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Standards and Incentives under Moral Hazard with Limited Liability

Standards and Incentives under Moral Hazard with Limited Liability

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Abstract

We consider a model of moral hazard with limited liability of the agent and effort that is two-dimensional. One dimension of the agent's effort is observable and the other is not. The principal can thus make the contract conditional not only on outcome but also on observable effort. The principal's optimal contract gives the agent no rent and – in contrast to the first-best allocation – uses too much observable effort and too little unobservable effort. This distortion in the relative use of the two kinds of effort increases if the agent's liability becomes more limited.

JEL classification: D82, D86, K32

Keywords: moral hazard, two-dimensional effort, regulation

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1. Introduction

Consider a principal-agent relationship with moral hazard. There will probably be many actions that the agent can take to further the principal's project. Some of these actions will be observable, some not. In the following, we will subsume all actions that are observable under the term observable effort, and all actions that are not observable under the term unobservable effort. In the first-best, without moral hazard, the optimal mix of efforts will in general include a mix of both kinds of effort. The contract that is usually assumed in situations of moral hazard is conditional on the observed *outcome* only. In this paper we will look at a contract that is also conditional on the level of observable *effort*. This means that the contract will stipulate a specific level of observable effort and the principal will only pay if he observes at least this level of observable effort.

Our main interest in this paper is the level of the contractually specified observable effort and its relation to the induced level of unobservable effort. We assume that there is no direct interaction between the costs or returns of the two kinds of effort; nevertheless, the limited liability of the agent will influence the levels of both kinds of effort. Moral hazard problems with limited liability of the agent usually have the following outcome: if the principal cannot extract the whole surplus at the first-best level of effort, he will lower the implemented effort below the first-best level.[1] In contrast, in our model the specified level of observable effort will be above the first-best level, while unobservable effort will be below the first-best level. This also means that the combination of observable and unobservable effort will not be cost-minimizing, i.e. the given amount of total effort is produced with too much observable effort and too little unobservable effort. In other words, the agent would be able to produce the same level of total effort with lower costs.

For an application, think about a situation where the principal wants the agent to undertake a project that can fail with catastrophic consequences. Consider a government that licenses a firm to operate an hazardous technology, like a chemical factory or a nuclear reactor. The government wants the firm to undertake effort that increases the probability that the firm operates safely. Some of this effort, like the compliance with technical regulations for the construction of the plant, or the education level of the operating personnel can be controlled rather easily. But other elements essential to safe operation will be very hard to observe, like the workload and alertness of the personnel or whether the firm's management exerts pressure on them to "bend the rules". The "regulatory contract" in such situations usually includes both standards for ob-

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[1]This may or may not imply a rent for the agent.

servable effort ("regulation") and monetary payments that depend on the outcome of the project ("fines" and "liability"). The compliance with the standards can and will be enforced ex-ante, while the ex-post payments give the firm incentives to undertake unobservable effort. A similar problem exists if a big firm subcontracts part of a project to a small firm. If the small firm produces bad quality, the damage for the big firm might be immense. Contractual arrangements in such situations will usually not only include payments that are conditional on final outcomes but will also authorise the big firm to monitor whether the work of the small firm is in compliance with contractual standards. In addition, the big firm might demand that the small firm will have its operations "certified" by a third party.

Our results suggest that in such situations the principal will set standards that demand observable effort which is above the first-best level, while the level of unobservable effort will be below the first-best. For example, the work of a small subcontractor will be more oriented toward observable effort compared to the case where the big firm would do the work itself. To generalize, we suggest a possible inefficiency existing under moral hazard with limited liability, which does not lie in the amount of total effort but in the way this effort is produced. This inefficiency has seen scant attention in theory but is often complained about in practice.

Many employees of big organizations complain about "bureaucracy". They feel that their work is inefficiently organized – it would be more productive if there were fewer regulations to observe and more time could be spend on doing "real work". Regulatory regimes for hazardous activities are criticized for putting too much emphasis on compliance with technical standards rather than on soft factors like "safety culture". And many observers question whether a firm's decision to seek certification for use of a "quality management systems" is mainly motivated by customer pressure, while the real effect on quality is questionable.[2]

This work is related to a number of papers which all exploit a similar effect: if the solution to the moral hazard problem calls for granting the agent a rent, the principal will try to expropriate this rent by forcing the agent to undertake some other activity that benefits the principal. This activity might be socially inefficient, but because its costs come out of the agent's rent, it is still advantageous for the principal to implement it. The activity in question might be another principal-agent project (Laux, 2001), reporting activities like "paperwork" (Strausz, 2006) or the effort in a preceding period

