Ordinal Item Response Theory Models for the Evaluation of Heterogeneity in Attitudes

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Abstract: The contribution provides ordinal item response models to investigate heterogeneity in attitudes. In contrast to standard models, the approach accounts for covariate effects in studying two essential components of attitudes: direction and strength. An application to the evaluation of presidential candidate traits exemplifies the types of substantive insights that can be gained.

Keywords: item response theory; ordinal responses; heterogeneity; attitudes.

1 Introduction

Item response theory (IRT) modeling to study attitudes relies on a battery of items that present statements linked to the latent trait. The standard response options are ordered grades, indicating the respondent's level of (dis)agreement with the statement. Several ordinal IRT models have been proposed (see Tutz 2020). A widely used one is the *Partial Credit Model* (PCM) (Masters 2016), which measures attitudes but does not account for heterogeneity linked to covariates. We present extensions of the PCM that incorporate covariate effects (Schauberger and Tutz 2022) to study heterogeneity in two attitude features: direction and strength. Direction refers to the substantive stances different individuals have about topics or persons. Strength refers to the intensity of the stances (Petty et al. 2023).

2 Modeling Heterogeneity in Attitudes

Let Y_{pi} denote the response of person p on item i and the data be given by $Y_{pi} \in \{0, 1, \ldots, k\}$, $p = 1, \ldots, P$, $i = 1, \ldots, I$, with $\{0, 1, \ldots, k\}$ representing ordered categories. The basic PCM can be given by

$$\log\left(\frac{P(Y_{pi} = r)}{P(Y_{pi} = r - 1)}\right) = \theta_p - \delta_{ir}, \quad r = 1, \ldots, k,$$

This paper was published as a part of the proceedings of the 39th International Workshop on Statistical Modelling (IWSM), Limerick, Ireland, 13–18 July 2025.

where person parameters θ_p give the location of person p and item parameters δ_{ir} indicate the location of the item on the latent attitude scale.
An extended model accounting for the effects of covariates $\boldsymbol{x}_p$ is

$$\log\left(\frac{P(Y_{pi}=r|\boldsymbol{x}_p)}{P(Y_{pi}=r-1|\boldsymbol{x}_p)}\right)=\theta_p+\boldsymbol{x}_p^{\mathrm{T}}\boldsymbol{\gamma}+(k/2-r+0.5)(\tilde{\theta}_p+\boldsymbol{x}_p^{\mathrm{T}}\boldsymbol{\alpha})-\delta_{ir}.$$

The term $\boldsymbol{x}_p^{\mathrm{T}}\boldsymbol{\gamma}$ specifies the *location effects* of covariates and captures attitude direction by modeling the level of (dis)agreement. If $\gamma_j > 0$, higher (agreement) categories are more likely when x_{pj} increases by one unit; if $\gamma_j < 0$, lower (disagreement) categories are more likely. $\tilde{\theta}_p$ is a person scaling parameter, and the factor $(k/2-r+0.5)$ causes the probabilities for either middle or extreme categories to increase by shifting the item parameters δ_{ir} (see Figure 1). If $\tilde{\theta}_p > 0$, the probability mass in the middle categories increases. Thus, individuals selecting middle categories show weak attitudes. If $\tilde{\theta}_p < 0$, the probabilities for extreme categories increase. Thus, individuals choosing extreme categories show strong attitudes. The term $\boldsymbol{x}_p^{\mathrm{T}}\boldsymbol{\alpha}$ specifies the *scaling effects* of covariates, representing attitude strength. If $\alpha_j > 0$ ($\alpha_j < 0$), middle (extreme) categories are more likely.

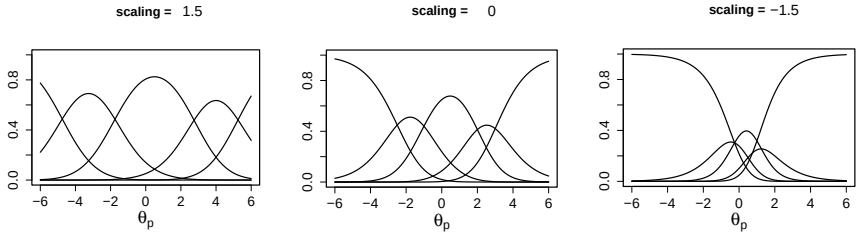

FIGURE 1. Probabilities with differing scaling.

