Speculative Attacks:
Unique Sunspot Equilibrium and Transparency*

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Abstract Models with multiple equilibria are a popular way to explain currency attacks. Morris and Shin (1998) have shown that, in the context of those models, unique equilibria may prevail once noisy private information is introduced. In this paper, we generalize the results of Morris and Shin to a broader class of probability distributions and show — using the technique of iterated elimination of dominated strategies — that uniqueness will hold, even if we allow for sunspots and individual uncertainty about strategic behavior of other agents. We provide a clear exposition of the logic of this model and we analyse the impact of transparency on the probability of a speculative attack. For the case of uniform distribution of noisy signals, we show that increased transparency of government policy reduces the likelihood of attacks.

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1 Introduction

Recent experience seems to confirm the impression that currency crises, triggered by speculative attacks, are an inevitable characteristic of fixed exchange rate regimes. Within the economics profession there is, however, a strong controversy about the causes and nature of such attacks. Panglossian economists like to argue that they are ultimately driven by weak fundamentals. In this view, such attacks should be welcomed: By attacking when fundamentals are weak, speculation imposes the market discipline required to correct an unsustainable policy regime of fixed exchange rates.[1]

On the other hand, countries suffering from such attacks usually like to blame irresponsible speculators: They claim that attacks of greedy international capitalists forced the defeat of their otherwise intrinsically sound economies. Recently, the latter view has found theoretical support by work of Obstfeld (1996) demonstrating that multiple equilibria may prevail under fixed exchange rate regimes: If everybody believes that a currency peg can be sustained, nobody dares to attack, and so the rate indeed stays fixed. On the other hand, if speculators believe that devaluation is likely to occur, they are inclined to attack, and this attack will trigger the devaluation. Under multiple equilibria expectations are self-fulfilling. As Obstfeld shows, fundamentals play some role for the occurrence of multiple equilibria: If fundamentals are really bad (below some critical minimum level $\underline{\theta}$) the fixed rate can never be sustained, similarly, if the fundamentals are really good (above some critical level $\bar{\theta}$), an attack will be unrewarding. But there is an intermediate range of fundamentals $\underline{\theta} < \theta < \bar{\theta}$ for which both "attack" and "no attack" are Nash equilibria.

The trouble with indeterminate equilibria is that the outbreak of an attack appears to be completely arbitrary: A sudden change in mood, caused by some unrelated event (such as a sunspot) can trigger the attack, as long as all speculators align with the sunspot - that is, as long as it is common knowledge that all agents coordinate their expectations after observing the fundamentals (and possibly some sunspot

[1] See Krugman (1979) for the first model of speculative attacks with a unique equilibrium.

variable). Thus, there exists a continuum of equilibria with self fulfilling beliefs. The fate of the economy seems to be in the hands of arbitrary expectations of speculators - irrespective of the soundness of policy variables. There is no reason why an attempt to guide speculators expectations to the good equilibrium should be successful.

This view has come under attack recently by papers of Morris and Shin (1998a,b). They show that the indeterminacy of equilibria can be completely removed, once small uncertainty of agents about the true fundamentals is introduced. More precisely, it turns out that there is a unique switching point $\theta^* \geq \underline{\theta}$: If fundamentals are worse than θ^*, there will be an attack with probability one. There will be no speculative attack when fundamentals are better than θ^*. While Morris and Shin (1998a,b) assume that fundamentals and signals have either uniform or normal distribution, we show that their method is applicable to a broader class of probability distributions.

The additional uncertainty introduced by noisy private information about fundamentals, rather than worsening the multiplicity problem, is sufficient to eliminate all indeterminacy - at first sight a paradoxical result. When agents get such noisy signals, common knowledge about the state of the economy no longer exists. Thus, there is no longer a correlating device on which agents can orientate their attacks. Even if agents have relatively precise information about the fundamental state and can deduce that rewards from a successful attack would pay off transaction costs, they cannot be sure how many other agents get such signals. Hence, the success of an attack seems uncertain to them. Now, agents have to compare potential gains and losses from success or failure of an attack. This imposes an additional equilibrium condition that did not exist under perfect information. Depending on the signals' distribution, it may eliminate all but one equilibrium. In the paper, we present detailed intuition for this result and for the conditions under which there is a unique equilibrium.

Lack of common knowledge is the driving force for the results in Morris and Shin. Therefore, one may doubt whether uniqueness still holds when all agents can commonly observe an additional variable. Such a variable, which may be uncorrelated with fundamentals (pure sunspots) or correlated (think of financial news), might serve as a correlating device for expectations and so substitute common knowledge of fundamentals. We will show that the introduction of sunspots does not restore multiplicity of equilibria. It is not the lack of a correlating device per se, but the uncertainty about other agents' information that causes the additional equilibrium condition.

In general, there are two kinds of uncertainty: Uncertainty about the fundamentals of the economy and uncertainty about the behavior of other agents. Uncertainty about behavior is a Knightian uncertainty to which we cannot assign any probabilities. In Nash equilibria, the latter type of uncertainty - so called 'market uncertainty' (Shell, 1987, p. 549) - is assumed away: All agents are assumed to know the strategies of all other agents. One way to model 'market uncertainty' is to consider a–posteriori equilibria (Aumann, 1974) or 'rationalizable expectations' as they have been called by Bernheim (1984) and Guesnerie (1992). Those are equilibria that can be inferred from the model by assuming rationality alone without imposing any further restrictions on beliefs. Rationalizable expectations equilibria (REE) are ascertained by iterated elimination of dominated strategies.[2] In general, the set of REE exceeds the set of Nash equilibria. While there is a continuum of Nash and REE under perfect information, we show that with uncertainty about fundamentals, equilibrium may be unique even if we allow for uncertainty about others' strategies.

A crucial issue in the policy discussion about currency attacks is transparency. Often, it is claimed that a more transparent policy (giving more precise information about fundamentals) will lead to a better outcome. The notion of transparency, however, is rarely made precise. In the paper we show that, for a specific example, increased transparency of government policy will indeed reduce the likelihood of attacks. Following Cukierman and Meltzer (1985), Faust and Svensson (1998), and Illing (1998) we model transparency in the following way: For all states, increased transparency reduces the noisiness of private signals (the more transparent government policy, the more precise private agents can infer the fundamentals from their information).

For a specific example (the case of uniform distribution of states and signals), we analyze how transparency affects the outburst of speculative attacks. A more transparent policy turns out to shift the range of fundamentals under which speculative

[2]This procedure has first been described by Morgenstern (1935). Brandenburger (1992) gives an overview over its relation to other equilibrium concepts and their decision theoretic foundations. The close relationship to sunspot equilibria has been analyzed by Heinemann (1997).

attacks occur in a predictable way. We show that the switching point θ^* increases when policy becomes less transparent: The lower transparency, the higher the likelihood of an attack. An intransparent policy may even trigger attacks when fundamentals are really good ($\theta^* > \bar{\theta}$).

