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Scaffolding of learning activities: Aptitude-treatment-interaction effects in math?

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ABSTRACT

Background: Due to differences in learners' resources in specific learning situations, they may not profit equally from learning activities such as task practice or learning with analogies. Scaffolds can help to adapt learning activities to learners' needs.
Aims: We want to answer the question who benefits from what scaffold when learning about fractions on the number line in digitally-enriched mathematics instruction in the classroom.
Sample: Participants were 332 6th-Graders.
Methods: Dynamic visualizations were used as *information-processing scaffold* when learning with analogies. During repeated practice, adaptive task difficulty was implemented as *motivational scaffold* and individualized explanations based on typical mistakes were offered as *information scaffold*. Students were randomly assigned to one of the scaffold conditions or a control group without scaffolds. As characteristics potentially affecting learning processes during learning activities, we assessed prior knowledge, sustained attention, general reasoning, visual-spatial abilities as well as interest and self-concept in mathematics. These learner characteristics were included as predictors in Generalized Linear Mixed Models, together with the experimental conditions. Due to the nature of the multi-track school system in Germany, advanced placement school (APS; *Gymnasium*) students and vocational school (VS; *Mittelschule*) students were considered separately.
Results and conclusion: For APS students, the different scaffolds yielded minimal effects. For VS students, dynamic visualizations could compensate for lower general reasoning and visual-spatial ability when learning with analogies and adaptive task difficulty seemed to successfully counteract lower interest during practice. Instruction can be individualized based on the conditioning of scaffolds on the specific mechanisms underlying different learning activities.

1. Introduction

There is broad evidence for the effectiveness of one-on-one tutoring with substantial positive effects (e.g., Cohen et al., 1982; Nickow et al., 2024; Pellegrini et al., 2021; Rheinheimer et al., 2010). Catering to the needs of the individual during the learning process is at the core of several instructional approaches, such as personalized, individualized, and adaptive instruction (e.g., Dockterman, 2018; Hillmayr et al., 2020; Plass & Pawar, 2020). While research utilizing techniques from educational data mining and learning analytics can uncover individual variations in learning processes and offer personalization in terms of content and difficulty level (Goldhammer & Zehner, 2017), we still lack answers to the question who profits from what kind of treatment (e.g., Chernikova et al., 2023; Plass & Pawar, 2020; Tetzlaff et al., 2021). In this study, we address this gap by starting with specific learning activities and adding different scaffolds to examine what works best for which learners — with the ultimate goal of providing all students with cognitively stimulating mathematics instruction at the secondary school level.

Traditional aptitude-treatment-interaction (ATI) research (e.g., Cronbach & Snow, 1977; Snow, 1991) investigates the fit between individual learner prerequisites and the learning situation, mostly focusing on group differences based on pretest scores (or expertise levels; e.g., Kalyuga, 2007) and on the effects of varying levels of guidance (e.g., Fukuda et al., 2022; Lazonder & Harmsen, 2016).

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Differential effectiveness research (Hunt, 1975) has broadened the scope of traditional ATI research by moving beyond the exclusive reliance on strictly experimental designs and the restrictive statistical requirement of a significant disordinal interaction (e.g., Faddar & Kjeldsen, 2022). In the broader context of social and behavioral research, the explicit recognition that treatments vary in effectiveness across different populations and contexts has recently been termed the 'Heterogeneity Revolution' (Bryan et al., 2021). Based on theories of causal mechanisms, behavioral researchers try to demonstrate that a treatment effect diminishes or disappears as a function of critical factors, such as specific individual characteristics (e.g., Bryan et al., 2021; Krefeld-Schwalb et al., 2024; Veltri, 2023). But which individual characteristics may be relevant for learning in a specific learning situation (treatment) based on theories of causal mechanisms? This question cannot be answered without considering the learning activities within the learning situation and their affordances.

1.1. How the affordances of learning activities can inform scaffolding

The last decades of research on learning and instruction resulted in a number of well-documented, established learning activities that are based on largely accepted general principles of learning (see e.g., Ainsworth, 2006; Eccles & Wigfield, 2020; Gentner, 1983; Georghiades, 2000). We define activities as learning activities when they prompt learners to engage with, direct, shape, or optimize cognitive, metacognitive, and/or motivational processes to acquire knowledge (by association or differentiation, integration, and restructuring; e.g., Carey, 2000) or to practice retrieval or the application of knowledge.

Learning activities that can support knowledge acquisition include, for example, *generating solutions to novel problems* prior to instruction, *engaging with contrasting cases*, *learning with analogies*, and *comparing superficially different examples* to identify a common underlying concept (e.g., Firth et al., 2021; Hofer et al., 2018; Loibl et al., 2024; Reinhold et al., 2024; Rittle-Johnson, 2017; Roy & Chi, 2005; Schwartz et al., 2011; Zepeda et al., 2015). Likewise, several learning activities have been investigated that support the practice and application of knowledge. These include *comparing problem types or solution methods* using alternative ordering tasks to foster procedural flexibility (e.g., Rittle-Johnson, 2017; Star, 2005). Other examples are employing *motor imagery* to enhance motor performance (e.g., Guillot & Collet, 2008), using *retrieval practice* to promote long-term retention of procedural and declarative knowledge (e.g., Roediger & Butler, 2011), and engaging in *problem-solving* within simulated real-world scenarios to support knowledge application (e.g., Fischer et al., 2022).

Yet, students often do not profit equally from different learning activities due to variation in their individual resources related to the cognitive, metacognitive, or motivational processes required for learning (e.g., Hofer et al., 2018; Hofer & Stern, 2016; Reinhold, Hofer, Berkowitz, et al., 2020; Reinhold, Hofer, Hoch, et al., 2020; Stern, 2017). For fact learning, for instance, there is evidence for age-related differences in performance between the learning activities *prediction generation* and *example-based learning*. The latter seems to correlate with the learners' ability for analogical reasoning (Breitwieser & Brod, 2021). Similar results are reported for *generating drawings* and *generating questions* (Brod, 2020). Likewise, *comparing solution methods* seems to be less effective than *comparing problem types* of studying examples sequentially, if students do not possess sufficient prior knowledge (Rittle-Johnson et al., 2009). In a collaborative learning setting, activities that involved *comparing only one stochastic concept* during the individual learning phase were less effective for learners with low prior knowledge than those that *contrasted three different stochastic concepts* (Deiglmayr & Schalk, 2015). Regarding non-cognitive learner resources, beneficial motivational and emotional orientations, such as learners' self-concept, and emotional engagement, including intrinsic motivation and interest, turned out to be relevant prerequisites for getting the most out of *practice* tasks in mathematics (Reinhold et al., 2021). Across studies, we see that sometimes students cannot make use of a specific learning activity as intended—for example, due to a lack of intrinsic motivation or insufficient prior knowledge. This is were scaffolding could come into play (see Fig. 1).

Wood et al. (1976) broadly described scaffolding as the process of guiding somebody through a task that would be impossible to complete without calibrated support. According to most conceptualizations of scaffolding, it refers to additional information that is provided on top of a learning activity supporting the learner (e.g., Belland, 2017; Dumont, 2019; Heitzmann et al., 2019; Nguyen, 2021; Pea, 2004; Puntambekar, 2022). Building on the concept of representational scaffolding (Fischer et al., 2022), we expand the definition of scaffolding beyond the provision of additional information (e.g., providing individualized explanations in addition to corrective feedback) to also include modifications in the implementation of the learning activity itself, with the goal of altering its demands. These modifications may involve, for example, using dynamic rather than static visualizations of content or adapting the difficulty level of practice tasks based on learner performance instead of predefined difficulty levels during practice. Scaffolds can specifically address cognitive, metacognitive, or motivational processes relevant for the learning activity to work as intended and hence help to adapt those activities to the needs of individual learners. The distinction between learning activities and scaffolding is critical, as it clarifies the nature of scaffolds as tools designed to enhance or support particular learning processes required to realize the potential of specific learning activities (see Fig. 1) allowing for a more targeted and purposeful use of scaffolding in research and practice.

To summarize, we consider all systematic amendments to and modifications in the implementation of a learning activity as scaffolding as long as they tap underlying cognitive, motivational, or metacognitive processes. Since these classifications are not supposed to be deterministic in the sense that a specific scaffold can only fulfill one function in the learning process (e.g., motivational), we refer to cognitive, motivational and metacognitive scaffolding as 'scaffolding intentions' (van de Pol et al., 2010).

