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Consumer Preferences in Monopolistic Competition Models

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Abstract

This paper develops a novel approach to modeling preferences in monopolistic competition models with a continuum of goods. In contrast to the commonly used CES preferences, which do not capture the effects of consumer income and the intensity of competition on equilibrium prices, the present preferences can capture both effects. I show that under an unrestrictive regularity assumption, the equilibrium prices decrease with the total mass of available goods (which represents the intensity of competition in the model) and increase with consumer income. The former implies that the entry of firms in the market or opening a country to international trade has a pro-competitive effect that decreases equilibrium prices.

*Keywords: firm prices; intensity of competition; consumer income.*

*JEL classification: D4*
1 Introduction

Starting with Dixit and Stiglitz (1977), the monopolistic competition framework has been widely used in the economic literature.\footnote{See for instance the international trade literature.} The most common assumption about preferences in this framework is the constant elasticity of substitution (CES) utility function. This is greatly owing to high analytical tractability of this particular functional form. Despite such a desirable property, the CES utility function has a shortcoming. One of the implications of the CES functional form is that prices set by firms depend only on marginal cost of those firms and the elasticity of substitution. This in turn implies that changes in the intensity of competition (that might follow as a result of opening a country to international trade) or changes in consumer income do not affect the prices that firms set. Meanwhile, the literature on pricing-to-market (see for instance Hummels and Lugovskyy (2008) and Simonovska (2009)) has demonstrated that prices of the same goods vary with characteristics of the importing markets. Hence, it seems desirable to have a tractable monopolistic competition model where prices would depend not only on marginal cost, but also on other relevant factors such as the intensity of competition or consumer income.

In this paper, I develop a novel approach to modeling preferences in monopolistic competition models with a continuum of good. I construct a general form of consumer preferences (for instance, the CES preferences are a special case of the preferences developed in this paper), which is analytically manageable and at the same time, captures the effects of income and the intensity of competition on equilibrium prices. I show that under a standard regularity condition, the equilibrium prices negatively depend on the intensity of competition in the market and positively depend on consumer income. The former implies that the entry of firms in the market or the opening a country to international trade has a pro-competitive effect decreasing the equilibrium prices, while the latter means that economy with richer consumers tends to have higher prices. These results are consistent with empirical findings in Hummels and Lugovskyy (2008) and Simonovska (2009).

I consider a framework where all potentially available goods are indivisible and con-
sumers purchase at most one unit of each good. Consumers have identical incomes but
differ in their tastes for a certain good. A taste for a certain good is a realization of a
random variable, which is independently drawn for each consumer and each good from a
common distribution. The utility function implies that given prices and consumer tastes,
goods are arranged so that consumers can be considered as moving down some list in
choosing what to purchase. That is, consumers first purchase a good they like best, then
move to the second best, and keep on until their income is exhausted. This list of goods
is consumer specific and depends on consumer income and tastes. Hence, demand for
a certain good is equal to the fraction of consumers who decide to purchase this good
multiplied by the total mass of consumers.

There are several advantages of this approach to modeling preferences. First, it is
highly tractable and eminently suitable for monopolistic competition models with a con-
tinuum of goods. Second, in the paper I show that the fraction of consumers who purchase
a certain good is endogenous and depends not only on the price of the good, but also on
the intensity of competition and consumer income. As a result, equilibrium prices de-
pend on the intensity of competition and consumer income as well. In particular, I show
that if the distribution of tastes satisfies the increasing proportionate failure rate (IPFR)
property, then the equilibrium prices decrease with the total mass of available goods
(which represents the intensity of competition in the model) and increase with consumer
income. Finally, this approach can be considered as a unifying way of modeling consumer
preferences in monopolistic competition framework. By choosing different distributions
of consumer tastes, one can generate different demand functions. In particular, a Pare-
to distribution leads to isoelastic demand (the CES preferences) with the possibility of
demand satiation.

