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Limiting Profit Shifting in a Model with Heterogeneous Firm Productivity*

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Abstract

This paper analyzes measures that limit firms' profit shifting activities in a model that incorporates heterogeneous firm productivity and monopolistic competition. Such measures, e.g. thin capitalization rules, have become increasingly widespread as governments have reacted to growing profit shifting activities of multinational companies. However, besides limiting profit shifting, such rules entail costs. As the regulations can only focus on the means to shift profits, not on profit shifting itself, they impose costs on all firms, no matter whether these firms shift profits abroad or not. In the model, these costs force some firms to exit the market. Thus, as this makes the remaining firms more profitable, regulations to limit profit shifting may even increase the aggregate amount of profits shifted abroad. From a welfare point of view, it may even be optimal no to limit profit shifting at all.

Keywords: profit shifting, heterogeneous firms, tax competition

JEL Classification: H 25, H 73, F 23

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1 Introduction

With growing financial integration during the last years, multinational companies have increasingly shifted profits abroad in order to reduce their tax payments. Thus, multinationals earn significantly lower profits than comparable domestic firms in high tax countries, paying over 50 % less taxes than similar domestic firms in high tax countries (see Egger et al., 2010).[1]

Governments have reacted: With the help of targeted changes to the tax code they have tried to secure their respective tax bases. Examples for such measures are stricter transfer pricing rules or thin capitalization rules. Such regulations have become widespread during the last years. Between 1996 and 2005, for example, the share of EU-27 countries that imposes thin capitalization rules has doubled, from 30 % to 60 % (see Buettner et al., 2008).

Such thin capitalization rules restrict the deduction of interest payments for tax purposes to a certain percentage of earnings. In Germany, for example, interest payments (net of interest expenses) can only be deducted if their value is less than 30 % of earnings before interest, taxes, depreciation and amortization (EBITDA). Such rules are not restricted to borrowing from affiliates, but comprise all kinds of debt finance. Due to non-discrimination rules, they apply to most corporations, even if they are not active internationally. The benefit of such a broad thin capitalization rule is that it effectively limits profit shifting via debt finance.[2]

However, such broadly applicable regulations also have disadvantages. They are badly targeted, as they also apply in cases that have nothing to do with profit shifting.[3] In extreme cases, it is even possible that tax payments accrue under such rules even if the firm makes a loss (see Homburg, 2007).[4] Further costs arise as firms hire consultants

[1] For further empirical evidence on profit shifting see, for example, Hines and Rice (1994), Huizinga and Laeven (2008) or Weichenrieder (2009).

[2] There are several empirical analyses of thin capitalization rules, which confirm that such rules indeed have a significant effect on firms' decisions. Buettner et al. (2008) confirm that higher taxes lead to a higher debt ratio of subsidiaries in that country. Due to thin capitalization rules, the debt ratio decreases and is less dependent on the tax rate. They find no evidence for negative effects of thin capitalization rules in the form of a lower capital stock. Weichenrieder and Windischbauer (2008) reach similar conclusions, as do Overesch and Wamser (2010) who use a difference-in-difference approach.

[3] For example, the German thin capitalization rule applies also if a group is only active nationally, and debt mostly stems from bank financing.

[4] Homburg (2007) gives the following example of a corporation making a loss of 20 million euros,

or choose inefficient strategies in order to comply with the regulations.[5]

In this paper, a model is set up that incorporates such costs along with the (beneficial) limitation of profit shifting. Then, the effects that limiting profit shifting has on welfare and on the aggregate sum of profits shifted abroad are analyzed. Firms in the model are heterogeneous in their productivity and compete under monopolistic competition.

The key result of the model is that strengthening a limitation on profit shifting does not necessarily lead to less profit being shifted abroad. As the costs of such regulations force some firms out of the market, there is less competition, so that the remaining firms become more profitable. It is therefore possible that the absolute amount of profits shifted abroad increases, even though only a smaller percentage of profits can be transferred. Furthermore, additional firms may start to shift profits abroad.

There are further effects of regulations to limit profit shifting besides the ambiguous effect on profits shifted itself. As such rules force some firms to exit the market, consumers have fewer varieties from which to choose, which implies a welfare loss. The overall welfare effect depends on the market situation: If firms have high market power, it is best if governments do not limit profit shifting possibilities. If firm productivity is very heterogeneously distributed, profit shifting should be limited, as there are relatively many firms that engage in profit shifting activities to begin with.

Limiting profit shifting is also more likely to be favorable if the costs of profit shifting are relatively low. As such costs have fallen during the last decades due to increasing global integration, this result is in line with the empirical evidence of increased regulation against profit shifting presented above.

Including heterogeneous productivity is crucial to this analysis of limiting profit shifting, as it allows to model that the effects of this specific tax policy differ among firms with different productivity levels. The literature on firm heterogeneity originating with Melitz (2003) has recently been adapted to analyze the effects of taxation. However, as of yet, there have been only few uses of heterogeneous productivity in the public finance literature.

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with net interest expenses of 60 million euros and 10 million euros of depreciation allowances. The German thin capitalization rule limits deductible interest expenses to 15 million euros, resulting in a taxable profit of 25 million euros (implying a tax payment of about 7.5 million euros despite making a loss).

[5]Under a thin capitalization rule, firms may abstain from internal debt financing even when it would otherwise be optimal (e.g. for investments by affiliates who face high interest rates).