The mechanisms underlying cognitive scaffolding are grounded in the idea of a multi-memory model cognitive architecture and the processes it describes as central to learning (e.g., Sweller et al., 2019). We speak of *cognitive information scaffolding* whenever scaffolding provides missing knowledge that is, however, important to process information. We speak of *cognitive information-processing scaffolding* whenever scaffolding supports activation of existing knowledge, selection, processing in working memory, encoding, or integration. Examples are provided in Fig. 2.

While cognitive scaffolding supports the cognitive processes involved in learning, *motivational scaffolding* targets the motivational factors that influence the depth and duration of volitional engagement in cognitive and metacognitive learning processes. Synthesizing what the various existing theories of learning motivation have in common, a learner's current motivation comprises an expectancy and a value component related to the current activity that is initiated, directed, controlled, regulated, maintained, and evaluated depending on that motivation. Both the situation and individual characteristics affect the expectancy and value component and hence current motivation (see Dresel & Lämmle, 2011; Eccles et al., 1983; Wigfield & Eccles, 2000). Accordingly, motivational scaffolding may activate relevant existing motivational traits, support (situational) feelings of competence, autonomy, and relatedness, promote self-efficacy and a positive self-concept and hence increase the expectancy of success, e.g., via supporting beneficial attributions, and stress the value of a task (e.g., Deci & Ryan, 2008; Weiner, 1985; see Fig. 2 for examples). Belland and colleagues (2013) provide a comprehensive framework for motivational scaffolding.

Finally, *metacognitive scaffolding*, which is not investigated in the present study, is intended to support metacognitive processes such as analyzing, planning, executing, and monitoring own learning behavior (Bannert et al., 2014; Lim et al., 2023).

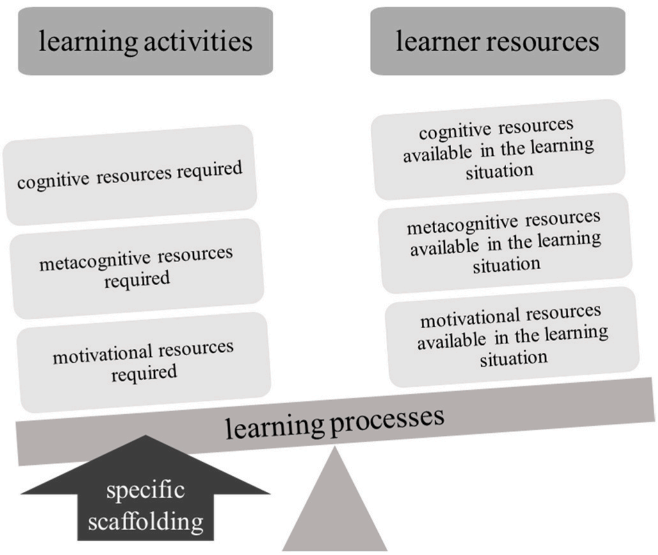

Fig. 1. Illustration of the interplay between learning activities, learner resources, and scaffolding to optimize learning processes.

1.2. Understanding the domain of fractions to derive learning activities

Before considering which scaffolds to implement, it is essential to first identify which learning activities are appropriate for the specific content domain. A solid understanding of fractions is a foundational aspect of mathematics learning (Booth & Newton, 2012), yet remains highly challenging for many students (Bailey et al., 2012; Lortie-Forgues et al., 2015; Reinhold et al., 2025; Reinhold, Hofer, Hoch, et al., 2020; Reinhold, Obersteiner, Hoch, et al., 2020). One concept that is particularly challenging to grasp is fraction measurement, which involves understanding the placement of fractions on a structured number line (Behr et al., 1993; Kieren, 1976; Wong & Evans, 2008).

Two specific features contribute to the difficulty of number line tasks as illustrated in Fig. 3: first, variation in how the unit is subdivided; and second, variation in the total length of the number line, which may not be identical to the defined unit (Behr et al., 1983). These variations create task formats in which naïve counting strategies based on natural number knowledge often lead to systematic errors (Hannula, 2003; Novillis-Larson, 1980). To solve such tasks correctly, students should interpret a fraction a/b as a measure of a out of b congruent parts of the defined unit (Kieren, 1976; Wong & Evans, 2008). As it is often the case in mathematics, the ability to use the number line as a tool to represent fractions can be described as a mixture of interrelated conceptual and procedural knowledge (Crooks & Alibali, 2014; Nuraydin et al., 2023; Rittle-Johnson, 2017). By building on students' prior knowledge, instruction can guide them to relate familiar components of a rectangle to corresponding fractions on the number line, thereby supporting their understanding of the challenging concept of fraction measurement (see Fig. 4). Through repeated practice with placing fractions on number lines that systematically vary in relevant difficulty-generating features (as described above), students can further develop both conceptual and procedural knowledge. Accordingly, we chose *learning with analogies* and *repeated practice* as well-established learning activities in mathematics education that support the acquisition and practice of conceptual and procedural knowledge and combined them in tablet-based instruction in the classroom.

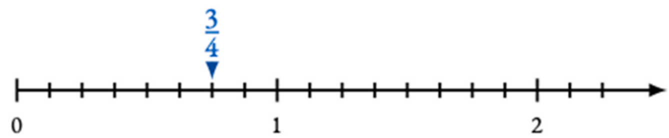

Fig. 3. Typical task involving the placement of 3/4 on a structured number line.

1.3. Learning with analogies with dynamic visualizations as cognitive information-processing scaffold

The learning activity *learning with analogies* can help us to infer new abstractions from what we already know and to understand concepts as systems of relationships that can be connected and flexibly manipulated (e.g., Gentner & Holyoak, 1997; Richland & Simms, 2015). The process of analogical reasoning is demanding and it often does not happen without support (e.g., Gentner et al., 2003; Kubricht, Lu, & Holyoak, 2017; Starr et al., 2018). Depending on their processing resources, individual learners differ in their capability to mentally hold and manipulate relationships in the analogy (e.g., Gray & Holyoak, 2020; Richland & Simms, 2015). Moreover, learners have to recognize similarities and differences between the target and the source of the analogy. This structure mapping process is affected by the learners' cognitive resources including general reasoning or spatial abilities (e.g., Begolli

for example …			learning activity	
			knowledge acquisition	**practice**
scaffolding intention	**cognitive scaffolding**	**information scaffolding**	during comparisons / provision of term definitions	*during repeated practice* / *explanations based on typical mistakes*
		information-processing scaffolding	*when learning with analogy* / *dynamic visualization*	during vocabulary training / automatic word recognition
	motivational scaffolding		when generating predictions / wording that stimulates mastery orientation	*during repeated practice* / *adaptive task difficulty*

Fig. 2. Grid showing examples of combinations of learning activities with scaffolds differing in their intentions. Italic typeface indicates examples implemented in this study.

et al., 2015; Braasch & Goldman, 2010; Krawczyk et al., 2008). Accordingly, learners with less resources in that regard are less likely to make use of analogies as learning activity (Gray & Holyoak, 2021).

We hence suggest to support *learning with analogies* with *dynamic visualizations* as scaffold (acknowledging that there are other options for scaffolding). The representation of relations via dynamic visualizations can support the integration of knowledge and structure mapping (Hegarty & Kriz, 2008). They provide an external representation to help learners build an adequate mental model of the relationships in the analogy (see Hegarty & Kriz, 2008; Lichti & Roth, 2018; Lowe & Schnotz, 2014). Accordingly, dynamic visualizations (instead of static visualizations of the analogy) can act as an information-processing scaffold supporting cognitive processes during knowledge acquisition with analogies.

There is evidence that these visualizations might be especially beneficial for learners with less cognitive resources, including visual-spatial ability (Höffler, 2010; Kubricht et al., 2017; Kühl et al., 2022). Yet, the evidence for visual-spatial ability as cognitive resource is, in parts, contradictory. Visual-spatial ability might help to follow a dynamic visualization, especially for highly complex content (Kühl et al., 2022). In that case, visual-spatial ability plays the role of an enhancer. However, there seems to be more support from recent research for another hypothesis, stating that learners with low visual-spatial ability might, in particular, be supported by the external representation provided by the visualization, which helps them to build an adequate mental model they would not have been able to build themselves (Höffler, 2010). We hence assume dynamic visualizations to have the potential to compensate for deficits in visual-spatial ability. To sum up, as a cognitive information-processing scaffold, *dynamic visualizations* might help students with less cognitive resources, especially visual-spatial and general reasoning ability, to make use of the learning activity *learning with analogies*.