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[2]The question whether firms introducing ISO 9000 quality management systems are mainly motivated by external reasons (customer pressure etc.) or by internal reasons (concern for quality and cost improvements) has been the subject of numerous studies, which have come to conflicting results. An overview of previous studies can be found in Heras Saizarbitoria et al. (2006); the Delphi study described in their paper finds that external reasons are dominating. In a similar vein, Buttle (1997) describes a survey of ISO 9000 certified firms; the highest scoring motivation for certification is "anticipated demand from future customers for ISO 9000".

of the principal-agent relationship (Kräkel and Schöttner, 2010). Our model is the first that applies this effect to the choice between observable effort and unobservable effort. This setting is not only of great practical importance, it does also allow for a sharp characterization of the trade-off that is responsible for the implementation of a socially inefficient activity.

In the "Law & Economics" literature, Bhole and Wagner (2008) analyze a setting where a firm can take observable effort as well as unobservable effort to prevent an accident.[3] They find that in many situations only the combined use of both liability and regulation will lead to optimal levels of effort in both dimensions. There are two important differences to our approach. First, in a tort law setting the principal has a different objective function (total welfare) and usually a restricted choice of policy measures. Second, Bhole and Wagner only consider a binary choice of observable effort; because in their model a high level of observable effort is first-best, the question of excessive regulation of observable effort is ruled out by assumption.

Multi-dimensional effort has been studied in number of other settings in the literature. In the most prominent treatment by Holmstrom and Milgrom (1991), different dimensions of effort interact through the agent's cost function. In our model, there is no such interaction; observable effort and unobservable effort influence each other only because of the shared limited liability constraint.

The rest of the paper is structured as follows: Section 2. sets up the model. In section 3., we discuss a benchmark case, namely a contract that is conditional on outcome only. The main part of the paper is section 4., which analyzes a contract that does also regulate the agents effort, while section 5. concludes. Proofs can be found in the Appendix.

2.   Setup of the Model

There are two kinds of effort, observable effort $o \in [0, o_{max}]$ and unobservable effort $u \in [0, u_{max}]$ with $o_{max}, u_{max} > 0$ and $o_{max} + u_{max} \leq 1$. The agent's project has two outcomes, it can either succeed or fail, $s \in \{0, 1\}$. The probability of success ($s = 1$) depends on the agents effort and is given by $p(o, u) = o + u$. At times we will denote this probability as *total effort.* If the agent exerts effort, he suffers costs of $c_o(o) + c_u(u)$. Note that under this setup there is no direct interaction between the two kinds of effort: the level of one kind of effort does not influence the marginal cost or the marginal return of the

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[3]In an article on liability for nuclear accidents, Trebilcock and Winter (1997) sketch a tort-law model with observable and unobservable effort but do not fully solve it.

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other kind of effort.[4] We further need the following technical assumptions for the cost functions:

Assumption 1. $c_o(o)$ *and* $c_u(u)$ *are continuous, three times differentiable, strictly increasing and strictly convex.*

Assumption 2. $c_o(o_{max}) = c_u(u_{max}) = \infty$.

Assumption 3. $c_o'(0) = c_u'(0) = 0$.

Assumption 4. $c_o'''(o), c_u'''(u) > 0$.

Assumption 5. $c_o(0) = c_u(0) = 0$.

Assumptions 2 and 3 ensure that the agent's problem has an interior solution, while Assumption 4 makes the principal's problem concave (the condition on $c_o'''(o)$ is only needed for the benchmark case).

The benefit for the principal if the project succeeds is set to $B > 0$. Both parties are risk neutral. To induce effort, the principal will write a contract that specifies a transfer scheme $t(s, o)$ that can depend on the outcome of the project and the observed effort. The agent faces a liability limit $L \geq 0$, which can either be interpreted as the maximum fine that can be imposed on the agent ex-post, or the maximum bond that can be posted by the agent ex-ante.[5] This liability limit is expressed by:

Assumption 6. $t(s, o) \geq -L \; \forall s \in \{0, 1\}, o \in [0, o_{max}]$.