A first paper is Davies and Eckel (2010), who show in such a model the classic result that tax competition leads to lower-than-optimal tax rates, and that too little of the public good is produced, so that a welfare loss arises. Haufler and Stähler (2009) also offer an explanation for the international race-to-the-bottom in profit tax rates by means of a model with heterogeneous productivity. They show that an increase in market sizes of both countries leads to lower equilibrium tax rates. Baldwin and Okubo (2009a) also predict a race-to-the-bottom in tax rates when the integration of goods markets increases. In another paper, Baldwin and Okubo (2009b), the same authors show that a tax-cut-cum-base-broadening tax reform can increase tax revenue.

Closest to this present paper is Krautheim and Schmidt-Eisenlohr (2011). They derive equilibrium tax rates in a model with monopolistic competition among heterogeneous firms. Firms make positive profits, which are taxed by the government. To avoid taxation to some extent, some firms shift *all* of their profits from the home country to the tax haven. In equilibrium, only the most productive firms shift profits.

The papers described above propose general tax models with heterogeneous productivity. None of them examines a specific policy measure. The present paper specifically asks about the effects of a regulation that limits profit shifting (e.g. by a thin capitalization rule).[6]

This paper is structured as follows. The next section introduces the reader to the model and derives a first result on the aggregate amount of profits shifted abroad. Section 3 analyzes the optimization problems of the two countries more in detail. Some numerical simulations in section 4 clarify the theoretical results. Section 5 concludes.

2 Model

The model consists of two countries, the "home market" and the "tax haven". The tax haven is small; all production takes place in the home market. The economy of the home market comprises two sectors. One of them is a numeraire sector, in which a homogeneous good is produced with a single factor (labor) under perfect competition using a

[6]Another paper specifically analyzing such a measure, namely a thin capitalization rule, is Haufler and Runkel (2008). In contrast to this paper, the firms' internationalization decision is not endogenous in the model; instead, it is assumed that only some firms are active internationally. It is shown that in a symmetric equilibrium, each country chooses inefficiently low tax rates and lax thin capitalization rules.

technology with constant returns to scale. The price of the final good in this sector is normalized to unity. In the second sector, differentiated goods are manufactured under monopolistic competition by firms which differ in their productivity. Higher productivity is represented by lower marginal cost, symbolized by the firm-specific efficiency coefficient a_i. The cost of production consists of constant, firm-specific marginal costs and fixed production costs, c. Marginal costs of all potentially active firms follow a Pareto distribution in the interval $[0, a_0]$.[7] The cumulative distribution function of the marginal cost is given by

$$G(a) = \left(\frac{a}{a_0}\right)^{\gamma}, \qquad \gamma > 1. \tag{1}$$

The Pareto distribution implies that higher values of a_i are more likely than lower values, i.e. that there are relatively few very productive firms. Firms are more heterogeneous when the shape parameter γ is lower.[8] To simplify, a_0 is set to one in the following.

Firms in the differentiated goods sector compete under Dixit-Stiglitz monopolistic competition: Each firm offers a product which is, from the consumers' point of view, only imperfectly substitutable by other goods. Therefore, firms have some market power. Consumers' preferences are given by

$$U = \mu \ln X_I + \beta X_G + X_N, \quad with \quad X_I = \left(\int_{i\in\Theta} x_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1 > \mu > 0. \tag{2}$$

X_I and X_N represent the quantities consumed of the differentiated and numeraire goods, respectively. X_G is a public good, financed by tax revenue, which enters the utility function weighted by a factor $\beta > 1$. μ shows the importance of the differentiated goods relative to the numeraire good. Θ is the set of all differentiated goods. The Dixit-Stiglitz parameter σ can be interpreted as the (constant) elasticity of substitution.

Maximizing the utility function (2) subject to the budget constraint, the demand for a particular variety of the differentiated good is given by

$$x_i = \frac{\mu}{\int p_i^{-(\sigma-1)}}\, p_i^{-\sigma} = \frac{\mu}{p_i^{\sigma}} P^{\sigma-1}, \qquad \text{with} \quad P = \left(\int_{i\in\Theta} p_i^{-(\sigma-1)} di\right)^{-\frac{1}{\sigma-1}}, \tag{3}$$

[7] The Pareto distribution is a good approximation of the empirically observed distribution of firm sizes (see Axtell, 2001). Its use is common in the literature, see e.g. Helpman et al. (2004) or Krautheim and Schmidt-Eisenlohr (2011).

[8] With a lower value of the parameter γ, more low-cost firms exist. If $\gamma \to 1$, marginal cost is uniformly distributed, which represents the highest degree of heterogeneity possible in this model. In the opposite case of $\gamma \to \infty$, firms are homogeneous (with marginal costs equal to a_0).

where p_i is the price of variety i, and P is the CES price index. Aggregate demand for X_I is thus given by $X_I = \frac{\mu}{P}$.

Firms facing this demand function maximize their profits. Therefore they set their prices as a constant mark-up over marginal cost

$$p(a_i) = \frac{\sigma}{\sigma - 1} a_i. \tag{4}$$

The price is higher if the elasticity of substitution is lower, i.e. if firms have more monopoly power.

Thus, pre-tax profits of firms are given by the following equation, whereby the second equality takes optimal price and quantity decisions into account[9]

$$\begin{aligned} \pi_i - c &= (p_i - a_i) \cdot x_i - c \\ &= \frac{\mu}{\sigma} \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma - 1} \left(\frac{P}{a} \right)^{\sigma - 1} - c. \end{aligned} \tag{5}$$

The most efficient firms (i. e. the firms with the lowest marginal cost a_i) are the most profitable.

