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# Derivatives and Default Risk\*

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## Abstract

Upstream producers that possess market power, sell forwards with a lengthy duration to regional electricity companies (REC). As part of the liberalization of the electricity market, RECs have been privatized and exposed to a possible bankruptcy threat if spot prices have fallen below their expected value. The downstream firms' expected profit is larger, when it is less likely to be bailed out, the effect on upstream profits is ambiguous while consumers loose. Options are less welfare increasing than forwards, but the difference is minimal. In the presence of bankruptcy, options are the preferred welfare maximizing market instrument.

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*JEL Classifications: D43, G33, G34, G35*

## 1 Introduction

### Motivation

The European Commission and the USA want to regulate the off-market trade of derivatives that covers 592,000 billion US-\$. This reform is one of the largest tasks for governments and regulators to come. After the insurance company American International Group (AIG) had to be backed up by the US government, due to its risky bets with derivatives in September 2008, the USA and Europe have been working on stricter regulations. Fundamental elements of the reform are Central Counter Parties (CCPs) that take over the risk in

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case of liquidity shortages. According to EU and US regulatory suggestions, standardized derivative contracts need to go through CCPs. Derivatives of this kind are often used by energy producers. Thus, it is not surprising that Eon, one of Europe's largest electricity and gas suppliers claims that it needs an additional 7.5 billion US\$ in capital, when the CCP requirements are enforced. (Financial Times, 7/10/ 2009).

This paper is a first attempt to evaluate defaults and forwards in the presence of an upstream oligopoly and downstream firms, operating in a competitive environment. In addition to potential government bailouts, the model shows that welfare decreases for another reason: the threat of market exit through insolvency affects the market equilibrium in itself. If an upstream oligopolist has sold forwards to a downstream firm, and the spot price has unexpectedly fallen, then the downstream firm might not be able to discharge its payment obligations to the oligopolist. An oligopolist reduces this risk by increasing the spot price, which has an immediate negative effect on customers. This model is applied to the electricity sector, and uses parameters that are based on historic data from England and Wales. It is based on regional electricity companies (RECs) that purchase electricity from oligopolistic generators. Before liberalization took place, RECs had local monopolies to supply residential customers. The interaction between producers and RECs takes place on a contract and spot market.

## Literature

The market environment of this model can be well framed into a branch of the industrial organization literature that was initiated by Allaz and Villa (1993) [AV], and is summarized in the following. AV's influential article shows that the presence of a contract market increases welfare, because the competition among firms is intensified. It creates a prisoner's dilemma, in which firms voluntarily sell forward some of their production on the contract market. Once they have engaged on the contract market, they find it profitable to extend production on the spot market; the marginal revenue increases with the amount that has been contracted before. Sustaining from contracting is a dominated strategy, because the other firm could increase its profits by writing contracts alone, to then become the Stackelberg leader of the game.

Mahenc and Salanié (2004) [MS] challenge the view that contract markets increase welfare. If risk neutral producers are allowed to buy their own quantity on the contract market, then it is a dominant strategy to do so in order to increase prices on the spot market. The intuition here is that producers want to increase their profits on the contract market, by increasing the spot price. In AV, producers compete in quantities on the contract and spot market, but in MS, producers compete in quantities on the future market and prices on the spot market. It is a necessary assumption that the spot market is modeled as a differentiated goods Bertrand model to ensure the strategic complementarity of prices. Another well-known method to avoid the prisoner's dilemma is to increase the time horizon, either to infinity or to a finite number of periods, where firms use trigger strategies. This has been done by Liski and Montero

(2006) [LM], who extend the two-stage model of AV and MS to a multiple period game with Bertrand and Cournot competition. Contracts are traded first, the corresponding spot market takes place one period later. In their model firms can use a trigger strategy to sustain collusion: they have to charge the monopoly price on both markets, or the price is otherwise set equal to marginal costs for all subsequent periods. Contract markets help to sustain collusion, because the spot market share decreases. Furthermore firms sell more forward, when they compete in prices, and less, when they compete in quantities to stay on the collusion path. Le Coq (2004) exhibits similar results under a different setting. Firms trade on the future market once. Quantities are delivered at multiple subsequent spot markets.

Newbery (1998) introduces contracts in a supply function model, which is more suitable to picture the electricity market. He shows that contracts that drive down the expected spot price, reduce the incentive for competitors to enter the market. Entry can thus be deterred, if incumbents hold sufficient capacity; a conception first illustrated by Dixit (1980). Murphy and Smeers (2005) [MS] introduce investment decisions in the two-markets setup. They prove that the equilibrium of a model with, and without a contract market is the same, when players have to choose capacities before they produce. The intuition behind the result is the same as in the Kreps and Scheinkman (1983) model; firms choose low capacities to avoid destructive competition and restore the Cournot equilibrium. Bushnell (2007) extends AV's model to  $n$  firms that face increasing marginal costs. He demonstrates, how the equilibrium changes, when an additional firm enters the market in the presence of a contract market, as opposed to the change in the absence of a contract market. Grimm and Zoettl (2006) [GZ] establish that a contract market decreases investment capacity in a time-varying demand model. Capacity choices decrease the positive competition effect of contract markets. Firms choose lower capacities to avoid competition, but when demand is low and capacity is not a binding constraint, then contracts do increase competition. Only when demand is certain and capacity binds, then contracts do not affect the efficiency outcome. The model of MS shows that capacity investment decisions under perfect foresight yield the same market outcome with and without a contract market. Newbery (2008) studies the effect of mergers in the presence of a contract market. He demonstrates that market power increases more after a merger, when a contract market is present. Furthermore he proves that contracts reduce capacity, which is consistent with GZ. They also increase the fraction of time that capacity is constrained, but still lower the time-weighted average price. The later finding shows that future markets increase at least consumer rents, in the presence of capacity investments, and come therefore closest to a positive contract market welfare analysis, even in the presence of capacity constraints.

GZ and Newbery (1998) are the only papers that reasonably allude, future markets could possibly be welfare decreasing, because producers scale down their installed capacity. All other model that claim, forwards are welfare decreasing, rest on very strict assumptions: differentiated goods, perfect information and collusion (LM) or allowing producers to buy forward (MS). However these as-

sumptions can be counteracted by a regulator, if they prove to be realistic. In line with the majority of articles, forwards reduce the spot price in this essay, however, the efficiency gain is lower, when buyers face an insolvency risk for low spot prices. The models examined so far, assume "no-arbitrage profits" from futures. They model the upstream market and assume that buyers accept any forward price, as long as it is not below the expected spot price. In equilibrium, the forward price equals the spot price, an assumption that often does not hold empirically.<sup>1</sup> This essay models both market participants, and allows the forward price to be different from the spot price.

### **The electricity sector**

Few papers have studied the impact of retail competition on contracts. Exceptions are Powell (1993) and Green (2004), on which this model is closely based. Powell shows that there are more forwards sold, when producers coordinate on the forward and spot market, as opposed to a market, where producers exclusively coordinate on the forward market. Green finds that the number of contracts sold is higher in an industry, where an incumbent does not face any competition (in the presence of yardstick regulation) as compared to an incumbent that is faced by a competitive fringe, which always charges the spot price (in the presence of switching costs).

After the electricity sector has been liberalized, incumbent retailers have faced fierce competition as opposed to producers, which have remained in an oligopoly position. A famous retail bankruptcy example for the British market is the failure of 'Independent Energy' that collapsed in 2000. Thus, RECs have become vulnerable to the risk of spot prices that have fallen below the expected level at the time, when contracts were written. If they charge a retail price that exceeds the spot price substantially, then some of their clients leave their previous electricity supplier to be supplied by a competitive fringe, which buys and re-sells electricity for the current spot price. Green takes account of the market reforms and calibrates his model with historic data from the English/Welsh electricity sector in the 1990s that this model utilizes.

In the course of the 1990 electricity market liberalization of the UK, the RECs were privatized. They became either public limited companies (plc) or they were bought by large domestic producers (e.g. Powergen and Scottish Power) and foreign firms (e.g. Eon and EDF). According to the Utilities Act in 2000, all former RECs had to separate their supply and distribution businesses. The forwards studied here, are "over-the-counter" (OTC) contracts that exclusively concern the supply business part, which supplies management services, such as billing, customer service, metering, debt collection and administration. There is not much capital bound in the newly formed retailer's business, a miscalculation of past forward purchases can easily destabilize the financial condition and force a retailer to exit the market.

