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Abstract

Risk and, thus, the volatility of financial asset prices plays a major role in financial decision making and financial regulation. Therefore, understanding and predicting the volatility of financial instruments, asset classes or financial markets in general is of utmost importance for individual and institutional investors as well as for central bankers and financial regulators.

In this paper we investigate new strategies for understanding and predicting financial risk. Specifically, we use componentwise, gradient boosting techniques to identify factors that drive financial–market risk and to assess the specific nature with which these factors affect future volatility. Componentwise boosting is a sequential learning method, which has the advantages that it can handle a large number of predictors and that it—in contrast to other machine-learning techniques—preserves interpretation.

Adopting an EGARCH framework and employing a wide range of potential risk drivers, we derive monthly volatility predictions for stock, bond, commodity, and foreign exchange markets. Comparisons with alternative benchmark models show that boosting techniques improve out–of–sample volatility forecasts, especially for medium– and long–run horizons. Another finding is that a number of risk drivers affect volatility in a nonlinear fashion.

Keywords: volatility, componentwise boosting, forecasting, GARCH, lag selection.

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1 Introduction

The relevance of understanding and adequately modeling the volatility of financial markets has – again – become evident in view of the recent market turbulences. The purpose of this paper is to use boosting techniques in order to identify forces driving market volatility, to assess the nature of their impact and to improve prediction. Our analysis is based on a monthly frequency and uses a wide range of potential predictors to model volatility.

In finance and macroeconomics, volatility forecasts are of exceptional importance: They are needed for calculation of common risk measures, such as the Value at Risk (VaR) and Expected Shortfall (ES), for the calculation of time-varying betas, conditional Sharpe ratios, and time-varying covariances for portfolio optimization and asset allocation and for the determination of the dynamic volatility in option valuation. A survey showing the importance of volatility beyond economics can be found in Andersen *et al.* (2006); and the survey of VaR forecasting strategies as given in Kuester *et al.* (2006).

The purpose of this paper is twofold: First, we identify the financial and macroeconomic factors which influence the volatility in different markets and assess their impact on volatility. Second, we forecast the volatility for different horizons and compare our method with a GARCH(1,1) model (Bollerslev, 1986), the benchmark in this field.

Whereas attempts to predict returns have a long tradition in the literature (e.g. Goyal and Welch, 2003, Welch and Goyal, 2008, Cochrane and Piazzesi, 2005, Lustig *et al.*, 2011), the prediction of volatility using macroeconomic and financial variables has received less attention.

Explicit modeling of volatility started with Engle (1982) and Bollerslev (1986) and has become an important and fruitful field in finance. Still, only few studies analyze the influence of financial and macroeconomic variables on the predictability of volatility. Examples are Schwert (1989), Engle *et al.* (2008), Paye (2012) and Christiansen *et al.* (2010). Schwert (1989) analyzes the relation of stock volatility and macroeconomic factors, like GDP fluctuations, economic activity and financial leverage, by employing autoregressive models. Engle *et al.* (2008) use inflation and industrial production by combining a daily GARCH process with a mixed data sampling (MIDAS) polynomial applied to monthly, quarterly, or bi-annual macroeconomic variables. Paye (2012) and especially Christiansen *et al.* (2010) extend the set of macroeconomic factors and asset classes. Both focus on the conventional linear model with log-transformed realized volatility being used as a *normalized* response. Lagged volatility, financial and macroeconomic factors are included as regressors. Christiansen *et al.* (2010) additionally use a Bayesian model averaging (BMA) approach but restrict the set of potential models again to the linear case. All these studies suggest that there is no general agreement on how to meaningfully predict volatility using financial and macroeconomic explanatory variables. The fact that different sample sizes, periods, models, forecasting evaluation criteria were employed in these analyses also contributes to the opaqueness.

Due to the unobservable nature of the variance we can either use the realized volatility as proxy and run conventional mean regression models, as done in the examples above, or estimate it latently.

The adequacy of the two strategies has been investigated in Claessen and Mittnik (2002) and Corsi *et al.* (2008). We follow the second strategy in this paper. Based on a broad set of macroeconomic and financial factors, we set up a flexible econometric model capable of directly modeling nonlinear influences of the factors on the volatility. Therefore, our approach is much in the spirit of GARCH modeling of the conditional variance (through the returns) rather than the linear modeling of the conditional mean (trough some transformation of the realized volatility).

Whereas the previous literature has mainly concentrated on stock market volatility (one exception is Christiansen *et al.*, 2010), we analyze four asset classes, namely stocks, bonds, commodities, and foreign exchange. We identify sets of driving forces, which, in part, affect volatility in a highly nonlinear fashion. This also extends the previous literature, which exclusively focused on linear influences of the predictors.

In our approach we use a version of the componentwise, gradient boosting (Bühlmann and Yu, 2003, Bühlmann and Hothorn, 2007), tailored to simultaneously select relevant factors and estimate the nature of their impact on the volatility. Boosting is especially suitable in this context since it addresses multicollinearity problems by shrinking effects towards zero and is, therefore, expected to improve out-of-sample predictions.

Volatility estimation with gradient boosting was first proposed by Audrino and Bühlmann (2003), who adopted a GARCH-type prediction model. They assume a stationary return process, $y_t = \sigma_t \varepsilon_t$ with $\varepsilon_t \sim N(0, 1)$, and an extremely general dependence structure between σ_t and past returns which lacks, however, any interpretability. Thus, their approach is suited only for prediction. A fairly similar model with neural networks as base-learners and the notable extension of a simultaneous conditional mean estimation was proposed by Matías *et al.* (2010). Bühlmann and McNeil (2002) developed an alternative nonparametric first-order GARCH solution. They propose another strategy for GARCH(1,1) modeling which allows interpretation.

The fundamental idea of volatility estimation by boosting, which is at the heart of our paper, was originally proposed by Audrino and Bühlmann (2009). Our model differs in several aspects, two of which are particularly important. First, we go beyond the GARCH(1,1) specification by allowing longer history and exogenous factors to enter the model. It turns out that inclusion of macroeconomic factors considerably improves the understanding of the volatility dynamics. Second, we employ componentwise predictor selection instead of the componentwise knot selection in a tensor spline estimation technique in Audrino and Bühlmann (2009). This leads to a genuinely different model, which has the positive side effect of no subjective decisions as the order of the penalized B-Splines that model the past returns and variance. More recently, Mayr *et al.* (2012) developed a boosting technique in the spirit of the Generalized Additive Models for Location Scale and Shape (GAMLSS, Rigby and Stasinopoulos, 2005) which extends the estimation even beyond the first two moments.

The paper is structured as follows. In Section 2, the boosting algorithm for estimating the variance is explained in some detail. We additionally present a short illustration. Section 3 contains a description of the data. In Section 4, we model the comovement between the regressors and the volatility and conduct an extensive forecasting study. Section 5 concludes.

2 Econometric Model

2.1 Gradient Boosting

We model along the lines of Nelson (1991) and adopt the exponential ARCH framework, allowing, however, for flexible relationships between volatility and a large number of potential risk drivers (plus seasonal components), so that the number of components may even exceed the number of the available observations.

