^1 x^{\frac{1}{\beta}} F''(N_i x)\, dx\right] \\ &= \frac{\beta\kappa}{N_i}\left[\alpha N^{\alpha-1} - N^{\alpha-1} + \alpha\,[1-\alpha]\,\frac{N_i^{\alpha-1}}{\beta}\int_0^1 x^{\frac{1}{\beta}+\alpha-2} dx\right]. \end{aligned} \tag{68}$$

Collecting terms and integrating gives (25):

$$\begin{aligned} \delta &= \beta\kappa N^{\alpha-2}(1-\alpha)\left[-1 + \frac{\alpha}{\beta}\left[\frac{x^{\frac{1}{\beta}+\alpha-1}}{\frac{1}{\beta}+\alpha-1}\right]_0^1\right] \\ &= -\beta\kappa N_i^{\alpha-2}(1-\alpha)^2 \frac{1-\beta}{1-[1-\alpha]\,\beta} \quad (< 0). \end{aligned} \tag{69}$$

7.5 Derivation of equation (29)

Eliminating the wage from the first equation in (19) and (26) gives

$$F'(N_i) - (1-\beta) rW^b = \frac{r+\lambda^s}{\lambda^m(\theta)}c + N_i^{-\frac{1}{\beta}} \int_0^{N_i} x^{\frac{1}{\beta}-1} F'(x)\, dx + N_i \int_0^1 x^{\frac{1}{\beta}} F''(N_i x)\, dx. \tag{70}$$

Integration by parts implies (footnote (18))

$$N_i^{-\frac{1}{\beta}} \int_0^{N_i} x^{\frac{1}{\beta}-1} F'(x)\, dx = \beta F'(N_i) - \beta N_i \int_0^1 x^{\frac{1}{\beta}} F''(N_i x)\, dx. \tag{71}$$

Substituting with (71) in (70) and canceling $(1-\beta)$ yields (29).

7.6 Derivation of equation (34)

The wage in (33) is obtained by combining (31) and (32) and collecting terms:

$$w_i^C = (1-\beta) rW^b + \frac{\beta}{\alpha\left(N_i^C\right)} \left[rW^b + \frac{r+\lambda^s}{1-\beta} \frac{c}{\lambda^m(\theta)} \right].$$

Expanding this expression gives

$$w_i^C = rW^b + \beta \left(\frac{1}{\alpha\left(N_i^C\right)} - 1 \right) rW^b + \frac{\beta}{1-\beta} \frac{r+\lambda^s}{\lambda^m(\theta)} c + \frac{\beta}{1-\beta} \frac{r+\lambda^s}{\lambda^m(\theta)} \left(\frac{1}{\alpha\left(N_i^C\right)} - 1 \right).$$

Collecting terms yields (34):

$$w_i^C = rW^b + \frac{\beta}{1-\beta} \frac{r+\lambda^s}{\lambda^m(\theta)} c + \beta \left(\frac{1}{\alpha\left(N_i^C\right)} - 1 \right) \left(rW^b + \frac{r+\lambda^s}{\lambda^m(\theta)} \frac{c}{1-\beta} \right).$$

The first two terms on the right-hand side are equal to w_i^I. As F/N is decreasing (concavity), $F' < F/N$ $(\alpha < 1)$, hence the third term is positive.

7.7 Derivation of equation (40)

Combining the reservation wage equation (37) with the partial equilibrium wage under individual bargaining (30) gives the reservation wage as a function of θ:

$$rW^b = b + \frac{\beta}{1-\beta} \frac{c}{\lambda^m(\theta)} \left[r + \lambda^s + p(\theta) \right]. \tag{72}$$

The aggregate policy function follows after plugging the reservation wage in the partial equilibrium employment condition and dropping the firm subscript:

$$F' - N \int_0^1 x^{\frac{1}{\beta}} F''(Nx)\, dx = b + \left[p(\theta) + \frac{r+\lambda^s}{\beta} \right] \frac{\beta}{1-\beta} \frac{c}{\lambda^m(\theta)}. \tag{73}$$

