GOVERNANCE AND THE EFFICIENCY
OF ECONOMIC SYSTEMS
GESY

Discussion Paper No. 321

On Liability Insurance for Automobiles

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May 2010

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

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May 11, 2010

Abstract. Car owners are liable for property damage inflicted on other motorists. In most countries such liability must be insured by law. That law may favor expensive or heavy vehicles, prone to suffer or inflict large losses. This paper explores links between liability rules and vehicle choice. It presumes cooperative insurance, but non-cooperative acquisition of vehicles. Thus, the Nash equilibrium and its degree of efficiency depend on the liability regime.

Keywords: liability, mutual insurance, core, pure Nash equilibrium, anonymous games, non-atomic measure.

JEL Classification: C71, C72, D61, K13.

1. Introduction

As motivation for the present inquiry, recall the following somewhat stylized story: Until unification of Germany, the East-German car pool comprised mostly inexpensive species - such as the two-cylinder "Trabbi". Soon thereafter, many a well-to-do eastern citizen replaced his Trabbi with an expensive Mercedes Benz. As a result, the risk exposure of Trabbi owners worsened, whence their insurance rates went up. At that time, East-German owners of Mercedes Benz enjoyed low liability risk and thereby favorable premiums. The implicit transfer from the first group to the second contributed to the speedy change of the East-German car pool - and in part, to the accelerated extinction of the infamous Trabbi.

Albeit rather unique, these events indicate some lack of efficiency in quite common liability rules, a lack that calls for closer analysis and maybe legal reform. Three interrelated issues occupy center stage in this regard. One, already mentioned, concerns monetary transfers from good to bad risks - a fairly frequent problem of insurance.

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The second issue revolves around internalization of costs. Third, there is the question whether established policies favor introduction of relatively expensive or heavy cars, prone to suffer or inflict great damage.

Insurance is modelled here as a mutual company. Given the collection of cars, liability risks are pooled so as to provide efficient and acceptable risk sharing. The availability of such sharing, and its specific form, may, however, tilt the choice of automobile toward species that cause large property losses.

To bring out that argument, the analysis below combines various concepts of game theory. Risk sharing is a cooperative business, yielding a *core solution* that captures all gains from exchange. In contrast, choice of car is non-cooperative - hence part of *Nash equilibrium* in a large, anonymous game where every player behaves as though he does not affect insurance premiums.

Thus aggregation operates twice, but at different levels: first, by the sharing of pooled risks; second, by affecting vehicle choice via insurance premiums. Aggregation assures existence of equilibrium at both levels. In fact, risk exchange between numerous agents - each bringing a moderate-size, almost independent risk - easily generates a price-supported core imputation. Further, since individual choice of vehicle has negligible overall effects, a pure-strategy Nash equilibrium exists under fairly weak assumptions. In short, solutions are available, but they might be unstable, inefficient, or unfair.

To argue why, the paper is planned as follows. For motivation Section 2 depicts two examples. Common to these are negative externalities caused by particular automobiles or drivers. Section 3 outlines a model in which vehicle acquisition depends on - and affects - liabilities. Section 4 formalizes liabilities as random variables and casts insurance as mutual. Thus policies and premiums become endogenous and built into car owners' objectives. Upon facing the pool of automobiles, each buyer chooses a vehicle so as to maximize his reduced form of objective function. Section 5 considers existence of pure Nash equilibrium. A few properties of such equilibrium are singled out in Section 6, and Section 7 concludes.

2. Motivating Examples

We consider property damage caused by collisions between cars. The chances of such accidents - and the attending losses - depend on numerous factors, including traffic density and vehicle types. The following examples focus on liability and insurance, but ignore personal injuries, supposing that each car perfectly protects its driver.[1]

[1] Some carmakers promise extra safety for drivers of their vehicles. It is hard, however, to protect fully against material damages. In fact, there appears to be a tradeoff between the two sorts of protection.