The proposed model is of the form

$$\begin{aligned} y_t &= \exp(\eta_t/2)\varepsilon_t \\ \eta_t &= \beta_0 + f_{\text{time}}(t) + f_{\text{year}}(n_t) + f_{\text{month}}(m_t) + \sum_{j=1}^{s} f_j(y_{t-j}) + \sum_{k=1}^{q}\sum_{j=1}^{p} f_{k,j}(x_{k,t-j}) \qquad (2.1) \\ &= \eta(\mathbf{z}_{t-1}), \end{aligned}$$

where returns $y_t = \log(\mathrm{P}_t/\mathrm{P}_{t-1})$ are derived from the observed prices $\mathrm{P}_1, \dots, \mathrm{P}_T$, $\varepsilon_t \sim N(0,1)$, $\mathbf{z}_{t-1} = (1, t, n_t, m_t, y_{t-1}, \dots, y_{t-s}, x_{1,t-1}, \dots, x_{1,t-p}, \dots, x_{q,t-1}, \dots, x_{q,t-p})^\top \in \mathbb{R}^r$ is the realization of the r-dimensional random variable Z, $r = s + qp + 4$, with none of the functions, $f_{\cdot}(\cdot)$, being specified in advance. The function $f_{\text{month}}(m_t)$, $m_t \in \{1, 2, \dots, 12\}$, is intended to capture deterministic seasonal patterns in volatility, $f_{\text{year}}(n_t)$, n_t describes the typical annual fluctuations which occur throughout the sample period, and $f_{\text{time}}(t)$, $t \in \{1, \dots, T\}$, models the volatility trend. The functions $f_j(y_{t-j}), j = 1, \dots, s$, capture the influence of past returns, and $f_{k,j}(x_{k,t-j}), j = 1, \dots, q$ are functions of additional, exogenous, lagged factors. The structure of all $f_{\cdot}(.)$ is very general and can be chosen to flexibly suit the circumstances: in our case, regression trees fit the abrupt volatility changes well. Regression trees are a nonparametric approach which essentially accomplishes two tasks: a) to recursively partition the predictor domain into groups with similar response values, and b) to assign a constant value for the response within each group. A good explanation of the underlying algorithms behind regression trees can be found in Breiman *et al.* (1984). We use conditional inference trees (Hothorn *et al.*, 2006) because they guarantee unbiased variable selection. Therefore, the model can be interpreted as a regime volatility model which partitions the predictor space according to the levels of the conditional variance. Linear estimations, non-parametric smooth estimations of $f_{\cdot}(.)$ or combinations of both extend the scope of the model considerably and are easily accessible in the R add–on package *mboost* (R Development Core Team, 2012, Hothorn *et al.*, 2011).

The estimation of η is done non-parametrically via componentwise, gradient boosting. Boosting is a general method to estimate a single complex model, step by step, in a series of models that are to advantage combined into one. The model should not immediately produce an overfitted estimate in the first iteration. Therefore, we control the bias-variance mean squared error (MSE) trade-off by using a low-variance high-bias model. Subsequent iterations reduce this bias while the variance increases at a slower rate (Bühlmann and Yu, 2003).

Boosting, in its original form as proposed by Freund and Schapire (1996), was intended to solve two–class classification problems by maximizing the confidence, or the "margins", of a binary classificator. It suffices that the classificator, called *base-learner* is performing only slightly better than random guessing in order to form arbitrary good accuracy (Kearns and Valiant, 1994, Schapire *et al.*, 1998).

Later, boosting was placed into a regression framework by Friedman (2001) who explained it as a functional gradient descent (FGD) technique. This interpretation of boosting is also shared by Breiman (1998, 1999), Friedman *et al.* (2000) and Mason *et al.* (2000) who view it as a function optimization approach strikingly similar to the well known steepest-descent optimization. In our case, boosting estimates η by minimizing the expectation of a loss function L, such that

$$\eta^* = \arg\min_{\eta} \mathbb{E} L(y_t, \eta(\mathbf{z}_{t-1})) \tag{2.2}$$

and $\exp(\eta^*(\mathbf{z}_{t-1})) = \mathbb{V}(y_t|Z = \mathbf{z}_{t-1})$. Since we use the derivative of L (see (2.5) below), L is assumed to be differentiable with respect to η. In practice, our goal is to seek for a solution of (2.2) not in the function space, but in the space spanned by the data. This requires the parametrization of η of the form

$$\eta^* = \eta(\mathbf{z}; \hat{\boldsymbol{\beta}}) = \arg\min_{\boldsymbol{\beta}} \frac{1}{T} \sum_{t=1}^{T} L(y_t, \eta(\mathbf{z}_t; \boldsymbol{\beta})). \tag{2.3}$$

The solution of (2.3) is found by successively reducing the empirical loss. Boosting iteratively builds up the solution in small steps, where each step is based on the previous ones. The final parameter estimates can be expressed as an additive sum of the former estimates. Hence, boosting is a sequential learning method that preserves interpretation—a property which does not apply to other parallel–learning techniques, such as bagging or random forests.

A careful specification of the loss function, L, leads to an estimation of the desired characteristic of the conditional distribution, here, the conditional variance. In the application below, we assume $y_t|\mathbf{z}_{t-1} \sim N(0, e^{\eta_t})$, so that the negative conditional log-likelihood function is the empirical loss function given by

$$L_t = \frac{1}{2}\left[\eta_t + \frac{y_t^2}{e^{\eta_t}}\right] \tag{2.4}$$

(after some simplifications) with the corresponding negative gradient given by

$$g_t = -\frac{\partial L_t}{\partial \eta_t} = \frac{1}{2}\left[-1 + \frac{y_t^2}{e^{\eta_t}}\right]. \tag{2.5}$$

Boosting favors the direction that reduces the empirical loss most, i.e., the direction specified by the negative gradient. This means that we seek the solution in the data space by fitting the covariates against the negative gradient. Instead of fitting all covariates simultaneously, they are fitted separately against the gradient through base-learners. This is typically, but not necessarily, a well-known statistical model, such as linear regression, GAM or regression–tree which specify the connection between the response and the covariates. At each boosting step only one covariate is included, namely the one which most correlates with the negative gradient, i.e., the steepest direction to the loss minimum.

We fit the separate covariates against the negative gradient via regression–trees with two nodes, i.e., stumps. Stumps being naturally inflexible cannot fit the whole signal, coming even from a single covariate, at once. This is especially true when the fitted coefficients are additionally shrunken against zero as proposed by Friedman (2001). The "right" amount of shrinkage is justified empirically and can be safely varied between 1% and 10%, essentially altering only the computational time. Fitting the base learner changes the next evaluation of the gradient, and repeating this procedure makes the covariates and the gradient more and more orthogonal.

Note, however, that the proposed method is extensible with any modelling procedure. The choice of stumps as base-learners is not mandatory but, due to the abrupt changes that are typical for the application below, was found to be especially advantageous over the smooth P-Spline or the simple linear relationship between the covariates and the response.

Boosting forever with stumps would inevitably lead to a zero-bias model, not very useful for prediction. This is why early stopping is crucial. Terminating the process on time means that we exclude the covariates not selected by that time. An optimal number of boosting steps is determined by bootstrapping which samples from the data points as if they were originating from a multinomial distribution with probabilities $1/T$. Thus, each sample uses roughly 64% percent of the original data (with replacement) for training and the remaining, unselected, data points are used for evaluation. We repeat this twenty five times for a large number of boosting steps and choose the step number that produces the lowest out-of-sample loss on average. In summary, the boosting algorithm reads as follows:

1. Initialize the function estimate $\hat{\eta}_t^{[0]} = \log\left(\frac{1}{T-1}\sum_{t=1}^{T}(y_t - \bar{y})^2\right), \bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t,\ t = 1, \ldots, T$.

2. Specify the set of base-learners via regression trees: $f_.(z_t) = \sum_{j=1}^{J} \gamma_j I_{R_j}(z_t), \forall z_t \in \mathbf{z}_t$. We use stumps so each tree has only two leaves, i.e., $J = 2$. Denote the number of base-learners by r and set $m = 0$.

3. Increase m by one.

4. (a) Compute the negative gradient (2.5) and evaluate $\hat{\eta}^{[m-1]}(z_t), i = 1, \ldots, T$.
   (b) Estimate the negative gradient by using the stumps specified in Step 2. This yields r vectors where each vector is an estimate of the gradient.
   (c) Select the base-learner $\hat{f}^{[m]}$ that most correlates with the gradient according to the residual-sum-of-squares criterion. Therefore, $\hat{f}^{[m]}$ is the selected estimate of the gradient vector.
   (d) Update the current estimate by setting $\hat{\eta}^{[m]} = \hat{\eta}^{[m-1]} + \nu\hat{f}^{[m]}$, where ν is regarded as a *shrinkage* parameter or as a step size.

