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Munich Discussion Paper No. 2010-34

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Online at https://doi.org/10.5282/ubm/epub.11878

The appeal of risky assets

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This version: October 2010

Abstract

A fund's performance is usually compared to the performance of an index or other funds. If a fund trails the benchmark, the fund manager is often replaced. We argue that this may lead to excessive risk-taking if fund managers differ in ability and have the opportunity to take excessive risk. To match the benchmark, fund managers may increase the risk of their portfolio even if this decreases the expected return on the portfolio.

JEL classification: G01; G11; G23
Keywords: Benchmarking; risk taking

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1 Introduction

In the wake of the recent financial crisis, it has often been argued that fund managers did not know about the risks in their portfolio; otherwise, it has been argued, they would not have invested in such risky assets. We argue, however, that fund managers might have known about the risks and invested in the risky assets nonetheless. We suggest that it might even have been individually optimal to invest in the risky assets if the expected return on these assets was lower than the expected return on other, less risky assets.

A fund's performance is usually compared to the performance of an index or other funds. If a fund trails the benchmark, the fund manager is often replaced. We argue that this may lead to excessive risk-taking if fund managers differ in ability and investment restrictions are inadequate. If investment restrictions are adequate, benchmarking may sort out low-ability from high-ability fund managers. If, however, investment restrictions are inadequate, benchmarking may lead to excessive risk-taking. To match the benchmark, fund managers may increase the risk of their portfolio even if this decreases the expected return on the portfolio.

Some fund managers, for instance, are restricted to invest in AAA-rated bonds. If all AAA-rated bonds offer similar yields at a similar risk, benchmarking may sort out low-ability from high-ability fund managers. If, however, some AAA-rated bonds (e.g. AAA-rated mortgage-backed securities) offer higher yields at a higher risk, fund managers may invest in these bonds to match the benchmark even if their expected return is lower than the expected return on other, less risky AAA-rated bonds.

We consider a model in which fund managers differ in ability and are fired if they miss the benchmark. Fund managers can create a perfectly diversified portfolio and, in addition, gamble. High-ability fund managers can create a perfectly diversified portfolio with a higher return than low-ability fund managers.[1] In addition, fund managers have the opportunity to invest in a risky asset which increases the risk of the overall portfolio and decreases the expected return on the overall portfolio. Fund managers are fired if the realized return is lower than the average realized return.[2] If a fund manager is fired, the fund manager incurs

[1]Suppose, for instance, high-ability managers may create a perfectly diversified portfolio at lower costs than low-ability managers, and investors receive the return of the portfolio net of these costs. Then, high-ability managers can create such a portfolio with a higher net return than low-ability managers.

[2]The firing rule is taken as given. If investment restrictions were adequate and fund managers could not gamble, such a firing rule would sort out low-ability from high-ability fund managers. The main purpose here, however, is not to design an optimal incentive or sorting scheme; it is to show that benchmarking may lead to excessive risk-taking if investment restrictions are inadequate.

costs. Besides, fund managers get a fixed wage, receive a share of profits and also bear a share of losses.

We find that if the costs of being fired are sufficiently large in relation to the share of the realized return, there exists no equilibrium in which no fund manager invests in the risky asset. If there exists a symmetric[3] equilibrium in pure strategies, low-ability fund managers, at least, invest in the risky asset. We find that there exists at least one such equilibrium if and only if the returns of the risky asset or the probability that the risky asset generates a high return are sufficiently large.[4] (If fund managers do not receive a share of the realized return, our results hold if fund managers incur positive costs if they are fired.)

There is a substantial literature on incentives in funds management.[5] Brown et al. (1996), for instance, show empirically that fund managers with a bad relative performance increase risk relative to fund managers with a good relative performance. They informally argue that fund managers may increase risk because fund inflows are convex in relative performance.[6] Taylor (2003) develops a theoretical model in which fund managers compete for inflows in a winner-takes-all tournament. Taylor finds, however, that in such a setting, fund managers with a good relative performance are more likely to increase risk than managers with a bad relative performance. Our model differs from the prevalent point of view. We do not consider a situation in which fund managers try to be a 'winner'. We consider a situation in which fund managers try not to be a 'loser'. Khorana (1996) documents that fund managers with a bad relative performance are more likely to be replaced. We suppose that this influences investment decisions. And in contrast to the predictions of Taylor's model, the predictions of our model are consistent with Brown et al.'s empirical finding.