This extends the analysis of Heinemann (2000) who corrects a faulty expression in Morris and Shin (1998a) and shows in which way the probability of attacks depends on the critical mass needed for success when fundamentals are fairly transparent to all market participants.

In the next section, we present a generalized version of Morris and Shin. Section 3 solves the model using the technique of iterated elimination of dominated strategies. We get the same solution that Morris and Shin (1998a) obtained under more special assumptions. Our proof shows that the solution does not change even if we introduce sunspots and uncertainty about the behavior of other agents. In section 4 we give an intuitive and graphical explanation of the results. In section 5 we analyze the impact of transparency on the probability of a speculative attack. For the case of uniform distribution of state and signals, we show that increased transparency helps to reduce the probability of attacks. Section 6 concludes this paper and gives an outlook on future research.

2 The Basic Model

Using essentially the same set up as Morris and Shin (1998a), we now lay out a generalized version of their model. As in Obstfeld (1996), the fundamentals of the economy are characterized by some parameter $\theta \in \mathbb{R}$. If an attack is successful, agents get a reward $R(\theta)$. $R(\theta)$ is non–increasing in θ: Usually, the better the fundamentals, the lower the return of an attack. Any attack imposes transaction costs t. If θ is low, fundamentals are weak, and a speculative attack would be successful, even if undertaken by only one single agent, giving him a reward (net of transaction costs) $R(\theta) - t$. On the other hand, if θ is high enough, the fundamentals of the economy are so sound, that a speculative attack would never pay off, even if all agents would attack the currency.

$\theta \in \mathbb{R}$ is a random variable not known to the agents. Only the density function h is common knowledge. In addition, each agent observes a private signal $x^i \in \mathbb{R}$. There is a continuum of agents $i \in [0,1]$. The signals x^i are i.i.d. random variables distributed around θ with finite variance and expected value. Their density function g is common knowledge as well. The cumulative distribution of x^i given θ is denoted by $G(x^i \,|\, \theta)$.

We assume that $\partial G/\partial\theta < 0$ for all x^i and θ with $0 < G(x^i \,|\, \theta) < 1$, i.e. a better state leads to a smaller proportion of speculators getting signals worse than some given x^i. In addition, we assume that $G(x^i|\theta)$ approaches one [zero] for $\theta \to -\infty$ [$+\infty$] for each finite x^i. Similarly, we assume that conditional cumulative distribution of θ given signal x^i, denoted by $H(\theta \,|\, x^i)$ decreases in x^i whenever $0 < H(\theta \,|\, x^i) < 1$. This says that posterior probability for the state being worse than some given θ decreases if the signal gets better. Both assumptions imply that x^i and θ are positively correlated. They characterize the kind of signals that we consider here. Uniform and normal distributions as in Morris and Shin (1998a,b) fulfill these requirements.

Agents must decide on whether to attack the currency or not. An individual strategy is a function $\pi^i : \mathbb{R} \to \{0,1\}$ with the interpretation that agent i attacks the currency after getting signal x^i if $\pi^i(x^i) = 1$. If all agents get the same signal x, a fraction

$$\pi(x) := \int_0^1 \pi^i(x)\,\mathrm{d}i \tag{1}$$

will attack the currency. If the fundamental state is θ, a fraction

$$s(\theta,\pi) := \int_{\mathbb{R}} \pi(x)\,g(x|\theta)\,\mathrm{d}x \tag{2}$$

will attack with probability one, because signals are independent.

There is a function $a : \mathbb{R} \to [0,1]$ that assigns to each state of the world the proportion of attacking agents necessary for an attack to be successful. We assume that a is continuous and non–decreasing and there is a $\underline{\theta} \in \mathbb{R}$ with $a(\theta) = 0$ if and only if $\theta \leq \underline{\theta}$,

$$A(\pi) := \{\theta \mid s(\theta,\pi) \geq a(\theta)\} \tag{3}$$

is the event where attacks are successful with probability one.

If an agent attacks, she must pay transaction costs $t > 0$. If the attack is successful the attacking agents get a reward of $R(\theta)$. We assume that R is non–increasing, $R(\theta) > 0$ for all θ, there exists a unique $\bar{\theta} \in \mathbb{R}$ with $R(\bar{\theta}) = t$, and $\underline{\theta} < \bar{\theta}$.

The expected payoff of an attack for an agent who gets signal x^i is

$$u(x^i, \pi) := \int_{A(\pi)} R(\theta)\, h(\theta|x^i)\, \mathrm{d}\theta - t. \tag{4}$$

It is crucial for uniqueness that there are "bad" ["good"] states at which attacking [non–attacking] is a dominant strategy. Morris and Shin (1998a) assume limited support of the signal's distribution. But, it is sufficient to assume that the expected payoff of an attack, provided that none of the other agents follows the attack, is positive for some signals while the expected payoff of an attack by all speculators is negative for some other signals. Formally, we assume that there exist signals $\underline{x}, \bar{x} \in \mathbb{R}$, such that

$$u(\underline{x}, 0) := \int_{-\infty}^{\underline{\theta}} R(\theta)\, h(\theta \,|\, \underline{x})\, \mathrm{d}\theta - t = 0 \tag{5}$$

and

$$u(\bar{x}, 1) := \int_{-\infty}^{+\infty} R(\theta)\, h(\theta \,|\, \bar{x})\, \mathrm{d}\theta - t = 0. \tag{6}$$

As will be shown in an appendix at the end of the paper, $u(x^i, 0)$ and $u(x^i, 1)$ are decreasing in x^i. Thus attacking is a dominant strategy for all signals $x^i < \underline{x}$, while non–attacking is dominant for $x^i > \bar{x}$.

3 Rationality and Common Knowledge

In this section we solve the model by assuming rationality, common knowledge of the game, and mutual knowledge of rationality. We do not assume that agents know the strategies of each other. We adopt the method of iterated elimination of dominated strategies that yields the set of rationalizable expectations equilibria[3]. This includes all Nash equilibria but also all a–posteriori or sunspot equilibria. We get the same condition for uniqueness that has been obtained by Morris and Shin (1998a,b) for special distributions. Thus, their result is robust, even if sunspots as a correlating device or differing subjective beliefs about the strategies of other players are considered[4]. It is one intriguing feature of this game that it has a unique rationalizable expectations equilibrium under fairly general conditions when there is uncertainty about fundamentals while there is a continuum of Nash equilibria under perfect information.