5. Iterate Steps 3 and 4 until the final step determined by the stopping condition.

2.2 An Illustration

Even though we restrain the simulation from being too exhaustive, it is worth illustrating volatility boosting with a small artificial example. Using the following model

$$\begin{aligned} y_t &= \exp(\eta_t/2)\varepsilon_t \\ \eta_t &= 0.2 + 0.5 \cdot x_{1,t} - 0.4 \cdot x_{2,t} + 1.2 \cdot I_{[1,2]}(x_{3,t}) + \\ &\quad 0 \cdot x_{4,t} + 0 \cdot x_{5,t} + 0 \cdot x_{6,t} \end{aligned} \tag{2.6}$$

with $\varepsilon_t \sim N(0,1)$ and $x_{i,t}$ being the t-th observation of $X_i \sim U[0,4]$, $i = 1, \ldots, 6$, and $t = 1, \ldots, T$ with $T = 800$. Note that only the first three covariates contribute to the volatility — the first two linearly, the third via a jump. The last three covariates are irrelevant. We choose linear base-learners for all but the third predictor which is fitted with a regression-tree base-learner. The model for boosting is then given by

$$\begin{aligned} y_t &= \exp(\eta_t/2)\varepsilon_t \\ \eta_t &= \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \sum_{j=1}^{J} \gamma_j I_{R_j}(x_{3,t}) + \beta_4 x_{4,t} + \beta_5 x_{5,t} + \beta_6 x_{6,t}. \end{aligned} \tag{2.7}$$

The aim is to estimate the β's and γ's as close as possible to the true values. This means that X_4, X_5 and X_6 are preferably not selected by the boosting algorithm, requiring the

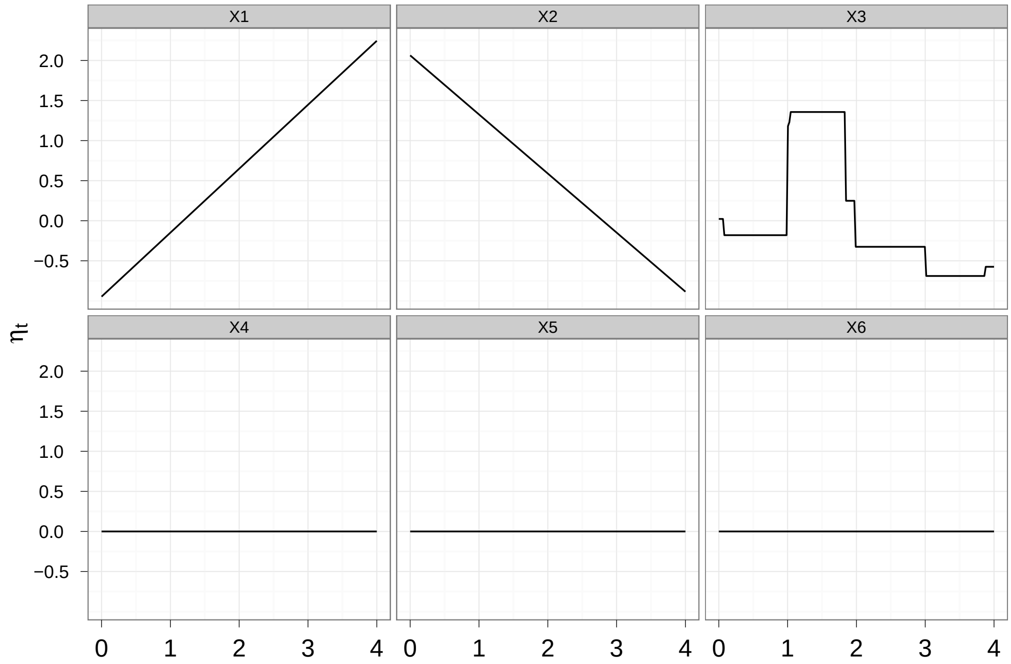


Figure 1: Partial volatility estimation of model (2.6). The volatility is measured by η_t (i.e., the log scale), depending on each covariate separately. The non-parametric estimation of the partial volatility in X_3 is zero-mean centered.

values for β_4, β_5 and β_6 to be exactly zero. Further, we care for partitioning the domain of X_3 at the right places, i.e., identifying the region $[1, 2]$, in order to discover the true dynamics.

Figure 1 shows the estimated partial volatility on the log scale. Without any fine tuning or expert calibration, the underlying volatility dynamics are modeled well. Early stopping of boosting ensures that the redundant predictors are never selected: $\beta_4 = \beta_5 = \beta_6 = 0$. With regard to the estimated parameters, $\hat{\beta}_1 = 0.46$, $\hat{\beta}_2 = -0.39$ and X_3 shows the largest jump in $[1, 2]$ in its domain. The result in Figure 1 is typical in the sense that the deviations of several hundred repetitions were negligibly small.

If we translate the log scale from Figure 1 into the standard deviation we get an estimation of the whole conditional density. Figure 2 shows the theoretical partial densities for the first three covariates with the central 95% interquantile range depicted in darker color. Figure 3 shows the empirical conditional density for simulated observations. It is evident that the changing volatility is captured very well. This is confirmed by a 95.12% coverage rate. The contribution of each covariate is readily observable and interpretable:

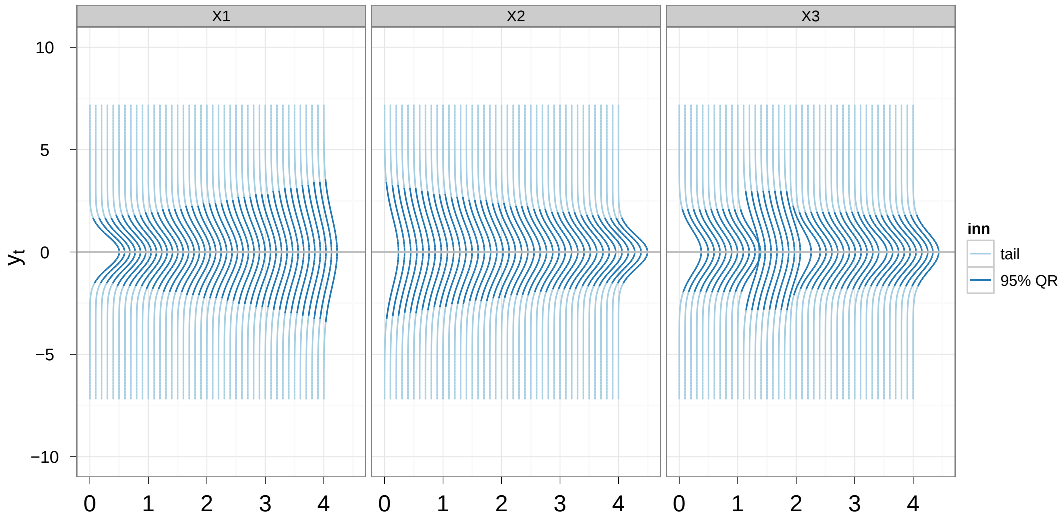


Figure 2: Partial conditional density estimations for the first three covariates. The dark lines show the estimated 95% interquantile range, the lighter colored lines represent the tails.

an increase in X_1 causes larger variance; X_2 negatively correlates with the variance; the variance markedly grows for $X_3 \in [1, 2]$; all other components do not affect the variance and the conditional density remains indifferent when linked to X_4, X_5 or X_6. Clearly, such detailed insight into the volatility dynamics greatly improves our understanding about factors driving the volatility in financial markets and should give rise to better forecasts. This intuition is thoroughly investigated in the subsequent sections.