The utility function considered in this paper is reminiscent of the stochastic utility
functions developed in Perloff and Salop (1985) and later in Anderson et al. (1992).

\[\text{2The IPFR property was first established in Singh and Maddala (1976), who describe the size distri-
ution of incomes. The property means that the hazard rate of the distribution does not decrease too fast.
A very wide family of distributions (including lognormal, power, and exponential distributions) satisfies
this property. See Van den Berg (2007) for details.}\]
However, my approach is different in at least two ways. First, in Perloff and Salop (1985) and Anderson et al. (1992), consumers are allowed to purchase only one unit of the good they like most, which is a rather simplifying way of describing individual demand. In contrast, in my paper consumers are not limited to buying only one good. Second, in Perloff and Salop (1985) and Anderson et al. (1992), there are no income effects. In these works, the marginal utility of income is just a parameter in the model. In my approach, the marginal utility of income is an endogenous variable and depends on the observable characteristics of economic environment including consumer income.

The present paper is not the only one that explores the dependence of prices on the characteristics of the economic environment. Melitz and Ottawiano (2008) use quasi-linear preferences to derive similar predictions regarding the relationship between prices and the mass of available goods. However, in their paper, the presence of a numeraire good eliminates all income effects. Hummels and Lugovskyy (2008) consider a generalized version of Lancaster’s "ideal variety" model (that allows for income effects operating through an intensity of preferences for the ideal variety) and establish a positive correlation between prices and consumer income. Nevertheless, they limit their analysis to a symmetric equilibrium and, therefore, do not allow for firm heterogeneity and hence differences in prices chosen by firms. In the present paper, the model remains analytically tractable even in the case of the presence of firm heterogeneity, which is for instance important for applications in the international trade literature. To capture the impact of consumer income and the intensity of competition on prices, Saure (2009) and Simonovska (2009) use the non-homothetic log-utility function that assumes the upper bound on the marginal utility from consumption. While my approach leads to the same predictions about firm prices as in these papers, it is based on different assumptions and represents a more general and natural way of modeling consumer preferences.

The remainder of the paper is organized as follows. Section 2 introduces the basic

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3In fact, Melitz and Ottawiano (2008) consider the effect of market size on firm prices and markups. However, due to free entry, a larger market size leads to higher number of available goods, which in turn affects the prices.
concepts of the model and formulates equilibrium conditions. In Section 3, I consider comparative statics of the model. Section 4 examines a special case of the model when the distribution of consumer tastes is Pareto. Section 5 concludes.

2 The Model

I consider a monopolistic competition model with a continuum of consumers and goods indexed by $i$ and $\omega$, respectively. I assume that each good $\omega$ is produced by a distinct firm and the set of firms in the economy denoted by $\bar{\Omega}$ is exogenously given and has a finite measure. It might be the case that in equilibrium certain firms choose not to produce at all, as producing any positive amount of the good would lead to negative profits. Therefore, I denote $\Omega \subseteq \bar{\Omega}$ as the set of available goods (goods that are actually produced and sold to consumers).

I assume that all goods are indivisible and consumers purchase at most one unit of each good. In particular, taken $\Omega$ as given, consumer $i$ chooses $\{x(\omega) \in \{0, 1\}\}_{\omega \in \Omega}$ to maximize the following utility function:

$$U_i = \int_{\omega \in \Omega} \varepsilon_i(\omega)x(\omega)d\omega$$

subject to

$$\int_{\omega \in \Omega} p(\omega)x(\omega)d\omega = y,$$

where $x(\omega)$ is the consumption of good $\omega$, $\varepsilon_i(\omega)$ is a consumer-specific taste for $\omega$, $p(\omega)$ is the price, and $y$ is consumer income (which is identical for all consumers). I assume that for any $i$ and $\omega$, $\varepsilon_i(\omega)$ is independently drawn from a common distribution. That is,

$$\Pr(\varepsilon_i(\omega) \leq \varepsilon) = F(\varepsilon),$$

where $F(\varepsilon)$ (common for all consumers and goods) is a differentiable function with the support on $[\varepsilon_L, \varepsilon_H]$. Here, $\varepsilon_L > 0$.