We have to distinguish two concepts. On the one hand, we have the socially optimal *first-best* effort levels o^* and u^*, which are given by $c_o'(o^*) = B$ and $c_u'(u^*) = B$. On the other hand, for a given level of total effort $\overline{p}$, we can find the least expensive combination of observable and unobservable effort that produces $\overline{p}$. Such a *cost-minimizing* combination of efforts will be characterized by $c_o'(o) = c_u'(u)$.[6] It is easy to see that first-best effort levels are also a cost-minimizing combination of efforts, but that there are also many other cost-minimizing combinations of efforts that are not first-best.

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[4]In reality those direct interaction will often exist, making the two kinds of efforts either complements or substitutes. In this paper, we assume no direct interaction to isolate those effects that are due to limited liability.

[5]We assume that the liability limit does not depend on the level of efforts.

[6]This condition results from $\min_{o,u} c_o(o) + c_u(u)$, subject to $p(o, u) = \overline{p}$. Formally, the marginal rate of technical substitution between these two kinds of effort must be equal to the ratio of respective marginal costs.

3. Benchmark Case: Incentives only

To establish a benchmark case, we will first consider a contract that conditions only on *outcome*. This contract can be described by the transfer scheme:

$$
t(s,o) = \begin{cases} b + w & \text{if } s = 1 \\ w & \text{if } s = 0 \end{cases}
$$

It has the usual property that the principal sets a base wage w and a bonus b. It follows that the profit function of the principal is given by $\Pi(o,u,b,w) = (B-b) \cdot p(o,u) - w$, while the payoff function of the agent is $V(o,u,b,w) = bp(o,u) + w - c_o(o) - c_u(u)$. The principal has to solve the problem:

$$
\begin{aligned}
&\max_{o,u,b,w}\ \Pi(o,u,b,w) \\
&\textbf{subject to:} \\
&V(o,u,b,w) \geq 0 && \text{PC} \\
&w \geq -L,\ w+b \geq -L && \text{LLCs} \\
&(o,u) \in \operatorname*{argmax}_{(o,u)} V(o,u,b,w) && \text{IC}
\end{aligned}
\tag{1}
$$

The fact that u is unobservable does not necessarily mean that the first-best will not be implemented. In fact, if the principal sets $b = B$, the agent will deliver effort levels o^* and u^*. The wage w^* that extracts all the agent's surplus is then given by $V(o^*, u^*, B, w^*) = 0$, which can be written as $w^* = c_o(o^*) - c_u(u^*) - Bp(o^*, u^*)$.

But this extraction of surplus is feasible only if $w^* \geq -L$; in this case, the principal can "sell the project" to the agent. If $w^* < -L$, the principal faces a tradeoff between incentivizing effort and extracting rent. In the following, we will always assume that the first-best will not be implemented, namely

Assumption 7. $w^* < -L$.

We will find the optimal effort levels o_{bm} and u_{bm} by using the so-called first-order approach. The following proposition shows that this approach is valid in our setting because the agent's optimal choice of effort levels is at a stationary point.

Proposition 1. *The optimal solution to (1) has $b > 0$ and o_{bm}, u_{bm} will be given by the agent's first-order order conditions $b - c'_o(o) = 0$ and $b - c'_u(u) = 0$, with $o_{bm} \in (0, o_{max})$, $u_{bm} \in (0, u_{max})$ and total effort $p(o,u) > 0$.*

We can therefore replace the incentive constraint with the agent's first-order conditions. Additionally, because $b > 0$, one of the limited liability constraints, $w + b \geq -L$, is superfluous. The Lagrangian for the principal's problem can now be written as:

$$
\begin{aligned}
&\mathcal{L}(o, u, b, w, \lambda, \eta, \mu_o, \mu_u) = \\
&(B - b) \cdot p(o, u) - w + \lambda \left(b \cdot p(o, u) + w - c_o(o) - c_u(u) \right) \\
&+ \eta \left(w + L \right) + \mu_o \left(b - c_o'(o) \right) + \mu_u \left(b - c_u'(u) \right)
\end{aligned} \tag{2}
$$

In the optimal solution, the limited liability constraint $w \geq -L$ will always be binding, while the participation constraint may be binding or not.

Proposition 2. *The optimal solution to (2) has $w = -L$ and*

$$
b = B - \frac{(1 - \lambda) p(o, u)}{\frac{1}{c_o''(o)} + \frac{1}{c_u''(u)}} \tag{3}
$$

with $0 \leq \lambda < 1$. If the agent will get a rent, we have $\lambda = 0$.