There is a number of models in which low-ability fund managers try to mimic the actions of high-ability fund managers. Trueman (1988) and Dasgupta and Prat (2006), for instance, show that fund managers may trade excessively (termed 'noise trading' or 'churning') because of career concerns. They argue that low-ability fund managers (who are uninformed about the future return of a risky asset) may trade in order to appear to have high ability (i.e. to be informed about the future return of a risky asset). In our model, however, fund managers do not try to mimic portfolio choices. Instead, they try to mimic portfolio returns.

[3]That is, all low-ability fund managers make the same investment decision and all high-ability fund managers make the same investment decision.

[4]The assumption, however, that an investment in the risky asset decreases the expected return on the overall portfolio still holds.

[5]Bhattacharya et al. (2008) provide a good literature overview.

[6]Sirri and Tufano (1998) among others, show empirically that fund inflows are indeed convex in relative performance.

The rest of the paper is organized as follows. In section 2, we present a model in which fund managers differ in ability, have the opportunity to take excessive risk, and are fired if they miss the benchmark. In section 3, we examine investment decisions and characterize possible equilibria. In section 4, we conclude and highlight the importance of adequate investment restrictions. Proofs are provided in the appendix.

2 The model

Consider a model with many fund managers. The managers differ in ability, have the opportunity to take excessive risk, and are fired if they miss the benchmark.

There are two types of managers who differ in their ability θ, $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$. Of each type, there is a continuum of mass 1. (That is, half of managers have high ability. Qualitative results do not change if, instead, a fraction λ, $\lambda \in (0, 1)$, of managers have high ability.) Managers with ability θ_L are indexed by l, $l \in [0, 1]$. Managers with ability θ_H are indexed by h, $h \in [0, 1]$.

At date 1, managers make their investment decisions. Managers can create a perfectly diversified portfolio and, in addition, gamble. They have the opportunity to invest in a risky asset which increases the risk of the overall portfolio and decreases the expected return on the overall portfolio. (For instance, a manager who is restricted to invest in AAA-rated bonds may have the opportunity to invest in AAA-rated mortgage-backed securities.) To simplify notation, we normalize the amount of capital available to a manager to 1. Manager l invests a_l, $a_l \in [0, 1]$, in the risky asset. Manager h invests a_h, $a_h \in [0, 1]$, in the risky asset.

Managers differ in their ability to create a perfectly diversified portfolio. (Suppose, for instance, high-ability managers may create a perfectly diversified portfolio at lower costs than low-ability managers, and investors receive the return of the portfolio net of these costs. Then, high-ability managers can create such a portfolio with a higher net return than low-ability managers.) To simplify the model, we assume managers with ability θ_L can create a perfectly diversified portfolio which generates a return $r^0 + \theta_L$. Managers with ability θ_H can create a perfectly diversified portfolio which generates a return $r^0 + \theta_H$. (Results do not change if the perfectly diversified portfolios are not risk-free, as long as the returns are perfectly correlated.)

Managers do not, however, differ in their ability to gamble. (Qualitative results do not change if managers who have a higher ability to create a perfectly diversified portfolio also have a higher ability to gamble.) Each manager can invest in a risky asset with a return R,

$$R = \begin{cases} r^+ & \text{with probability } p \\ r^- & \text{with probability } 1-p. \end{cases} \tag{1}$$

We assume

$$r^+ > r^0 + \theta_H \tag{2}$$

and

$$E[R] = pr^+ + (1-p)r^- < r^0 + \theta_L. \tag{3}$$

That is, if the risky asset generates a high return, then the return on the risky asset is higher than the return on the perfectly diversified portfolio for each manager. (For instance, AAA-rated mortgage-backed securities offered a higher yield than other, less risky AAA-rated bonds.) However, the expected return on the risky asset is lower than the return on the perfectly diversified portfolio for each manager. (The purpose here is to show that it may be individually optimal to increase the risk of the portfolio *even if* this decreases the expected return on the portfolio. For instance, we want to argue that it might even have been individually optimal to invest in AAA-rated mortgage-backed securities if the expected return on these bonds was lower than on other, less risky bonds.)