The utility maximization problem implies that consumer $i$ purchases good $\omega$ if and only if

$$\frac{\varepsilon_i(\omega)}{p(\omega)} \geq Q,$$
where $Q$ is the Lagrange multiplier associated with the maximization problem and represents the endogenous marginal utility of income. Since $\varepsilon_i(\omega)$ are independently distributed, the proportion of consumers, who purchase good $\omega$, is equal to $1 - F(p(\omega)Q)$. Notice that if the price of $\omega$ is sufficiently low (namely, $p(\omega)Q \leq \varepsilon_L$), then all consumers purchase the good. Similarly, if the price is high enough ($p(\omega)Q > \varepsilon_H$), then nobody purchases the good $\omega$. Hence, the demand for good $\omega$ is given by

$$D(p(\omega)) = \begin{cases} 
L, & \text{if } p(\omega) \leq \frac{\varepsilon_L}{Q}, \\
(1 - F(p(\omega)Q))L, & \text{if } \frac{\varepsilon_H}{Q} \geq p(\omega) > \frac{\varepsilon_L}{Q}, \\
0, & \text{if } p(\omega) > \frac{\varepsilon_H}{Q},
\end{cases}$$

where $L$ is the total mass of consumers.

The marginal utility of income $Q$ can be found from the budget constraint (2) in the consumer maximization problem. Namely,

$$\int_{\omega \in \Omega} p(\omega) \Pr (\varepsilon_i(\omega) \geq p(\omega)Q) \, d\omega = y,$$

which is equivalent to

$$\int_{\omega \in \Omega} p(\omega) (1 - F(p(\omega)Q)) \, d\omega = y. \quad (5)$$

### 2.1 Equilibrium

I consider a partial equilibrium in the model. Firms choose prices $p(\omega)$ to maximize their profits. I assume that a firm producing good $\omega$ incurs marginal cost of $c(\omega)$.\(^4\) Hence, the firm maximization problem is as follows:

$$\max_p \{ (p - c(\omega)) D(p) \}, \quad (6)$$

where $D(p)$ is defined by (4).

Notice that the demand function $D(p(\omega))$ has a kink at $p(\omega) = \frac{\varepsilon_H}{Q}$. This implies that for some $\omega$, the maximization problem (6) results in the corner solution with $p(\omega) = \frac{\varepsilon_L}{Q}$. While for the other goods, the solution of (6) is interior and satisfies

$$\frac{c(\omega)}{p} = 1 - \frac{1 - F(pQ)}{pQf(pQ)}. \quad (7)$$

\(^4\)To simplify the analysis, I assume that there are no fixed costs of production. However, the model can be easily extended to the case when firms incur fixed costs as well.
To guarantee the uniqueness of the solution of (7), I assume that the distribution of tastes satisfies the increasing proportionate failure rate (IPFR) property. Namely, \( \frac{\epsilon f(\epsilon)}{1 - F(\epsilon)} \) is strictly increasing in \( \epsilon \) on \([\epsilon_L, \epsilon_H]\) (where \( f(\cdot) \) is a density function associated with \( F(\cdot) \)). Notice that this property is weaker than the increasing hazard rate property and holds for many distribution families (see Van den Berg (2007)).

The IPFR property implies that the right-hand side of the equation (7) is strictly increasing in \( p \), while the left-hand side is strictly decreasing. Hence, if the solution of (7) exists, then it is unique. It is straightforward to show that the necessary and sufficient condition for existence of the solution is

\[
c(\omega) \in \left[ \frac{f(\epsilon_L)\epsilon_L - 1}{f(\epsilon_L)Q}, \frac{\epsilon_H}{Q} \right].
\]