The optimal contract can be found be trying out two cases. In the first case with $\lambda = 0$, the optimal effort levels are given by the trade-off between the costs of incentives and the principal's benefit from having more effort, ignoring the PC (this will usually mean a rent for the agent). But if those effort levels and $w = -L$ do not satisfy the PC, we have the case $\lambda > 0$. The principal sets $w = -L$ and chooses the unique level of b that makes the PC binding. This will mean higher effort levels than in the first case and no rent for the agent.[7]

In both cases we will have $c_o'(o_{bm}) = c_u'(u_{bm}) = b < B$. This implies that both kinds of effort are below the first-best level ($o_{bm} < o^*$ and $u_{bm} < u^*$), but because $c_o'(o_{bm}) = c_u'(u_{bm})$, they form a cost-minimizing combination.

4.   Joint Use of Incentives and Standards

We now look at a contract that makes the principal's payments conditional not only on outcome, but also on observable effort. At first glance the problem of finding the optimal contract looks quite simple: set the observable effort to o^* and optimize over u (because we assume $p(o, u) = o + u$, there is no interaction between the two kinds of effort). But it will turn out that the optimal contract will have a level of observable effort that is above o^*.

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[7]Which case obtains depends on the severity of the liability limit. Define L^* by $V(o^*, u^*, B, -L^*) = 0$ and $\tilde{L}$ by $V(o_{bm}, u_{bm}, B, -\tilde{L}) = 0$ (where o_{bm} and u_{bm} are given by (3) with $\lambda = 0$). If $0 \leq L < \tilde{L}$ the agent gets a rent, if $\tilde{L} \leq L < L^*$ there will be no rent.

We consider contracts of the following form:[8]

$$t(s, o) = \begin{cases} b + w & \text{if } s = 1 \text{ and } o \geq \underline{o} \\ w & \text{if } s = 0 \text{ and } o \geq \underline{o} \\ -L & \text{if } o < \underline{o} \end{cases}$$

where $\underline{o}$ is contractually specified level of observable care. The principal's expected profit is given by

$$\Pi_{\underline{o}}(o, u, b, w) = \begin{cases} (B - b) \cdot p(o, u) - w & \text{if } o \geq \underline{o} \\ B \cdot p(o, u) + L & \text{if } o < \underline{o} \end{cases}$$

while the agent's payoff has the form:

$$V_{\underline{o}}(o, u, b, w) = \begin{cases} bp(o, u) + w - c_o(o) - c_u(u) & \text{if } o \geq \underline{o} \\ -L - c_o(o) - c_u(u) & \text{if } o < \underline{o} \end{cases}$$

The principal's problem is given by:

$$\max_{o, u, b, w, \underline{o}} \Pi_{\underline{o}}(o, u, b, w)$$

subject to:

$$V_{\underline{o}}(o, u, b, w) \geq 0 \qquad \text{PC} \tag{4}$$

$$w \geq -L, \; w + b \geq -L \qquad \text{LLCs}$$

$$(o, u) \in \operatorname*{argmax}_{(o, u)} V_{\underline{o}}(o, u, b, w) \quad \text{IC}$$

Denote by $\hat{o}$ and $\hat{u}$ the effort levels that are implemented in the optimum. The first problem is again to show that the first-order approach is valid here.

Proposition 3. *The optimal solution to (4) has $\hat{o} = \underline{o}$ and $b > 0$. Effort level $\hat{u}$ will be given by the agent's first-order order condition $b - c'_u(u) = 0$, with $\hat{o} \in (0, o_{max})$, $\hat{u} \in (0, u_{max})$ and total effort $p(\hat{o}, \hat{u}) > 0$.*

We can again use the agent's first order condition for u and ignore the constraint $w + b \geq 0$. The Lagrangian for the principal's problem can be written as:

$$\mathcal{L}(o, u, b, w, \lambda, \eta, \mu) =$$
$$(B - b) \cdot p(o, u) - w + \lambda \left(b \cdot p(o, u) + w - c_o(o) - c_u(u) \right) \tag{5}$$
$$+ \eta (w + L) + \mu \left(b - c'_u(u) \right)$$

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[8]The principal cannot improve his profit by using a more general contract that distinguishes between more levels of o, because, besides his effort level, the agent has no other private information.