The return on a manager's overall portfolio depends on the return on the perfectly diversified portfolio, the return on the risky asset and the composition of the overall portfolio. Manager l's portfolio generates a return Π_l,

$$\Pi_l = r^0 + \theta_L + [R - (r^0 + \theta_L)]a_l. \tag{4}$$

Manager h's portfolio generates a return Π_h,

$$\Pi_h = r^0 + \theta_H + [R - (r^0 + \theta_H)]a_h. \tag{5}$$

The average portfolio return $\bar{\Pi}$ depends on the managers' investment decisions. An individual manager's investment decision, however, has no influence on the average portfolio return because there is a continuum of managers. The average portfolio return is given by

$$\bar{\Pi} = \frac{1}{2}\left(\int_{l=0}^{1} \Pi_l dl + \int_{h=0}^{1} \Pi_h dh\right) \tag{6}$$

or

$$\bar{\Pi} = r^0 + \frac{1}{2}\left\{\theta_L + \theta_H + [R - (r^0 + \theta_L)]\int_{l=0}^{1} a_l dl + [R - (r^0 + \theta_H)]\int_{h=0}^{1} a_h dh\right\}. \tag{7}$$

At date 2, returns and payoffs are realized. Managers receive a fixed wage w and a share s, $s \in [0,1]$, of the realized return. Manager l (h) is fired, if the realized return π_l (π_h)

is lower than the average realized return $\bar{\pi}$. If a manager is fired, the manager incurs costs c, $c \in \mathbb{R}_0^+$. (These costs may be interpreted as costs of finding a new job.)

The compensation contract and the firing rule are taken as given. If investment restrictions were adequate and managers could not gamble, the firing rule would sort out low-ability from high-ability managers. The main purpose here, however, is not to design an optimal incentive or sorting scheme; it is to show that if investment restrictions are inadequate and managers can gamble, such a firing rule may lead to excessive risk taking.

The probability that manager l (h) misses the benchmark depends on the amount a_l (a_h) the manager invests in the risky asset, and the amount a_{-l} (a_{-h}) the other managers invest in the risky asset. Let $P[\Pi_l < \bar{\Pi}|a_l, a_{-l}]$ ($P[\Pi_h < \bar{\Pi}|a_h, a_{-h}]$) denote the probability that manager l (h) misses the benchmark.

Managers are risk-neutral and maximize their expected utility. Manager l chooses a_l to solve

$$\max_{a_l \in [0,1]} E[U] = w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]a_l\} - P[\Pi_l < \bar{\Pi}|a_l, a_{-l}]c. \tag{8}$$

Manager h chooses a_h to solve

$$\max_{a_h \in [0,1]} E[U] = w + s\{r^0 + \theta_H + [E[R] - (r^0 + \theta_H)]a_h\} - P[\Pi_h < \bar{\Pi}|a_h, a_{-h}]c. \tag{9}$$

3 The appeal of risky assets

Managers may not maximize the expected return on the overall portfolio because they incur costs if the realized return is lower than the average realized return. We now examine investment decisions and characterize possible equilibria. We focus on symmetric[7] equilibria in pure strategies.

Proposition 1 *If costs c are sufficiently large in relation to the share s of the realized return, there exists no equilibrium in which no manager invests in the risky asset.*

The intuition is straightforward. Consider a manager with low ability and suppose all other managers do not invest in the risky asset. If the manager does not invest in the risky asset, the manager misses the benchmark with probability 1. If, by contrast, the manager

[7]That is, we examine equilibria in which all managers with low ability θ_L make the same investment decision and all managers with high ability θ_H make the same investment decision.

invests sufficiently in the risky asset, there are two effects: On the one hand, the expected return on the overall portfolio decreases. On the other hand, the probability of matching the benchmark increases. The manager matches the benchmark if the risky asset generates a high return. If costs c are sufficiently large in relation to the share s of the realized return, the manager hence invests in the risky asset.

Note that the threshold for costs c decreases with the share s of the realized return. If managers do not receive a share of the realized return ($s = 0$), a low-ability manager always invests in the risky asset (i.e. if $c \geq 0$ which is satisfied by assumption).