Profits are taxed at a constant marginal rate t_H in the home country. However, firms have the possibility to shift profits to the tax haven, where the profit tax rate t_X is lower than in the home country ($0 < t_X < t_H < 1$).[10] In order to be able to do so, firms incur a fixed cost, f. It is assumed that this cost does not reduce the taxable base. As the cost of profit shifting is fixed, only the most profitable firms choose to shift their profits abroad (see Krautheim and Schmidt-Eisenlohr, 2011).

In reality, many regulations limit profit shifting activities. Governments can curb the possibilities of (legal) tax avoidance, but such regulations entail costs for many firms, as they have to be generally applicable and can only target the means of profit shifting, i.e. transfer prices or intra-firm debt, not profit shifting itself. An example are thin-capitalization rules, which dictate that only interest expenses of up to a certain fraction of profits can be subtracted from earnings for tax purposes. This limits the possibilities for profit shifting via debt, but also increases firm's financing costs. In the model, the government of the home country can, with the help of such regulations, limit the maximum percentage of profits α which firms are able to shift abroad.

[9] In order to keep profits finite, it is assumed that $\gamma - \sigma + 1 > 0$.

[10] As the tax haven has no firms of its own, it collects tax revenue only if it sets a lower tax rate than the home market. Otherwise, no firm would be willing to shift profits.

This, however, imposes costs on all firms. These costs can be interpreted in manifold ways: Firms may choose inefficient strategies in order to comply with the regulations, or have to hire costly consultants. If thin capitalization rules are used, financing costs may rise. Such costs arise even if the firm does not engage in profit shifting activities – it still has to comply with the regulations, e.g. limit its debt financing.

While all firms have some additional costs due to such a regulation (such as consultants and investment distortions), firms with relatively low profits bear the highest cost. For example, if profit shifting is limited using a thin capitalization rule, unproductive firms are hit harder as they have fewer self-financing possibilities and are more dependent on debt finance. When their interest expenses, which cause real case-flows, are no longer deductible for tax purposes, it is possible that firms which were marginally profitable without the thin capitalization rule are no longer profitable after its introduction and are therefore forced to exit the market.

It is clear that the model should thus include that the costs imposed by such regulations on all firms should be highest relative to profits for low-productivity firms. Using that such firms also realize lower profits than more productive firms, the model imposes the same *absolute* costs on all firms. However, importantly, this burden is thus highest for low-productivity firms in *relative* terms. While this is surely a very simplifying assumption, assuming that this cost changes with firm productivity would bring no additional effects, but would make the exposition much more difficult. Thus I opt for the simple version in order to preserve clarity.

These compliance costs are represented by $(1-\alpha)\tau$ in the model. The parameter τ scales the severity of the burden, which also depends on the strictness of the limitation on profit shifting, $(1-\alpha)$. Due to this burden, and to fixed costs, not all potential firms are productive enough to be in business. A zero-profit condition determines the cut-off value a_τ, that is, the cost coefficient of the least productive firm in the market:[11]

$$\left[p\left(a_\tau\right)x\left(a_\tau\right) - a_\tau x\left(a_\tau\right) - c\right]\left(1-t_H\right) - (1-\alpha)\tau = 0. \tag{6}$$

Solving this condition for a_τ yields

$$a_\tau = \frac{\sigma-1}{\sigma}\left(\frac{\mu}{\sigma}\frac{1-t_H}{c\left(1-t_H\right)+(1-\alpha)\tau}\right)^{\frac{1}{\sigma-1}}P. \tag{7}$$

[11]Note that, as profit shifting causes a fixed cost, the more productive firms have a stronger incentive to shift profits. I assume that fixed costs are such that not all firms engage in profit shifting, so that the least productive firm in the market (i.e. the firm with marginal costs of a_τ) is not avoiding taxes.

If the tax rate in the home country or the cost of the limitation on profit shifting is higher, fewer firms are in the market. More firms are active in larger markets (as measured by μ). For $\alpha = 1$ equation (7) collapses to the case without a limitation on profit shifting.

If a firm is sufficiently productive, it incurs the fixed cost f in order to shift some of its profits abroad. As the most profitable firms have the most to gain from avoiding taxes, while the cost of profit shifting is fixed, only firms with marginal costs below a level a_P shift profits abroad. This value is determined by the following indifference condition, which already takes into account that fixed costs c and the burden of the limitation on profit shifting $(1-\alpha)\tau$ have to be borne in both cases:[12]

$$(1-t_H)\,\pi\,(a_P) = (1-t_H)\,(1-\alpha)\,\pi\,(a_P) + (1-t_X)\,\alpha\pi\,(a_P) - f. \tag{8}$$

The left hand side of equation (8) represents the case in which the firm pays taxes only in the home country. On the right hand side, it shifts profits into the tax haven. As the cost of profit shifting, f, is fixed, the firm always shifts as much of its profit abroad as is possible. It is assumed that the cost of profit shifting is not deductible from the firm's taxable base.[13]

Inserting equation (5) for the profits, the marginal cost level a_P is given by

$$a_P = \frac{\sigma-1}{\sigma}\left(\frac{\mu}{\sigma}\frac{\alpha\,(t_H-t_X)}{f+\alpha c\,(t_H-t_X)}\right)^{\frac{1}{\sigma-1}} P. \tag{9}$$

Firms with marginal costs under a_P shift as much of their profits as possible abroad; the other firms (with marginal costs $a_i \in [a_P,\ a_\tau]$) prefer to pay taxes on all profits in the home country, as the costs of profit shifting are – for them – too high relative to their earnings. A higher tax rate in the home country or a lower tax rate abroad induce more firms to shift profits abroad. Note that a_P depends only indirectly (via the price index) on the costs that profit shifting limitations impose on all firms.