3 Data Set

We analyze the predictability of volatility in four different markets, namely, stocks, bonds, commodities, and foreign exchange. The equity market is represented by the S&P500 futures contract traded on the Chicago Mercantile Exchange. For the bond market, we use 10-year Treasury note futures contracts traded on the Chicago Board of Trade (CBOT). The commodity market is represented by Standard & Poor's GSCI commodity index. As a proxy for the foreign exchange market we use a trade-weighted portfolio provided by the Federal Reserve Bank of St. Louis. It is a weighted average of the foreign exchange value of the U.S. dollar against a subset of the broad index currencies that circulate widely outside the country of issue, including the Euro Area, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden. Our data set covers 332 months and spans the period

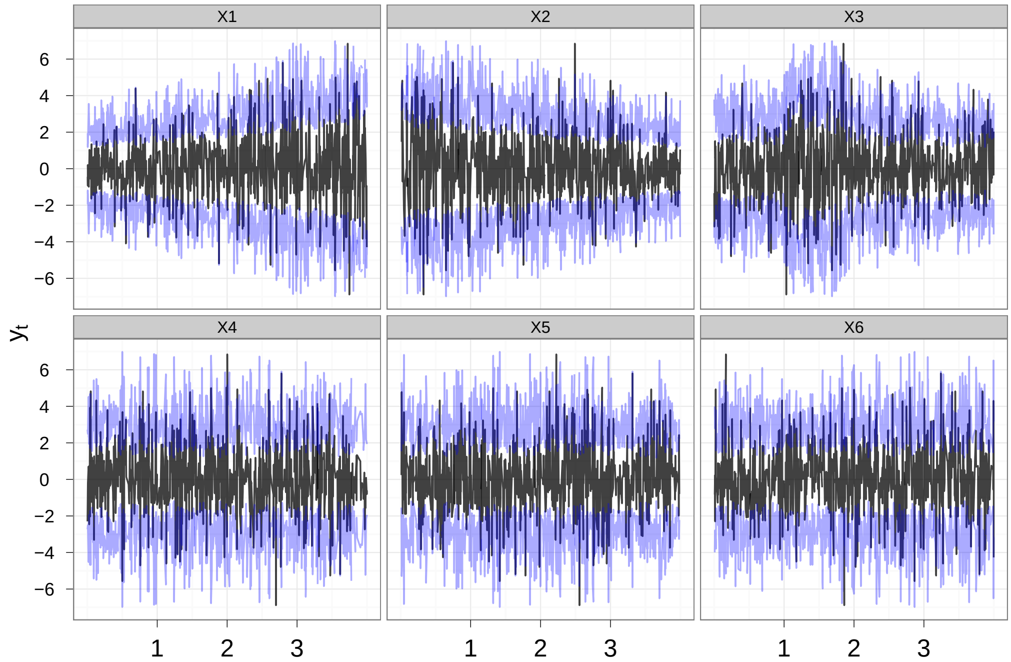


Figure 3: The observations y_t are depicted in black. The blue lines represent the 95%-interquantile range of the conditional density.

February 1983 to September 2010.

3.1 Measures for Volatility

Volatility is inherently unobserved, or latent, and its measurement is a challenge. Since the work of Andersen *et al.* (2003), realized volatility has become an accepted proxy for the true, but latent, integrated volatility. In this paper, monthly realized volatility is calculated by lower frequent, i.e., daily, returns. The realized volatility $RV_{i,t}$ for asset class i in month t is given by

$$RV_{i,t} = \log \sum_{\tau=1}^{M_t} r_{i,t,\tau}^2, \quad t = 1, \ldots, T \tag{3.1}$$

where $r_{i,t,\tau}$ denotes the τth daily return of asset i in month t, and M_t the number of trading days in month t. The realized volatility for all markets in our study is depicted in Figure 4. We regard the realized volatility as the unobservable, "true" volatility. A justification and review of this concept is given by Andersen *et al.* (2006).

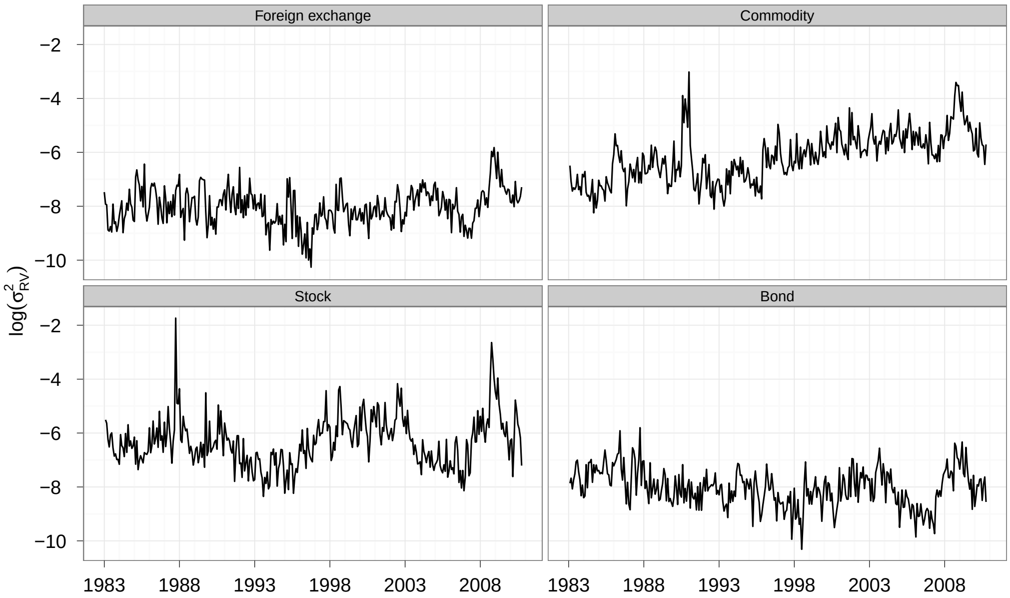


Figure 4: The logarithm of the monthly realized volatility as defined in Equation (3.1).

3.2 Financial and Macroeconomic Factors

We use an exhaustive set of macroeconomic and financial factors as potential explanatory variables. Our set of predictors is compiled from different sources. We include (transformations of) the explanatory variables used by Welch and Goyal (2008) for predicting stock market returns. These variables are included in Table 1 and involve the following factors: dividend price ratio, book to market ratio, net equity expansion, cross sectional premium, term spread, relative T-bill rate, relative Bond rate, long term bond return, T-bill rate, default spread. In addition, we include the Fama French factors: U.S. market excess return, size factor and value factor.

The set of predictors contains also the Pastor and Stambaugh (2003) liquidity factor, return on the MSCI world index, TED spread, i.e., the difference between 3 month LIBOR rate and T-Bill rate, the Cochrane and Piazzesi (2005) bond factor, the return on the CRB spot index, carry trade factor and return on dollar risk factor introduced by Lustig *et al.* (2011), FX average bid-ask spread (Menkhoff *et al.* (2011)).