Suppose costs c are sufficiently large in relation to the share s of the realized return and a symmetric equilibrium in pure strategies exists. Then, there is a linear relationship between the amount managers with low ability and managers with high ability invest in the risky asset:

Proposition 2 *Suppose costs c are sufficiently large in relation to the share s of the realized return, and a symmetric equilibrium in pure strategies exists. Then,*

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h. \tag{10}$$

The intuition is as follows: Suppose costs c are sufficiently large in relation to the share s of the realized return and a symmetric equilibrium in pure strategies exists. Then, both low- and high-ability managers must invest just as much in the risky asset as is necessary to match the benchmark if the risky asset generates a high return.

There may exist an equilibrium in which only low-ability managers invest in the risky asset. If high-ability managers invest

$$a_h = 0 \tag{11}$$

in the risky asset, and low-ability managers invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} \tag{12}$$

in the risky asset, both low- and high-ability managers match the benchmark if the risky asset generates a high return.

However, there may also exist equilibria in which high-ability managers invest in the risky asset. If high-ability managers invest

$$a_h = x \tag{13}$$

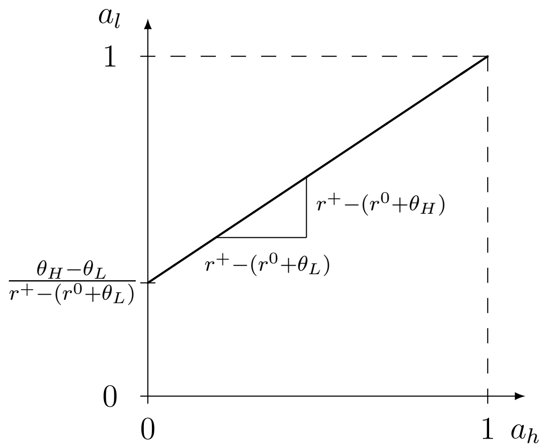

Figure 1: Possible equilibria.

in the risky asset, where $0 < x \leq 1$, low-ability managers have to invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} x \tag{14}$$

in the risky asset to match the benchmark if the risky asset generates a high return. And if, in turn, low-ability managers invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} x \tag{15}$$

in the risky asset, high-ability managers have to invest

$$a_h = x \tag{16}$$

in the risky asset to match the benchmark if the risky asset generates a high return.

Hence, if costs c are sufficiently large in relation to the share s of the realized return and a symmetric equilibrium exists, there is a linear relationship between the amount low- and high-ability managers invest in the risky asset:

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h. \tag{17}$$

The more high-ability managers invest in the risky asset, the more low-ability managers invest in the risky asset, and vice versa.

Possible symmetric equilibria in pure strategies are depicted in figure 1. Note that low-ability managers, at least, invest in the risky asset ($a_l > 0$), and they invest at least as much as high-ability managers ($a_l \geq a_h$).

If costs c are sufficiently large in relation to the share s of the realized return, at least one symmetric equilibrium exists under certain conditions:

Proposition 3 *Suppose costs c are sufficiently large in relation to the share s of the realized return. Then, there exists at least one symmetric equilibrium in pure strategies if and only if*

- *the returns r^+ and r^- of the risky asset are sufficiently large, or*
- *the probability p that the risky asset generates a high return is sufficiently large in relation to the share s of the realized return.*

The intuition is as follows. There exists at least one equilibrium if and only if there exists an equilibrium in which high-ability managers invest

$$a_h = 0 \tag{18}$$

in the risky asset, and low-ability managers invest

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} \tag{19}$$

in the risky asset. The 'if' part is obvious. For the 'only if' part, note that if there does not exist an equilibrium in which only low-ability managers invest in the risky asset, there does not exist an equilibrium in which both low- and high-ability managers invest in the risky asset.

There exists an equilibrium in which high-ability managers invest 0 in the risky asset, and low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, if and only if the returns r^+ and r^- are sufficiently large, or the probability p is sufficiently large in relation to the share s of the realized return.[8] To see this, consider the investment decisions of high- and low-ability managers.

First, consider the investment decision of a high-ability manager. Suppose all low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset and all other high-ability managers invest 0 in the risky asset. If the high-ability manager also invests 0 in the risky asset, the manager misses the benchmark with probability 0. If, by contrast, the high-ability manager invests more than 0 in the risky asset, there are two negative effects. First, the expected return on the overall portfolio decreases. And second, the probability of missing the benchmark may increase. Hence, the high-ability manager also invests 0 in the risky asset.