The cut-off values a_τ and a_P depend on the price index. Using the definition of the price index and combining it with equations (4) and (7), the equilibrium value of the

[12]The tax-deductible fixed cost of production, c, is always deducted in the home country. Due to the higher tax rate there, this is optimal for the firm.

[13]The assumption of no deductibility is justified if the costs of profit shifting lie in distortions or soft costs (such as language barriers or an inability to effectively monitor employees) that arise because an investment (e.g. a sales and distribution office) is undertaken in a tax haven instead of in a high-tax country. An alternative assumption would be that these costs are deductible in the tax haven, which would not change the analysis qualitatively.

price index is

$$P = \left(\int_0^{a_\tau} p_i^{-(\sigma-1)} di \right)^{-\frac{1}{\sigma-1}} = \frac{\sigma}{\sigma-1} \left(\frac{\gamma-\sigma+1}{\gamma} \right)^{\frac{1}{\gamma}} \left(\frac{1-t_H}{c\,(1-t_H)+(1-\alpha)\,\tau} \right)^{-\frac{\gamma-\sigma+1}{\gamma(\sigma-1)}}. \tag{10}$$

Lastly, let us consider optimal quantities of the numeraire good X_N and the public good X_G. Demand for the numeraire good is given by $X_N = I - \mu$, whereby the income I consists of labor income L and profit income. The public good is financed by tax revenue T, $X_G = T$. Thus, optimal quantities of both goods depend on aggregate profits and their taxation. To look at this in more detail, consider the tax base in both countries (i.e. aggregate profits without deducting the burden imposed by the limitation on profit shifting).These are, for the home country (Π) and tax haven (Π^*) respectively:

$$\begin{aligned} \Pi &= \int_{a_P}^{a_\tau} (\pi_i - c)\, dG(a) + \int_0^{a_P} [(1-\alpha)\,\pi_i - c]\, dG(a) \qquad (11) \\ &= \frac{\mu}{\sigma} \left[1 - \alpha \left[\frac{c\,(1-t_H)+(1-\alpha)\,\tau}{1-t_H} \frac{\alpha\,(t_H - t_X)}{f + \alpha\,(t_H - t_X)\,c} \right]^{\frac{\gamma-\sigma+1}{\sigma-1}} - c\frac{\gamma-\sigma+1}{\gamma} \frac{1-t_H}{c\,(1-t_H)+(1-\alpha)\,\tau} \right], \end{aligned}$$

$$\Pi^* = \int_0^{a_P} \alpha \pi_i dG(a) = \alpha \frac{\mu}{\sigma} \left[\frac{c\,(1-t_H)+(1-\alpha)\,\tau}{1-t_H} \frac{\alpha\,(t_H - t_X)}{f + \alpha\,(t_H - t_X)\,c} \right]^{\frac{\gamma-\sigma+1}{\sigma-1}}. \tag{12}$$

The tax base in the home country, equation (11), can be interpreted as the sum of all profits (i. e. the tax base without any profit shifting, $\frac{\mu}{\sigma}$) less the profits shifted to the tax haven (the second term) and aggregate fixed costs.

What determines how much profit is shifted abroad on aggregate? The tax base in the tax haven rises if the tax difference between the two countries increases. It falls if firms have higher costs to shift profits (higher f) or if the demand for differentiated goods in the home market is lower (lower μ). If the burden imposed by profit shifting limitations is greater, the tax base in lower, as there are fewer firms in the market. The tax base in the haven is also smaller if the firms are more heterogeneous (lower γ).

Importantly, it is not always the case that a limitation on profit shifting leads indeed to less profit being shifted abroad on aggregate. Differentiating equation (12) with respect to α, it becomes clear that counteracting effects are at work:

$$\frac{\partial \Pi^*}{\partial \alpha} = \int_0^{a_P} \left(\pi_i + \alpha \frac{\partial \pi_i}{\partial \alpha} \right) dG(a) + \frac{\partial a_P}{\partial \alpha} \pi(a_P). \tag{13}$$

The first term reflects the effects on the intensive margin, that is, the change in the amount of profit each firm shifts abroad. First, there is a direct effect $\left(\int_0^{a_P} \pi_i dG(a) \right)$, as

a change in α changes the percentage of profits that each firm is allowed to shift abroad. Secondly, there is an indirect effect: By strengthening a limitation on profit shifting (lowering α), the government crowds some firms out of the market. For the remaining firms, the market becomes less competitive, thus rendering them more profitable. Thus, possibly, if the increase in profitability is strong enough, these firms shift *more* profits abroad despite the profit shifting regulation. The second term captures an effect on the extensive margin, that is, on the number of firms that shift profits. As all active firms become more profitable, it is possible that firms that did not shift profits abroad before start to do so after it is strengthened.[14] The following proposition summarizes these effects.