### **Objectives**

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<sup>1</sup>One of the first empirical essays on this issue is Protopapadakis and Stoll (1983).

This model does not reconstruct bankruptcy probabilities for the electricity market in England and Wales, it uses the noise that was generated by its liberalization to justify the assumption that the spot market alone is affected by the threat of insolvency. Before the liberalization, incumbent retailers held monopoly positions and bankruptcies were highly unlikely. The forward market, described in this model, has a very long time horizon, such that the liberalization was not anticipated, when the contract market opened. I study the market equilibrium, where the bankruptcy threat is anticipated, in a different paper (Scholz, 2009). That model uses the same assumptions as the literature described in the beginning; retailers are modeled just implicitly and buy any number of forwards, offered by producers, but it lacks the adoptability to the electricity market. Furthermore closed-form solutions cannot be derived, when the default risk is endogenous. It shows that the anticipation of bankruptcy at the closure of contracts reduces the number of contracts. This induces the negative welfare effect of the insolvency risk to be even larger. The results presented here, can thus be interpreted as being a conservative estimation.

Furthermore this model compares welfare effects between forwards and options. It demonstrates that options yield a slightly lower welfare, but are easily the preferred instrument in the presence of bankruptcy. It is the first model that allows a welfare analysis, in which forwards are compared to options, whose strike price is endogenous. The model of this essay has two parts; the first part (section 2) studies the impact of bankruptcy in the presence of a forward market. The second part (section 3) compares the market equilibrium in the presence of forwards, with the one in the presence of options. Section 4 concludes.

## 2 Forwards

### 2.1 Pre-liberalization period

There is an upstream market with producers, who sell forwards to incumbent regional electricity companies. Producers set the price on the forward market and the quantities on the spot market.<sup>2</sup> RECs decide, how many forwards they want to buy.

Producers cooperate on the forward market, but do not coordinate on the spot market. If there was no coordination among producers on the forward market, the price would equal marginal costs, which is unrealistic for the electricity market. If there was coordination on the forward and spot market, such that the total spot quantity is the monopoly quantity, then the number of contracts increases compared to a market, where coordination is restricted to the forward market.<sup>3</sup> When a REC has paid a high forward price  $p^f$ , and the spot price is

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<sup>2</sup>Until 1995, the generation duopoly in the UK, even though it held less than 50% of generation capacity, set the price 90% of the time, see Wolfram (1999). Section 2.2 explores this issue in greater depth.

<sup>3</sup>The proofs are given in Powell (1993) on p. 449-450.

unexpectedly low, then the REC makes a loss. The more contracts have been traded in the past, the larger the loss and the bankruptcy probability; a positive correlation of these two variables is assumed. Thus if producers cooperate on the spot market, retailers would have bought more contracts, and the default probability would be even larger. The results presented here, can then again be interpreted, as being a conservative estimation.

### 2.1.1 Production sector

Producers maximize their expected profits  $E\pi^P$ , while RECs maximize a mean-variance utility function of their profit  $\pi^R$ . There are two symmetric producers and RECs, such that in equilibrium the production quantity of producer  $i$  equals that of producer  $j$  and the number of forwards sold to each REC is equal. Call  $f_i(f_j)$  the number of forwards sold by producer  $i$  ( $j$ ) and purchased by REC  $i$  ( $j$ ).<sup>4</sup> The game is solved by backward induction. When producers set their spot market quantities, they do so given the number of forwards  $f$  sold. Producers maximize their *expected* profits, as they face an uncertain demand. Producer  $i$ 's objective is

$$\max_{Eq_i} E\pi^P = (Ep - c)Eq_i - f_i(Ep - p^f) + Cov(p, q_i) \quad (1)$$

where  $q_i$  is the spot quantity of producer  $i$ ,  $c$  is marginal cost and  $p^f$  is the forward price. The first part of (1) is the spot market profit, and the second the contract market profit.  $Cov(p, q)$  is the constant covariance of the spot price and quantity. The linear inverse residual demand function with an intercept  $A$  and slope  $-b$  can be expressed as

$$p = A - bq_i - bq_j + \epsilon \quad (2)$$

where  $\epsilon \sim N(0, \sigma^2)$ . All customers that do not pay the retail price are described by the term "residual"; in particular large industrial customers, who can buy electricity from the production sector directly. It is straightforward to solve (1) for the expected spot quantity of producer  $i$ ;

$$Eq_i = \frac{A - c + 2bf_i - bf_j}{3b}. \quad (3)$$

As producers are symmetric, the expected spot price is

$$Ep = \frac{A + 2c - bf_i - bf_j}{3} \quad (4)$$

As mentioned before, producers set the forward price and maximize their objective accordingly.

$$\max_{p^f} E\pi^P = (Ep - c)Eq - f(Ep - p^f) + Cov(p, q) \quad (5)$$

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<sup>4</sup>In Germany RECs ("Stadtwerke") often still buy all electricity exclusively from one generator, even though they are not owned by them anymore.

The first order condition of (5) can be solved for  $p^f$

$$p^f = Ep + \left( \frac{\partial f_i}{\partial p^f} \right)^{-1} \left[ f_i - (Eq_i - f_i) \frac{\partial Ep}{\partial p^f} - (Ep - c) \frac{\partial Eq_i}{\partial p^f} \right] \quad (6)$$

Powell (1993) shows that the forward price is larger than the expected spot price. The capacity literature can be viewed parallel to this observation; in order to mitigate the negative effect of forwards on their market power, producers charge a higher price than  $Ep$ , whereas in the capacity literature, incumbents might have an incentive to over-invest in capacity as a strategic device; see Spence (1977), Dixit (1980) and Newbery (1998) as a more recent application to the electricity market. In order to find  $\frac{\partial f}{\partial p^f}$ , RECs are modeled that choose the optimal number of contracts, given the forward price that is offered by the production sector.

### 2.1.2 Retail sector

A fix number of customers served by an incumbent REC,  $V$  purchases electricity for a regulated price  $r$  before the market was reformed.

The two RECs that have been characterized by subindexes  $i$  and  $j$  in the last section, operate in separate markets but are symmetric. In reality there were 12, and not two heterogeneous risk averse RECs in England and Wales; cooperation would thus have been very difficult to implement, and is not assumed in this model. A REC maximizes a mean-variance function applied to its profit as in Powell (1993),

$$U_i = E(\pi_i^R) - \frac{1}{2} \lambda \text{Var}(\pi_i^R) \quad (7)$$

where the expected profit is

$$E(\pi_i^R) = V[r - E(p)] + f_i[E(p) - p^f] \quad (8)$$

The variance is  $\text{Var}(\pi_i^R) = \text{Var}[V(r - p) + f_i(p - p^f)] = \text{Var}[p(f_i - V)] = (V - f_i)^2 \sigma^2$ , where the only variable part is the price. RECs choose the optimal number of contracts, they purchase. REC  $i$ 's objective is  $\max_{f_i} U_i = E(\pi_i^R) - \frac{1}{2} \lambda \text{Var}(\pi_i^R)$ , which is solved for  $f_i$

$$f_i = V + \frac{E(p) - p^f}{\lambda \sigma^2 - [\partial E(p) / \partial f_i]} \quad (9)$$

(4) is used to manipulate (9), in order to derive REC  $i$ 's demand for contracts as a function of the number of contracts bought by the other REC.

$$f_i(f_j) = \frac{V(b + 3\lambda\sigma^2) + A + 2c - f_j - 3p^f}{2b + 3\lambda\sigma^2} \quad (10)$$

Due to symmetry,  $f_i(f_j)$  and  $f_j(f_i)$  solve for REC's demand function of contracts, given the forward price:



$$f(p^f) = \frac{V(b + 3\lambda\sigma^2) + A + 2c - 3p^f}{3(b + \lambda\sigma^2)} \quad (11)$$

Equation (6) and (11) can be solved for the optimal number of forwards,  $f_i^* = f_j^* = f^*$ , based on the underlying parameters. The derivatives in (6) can easily be derived, using (3), (4) and (11).

$$f^* = \frac{V(\frac{b}{3} + \lambda\sigma^2) - \frac{1}{9}(A - c)}{\frac{10}{9}b + 2\lambda\sigma^2} \quad (12)$$

Furthermore the first order condition of  $p^f$ , (6) is solved with (3), (4), (11) and (12) to express  $p^f$  based on the expected spot price and the number of contracts signed

$$p^f = Ep + \frac{1}{9}(A - c) + f^*(\frac{7}{9}b + \lambda\sigma^2) \quad (13)$$

This shows that the forward price exceeds the level of the expected spot price. The difference increases with the risk aversion parameter. Even for  $\lambda = 0$ , the forward price exceeds the expected price, because contracts decrease future spot prices (see also Powell, 1993). The demand for contracts decreases with the number of contracts the other REC purchases, (10), which is a justified result, as RECs were of considerable size. The larger the demand elasticity, the more RECs hedge, because the negative impact on the spot price per forward contract, increases with  $b$ . (see (4)) Another reason for price divergence is the large percentage of OTC trade in the electricity sector, which implicates non-transparent pricing.<sup>5</sup>

## 2.2 Post liberalization period

Since the market was reformed, residential customers have been able to choose their electricity supplier. If a customer chooses to find a new supplier in this model, then she would receive her electricity from the competitive fringe. Customers are assumed to face switching costs, such that some are willing to remain with their regional electricity company and pay a higher price.