Now, consider the investment decision of a low-ability manager. Suppose all high-ability managers invest 0 in the risky asset and all other low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the

[8] The assumption, however, that the expected return on the risky asset is lower than the return on the perfectly diversified portfolio still holds.

risky asset. If the low-ability manager also invests $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, the manager matches the benchmark if the risky asset generates a high return, but misses the benchmark if the risky asset generates a low return. That is, the manager misses the benchmark with probability $1 - p$. If, by contrast, the low-ability manager invests 0 in the risky asset, there are again two effects. First, the expected return on the overall portfolio increases. Second, the probability of missing the benchmark changes. The probabiliy of missing the benchmark now depends on the returns r^+ and r^- of the risky asset. If r^+ and r^- are sufficiently large, the low-ability manager misses the benchmark with probability 1. If r^+ and r^- are not sufficiently large, the low-ability manager only misses the benchmark with probability p.

The reason is as follows. (Consider again the investment decision of a low-ability manager, and suppose again the manager invests 0 in the risky asset, while all other low-ability managers invest $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and all high-ability managers invest 0 in the risky asset.) If the risky asset generates a high return, the low-ability manager misses the benchmark in any case. If the risky asset generates a low return, the manager also misses the benchmark if the average portfolio return does not suffer much. This is the case if r^+ and r^- are sufficiently large. If the return r^+ is large, the other low-ability managers do not have to invest much in the risky asset to match the benchmark if the risky asset generates a high return (i.e. $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ is small). And if the return r^- is large, portfolio returns do not suffer much for a given amount invested in the risky asset if the risky asset generates a low return. Hence, if r^+ and r^- are sufficiently large, the low-ability manager misses the benchmark with probability 1 if the manager does not invest in the risky asset. If r^+ and r^- are not sufficiently large, the low-ability manager only misses the benchmark with probability p if the manager does not invest in the risky asset.

Suppose now the returns r^+ and r^- are sufficiently large. Then, the low-ability manager misses the benchmark with probability 1 if the manager does not invest in the risky asset. Hence, the low-ability manager invests $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset if costs c are sufficiently large in relation to the share s of the realized return (which is satisfied by assumption).

Note that the threshold for costs c decreases with the share s of the realized return. If managers do not receive a share of the realized return ($s = 0$), a low-ability manager invests in the risky asset if managers incur positive costs if they are fired ($c \geq 0$).

Now, suppose the returns r^+ and r^- are not sufficiently large. Then, the low-ability manager misses the benchmark with probability p if the manager does not invest in the risky asset. Hence, the low-ability manager invests $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset if the probability p that the risky asset generates a high return is sufficiently large in relation to the share s of the realized return.

Note that the threshold for probability p decreases with the share s of the realized return. If managers do not receive a share of the realized return ($s = 0$), a low-ability manager invests in the risky asset if a high return is at least as likely as a low return ($p \geq \frac{1}{2}$).

There may also exist equilibria in which both low- and high-ability managers invest in the risky asset if the returns r^+ and r^- are sufficiently large, or the probability p is sufficiently large in relation to the share s of the realized return. If high-ability managers invest more than 0 in the risky asset, low-ability managers face a trade-off similar to those described above. If low-ability managers invest more than $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, high-ability managers also face a similar trade-off. Suppose low-ability managers invest more than $\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset. Then, if a high-ability manager invests 0 in the risky asset, the manager misses the benchmark if the risky asset generates a high return. If, by contrast, the high-ability manager invests sufficiently in the risky asset, the manager misses the benchmark with probability 0. If costs c are sufficiently large in relation to the share s of the realized return, the high-ability manager hence invests in the risky asset.

As a result, managers may face a coordination problem. They would be best off if only low-ability managers invested in the risky asset. However, they may end up in an equilibrium in which both low- and high-ability managers invest in the risky asset.

4 Conclusion

We argue that fund managers may take excessive risk if they differ in ability, are fired if they miss the benchmark, and have the opportunity to gamble. If investment restrictions are adequate, benchmarking may sort out low-ability from high-ability fund managers. If, however, fund managers have the opportunity to gamble, they may take excessive risk to match the benchmark.

Investment restrictions are often based on ratings. It can be argued that inflated ratings increase fund managers' ability to gamble. Hence, inflated ratings may lead to excessive risk-taking even if fund managers do not take ratings at face value and know about the risks. Therefore, it is indeed important that credit rating agencies assign correct ratings.