Proposition 1 (Effectiveness of Limits on Profit Shifting)**.** *Stricter limitations on profit shifting do not necessarily lead to less profit shifted abroad on aggregate. Such regulations are only effective if the burden associated with them is relatively small.*

Proof. By inspection of equation (13) and using equations (5), (9) and (10) it follows that $\frac{\partial \Pi^*}{\partial \alpha} > 0 \Leftrightarrow \tau < \frac{c(1-t_H)(\sigma-1)\left[c\alpha(t_H-t_X)+f+\frac{\gamma-\sigma+1}{\sigma-1}\right]}{[c\alpha(t_H-t_X)+f](\alpha\gamma-\sigma+1)-(1-\alpha)(\gamma-\sigma+1)}$ ■

This proposition is illustrated in a numerical simulation in section 4. First, however, I will derive and discuss the conditions that determine the optimal tax rates and limitation on profit shifting in the following section.

3 Optimal Tax Policies

3.1 Optimization of the Tax Haven

The tax haven sets its tax rate t_X to maximize its tax revenue. As it has no firms of its own, maximizing tax revenue is optimal from a welfare point of view as well. Thus, its optimization problem is

$$\max t_X \Pi^*. \tag{14}$$

Solving this yields the tax haven's best response function,

$$t_X = t_H - \frac{1}{2}\frac{\sqrt{4t_H cf\alpha(\sigma-1)(\gamma-\sigma+1)+f^2\gamma^2}-f\gamma}{c\alpha(\sigma-1)}. \tag{15}$$

[14] Again, two counteracting effects are at work on the extensive margin, as can be seen by inspection of equation (9). A stricter regulation makes profit shifting less attractive per se, but the increase in profits (due to less competition, i.e. a higher price index P) may change that.

The tax haven reacts to stricter limitations on profit shifting (lower α) by lowering its tax rate. If there is an increase of tax rate in the home country, the tax haven responds in kind, but raises its tax rate by less. If firms are very heterogeneous (low γ), the tax rate is higher: There are more productive firms, and those firms are the first to shift profits. Therefore, the tax haven can attract quite a lot of firms even if its tax rate is not that low. If the elasticity of substitution, σ, is lower, firms have more monopoly power and realize higher profits. In this case, the tax haven sets a lower tax rate, as it has a stronger incentive to attract profits.

3.2 Optimization of the Home Country

The home country faces a more complicated optimization problem. It can decide about two policy instruments, the tax rate and the degree to which it restricts profit shifting. The government sets these in order to maximize social welfare, as given by the indirect utility function of the representative consumer. Indirect utility follows from using $X_I = \frac{\mu}{P}$, $X_G = T = t_H \Pi$ and $X_N = I - \mu$ in the utility function (2),

$$V = L - \mu + \mu \ln\left(\frac{\mu}{P}\right) + (1 + (\beta - 1)\, t_H)\, \Pi + (1 - t_X)\, \Pi^* - N^* f - N_\tau\, (1 - \alpha)\, \tau. \quad (16)$$

Note that income I consists of labor income L and (after-tax) profit income. The fixed costs of profit shifting, f, are paid by all $N^* = a_P^\gamma$ firms that shift profits abroad. $N_\tau = a_\tau^\gamma$ marks the mass of firms in the market. Fixed costs of production, c, are already deducted from aggregate profits.

The first order conditions for the optimal limitation on profit shifting and the optimal tax rate are

$$\frac{\partial V}{\partial \alpha} = \frac{-\mu}{P}\frac{\partial P}{\partial \alpha} + [1 + (\beta - 1)\, t_H]\, \frac{\partial \Pi}{\partial \alpha} + (1 - t_X)\, \frac{\partial \Pi^*}{\partial \alpha} - f\frac{\partial N^*}{\partial \alpha} - \frac{\partial N_\tau}{\partial \alpha}\, (1 - \alpha)\, \tau + N_\tau \tau = 0, \quad (17)$$

$$\frac{\partial V}{\partial t_H} = \frac{-\mu}{P}\frac{\partial P}{\partial t_H} + [1 + (\beta - 1)\, t_H]\, \frac{\partial \Pi}{\partial t_H} + \Pi\, (\beta - 1) + (1 - t_X)\, \frac{\partial \Pi^*}{\partial t_H} - f\frac{\partial N^*}{\partial t_H} - \frac{\partial N_\tau}{\partial t_H}\, (1 - \alpha)\, \tau = 0. \quad (18)$$

Due to the analytical complexity of the model, these first order conditions cannot be solved explicitly for t_H and α. In section 4, numerical solutions will be derived and shown graphically. However, before doing so, I will give some intuition for the various effects a limitation on profit shifting has on welfare.

First, to interpret the effects of such a regulation better, equation (17) can be rewritten as

$$\frac{\partial V}{\partial \alpha} = \frac{-\mu}{P}\frac{\partial P}{\partial \alpha} - [t_H\,(\beta-1) + t_X]\,\frac{\partial \Pi^*}{\partial \alpha} - f\frac{\partial N^*}{\partial \alpha} - [1 + (\beta-1)\,t_H]\,c\frac{\partial N_\tau}{\partial \alpha} - \left[(1-\alpha)\,\frac{\partial N_\tau}{\partial \alpha} - N_\tau\right]\tau \tag{19}$$

using that $\frac{\partial \Pi}{\partial \alpha} = -\frac{\partial \Pi^*}{\partial \alpha} - c\frac{\partial N_\tau}{\partial \alpha}$. The interpretation will be done from the point of view of tightening regulation that limits profit shifting, i.e. of lowering α.