Example 2.1 (*One Mercedes Benz among Trabbis*) Suppose there is *one* quite expensive vehicle, called Mercedes. Further suppose its driver is so marvelously skilled as to never cause any accident. Chances are, however, that other motorists, all circulating in utterly cheap cars, will collide with the one and only Mercedes. Even the lightest of such collisions causes very expensive repairs on that vehicle. Clearly, if standard liability is in vigor, the Mercedes owner needs no insurance coverage. Yet he exposes other motorists to greater risk. If liability insurance is mandatory, and standard policies are offered at reasonably fair premiums, the owner of the exceptional car gets a bargain. Since all others suffer, they might think tort and liability law sorely in need of reform. Instead of standard liability they would prefer a pure *no fault regime*, meaning one that merely covers damages on own car. $\diamondsuit$

Example 2.2 (*One SUV among Trabbis*) Now replace the Mercedes with a heavy, quite solid car, called SUV. As before, all other vehicles are light and fragile. But driving skills are reversed. That is, suppose no driver except the one who owns the SUV can cause any accident. Whenever he collides, his vehicle suffers negligible damage but totally scraps the countering car. Clearly, only the exceptional driver needs liability insurance. All others would vote for a *standard liability regime* which covers damages inflicted on the other car. Otherwise compulsory insurance might appear unfair and inefficient. $\diamondsuit$

Thus, either liability regime, standard or no fault, produces undesirable incentives. Common to these regimes is the negligence of vehicle brands and drivers' behavior or skills. In fact, after collision, liability issues mainly hinge upon damages and responsibilities. Plainly, so narrow a perspective ex post induces distortions ex ante. While a no fault regime favors cheap cars, the standard regime offers advantages for heavy lorries and SUVs. To mitigate such lack of efficiency and fairness it appears prudent to condition liability on vehicle and driver type. A setting for doing so is spelled out in Section 4. But first comes the main structure of our approach.

3. Structure of the Model

This section outlines the model, deferring details on liabilities to the subsequent section. There are *two* stages, recursively connected and brought together in Section 5. At the second stage, after cars have been chosen, liabilities are shared and insured. At the first stage, while anticipating insurance premiums, each agent chooses a car. We begin with the second stage.

Insurance. Automobile owners constitute a large ensemble $\mathbb{A}$, comprising n members, referred to as agents.[2] For a start, suppose everybody has already chosen his car. Agent $a \in \mathbb{A}$ thereafter faces liability L_a and gets expected utility $Eu_a(-X_a)$ by

[2] If a household owns several autos, they could be assigned to different members.

retaining risk X_a and pays premium $\rho(L_a - X_a)$ for risk shed. Assume each utility function $u_a(\cdot)$ is concave, strictly increasing and, for now, state-independent.

Suppose agents measure utilities and liabilities in money, and regard either entity as perfectly transferable. Accordingly, for the purpose of mutual risk-sharing, they

$$\text{maximize } \sum_a Eu_a(-X_a) \text{ subject to } \sum_a X_a \geq \sum_a L_a,$$

and agree ex ante to evaluate state-contingent risk transfers by a shadow price density P, associated to the above inequality. Thus, when $\rho(L_a - X_a) = E\left[P \cdot (L_a - X_a)\right]$, agent a gets the *net payoff*

$$\pi_a := \sup_{X_a} E\left[u_a(-X_a) - P \cdot (L_a - X_a)\right]. \tag{1}$$

From Wilson (1968), the solution to this maximization problem has the following properties:

Proposition 3.1 (On mutual risk-sharing)
• *The sum* $\sum_a \pi_a$ *of net payoffs* (1) *depends only on the aggregate liability* $L := \sum_a L_a$.
• *Likewise, agent a gets a gross payoff*

$$\Pi_a := \sup_{X_a} E\left[u_a(-X_a) + P \cdot X_a\right]$$

that depends only on the said aggregate L.
• *Granted differentiability and interior solutions, Borch's (1962) rule holds*:

$$P = u_a'(-X_a) \text{ for each } a \text{ and optimal } X_a.$$

• *If agents have the same utility function, their gross payoffs Π_a coincide, and the net payoffs π_a differ only in the gross premiums $E\left[P \cdot L_a\right]$.*
• *Moreover, in that case, everyone retains the same share $X_a = L/n$ of the aggregate liability.* □

Acquisition of car. Stepping back to the first stage, suppose agent a contemplates to buy a vehicle from a finite list V of different brands. Most likely, his choice $v \in V$ directly affects his utility function, as indicated by expressly writing u_{av} in lieu of u_a. Thus, with reference to (1), agent a should

$$\text{maximize } \pi_{av} := \sup_{X_a} E\left[u_{av}(-X_a) - P \cdot (L_a - X_a)\right] \text{ over } V. \tag{2}$$

Our main interest is to establish existence of an equilibrium vehicle profile and explore some of its properties. To address these issues we must spell out how agents interact.

Doing so amounts to close and complete the model by describing how liabilities depend on underlying data and the profile of automobile acquisitions.