Appendix

Proof of proposition 1

Consider manager l's investment decision (choice of a_l) and suppose all other managers do not invest in the risky asset ($a_{-l} = 0$). Then, the average portfolio return is

$$\bar{\Pi} = r^0 + \frac{1}{2}(\theta_L + \theta_H). \tag{20}$$

If the manager does not invest in the risky asset ($a_l = 0$), the portfolio return is

$$\Pi_l = r^0 + \theta_L. \tag{21}$$

The manager hence misses the benchmark with probability 1 and gets the utility

$$U = w + s(r^0 + \theta_L) - c. \tag{22}$$

The manager's utility decreases with the amount a_l invested in the risky asset until a_l is sufficiently large such that the portfolio return matches the average portfolio return if the risky asset generates a return r^+. The portfolio return matches the average portfolio return if $R = r^+$ and

$$r^0 + \theta_L + [r^+ - (r^0 + \theta_L)]a_l = r^0 + \frac{1}{2}(\theta_L + \theta_H) \tag{23}$$

or

$$a_l = \frac{\theta_H - \theta_L}{2[r^+ - (r^0 + \theta_L)]}. \tag{24}$$

Let

$$\hat{a} \equiv \frac{\theta_H - \theta_L}{2[r^+ - (r^0 + \theta_L)]} \tag{25}$$

and note that

$$0 < \hat{a} < 1. \tag{26}$$

If the manager chooses $a_l = \hat{a}$, the manager only misses the benchmark with probability $1 - p$. The manager hence gets the utility

$$E[U] = w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]\hat{a}\} - (1 - p)c. \tag{27}$$

Starting from $a_l = \hat{a}$, the manager's utility again decreases with the amount a_l invested in the risky asset.

The manager strictly prefers $a_l = \hat{a}$ to $a_l = 0$ if

$$w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]\hat{a}\} - (1-p)c > w + s(r^0 + \theta_L) - c. \tag{28}$$

This can be rearranged to get

$$pc > s(r^0 + \theta_L - E[R])\hat{a}. \tag{29}$$

Substituting $\hat{a}$ and rearranging gives

$$c > s\frac{\theta_H - \theta_L}{2p}\frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}. \tag{30}$$

Hence, if costs c are sufficiently large in relation to the share s of the realized return, there exists no equilibrium in which no manager invests in the risky asset. ∎

Proof of proposition 2

Suppose

$$c > s\frac{\theta_H - \theta_L}{2p}\frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}, \tag{31}$$

and all managers with low ability θ_L invest the same amount a_l in the risky asset and all managers with high ability θ_H invest the same amount a_h in the risky asset.

First, consider managers with low ability θ_L. Note that they match the benchmark if they match managers with high ability θ_H. If the risky asset generates a high return r^+, managers with low ability θ_L match the benchmark if

$$r^0 + \theta_L + [r^+ - (r^0 + \theta_L)]a_l = r^0 + \theta_H + [r^+ - (r^0 + \theta_H)]a_h \tag{32}$$

or

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)}a_h. \tag{33}$$

Let

$$\hat{a}_l(a_h) \equiv \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)}a_h, \tag{34}$$

and note that

$$0 < \hat{a}_l(a_h) \leq 1. \tag{35}$$

There cannot be an equilibrium in which managers with low ability θ_L invest $0 < a_l < \hat{a}_l(a_h)$ in the risky asset. Each manager with low ability θ_L would be better off investing $a_l = 0$ in the risky asset.

There cannot be an equilibrium in which managers with low ability θ_L invest $a_l > \hat{a}_l(a_h)$ in the risky asset. Each manager with low ability θ_L would be better off investing $a_l = \hat{a}_l(a_h)$ in the risky asset.

Therefore, if there is an equilibrium, managers with low ability θ_L must invest either $a_l = 0$ or $a_l = \hat{a}_l(a_h)$ in the risky asset.

Now, consider managers with high ability θ_H. Note that they fall behind the benchmark if they fall behind managers with low ability θ_L. If managers with low ability θ_L invest $a_l \leq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, managers with high ability θ_H do not fall behind the benchmark if they invest $a_h = 0$ in the risky asset. If, by contrast, managers with low ability θ_L invest $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, managers with high ability θ_H fall behind the benchmark if they invest $a_h = 0$ in the risky asset and the risky asset generates a high return r^+.