1.4. Repeated practice with adaptive task difficulty as motivational scaffold

A learning activity that is especially suited to improve learners' procedural knowledge is *repeated practice*, which is closely related to the concepts of repeated testing or practice testing (e.g., Larsen et al., 2009; Stenlund et al., 2016). It has proven to be important to consolidate learning, especially if it involves active processing, problem solving, and reflection (e.g., Karpicke & Roediger III, 2007; Lehtinen et al., 2017) and if effortful retrieval is involved (Adesope et al., 2017; Roediger & Butler, 2011; Stenlund et al., 2016). Yet, learners may feel overwhelmed or bored by inappropriate (too difficult or too easy) practice problems, which can harm the learning progress (Su, 2017; Tanaka & Murayama, 2014).

One solution is to implement *adaptive task difficulty* (instead of predetermined difficulty levels) as scaffold with the difficulty level being adapted to learners' previous performance (e.g., Reinhold, Hoch, Werner, et al., 2020; Reinhold, Hofer, Hoch, et al., 2020; Sjaastad & Tømte, 2018). Accordingly, in the present study, task difficulty is adapted to a sequence of students' previous performance referring to domain-specific difficulty generating factors (Kieren, 1976; Hannula,

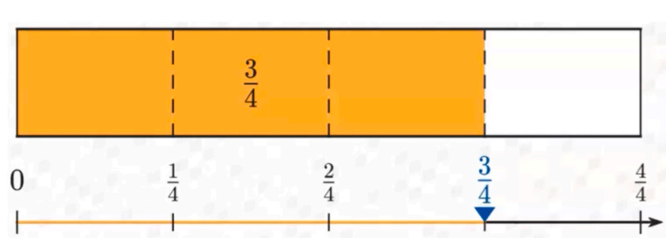

Fig. 4. Analogy showing how parts of a rectangle relate to fractions on the number line.

2003; Reinhold et al., 2025). This allows all students to learn with optimally challenging tasks (e.g., Wood et al., 1976).

In line with self-determination theory (e.g., Deci & Ryan, 2008), *adaptive task difficulty* as motivational scaffold during *repeated practice* can promote a sense of achievement and competence by experiencing tasks as challenging but manageable. A boosted sense of competence might be especially critical and hence increase learning motivation for students with below-average self-concept and interest in mathematics (Reinhold et al., 2021).

1.5. Repeated practice with individualized explanations based on typical mistakes as cognitive information scaffold

Repeated practice activities, described in section 1.4, often incorporate some form of feedback, which plays a crucial role in learning through repeated practice (e.g., Abbott et al., 2017; Bosse et al., 2015; Wang & Yang, 2023). Feedback refers to information given to learners concerning a gap between their performance or comprehension and the desired instructional outcome. Extensive research has shown that feedback is instrumental in advancing learners' performance (e.g., Hattie & Timperley, 2007; Krapp, 2005; Narciss et al., 2014), allowing students to benefit from targeted generative processing (e.g., Moreno, 2004). The effectiveness of feedback, however, is contingent upon learners establishing an internal mental representation of the practice task. Learners' prior knowledge about the task and the learning content influence the ease of mentally representing the practice task. Processing of feedback can hence overwhelm an individual's cognitive resources (e.g., Lam et al., 2011).

Feedback as part of a repeated practice learning activity can be provided in many different ways, among others, regarding feedback content and timing (see Panadero & Lipnevich, 2022). In the present study, basic corrective feedback is complemented by additional *individualized explanations based on typical mistakes* as a cognitive information scaffold in the context of repeated practice. These additional individualized explanations are derived from an analysis of the learners' specific mistakes compared to empirically-validated typical mistakes on the task (Reinhold et al., 2025). Individualized explanations addressing domain-specific errors (e.g., Asterhan & Dotan, 2018) may facilitate targeted generative processing during feedback reception. Providing explicit and timely information about mistakes can help integrate new insights, even into less developed mental representations (e.g., Mason & Bruning, 2001; Moreno, 2004).

Low-prior knowledge learners tend to benefit more from feedback that includes immediate explanations, rather than just indicating correct or incorrect answers (e.g., Moreno, 2004). For high-prior knowledge learners, processing of comprehensive feedback has been found to cause unnecessary cognitive load in line with the redundancy principle and expertise reversal effects (see e.g., Fyfe, 2016; Fyfe et al., 2012; Fyfe & Rittle-Johnson, 2016; Kalyuga, 2007). The individualized explanations implemented in this study, however, are delivered solely in response to actual errors and are precisely targeted to address those specific mistakes. Accordingly, high-prior knowledge learners could still benefit from individualized explanations when they do make a mistake, although they might be able to understand and resolve errors based on their existing knowledge as well.

At the same time, sufficient cognitive resources (Fyfe et al., 2015), such as sustained attention, may be essential to adequately process explanations that are provided as feedback during repeated practice. Accordingly, students with lower prior knowledge might especially profit from *individualized explanations based on typical mistakes* as cognitive information scaffold during *repeated practice* (e.g., Fyfe, 2016) – and higher sustained attention may help to make use of the scaffold.

2. The present study

The present study focused on students' knowledge about fractions on the structured number line in tablet-based mathematics instruction in the classroom. We aimed to answer the following research question: *During instruction about fractions on the number line based on established learning activities, who benefits from what type of scaffold linked to those learning activities?*

Referring to existing research, dynamic visualizations were used as a cognitive information-processing scaffold when learning new information with analogies. During repeated practice, adaptive task difficulty was implemented as motivational scaffold and individualized explanations based on typical mistakes were offered as cognitive information scaffold. Accordingly, 6th-Grade students from German secondary schools were randomly assigned to one of the scaffold conditions or a control group without any scaffolds. To examine characteristics that may influence cognitive and motivational processes during learning—and thus determine for whom the different scaffold conditions might be effective—prior domain knowledge, sustained attention, general reasoning, visual-spatial abilities, self-concept in mathematics, and interest in mathematics were assessed and included as predictors in Generalized Linear Mixed Models. These predictors, along with the three scaffold conditions and the control group, were used to predict knowledge of fractions on the number line.

We expected the cognitive information-processing scaffold dynamic visualizations to help students with less cognitive resources, i.e., general reasoning (H1) and visual-spatial ability (H2), to make use of the learning activity learning with analogies—and to be of no additional help for students with high cognitive resources.

Similarly, we hypothesized that the motivational scaffold adaptive task difficulty would be especially effective in supporting motivational processes, thereby enhancing learning during the learning activity repeated practice, more so for students with low interest (H3) and self-concept in mathematics (H4) than for those with high interest and self-concept.

Finally, the cognitive information scaffold—individualized explanations in response to domain-specific errors—was assumed to be particularly beneficial for learners with low prior knowledge. It was expected to allow them to learn from their mistakes during repeated practice. The scaffold was considered less essential for high-prior knowledge learners, as these learners are likely to make fewer mistakes and to use their existing knowledge to understand and resolve errors (H5). In addition, we expected sufficient cognitive resources, especially sustained attention, to be a necessary prerequisite for students to process the individualized explanations—with students lacking sustained attention being 'distracted' by this additional information (H6). A visual summary of the six underlying hypotheses is given in Fig. 5.

3. Methods

3.1. Sample

A total of $N = 332$ 6th-Grade students (146 female, 179 male, 7 preferred not to tell) from $k = 16$ German secondary school classrooms participated in the study. Of those participants, 192 students attended advanced placement schools (APS; German Gymnasium, all in 2021) and 140 students attended vocational schools (VS; German Mittelschule, 62 in 2021 and 78 in 2022 with no significant cohort differences on central study variables). There is broad evidence for significant differences between students from those school types with regard to performance levels and learning conditions (e.g., Baumert et al., 2006, 2009; Dumont et al., 2013; Hofer et al., 2024; Reinhold, Hofer, Hoch, et al., 2020). In both types of schools, fractions are part of the 6th-Grade curriculum. Accordingly, the students were formally introduced to fractions in advance of their participation in the study. This study focuses on the specific topic of fractions on the number line, which the participants have had limited exposure to in their coursework. Students were randomly assigned to one of the four experimental conditions; the final resulting distribution is shown in Table 1.