If \( c(\omega) < \frac{f(\epsilon_L)\epsilon_L - 1}{f(\epsilon_L)Q} \), then the firm maximization problem (6) has a corner solution with \( p(\omega) = \frac{\epsilon_H}{Q} \). In words, firms with sufficiently low marginal cost choose such the price that all consumers purchase their goods. This is explained by the fact that demand is inelastic if price is lower than \( \frac{\epsilon_H}{Q} \). Note that if the marginal cost \( c(\omega) \) is high enough (\( c(\omega) > \frac{\epsilon_H}{Q} \)), then the production of \( \omega \) yields negative profits. That is, firms with \( c(\omega) > \frac{\epsilon_H}{Q} \) do not operate in the market. This means that the set \( \Omega \) is given by \( \{ \omega \in \bar{\Omega} : c(\omega) \leq \frac{\epsilon_H}{Q} \} \). The following lemma summarizes the findings above.

**Lemma 1** If \( F(\epsilon) \) satisfies the IPFR property, then there exists a unique solution of the firm maximization problem (6). Furthermore, if \( c(\omega) < \frac{f(\epsilon_L)\epsilon_L - 1}{f(\epsilon_L)Q} \), then

\[
p(\omega) = \frac{\epsilon_L}{Q},
\]

while if \( c(\omega) \in \left[ \frac{f(\epsilon_L)\epsilon_L - 1}{f(\epsilon_L)Q}, \frac{\epsilon_H}{Q} \right] \), \( p(\omega) \) satisfies

\[
\frac{c(\omega)}{p(\omega)} = 1 - \frac{1 - F(p(\omega)Q)}{p(\omega)Qf(p(\omega)Q)}.
\]

**Proof.** See above. ■

Next, I define the equilibrium in the model.
Definition 1 Given the set of parameters \((y, L, F(\cdot), \{c(\omega)\})\), the equilibrium in the model is defined by \(\{p(\omega)\}_{\omega \in \Omega}, Q, \Omega\) such that

1) \(\{p(\omega)\}_{\omega \in \Omega}\) are determined by the firm maximization problem.

2) \(Q\) satisfies the budget constraint (5).

3) \(\Omega = \{\omega \in \Omega : c(\omega) \leq \frac{\varepsilon y}{Q}\}\).

The next section focuses on comparative statics.

3 Comparative Statics

This section explores how consumer income and the mass of available goods affect the equilibrium prices. Recall that the pricing rule determined in the previous section (see Lemma 1) implies that \(p(\omega)\) depends on \(c(\omega)\) and \(Q\). Though I do not change the notation, in the analysis below I implicitly assume that \(p(\omega)\) is in fact \(p(\omega, Q)\) and consider all expressions as a function of \(Q\). Next, I formulate two properties of \(p(\omega)\) assuming that \(F(\varepsilon)\) satisfies the IPFR property.

Lemma 2 For any \(\omega \in \Omega\), \(p(\omega)\) is strictly decreasing in \(Q\).

Proof. See the proof in the Appendix.

The lemma states that higher marginal utility of income results in lower prices set by firms. In other words, higher \(Q\) implies that consumers become more "fastidious" in choosing which goods to purchase. As a result, firms reduce their prices in order to increase their profits. Furthermore, in the next lemma, I show that higher marginal utility of income reduces not only prices, but also demand for some goods.

Lemma 3 For any \(\omega \in \Omega\), \(c(\omega) \in \left[\frac{f(\varepsilon)}{f(\varepsilon L)Q}, \frac{\varepsilon y}{Q}\right]\), \(p(\omega)Q\) is increasing in \(Q\).

Proof. See the proof in the Appendix.

Remember that demand for \(\omega\) is given by \((1 - F(p(\omega)Q))L\). Therefore, the direct implication of Lemma 3 is that demand for goods with sufficiently high marginal cost decreases with a rise in \(Q\).
3.1 Consumer Income

Note the expected spendings on good $\omega$ are equal to $p(\omega) (1 - F (p(\omega)Q))$. The results of the previous lemmas imply that for any $\omega \in \Omega$, $p(\omega) (1 - F (p(\omega)Q))$ is strictly decreasing in $Q$. That is, higher marginal utility of income reduces consumer spendings on all available goods. In addition, higher $Q$ decreases the mass of available goods measured by $M = \int_{\omega \in \Omega} 1 d\omega$, as $\Omega = \{ \omega \in \hat{\Omega} : c(\omega) \leq \frac{y}{Q} \}$. These two properties allow us to establish a relationship between the consumer income $y$ and the equilibrium prices. Namely, the following proposition holds.