The market share of the incumbent retailer decreases, when the retail price, which is assumed not to be regulated after liberalization, is above the current spot price. Green's (2004) simple demand expression that an incumbent retailer faces, after a competitive fringe has entered the market, is also useful for this work

$$V^{NEW} = V - h(r - p) \quad (14)$$

A high constant parameter  $h$  is interpreted by low switching costs. When switching costs are low, the incumbent's market share decreases more for  $(r - p) > 0$ .

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<sup>5</sup>In Germany for instance the liberalization of the electricity market has not yet reached the same level as in the UK, because RECs ("Stadtwerke") still hold both: distribution and supply. Over 80% of electricity is sold through bilateral contracts, most with a single incumbent generator based on historical ties. Due to commercial confidentiality, neither price nor quality information are revealed. (WIK, 2008)

### 2.2.1 Retail sector

The former RECs are allowed to choose the retail price  $r$ , which has been dictated by a regulator before the liberalization. Thus the new retail objective becomes

$$\max_r \pi^R = V^{NEW}(r - p) + f(p - p^f) \quad (15)$$

The optimal new retail price is

$$r^* = p + \frac{V}{2h} \quad (16)$$

There is no expectation operator in (15), because retailers know the realization of  $\epsilon$ , when they choose  $r^*$ . In the past, forward contracts were written to protect RECs from volatile pool prices, because they had to sell into a regulated market with a formerly fix retail price. After the liberalization, this alleged protection has jeopardized retailers that now have to act in a volatile retail price environment. Meanwhile the market has become more competitive and retailers have to carry the burden of contracts. This model assumes that there is a positive probability of bankruptcy, when a retailer incurs a loss based on the contract of differences. The return of forwards is negative, when the spot price is below the forward price, otherwise contracts yield positive returns. If  $p < p^f$  the situation worsens with low switching costs (large  $h$ ), because in that case, retailers can just charge a low mark-up, see (16). The bankruptcy probability consists of an exogenous part  $s$ , which contains information about its ownership structure, how likely the retailer is able to raise loans from banks, and how much savings it holds. Incumbent retailers might also be bailed out by their owners, when these are able to raise sufficient funds. Owners are generally less willing to vouch for the retailers, when the loss  $-\pi^R$  is very large, which is incorporated in the bankruptcy probability. But there are also different warrantors as such; public entities are generally more willing to burn (taxpayer's) money than private entities, to preserve trust. In the English/ Welsh market all RECs were bought by private companies, some of them very large and operating worldwide, thus they would be reluctant not to act as a guarantor for their retailer, registered as public limited company, to maintain their reputation. The different owner types are expressed by the exogenous multiplier  $s$ . Thus the default probability is defined as

$$\alpha = \alpha(s\pi^R) = \begin{cases} 0 & \text{if } \pi^R > 0 \\ -s\pi^R & \text{if } \pi^R < 0 \end{cases} \quad (17)$$

The survival probability is denoted by  $\eta(\pi^R) = 1 - \alpha(\pi^R)$ . If a retailer has a low  $s$  then it is owned by an entity that is more likely to guarantee for its retailer's payments, when  $\pi^R < 0$ . If  $\pi^R$  is positive, then the bankruptcy threat is absent,  $\alpha = 0$  and  $\eta = 1$ . If bankruptcy occurs or not, is irrelevant in this model; it is the risk that affects the spot market equilibrium.

### 2.2.2 Production sector

The retailer's ownership structure is known in the UK, hence  $s$  can be estimated. Furthermore the number of contracts can be assessed, based on the market that the former REC operated, allowing the probability of default to be derived. Producers maximize their expected profit by choosing an optimal production quantity, where the expectation is based on, how likely it is that the retailer manages to transfer  $p^f - p$ , for the contracts signed. There is no uncertainty about the demand intercept at this stage. If a retailer fails, contracts become worthless, but producers still sell an unconstrained quantity on the spot market. The residual demand is not affected by bankruptcy, because there are other generators that can absorb customers from bankrupt, incumbent retailers. The generation capacity of the duopoly, which covered 73% of total capacity in 1990-91 decreased to 46% in 1995-96 and an estimated 38% in 2000-01 (Monopolies and Merger Commission, 1996). But until 1995, the duopoly set the pool's electricity price 90% of the time, which justifies this model's assumption that the duopoly sets a quantity that reflects the market price.<sup>6</sup>

$$\max_{q_i} E\pi_i^P = pq_i - cq_i - f_i(p - p^f)\eta_i(\pi_i^R) \quad (18)$$

To find the spot quantity of (18), a function is maximized that depends on the optimal outcome, as  $\pi^R(q)$  is a function of  $q = (q_i, q_j)$ . The optimal value of  $q$  is found by solving two separate maximization problems with  $\eta(\pi^R) < 1$  and  $\eta(\pi^R) = 1$  respectively, because  $\eta(\pi^R)$  is not continuous. Define  $\pi^{P,B}(q^B)$  to be the producer's objective, when  $\eta(\pi^R) < 1$  and  $\pi^{P,NB}(q^{NB})$  the objective, when  $\eta(\pi^R) = 1$ .<sup>7</sup>  $q^{B*}$  and  $q^{NB*}$  are the corresponding optimal values, which are compared in the four different equilibria, possible;

1.  $q^{max} = q^{B*}$  if  $E\pi^{P,B}(q^{B*}) > \pi^{P,NB}(q^{NB*})$  and  $E\pi^R(q^{B*}) < 0$ .<sup>8</sup>
2.  $q^{max} = q^{NB*}$  if  $\pi^{P,NB}(q^{NB*}) > E\pi^{P,B}(q^{B*})$   $E\pi^R(q^{B*}) > 0$ .
3.  $\pi^{P,NB}(q^{NB*}) > E\pi^{P,B}(q^{B*})$  and  $\pi^R(q^{NB*}) < 0$ : when this outcome occurs, producers prefer that retailers have a zero probability to go bust. If  $\pi^{P,NB}(q^{NB*}) + \pi^{R,NB}(q^{NB*}) > E\pi^{P,B}(q^{B*}) + E\pi^{R,B}(q^{B*})$ , producers and retailers might consider to either merge or renegotiate their contracts.
4.  $E\pi^{P,B}(q^{B*}) > \pi^{P,NB}(q^{NB*})$  and  $\pi^R(q^{B*}) > 0$ : In this case producers rather maximize the objective when their retailers could possibly default. Producers produce  $q^{max} = q^{NB}$  as they cannot force retailers to go bankrupt, when  $\pi^R(q^{B*}) > 0$ . Furthermore computing  $E\pi^{P,B}(q^{B*})$  does not make sense, because one would assume that  $\eta > 1$ . Thus this equilibrium is not realistic.

<sup>6</sup>For background information see Wolfram (1999) and Newbery (1995, 1998).

<sup>7</sup>Throughout the rest of this chapter, the superscript  $B$  stands for, "there exists a bankruptcy risk", and  $NB$  stands for, "there exists no bankruptcy risk".

<sup>8</sup>The expectation operator for profits applies to the  $B$ - case only, because there is no uncertainty, when retailers cannot possibly go bankrupt.