Rationality requires the agents not to play a dominated strategy. Agents who receive signals below $\underline{x}$ will attack the currency, agents who receive signals above $\bar{x}$ will not. An agent who knows the game and knows that other players are rational and know the game as well, can deduce that none of them will play a dominated strategy. The expected payoff of an attacking agent rises with the probability of success. Success is more likely, the more agents attack at any given distribution of private signals. Hence, the worst [best] strategy an attacking agent must fear [can hope for] to be played by her colleagues is $I_{\underline{x}}$ $[I_{\bar{x}}]$, where I_k is defined by

$$I_k(x) := \begin{cases} 1 & \text{if} \quad x < k \\ 0 & \text{if} \quad x \geq k \end{cases} . \tag{7}$$

If all agents play strategy I_k then all agents getting signals below k will attack the currency. Thus,

$$s(\theta, I_k) = \int_{-\infty}^{k} g(x|\theta)\,\mathrm{d}x = G(k|\theta). \tag{8}$$

[3]For an application of this method to a game of similar structure see Carlsson and van Damme (1993).

[4]This can also be shown by application of a result by Milgrom and Roberts (1990), who proved that for supermodular games as ours the sets of pure Nash equilibria and rationalizable strategies have identical bounds.

Since $a(\theta)$ is increasing from zero at $\underline{\theta}$ and $G(k|\theta)$ is non–increasing in θ and ranges from 1 to 0, there is a unique $\hat{\theta}(k) \geq \underline{\theta}$, defined by

$$\hat{\theta}(k) := \sup\{\theta \,|\, a(\hat{\theta}) = s(\hat{\theta}, I_k)\} \tag{9}$$

such that

$$A(I_k) = (-\infty, \hat{\theta}(k)]. \tag{10}$$

The expected payoff to an attacking agent i when others play strategy I_k is given by

$$u(x^i, I_k) = \int_{-\infty}^{\hat{\theta}(k)} R(\theta)\, h(\theta \,|\, x^i)\, \mathrm{d}\theta - t. \tag{11}$$

After eliminating all dominated strategies, it unambiguously pays to attack the currency when $u(x^i, I_{\underline{x}}) > 0$, it does not pay to attack when $u(x^i, I_{\bar{x}}) < 0$. As will be shown in the appendix, $u(x^i, I_k)$ is decreasing in x^i. Hence, there are unique values $\underline{x}^1, \bar{x}^1 \in (\underline{x}, \bar{x})$ for which

$$u(\underline{x}^1, I_{\underline{x}}) = 0 \qquad \text{and} \qquad u(\bar{x}^1, I_{\bar{x}}) = 0, \tag{12}$$

and we may eliminate all strategies that assign a positive fraction of non–attackers to signals below $\underline{x}^1$ or a positive fraction of attackers to signals above $\bar{x}^1$

Now we are ready to define step $n > 1$ of the iterative process $\underline{x}^n, \bar{x}^n \in \mathbb{R}$ by

$$u(\underline{x}^n, I_{\underline{x}^{n-1}}) = 0 \qquad \text{and} \qquad u(\bar{x}^n, I_{\bar{x}^{n-1}}) = 0. \tag{13}$$

Given that agents are rational and rationality is mutual knowledge, strategies with $\pi^i(x) < 1$ for $x < \underline{x}^n$ or $\pi^i(x) > 0$ for $x > \bar{x}^n$ are inconsistent with n–order knowledge of the game. Common knowledge takes the process to its limit. Define $\underline{x}^\infty = \lim_{n\to\infty} \underline{x}^n$ and $\bar{x}^\infty = \lim_{n\to\infty} \bar{x}^n$.

Because $\underline{x}^n \geq \underline{x}^{n-1}$ and $\bar{x}^n \leq \bar{x}^{n-1}$ for all n, the limit points are given by

$$\underline{x}^\infty = \inf\{x \mid u(x, I_x) = 0\} \qquad \text{and} \qquad \bar{x}^\infty = \sup\{x \mid u(x, I_x) = 0\}. \tag{14}$$

Since $u(x, I_k)$ is decreasing in x, strategy combinations in which all agents play $I_{\underline{x}^\infty}$ or in which all agents play $I_{\bar{x}^\infty}$ are Nash equilibria. Thus, we have multiple equilibria if $\underline{x}^\infty < \bar{x}^\infty$.

On the other hand, if $\underline{x}^\infty = \bar{x}^\infty =: x^*$, a rationalizable expectations equilibrium is a strategy profile with

$$\pi^i(x) = \begin{cases} 1 & \text{if} \quad x < x^* \\ 0 & \text{if} \quad x > x^* \end{cases} . \tag{15}$$

for all i. Agents who get signal x^* are indifferent between attacking and non–attacking. But they have mass zero with probability one. $\theta^* := \hat{\theta}(x^*)$ is the threshold of the fundamental state up to [above] which a currency attack will occur with probability one [zero]. I.e., in all equilibria $A(\pi) = (-\infty, \theta^*]$.

If $u(x, I_x)$ is decreasing in x, there can only be one point x^* at which $u(x^*, I_{x^*}) = 0$, so that there is a unique threshold as described above. These results are in line with Morris and Shin (1998a,b), who proved that $u(x, I_x)$ is decreasing in x if g and h are either uniform distributions or normal with a sufficiently small variance $\mathrm{Var}(x^i|\theta)$. But note that the influence of x on u via I_x is positive and may exceed the negative *partial* derivative $\partial u/\partial x$. In section 4 we attempt an illustration of this case.

Suppose now, that we have yet another random variable $s \in \mathbb{R}$ which is independent from θ and x^i and commonly observable. It may be viewed at as sunspots. We have shown that the iterated elimination of dominated strategies may lead to a unique threshold θ^* for all rationalizable expectations equilibria. This threshold cannot depend on sunspots. To see why, think of agents who observe a certain realization of s. Now, a strategy is a function $\tilde{\pi}(s, x^i) \in \{0, 1\}$ that assigns one of the two possible actions to each combination of sunspots and individual signal. A dominated strategy remains dominated for all realizations of s. Therefore, we can eliminate all strategies that assign positive probability to agents who do [not] attack for some sunspots when they get a signal above $\bar{x}$ [below $\underline{x}$]. The iterated elimination can be pursued just the same way as before and leads to the same switching point x^* for each s. Hence, sunspots can only matter for agents who get signal x^* which is a proportion of mass zero with probability one. Thus, θ^* is independent of s.

If $\underline{x}^\infty < \bar{x}^\infty$, we have multiple Nash equilibria. It is well known that multiplicity of Nash equilibria is sufficient for the existence of equilibria in which sunspots matter. Since $I_{\underline{x}^\infty}$ and $I_{\bar{x}^\infty}$ are equilibrium strategies, we can construct a sunspot equilibrium easily by defining

$$\tilde{\pi}(s, x^i) = \begin{cases} I_{\underline{x}^\infty}(x^i) & \text{if} \quad s < \bar{s} \\ I_{\bar{x}^\infty}(x^i) & \text{if} \quad s \geq \bar{s} \end{cases}$$

for some arbitrary number of sunspots $\bar{s}$. Rationalizable expectations allow even for a wider variety of strategies. Since strategies are not known by other agents, some agents might play $I_{\underline{x}^\infty}$ while others play $I_{\bar{x}^\infty}$.

