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Abstract

This note relates the mechanisms that are based on mediated contracts of Rahman and Obara (2010) to the mechanisms of Myerson (1982). It shows that the mechanisms in Myerson (1982) are more general in that they encompass the mechanisms based on mediated contracts. It establishes an equivalence between the two classes if mediated contracts are allowed to be stochastic.

Keywords: mediated contract, mechanism design, revelation principle, moral hazard

1 Introduction

In an inspiring paper, Rahman and Obara (2010) introduce the concept of *mediated contracts*.[1] The authors clearly show that, in team problems with balanced budgets, mediated contracts outperform standard, non–mediated ones and even enable a virtual implementation of the first best. What remains less clear is how their concept of mediation relates to other concepts of mediation in earlier work on mechanism design (e.g., Myerson 1986, 1991, Forges 1986). This note tries to clarify this link by contrasting mediated contracts to the mechanisms of Myerson (1982), who extends the revelation principle to settings with moral hazard.[2] It argues that the

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[1]See also Rahman (2009) for the use of mediated contracts in a monitoring context.

[2]Both Myerson (1982) and Rahman and Obara (2010) allow, next to moral hazard, also for ex ante private information, but because the conceptual difference relates to the moral hazard problem, this notes focuses on the moral hazard and abstracts from ex ante private information.

mechanisms in Myerson encompass the mechanisms based on mediated contracts and are more general in that they allow for a larger degree of randomization. An equivalence between the two classes of mechanisms obtains if one extends mediated contracts to stochastic mediated contracts that also condition on purely random signals. The equivalence implies that there is no loss of generality in restricting attention to incentive compatible mediated contracts in the sense of Rahman and Obara (2010) if such contracts are allowed to be stochastic.

2 Setup

I illustrate my arguments in a version of Holmström (1982)'s seminal moral hazard in teams problem. There are two agents who each have to pick an effort level $e \in R_+$.[3] The cost of effort is $c(e) = e^2/2$. The output's value is linear in effort: $y(e_1, e_2) = e_1 + e_2$. Agents are identical and have the quasi–linear utility function

$$u(t, e) = t - c(e),$$

where t represents a monetary transfer. In this quasi–linear framework, the Pareto efficient pair of effort levels (e_1^*, e_2^*) is unique. It maximizes $y(e_1, e_2) - c(e_1) - c(e_2)$ and exhibits $e_1^* = e_2^* = 1$.

When effort levels are unobservable, the agents cannot implement the efficient effort levels directly. When output is observable, agents can however write binding contracts how to share the proceeds from the final output. These contracts induce a simultaneous move game in which the two agents each pick an effort level non–cooperatively. Holmström (1982) shows however that there does not exist a contract pair $(t_1(y), t_2(y))$ that satisfies the budget balance condition

$$t_1(y) + t_2(y) = y$$

and induces a simultaneous move game with a Nash–equilibrium in which the agents pick the efficient effort levels $e_1^* = e_2^* = 1$.

[3] To circumvent measure theoretical complications, Rahman and Obara (2010) derive their formal results for finite action spaces, but this example shows their construction works just as well with infinite action spaces.

3 Mediated Contracts

Rahman and Obara (2010) extend the concept of a contract to mediated contracts by introducing a mediator. In line with earlier literature, the mediator merely facilitates communication between the two agents; he gives only non–binding recommendations about effort levels. The new idea of Rahman and Obara (2010) is to condition the contract on the mediator's recommendations.[4] Hence, the contract no longer conditions transfers only on the output level, but also on the mediator's recommendations. In the Holmström example above, this means that, instead of $t_i(y)$, contracts are expressed by $t_i(y, e_1^r, e_2^r)$ where e_i^r represents the mediator's recommendation to agent i.