### 2.2.3 For a retailer's survival probability of $\eta(\pi^R) < 1$

First, the optimal spot market quantity is solved, which is set by producers. When an incumbent retailer goes bust, then the producer does not receive the forward price for the contract coverage, but sells to customers directly or through the competitive fringe. A retailer is threatened by a loss when  $p \ll p^f$ , where the price difference has to be sufficient, because retailers realize a profit from those customers who do not switch, and pay a retail price above the spot price, see (16). After the bankruptcy of a retailer, whom a producer has written contracts with, the positive transfer of  $f(p - p^f)$  would not be obtained. Thus producers minimize the default risk, by keeping the spot price up. (15) and (17) are converted, to rewrite the producer's profit as a function of the retailer loss

$$\pi_1^{P,B} = (p - c)q_1 - f_1(p - p^f) \{1 + s [V(r^* - p) + f_1(p - p^f)]\}$$

The first term is the spot market profit, the second term is the expected contract market return. The later simple contains the survival probability,  $\eta(\pi^R)$  as a multiplier. Substituting  $r^*$

$$\pi_1^{P,B} = (p - c)q_1 - f_1(p - p^f) \left\{ 1 + s \left[ \frac{V^2}{4h} + f_1(p - p^f) \right] \right\}$$

The spot price  $p$  contains  $A^* = A + \epsilon$ . The demand is known at this point and producers play the Cournot game on the spot market. At the contract market though conjectural variations  $\frac{\partial f_i}{\partial f_j}$  are assumed to equal zero.

$$\frac{\partial \pi_1^{P,B}}{\partial q_i} = A^* - 2bq_i - bq_j - c + bf_i \left\{ 1 + s \frac{V^2}{4h} + sf_i [(A^* - bq_i - bq_j) - p^f] \right\} + sbf_i^2 [(A^* - bq_i - bq_j) - p^f] \stackrel{!}{=} 0$$

The foc can be rewritten for identical retailers and producers,  $f_i = f_j = f$  and  $q_i^B = q_j^B = q^B$

$$q^{B*} = \frac{A^* - c + bf \left( 1 + s \frac{V^2}{4h} \right) + 2sbf^2(A^* - p^f)}{3b + 4sb^2f^2} \quad (19)$$

If  $q^{B*}$  is realized, then the spot price is equal to

$$p^{B*} = A^* - 2bq^{B*} = \frac{A^* + 2c - bf \left( 2 + s \frac{V^2}{2h} \right) + 4sbf^2p^f}{3 + 4sbf^2} \quad (20)$$

(20) is the optimal production quantity, when bankruptcy is possible. One can substitute (12), (16), (19) and (20) in (15) and (18) to derive  $\pi^R(q^{B*})$  and  $E\pi^P(q^{B*})$  that only depend on the underlying parameters of the model. If  $\pi^R < 0$  and  $\pi^{P,B}(q^{B*}) > \pi^{P,NB}(q^{NB*})$ , then it is for generators optimal to take the risk, that incumbent retailers are exposed to the bankruptcy threat.

#### 2.2.4

#### 2.2.5 For a retailer's survival probability of $\eta(\pi^R) = 1$

If a former REC realizes a profit, then it survives by definition and the survival probability equals one. Again, producers play the Cournot game on the spot market, while conjectural variations at the forward market are zero. The equilibrium is described through (3) and (4), where  $A$  is substituted for the realized intercept  $A^*$ .

$\pi^R(q^{AV*})$  and  $\pi^R(q^{AV*})$  can thus also be expressed by the model's parameters. When  $\pi^R > 0$  and  $\pi^{P,C}(q^{C*}) < \pi(q^{AV*})$ , generators choose a spot quantity such that the incumbent retailer survives with certainty.

### 2.3 Results

This section presents a numerical solution of this model, based on data of the electricity sector in England and Wales. In the early 1990s, there was a generation duopoly, and there were 12 incumbents in the retail sector. The two privatized firms, National Power and Powergen, held respectively 50% and 30% of the total generation capacity. The Electricity Supply Industry in England and Wales was reformed in 1990. Before its restructuring took place, there had been a state-owned Central Electricity Generating Board, responsible for generation and transmission, selling to 12 state-owned Area Electricity Boards, which were responsible for distribution. Nearly 80% of the industry's generation came from coal-fired stations, and most of the remaining electricity from nuclear power. Green's (2004) parameter values are applied in this model. He assumes marginal cost  $c$  being equal to £20/ MWh. The parameters of the residual demand curve are set to  $A = 50$  and  $b = \frac{2}{3}$ .

The welfare analysis, which is conducted later, estimates the consumer surplus based on "residual" demand.<sup>9</sup> Customers that remained with the incumbent are ignored, because this is not a general welfare analysis of the liberalization process as such. This model rather analyzes, how the bankruptcy aspect affects the market equilibrium. In the 1990s, there were already some small generators on the market, which were price takers. Therefore the profit and expected profit that are derived are "residuals", too.

Green sets the sales volume per REC to  $V=2.5$  GW representing the total sales to small customers of the 12 RECs equal to 30 GW. There are two retailers in this model; each writes contracts with one generator, nevertheless the same volume per REC of  $V=2.5$  GW is adopted, as it can be shown that even with a relatively small contract coverage, the market equilibrium is changed by the risk of default, significantly. This makes this model's findings even more meaningful. The switching cost parameter is set to  $h = 0.15$ , because incumbents lost approximately one third of their market share or 0.9 GW of sales due

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<sup>9</sup>In addition to consumer rents, welfare includes expected producer and retailer profits. Consumer rents equal the area between the inverse demand function and the spot price.

to a 10% retail price difference at that time, when the retail price was around £60/MWh. Green claims that the variance of the annual pool's price,  $\sigma^2 = 5.76$  (from 01/1990 to 01/2000) was distinctively low, because of high level contracting, market power and regulatory pressure. It is contrary to the volatility in Nordic countries, which depend heavily on rainfall, due to the importance of hydro power plants. There, the variance was equal to 34.9 between 1993 and 2003. A volatility of somewhere in between is used;  $\sigma^2 = 30$ . Green derives a risk aversion parameter of  $\lambda = 0.178$ .<sup>10</sup>

The default probability consists of the retailer's profit  $\pi^R$  and the exogenous parameter  $s$ .<sup>11</sup> In the first numerical simulation, which is summarized by figure 1, the intercept is 20% below the expected value, thus  $\epsilon = -10$ . In order to analyze bankruptcy, it must occur with positive probability. Accordingly, the necessary condition is that retailers incur a loss. Instead of choosing a small  $\epsilon$ , one could have lowered switching costs though increasing  $h$ . Former RECs make a small profit, when the switching parameter is lifted from  $h = 0.15$  to  $h = 0.14$ . Thus in the absence of the unexpected market entry, RECs would have never had to face a loss, which justifies the assumption that default was never contemplated, when the contracts were signed. Finally  $s$ , the multiplier of  $\pi^R$  for  $\pi^R < 0$  is chosen to arrive at the bankruptcy probability  $\alpha$ . This essay is not interested in finding a potential default rate of former UK RECs, however in the question, how the market equilibrium is affected, when different  $s$ -parameters are considered. Different values of  $s$  are chosen that determine a reasonably bankruptcy rate  $\alpha$ .

In this setup, the number of forwards is not affected by the default probability. It is just the production quantity that producers can influence. The number of contracts has already been chosen before market entry took place, when RECs made profits even when the spot price was below the forward price. Based on the underlying parameters, the expected price is  $Ep = 29.59$  by (4) and the optimal number of contracts  $f^* = 0.93$ , see (12). Thus 37% of total expected sales are bought on the forward market for a forward price of  $p^f = 38.35$ , see (13). One can alter the multiplier  $s$  to show, how total welfare, producer profits,  $\alpha$  and thus the retailer's loss are affected. When  $s = 0$ , then  $q^{B*} = q^{NB*}$ , because the objective (18) reduces to objective (1). When the exogenous multiplier  $s$  of the bankruptcy probability  $\alpha(\pi^{R*})$  increases by some percentage, then the default probability strictly increases by less or even decreases, as long as  $b < 1$ . Producers lower the difference of  $p^f$  and  $p$  by increasing  $p$ , (which lowers the retailer's loss) to scale down the probability that retailers fail. The intuition behind this result is simple; when a retailer goes bust, producers do not receive the transfer from contracts.

The expected producer profit increases with  $s$  as the spot price goes up. The retailer's expected loss is largest when  $s = 0$  and  $E\pi^R = -0.775$ . Retailers

<sup>10</sup>Green (2004) applies Grinold (1996)'s "grapes from wine" method, pp. 16-17.