3.2. Intervention procedure and experimental conditions

This study was conducted at two time points in July 2021 and 2022. The Bavaria n Ministry of Education (reference IV.7-BO4106.2019/52/9) granted its approval including ethical clearance. After obtaining consent from school principals, teachers, and parents, researcher-provided tablets were used by the students during the 2-h intervention, conducted within their classrooms during regular school days and under the guidance of a trained investigator who was not the classroom teacher.

The digital learning environment utilized in this study is in parts based on the ALICE:fractions environment, which was developed in a collaborative effort between the Chair of Geometry and Visualization at the Department of Mathematics and the Heinz Nixdorf Foundation Chair of Mathematics Education, both at TU Munich (Reinhold, Hoch, Werner, et al., 2020; Reinhold, Hofer, Hoch, et al., 2020). In this digital environment, students first completed a prior knowledge test before individually working through four identically structured content blocks, all following exactly the same procedure but differing in the specific content covered. Each block concluded with a block-specific post-test. The procedure is explained in more detail below.

Initially, students received concise instructional material on basic knowledge about fractions before completing a prior knowledge test

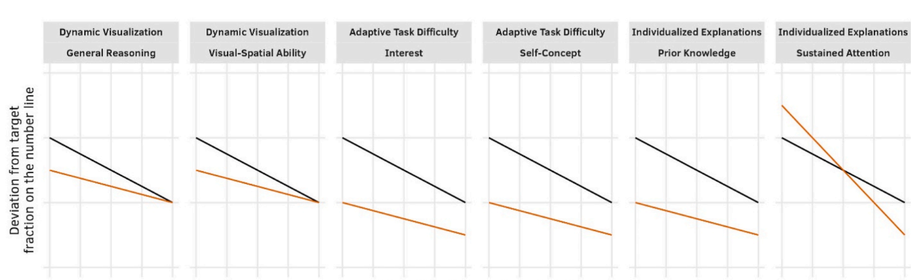

Fig. 5. Interaction hypotheses regarding the effectiveness of the different scaffolds with varying learner characteristics. Black = Control condition; Orange = Experimental condition with the scaffold and learner characteristic given in the respective headline. Performance on the number line tasks is measured as deviation from the target fraction on the number line (the smaller the deviation the higher the performance). (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

Table 1
Distribution of the sample to the four experimental conditions.

School type	Control	Ind. Explanations	Adapt. Difficulty	Dyn. Visualization	Total
APS	46	51	47	48	192
VS	31	37	38	34	140
Total	77	88	85	82	332

comprising four sections of increasing difficulty, assessing their comprehension of fractions on the number line. Subsequently, students engaged in the intervention, which consisted of four identically structured sequential blocks of content and tasks – i.e., knowledge acquisition via learning with an analogy followed by practice of retrieval and application of knowledge with repeated practice – escalating in complexity. All blocks addressed understanding of fractions on the number line. Accordingly, the first block targeted the standard number line from 0 to 1 and segmentation had to be refined or coarsened. In the second block, students were introduced to fractions on a number line exceeding 1. In the third block, students had the opportunity to enhance their understanding of fractions on number lines by engaging with segmentations that did not align with the fractions to be placed on the number line. Finally, the fourth block dealt with placing fractions on number lines with no segmentation at all. Within all four blocks, the three experimental scaffolding conditions (illustrating screenshots are available as supplemental material) were embedded within the learning activities *learning with an analogy* or *repeated practice* to support students to acquire and practice retrieval and the application of knowledge related to fractions on the number line, as detailed in the following sections.

3.2.1. Dynamic visualizations as cognitive information-processing scaffold

Each block started with written information on the topic using analogies accompanied by static or dynamic (depending on the experimental condition) visualizations to support knowledge acquisition. Those visualizations represented block-specific analogies (i.e., learning with an analogy). For instance, they depicted how parts of a rectangle relate to fractions on the number line (see Fig. 4) to visualize the process of refining and coarsening. In the dynamic visualization condition, students could interact with the visualization by changing the denominator of the fraction and observing the consequences for the source relationship (e.g., partition of a rectangle), while in all other conditions, students were provided with static visualizations of the analogy for different fractions. Students across all conditions were asked to summarize the knowledge acquired, facilitated by two reflection prompts.

3.2.2. Adaptive task difficulty as motivational scaffold

Subsequently, they practiced placing fractions accurately on a number line (see Fig. 3) for a duration of 5 min (i.e., repeated practice). During the practice phase, task difficulty was adjusted in accordance with the students' performance in the adaptive task difficulty condition, utilizing empirically supported factors for generating difficulty levels (Reinhold et al., 2025): A random set of five tasks was generated on the pre-defined set of difficulty-generating factors. Students who completed three or fewer of those tasks correctly received another set within the same difficulty level, while those who completed more than three tasks correctly were presented with a set of increased difficulty. In all other experimental conditions, difficulty escalated after each set regardless of student performance.

3.2.3. Individualized explanations based on typical mistakes as cognitive information scaffold

After each practice task, students received corrective feedback indicating the accuracy of their response and, if necessary, providing the correct solution. Only in the individualized explanation condition, students received additional information, including an error analysis (based on their input and the most probable underlying error according to Reinhold et al., 2025) and guidance on reaching the correct solution.

Finally, students completed a post-test specific to each block, assessing their understanding of fractions on the number line (without feedback). The same procedure (knowledge acquisition via learning with an analogy, practice of retrieval and application of knowledge with repeated practice, block-specific post-test) was repeated across all four blocks. Each block was designed to take up 15 min. Subsequently, learner characteristics and demographic data were gathered through an online questionnaire (Lime Survey).

3.3. Instruments and scales

Table 2 provides information about the instruments and scales used in this study, with the exception of the assessment of knowledge about fractions on the number line, which is described in greater detail below. While the visuo-spatial ability test was conducted in a paper-based format, all other cognitive learner characteristics were evaluated using established tests administered via tablets. Motivational characteristics were gauged through self-report scales, employing response options structured on a Likert scale ranging from 1 = "rarely" to 4 = "mostly" or 1 = "do not agree at all" to 4 = "completely agree". Beyond the learner characteristics outlined in Table 2, several additional scales were employed as part of a broader data collection effort. These encompassed the trait assessments of mathematics anxiety, excessive demand, and behavioral and cognitive engagement as well as an optimal learning moments survey. However, these scales did not align with the focus of the current study.

Math educators devised the prior knowledge test on fractions on the number line and all block-specific post-tests, drawing from their classroom teaching expertise and existing research on fraction learning. Each item of the prior knowledge and the post-tests required learners to position fractions on a number line (see Fig. 3; a screenshot is available as supplemental material), with variations introduced to reflect difficulty-generating factors as outlined by Reinhold, Hoch, and Hofer (2025; i.e., alignment of the denominator with the number line unit, length of the number line, presence or absence of specific segmentation). To assess prior knowledge, students' baseline understanding of fractions on number lines was evaluated through eight items. Following the completion of each block, a post-test consisting of six to eight items was administered. In all of those assessments, lower scores indicated higher proficiency, as the results represented the deviation from the correct solutions (i.e., perfect performance would correspond to a score asymptotically approaching zero). Whenever prior knowledge was used as learner characteristic in interaction terms (e.g., Fig. 5), the resulting value was multiplied by (-1) to create an equal scaling across all learner characteristics (i.e., to ensure that higher values consistently indicate higher manifestations on the respective learner characteristic) and hence facilitate interpretation.

Regarding the reliability of the instrument to assess students' knowledge about fractions on the number line, the prior knowledge test (8 items) showed a near-to-perfect Cronbach's alpha, $\alpha = .977$, 95 % CI [0.973, 0.980]. All 28 post-test items from all four blocks also showed a near-to-perfect Cronbach's alpha, $\alpha = .978$, 95 % CI [0.975, 0.982]. Moreover, the nesting of post-test items in the four blocks was measurable with those post-test items; McDonald's Omega (which automatically performs an explorative factor analysis, here with $k = 4$ factors) perfectly mapped the four blocks in the factor structure, resulting in a hierarchical $\omega_h = 0.83$, an asymptotic $\omega_a = 0.83$, and a total $\omega_t = 0.99$. All other reliability estimates are provided in Table 3. Note that the coefficients for the visual-spatial ability test are low. However, for speeded tests, such as the visual-spatial ability test used in this study, conventional reliability estimates may not provide a valid indicator of reliability and parallel forms should be used instead (e.g., Gulliksen, 1950). Reliability has been ensured following this approach based on other samples (Jäger et al., 1997).