Proposition 4. *In the optimal solution to (5), both the participation constraint $V_{\underline{o}}(o, u, b, w) \geq 0$ and the limited liability constraint $w \geq -L$ are binding. The optimal effort levels $\hat{o}$ and $\hat{u}$ are given by*

$$c_o'(o) = B + \frac{1-\lambda}{\lambda}(B - c_u'(u)) \tag{6}$$

and $c_u'(u) = B - (1-\lambda) \cdot p(o, u) \cdot c_u''(u)$ $\tag{7}$

with $0 < \lambda < 1$.

It is quite intuitive that the principal will not give the agent a rent. Suppose the principal would choose some o and some $b < B$ so that the agent gets a rent. The principal could then increase observable effort and get a marginal benefit of $B - b$ while letting the agent take the additional costs out of his rent. So the principal will transform the agent's rent into his own benefit.

From (6) and (7) and $0 < \lambda < 1$ we can conclude that $c_u'(u) < B$ and $c_o'(o) > B$. This implies that $\hat{o} > o^*$ and $\hat{u} < u^*$, so observable effort is above and unobservable effort is below the first-best level. We also note that $\hat{o}$ and $\hat{u}$ are not a cost-minimizing combination of efforts (because $c_o'(\hat{o}) \neq c_u'(\hat{u})$), meaning that $p(\hat{o}, \hat{u})$ could be produced less costly by a different combination of efforts. It is also clear that $\hat{o} > o_{bm}$, but we cannot tell whether $\hat{u}$ is greater or smaller than u_{bm}. In fact, numerical simulations show that both cases can occur.

The principal is willing to set observable effort above the first-best level because stipulating more observable effort has the additional benefit of inducing more unobservable effort. When the principal demands additional observable effort, he must compensate the agent for the additional cost (because the PC is binding), but does so by increasing b, thereby increasing the agent's incentive for providing unobservable effort. This can bee seen if we combine the two implicit equations (6) and (7) by eliminating λ:

$$c_o'(o) - B = (B - b) \cdot \frac{1}{c_u''(u)} \cdot \frac{1}{p(o, u)}(c_o'(o) - b) \tag{8}$$

Equation (8) can interpreted as the trade-off facing the principal at the margin when he increases $\hat{o}$ beyond o^*. The term on the left-hand-side is the principal's cost of increasing observable effort further above the first-best level. Because the PC is binding, he has to compensate the agent for the marginal cost of additional effort but receives additional expected benefit of only B (which is smaller than $c_o'(o)$ because $\hat{o} > o^*$). The right hand side is his marginal benefit and can be interpreted as follows (read from right to left): if o is increased, the agent has marginal costs of $c_o'(o)$ but receives a marginal increase in expected payoff of only b. To compensate the agent for a small

loss in payoff, the principal has to marginally increase b by $\frac{1}{p(o,u)}$. An marginal increase in b will increase unobservable effort by $\frac{1}{c_u''(u)}$, while a marginal increase in u will give the principal an marginal benefit of $B - b$. These effects can be labeled as follows:

$$\underbrace{c_o'(o) - B}_{-\dfrac{d\Pi}{do}} = \underbrace{(B - b)}_{\dfrac{d\Pi}{du}} \cdot \underbrace{\dfrac{1}{c_u''(u)}}_{\dfrac{du}{db}} \cdot \underbrace{\dfrac{1}{p(o,u)}}_{\dfrac{db}{dV}} \underbrace{(c_o'(o) - b)}_{-\dfrac{dV}{do}}$$

In our model, the agent's limited liability causes a combination of the two kinds of effort that is not cost-minimizing, namely too much observable and too little unobservable effort. This suggests that a decrease in L – the problem of limited liability becomes worse – will increase this distortion. The next proposition shows that this is indeed the case.

Proposition 5. *If L decreases (the agent's liability becomes more limited), $\hat{o}$ increases and $\hat{u}$ decreases.*

This result looks more obvious than it is. Because if L decreases, it changes not only the optimal combination of o and u that implements a given level of $p(o,u)$ (substitution effect), but it may also change the level of $p(o,u)$ that is optimal for the principal to implement (scale effect).[9] Proposition 5 shows that the first effect dominates. This result also suggests a possible way to test our theory: for agents with a stricter liability limit we should observe standards that prescribe a higher level of observable effort.