<sup>11</sup>In order to derive profits and consumer surplus in monetary values one multiplies the values we derive by 8.780 million. Prices are in MWh and based on hourly consumption, while  $V$  is an annual capacity measured in GW. 1 GW annual capacity of electricity is equivalent to 8,780 GWh and 1 GWh=1000 MWh.

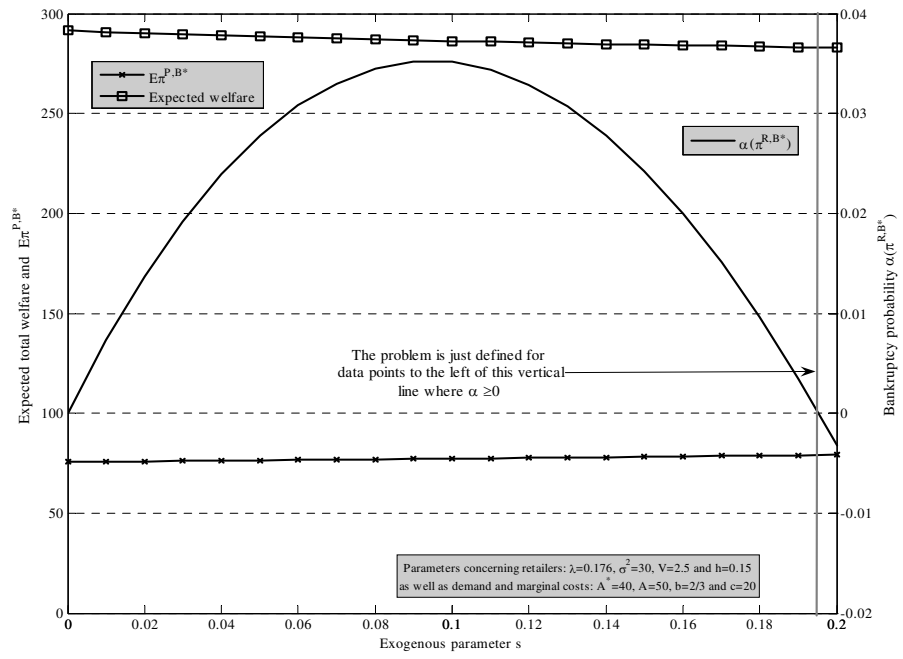


Figure 1: The effect of  $s$  on  $\alpha$ , expected welfare and profits

break even at  $s \approx 0.195$ , where producers have their largest profit of  $E\pi^P = 79.1$  up by 4.5% compared to its profit at  $s = 0$ . The bankruptcy probability reaches its peak for intermediate values of  $s$ . Total expected welfare, consisting of consumer surplus, producer and incumbent retail profits decreases from 291.5 to 283.1, when  $s$  increases from zero to 0.195 (and  $\alpha = 0$  for the second time after  $s = 0$ ) at the expense of the residual consumer surplus that decreases when  $p$  goes up. Thus a producer prefers a less solvent retailer, as producers are then committed to set a lower production quantity to keep the possible loss from retailers low. This has been the realization of equilibrium 1 at each data point on the left hand side of the gray vertical line. Equilibrium 4, which is not reasonable in reality occurs to the right of the vertical line. Next, examples for equilibrium 3, which is realized when  $\epsilon$  is smaller, are demonstrated.

The parameters of the simulation that figure 2 is based on are the same as before, except that  $\epsilon = -15$  (upper part) and  $\epsilon = -25$  (lower part). In the upper part of figure 2, one can notice again that  $\alpha$  is a concave function of  $s$ . Producers reduce the spot price when they face a less solvent retailer to increase the probability that they receive a payment when  $\epsilon < 0$  and  $p^f \gg p$ . Retailers benefit from increasing  $s$ , thus less solvent retailers have a lower loss. Producers do not benefit from low retailer reserves.  $\pi^{P,B}(q^{B*})$  falls with  $s$  up to a level where  $s = 0.12$ , which is equivalent to a bankruptcy probability of 20%. The sum of expected producer and retailer profits is illustrated. It is a convex increasing function of  $s$ . The sum decreases with  $s$  for small  $s$  and increases once  $s \approx 0.04$ , which corresponds to  $\alpha \approx 9\%$ . Thus in a range of  $s \in (0, 0.04]$ , producers and retailers prefer to merge, as  $E\pi^{P,B}(q^{B*}) + E\pi^R(q^{B*}) < \pi^{P,NB}(q^{NB*}) + \pi^{R*}(q^{NB*})$ , which is equilibrium 3.

The lower part of figure 2, where  $\epsilon$  is even smaller, shows that for any  $\alpha > 0$  or  $s > 0$ , the sum of the retail profit and expected producer profit is lower as when  $\alpha = 0$  and  $s = 0$ . Thus a merger might always be a preferable solution. It is straightforward to derive the solution for asymmetric retailers with respect to  $h$  and  $s$ . A producer, who has written contracts with a retailer that is less likely to be bailed out (high  $s$ ), or one that has customers that are more likely to switch (high  $h$ ), is more prepared to adjust the spot price downward. The other producer increases its production quantity, due to the strategic substitutability. Accordingly, the profit of the producer with the more solvent retailer, rises, while that of its competitor falls. The same holds for producers, who wrote contracts with retailers that used to cover a larger market or were more risk averse. These retailers purchased larger stakes in production plants and hold more forwards; thus they are more exposed to the risk of low spot prices today.

### 3 Options

'Contracts for differences' (CfDs) are pure financial contracts that resemble two-way forward contracts, which have been examined in section 2. One-way CfDs, which are call options are examined next. The first type has been studied by a broad literature mentioned in the introduction, the second type only by very few



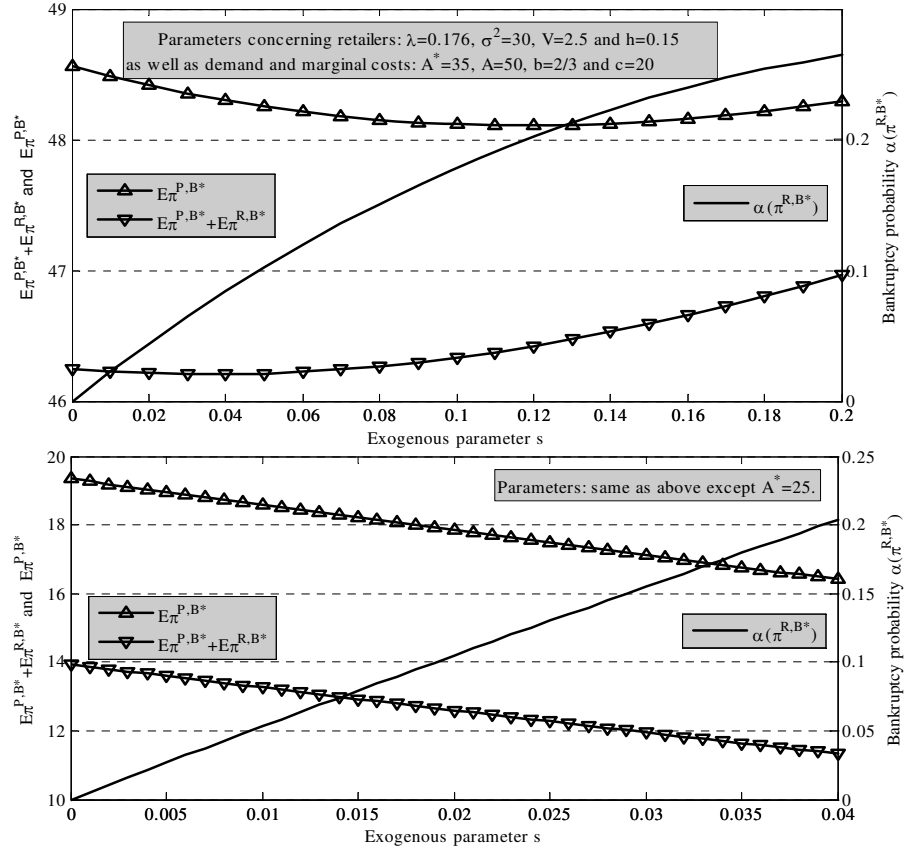


Figure 2: When renegotiations or mergers are preferable

authors. This model demonstrates, how options can reduce market power, just in the same way, as two-way contracts can. Retailers do not transfer  $(p^f - p)f$ , when the spot price  $p$  is low, thus they have the advantage that an incumbent retailer cannot be underbid by a competitive fringe, which might enter the market after privatization takes place. When the spot price is low, the option holder purchases its demand at the spot market, when the price is high, the option holder pays the lower strike price  $p^s$ . The cost of an option, paid in any demand state, is  $p^o$ . Indeed if the spot price is lower than the strike price, the REC loses  $p^o$  on each option bought. The option price is generally paid before the spot market opens, thus illiquidity does not occur when the spot price is lower than expected.<sup>12</sup>