**Proposition 1** If $F(\varepsilon)$ satisfies the IPFR property, then higher consumer income leads to higher equilibrium prices.

**Proof.** From the previous consideration, $\int_{\omega \in \Omega} p(\omega) (1 - F (p(\omega)Q)) d\omega$ is strictly decreasing in $Q$. Remember that the equilibrium value of $Q$ is determined from the following equation:

$$\int_{\omega \in \Omega} p(\omega) (1 - F (p(\omega)Q)) d\omega = y.$$

Since the left-hand side of the equation is strictly decreasing in $Q$, a rise in $y$ leads to lower equilibrium value of $Q$. From Lemma 2, lower $Q$ results in higher equilibrium prices set by firms. Q.E.D. ■

The proposition implies that given other things equal, economies with richer consumers tend to have less elastic demand and, thereby, higher prices. Notice that this result holds for any distribution of consumer tastes satisfying the IPFR property. Another implication of the proposition is that richer economies have greater number of available goods, since the measure of $\Omega$ is increasing in $y$.

3.2 The Mass of Available Goods

In this section, I show that all else equal, higher mass of available goods leads to lower equilibrium prices. In particular, I consider such changes in the set $\hat{\Omega}$ that for any $Q$, the
new set of available goods $\Omega_{new}$ can be decomposed into the sum of $\Omega_{old}$ and $\Omega'$, where $\Omega_{old}$ is the set of old goods and $\Omega'$ represents some new goods. In other words, for any $Q$,

$$\Omega_{new} = \Omega_{old} \sqcup \Omega'.$$

It is straightforward to see that the measure of $\Omega_{new}$ is strictly higher than that of $\Omega_{old}$. As examples of such a comparative static, one can consider the additional entry of firms into the market or opening a country to international trade where the role of new goods is played by imports. I do not construct a particular mechanism, since it is beyond the scope of the paper.

In this case, the equilibrium equation (5) can be rewritten as follows:

$$\int_{\omega \in \Omega_{old}} p(\omega) \left(1 - F(p(\omega)Q)\right) d\omega + \int_{\omega \in \Omega'} p(\omega) \left(1 - F(p(\omega)Q)\right) d\omega = y. \quad (8)$$

As the measure of $\Omega'$ is positive,

$$\int_{\omega \in \Omega'} p(\omega) \left(1 - F(p(\omega)Q)\right) d\omega > 0.$$

Therefore, in order the equality (8) holds (I assume that consumer income does not change), $\int_{\omega \in \Omega_{old}} p(\omega) \left(1 - F(p(\omega)Q)\right) d\omega$ must decrease.\(^5\) Since $\int_{\omega \in \Omega_{old}} p(\omega) \left(1 - F(p(\omega)Q)\right) d\omega$ is decreasing in $Q$, this implies that $Q$ rises. This in turn decreases the equilibrium prices (see Lemma 2). The intuition behind this result is quite straightforward. The higher mass of available goods induces tougher competition and, therefore, leads to lower prices. The next proposition summarizes these findings.

**Proposition 2** If $F(\varepsilon)$ satisfies the IPFR property, then the availability of additional new goods reduces the equilibrium prices.

**Proof.** See above. \(\blacksquare\)

\(^5\)Note that before changes in the mass of available goods, the equilibrium condition was

$$\int_{\omega \in \Omega_{old}} p(\omega) \left(1 - F(p(\omega)Q)\right) d\omega = y.$$
In the paper, the mass of consumers $L$ does not affect the equilibrium outcomes. There are at least two standard ways to incorporate the effects of $L$ in the model. First, one can assume free entry into the market. In this case, higher $L$ would lead to more entry and, thereby, higher mass of available goods and Proposition 2 can be applied. Second, one can assume that firms incur fixed costs of production as well. Then, changes in the mass of consumers $L$ affect the set of available goods $\Omega$ (higher $L$ increases the measure of $\Omega$) and Proposition 2 can be applied again. In other words, changes in the mass of consumers mainly affect the equilibrium through changes in the mass of available goods. Thus, to some extent, the effects of $L$ are described by Proposition 2.