Parameter estimates can be obtained by the Marginal Maximum Likelihood estimation, which assumes a fixed distribution for the person parameters. Let $f(\theta_p, \tilde{\theta}_p)$ be the density of the assumed distribution. Typically, a normal distribution with mean zero and covariance $\boldsymbol{\Sigma}$, indicated by $f_{0,\boldsymbol{\Sigma}}(.)$, is assumed. Then, the marginal log-likelihood is

$$l(\boldsymbol{\gamma},\boldsymbol{\alpha},\{\delta_{ir}\})=\sum_{p=1}^{P}\log(\int\prod_{i=1}^{I}\prod_{r=1}^{k}P(Y_{pi}=r)^{y_{pir}}f_{0,\boldsymbol{\Sigma}}(\theta_p,\tilde{\theta}_p)d\theta_p d\tilde{\theta}_p).$$

3 Application

We investigate the evaluation of presidential candidate traits, also referred to as valence (e.g., Mauerer and Tutz 2024): strong leadership, cares, knowledgeable, honest, speaks mind, even-tempered with five response options from 'not at all' to 'extremely' (American National Election Study). We analyze the candidates Clinton and Trump running in 2016 and consider standard demographics: gender (1 female, 0 male), age, self-identifications as African American (binary) or Latino (binary), and education (from 8 to 16 years); variables are standardized; N=2257. We estimated two models: The first specifies location effects (attitude

direction) only; the second adds scaling effects (attitude strength). The covariates are very useful in explaining both heterogeneity in attitude direction and strength (Table 1). Likelihood ratio tests demonstrate that considerable scaling effects are present that should not be neglected to avoid biased location effects. The results also indicate that the models fit way better for Trump than Clinton.

TABLE 1. Effect Parameter Estimates.

	Location		Location		Scaling	
Clinton	γ	s.e.	γ	s.e.	α	s.e.
female	0.26	0.04	0.25	0.04	0.03	0.02
age	-0.02	0.04	0.01	0.04	-0.02	0.02
African American	0.50	0.03	0.53	0.04	0.13	0.03
Latino	0.29	0.04	0.30	0.04	0.08	0.02
education	0.24	0.04	0.27	0.04	0.06	0.02
Loglik. (df)	-16222.80 (30)		-16016.47 (37)			
LR test	$\chi^2(7) = 412.66$					
Trump	γ	s.e.	γ	s.e.	α	s.e.
female	-0.12	0.03	-0.13	0.04	-0.16	0.03
age	0.12	0.03	0.10	0.04	0.11	0.03
African American	-0.46	0.02	-0.40	0.04	-0.26	0.03
Latino	-0.23	0.04	-0.33	0.02	-0.22	0.03
education	-0.28	0.03	-0.31	0.04	-0.16	0.03
Loglik. (df)	-14426.60 (30)		-13738.53 (37)			
LR test	$\chi^2(7) = 1376.15$					

Almost all location effects are significant and indicate markedly polarized attitudes. Females, African Americans, Latinos, and higher educated have a positive orientation (valence) toward Clinton; the same subgroups have a negative orientation toward Trump. The largest differences are observed for African Americans. In addition, older people evaluate Trump more positively, whereas there are no significant age differences when evaluating Clinton. All location effects are also more considerable for Clinton than Trump, suggesting the subgroups that positively (or negatively) evaluate the candidates do so much more toward Clinton than Trump.

Most scaling effects are significant as well, except gender and age for Clinton. We also see pronounced polarization here and the reversed pattern as for the location effects. All scaling effects are larger for Trump than Clinton. All subgroups with a positive valence toward Clinton hold weaker attitudes, and all subgroups with a negative valence toward Trump hold very strong attitudes. Only older people show weak attitudes in their evaluation of Trump. Especially African Americans, with a more negative orientation toward Trump than the others, are the ones with the highest attitude strength. The same subgroup, who more positively evaluate Clinton than the others, holds the weakest attitude strength toward Clinton.

4 Conclusion

We presented models for studying heterogeneity in attitudes. The more we understand how attitudes vary in the population, the better we understand their impacts on crucial aspects of representative politics, such as voting (e.g., Mauerer and Tutz 2023). We contribute to this goal through explanatory ordinal item response modeling.

Acknowledgments: Ingrid Mauerer acknowledges financial support from the program EMERGIA (EMC21-00256), Junta de Andalucia.

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