Table 2
Overview of all instruments and scales used in this study (adapted from Bach et al., 2024).

Learner Characteristic	Operationalization	Adapted From	Sample Items
Cognitive Characteristics			
Sustained Attention	For 3 min, participants have to respond to stimuli (pictorial flowers) and sort them up or down, based on two memorized rules. The number of z-standardized hits, omissions, mistakes, and dismissals is collected, and a score is built by subtracting the number of mistakes and omissions from the hits (score ranging between −1 and 1).	Attention Swiping Task (AST; Koch et al., 2021)	–
General Reasoning	As in classical matrices tests, nine fields are shown, each following a specific rule. Following this rule, students must fill in the ninth field by composing their answers from a selection of 20 elements. Students have 16 min to complete 16 matrices tasks (maximum score = 16).	DESIGMA construction-based figural matrices task (Koch et al., 2022)	–
Visual-Spatial Ability	In a total of five tasks, students are presented with a paper folding template and have to choose from a selection of five objects the one that can be folded out of this (maximum score = 5).	Paper-Folding-Test (Jäger et al., 1997) of the Berliner Intelligenz-Struktur-Test (BIS-Test)	–
Motivational Characteristics			
Math Self-Concept	Five items ask about students' beliefs regarding their math abilities and skills.	SCMAT survey, used in the PISA 2012 assessment (OECD, 2013)	"I have always been convinced that math is one of my best subjects."
Interest in Math	Four items assess interest as a trait.	INTMAT survey, used in the PISA 2012 assessment (OECD, 2013)	"I like math books."

3.4. Statistical analyses

The statistical analyses were performed in R (R Core Team, 2021), version 4.3.1. Based on prior studies on systematic differences between different school tracks in the German school system (e.g., Guill et al., 2017; Reinhold, Hoch, Werner, et al., 2020; Reinhold, Hofer, Hoch, et al., 2020), APS students can be expected to outperform VS students on cognitive and motivational learner characteristics. To adequately consider the specifics of our student sample before addressing our research question, we conducted Welsh t-tests comparing students from the two school tracks regarding the learner characteristics; Welsh-corrected effect sizes were estimated using the {rstatix}-package, version 0.7.2, (Kassambara, 2023). Expecting school track-related subgroups within our sample, we further had to show that there were no significant differences in any of the learner characteristics between the randomly assigned experimental conditions within each subgroup. We accordingly used Kruskal-Wallis tests comparing students between the four conditions; tests were performed for each subgroup separately; effect sizes were estimated using the {rstatix}-package.

Finally, our research question was answered with Generalized Linear Mixed Models (GLMM) for each learner characteristic separately using the {glmmTMB}-package, version 1.1.8, (Brooks et al., 2017). To appropriately model the absolute deviation from the correct value for fractions placed on the number line as the dependent variable, we assumed a lognormal distribution for the response variable; the lognormal distribution is particularly suited for positively skewed non-negative data.

To investigate the hypothesized interaction effects depicted in Fig. 5, we used the following fixed effect structure in all models: We included interaction terms between *school type* (baseline: VS; vs. APS), *experimental condition* (baseline: control group; vs. Dyn. Visualization/Ada. Difficulty (i.e., adaptive task difficulty)/Ind. Explanations), and *learner characteristic* (interest, self-concept, general reasoning, visual-spatial ability, sustained attention, and prior knowledge)—with each learner characteristic considered in a separate model. Aiming at the comparison of the learner characteristics *within* the respective subgroup (VS and APS), all learner characteristics were cluster mean centered within students of the same school track (Enders & Tofighi, 2007).

The multilevel structure of the data was considered in the random effects. Models included *students* nested in *classrooms* as random intercepts, as well as *items* nested in *blocks*; as the number of blocks was low (four), we only included an item random intercept and modelled the dependence of the four blocks of items as an additional fixed effect (Brauer & Curtin, 2018). Significance testing was conducted via model comparison, comparing a full model with all interaction terms to a restricted model without interaction terms between the learner characteristic and the scaffold of interest.

When significant interaction terms were identified, we examined the underlying subgroup interaction graphs using the hypothesized interactions (Fig. 5) as a reference point. This approach allowed us to interpret interactions between different learner characteristics and scaffolds *within* the respective subgroup, ensuring a comprehensive understanding of potential ATI effects.

Only the estimates relevant for our hypotheses are reported in the results section; the full model tables are given as supplemental material to the present article. To ease the interpretation of the complex interaction terms, estimated marginal means were calculated using the {emmeans}-package, version 1.8.8, (Lenth, 2023), and plotted using the {ggplot2}-package, version 3.5.0, (Wickham, 2016).

We used an alpha-level of 0.05 to investigate the six interaction hypotheses regarding the effectiveness of the different scaffolds with varying learner characteristics. The results of all other interactions are reported as explorative, tentative findings that might stimulate research and require further testing. Prioritizing the detection of potential interaction effects and minimizing the risk of missing them, for those explorative analyses, we also indicate results with a significance level of 0.10 (e.g., Peteranderl et al., 2023). However, we point out that these results involve a high level of uncertainty and need to be confirmed

Table 3
Reliability estimates.

	Reliability	
	α	ω
General reasoning	.89	–
Sustained attention	.88[a]	–
Visual-spatial ability	.45	.47
Self-concept	.86	.87
Interest	.84	.84

[a] For sustained attention, a split-half reliability was estimated (see Hofer et al., 2022 for more details).

through further studies with sufficiently large sample sizes.

4. Results

4.1. Descriptive results

Before addressing our research question, we had to consider the specifics of our student sample. We expected APS students to perform better on all learner characteristics than VS students. Table 4 shows that this was true for all learner characteristics except interest in mathematics. According to Welch t-tests, APS students outperformed VS students significantly in terms of prior domain knowledge, $d = 1.20$, general reasoning, $d = 1.19$, sustained attention, $d = 1.04$, and visual-spatial ability, $d = 1.00$, with large effect sizes, and in terms of self-concept, $d = 0.46$, with a medium-sized effect (Table 4). As for these results, we considered the two school tracks as subgroups within our sample in all analyses, indicating systematic differences in terms of the learning resources available to students.

Table 5 shows that the randomized distribution of students to the four conditions led to comparable groups in terms of the learner characteristics both in the APS and VS subgroup. Kruskal Wallis tests indicated that there was no significant difference in any of the learner characteristics in neither the APS nor the VS students (Table 5). We thus considered the experimental conditions comparable in both subgroups. Correlations of the learner characteristics for APS and VS students can be derived from Table 6. At a descriptive level, students from APS showed an estimated marginal mean deviation from the target fractions on the number line in the post-test knowledge measures of EMM = 0.078, 95 % CI [0.071, 0.086], while students from VS showed a higher estimated marginal mean deviation of EMM = 0.161, 95 % CI [0.146, 0.179].

4.2. Interactions between learner characteristics and scaffolds

All learner characteristics used in the following analyses are cluster mean centered with respect to the APS or VS subgroup (i.e., a value of 0 represents the average within the specific subgroup). The means and standard deviations used for cluster mean centering are presented in Table 4.

We expected specific interactions between learner characteristics and type of scaffold (compared to the control group) in the digital learning environment (Fig. 5). Except for individualized explanations and sustained attention, we hypothesized compensatory effects of the different scaffolds, expecting them to compensate for low manifestations on the specific learner characteristics considered to be necessary to profit from the respective learning activity. Given the significantly lower manifestations on most of the learner characteristics in the VS subgroup compared to the APS subgroup (Table 4), effects might be more pronounced in the former subgroup with more potential for compensation at the lower end. Relevant parameter estimates for the GLMMs can be found in Table 7 (all estimated parameters are available as supplemental material).