Moreover, we include the following macroeconomic variables: M1 growth, investor sentiment, purchasing manager index, housing starts, inflation, industrial production growth and orders. Finally, we take up the Financial Stability Index (FSI) for advanced economies, which was developed by the International Monetary Fund. A detailed overview of the in-

Table 1: Description of the predictor variables

Variable	Abbrev.	Source	Description
US Market Execess Return	MKTRF	Homepage FF	Fama-French's market factor: U.S. stock return minus one-month T-bill rate
Size Factor	SMB	Homepage FF	Fama-French's SMB factor: return on value small minus return on big stocks
Value Factor	HML	Homepage FF	Fama-French's HML factor: return on value stocks minus return on growth stocks
US IP growth	IPGR (IPGR)	Datastream	Year-over year (log) growth rate of U.S. industrial production
New orders consumer growth	ORDc (ORD)	Datastream	New orders of consumer goods and materials; year-over year (log) growth rate
US M1 change	M1c (M1)	Datastream	Year-over-year (log) growth rate of U.S. M1
Consumer Sentiment change	SENTc (SENT)	Datastream	Monthly change in consumer sentiment
Purchasing Manager change	PMIc (PMI)	Datastream	Monthly change in purchasing manager index
Housing Starts	HSc (HS)	Datastream	Monthly change in housing started
TED spread	TED	Datastream	Measure of illiquidity: LIBOR minus T-Bill rate
MSCI World	MSCIc (MSCI)	Datastream	Return on the MSCI world stock market index.
CRB Spot Index annual ret	CRBc (CRB)	Datastream	Measure of growth in commodity prices: annual log difference of CRB spot index
Book to market Ratio	b/m (BM)	Goyal-Welch	Ratio of book value to market value for the Dow Jones Industrial Average.
Net equity expansion	ntis (NTIS)	Goyal-Welch	Ratio of 12-month moving sums of net issues by NYSE listed stocks divided by total NYSE market capitalization (end-of-year)
Inflation	infl	Goyal-Welch	Year-over year (log) growth rate of the U.S. consumer price index
Long term rate of return	ltr (LTR)	Goyal-Welch	Rate of return on long term government bonds
Default Spread	DEF	Goyal-Welch	Measure of default risk of corporate bonds: BAA bond yields minus AAA bond yields
Rel. T-Bill rate	RTB	Goyal-Welch	T-Bill rate minus its 12 month moving average
Rel. Bond rate	RBR	Goyal-Welch	Long-term bond yield minus its 12 month moving average
Term spread	TS	Goyal-Welch	Long-term bond yield minus three-month T-Bill rate
Pastor-Stambaugh Liq. Fac.	liq_fac (LIQ)	Pastor Stambaugh (2003)	Measure of stock market liquidity based on price reversals. Pastor and Stambaugh (2003)
FX Average Bid-Ask spread	BAS	Menkhoff *et al.* (2011)	Measure of illiquidity in the foreign exchange market calculated from bid-ask spreads
Return on Dollar Risk Factor	RX (DOL)	Lustig *et al.* (2011)	FX risk premium measure: average premium for bearing FX risk
Carry Trade Factor	HML (CT)	Lustig *et al.* (2011)	Return on high interest rate currencies minus return on low interest currencies
Cochrance Piazessi Factor	CP		Measure of bond risk premia; recursively estimated based on Fama-Bliss file according to Cochrane and Piazzesi (2005)
Financial Stress Index	FSI	IMF	The FSI for advanced economies comprises seven variables which serve to capture three financial market segments.

cluded macroeconomic and financial factors can be found in Table 1.

3.3 Forecasting Strategy

Our forecasted period covers June 2002 to September 2010. We use a rolling window scheme for forecasting. Starting with a time window of size 230, we shift it onwards in order to produce 100 one-period ahead forecasts.[1] Therefore, each volatility forecast is carried out by a "new model," based only on the past 230 observations. Further, applying a direct forecasting approach[2] we produce multi-period forecasts for one through six months.

For all 26 factors we include the first and second lag in our model, i.e., $q = 26$ and $p = 2$ in (2.7). Our regressors are augmented by the first and second lag of the realized volatility ($s = 2$) to capture state dependence and autoregressive behavior. We consider all factors with their lags simultaneously in addition to the seasonal components. This results in $r = 58$ predictors in total.

Since the true volatility is latent, it is common to utilize the squared returns y_t^2 as a noisy estimator for the squared volatility σ_t^2. We, however, avoid this due to the extremely low signal to noise ratio. Therefore, we evaluate the forecasting performance in terms of the mean squared error between the "true" (realized) volatility, as defined in (3.1), and our forecasts. The t-th squared error in the i-th market is defined as

$$\mathrm{ERR}_t = (RV_{i,t} - \eta_t)^2, \quad t = 1, \dots, T. \tag{3.2}$$

We conduct direct forecasting which slightly alters the model given in (2.7) such that

$$\begin{aligned} y_{t+h} &= \exp(\eta_t/2)\varepsilon_{t+h} \\ \eta_t &= \beta_0 + f_{\text{time}}(t) + f_{\text{year}}(n_t) + f_{\text{month}}(m_t) + \sum_{j=1}^{s} f_j(y_{t-j}) + \sum_{k=1}^{q}\sum_{j=1}^{p} f_{k,j}(x_{k,t-j}) \qquad (3.3) \\ &= \eta(\mathbf{z}_{t-1}), \end{aligned}$$

for $h = 0, \dots, 5$. Note that h refers to a $h+1$ period–ahead forecast.

4 Empirical Results

In this section we present the results for all markets in detail. Specifically we report which factors influence the realized volatility. One important finding is that the influence is highly nonlinear. This is confirmed by a forecasting exercise similar to the one proposed here with

[1] Two observations are "lost" due to the lag operator.

[2] Direct forecasting in nonlinear time series context via boosting can be found in Robinzonov *et al.* (2012).

Table 2: Theil's U

Horizon	FX_major	GSCI	SP	TNOTE
1	1.032	1.072	0.998	1.047
2	1.011	0.866	0.988	1.017
3	0.918	0.923	0.893	1.000
4	0.931	0.901	0.852	0.981
5	0.968	0.790	0.870	0.943
6	0.959	0.714	0.835	0.885

Table 3: Out-of-sample R^2

FX_major	GSCI	SP	TNOTE
-0.066	-0.150	0.004	-0.096
-0.023	0.250	0.024	-0.034
0.158	0.149	0.202	0.000
0.133	0.189	0.274	0.038
0.063	0.376	0.243	0.111
0.080	0.491	0.303	0.218

the sole difference in the base-learners being linear models (instead of regression trees). The resulting forecasts (not shown here) were of notably lower accuracy. It appears that this phenomenon has not been analyzed in the literature before and offers valuables insights into the nature of the impact of the risk drivers.

In addition to detailing the "anatomy of volatility," we compare the out-of-sample forecasts of our boosting procedure with the forecasts of a GARCH specification for different horizons, an application which is of great importance in practice.

4.1 Forecast Evaluation

In order to evaluate the out-of-sample forecasts we report descriptive statistics like Theil's U, the out-of-sample R^2 as in Campbell and Thompson (2008). In the subsequent sections we provide a more in-depth view of each market's volatility drivers and a graphical representation of the forecasting study.

In the following, $\eta^M_{i,t+1}$ denotes the forecasts of our model, $\eta^B_{i,t+1}$ the forecasts of the benchmark model. Theil's U is defined as the quotient of the root mean squared error (RMSE) of our model and the RMSE of the benchmark model. A value smaller than one indicates that our model outperforms the benchmark model in terms of forecasting accuracy. The results are given in Table 2. For the commodity and stock markets nearly all values are smaller than 1 and indicate advantageous prediction for our model. For the bond and foreign exchange markets this is the case for horizons longer than two months. As confirmed by the tests later, even the first two months can be regarded as indistinguishable in terms of forecasting accuracy. Thus, our boosting strategy is at least as good as GARCH in the short run and typically better in medium– and long–term horizons.