In this section, a model is introduced that compares the market equilibrium with forwards, to one with options. It is also shown, if welfare is higher in the forward or option model. This is the first model that allows a welfare analysis between an option and a forward market in the Allaz Vila (1993) framework, where the strike price is endogenously determined. Vázquez et al. (2002) propose options as a long-term security of supply mechanism. Few papers have compared welfare effects between markets that use options and those that use forwards. Exceptions are Chao/ Wilson and Willems (both 2004). The first of the two papers proposes

“an annual auction of a specified quantity of multi-year option contracts at each strike price in a specified range. Each contract is an option on physical capacity since it requires the supplier to back the contract with available capacity, to submit a standing bid at the ISO for the contracted quantity at a price no higher than the strike price, and to be dispatchable for either energy or reserve capacity.” (p.3)

Willems took the idea of using auctions and embedded this into a model that can be compared to the AV model. Willems introduced an exogenous strike price and retrieved the following results from his model: When the strike price is above the Cournot price, options are out of money and the producers settle at the Cournot equilibrium. For strike prices below the spot price achieved in AV, the AV price is achieved. For intermediate strike prices, producers flood the market until the market price reaches the strike price. The main result of Willems (2004) was that the market price is never lower in an option market than in a forward market. This model supports this view, but shows that if a model is extended by a possible bankruptcy threat, an option market is preferred to a forward market from a welfare perspective.

One needs to make some changes to the setup, to be able to compare the option with the forward market, while keeping it simple. If one continues to use an  $\epsilon \sim N(0, \sigma^2)$ , the spot price, strike price and the optimal number of

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<sup>12</sup>Schmidt (1997) shows that liquidation risk increases managerial incentives. If a firm would have had a low liquidity in the past, previous payments would have increased managerial incentives. Hence generally payments, made in the past are less harmful to a firm as they could be balanced through managerial effort in consecutive periods.

contracts would depend on the probability  $Pr(p > p^o)$  and its derivative with respect to these variables. To avoid the resulting complications that do not add further insight to our questions, it helps to assume that  $\epsilon$  takes on discrete values, which act as demand shocks;  $\epsilon \in \{0, H\}$  with probabilities  $\phi$  for  $\epsilon = H$  and  $(1 - \phi)$  for  $\epsilon = 0$ . Thus  $E\epsilon = \phi H$  and  $Var(\epsilon) = \sigma^2 = \phi(1 - \phi)H^2$ . If the risk aversion parameter in (12) is set equal to zero, the optimal number of forwards becomes:  $f^* = \frac{3Vb - (A - c)}{10b}$ . Just for  $3Vb = A - c$ ,  $f^* = 0$  otherwise hedging still takes place; there are short hedges when  $3Vb > A - c$  and long hedges when  $3Vb < A - c$ . Powell (1993) shows that in the absence of risk aversion and when generators sell the monopoly quantity on the spot market;  $f^* = \frac{1}{2}V > 0$ , which implies that there is always short, never long hedging. Meaning even risk neutral RECs hedge to keep the future spot price low, assuming that they are not too small in relative size to the market. It is legitimate to assume that RECs buy contracts to lower the future spot price, as their size was significant in the UK, before they were privatized. Producers coordinate on the future market as before. This model has two parts again. First, the market equilibrium before the liberalization is solved, second, it is modified to account for bankruptcy after the liberalization. The first part of the model is solved for producers that coordinate on the spot market, as only then the number of forwards and options is guaranteed to be positive. For a study on market power of UK's generation duopoly, see Wolfram (1999).<sup>13</sup> The equilibrium is shown, when there is no coordination likewise. After the liberalization of the market, producers do not coordinate, as they did in section 2. The liberalization of the electricity market has come along with strict actions by regulators against price agreements among incumbent generators.<sup>14</sup> RECs are risk neutral,  $\lambda = 0$ , which reduces the demand for options and forwards in the same way, a comparison of the two models is thus still possible. They maximize their expected profit instead of a mean-variance utility function. First, the forward market has to be solved under the changed market setup to be able to compare it to the option market, then the option market is solved.

### 3.1 Pre-liberalization period

#### 3.1.1 Forward Market

##### Production sector

Producers maximize the expected monopoly profit  $E\pi^M$  by choosing the expected monopoly spot quantity  $EQ$ , taking the total number of forwards  $f_\Sigma$  as given;  $\max_{EQ} E\pi^M = (Ep - c)EQ - f_\Sigma(Ep - p^f) + Cov(p, Q)$  where  $Ep = A + E\epsilon - bEQ$  and  $Cov(p, Q) = \frac{\sigma^2}{4b}$ . Thus the expected forward price and monopoly quantity are

<sup>13</sup>Müsgens (2006) shows that there have been price agreements among German producers in particular during peak periods.

<sup>14</sup>The former generation duopoly National Power and Powergen was forced to sell generation units to reduce their market power. Finally they were bought by foreign competitors after National Power demerged in 2001.

$$Ep = \frac{A + \phi H + c - bf_{\Sigma}}{2} \quad (21)$$

$$EQ = \frac{A + \phi H - c + bf_{\Sigma}}{2b} \quad (22)$$

The first order condition of the forward price (6), is thus

$$p^f = Ep + \left( \frac{\partial f_{\Sigma}}{\partial p^f} \right)^{-1} \left[ f_{\Sigma} - (EQ - f_{\Sigma}) \frac{\partial Ep}{\partial p^f} - (Ep - c) \frac{\partial EQ}{\partial p^f} \right]$$

### Retail sector

RECs maximize their utility functions of the form (7) (with  $\lambda = 0$ ) where the foc can be solved for the number of contracts  $f$  (see (9)), which can be further transformed to

$$f^*(p^f) = \frac{Vb + A + \phi H + c - 2p^f}{3b} \quad (23)$$

This is the number of forwards as a function of  $p^f$ , which each REC requests. One can now transform the  $p^f$ -foc using (21), (22) and (23) to solve for the future spot price premium,

$$p^f - Ep = \frac{3bf_{\Sigma}}{4} \quad (24)$$

and the optimal number of contracts, which depends on the REC's market size alone

$$f_{\Sigma}^* = \frac{V}{2} \quad (25)$$

### 3.1.2 Option Market

#### Production sector

The spot price is defined to exceed the strike price,  $p > p^s$  when  $\epsilon = H$ , otherwise a REC would never want to exercise its option. Thus  $H$  must be sufficiently large, because  $p^f > Ep$ . Later it is proven that the same holds for options; the expected unit price, covered by an option is larger than the expected spot price:  $(1 - \phi)p^L + \phi p^s + p^o > Ep$ . RECs are willing to pay an option price premium, just as they pay a forward price premium. In return, to receive one unit for the lower strike price, the REC pays an option price  $p^o$  to the producer in any state of the world. The total number of options sold to both RECs is  $o_{\Sigma}$ . The generator's monopoly profit is  $\pi^M = (p^H - c)Q^H - \phi o_{\Sigma}(p^H - p^s) + o_{\Sigma}p^o$  with probability  $\phi$  where  $p^H = A + H - bQ^H$ . It is  $\pi^M = (p^L - c)Q^L + o_{\Sigma}p^o$  with probability  $(1 - \phi)$  and  $p^L = A - bQ^L$ , thus the expected profit function is  $E\pi^M = (Ep - c)EQ - \phi o_{\Sigma}(p^H - p^s) + o_{\Sigma}p^o + Cov(p, Q)$ , where  $Ep$  and  $Cov(p, Q)$  are defined in the forward model. The expected price, the corresponding spot quantity and the high spot price are

$$Ep = \frac{A + \phi H + c - \phi b o_{\Sigma}}{2} \quad (26)$$

$$EQ = \frac{A + \phi H - c + \phi b o_{\Sigma}}{2b} \quad (27)$$

$$p^H = \frac{A + H + c - b o_{\Sigma}}{2} \quad (28)$$

Besides the spot quantity, producers choose  $p^o$  and  $p^s$ , while RECs choose the number of options they want to buy. The first order conditions are

$$\frac{\partial E\pi^M}{\partial p^o} = \frac{\partial Ep}{\partial p^o}EQ + (Ep - c)\frac{\partial EQ}{\partial p^o} + \frac{\partial o_{\Sigma}}{\partial p^o} [p^o - \phi(p^H - p^s)] + o_{\Sigma} \left(1 - \phi \frac{\partial p^H}{\partial p^o}\right) \doteq 0 \quad (29)$$

$$\frac{\partial E\pi^M}{\partial p^s} = \frac{\partial Ep}{\partial p^s}EQ + (Ep - c)\frac{\partial EQ}{\partial p^s} + \frac{\partial o_{\Sigma}}{\partial p^s} p^o - \phi \frac{\partial o_{\Sigma}}{\partial p^s} (p^H - p^s) + \phi o_{\Sigma} \left(1 - \phi \frac{\partial p^H}{\partial p^s}\right) \doteq 0 \quad (30)$$