Using integration by parts, i.e. footnote (18), the left-hand side is equal to

$$\frac{1}{\beta} N^{-\frac{1}{\beta}} \int_0^N z^{\frac{1}{\beta}-1} F'(z)\, dz = \frac{1}{\beta} N^{-\frac{1}{\beta}} \int_0^1 (Nx)^{\frac{1}{\beta}-1} F'(Nx)\, N dx, \tag{74}$$

where the second equality makes use of the substitution $z = Nx$. Substituting this expression back in (73) and multiplying by β yields an implicit expression for the policy function:

$$\int_0^1 x^{\frac{1}{\beta}-1} F'(Nx)\, dx = \beta b + [r + \lambda^s + \beta p(\theta)] \frac{\beta}{1-\beta} \frac{c}{\lambda^m(\theta)}.$$

7.8 Derivation of the hypothetical $w' = 0$ model

We isolate the employment effect by simply imposing $w' = 0$ in the job creation curve $\bar{J}_i = \Pi'_i|_{w'=0}$:

$$w = F' - \frac{r + \lambda^s}{\lambda^m(\theta)} c. \tag{75}$$

The wage setting curve, derived from $w_i = \arg\max \left(\tilde{W}_i^{\beta} \bar{J}_i^{1-\beta}\right)$, is given by

$$\bar{w}_i = (1-\beta)\, rW^b + \beta F'(N_i). \tag{76}$$

We can compare this to the wage schedules under both individual and collective bargaining where $w' > 0$. Under individual bargaining, we had

$$\begin{aligned} w_i &= (1-\beta)\, rW^b + \int_0^1 x^{\frac{1}{\beta}-1} F'(N_i x)\, dx = N^{-\frac{1}{\beta}} \int_0^N x^{\frac{1}{\beta}-1} F'(x)\, dx \\ &= \beta F'(N_i) - N_i \int_0^1 x^{\frac{1}{\beta}} F''(N_i x)\, dx \quad \left(> \beta F'(N_i)\right). \end{aligned} \tag{77}$$

The wage outcome under collective bargaining was $w_i = (1-\beta)\, rW^b + \beta \frac{F}{N_i}$. If $\alpha' = 0$, concavity requires $\alpha < 1$. Hence, the wage schedule moves further outwards if we go from the $w' = 0$ model to the individual model and to the collective model where $w_i = (1-\beta)\, rW^b + \frac{\beta}{\alpha} F' N_i$.

Combining $\bar{J}_i$ and $\bar{w}_i$ gives the hypothetical employment at the firm level:

$$F'\left(\bar{N}_i\right) = rW^b + \frac{r + \lambda^s}{\lambda^m(\theta)} \frac{c}{1-\beta}.$$

Eliminating rW^b using the reservation wage gives

$$F'\left(\bar{N}_i\right) = \frac{r + \lambda^s + \beta p(\theta)}{\lambda^m(\theta)} \frac{c}{1-\beta} + b. \tag{78}$$

If $\alpha' = 0$, we can explicitly solve this expression and compare it to the one in the main text:

$$\bar{N}_i = \left[\frac{\alpha}{b + \left(\frac{r+\lambda^s}{\lambda^m(\theta)}\right) \frac{\beta}{1-\beta} c} \right]^{\frac{1}{1-\alpha}}. \tag{79}$$

The wage that solves $\bar{J}_i$ and $\bar{w}_i$ is $\bar{w}_0 = rW^b + \frac{\beta}{1-\beta} \frac{r+\lambda^s}{\lambda^m(\theta)} c$, i.e., the same as in the individual bargaining case with $w' > 0$. Consequently, eliminating the reservation wage yields again (39).