Another descriptive forecast evaluation criterion is the out-of-sample R^2 as proposed by Campbell and Thompson (2008). The out-of-sample R^2 is calculated as

$$R^2_{OOS} = 1 - \frac{\sum_{t=R}^{T-1} \left(RV_{i,t+1} - \eta^M_{i,t+1} \right)^2}{\sum_{t=R}^{T-1} \left(RV_{i,t+1} - \eta^B_{i,t+1} \right)^2} \tag{4.1}$$

Table 4: Modified Diebold-Mariano test results

Horizon	FX_major	GSCI	SP	TNOTE
1	0.630	0.790	0.4901	0.8585
2	0.555	0.011 **	0.4407	0.6330
3	0.215	0.114	0.1039	0.5011
4	0.288	0.071 *	0.0519 *	0.3568
5	0.370	0.016 **	0.0496 **	0.1542
6	0.355	<0.01 ***	0.0112 **	0.0297 **

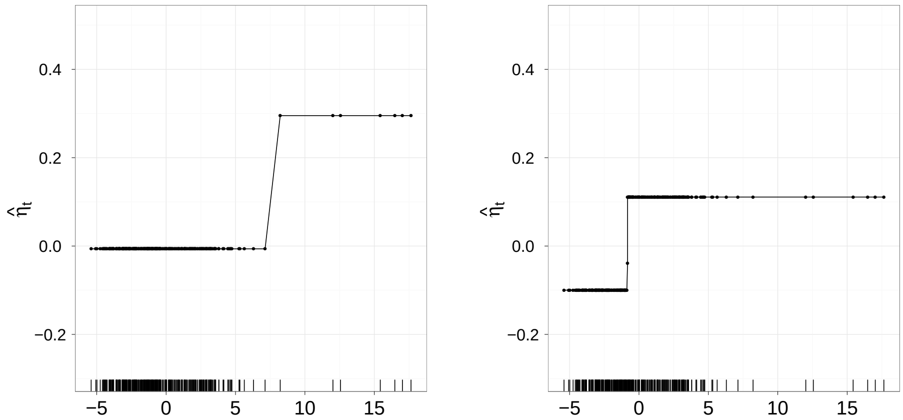
where T denotes the total sample size and R the initialization period. We show the results for this criterion in Table 3 and the interpretation remains essentially the same as for Theil's U statistic.

Our model and the benchmark model are non-nested which allows us to apply the test of Diebold and Mariano (1995) resp. the modified version of Harvey *et al.* (1997) in order to compare forecasting accuracies. The null hypothesis is that the GARCH forecasting error is smaller than the boosting forecasting error. Therefore, rejection of the null hypothesis favours our method. In Table 4 we provide the p-values of the modified Diebold-Mariano test for different forecasting horizons. The additional information of the exogenous factors, as well as the regime estimation obtained by boosting clearly improves the forecasting accuracy in the commodity market for horizons longer than one month. This is also evident in the stock market for medium– and long–term forecasting. We would also like to mention that all cases, especially the insignificant ones, remain insignificant when we invert the null hypothesis, namely that boosting performs better than GARCH. This confirms our finding that boosting forecasts at least as good as GARCH in short–terms and better in medium– and long–terms.

4.2 Stock Market

4.2.1 Driving Factors

The stock market is represented by the S&P500. When modeling the influence of the explanatory variables via regression trees, we identify the financial stability index (FSI), the lagged realized volatility, relative bond rate (RBR), the lagged returns, the U.S. market excess return and the CRB spot index as main drivers. Due to the built-in lag and factor selection in our approach, all other factors were excluded. Furthermore, not all lags are considered as influential. For example, the lagged returns, U.S. market excess return, CRB spot index enter through their first lag in the model, whereas FSI (Figure 5) and realized volatility (Figure 6) have a greater, long-lived impact and enter with the additional second lag.



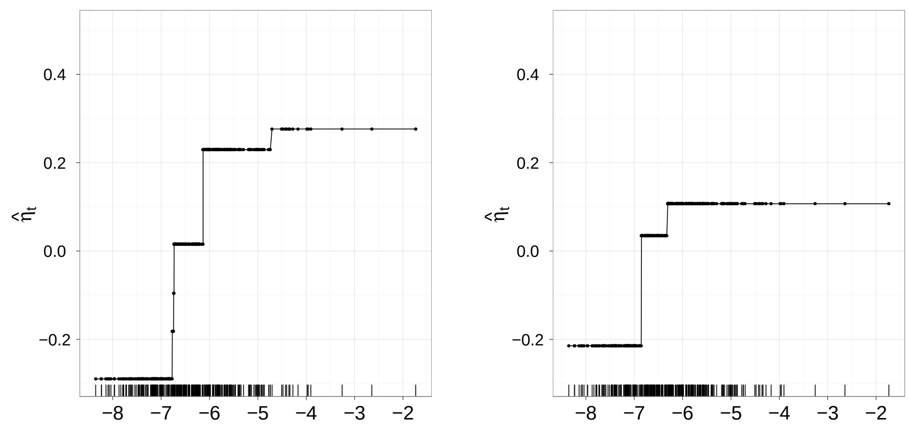
(a) Fitted function for FSI (first lag) (b) Fitted function for FSI (second lag)

Figure 5: Financial stability index (FSI)

The Financial Stress Index for advanced economies was developed by the IMF and comprises seven variables capturing three financial market segments, i.e., banking, securities markets and exchange markets. The motivation for and detailed composition of the FSI is given in Cardarelli *et al.* (2009). Figure 5 clearly shows several regimes for the FSI index when related to volatility. Values of the FSI above 7.5 lead to a considerable volatility increase in the following month — roughly 0.3 on the log scale as shown in Figure 5(a) which corresponds to 16% increase in the volatility. Furthermore, the considerable influence of the FSI is confirmed by a long lived impact of its movements. Positive changes in FSI moderately increase the volatility even two months from now, while negative changes reduce it.

Past realized volatility is also found to be very influential. Small values of the realized volatility, i.e., $\log(\hat{\sigma}^2_{\mathrm{RV}}) < -7$ shown in Figure 6(a) or $\hat{\sigma}_{\mathrm{RV}} < 0.03$, cause a decrease in the volatility next month. From approximately $\log(\hat{\sigma}^2_{\mathrm{RV}}) > -6$, or $\hat{\sigma}_{\mathrm{RV}} > 0.05$, onwards the influence becomes positive, i.e., the volatility is expected to increase in a highly nonlinear fashion. The long lived impact of the realized volatility, again, is confirmed by the selection of the second lag as shown in Figure 6(b).

We further findd that positive changes of the S&P500 slightly decrease volatility, while negative changes below 10% drastically increase it by 10%. Small negative changes, between -10% and zero, of the S&P500 mildly increase volatility. The relative bond rate (RBR) causes a considerable increase in volatility by 28% when it increases above one percent. Positive U.S. market excess returns have a moderate calming effect on the market while negative values below -2.5% increase volatility by 2%.



(a) Fitted function for the realized volatility (first lag).

(b) Fitted function for the realized volatility (second lag).

Figure 6: Lagged realized volatility as defined in Equation 3.1.

4.2.2 Forecasting

Another important question is how boosting performs in terms of forecasting accuracy as compared to the GARCH model, which is a common benchmark and hardly outperformed.

Figure 7 summarizes the mean squared errors of our boosting approach and the GARCH for 100 forecasts. The general impression is that boosting performs at least as good as GARCH in terms of forecasting accuracy. Even though having smaller error variance, boosting does not seem to outperform GARCH(1,1) for one– or two period–ahead forecasts. This is confirmed by the modified Diebold-Mariano test (Harvey *et al.*, 1997) shown in Section 4.1, Table 4. However, we find boosting to significantly outperform GARCH for medium– to long–term forecasts and Figure 7 offers the graphical explanation behind this statement.