The timing with mediated contracts is as follows: First, the agents agree on some mediated contract pair $(t_1(y, e_1^r, e_2^r), t_2(y, e_1^r, e_2^r))$ and a probability distribution over effort pairs (e_1, e_2). The mediator then draws an effort pair (e_1^r, e_2^r) according to this pre–specified distribution and reports to each agent i the drawn effort level e_i^r as a confidential recommendation. Only knowing their own recommendation, the two agents choose their effort levels simultaneously. Finally, the chosen effort levels result in a final output y and leads to transfers according to the pre–determined contract pair $(t_1(y, e_1^r, e_2^r), t_2(y, e_1^r, e_2^r))$. Rahman and Obara (2010) consider mediated contracts that balance the budget and are Bayes' incentive compatible in the sense that each agent has an incentive to follow the recommendation if the other agent follows the recommendation.

In order to illustrate the power of mediated contracts in the Holmström framework, consider the following specific one. With probability $1 - \varepsilon$ the mediator recommends each agent the efficient effort level: $(e_1^r, e_2^r) = (1, 1)$. With probability $\varepsilon/2$ the mediator recommends agent 1 the efficient effort level and agent 2 to shirk: $(e_1^r, e_2^r) = (1, 0)$. Finally, with probability $\varepsilon/2$ the mediator recommends agent 1 to shirk and agent 2 to work efficiently: $(e_1^r, e_2^r) = (0, 1)$. Crucially, recommendations are given confidentially so that when agent i receives the recommendation $e_i^r = 1$, he is unsure about the recommendation e_j^r of the other agent $j \neq i$. He, however, knows the probability distribution according to which the mediator gives

[4] The recommendations, therefore, need to be verifiable, but this seems unproblematic: The mediator writes each recommendation on a piece of paper and sends each piece of paper to the respective agent. After the output occurs, agents are obliged to reveal the pieces of paper publicly.

recommendations. Using Bayes' rule, agent i, therefore, expects that $e_j^r = 0$ with probability

$$\mu_0 = \Pr\{e^j = 0 | e^i = 1\} = \frac{\varepsilon/2}{1-\varepsilon+\varepsilon/2} = \frac{\varepsilon}{2-\varepsilon}$$

and $e_j^r = 1$ with probability

$$\mu_1 = \Pr\{e^j = 1 | e^i = 1\} = 1 - \mu_0 = \frac{2-2\varepsilon}{2-\varepsilon}.$$

In addition, let the contract depend on the recommendation (e_1^r, e_2^r) as follows. For any pair of recommendations the output is shared equally, except for the recommendation $(1,0)$ and $(0,1)$. For these two special cases, the agent who is given the recommendation to work efficiently ($e^r = 1$) receives $1/\varepsilon$ times the output as a transfer, whereas the agent who is told to shirk ($e^r = 0$) receives the (negative) transfer $(\varepsilon - 1)y/\varepsilon$. Formally,

$$t_1(y, e_1^r, e_2^r) = \begin{cases} y/\varepsilon, & \text{if } (e_1^r, e_2^r) = (1,0), \\ (\varepsilon-1)y/\varepsilon, & \text{if } (e_1^r, e_2^r) = (0,1), \\ y/2, & \text{otherwise;} \end{cases} \quad t_2(y, e_1^r, e_2^r) = \begin{cases} (\varepsilon-1)y/\varepsilon, & \text{if } (e_1^r, e_2^r) = (1,0), \\ y/\varepsilon, & \text{if } (e_1^r, e_2^r) = (0,1), \\ y/2, & \text{otherwise.} \end{cases}$$

By construction, the contract's budget is balanced.