Whatever combination of strategies is played, the ones that have been eliminated can neither occur in sunspot equilibria, nor under rationalizable expectations. So, the worst combination of strategies for the currency threatened by attack is $I_{\bar{x}^\infty}$, the most harmless is $I_{\underline{x}^\infty}$ and the set of states at which an attack must be expected for some equilibria but not for others is given by $\left(\hat{\theta}(\underline{x}^\infty), \hat{\theta}(\bar{x}^\infty)\right]$.

4 Intuitive Explanation

In this section we give an intuitive and graphic explanation why there may be a unique equilibrium under uncertainty about fundamentals. We also explain under which conditions there are multiple equilibria.

Let us first describe a Nash equilibrium under perfect information. For any fundamental state $\theta \in (\underline{\theta}, \bar{\theta})$ it pays to attack if and only if a sufficient proportion of speculators $a(\theta) \in [0, 1]$ follows the attack. Hence, any combination of strategies $(\pi^i)_{i\in[0,1]}$ with

$$\pi^i(\theta) = \pi(\theta) \quad \text{for all } \theta \neq \bar{\theta},$$

$$
\begin{aligned}
&\pi^i(\theta) = 1 && \text{for } \theta \leq \underline{\theta}, \\
&\pi^i(\theta) = 0 && \text{for } \theta > \bar{\theta}, \quad \text{and} \\
&\Big(\ \pi(\bar{\theta}) \geq a(\bar{\theta}) \quad \text{or} \quad \pi(\bar{\theta}) = 0\ \Big)
\end{aligned}
$$

is a Nash equilibrium.

Within the interval $(\underline{\theta}, \bar{\theta})$ there are two possible outcomes associated with each realization of θ: attack or no attack. Since the fundamental state is common knowledge, either all agents or none will attack the currency.[5] In equilibrium an attack is always successful. Thus, agents do not need to compare the potential reward from a successful attack with the potential loss from an attack that would fail. Even for relatively good states, close to $\bar{\theta}$, where rewards are rather low compared to transactions costs, agents may attack the currency.

The situation changes dramatically if there is some uncertainty about θ. Given that there are some states at which attacking and some others at which non–attacking is a dominant strategy, there must be states at which the success of an attack is uncertain. At those states agents have to weigh potential gains and losses in the two events that an attack succeeds or fails. This imposes an additional equilibrium condition that did not exist in the perfect information case.

Consider a state close to $\bar{x}$. Many agents will receive a signal above $\bar{x}$ for which non–attacking is a dominant action. They will not attack. Agents who receive signals below $\bar{x}$ expect many others to get signals above, so they attach a high probability to failure and abstain from attacking too, even if they expected a reward in the case of success. In equilibrium switching points, rewards of an attack weighted with the probability of success must equal the expected losses in the case of failure.

Figure 1 shows a Nash equilibrium under perfect information with four points at which agents switch between attacking and non–attacking. For the illustration we assume that state θ has uniform distribution in the relevant area and signals x^i are uniformly distributed in $[\theta - \epsilon, \theta + \epsilon]$. The event that an attack succeeds [fails] given some signal x is that part of the interval $[x - \epsilon, x + \epsilon]$ for which $s(\theta, \pi) \geq$ $[<]$ $a(\theta)$. The lower part of Figure 1 shows that only one of the switching points fulfills the equilibrium condition that gains from a successful attack weighted with

[5] except for $\bar{\theta}$, at which agents may be indifferent.

the probability of success equal the losses from a failed attack weighted with the probability of failure.

Insert Figure 1 about here!

Consider an agent receiving signal x^*. Since conditional distribution of θ given x^i is uniform, the agent attaches equal weight to all fundamental states within $(x^* - \epsilon, x^* + \epsilon)$. At any state θ, individual signals are dispersed in $(\theta - \epsilon, \theta + \epsilon)$. If all agents pursue the strategy to attack if and only if their signal is below x^*, the proportion of attacking agents at any of these states is $s(\theta, I_{x^*}) = \frac{x^* - \theta + \epsilon}{2\,\epsilon} \in [0, 1]$. The agent can expect an attack to succeed whenever this proportion exceeds $a(\theta)$. The conditional probability of this event, given signal x^*, is $1 - a$.

The agent weights expected return under success against expected loss under failure. For the marginal attacker, who gets signal x^*, expected return under success $\int_{x^*-\epsilon}^{\theta^*} R(\theta)\, \mathrm{d}\theta - t$, weighted with probability $1 - a(\theta^*)$, will just compensate loss t weighted with the probability of failure $a(\theta^*)$.

Consider the case (illustrated in Figure 1) that $a(x^*) > 1/2$. If the true state were x^* then only half of all agents would get a signal below x^* and an attack would fail. The agent getting signal x^* knows that the true state must be sufficiently worse $(\theta \leq \theta^*)$ to bring about a distribution of signals, such that at least $a(\theta^*) > 1/2$ agents get signals below x^* and attack. To compensate for the low subjective probability of this event, rewards $R(\theta)$ must be correspondingly higher than transaction costs.

If uncertainty shrinks to almost zero (e.g. by reducing the variance), the additional equilibrium condition converges to the equality of gains and losses at x^*, each weighted with posterior probability of θ being higher or lower than $\hat{\theta}(x^*)$ given that signal.

It is obvious that there can only be one equilibrium under the distributional presumptions made in Figure 1. This gives an intuitive insight that may be misleading once we allow for more general distributions. As Morris and Shin (1998b) have shown, there may be multiple equilibria if there is a strong dispersion of individual signals. They demonstrated that $u(x, I_x)$ may be rising in x if the variance associated with $G(x|\theta)$ is big enough.

In Figure 2 we illustrate such a case for which the expected payoff of an attack at some signal x, given that others play strategy I_x, is smaller than the expected payoff of an attack at $x' > x$, given that others play $I_{x'}$. After observing a signal, agents update their initial beliefs about the fundamental. If the signal is low, conditional density of θ shifts to the left, whereas it shifts to the right for high signals. The more precise the signal, the stronger this shift. With decreasing precision of the signal, agents attach more and more weight to the a–priori (unconditional) density $h(\theta)$ relative to the signal itself. The larger the signals' dispersion, the less moves conditional density of θ given signal x relative to the signal itself. In the extreme case where the signal is completely unrelated to θ the conditional density equals the unconditional one. So inevitably with high dispersion, conditional probability of the state being worse than the signal rises with better signals: $H(x|x) << H(x'|x')$.