Our first two hypotheses posited that dynamic visualizations would be particularly beneficial for students showing low general reasoning and low visual-spatial ability (Fig. 5). This was true for VS students. While no notable interaction effect was found for APS students (Fig. 6), there was a positive effect of dynamic visualization for low levels of general reasoning and no effect for high levels of general reasoning in the VS subgroup (Fig. 7). Regarding visual-spatial ability, again the interaction was present for VS students: For higher visual-spatial ability, dynamic visualization was not as beneficial as for lower visual-spatial ability, but it was still not worse than the control group (Fig. 7). The respective complex interaction terms were significant for both general reasoning, $X^2(8,68) = 24.96$, $p < .01$, and visual-spatial ability, $X^2(8,68) = 20.15$, $p < .01$. Exploratively, we found that dynamic visualization was more beneficial for highly-interested VS students than for highly-interested APS students, $X^2(8,68) = 24.32$, $p < .01$, and that prior domain knowledge increased the positive effect of dynamic visualization for VS students, $X^2(8,68) = 23.08$, $p < .01$.

Next, we hypothesized that adaptive task difficulty would be beneficial especially for low-interested students and students with low self-concept (Fig. 5). This was (in tendency) true for self-concept in APS students (Fig. 6). Contrary to our hypothesis, a positive effect of adaptive task difficulty was present for VS students with high self-concept (Fig. 7). For interest, the hypothesized interaction could be found for VS students with a decline of the positive effect of adaptive task difficulty for highly-interested VS students (Fig. 7). The respective complex interaction terms were significant for both interest, $X^2(8,68) = 28.51$, $p < .001$, and self-concept, $X^2(8,68) = 18.03$, $p < .05$. Exploratively, we found that higher visual-spatial ability increased the effect of adaptive task difficulty for VS students, $X^2(8,68) = 19.35$, $p < .05$, and that adaptive task difficulty had a compensatory effect for VS students with lower prior knowledge compared to the control group, $X^2(8,68) = 13.40$, $p < .10$.

Our fifth and sixth hypotheses posited that individualized explanations would be particularly helpful for students with low prior knowledge, and that they would benefit students with high sustained attention while potentially hindering those with low sustained attention (Fig. 5). Figs. 6 and 7 showed that neither of these was true for our study. However, we exploratively found individualized explanations to be especially helpful for VS students with high interest, $X^2(8,68) = 16.29$, $p < .05$, high self-concept, $X^2(8,68) = 27.23$, $p < .001$, and high visual-spatial ability, $X^2(8,68) = 23.21$, $p < .01$ (Fig. 7).

To summarize, variation both in learner characteristics and in experimental conditions within the APS subgroup had little impact on knowledge about fractions on the number line (Fig. 6). The estimates for the VS students depicted in Fig. 7, however, indicate the potential of the different scaffolds to increase knowledge about fractions on the number line compared to the control condition and point to ATI effects within this subgroup.

5. Discussion

5.1. Summary of results

In this study, we propose an approach that provides insights needed to implement adaptive instruction by connecting the concept of scaffolding with well-established learning activities. The scaffolding is meant to enhance their effectiveness to offer cognitively stimulating

Table 4
Comparison of APS and VS students regarding the learner characteristics.

	APS students		VS students					
	M	*SD*	*M*	*SD*	*t*	*df*	*p*	*d*
Prior knowledge	0.12	0.12	0.28	0.15	10.27	232.89	0.00	1.20
General reasoning	5.63	4.27	1.76	1.66	−11.43	259.99	0.00	1.19
Sustained attention	0.29	0.49	−0.26	0.57	−9.18	268.17	0.00	1.04
Visual-spatial ability	2.06	1.19	0.97	0.98	−9.26	332.43	0.00	1.00
Interest	2.15	0.75	2.16	0.78	0.07	296.25	0.94	0.01
Self-concept	2.61	0.76	2.26	0.73	−4.21	311.15	0.00	0.46

Table 5
Comparison of the experimental conditions regarding the learner characteristics for APS und VS students.

	Control		Ind. Explanations		Ada. Difficulty		Dyn. Visualization					
APS students	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*KW*	*df*	*p*	η^2
Prior knowledge	0.13	0.14	0.12	0.13	0.11	0.11	0.11	0.12	0.53	3.00	0.91	−0.01
General reasoning	5.59	4.47	5.60	4.21	5.79	4.19	5.54	4.34	0.26	3.00	0.97	−0.01
Sustained attention	0.14	0.60	0.38	0.38	0.36	0.44	0.29	0.51	6.36	3.00	0.10	0.02
Visual-spatial ability	2.07	1.14	2.10	1.27	2.09	1.02	2.00	1.34	0.05	3.00	1.00	−0.02
Interest	2.21	0.65	2.16	0.78	2.05	0.71	2.19	0.83	1.52	3.00	0.68	−0.01
Self-concept	2.62	0.78	2.63	0.80	2.54	0.71	2.64	0.77	0.54	3.00	0.91	−0.01
VS students												
Prior knowledge	0.26	0.18	0.29	0.14	0.30	0.15	0.26	0.15	1.77	3.00	0.62	−0.01
General reasoning	2.00	1.90	1.59	1.21	1.82	1.78	1.65	1.77	0.82	3.00	0.84	−0.02
Sustained attention	−0.30	0.52	−0.19	0.59	−0.33	0.55	−0.21	0.61	1.39	3.00	0.71	−0.01
Visual-spatial ability	1.13	1.09	0.78	0.92	1.07	1.00	0.91	0.93	2.95	3.00	0.40	−0.00
Interest	2.39	0.86	2.10	0.60	1.95	0.75	2.26	0.88	5.57	3.00	0.14	0.02
Self-concept	2.41	0.60	2.12	0.71	2.11	0.74	2.46	0.80	7.56	3.00	0.06	0.03

Table 6
Correlations of the learner characteristics for APS und VS students.

	1.	2.	3.	4.	5.	6.
1. General reasoning	–	**0.15**	**−0.45**	**0.41**	**0.28**	**0.35**
2. Interest	**0.27**	–	−0.01	**0.46**	**0.15**	0.07
3. Prior knowledge	**−0.29**	**−0.21**	–	**−0.28**	−0.08	−0.07
4. Self-concept	**0.21**	**0.62**	**−0.26**	–	**0.22**	**0.27**
5. Sustained attention	0.10	0.12	−0.05	0.06	–	0.02
6. Visual-spatial ability	−0.03	0.13	−0.16	**0.17**	0.17	–

APS students = above diagonal; VS students = below diagonal. Significant correlations at level $p < .05$ are printed in **bold**.

instruction for a broader range of students. This involves considering how these learning activities work and incorporating insights from research on how different students engage with them. Accordingly, dynamic visualizations were used as a cognitive information-processing scaffold when learning new information with analogies. During repeated practice, adaptive task difficulty was implemented as a motivational scaffold and individualized explanations based on typical mistakes were offered as a cognitive information scaffold. In Table 8, we summarize the findings according to our hypotheses (see Fig. 5) and only interpret the result (see table note) that is not discussed in more detail in the subsequent sections.

Given the large discrepancy in prior learning resources between APS and VS students (Table 4), it is reasonable to assume that the intervention was more challenging for the latter, who may therefore have had greater potential to benefit from scaffolding – especially those with less available prior learning resources even within this group (see Reinhold, Hofer, Hoch, et al., 2020; Schwartz et al., 2016).

Table 7
Relevant parameter estimates from the generalized linear mixed models.