The first term of equation (19) captures the effect of such a regulation on consumption. This term is always positive, showing that stricter regulations (lower α) have a negative effect on welfare: As some firms exit the market, fewer varieties are available to the consumer.[15]

The main advantage of a regulation that limits profit shifting is supposedly that less profits are shifted abroad. The change in the volume of profits shifted has two effects, which are captured in the second term of equation (19). First, less profit shifting increases tax revenues in the home country. Second, it decreases the loss of income due to the tax payments in the tax haven (from the home country's point of view, taxes paid on profits in the tax haven are a pure loss, as they neither generate tax revenue nor profit income). Moreover, as shown by the third term of equation (19), if fewer firms shift profits, less profit income is lost due to the fixed cost f which firms incur in order to shift profits. Note, however, that it is not clear whether such a rule really leads to less profits being shifted abroad (see proposition 1).

The fourth term of equation (19) reflects that as there are fewer firms in the economy, fewer firms incur the fixed costs of production, c. As this cost is tax-deductible, this also has implications for tax revenue. Lastly, the strictness of regulations influences the severity of the burden that is associated with such a limitation on profit shifting. First, lowering α implies that fewer firms are affected by this burden, as some firms exit the market. However, a stricter limit on profit shifting also implicates that this burden is higher for all firms. This second effect is always stronger than the first, implying that the total effect is negative (i.e. the last term of equation (19) is always positive).

Proposition 2 (Welfare effects of regulations to limit profit shifting)**.** *The welfare effects of strengthening a limitation on profit shifting are ambiguous and given by equation* (19)*. Besides the positive effect of keeping profits in the country, such a regulation*

[15] As the CES price index reflects the price of the optimized consumption bundle, it unambiguously falls when fewer varieties are available, even though these varieties were the most expensive in the market.

has further effects due to the market exit of some firms. This decreases competition and makes consumers worse off as they have fewer varieties from which to choose, but may increase tax revenue (see proposition 1).

Next, let us consider the effects of a change of the tax rate in the home country, t_H. Again, it is helpful to rewrite the first order condition (18) as

$$\frac{\partial V}{\partial t_H} = \frac{-\mu}{P}\frac{\partial P}{\partial t_H} - (\beta - 1)\left(\Pi + t_H \frac{\partial \Pi}{\partial t_H}\right) + t_X \frac{\partial \Pi^*}{\partial t_H} - f\frac{\partial N^*}{\partial t_H} - [c + (1-\alpha)\,\tau]\,\frac{\partial N_\tau}{\partial t_H} \qquad (20)$$

The first term again captures the effect on consumption: If the tax rate is higher, it is more difficult to be profitable enough to stay in the market despite the excess burden of regulations to limit profit shifting. Thus, a higher tax rate implies fewer varieties in the market, thereby decreasing welfare. The second term captures the effect of a tax rate increase on tax revenues. First, there is a direct effect: A higher tax rate implies higher revenues, everything else being equal. However, there is also a negative indirect effect, as the tax base decreases because the higher tax rate leads to more profit shifting. The additional profit shifting also implies that income is "lost" in the tax haven, because more profits are taxed there. This effect is represented by the third summand of equation (20). Additional income is lost as more firms incur the fixed costs of profit shifting, as shown by the fourth term. Lastly, there also is a positive effect of market exit due to higher tax rates: As there are fewer firms active, fewer firms are affected by the dead weight loss of the limitation on profit shifting or pay the fixed costs of production.

These various effects allow no clear conclusion whether limiting profit shifting is desirable, given that it imposes costs on all firms. To see the effects of such a limitation more clearly, the next section looks at some numerical simulations of the modeled economy.

4 Numerical Analysis

4.1 Simulation of Proposition 1

The theoretical model has shown that it is not clear that a regulation that limits profit shifting always succeeds in its aim of decreasing the amount of profits that is shifted to a tax haven (see proposition 1). In the following, this will be illustrated by numerical simulations. Its results are shown in figure 1.

The graphs clarify how the tax policy of the home country affects the aggregate amount of profits shifted abroad. It compares the aggregate value of profits shifted abroad in the case with a limitation on profit shifting (dark plane) and without such a limitation (light plane). The optimal response of the tax haven (i.e. the optimal t_X) is taken into account.

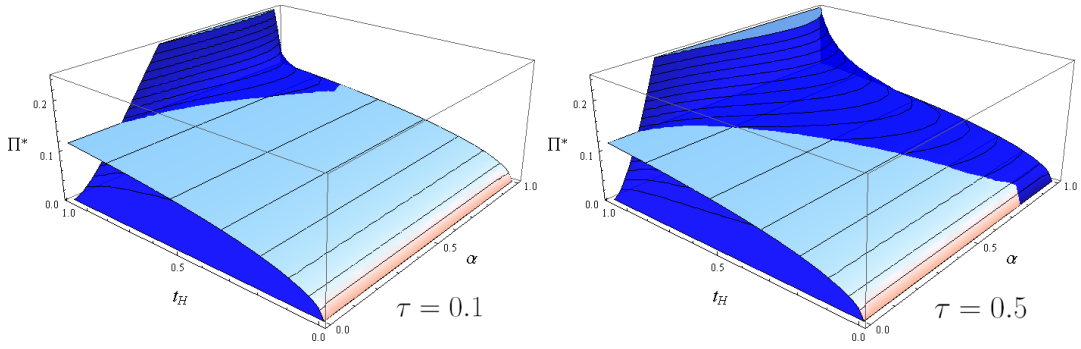

Figure 1: *Aggregate profits shifted abroad, with a limitation imposed (dark plane) and without such a limitation (light plane). Parameter values:* $L = 1, \mu = 0.5, \beta = 3, f = c = 0.1, \gamma = 1.5, \sigma = 2$. *On the left,* $\tau = 0.1$, *on the right,* $\tau = 0.5$.