5.   Conclusion

The paper analyzes a model of moral hazard with limited liability of the agent where the agent's effort has one observable and one unobservable dimension. For simplicity, we only consider the case where the two kinds of efforts do not interact with each other. We consider different contracts with regard to two questions: whether each of the two kinds of effort is above or below its first-best level and whether the two levels form a cost-minimizing combination. With a contract that is conditional on outcome only, both kinds of effort are below their first-best levels but they form a cost-minimizing combination. With a contract that is conditional on both outcome and observable effort, unobservable effort will still be below its first best level while observable effort

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[9]The terminology is taken from Nagatani (1978). It can be shown that if L decreases, the substitution effect is positive for o and negative for u. But if the optimal $p(o,u)$ decreases, the scale effect will be negative for both kinds of effort.

will be above the first-best level. This combination of efforts will not be cost-minimizing. The distortion between the two kinds of efforts increases if the agent's liability becomes more limited.

6. Appendix

Proof of Proposition 1

We first show that all $b \leq 0$ give the principal the same profit. If the principals sets $b \leq 0$, the agent will always choose $o = 0$ and $u = 0$, and $w = 0$ will make the PC binding. This will give the principal a profit $\Pi = 0$, for all $b \leq 0$. Thus to show that $b \leq 0$ is not optimal it is sufficient to show that $b = 0$ is not optimal

We now show that for a given $b \geq 0$ and w, the maximum of $V(o, u, b, w) = bp(o, u) + w - c_o(o) - c_u(u)$ will be characterized by the first-order conditions $b - c'_o(o) = 0$ and $b - c'_u(u) = 0$. Because $V(o, u, b, w)$ is strictly concave in o and u, an interior maximum will be characterized by the first-order conditions. As regards to corner solutions, $o = o_{max}$ or $u = u_{max}$ cannot be a maximum because the costs would be infinite, so zero effort would be better. A possible corner solution with $o = 0$ and $u = 0$ would have $b - c'_o(0) \leq 0$. Because of $b \geq 0$ and $c'_o(0) = 0$ this implies $b = 0$ and this maximum would also fulfill the first-order condition with equality.

Now we show that $b = 0$ cannot be an optimum. Suppose otherwise: then the agent would choose $o = 0$ and $u = 0$, and $w = 0$ would make the PC binding. If the principal would marginal increase b he would get:

$$
\frac{d\Pi}{db} = -p(o, u) + (B - b)\left(\frac{do}{db} + \frac{du}{db}\right) \tag{9}
$$

$$
= B\left(\frac{1}{c''_o(o)} + \frac{1}{c''_u(u)}\right) > 0 \tag{10}
$$

where the values of $\frac{do}{db}$ and $\frac{du}{db}$ come from implicitly differentiating the agent's first-order conditions; at the same time, at this point, $\frac{dV}{db} = p(0, 0) - c'_o(0)\frac{do}{db} - c'_u(0)\frac{du}{db} = 0$ so the PC will still be satisfied. Because this implies $o_{bm}, u_{bm} > 0$, we must have $p(o, u) > 0$. $\square$

Proof of Proposition 2

The first order conditions for a maximum are:

$$
\frac{\partial \mathscr{L}}{\partial o} = (B - b) + \lambda(b - c'_o(o)) - \mu_o c''_o(o) = 0 \tag{11}
$$

$$
\frac{\partial \mathscr{L}}{\partial u} = (B - b) + \lambda(b - c'_u(u)) - \mu_u c''_u(u) = 0 \tag{12}
$$

$$
\frac{\partial \mathscr{L}}{\partial b} = -p(o, u) + \lambda p(o, u) + \mu_o + \mu_u = 0 \tag{13}
$$

$$
\frac{\partial \mathscr{L}}{\partial w} = -1 + \lambda + \eta = 0 \tag{14}
$$

$\lambda, \eta, \mu_o, \mu_u \geq 0$ (with complementary slackness)

It cannot be the case that both the PC and the LLC are slack. In this case, the principal could always increase his profit by decreasing w (formally, equation (14) can never be fulfilled). Furthermore, it cannot be the case that the PC is binding and the LLC is not binding: Because this would imply $\eta = 0$, and from (14) we would get $\lambda = 1$. Then (13) and complementary slackness give us $\mu_o = 0, \mu_u = 0$ and from this we get $o = o^*$ and $u = u^*$ (using equations (11) and (12)). But this contradicts Assumption 7.

By using the agent's first order conditions we can simplify the equations to

$$b + \mu_o c_o''(o) = B$$

$$b + \mu_u c_u''(u) = B$$

$$\mu_o + \mu_u = (1 - \lambda) p(o, u)$$

with $0 \le \lambda < 1$. Solving this system of equations for b yields:

$$b = B - \frac{(1 - \lambda) p(o, u)}{\frac{1}{c_o''(o)} + \frac{1}{c_u''(u)}}.$$

This implies $b < B$ and $\mu_o, \mu_u > 0$.