The transfer schedule together with the mediator's probability distribution over recommendations is Bayes' incentive compatible: First, given a recommendation to shirk $e_i^r = 0$, agent i is certain that agent $j \neq i$ has received the recommendation $e_j^r = 1$ so that agent i expects a negative transfer $(\varepsilon - 1)y/\varepsilon$ from the output y. Given these expectations, it is a dominant strategy to follow the mediator's recommendation $e_i^r = 0$. Alternatively, given the recommendation $e_i^r = 1$, agent i believes that agent $j \neq i$ received and follows the recommendation $e_j^r = 0$ with probability μ_0 and the recommendation $e_j^r = 1$ with probability μ_1. In the former case, his effort level e_i yields him a transfer e_i/ε. In the latter case, the effort level e_i yields him a transfer $(1 + e_i)/2$. Hence, his expected utility from an effort level e_i is

$$\mu_0 e_i/\varepsilon + (1-\mu_0)(1+e_i)/2 - e_i^2/2 = \frac{1-\varepsilon}{2-\varepsilon} + e_i - e_i^2/2.$$

Clearly, $e_i = 1$ maximizes this expression so that it is a best response for agent i to follow the recommendation $e_i^r = 1$.

Because, for any $\varepsilon \in (0,1)$ the corresponding mediated contract implements the efficient effort choices $e_1 = e_2 = 1$ with probability $1-\varepsilon$, the mediated contract virtually implements the efficient outcome as ε approaches 0. This result contrasts sharply with Holmström's inefficiency result and demonstrates the power of mediated contracts.

4 Mechanism Design

The literature on communication and mechanism design has noted the beneficial role of mediators in implementation problems before. In particular, Myerson (1982) introduces a mediator in a simultaneous move game with moral hazard.[5] Similar to Rahman and Obara (2010), the mediator in Myerson makes confidential recommendations before the agents make their choices. In contrast, Myerson does not capture contractual commitments by a formal contract, but by the way the mediator is "programmed" to choose among the contractible variables.

In the context of the Holmström model, the contractible variable is the pair of conditional transfer schedules $(t_1(y), t_2(y))$. Hence, Myerson's interpretation is that the mediator chooses the recommendations *and* the transfer schedules, but the picked transfer schedules are revealed only after the agents have chosen their effort levels. The mechanism describes exactly how the mediator is to pick the recommendations and the pair of conditional transfer schedules $(t_1(y), t_2(y))$. This reveals the main conceptual difference between the two frameworks: In Myerson (1982), a mechanism is a probability distribution over the combination of both recommendations and the pairs of conditional transfer schedules $(t_1(y), t_2(y))$. In Rahman and Obara (2010), a mechanism is a specific pair of mediated transfer schedules $(t_1(y, e_1^r, e_2^r), t_2(y, e_1^r, e_2^r))$ and a probability distribution over only the recommendations.[6]

Hence, Myerson's framework seems, on the one hand, more general, because the mediator randomizes over both recommendations and pairs of conditional transfer schedules, whereas Rahman and Obara (2010) allow only randomizations over recommendations. On the other hand, Myerson's framework seems less general, because it allows only transfer schedules that condition on output, whereas Rahman and Obara (2010) allow transfer schedules that condition also on the mediator's recommendations.

Note however that the second suggestion is misleading. Myerson explicitly allows the mediator to randomize over *combinations* of recommendations and contracts. This, in particular, allows correlation between random draws of recommendations and random draws of transfer schedules. As a consequence, one can, for any mechanism in the framework of Rahman and

[5]See also Forges (1986) and Myerson (1986), (1991).

[6]When the action set of the agents is finite but transfers are non–countable, the definition of a mechanism in Rahman and Obara (2010) is, from a measure theoretical viewpoint, simpler than Myerson's view, because Myerson's definition also allows mixing over transfers.

Obara, find an equivalent one in the Myerson framework. In the specific example considered above, the equivalent mechanism in the sense of Myerson is a mediator who mixes over the three combinations $(e_1^r, e_2^r, t_1, t_2) = (1, 1, y/2, y/2)$, $(e_1^r, e_2^r, t_1, t_2) = (1, 0, y/\varepsilon, (\varepsilon - 1)y/\varepsilon)$, and $(e_1^r, e_2^r, t_1, t_2) = (0, 1, (\varepsilon - 1)y/\varepsilon, y/\varepsilon)$ with the respective probabilities $1 - \varepsilon$, $\varepsilon/2$, and $\varepsilon/2$.