	Learner Characteristics (LC)											
	Interest		Self-Concept		General Reasoning		Vis.-Spa. Ability		Sustained Attention		Prior Knowledge	
Fixed effects	*Est.*	*SE*	*Est.*	*SE*	*Est.*	*SE*	*Est.*	*SE*	*Est.*	*SE*	*Est.*	*SE*
School Type[a]	0.448	0.040	0.468	0.042	0.452	0.038	0.473	0.043	0.469	0.044	0.470	0.043
x Dyn. Vis.	1.103	0.127	1.028	0.113	1.115	0.126	1.058	0.126	1.057	0.124	1.085	0.128
x Ada. Difficulty	1.109	0.126	1.051	0.114	1.047	0.116	1.011	0.118	1.032	0.119	1.010	0.114
x Ind. Explanations	1.067	0.120	1.066	0.115	1.075	0.118	1.032	0.120	0.965	0.112	1.023	0.116
LC[b]	0.841	0.048	0.924	0.070	0.786	0.044	0.975	0.062	0.799	0.059	0.842	0.056
x Dyn. Vis.	1.041	0.082	0.860	0.078	1.178	0.093	1.096	0.103	1.116	0.107	0.948	0.097
x Ada. Difficulty	0.994	0.081	0.906	0.085	1.130	0.086	0.937	0.086	1.146	0.109	1.040	0.097
x Ind. Explanations	0.834	0.080	0.819	0.078	1.061	0.103	0.918	0.082	1.095	0.104	1.009	0.099
LC[b] x School Type[a]	1.111	0.092	0.950	0.086	1.150	0.086	0.979	0.084	1.206	0.103	1.103	0.090
x Dyn. Vis.	0.984	0.108	1.198	0.138	0.853	0.091	0.917	0.110	0.863	0.100	1.033	0.127
x Ada. Difficulty	1.017	0.118	1.149	0.137	0.855	0.090	1.113	0.140	0.899	0.107	0.917	0.109
x Ind. Explanations	1.265	0.157	1.318	0.154	0.927	0.111	1.087	0.127	0.904	0.112	0.962	0.114
Random effects	*Var.*	*SD*	*Var.*	*SD*	*Var.*	*SD*	*Var.*	*SD*	*Var.*	*SD*	*Var.*	*SD*
Students ($k = 332$)[c]	0.093	0.305	0.083	0.287	0.090	0.300	0.103	0.322	0.095	0.308	0.089	0.298
Classrooms ($k = 16$)	0.004	0.063	0.007	0.084	0.003	0.057	0.005	0.069	0.006	0.079	0.006	0.077
Items ($k = 28$)	0.019	0.137	0.018	0.133	0.017	0.132	0.018	0.134	0.017	0.132	0.019	0.137
Model characteristics												
Observations	8611		8611		8611		8639		8423		8299	
R^2_{marginal}	0.301		0.304		0.294		0.279		0.288		0.295	
$R^2_{\text{conditional}}$	0.441		0.436		0.431		0.434		0.434		0.441	

[a] Baseline for factor School Type is VS (vocational school), i.e., effects represent a shift from VS to APS students.
[b] Respective learner characteristics are given in the column head. All further estimates can be found in the supplemental material.
[c] Per model, between 0 and 17 students had to be excluded from the analyses due to randomly missing data resulting from technical problems in data processing (see supplemental material for the exact number per model).

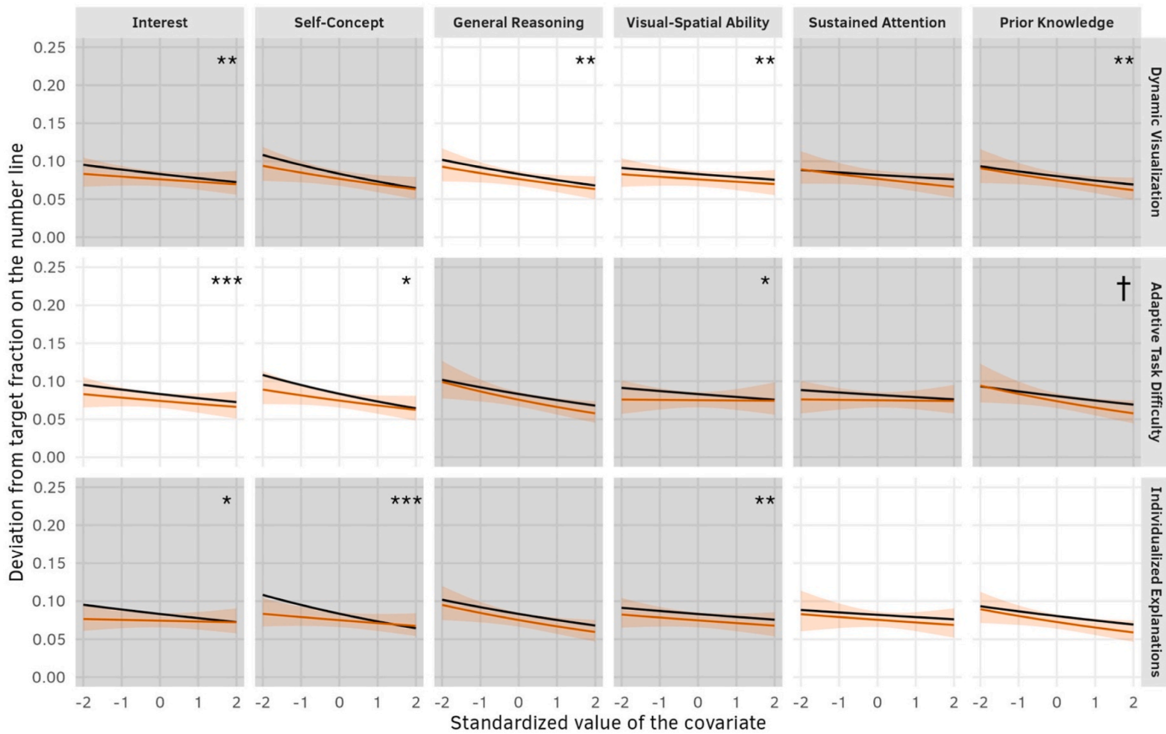

Fig. 6. Interactions between learner characteristics and scaffolds for APS students. Black = Control condition; Orange = Experimental condition with the specific scaffold indicated on the right; 95 % CI for the experimental condition is shown as orange ribbon. White background = Hypothesized interactions; Gray background = Explorative investigation. Levels of significance: $^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$, $^{\dagger}p < .10$. Values are estimated marginal means (EMMs) derived from the same GLMM reported in Table 7. The figure does not represent a separate model but a disaggregated visualization of the three-way interaction. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

5.2. Research implications: layered scaffolding

In the preceding section, we established that individuals are more likely to require scaffolding when confronted with challenging content. However, processing and utilizing scaffolds may also require certain individual prerequisites. Adaptive task difficulty, for instance, was only beneficial for VS students with a comparably high self-concept in mathematics. A positive self-concept might have helped them to work on tasks within one difficulty level until mastery without giving up. This is in line with studies showing associations between self-concept, self-assessment practice in learning situations, and academic performance (e.g., Guay et al., 2010; Xia et al., 2021; Yang et al., 2023). Likewise, in our exploratory analyses, prior knowledge seemed to help VS students process the dynamic visualizations, confirming research on the importance of existing knowledge for the integration of the new information provided by the dynamic visualization (e.g., Kalyuga, 2008) or for the targeted orientation of the learners' visual attention while working with the visualization (e.g., Hegarty & Kriz, 2008). In the explorative analyses, we also found catalytic effects of adaptive task difficulty and individualized explanations for VS students with higher visual-spatial ability. The number line task, in general, requires the learners to process visual-spatial information. Assuming that adaptive task difficulty motivates learners to practice with more engagement, this engagement could be better invested with higher spatial ability helping the learners process the number line task. In addition to the number line itself, the individualized explanations were in parts displayed visually, for instance, by indicating the correct segmentation if the denominator was ignored. Students with higher visual-spatial ability might have profited more from this kind of additional explanatory information. Finally, in the exploratory analyses, it was found that self-concept and interest in mathematics enhanced the effects of individualized explanations for VS students. This finding underscores the significance of adequate motivation in engaging with and learning from personalized feedback (see DePasque & Tricomi, 2015 from a neuropsychological perspective), an aspect that has not been investigated intensively in educational research so far (see Gan et al., 2021). Along a similar vein, Lam and colleagues (2011) demonstrated that affect influences the effectiveness with which learners process feedback, finding that individuals with higher levels of positive affect were not adversely impacted by either low or high feedback frequencies.

All those catalytic effects could also be used to inform the design of additional scaffolds: If, for instance, a certain level of motivation is required to process individualized explanations, they might be combined with a motivational scaffold, such as statements that stress the role of effort in learning (see Belland et al., 2013; Hamm et al., 2014). Likewise, a cognitive information-processing scaffold (e.g., signaling; Boucheix et al., 2013) could be added to dynamic visualizations to guide learners' attention. Graded assistance (Reinhold, Hofer, Hoch, et al., 2020) providing knowledge relevant for the processing of the visualization could be implemented as additional cognitive information scaffold, to give just a few examples. We refer to this instructional design process combining cognitive, motivational, and/or metacognitive scaffolds within a learning activity as layered scaffolding.