For this solution to be a maximum, the Lagrange function evaluated with the Lagrange-multipliers found above must be concave. This function is given by:

$$\mathcal{L}^*(o, u, b, w) = B p(o, u) + (1 - \lambda) L - \lambda [c_o(o) + c_u(u)] - \mu_o c_o'(o) - \mu_u c_u'(u)$$

which is concave in o, u, b and w (because $c_o'''(o), c_u'''(u) > 0$). $\square$

Proof of Proposition 3
We first show that $o = \underline{o}$ by contradiction. Consider the case $o < \underline{o}$. If the agent would like to disobey the contract, his optimal choice of efforts is $o = 0$ and $u = 0$, which would give him a payoff of $-L < 0$. But this cannot be optimal for the agent because obeying and delivering $o = \underline{o}$ would give him a non-negative payoff (because the principal has to fulfill the PC).

Now consider the case $o > \underline{o}$. This would be optimal for the agent if the effort level given by $c_o'(o) = b$ is higher than $\underline{o}$, or $c_o'(\underline{o}) < b$. To show the opposite first note that in the principal's optimum it must be the case that $c_o'(\underline{o}) \ge B$. If not, the principal could marginally increase o while holding the agent's payoff constant by increasing w. This would increase the principal's profit marginally by $B - c_o'(o) > 0$. Second, it cannot be optimal for the principal to set $b > B$. Consider

$$\frac{d\Pi}{db} = -p(o, u) + (B - b) \frac{1}{c_u''(u)}$$

which is negative for $b > B$. If the LLC is binding, this shows that decreasing b will increase Π. If the LLC is not binding, the principal could extract the increase in the agent's surplus

$$\frac{dV}{db} = p(o, u) + b \frac{1}{c_u''(u)} - c_u'(u) \frac{1}{c_u''(u)} = p(o, u)$$

where the last equality results from using the agent's first-order condition. So the principal's profit would increase by $\frac{d\Pi}{db} + \frac{dV}{db} = (B-b)\frac{1}{c_u''(u)}$ which is still negative for $b > B$. So we have $c_o'(\underline{o}) \geq B$ and $B \geq b$ which implies $c_o'(\underline{o}) \geq b$ which means that in the optimum $\underline{o}$ will be greater than the effort level implied by $c_o'(o) = b$.

We next show that all $b \leq 0$ give the principal the same profit. Suppose the principal chooses some $o \in [0, o_{max}]$ and sets $b \leq 0$. Then the agent will choose $u = 0$. For the PC to hold the principal has to set $w = -bo + c_o(o) > 0 \geq -L$. This will give him the same profit $\Pi = Bo - c_o(o)$ for all $b \leq 0$.

The proof that for all $b \geq 0$ the optimal u will be given by the agent's first-order condition $b - c_u'(u) = 0$ is analogous to the argument in the proof of Proposition 1. Because $\hat{o} > o^* > 0$ we will also have $p(o, u) > 0$. $\square$

Proof of Proposition 4
The first order conditions for a maximum are:

$$\frac{\partial \mathscr{L}}{\partial o} = (B - b) + \lambda \left(b - c_o'(o)\right) = 0 \tag{15}$$

$$\frac{\partial \mathscr{L}}{\partial u} = (B - b) + \lambda \left(b - c_u'(u)\right) - \mu c_u''(u) = 0 \tag{16}$$

$$\frac{\partial \mathscr{L}}{\partial b} = -p(o, u) + \lambda p(o, u) + \mu = 0 \tag{17}$$

$$\frac{\partial \mathscr{L}}{\partial w} = -1 + \lambda + \eta = 0 \tag{18}$$

$\lambda, \eta, \mu \geq 0$ (with complementary slackness)

We show that a solution to these conditions must have both the PC and the LLC binding. If the PC is not binding, we will have $\lambda = 0$. Then (15) gives us $b = B$. But from (17) we get $\mu = p(o, u)$ and plugging into (16) gives us $b = B - p(o, u)c_u''(u) < B$, a contradiction. If only the PC is binding but the LLC is not, we will have $\eta = 0$. From (18) we get $\lambda = 1$ and from (17) we get $\mu = 0$. Plugging into (15) and (16) gives us $c_o'(o) = B$ and $c_u'(u) = B$ respectively. This implies, that the first best can be achieved with a bonus contract without violating the LLC. But this contradicts Assumption 7.