4.3 Commodity Market

4.3.1 Driving Factors

The volatility of the commodity market is influenced by the past realized volatility, the net equity expansion, the Cochrane Piazessi factor and U.S. market excess return. The Cochrane Piazessi factor has a long lived impact through both of its lags, while the net equity expansion has an influence through its second lag. A remarkable finding for this

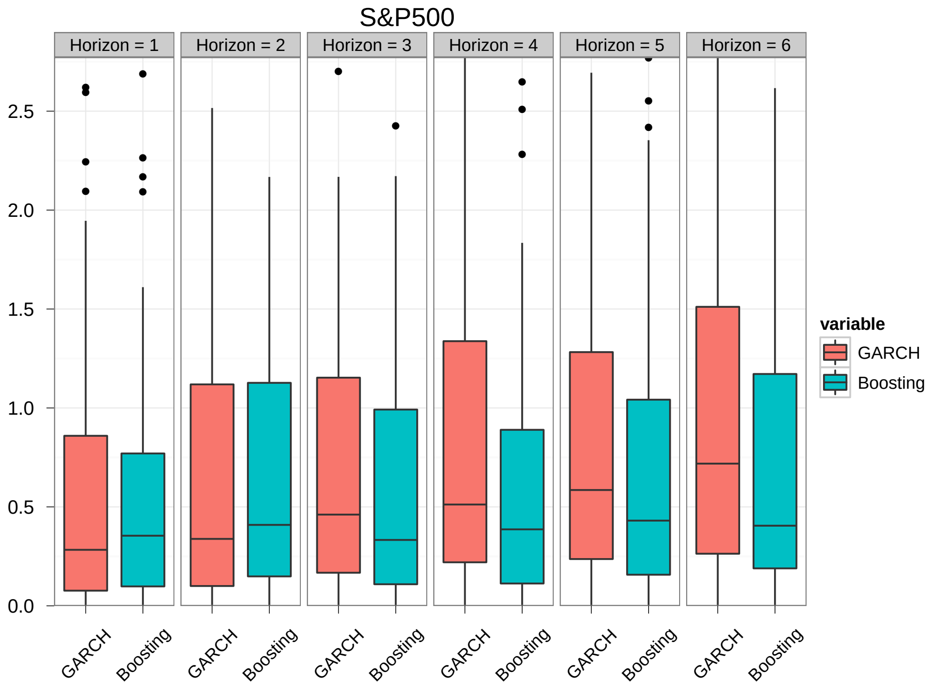


Figure 7: Forecasting comparison of the MSE between GARCH(1,1) and our model for the stock market.

market is the fact that since 1998 the volatility has, on average, increased by 12% compared to the preceding period 1983 to 1997.

The lagged realized volatility has a highly nonlinear influence as shown in Figure 8. Highly negative values of the lagged realized volatility, i.e., values below -6.5, reduce the volatility by roughly 0.2 on the log scale or -10% of σ. Values above -6.5 lead to an increase of the volatility in the commodity market, especially at -4 there is a jump and the volatility is increased by approximately 60%. The net equity expansion has an increasing effect on volatility if it is below -3%, otherwise it slightly decreases volatility. U.S. market excess returns above -2% lower the volatility, otherwise they increase it. The pattern is similar for the Cochrane Piazessi factor, except that the threshold is 2% in this case.

4.3.2 Forecasting

For the out-of-sample forecasting our method on average outperforms the GARCH model for all horizons as can be seen in Figure 9. Theil's U (see Section 4.1) supports this result.

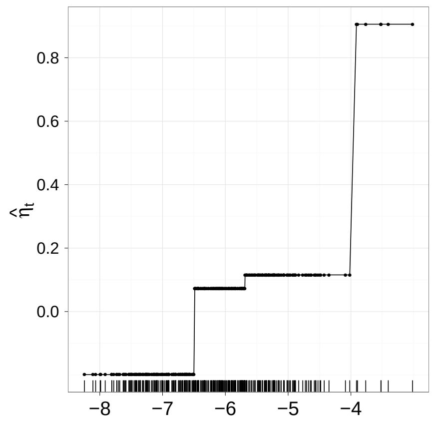


Figure 8: Lagged realized volatility.

For all horizons except for the first one Theil's U is below 1. For six months ahead forecasts Theil's U decreases to 0.714 which indicates that the boosting model is superior to the GARCH model regarding predictive performance especially for medium and long horizons. The modified Diebold and Mariano (1995) test confirms this.

4.4 Bond Market

4.4.1 Driving Factors

When modeling the base functions with regression trees we find that the default spread, change of the money supply (M1), change of the purchasing manager index, net equity expansion, relative bond rate, change of consumer sentiment and book to market ratio drive the volatility in the bond market.

For the default spread we find a clear threshold which identifies two volatility regimes. A default spread growth of 1.1% or more leads to an increase in volatility by 7%, otherwise it decreases it by roughly 4%. The relative bond rate has an effect on volatility only if it is higher than 1% which leads to a 10% higher volatility. The change of the consumer sentiment and the book to market ratio show a similar pattern: below a certain threshold, 5% growth of the consumer sentiment and 0.72 book to market ratio, they have no influence on the volatility at all. Only if they exceed this threshold the volatility grows. An expansive monetary policy, i.e. changes of the money supply above 5% increase the volatility in the bond market by approximately 10%. Smaller expansions, or reduction of the money supply, decrease the volatility by 6.8%.

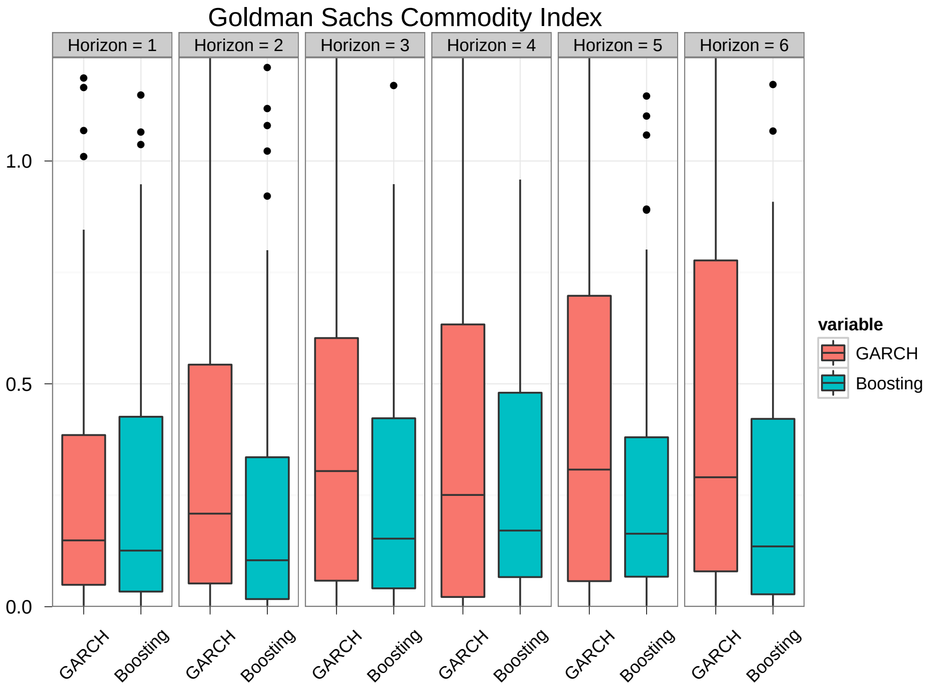


Figure 9: Forecasting comparison of the MSE between GARCH(1,1) and our model for the commodity market.

4.4.2 Forecasting

The results of the out-of-sample forecasts evaluated by the MSE are given in Figure 10. We use again direct forecasts for the one to six months ahead. For the short run predictions it is a neck-and-neck race, while for the longer horizons boosting tends to deliver better forecasts on average. This can also be seen in Table 2 with Theil's U being smaller than 1 for the horizons of four, five and six months. However, the only forecasting period where boosting is found to significantly outperform GARCH was six months in advance (Table 4). Our results also confirm that macroeconomic and financial factors are in particular useful for medium– and long–term forecasts.