7.9 Derivation of equation (43)

Collecting terms in the partial equilibrium expression for the wage under collective bargaining, (33), yields

$$w_i = \left(1 - \beta + \frac{\beta}{\alpha(N_i)}\right) rW^b + \frac{\beta}{\alpha(N_i)} \frac{r + \lambda^s}{1 - \beta} \frac{c}{\lambda^m(\theta)}. \tag{80}$$

Combining (80) with the reservation wage equation (37), we get the reservation wage as a function of θ:

$$rW^b = \frac{r + \lambda^s}{r + \lambda^s + \beta p(\theta)} b + \frac{\beta}{1 - \beta} \frac{p(\theta)(r + \lambda^s)}{\alpha(N_i)(r + \lambda^s) - [1 - \alpha(N_i)] \beta p(\theta)} \frac{c}{\lambda^m(\theta)}. \tag{81}$$

Combining this equation with the expression for the partial equilibrium employment (31) gives the policy function:

$$F'(N) \left[1 - \frac{\beta p(\theta)}{r + \lambda^s} \left[\frac{1}{\alpha(N)} - 1\right]\right] = b + \frac{r + \lambda^s + \beta p}{1 - \beta} \frac{c}{\lambda^m(\theta)} \tag{82}$$

7.10 Derivation of equation (48)

(48) is simply obtained by combining the partial equilibrium wage with the reservation wage equation. One way of doing this is to start from the partial equilibrium wage $w = (1 - \beta) rW^b + \frac{\beta}{\alpha}\left(rW^b + \frac{r+\lambda^s}{1-\beta} \frac{c}{\lambda^m(\theta)}\right)$, and combine it with the reservation wage equation in (37):

$$w = (1 - \beta) \left[\frac{r + \lambda^s}{r + \lambda^s + p(\theta)} b + \frac{p(\theta)}{r + \lambda^s + p(\theta)} w\right] + \frac{\beta}{\alpha(N_i)} \left(rW^b + \frac{r + \lambda^s}{1 - \beta} \frac{c}{\lambda^m(\theta)}\right).$$

Collecting terms yields

$$\begin{aligned} (r + \lambda^s + \beta p(\theta)) w &= (r + \lambda^s)(1 - \beta) b + \\ &+ \frac{\beta}{\alpha(N_i)} \left[rW^b (r + \lambda^s + p(\theta)) + \frac{r + \lambda^s}{1 - \beta} \frac{c}{\lambda^m(\theta)} (r + \lambda^s + p(\theta))\right]. \end{aligned}$$

Substituting once more with (37) and collecting terms gives (48).

7.11 Derivation of equation (51) and signs of equations (53) and (54)

We expand the equilibrium wage under collective bargaining, assuming $\alpha' = 0$:

$$\begin{aligned} w^C &= b + b\left(\frac{1 + \beta\left(\frac{1}{\alpha} - 1\right)}{1 - \beta\left(\frac{1}{\alpha} - 1\right)\frac{p(\theta^C)}{r+\lambda^s}} - 1\right) + \\ &+ \frac{\beta}{1 - \beta} \frac{c}{\lambda^m(\theta^C)} \left[r + \lambda^s + p\left(\theta^C\right)\right] \left(\frac{1}{\alpha} \frac{1}{1 - \beta\left(\frac{1}{\alpha} - 1\right)\frac{p(\theta^C)}{r+\lambda^s}} - 1\right). \end{aligned}$$

Rearranging and collecting terms gives (51). The sign of (53) is determined by the sign of the denominator:

$$1-\beta\left(\frac{1}{\alpha}-1\right)\frac{p\left(\theta\right)}{r+\lambda^{s}}>0. \tag{83}$$

The inequality follows because rearranging (83) yields

$$\alpha>\frac{\beta p}{r+\lambda^{s}+\beta p\left(\theta\right)}, \tag{84}$$

i.e. what is required for an equilibrium to exist, see (44). (84) also determines the sign of (54), which is negative if $\alpha-\beta\left(1-\alpha\right)\frac{p\left(\theta^{C}\right)}{r+\lambda^{s}}>1$, which, after rearranging, is equal to (84).