4 A Special Case: Pareto Distribution

Assume that the distribution of consumer tastes is Pareto. That is,

$$F(\varepsilon) = 1 - \left(\frac{\varepsilon L}{\varepsilon}\right)^{\sigma},$$

where $\sigma > 1$. In the case of a Pareto distribution, the upper bound of the distribution is infinity meaning that $\varepsilon_H = \infty$. This implies that all firms operate in the market and, thereby, $\Omega = \bar{\Omega}$.

As for a Pareto distribution, $\frac{\varepsilon f(\varepsilon)}{1-F(\varepsilon)}$ is equal to $\sigma$ and, therefore, does not depend on $\varepsilon$, the IPFR property is not valid for a Pareto distribution. However, it is possible to show that in the case of Pareto, there exists a unique solution of the firm maximization problem. Namely, if the distribution of tastes is Pareto, the demand function can be written as follows:

$$D(p(\omega)) = \left\{ \begin{array}{ll} L, & \text{if } p(\omega) \leq \frac{\varepsilon L}{Q}; \\ \left(\frac{\varepsilon L}{p(\omega)Q}\right)^{\sigma} L, & \text{if } p(\omega) > \frac{\varepsilon L}{Q}. \end{array} \right. \quad (9)$$

Hence, it is straightforward to obtain that the equilibrium prices are equal to

$$p(\omega) = \left\{ \begin{array}{ll} \frac{\varepsilon L}{Q}, & \text{if } c(\omega) < \frac{\sigma-1}{\sigma} \frac{\varepsilon L}{Q}; \\ \frac{\sigma}{\sigma-1} c(\omega), & \text{otherwise}. \end{array} \right. \quad (10)$$
Finally, the marginal utility of income $Q$ can be found from

$$\int_{\omega \in \Omega} p(\omega) (1 - F(p(\omega)Q)) \, d\omega = y \iff Q = \varepsilon_L \left( \frac{\int_{\omega \in \Omega} (p(\omega))^{1-\sigma} \, d\omega}{y} \right)^{1/\sigma}. \quad (11)$$

Hence, the equations (10) and (11) describe the equilibrium in the model.

Note that using the expression (11), the demand function in (9) can be rewritten as follows:

$$D(p(\omega)) = \begin{cases} L, & \text{if } p(\omega) \leq \frac{\varepsilon_L}{Q}, \\ \frac{yL}{\varepsilon L} \left( \frac{p(\omega)}{P} \right)^{-\sigma}, & \text{if } p(\omega) > \frac{\varepsilon_L}{Q}, \end{cases}$$

where $P$ equal $\left( \int_{\omega \in \Omega} (p(\omega))^{1-\sigma} \, d\omega \right)^{1/(1-\sigma)}$ is the CES price index. As it can be seen from above, a Pareto distribution leads to the CES preferences with possibility of satiated demand for goods with sufficiently low marginal cost of production. The role of the elasticity of substitution is played by the shape parameter $\sigma$. Higher $\sigma$ leads to lower variance of the distribution. As a result, consumer tastes become more similar and the elasticity of substitution increases. In the limit case when $\sigma = \infty$, all consumers have identical tastes and depending on $c(\omega)$, demand for $\omega$ is equal to either the mass of consumers $L$ or 0 (see Foellmi and Zweimueller (2006) or Tarasov (2007)).