On the two axes with the independent variables are t_H and α, which constitute the home countries tax policy. The graph in the benchmark case without a limitation on profit shifting (i.e. the light layer) is independent of α, which is drawn on the right-hand axis. Aggregate profits in the tax haven go to zero if either $\alpha \to 0$ or $t_H \to 0$, as then there is either no possibility or no incentive to shift profits abroad. Note, however, that this does not allow any inference about welfare.

The graphs clarify that less profit is shifted abroad only for some combinations of α and t_H. Even with relatively low costs of profit shifting regulation (left graph), a strict regulation may induce more profit shifting if the tax rate is high. If the burden is relatively high (right graph), very strict regulations to limit profit shifting may be counterproductive no matter what the tax rate is. As explained above, this happens because such regulation decreases competition in the market.

4.2 Simulation of Proposition 2

Lastly, let us consider the welfare effects which where described in section 3 and summarized in proposition 2. A numerical analysis of the model confirms that it is not always optimal to limit profit shifting if this entail costs for all firms.

However, the simulations also show that if a limitation is welfare-increasing at all, then the government should set is as strict as possible, i.e. set $\alpha \to 0$ (see appendix A).[16] Thus, the optimum in the model economy is always a corner solution, setting α either to 0 or to 1.

It depends on the characteristics of the economy (i.e. on parameters) whether a country chooses to prohibit profit shifting or not. The following two figures (figures 2 and 3) show how market and firm characteristics influence whether profit shifting should be barred.

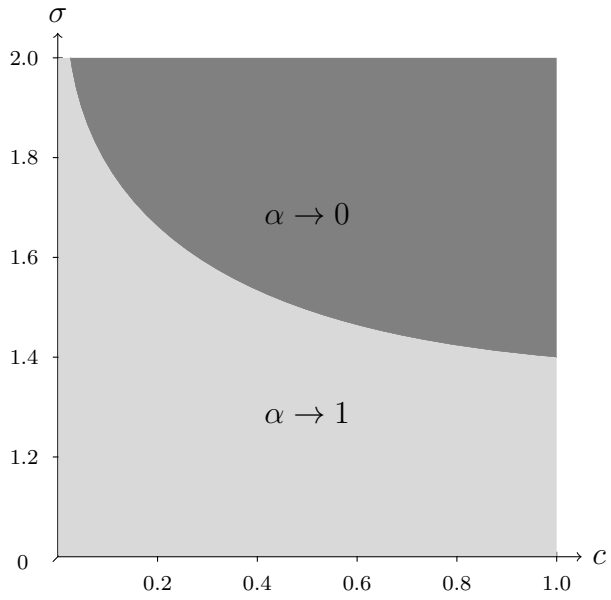

Figure 2: *Prohibiting profit shifting under different market characteristics. Parameter values:* $\gamma = 1.5, L = 1, \mu = 0.5, \beta = 3$ *and* $f = \tau = 0.1$.

Figure 2 summarizes the results of simulations comparing welfare without a limitation on profit shifting (i.e. $\alpha \to 1$) and after its introduction for different market characteristics. These are on the one hand the elasticity of substitution, σ, which is also a measure for competition in the market, and fixed costs, c, which represent barriers to entry into the market. The darker area represents parameter constellations under which it is favorable to prohibit profit shifting.

Profit shifting should not be limited if there are relatively many firms in a relatively

[16] A prohibition of all profit shifting possibilities is not what we observe in reality. Note, however, that in the model it is actually feasible to deter all profit shifting, which is hardly the case in reality. It should hence be interpreted as the government limiting profit shifting as much as it can, while in the other alternative the government chooses not to limit profit shifting at all.

uncompetitive market. If the elasticity of substitution is low, then it is more important for consumers to have as many firms in the market as possible. Hence, the utility loss of loosing additional varieties is higher. In contrast to what might be the first intuition, this effect is stronger when there are many firms in the market (low fixed cost c), because the additional fixed cost of limiting profit shifting becomes more important when other fixed costs are low.

A further interesting aspect is the interplay of the different firm characteristics, namely fixed costs c and the costs of profit shifting, f, which is depicted in figure 3. It is intuitive that the benefit from limiting profit shifting is smaller if few firms shift profits due to high costs f, especially because the burden imposed by regulation to hinder this falls on all firms. However, high fixed costs c make it more likely that profit shifting should be limited. If fixed costs are high, the market consists mainly of highly profitable firms which are more likely to shift profits abroad, thus increasing the benefit of limiting profit shifting.