Numerous studies have explored the integration of additional support within a particular instructional design. However, due to variations in language and descriptions of the instructional design process across these studies, alignment proves challenging. Using the framework of (layered) scaffolding within learning activities might be one way to help synthesize findings across studies in the future.

On a more general level, we have to consider the actual utilization of the scaffolds by students (Reinhold et al., 2024). We didn't find the expected effects for individualized explanations. Other than

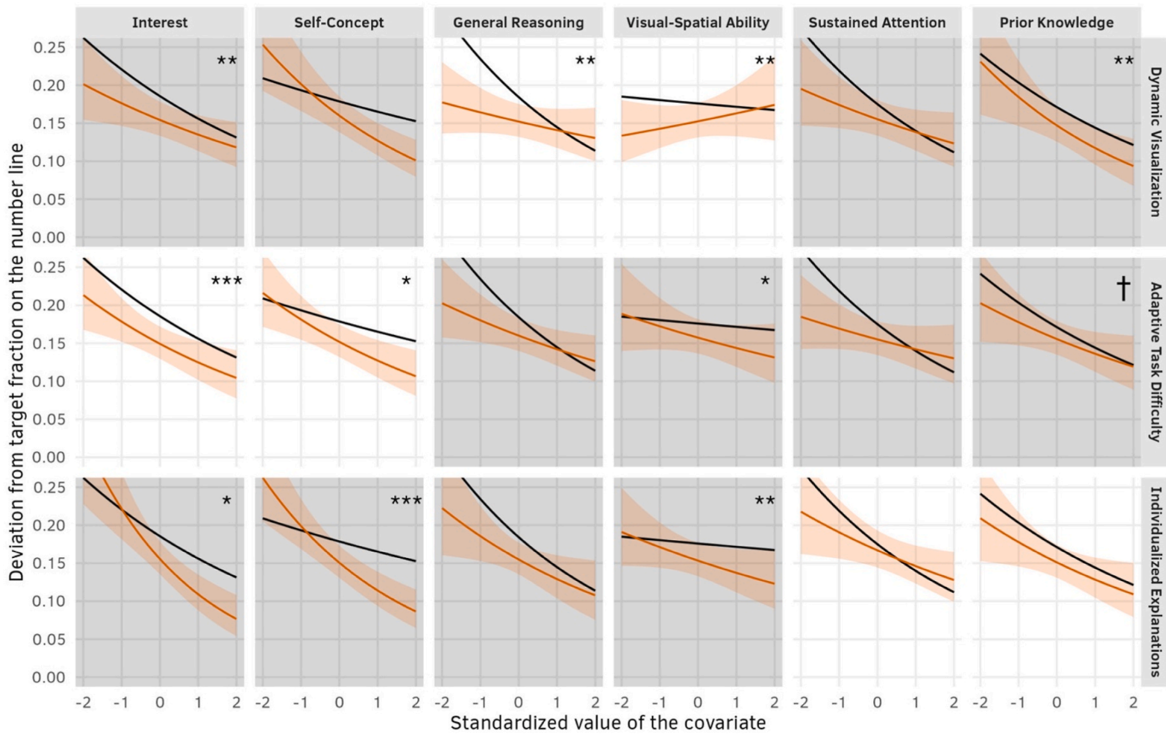

Fig. 7. Interactions between learner characteristics and scaffolds for VS students. Black = Control condition; Orange = Experimental condition with the specific scaffold indicated on the right; 95 % CI for the experimental condition is shown as orange ribbon. White background = Hypothesized interactions; Gray background = Explorative investigation. Levels of significance: $^{***}p < .001$, $^{**}p < .01$, $^{*}p < .05$, $^{\dagger}p < .10$. Values are estimated marginal means (EMMs) derived from the same GLMM reported in Table 7. The figure does not represent a separate model but a disaggregated visualization of the three-way interaction. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

hypothesized, this cognitive information scaffold did not compensate for lower prior knowledge and its effectiveness also didn't depend on sustained attention. However, lower prior knowledge does not imply that those students make use of the learning opportunity provided and the trait-measure of sustained attention does not imply that students actually attend to the individualized explanations in the learning situation. Self-report data on students' situational engagement as well as log data (e.g., time on task, time on feedback, reaction to feedback, number of tasks processed, number of tasks solved correctly, number of interactions with the visualization), documenting the actual interactions of students with each scaffold, can be used to categorize students depending on how they make use of each scaffold in follow-up studies. This information could then, in turn, be used to inform the design of layered scaffolding.

Table 8
Summary of findings for all hypotheses.

Hypothesis	Scaffold	Expected Effect for Students With …	Confirmed	Exploratory Findings
H1	Dynamic Visualization	Low General Reasoning	Yes (VS)	high interest → more beneficial
H2	Dynamic Visualization	Low Visual-Spatial Ability	Yes (VS)	(VS); high prior knowledge → more beneficial (VS)
H3	Adaptive Task Difficulty	Low Interest	Yes (VS)	high visual-spatial ability →
H4	Adaptive Task Difficulty	Low Self-Concept	No ↔ high self-concept → more beneficial (VS)	more beneficial (VS); low prior knowledge → more beneficial (VS)[a]
H5	Individualized Explanations	Low Prior Knowledge	No	high interest, self-concept, and
H6	Individualized Explanations	High Sustained Attention	No	visual-spatial ability → more beneficial (VS)

VS = vocational school; APS = advanced placement school. Exploratory findings were not hypothesized.

[a] Result might indicate that the less you know, the more you can learn within an adaptive practice situation, if you continue practicing.

5.3. Limitations

Several limitations should be acknowledged in interpreting the findings of this study. First, while our hypotheses are based on proposed underlying mechanisms, in the present study, no process data were analyzed to directly test these mechanisms. As mentioned above, process data from digital tools may serve as suitable indicators for student-tool-interactions that may operationalize how students engage with digital interventions (Reinhold et al., 2024). In the case of the present study, additional analyses of process data may shed light on (1) how scaffolds were utilized by individual students (e.g., usage of the dynamic visualization tools, processing of individualized explanations, pace in the adaptive task difficulty condition) and on (2) whether the scaffolds worked in the way we theoretically assumed.

Second, dynamic measurement approaches, including the consideration of trait-state distinctions, could offer a more nuanced understanding of the cognitive and motivational characteristics and processes and their interactions with scaffolds. Such a dynamic perspective acknowledges changes in learning prerequisites along the learning process (e.g., Chernikova et al., 2020; Engelmann et al., 2021; Tetzlaff et al., 2021).

Third, the assumption of linear aptitude-treatment interactions may oversimplify the complexity of the relationships observed in the data. Exploring non-linear interactions could provide a more accurate representation of how individual differences interact with instructional design. Person-centered analyses, including latent profile analyses, are also a promising approach for examining various configurations of learner characteristics and their interactions with instructional support.

Additionally, this study focused on testing mechanisms within specific learning activities using particular scaffolds. Future research could extend these investigations by testing the proposed mechanisms with alternative learning activities and scaffolds. For example, variation in pace or complexity of the presentation of the content embedded in different learning activities (also referred to as representational scaffolding, Fischer et al., 2022) could be considered in addition.

5.4. Practical implications

Students with less beneficial learning prerequisites appear to benefit from scaffolding in fraction instruction in secondary school mathematics. The scaffolds seem to enable them to make use of proven learning activities. Within this group of students, individual differences affect the effectiveness of scaffolds, underscoring the potential value of adaptive teaching, with or without digital tools, especially for these learners. The present findings can be used to help teachers adapt their regular classroom instruction. In particular, dynamic visualizations might be helpful for students with lower general reasoning and visual-spatial abilities to process analogies. Adapting task difficulty to students' performance seems to improve practice for students with lower interest. Since higher self-concept may increase this effect, teachers could try to strengthen students' competence beliefs and emphasize individual progress during practice, in line with the idea of layered scaffolding.

CRediT authorship contribution statement

Sarah I. Hofer: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Resources, Project administration, Methodology, Investigation, Funding acquisition, Data curation, Conceptualization. **Frank Reinhold:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Conceptualization.

Research data availability

Research data and materials are available upon request from the authors.

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Declarations of interest

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.learninstruc.2025.102177.

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