If $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and $R = r^+$, managers with high ability θ_H do not fall behind the benchmark if

$$r^0 + \theta_H + [r^+ - (r^0 + \theta_H)]a_h = r^0 + \theta_L + [r^+ - (r^0 + \theta_L)]a_l \tag{36}$$

or

$$a_h = -\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_H)} + \frac{r^+ - (r^0 + \theta_L)}{r^+ - (r^0 + \theta_H)} a_l. \tag{37}$$

Let

$$\hat{a}_h(a_l) \equiv -\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_H)} + \frac{r^+ - (r^0 + \theta_L)}{r^+ - (r^0 + \theta_H)} a_l, \tag{38}$$

and note that

$$0 < \hat{a}_h(a_l) \leq 1. \tag{39}$$

If $a_l \leq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$, there cannot be an equilibrium in which managers with high ability θ_H invest $a_h > 0$ in the risky asset. Each manager with high ability θ_H would be better off investing less in the risky asset.

Therefore, if $a_l \leq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and there is an equilibrium, managers with high ability θ_H must invest $a_h = 0$ in the risky asset.

If $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$, there cannot be an equilibrium in which managers with high ability θ_H invest $0 < a_h < \hat{a}_h(a_l)$ in the risky asset. Each manager with high ability θ_H would be better off investing $a_h = 0$ in the risky asset.

If $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$, there cannot be an equilibrium in which managers with high ability θ_H invest $a_h > \hat{a}_h(a_l)$ in the risky asset. Each manager with high ability θ_H would be better off investing $a_h = \hat{a}_h(a_l)$ in the risky asset.

Therefore, if $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ and there is an equilibrium, managers with high ability θ_H must invest either $a_h = 0$ or $a_h = \hat{a}_h(a_l)$ in the risky asset.

By proposition 1, $a_l = 0$ and $a_h = 0$ cannot be an equilibrium. Thus, if there is an equilibrium, $a_l \geq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$.

Note that

$$\hat{a}_h\left(\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}\right) = 0 \tag{40}$$

and

$$\hat{a}_h(a_l) = \hat{a}_l^{-1}(a_h). \tag{41}$$

Therefore, if there is an equilibrium, there is a linear relationship between the amount managers with low ability θ_L and those with high ability θ_H invest in the risky asset:

$$a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h. \quad \blacksquare \tag{42}$$

Proof of proposition 3

Suppose

$$c \geq s \frac{\theta_H - \theta_L}{p} \frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}. \tag{43}$$

Let

$$\hat{a}_l(a_h) \equiv \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + \frac{r^+ - (r^0 + \theta_H)}{r^+ - (r^0 + \theta_L)} a_h \tag{44}$$

and

$$\hat{a}_h(a_l) \equiv -\frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_H)} + \frac{r^+ - (r^0 + \theta_L)}{r^+ - (r^0 + \theta_H)} a_l. \tag{45}$$

There exists a symmetric equilibrium in pure strategies if and only if each manager with low ability θ_L prefers

$$a_l = \hat{a}_l(a_h) \tag{46}$$

to

$$a_l = 0, \tag{47}$$

and each manager with high ability θ_H prefers

$$a_h = \hat{a}_h(a_l) \tag{48}$$

to

$$a_h = 0. \tag{49}$$

First, consider the investment decision of a manager with high ability θ_H. Then, consider the investment decision of a manager with low ability θ_L.

If all managers with low ability θ_L invest $a_l = \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, $\hat{a}_h = 0$. If all managers with low ability θ_L invest $a_l > \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$ in the risky asset, $\hat{a}_h > 0$. In this case, each manager with high ability θ_H prefers $a_h = \hat{a}_h$ to $a_h = 0$ if

$$w + s\{r^0 + \theta_H + [E[R] - (r^0 + \theta_H)]\hat{a}_h\} \geq w + s(r^0 + \theta_H) - pc. \tag{50}$$