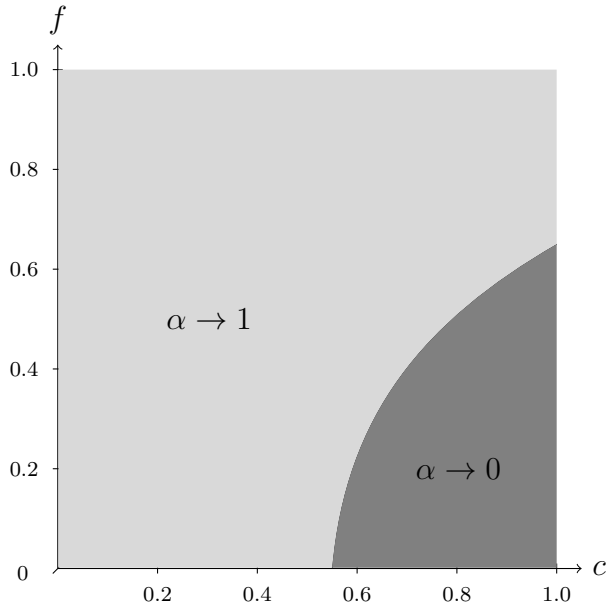

Figure 3: *Prohibiting profit shifting under different firm characteristics. Parameter values:* $\gamma = 1.5, L = 1, \mu = 0.5, \beta = 3, \tau = 0.1$ *and* $\sigma = 1.5$.

The degree of firm heterogeneity also influences whether profit shifting should be prohibited or not. Heterogeneity is measured by γ. High heterogeneity (a low γ) implies that the distribution of firm productivity is relatively even, i.e. there are many firms

with very high or low productivity levels.[17] The simulations also show that limiting profit shifting is more favorable if firms are very heterogeneous. In that case, there are relatively many large, productive firms who would shift all of their profits abroad otherwise, and relatively few small, unproductive firms which are affected negatively (or even forced out of the market) by the regulations.

5 Conclusion

This article has analyzed the various effects and welfare implications of limiting profit shifting. It points out that regulations which aim to limit profit shifting may curb competition by forcing some firms out of the market. By rendering the remaining firms more profitable, it is possible that *more* profits are shifted abroad on aggregate after the introduction of a regulation that is supposed to prohibit or limit profit shifting.

In the introduction it was mentioned that such measures, e.g. thin capitalization rules, have increasingly been introduced or strengthened during the last years. The model also offers explanations for this by clarifying the effect of different parameters on the likelihood that limiting profit shifting increases welfare. It becomes clear that lower costs of profit shifting, which may have resulted from increasing financial integration, make limiting profit shifting more beneficial.

[17] A low level of firm heterogeneity in this sense would be the case if there are many firms with similar (low) productivity levels and only very few highly productive firms.

A Appendix: Simulation Results

The following table 1 states some results of numerical simulations of the model. The fixed parameters are $\tau = 0.1, \mu = 0.5, \beta = 3$ and $L = 1$.

Parameters																
c	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5	0.1	0.5
f	0.1	0.1	0.5	0.5	0.1	0.1	0.5	0.5	0.1	0.1	0.5	0.5	0.1	0.1	0.5	0.5
γ	1.5	1.5	1.5	1.5	2	2	2	2	1.5	1.5	1.5	1.5	2	2	2	2
σ	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	2	2	2	2	2	2	2	2
t_H	0.99	0.20	0.99	0.87	0.97	0.26	0.96	0.85	0.79	0.88	0.94	0.89	0.80	0.85	0.89	0.86
t_X	0.43	0.09	0.36	0.37	0.35	0.10	0.26	0.29	0.53	0.60	0.64	0.59	0.46	0.45	0.46	0.43
α	1	1	1	1	1	1	1	1	0	0	1	0	1	0	1	0

Table 1: Results of the numerical simulation.

It becomes clear that is always either optimal not to limit profit shifting at all ($\alpha \to 1$) or to prohibit profit shifting completely ($\alpha \to 0$). This is also clarified by the following graphs (figure 4), which plot welfare depending on the tax rate t_H and on α.

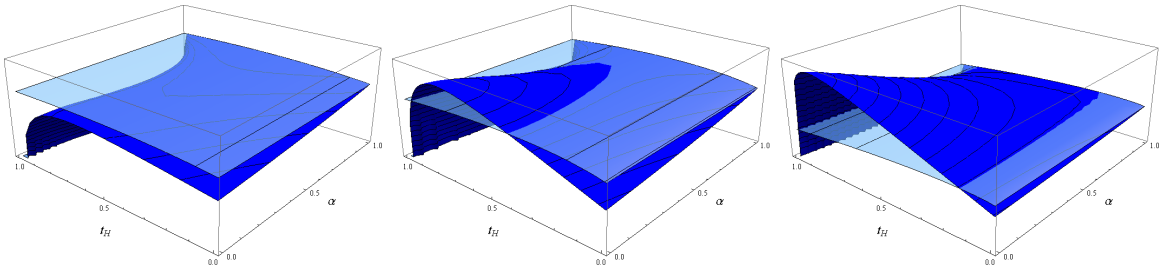

Figure 4: *3D-Plots of simulated welfare levels, varying* σ *(from left to right:* $\sigma = 1.75, \sigma = 2$ *and* $\sigma = 2.25$*). Parameter values:* $L = 1, \mu = 0.5, \beta = 3, f = c = \tau = 0.1$ *and* $\gamma = 1.5$.

The graphs clarify that it is not always welfare-increasing to introduce a limitation on profit shifting: In the graph on the left, welfare with such a limitation is always lower than in the benchmark case where profit shifting is not limited. If such a rule should be introduced, it is optimal to set it as strict as possible (that is, at the left side of the graph).

Welfare is depicted for different values of the elasticity of substitution, σ, in figure 4. It shows that limiting profit shifting becomes more favorable if the elasticity of substitution is higher. In that case, it is less important for the consumer to have different varieties available. Hence, the negative effect of a limitation on profit shifting (i.e. market exit by some firms and thus the loss of these varieties) is less pronounced if σ is high.

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