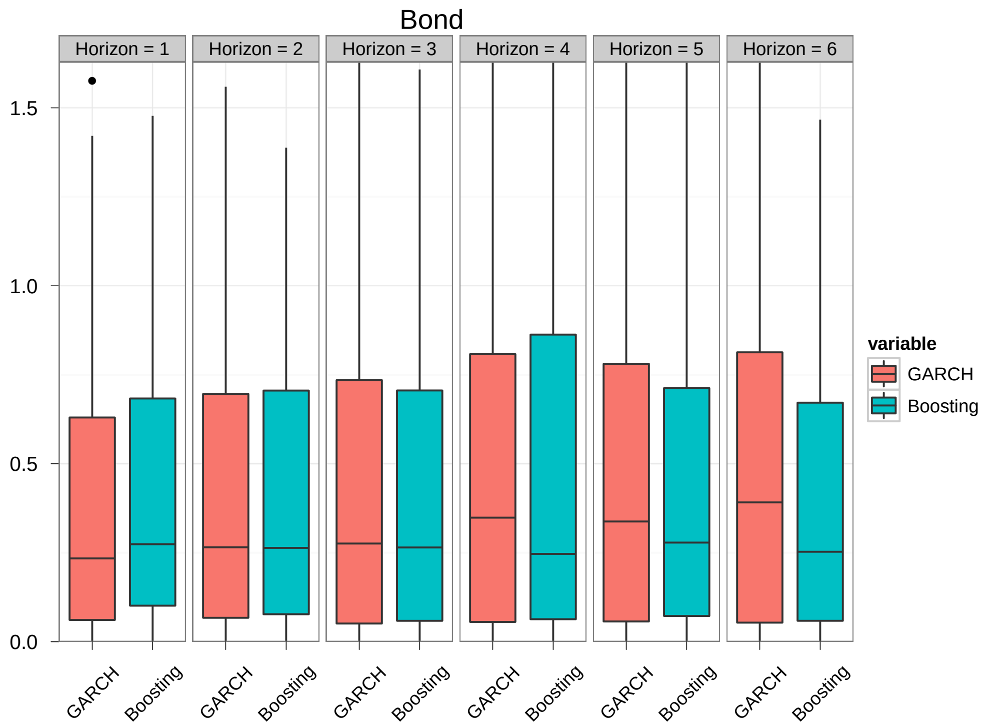


Figure 10: Forecasting comparison of the MSE between GARCH(1,1) and our model for the bond market.

4.5 Foreign Exchange Market

4.5.1 Driving Factors

The volatility of foreign exchange market is driven by the financial stability, default spread, realized volatility, TED spread, U.S. market excess return, long term rate of return, change of the money supply and several other factors. Periods of high financial stress, defined by values of the FSI index larger than five, drive up volatility by 12%, whereas low financial stress asymmetrically decreases volatility by a much smaller amount, i.e., less than 1%. Similarly to the other markets, values of the realized volatility in the past month which are smaller than -7 on the log scale, or $\hat{\sigma}_{\mathrm{RV}} < 0.03$, have a slight calming effect on volatility. Values above this cutoff considerably increase volatility by 15%. U.S. market returns seem to influence volatility only if they are below -10%. In this case they increase the volatility.

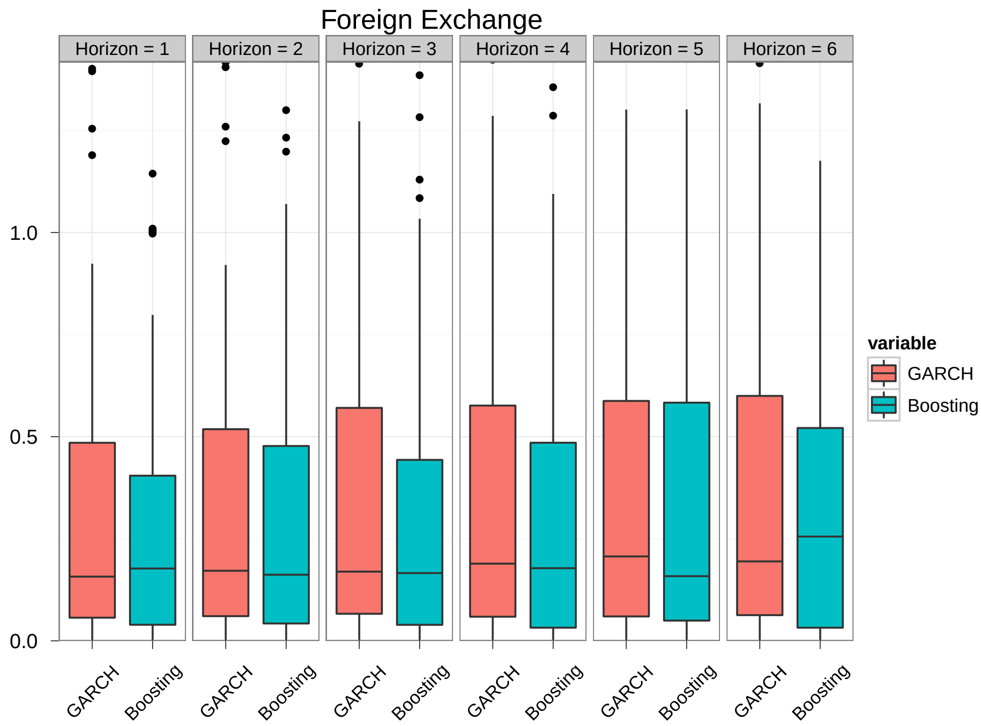


Figure 11: Forecasting comparison of the MSE between GARCH(1,1) and our model for the FX market.

4.5.2 Forecasting

In line with the previous literature, it appears to be very challenging providing useful forecasts of the foreign exchange volatility. This is especially true for the lower frequency suggested by the monthly observations. Still, our model predictions are on the level of the GARCH forecasts as can be seen in Figure 11. For the horizons of three to six months Theil's U is below 1 but none of the tests were significant.

5 Conclusions

In this paper, we analyze the volatility of four asset classes, namely, stocks, commodities, bonds and foreign exchange. This wide range of markets, coupled with an exhaustive set of macro and financial drivers, offers a fairly comprehensible overview on financial volatility.

Working with monthly data, we use boosting with regression trees as base-learners in order to identify influential macro and financial factors in different volatility regimes. Specifically, we use componentwise boosting tailored to sorting out irrelevant predictors, or some of their lags. Boosting is well suited for model selection and estimation in an unified framework with many (possibly highly correlated) potential predictors. Therefore, all drivers considered are simultaneously included with their first and second lag, along with some deterministic (seasonal) components in a regression-type of model. It is important to note that the proposed strategy can cope with "wide" data (**?**) models, i.e., situations in which the number of predictors exceed the number of observations. In our case, we typically have 58 predictors and 230 observations.

After introducing our modeling strategy we provided some elucidating examples, based on simulated data, in order to emphasize the understanding. Our empirical results give insight into the "anatomy" of volatility by: a) identifying small groups of influential drivers for each market and b) estimating thresholds for each driver which partition its domain into areas with similar volatility movements.

One finding is that the relationship between the financial drivers and volatility is highly nonlinear. This is an important finding, given that the literature almost exclusively concentrates on linear volatility dynamics.

We conduct out-of-sample forecasts for different horizons to compare our model with the benchmark model in this field, namely GARCH(1,1) model. The forecasting performance is evaluated in terms of mean squared prediction errors of the ex-ante estimates of both models relative to the ex-post realized volatility estimation. Realized volatility is, therefore, used as a proxy for the unobserved volatility. We gain favourable results for stocks and commodities. In these two markets, we significantly outperform the GARCH model. Especially for longer horizons our approach increases the predictive power when compared to GARCH.

In the bond and foreign exchange markets we obtain forecasts that are, for short horizons, at least as good as the GARCH benchmark and slightly better, although significantly unproven, in medium– to long–term periods.

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