#### Retail sector

RECs choose the number of options they buy, REC  $i$ 's objective (7) reduces to  $\max_{o_i} U_i = E(\pi_i^R)$ .<sup>15</sup> Retailer  $i$ 's expected profit is

$$E\pi_i^R = V(r - Ep) + \phi o_i(p^H - p^s) - o_i p^o \quad (31)$$

The first order condition of (31) is

$$o_1 = \frac{V \frac{\partial Ep}{\partial o_1} + p^o - \phi(p^H - p^s)}{\phi \frac{\partial p^H}{\partial o_1}} \quad (32)$$

As the denominator of (32) is negative, the REC's demand for options increases with  $p^H$  and decreases with the strike and option price. (26)-(28) solve for  $o_1(o_2) = \frac{1}{2} [V - o_2 + b^{-1} (A + H + c - 2p^s - 2\phi^{-1}p^o)]$ , taking  $p^s$  and  $p^o$  as given. The demand for options decreases with the number of options that the other REC buys. The public good attributes that are observed on the forward market also apply to options. Due to symmetry across RECs, the optimal number of options based on the underlying parameter set is

$$o(p^s, p^o) = \frac{1}{3} [V + b^{-1} (A + H + c - 2p^s - 2\phi^{-1}p^o)] \quad (33)$$

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<sup>15</sup> If we would allow  $\lambda > 0$ , the variance of the REC's profit is  $(\phi - \phi^2)[VH - o(p^H - p^s)]^2$ . The high demand price is a function of  $o$ . The derivative of  $Var(\pi^R)$  with respect to  $o$  depends on cubic and quadratic  $o_i$ - and  $o_j$ - terms, which would not allow us to have closed form solutions.

(26)-(28) and (33) transform (29) and (30) in order to find expressions for  $p^s$  and  $p^o$ , keeping in mind that  $o_\Sigma = 2o(p^s, p^o)$ . The first order condition for the strike price  $p^s$  can be expressed as

$$p^s(p^o, o_\Sigma) = \frac{1}{4} [2(A + H + c - 2\phi^{-1}p^o) + o_\Sigma(2b\phi - b)] \quad (34)$$

It is linearly dependent to the first order condition for the option price  $p^o$

$$p^o(p^s, o_\Sigma) = \frac{\phi}{4} [2(A + H + c - 2p^s) + o_\Sigma(2b\phi - b)] \quad (35)$$

This is not surprising as  $(p^H - p^s)$  is a transfer from the producer to the REC with probability  $\phi$ , and  $p^o$  is a transfer from the REC to the producer with certainty, while both pairs of players are risk neutral and maximize their expected profit,  $p^s$  must be negatively correlated to  $p^o$ , and one variable can be expressed through the other. One can write  $p^o(o, p^s)$  as  $p^o(p^s)$  using  $o(p^s, p^o)$  and substitute  $p^o(p^s)$  in  $o(p^s, p^o)$  to receive the optimal number of options  $o^*$ , independent of  $p^o$  and  $p^s$ , based on the underlying parameter set.  $p^o(p^s)$  can be transformed to

$$p^o + \phi p^s = \frac{\phi}{2}(A + H + c) + \frac{\phi b(2\phi - 1)}{2(\phi + 1)}V \quad (36)$$

which is the expected option payment made to the generator, to avoid paying  $p^H$ . Adding  $(1 - \phi)p^L$  to  $p^o + \phi p^s$ , gives an expected unit price; when that unit is covered with an option.  $(1 - \phi)p^L + p^o + \phi p^s - Ep = \frac{(4\phi+1)b\phi V}{6(1+\phi)} > 0$  does not depend on the size of the demand shock. This corresponds to the observation, first made by Powell (1993) for the forward market, who shows that  $p^f > Ep$ , which goes back to Allaz and Vila's (1993) article. But so far, it has not been shown for the option market. The reason behind this solution is the same; forwards and options lower the expected future spot price. Substituting  $p^o$  in (33) by (36) gives an expression for the optimal number of options

$$o_\Sigma^* = \frac{2}{3}V \frac{2 - \phi}{1 + \phi} \quad (37)$$

There are all the ingredients, one needs to compare the expected spot price for futures and options, stated here again for convenience:  $Ep^{Futures} = \frac{A + \phi H + c - b f_\Sigma}{2}$  with  $f_\Sigma^* = \frac{V}{2}$  and  $Ep^{Options} = \frac{A + \phi H + c - \phi b o_\Sigma}{2}$ . The expected spot price is larger in the presence of options than forwards when:  $\phi o_\Sigma^* < f_\Sigma^* \Leftrightarrow \frac{2\phi(2-\phi)}{3(1+\phi)} < 1$ , which holds for all  $0 \leq \phi \leq 1$ . The expected spot price is smaller in the forward model. For the simulation of both models, the same parameters are used as before:  $h = 0.15$ ,  $V = 2.5$ ,  $A = 50$ ,  $b = 0.67$  and  $c = 20$ . The retail price did not matter in the analysis of the first part of this article, here it equals  $r = \text{£}60/\text{MWh}$  as in Green (2004). Furthermore the probability of a positive demand shock,  $H = 20$ , is  $\phi = 50\%$ . Before the liberalization takes place, market participants choose the values given by table 1.

	$f_{\Sigma}^*, o_{\Sigma}^*$	$Ep$	$p^f, Ep^{w. Options}$	$EQ$	$E\pi^M$	$E\pi^R$	$E(CS)$
Forwards	1.25	39.48	40.21	30.63	609.90	50.65	159.51
Options	1.6	39.72	40.00	30.42	609.73	50.23	156.31

Table 1: Welfare in the absence of bankruptcy

Welfare is hardly smaller in the one-way contract model, residual consumers loose about 2% in expectation  $[E(CS)]$ , while expected producer and REC profits are barely different. Note that the expected price of a unit purchased with an option is less expensive than  $p^f$ , which holds when  $8\phi^2 - 5\phi - 1 < 0$ . One can repeat the derivations described in this section so far, when producers do not coordinate at the spot market. In that case, the analog equation for (36) and (37) are

$$p^o + \phi p^s = \frac{\phi}{18 + 12\phi} [A(9 + 4\phi) + c(9 + 8\phi) + H(6 + 7\phi) + Vb(-3 + 4\phi)] \quad (38)$$

and

$$\frac{o_{\Sigma}^*}{2} = \frac{3Vb + c - A - \phi H}{b(6 + 4\phi)} \quad (39)$$

One can easily see that there are long hedges possible, when producers do not cooperate at the spot market and  $3Vb + c < A + \phi H$ . The rest of the analysis uses the model, where producers cooperate, and shows under what circumstances a welfare maximizing regulator prefers one-way contracts over two-way contracts, when the default threat is included. In the absence of bankruptcy, one-way contracts have the disadvantage that just in the high demand state, a fix price is paid for the production that was covered. When options are “out-of-the-money” and producers play the Cournot game. Thus intuitively it is clear, why one-way contracts can not reduce market power to the same extend as two-way contracts do.

### 3.2 Post liberalization period

This section contains the same structure previously used. After a competitive fringe has entered the market, the new demand for incumbent retailers is (14) as before, and the optimal retail price is (16).