If we assume that all $\omega$ are such that $c(\omega) \geq \frac{\sigma - 1}{\sigma} \frac{\varepsilon_L}{Q}$, then we observe a standard CES framework where the equilibrium prices of all goods depend only on marginal cost and the elasticity of substitution. However, if there exist goods with sufficiently low marginal cost, then the prices of those goods depend on $Q$ and Propositions 1 and 2 can be applied. Namely, higher consumer income or lower mass of available goods results in lower $Q$ and, thereby, higher prices of goods with satiated demand ($\omega \in \tilde{\Omega} : c(\omega) < \frac{\sigma - 1}{\sigma} \frac{\varepsilon_L}{Q}$).

5 Conclusion

This paper develops a new family of consumer preferences in the monopolistic competition framework, which can capture the effects of consumer income and the intensity of
competition on equilibrium prices. The constructed preferences have two key features. First, goods are indivisible and consumers purchase at most one unit of each good. Second, consumers are allowed to have different tastes for a particular good. I show that if the distribution of tastes satisfies the increasing proportionate failure rate property, then the equilibrium prices positively depend on consumer income and negatively depend on the intensity of competition. The latter implies that the entry of firms into the market or opening a country to international trade has a pro-competitive effect decreasing the equilibrium prices.

The developed approach to modeling preferences is quite flexible and can be used in many various applications requiring variable firm markups. For instance, in the analysis of international trade, Verhoogen (2008) uses a variation of the multinomial-logit demand function with constant consumers’ willingness to pay for quality (the analogue of the marginal utility of income) resulting in constant firm markups. The quality of a product can be incorporated in the present model as well. Furthermore, an exponential distribution of consumer tastes results in the analogue of the multinomial-logit demand function. However, in this case, consumers’ willingness to pay for quality and, therefore, markups are endogenous.

The considered model can be extended to the case when consumers are different not only in their tastes, but also in their incomes. This would allow us to analyze the relationship between prices and income distribution. Unfortunately, the presence of income heterogeneity makes the model quite complicated. The closed-form solution can be derived only in some special cases and numerical analysis has to be applied. I leave these issues for future work.

\(^6\)It is sufficient to introduce some quality index in the utility function.
References


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Appendix

In the appendix, I provide the proofs of Lemmas 2 and 3.

The Proof of Lemma 2

From Lemma 1, if \( c(\omega) < \frac{f(\varepsilon_{L})\varepsilon_{L}-1}{f(\varepsilon_{L})Q} \), then \( p(\omega) \) is equal to \( \frac{\varepsilon_{L}}{Q} \) and, therefore, is decreasing in \( Q \). If \( c(\omega) \in \left[ \frac{f(\varepsilon_{L})\varepsilon_{L}-1}{f(\varepsilon_{L})Q}, \frac{\varepsilon_{H}}{Q} \right] \), then \( p(\omega) \) is the solution of

\[
\frac{c(\omega)}{p} = 1 - \frac{1 - F(pQ)}{pQf(pQ)}.
\]

As \( F(\varepsilon) \) satisfies the IPFR property, \( \frac{1 - F(pQ)}{pQf(pQ)} \) is decreasing in \( Q \) for any \( p \). This implies that for any \( p \), the right-hand side of the equation above is increasing in \( Q \). That is, higher \( Q \) shifts the function \( 1 - \frac{1 - F(pQ)}{pQf(pQ)} \) up. As a result, the value of \( p(\omega) \) decreases. Hence, I show that for all \( \omega \in \Omega \), \( p(\omega) \) is decreasing in \( Q \). Q.E.D.

The Proof of Lemma 3

Consider \( \omega : c(\omega) \in \left[ \frac{f(\varepsilon_{L})\varepsilon_{L}-1}{f(\varepsilon_{L})Q}, \frac{\varepsilon_{H}}{Q} \right] \). Then, \( p(\omega)Q \) solves

\[
\frac{Qc(\omega)}{x} = 1 - \frac{1 - F(x)}{xf(x)}
\]

with respect to \( x \). Higher \( Q \) shifts the left-hand side of the equation up. This means that the value of \( p(\omega)Q \) increases, as the right-hand side is increasing in \( x \). Q.E.D.