Now consider strategies I_x and $I_{x'}$. The events where attacks are successful are given by $(-\infty, \hat{\theta}(x)]$ and $(-\infty, \hat{\theta}(x')]$ respectively. u would unambiguously increase in x if both a and R were constant. Rising a and falling R dampen this effect and will eventually reverse it. But as long as both a and R are relatively flat, u can still be increasing. The dampening effects work in two ways: The steeper the reward function R, the larger is the direct negative effect $\partial u/\partial x$. An increase in the hurdle $a(\cdot)$ works in a more subtle way. It shifts $\hat{\theta}(x)$ to the left relative to x. This reduces the probability of attacks being successful and so tends to reduce expected rewards.

As is shown in Figure 2, an overall positive effect on u via I_x can easily dominate the negative direct effect. If a is rather flat, then $\hat{\theta}(x)$ does not fall too much behind a rising x, and posterior probability of an attack being successful given signal x and strategy I_x rises in x: $H(\hat{\theta}(x)|x) < H(\hat{\theta}(x')|x')$. If R is flat as well (or even constant), then changes in $u(x, I_x)$ are driven by changes in posterior probability of success. Hence, $u(x, I_x) < u(x, I_{x'})$.

Insert Figure 2 about here!

Now choose transaction costs t, such that $u(x, I_x) < 0 < u(x', I_{x'})$. Then there must be equilibrium switching points like $\underline{x}^\infty < x$ and $\bar{x}^\infty > x'$. Hence, we have multiple equilibria with different thresholds. From Figure 2 and the description above, it is clear that uniqueness is the easier to get the smaller the dispersion of signals or the steeper functions a and R.

On the other hand, if a and R are both constant then for any reasonable distribution there is an interval in which $u(x, I_x)$ is rising in x. If, in addition, the distribution of θ is about symmetric and peaks around the levels where currency attacks are to be expected from the values of a, R and t, then there are multiple equilibria whenever there is considerable dispersion in private information.

This leads to the next question we want to address: Does this theory give any insight in optimal information policy by the authorities?

5 Transparency

In the wake of the Asian crisis, much attention has been paid to the question how policy should be designed to make speculative attacks more difficult. Frequently, based on models with multiple equilibria, it is argued that policy should help to coordinate expectations such that they are guided to the "good" (non-attacking) equilibrium. Since within such models, expectations are rather arbitrary and there is no reason why sunspot events triggering a crisis should be related to policy variables, such reasoning is rather ad hoc. In sunspot models, there is no lack of coordination of expectations, and so it is hard to see why increased coordination should be able to improve upon the outcome.

As a response to the Asian crisis, international policy makers recently favor a different route: They suggest that increased transparency could help to avoid speculative crashes and, simultaneously, make sure that unsustainable pegs will be corrected in good time.[6] In this section we analyze to what extent the model set up in the last sections can shed light on this issue. In order to do that, we have to make precise the notion of transparency. Along the route suggested by work on monetary policy, such as Cukierman and Meltzer (1986), Faust and Svensson (1998) and Illing (1998), transparency is modeled in the following way: Higher transparency of government policy increases the precision of private signals. The more transparent the policy, the better private agents can infer the fundamentals from their information.

[6]See, for instance, recent calls by IMF (1998a,b) and BIS (1998) for more transparency.

In the model, government is informed about the fundamentals θ and observes the proportion $s(\theta, \pi)$ of all agents attacking. If $s(\theta, \pi) \geq a(\theta)$, the currency will be devalued. Government cannot convey information about the true state to the public, and so private agents receive only noisy signals. But by committing to a more transparent policy (e.g., allowing access to information by outside observers such as independent rating agencies), the noise in private signals can be reduced. We assume that government can commit in advance to pursue an information policy with a given degree of transparency. Thus, increased transparency reduces the noisiness of private signals in all states.[7]

The results obtained in the last section suggest that higher transparency in the sense modeled here may indeed help to reduce market uncertainty and so reduce incentives for speculative attacks: Multiplicity of equilibria is less likely the higher the precision of private signals. But here, we are interested in a more specific question: Can higher transparency help to reduce the likelihood of attacks even if equilibrium is unique? In order to focus on this issue, we concentrate on the case of uniform distributions of fundamentals and signals. For that specific case, we analyze how increased variance of private signals affects the probability of speculative attacks.

Assume that the fundamental state θ and private signals x^i have uniform distributions, such that

$$h(\theta \,|\, x^i) = \begin{cases} \frac{1}{2\,\epsilon} & \text{if } x^i - \epsilon \leq \theta \leq x^i + \epsilon \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

and

$$g(x^i \,|\, \theta) = \begin{cases} \frac{1}{2\,\epsilon} & \text{if } \theta - \epsilon \leq x^i \leq \theta + \epsilon \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

This is the case for which Morris and Shin (1998a) proved that there is a unique equilibrium with an associated switching point x^* and a threshold $\theta^* = \hat{\theta}(x^*)$ characterized by two equations:

$$u(x^*, I_{x^*}) = \frac{1}{2\,\epsilon} \int_{x^*-\epsilon}^{\theta^*} R(\theta)\, \mathrm{d}\theta - t = 0, \tag{18}$$

[7] Of course, ex post, once the true state is known, government may renege on its commitment, but analyzing these incentives is left for future research.

$$s(\theta^*, I_{x^*}) = \frac{x^* - \theta^* + \epsilon}{2\,\epsilon} = a(\theta^*). \tag{19}$$

Total differentiation of (18) and (19) and rearranging terms yields

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\epsilon} = 2\,\frac{(1 - a(\theta^*))\,R(x^* - \epsilon) - t}{R(x^* - \epsilon) - R(\theta^*) + 2\,\epsilon\, a'(\theta^*)\,R(x^* - \epsilon)}. \tag{20}$$

Since R is positive and non–increasing, the denominator is positive. The numerator is positive if and only if

$$t < (1 - a(\theta^*))\,R(x^* - \epsilon) \tag{21}$$

Suppose R is constant around equilibrium. Then (18) and (19) imply $1 - a(\theta^*) = t/R$. Threshold θ^* is determined by this equation and independent of ϵ. Switching point x^* shifts upwards [downwards] accordingly if $a(\theta^*) > [<]\,1/2$. So if R is constant, transparency has no effect on the probability of speculative attacks.