This can be rearranged to get

$$c \geq s\frac{r^0 + \theta_H - E[R]}{p}\hat{a}_h. \tag{51}$$

Now, consider a manager with low ability θ_L and suppose the manager invests $a_l = 0$ in the risky asset. Furthermore, suppose all other managers with low ability θ_L invest $\hat{a}_l$ in the risky asset and all managers with high ability θ_H invest $\hat{a}_h$ in the risky asset. Then, the low-ability manager misses the benchmark if the risky asset generates a high return r^+. If the risky asset generates a low return r^-, the manager also misses the benchmark if

$$r^0 + \theta_L < r^0 + \frac{1}{2}\left\{\theta_L + \theta_H + [r^- - (r^0 + \theta_L)]\hat{a}_l + [r^- - (r^0 + \theta_H)]\hat{a}_h\right\}. \tag{52}$$

This can be rearranged to get

$$(r^0 + \theta_L - r^-)\hat{a}_l + (r^0 + \theta_H - r^-)\hat{a}_h < \theta_H - \theta_L. \tag{53}$$

That is, if the risky asset generates a low return r^-, the low-ability manager misses the benchmark if the other managers do not invest 'too much' in the risky asset.

First, suppose

$$(r^0 + \theta_L - r^-)\hat{a}_l + (r^0 + \theta_H - r^-)\hat{a}_h < \theta_H - \theta_L. \tag{54}$$

Then, a manager with low ability θ_L misses the benchmark with probability 1. If, by contrast, the manager also invests $a_l = \hat{a}_l$ in the risky asset, the manager matches the benchmark if the risky asset generates a high return r^+. That is, the manager only misses the benchmark with probability $1 - p$. The manager prefers $a_l = \hat{a}_l$ to $a_l = 0$ if

$$w + s\{r^0 + \theta_L + [ER - (r^0 + \theta_L)]\hat{a}_l\} - (1 - p)c \geq w + s(r^0 + \theta_L) - c. \tag{55}$$

This can be rearranged to get

$$c \geq s\frac{r^0 + \theta_L - E[R]}{p}\hat{a}_l. \tag{56}$$

Note that $\hat{a}_h \geq 0$ and $\hat{a}_l \geq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$. Therefore, there exists at least one symmetric equilibrium in pure strategies if

$$c \geq s \frac{\theta_H - \theta_L}{p} \frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}, \tag{57}$$

which is satisfied by assumption, and

$$(r^0 + \theta_L - r^-) \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)} + (r^0 + \theta_H - r^-)0 < \theta_H - \theta_L. \tag{58}$$

The latter condition can be rearranged to get

$$r^+ + r^- > 2(r^0 + \theta_L). \tag{59}$$

Now, suppose

$$(r^0 + \theta_L - r^-)\hat{a}_l + (r^0 + \theta_H - r^-)\hat{a}_h \geq \theta_H - \theta_L. \tag{60}$$

Then, if a manager with low ability θ_L invests $a_l = 0$ in the risky asset, the manager only misses the benchmark if the risky asset generates a high return r^+. If, by contrast, the manager also invests $a_l = \hat{a}_l$ in the risky asset, the manager does not miss the benchmark if the risky asset generates a high return r^+. However, the manager misses the benchmark if the risky asset generates a low return r^-. The manager prefers $a_l = \hat{a}_l$ to $a_l = 0$ if

$$w + s\{r^0 + \theta_L + [E[R] - (r^0 + \theta_L)]\hat{a}_l\} - (1-p)c \geq w + s(r^0 + \theta_L) - pc. \tag{61}$$

This can be rearranged to get

$$p \geq \frac{1}{2} + s \frac{r^0 + \theta_L - E[R]}{2c} \hat{a}_l. \tag{62}$$

Note again that $\hat{a}_h \geq 0$ and $\hat{a}_l \geq \frac{\theta_H - \theta_L}{r^+ - (r^0 + \theta_L)}$. Therefore, there exists at least one symmetric equilibrium in pure strategies if

$$p \geq \frac{1}{2} + s \frac{\theta_H - \theta_L}{2c} \frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}. \tag{63}$$

Finally, note that if neither

$$r^+ + r^- > 2(r^0 + \theta_L) \tag{64}$$

nor

$$p \geq \frac{1}{2} + s \frac{\theta_H - \theta_L}{2c} \frac{r^0 + \theta_L - E[R]}{r^+ - (r^0 + \theta_L)}, \tag{65}$$

there exists no symmetric equilibrium in pure strategies. ∎

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