Considering the question whether the Myerson framework is more general, note that mediated contracts as defined in Rahman and Obara (2010) do not allow for transfer schedules that, conditional on the recommendation (e_1^r, e_2^r), are random. In contrast, Myerson explicitly allows for this.[7] Because Rahman and Obara (2010) consider a quasi–linear framework, where all agents are risk neutral with respect to transfers, this distinction does not matter. Any transfer schedule that, conditional on the recommendation, is random can be replaced by a deterministic transfer schedule that corresponds to its expected value. This does neither affect the agent's utilities nor the balanced budget constraint.

Rasmusen (1987), however, shows that, in teams with risk averse agents, Pareto efficient allocations are implementable, but this requires random transfers off–the–equilibrium. Because mechanisms in the sense of Rahman and Obara (2010) cannot capture such random transfers, whereas the mechanisms in the sense of Myerson (1982) can, the mechanisms of Rahman and Obara are, in the setup of Rasmusen (1987), suboptimal in comparison to Myerson.[8]

A straightforward remedy to the suboptimality is to consider *stochastic* mediated contracts that, in addition to recommendations, can also condition on a purely random signal.[9] This results in a class of mechanisms that is equivalent to Myerson (1982) in that there is a one–to–one correspondence between the two. Stochastic mediated contracts are, therefore, an alternative representation of the mechanisms in Myerson (1982).

[7] It is for this reason that, as noted in footnote 6, mechanisms in the sense of Rahman and Obara are simpler constructs.

[8] I thank Johannes Münster for pointing out that, with risk aversion, it is even not clear if mediated contracts allow a virtual implementation of Pareto efficient allocations. E.g., the above construction in Holmström's framework allows an implementation of the efficient effort levels with a probability arbitrarily close to one, but with an unbounded variance in the agents' utility.

[9] For stochastic mediated contracts see also Rahman (2005).

5 Conclusion

Mechanisms based on stochastic mediated contracts are formally equivalent to the mechanisms in Myerson (1982). They have, however, two interpretational advantages. First, they make more explicit the correlation between the mediator's recommendations and contract choice. Second, they seem more natural, because the contract choice is explicitly left to the economic agents and the mediator is limited to providing only recommendations. This representation is also closer to the fundamental idea of agency theory that contracts are used to provide incentives. Moreover, because Myerson (1982) derives his class of mechanisms from a revelation principle, the equivalence further implies that there is no loss of generality in restricting attention to incentive compatible mediated contracts in the sense of Rahman and Obara (2010) if such contracts are allowed to be stochastic.

References

[1] Forges, F. (1986), "An Approach to Communication Equilibria", *Econometrica* 54, 1375–1385.

[2] Holmström, B. (1982), "Moral Hazard in Teams", *Bell Journal of Economics*, 13, 324340

[3] Myerson, R. (1982), "Optimal Coordination Mechanism in Generalized Principal-Agent Problems", *Journal of Mathematical Economics* 10, 67–81.

[4] Myerson, R. (1986), "Multistage games with communication", *Econometrica* 54, 323-358

[5] Myerson (1991), Game Theory: Analysis of Conflict, Harvard University Press, Cambridge, Massachusetts, London, England.

[6] Rahman, D. (2005), "Optimum Contracts with Public and Private Monitoring", mimeo University of California, San Diego.

[7] Rahman, D. (2009), "But Who Will Monitor the Monitor?", mimeo University of Minnesota.

[8] Rahman, D. and Obara, I. (2010), "Mediated Partnerships", *Econometrica* 78, 285 – 308.

[9] Rasmusen, E. (1987), "Moral Hazard in Risk-Averse Teams", *The RAND Journal of Economics* 18, 428–435.