Now, suppose $R' < 0$. Equilibrium condition (18) implies

$$\frac{1}{2\,\epsilon}\int_{x^*-\epsilon}^{\theta^*} R(\theta)\mathrm{d}\theta = t < \frac{1}{2\,\epsilon}\int_{x^*-\epsilon}^{\theta*} R(x^* - \epsilon)\mathrm{d}\theta = \frac{\theta^* - x^* + \epsilon}{2\,\epsilon}\,R(x^* - \epsilon). \tag{22}$$

Using (19), we find that $\frac{\theta^* - x^* + \epsilon}{2\,\epsilon} = 1 - a(\theta^*)$, so $t < (1 - a(\theta^*))\,R(x^* - \epsilon)$. Hence $\partial\,\theta^*/\partial\epsilon > 0$ and transparancy decreases the probability of speculative attacks.

In Figure 3 we give a graphical illustration of this point. If dispersion increases from ϵ_1 to ϵ_2, the equilibrium switching point x^* must adjust in a way that gains from those additional states where attacks are successful (on the left margin of $[x^* - \epsilon, x^* + \epsilon]$) equal losses from additional states in which attacks fail (on the right margin of this interval). If x^* would adjust in a way such that θ^* remained constant (see Figure 3) then expected gains from an attack given signal x^* would exceed expected losses. Because of $R' < 0$, additional rewards on the left margin are ever increasing, while additional losses are always of the same magnitude t. Hence, an attack at x^* is promising, and the equilibrium switching point has to move further up and drags θ^* in the same direction. This effect vanishes if R is constant.

Insert Figure 3 about here!

An increase in the conditional variance of θ given some signal x puts a larger weight on states that are further left and, given $R' < 0$, associated with higher gains from successful attacks. Higher weight on states to the right does not change the expected payoff of an attack, because losses from a failed attack (transaction costs) are constant.

If dispersion of individual signals ϵ shrinks to almost zero, threshold θ^* approaches $\theta_0^* \in (\underline{\theta}, \bar{\theta})$ uniquely defined by

$$R(\theta_0^*)\,(1 - a(\theta_0^*)) = t \tag{23}$$

The l.h.s. of (23) are the gains from successful attacks weighted with the probability of success, which is $1 - a$. t are the costs that have to be borne with certainty. This result is proven in Heinemann (2000) and corrects a faulty expression in Morris and Shin (1998a). On the other hand, if dispersion ϵ is large, threshold θ^* may even exceed $\bar{\theta}$ which has been the upper bound on Nash equilibria under perfect information.

Since θ is the information on which the government's decisions are based, these results give us a first hint on how transparency might influence the probability of speculative attacks. Assuming uniform distribution and $R' < 0$ we find that θ^* rises with rising variance of individual information about θ. Transparent policy may reduce the dispersion of individual information *at all states*, i.e. the variance of $g(\cdot|\theta)$ for all θ. This lowers θ^* and thus, reduces the probability of speculative attacks.

Using the same example, Morris and Shin (1998a) claim that without common knowledge, speculative attacks may be triggered even though everyone knows that fundamentals are sound. They conclude that public announcements restoring common knowledge stabilize the market. This interpretation may be misleading. With common knowledge, a range of fundamental states $(\underline{\theta}, \bar{\theta}]$ exists, for which an attack occurs at some equilibria but fails to come about at others. If dispersion of individual information approaches zero, the unique equilibrium threshold θ_0^* is in the interior of this interval. Thus, a public announcement that restores common knowledge of fundamentals opens the way for attacks at states above θ_0^* and may, instead of preventing it, even trigger an attack.

Transparency in the sense that common knowledge is restored, as advocated by Morris and Shin, makes the onset of a currency crisis unpredictable. It may prevent crises in relatively bad states in $(\underline{\theta}, \theta_0^*)$, where they would happen with probability one if the states were not common knowledge. But common knowledge bears the danger that a crisis occurs at relatively good states in $(\theta_0^*, \bar{\theta})$ at which it could be prevented by an information policy that reveals fundamentals as precisely as possible, but stops short of common knowledge. One way to achieve this is in giving everybody reliable information on request without announcing it publicly. Then, speculators can never be certain that other agents have the same information at any given moment.

6 Conclusion and Outlook on Future Research

Building up on a reduced game developed by Morris and Shin (1998a) to model currency attacks under noisy private information, we have proven that the set of fundamental states for which currency attacks occur in some equilibria but not in others is robust against the introduction of sunspots and diverse beliefs about strategic behavior. Using the technique of iterated elimination of dominated strategies, the paper characterized conditions under which equilibrium is unique when agents receive noisy information about fundamentals. The noise in the signals imposes an additional equilibrium condition which — for specific distributions — leads to a unique outcome, whereas multiple equilibria prevail under common knowledge about fundamentals. In particular, the condition for uniqueness of equilibrium is the same whether Nash or rationalizable expectations equilibria are considered.

For a specific example, we showed that increased transparency reduces the probability of speculative attacks. It would be rash, however, to conclude from the model that policy should unambiguously aim for more transparency.

First, we should note that this result does depend on the assumption of uniform distributions. For the case of normal distributions of state and signals, Morris and Shin (1999) have shown that a decreasing variance of private signals need not monotonically reduce the probability of speculative attacks.

Second, it is an open question how government could commit to transparency. In the model, θ represents the government's information about fundamentals. It may have strong incentives to misrepresent and distort information in some states, whereas it may be inclined to give very precise signals for other states. In general, the authority may wish to minimize θ^* by choosing g, which may be interpreted as information policy of the government. This optimization might lead to different conditional variances of x^i for different states θ. The result of such an optimization problem depends on various assumptions about the shape of the functions used here, and the optimization problem itself is plagued by the multiplicity of equilibrium strategies for some information policies. In addition one should consider rational expectations about information policy. A more general analysis of optimal information policy is a promising task for future research.

Third, the model is not suitable to answer the question under what conditions speculative attacks should indeed be prevented. Based on Krugman (1979), the single objective of the government is to prevent currency attacks. The better the fundamentals, the more resources are available to defend the currency and the higher the hurdle for a successful attack. Both an increase in transactions costs (such as a Tobin tax) and the imposition of capital controls will reduce the likelihood of attacks. The model can be seen as a reduced form without specifying government objectives explicitly. Preventing attacks may not necessarily be good for the economy. When fundamentals are bad enough, a surrender of an unsustainable peg could be welfare improving. Obstfeld (1996) gave an explicit structure modeling government objectives. To analyse welfare implications of transparency by introducing lack of common knowledge in such a set up could be a promising extension of the current analysis.