From (18) we get $\lambda = 1 - \eta$ and with $\lambda, \eta > 0$, we must have $0 < \lambda < 1$. From (17) we get $\mu = (1 - \lambda)p(o, u) > 0$. Substituting for μ into (16) and rearranging gives us

$$c_u'(u) = b = B - (1 - \lambda) \cdot p(o, u) \cdot c_u''(u) < B$$

and substituting $b = c_u'(u)$ into (15) gives us:

$$c_o'(o) = B + \frac{1 - \lambda}{\lambda}(B - c_u'(u)) > B.$$

For this solution to be a maximum, the Lagrange function evaluated with the Lagrange-multipliers found above must be concave. This function is given by:

$$\mathscr{L}^*(o, u, b, w) = Bp(o, u) + (1 - \lambda)L - \lambda[c_o(o) + c_u(u)] - \mu c_u'(u)$$

which is concave in o, u, b and w (because $c_u'''(u) > 0$). $\square$

Proof of Proposition 5

We have to show that $\frac{do}{dL} < 0$ and $\frac{du}{dL} > 0$. The optimal values for o, u, b and w are given by the solution to the four equations:

$$b \cdot p(o, u) + w - c_o(o) - c_u(u) = 0 \tag{19}$$

$$L + w = 0 \tag{20}$$

$$b - c'_u = 0 \tag{21}$$

$$(c'_o - B)c''_u \cdot p(o, u) + (b - B)(c'_o - b) = 0 \tag{22}$$

where (22) is a rewritten form of (8). If we differentiate these four equation with respect to L, we get:

$$p(o, u)\frac{db}{dL} + 1 \cdot \frac{dw}{dL} + (b - c'_o)\frac{do}{dL} + (b - c'_u)\frac{du}{dL} = 0 \tag{23}$$

$$1 + \frac{dw}{dL} = 0 \tag{24}$$

$$\frac{db}{dL} - c''_u\frac{du}{dL} = 0 \tag{25}$$

$$((c'_o - b) + (B - b))\frac{db}{dL} + (c''_o c''_u p(o, u) + (c'_o - B)c''_u + (b - B)c''_o)\frac{do}{dL}$$
$$+ ((c'_o - B)c'''_u p(o, u) + (c'_o - B)c''_u)\frac{du}{dL} = 0 \tag{26}$$

We can now solve (24) for $\frac{dw}{dL} = -1$ and (25) for $\frac{db}{dL} = c''_u\frac{du}{dL}$. Plugging these results into (23) and using (21) gives us

$$p(o, u)c''_u\frac{du}{dL} + (b - c'_o)\frac{do}{dL} = 1 \tag{27}$$

while plugging the results into (26) gives us:

$$(c''_o c''_u p(o, u) + (c'_o - B)c''_u + (b - B)c''_o)\frac{do}{dL}$$
$$+ ((c'_o - B)c'''_u p(o, u) + 2(c'_o - b)c''_u)\frac{du}{dL} = 0 \tag{28}$$

To simplify calculations, we make the following substitutions:

$$e = b - c'_o$$

$$f = p(o, u)c''_u$$

$$g = [c''_o c''_u p(o, u) + (c'_o - B)c''_u + (b - B)c''_o]$$

$$h = [(c'_o - B)c'''_u p(o, u) + 2(c'_o - b)c''_u]$$

The two equations can then be written as

$$\begin{bmatrix} e & f \\ g & h \end{bmatrix} \cdot \begin{bmatrix} \frac{do}{dL} \\ \frac{du}{dL} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Using Cramer's Rule we can solve for

$$\frac{do}{dL} = \frac{h}{eh - fg} \quad \text{and} \quad \frac{du}{dL} = \frac{-g}{eh - fg}.$$

Because at the optimum $c_o' > B > b$, we will have $e < 0$, $f > 0$ and $h > 0$. To sign g, we rewrite (22) and get

$$\frac{B - b}{c_u'' p(o, u)} = \frac{c_o' - B}{c_o' - b} < 1$$

where the inequality follows again from $c_o' > B > b$. Because $c_u'' p(o, u) > 0$, this implies $c_u'' p(o, u) > B - b$. Now we can easily show $g > 0$. These results imply $eh - fg < 0$ and finally $\frac{do}{dL} < 0$, $\frac{du}{dL} > 0$. $\square$

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