#### Forwards

The retailer’s profit in the absence and presence of a demand shock are

$$\begin{aligned} \pi^R &= \frac{V}{4h} + f^*(p^H - p^f) & \text{if } \epsilon &= 0 \\ \pi^R &= \frac{V}{4h} + f^*(p^L - p^f) & \text{if } \epsilon &= H \end{aligned} \quad (40)$$

$\pi^R > 0$ , when  $\epsilon = H$  in the forward model, because  $p^f > Ep > p^L$ . Bankruptcy is possible, when  $\epsilon = 0$  in the forward model when switching costs are low (large value of  $h$ ),  $V$  is small and  $p^L$  is much smaller than  $p^f$ , such that  $\pi^R$  becomes negative. When  $\epsilon = H$ , and thus bankruptcy does not occur with certainty, (3) and (4) continue to hold, where  $A$  is substituted by the realized intercept  $A + H$ . Bankruptcy does not play any role. When  $\epsilon = L$ , then a retailer's profit is  $\pi_i^R = \frac{V}{4h} + f_i^*(p^L - p^f)$ , where  $f_i^* = \frac{f_i^*}{2}$ . Each producer maximizes its profit,  $\max_{q_i^L} \pi_i^P = (p^L - c)q_i^L - f_i^*(p^L - p^f)(1 + s\pi_i^R)$ , where  $(1 + s\pi_i^R)$  is the survival probability of retailer  $i$ , when  $\pi_i^R < 0$ . (19) and (20) describe the equilibrium, where the realized intercept is  $A^* = A$ .

### 3.2.1 Options

The possible retail profits in the option model are

$$\pi^R = \begin{cases} \frac{V}{4h} - o^*p^o & \text{if } \epsilon = 0 \\ \frac{V}{4h} + o^*[p^H - (p^s + p^o)] & \text{if } \epsilon = H \end{cases} \quad (41)$$

When  $\epsilon = 0$  the producer's objective is  $\max_{q^L} \pi^P = (p^L - c)q^L + op^o$  and the equilibrium values are  $q^{L*} = \frac{A-c}{3b}$  and  $p^{L*} = \frac{A+2c}{3}$ . Thus producers offer the regular Cournot price. It is a reasonable assumption that the option price has been paid in advance, thus a possible bankruptcy does not affect the producer's objective. When  $\epsilon = H$ , the producer's objective is

$$\max_{q_i^H} \pi_i^P = (p^H - c)q_i^H + o_i[p^o + p^s - p^H] \quad (42)$$

If  $\epsilon = H$ , options are "in the money" and the second term of  $\pi^R$  is strictly positive by definition, as  $p^o + p^s > p^H$ . Otherwise the option would never be exercised. In the preceding analysis, the relevant value has been,  $p^o + \phi p^s$ . The problem was defined in such a way that options are exercised when  $\epsilon = H$ . It would not have been a realistic assumption from the REC's point of view, when it would make a loss through exercising an option, when the demand is high. Options just lower producer's market power on the spot market if they are exercised for  $\epsilon = H$ . If it would not be in the REC's interest to exercise the option, it would be known to the producers. Thus options would not reduce market power and RECs would not purchase them in the first place.

The optimal values are  $q^{H*} = \frac{A+H-c+bo^*}{3b}$  and  $p^{H*} = \frac{A+H+2c-2bo^*}{3}$ , where  $o^* = \frac{o^*}{2}$ . They have the same structure as in the Allaz Vila equilibrium. The buyer of one-way contracts pays in either state of the world  $p^o$ . Thus the difference between profit and loss is smaller with options than with forwards.

### 3.3 Results

There are two different equilibria for forwards and options, respectively when  $\epsilon = 0$  and  $\epsilon = H$ . It has already been proven that forwards are preferable to options



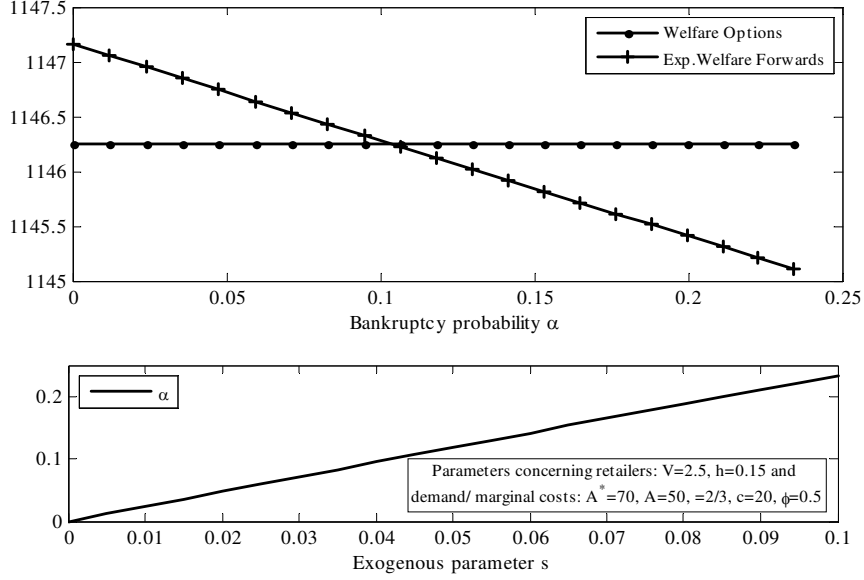


Figure 3: Welfare: Forwards vs. Options

in the absence of bankruptcy. After weighting the two possible outcomes of the forward model equilibrium, one can determine, if either options or forwards are preferable in the presence of the threat of bankruptcy.

The same parameters are used to generate figure 3, which summarizes the main result: in the absence of a default threat ( $s = \alpha = 0$ ), the option model yields a lower welfare than the forward model, which has been shown algebraically before. But note that the welfare difference is quite small. The lower diagram shows the relation between  $s$  and  $\alpha$ , the upper diagram between  $\alpha$  and welfare.<sup>16</sup> As the bankruptcy threat of former RECs just concerns producers in the forward model, welfare in the option model is not affected by the exogenous parameter  $s$ . The higher the bankruptcy probability  $\alpha = -s\pi^R$  becomes, the lower the welfare in the two-way contract model just as in figure 1 and 2. For a default probability of around 10%, a regulator that maximizes total welfare would prefer the option model over that of the forward model. One has to bear in mind that this estimate is very conservative, as in this model, default only has an effect on the spot, but not contract market.

<sup>16</sup>Welfare equals the residual consumer surplus, which is the area below the inverse demand function and above the spot price. In addition, the producer and retail profits are added.

## 4 Conclusion

This paper introduces a simple model, where downstream firms, operating in a competitive environment, may go bankrupt after incurring a loss on forward contracts that have been signed with upstream firms. The first part of the model shows that former RECs, which still hold long term forward contracts, benefit when they have an owner that is less likely to bail them out. The bankruptcy probability consists of the retailer's loss, and an exogenous multiplier that reflects the willingness for bail-out of the retailer's owner. Producers minimize the risk of a retailer not meeting its contract of difference payments by reducing the production quantity and hence increasing the spot price, which in return reduces the difference. For reasonable parameters, when an owner is less likely to bail its subsidiary out, the upward spot price shift is sufficiently large to *turn a loss* (in the absence of a default threat) *into a profit* (in the presence of a default threat). Depending on the extent of the upward price adoption, producers benefit or loose. Consumers always loose more than firms gain, thus welfare decreases.

The second part of the model introduces one-way contracts, and demonstrates that options lower the spot price, as forwards do. In the absence of risk aversion, the expected price of a unit that is bought with an option, exceeds the expected price. Once again, the parallel can be drawn to the forward market. Options reduce the profit of an upstream firm less than forwards do; thus the spot price is larger in an option model than a forward model. The model simulates the English/ Welsh electricity market and shows that the welfare difference is very small. Producer and retailer profits are hardly differently affected by the instruments, and consumer surplus decreases by a mere 2%. Including bankruptcy and comparing the option and forward model again, one-way contracts are preferred to two-way contracts for bankruptcy probabilities of 10%. If one includes other costs that are connected to bankruptcy, and considering that this estimate is very conservative, options might quickly become preferred to forwards.

When regulators decide how to treat off-market trade in the near future, they shall have to keep in mind that depending on the industry structure, insolvency might not only cost taxpayers' money, it may also reduce the market power mitigating effect of two-way contracts. If two-way contracts become more heavily regulated to avoid bankruptcies, and require a large amount of capital as market participants claim, one-way contracts could step in, and play the same role that forwards have done in the past.

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