Appendix

First, we show that $u(x, I_k)$ is decreasing in x for all k. Since $R(\theta)$ is non–increasing in θ, we can define an "inverse" function $R^{-1}(p) := \sup\{\theta \mid R(\theta) \geq p\}$, and then we have

$$u(x, I_k) = \int_{-\infty}^{\hat{\theta}(k)} R(\theta)\, h(\theta \,|\, x)\, \mathrm{d}\theta - t. \tag{24}$$

$$= R(\hat{\theta}(k))\, H(\hat{\theta}(k) \,|\, x) + \int_{R(\hat{\theta}(k))}^{R(-\infty)} H(R^{-1}(y) \,|\, x)\, \mathrm{d}y - t. \tag{25}$$

From this we get

$$\frac{\partial u}{\partial x} = R(\hat{\theta}(k))\, \frac{\partial H(\hat{\theta}(k) \,|\, x)}{\partial x} + \int_{R(\hat{\theta}(k))}^{R(-\infty)} \frac{\partial H(R^{-1}(y) \,|\, x)}{\partial x}\, \mathrm{d}y. \tag{26}$$

Since $R(\cdot) > 0$ and $H(\cdot|x)$ is decreasing in x whenever $0 < H < 1$, this derivative is negative for $u(\cdot) > -t$.

Now, consider $u(x, 0)$ and $u(x, 1)$ as defined by (5) and (6). Here, we can use the same argument if we replace $\hat{\theta}(k)$ by $\underline{\theta}$ and ∞ respectively.

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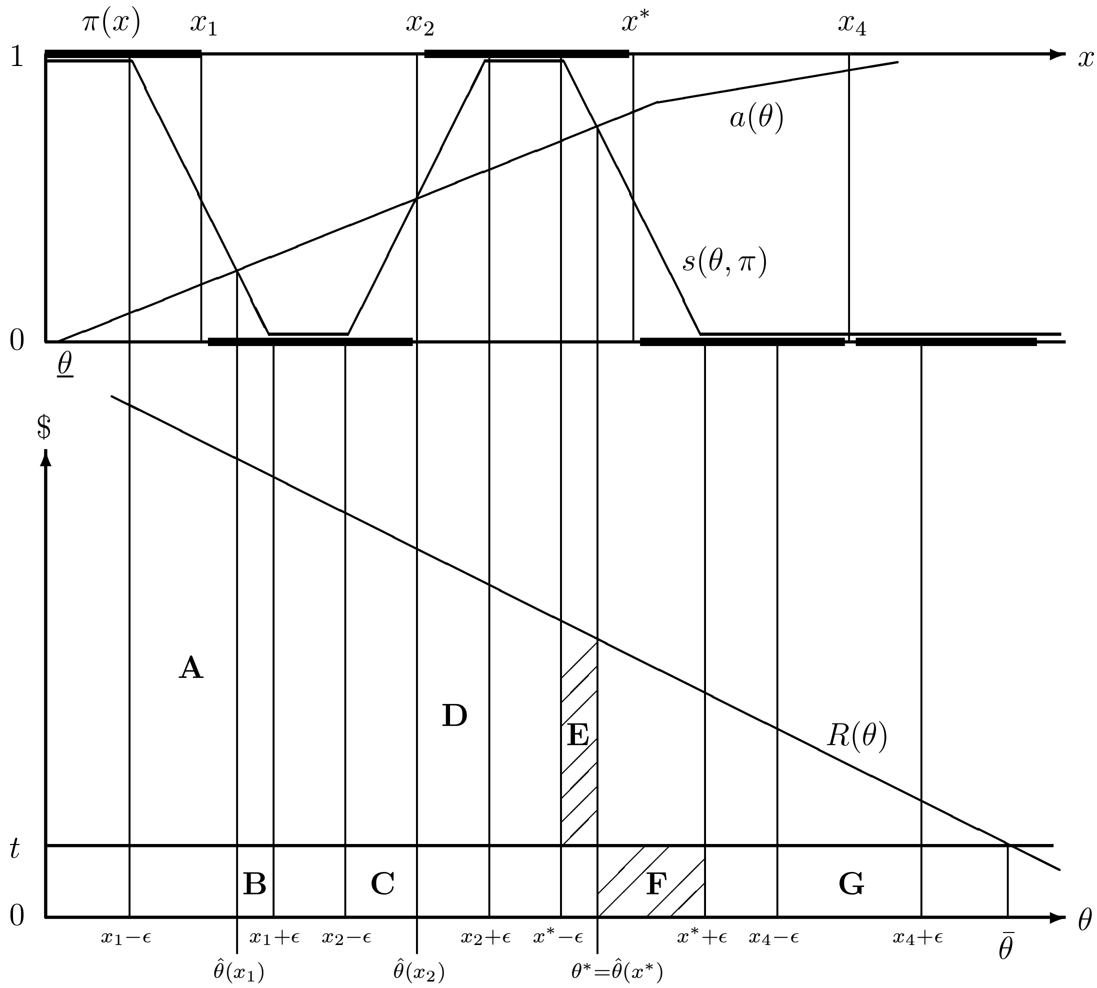

Figure 1 *The bold graph π is a Nash equilibrium under perfect information. There are four arbitrary switching points in $(\underline{\theta}, \bar{\theta})$. ϵ stands for the dispersion of individual signals. The fraction of agents getting a signal at which $\pi(x) = 1$ is given by $s(\theta, \pi)$. Expected gains from a successful attack at x_1 or x_2, given by areas* ***A*** *and* ***D*** *respectively, exceed expected losses of* ***B*** *and* ***C****. A point like x_4 cannot occur in an equilibrium under uncertainty, because it leads to loss t with probability one. x^* is the only possible switching point, characterized by the equality of expected gains* ***E*** *and losses* ***F****.*

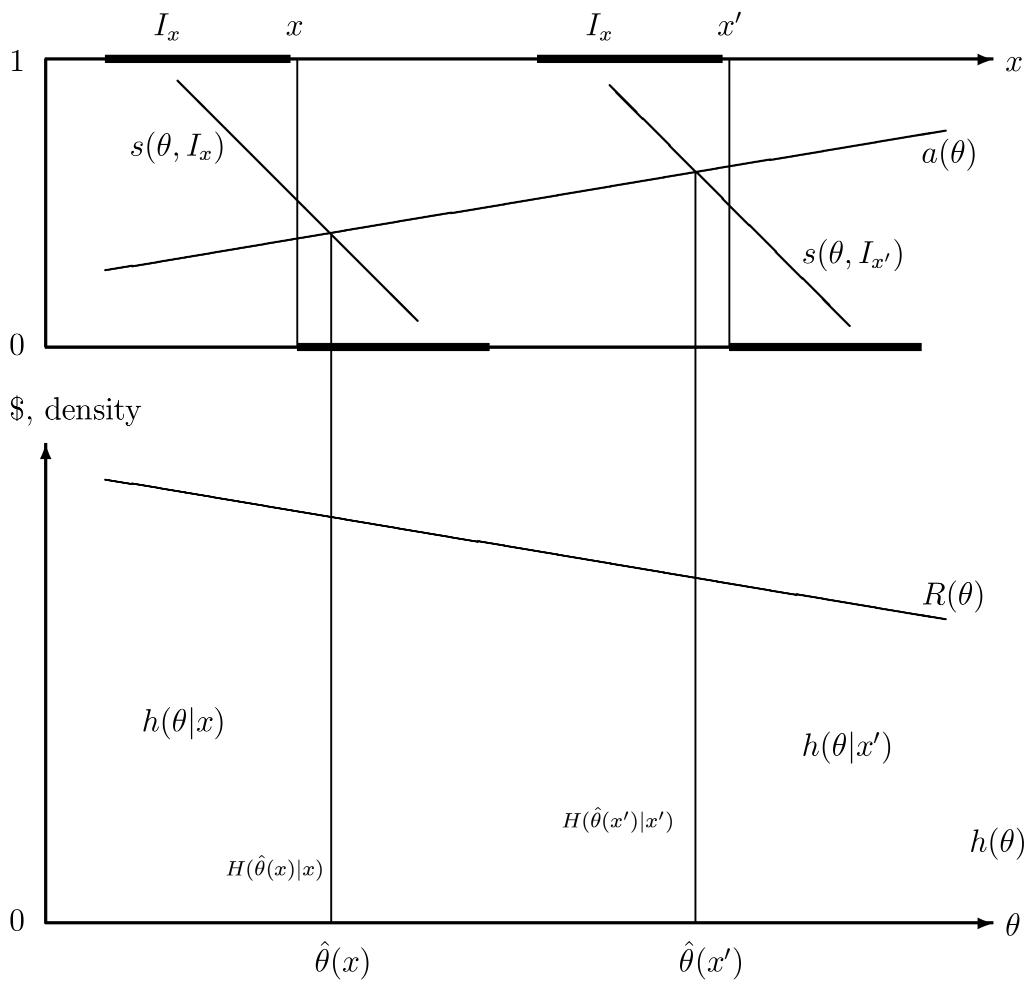

Figure 2 *Compare two signals $x < x'$. Conditional probabilities for the success of an attack given strategies I_x and $I_{x'}$, respectively, are $H(\hat{\theta}(x)|x) < H(\hat{\theta}(x')|x')$.*

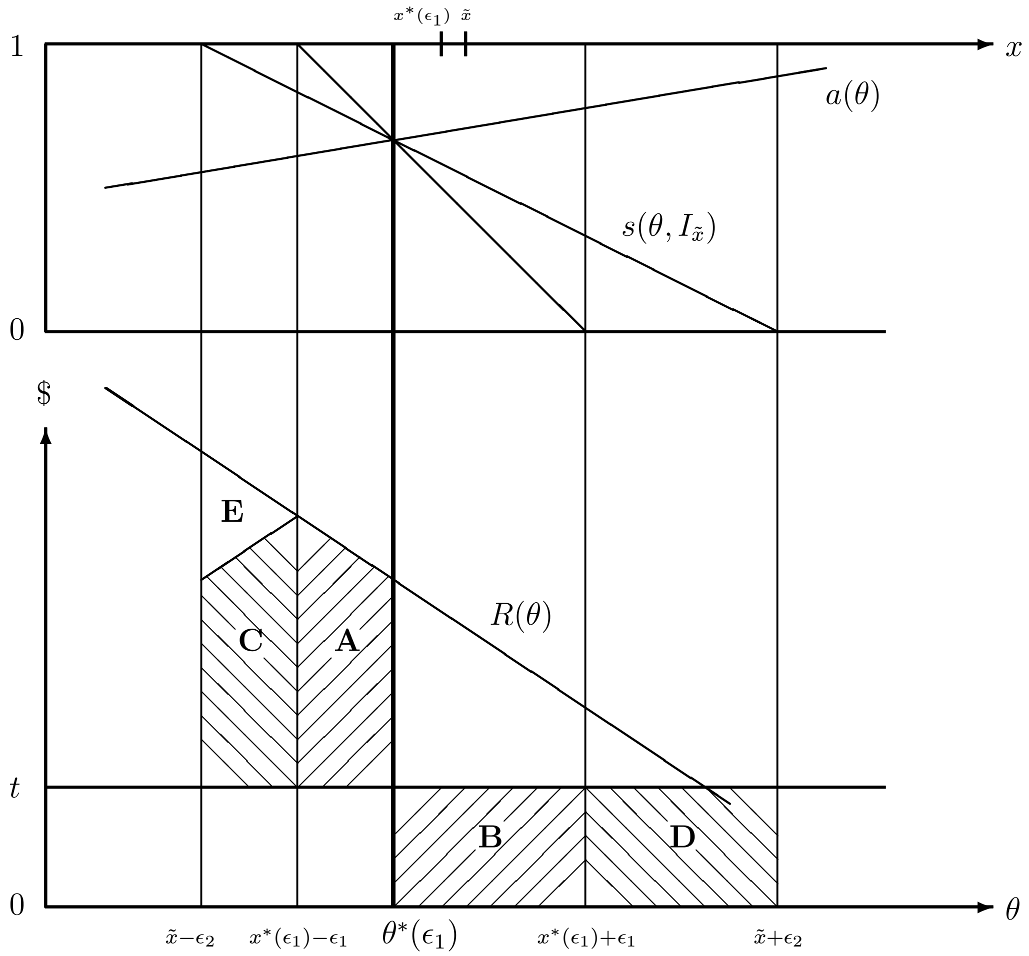

Figure 3 *Areas A and B are of equal size, so that $x^*(\epsilon_1)$ and $\theta^*(\epsilon_1)$ are the equilibrium switching point and threshold for signals distributed uniformly in an ϵ_1 surrounding of the true state. If dispersion is increased to ϵ_2 and strategies are adjusted to $I_{\tilde{x}}$, such that the threshold remains $\theta^*(\epsilon_1)$, then expected gains are given by $A+C+E$ and exceed expected losses $B+D$ by the triangular area E. Note that for $\epsilon_2 = 2\,\epsilon_1$, area $C = A$ and $D = B$.*

Footnotes

1 See Krugman (1979) for the first model of speculative attacks with a unique equilibrium.

2 This procedure has first been described by Morgenstern (1935). Brandenburger (1992) gives an overview over its relation to other equilibrium concepts and their decision theoretic foundations. The close relationship to sunspot equilibria has been analyzed by Heinemann (1997).

3 For an application of this method to a game of similar structure see Carlsson and van Damme (1993).

4 This can also be shown by application of a result by Milgrom and Roberts (1990), who proved that for supermodular games as ours the sets of pure Nash equilibria and rationalizable strategies have identical bounds.

5 except for $\bar{\theta}$, at which agents may be indifferent.

6 See, for instance, recent calls by IMF (1998a,b) and BIS (1998) for more transparency.

7 Of course, ex post, once the true state is known, government may renege on its commitment, but analyzing these incentives is left for future research.