Agents differ in *type* $t \in T$ and ownership of vehicle $v \in V$, the set T being fixed and finite. Any type-vehicle combination $c = (t, v) \in T \times V =: \mathbb{C}$ is referred to as a *characteristic*. By slight abuse of notation, we write $a \in c$ to indicate that agent a is of type t_a, and owns a vehicle v_a for which $c = (t_a, v_a)$. In principle, each characteristic $c = (t, v)$ is possible and perfectly observable. Further, characteristics are presumed so informative as to preclude concerns with adverse selection.

Let n_c be the number of car owners with characteristic $c \in \mathbb{C}$. These numbers define a frequency *distribution* $\delta = (\delta_c) := (n_c/n)$ across $\mathbb{C}$. In turn δ - alongside the liability regime - defines P, L_a and how these variables are distributed. The links are explained in the subsequent section.

For now, it suffices to stress that individual objectives, defined in (2), depend on the distribution δ. Accordingly, let $\pi_{av}(\delta)$ denote the expected net payoff - or *indirect expected utility* - that accrues to automobile owner a, upon choosing vehicle v, when faced with distribution δ.

A pure strategy, vehicle profile $a \in \mathbb{A} \mapsto v_a \in V$, denoted (v_a), is declared a *Nash equilibrium* iff it generates a distribution δ across $\mathbb{C}$ such that

$$\pi_{av_a}(\delta) \geq \pi_{av}(\delta) \text{ for all } a \in \mathbb{A} \text{ and } v \in V. \tag{3}$$

The crucial assumption behind this definition is that *no single driver affects the distribution* δ - or at least, each behaves as though he does not. Justification for assuming so derives from the size of the car pool; drivers are in millions and of diverse types, each type comprising thousands.[3] In short: the game is large; every player is anonymous and negligible; and interaction is perceived merely via aggregates; see Schmeidler (1973).

Clearly, ownership of a *single* car precludes randomized use of several such objects. Our interest is therefore with the existence and properties of *pure* equilibria. To address these issues we must formalize liabilities next - and thereafter spell out how insurance affects individual payoffs $\pi_{av}(\delta)$.

4. Liability and Insurance

This section deals with drivers' *behavior, liability, insurance,* and *indirect utility* - in that order. The aim is to outline how the function $\pi_{av}(\delta)$ in (3) could derive from underlying, more primitive data.

[3] Clearly, if all drivers of some type opt to change vehicle, the distribution is affected. But strategic coordination of such sort is neither plausible nor part of the setting.

Behavior. Independent of each other, each automobile owner $a \in c$ equally randomizes his driving behavior b across a fixed, finite set $\mathbb{B}$. Whenever he is party to a collision, suppose b is identifiable ex post. Such identification is likely to influence the apportioning of fault and liability. And clearly, after-collision reconstruction of behavioral modes helps mitigating moral hazards.

Liability. We consider automobile crashes that cause property damages. The basic event is a collision between two cars from a closed pool. So, implicitly we deal with a time span during which exactly *one* collision occurs.

There is a finite list $\mathbb{D}$ of possible damages, all measured in money and non-negative.[4] Loss of life or limb is regarded as a separate insurance problem and hence not dealt with. Alternatively, for the sake of a simpler argument, one may suppose that each automobile offers perfect protection against personal injuries.

After any collision, by assumption, there is no doubt as to the agents involved, their behaviors, characteristics, and damages.[5] Correspondingly, whatever be the apportioning of fault or liability, we assume that transaction costs be negligible. Put differently: by assumption, there are no litigation costs, and no return to effort, skill, or time in filing a lawsuit.[6]

Formally, to realize - or simulate - a collision amounts to random draw of an outcome

$$\omega = (a, b, c, d; a', b', c', d')$$

from the finite sample set $\Omega := (\mathbb{A} \times \mathbb{B} \times \mathbb{C} \times \mathbb{D})^2$. This particular draw, referred to as *state* $\omega \in \Omega$, means that agents a, a' - with respective behaviors b, b' and characteristics c, c' - have collided and thereby inflicted damages d, d' on their vehicles. Mentioning agent a first indicates that he *caused* the accident - maybe at fault, but not intentionally.

One should think of state ω as the result of a hypothetical experiment - or simulation - that proceeds in *four* steps: *First*, given the distribution δ, orderly select two categories $c, c' \in \mathbb{C}$ by their relative frequency and propensity to collide. *Second*, randomize behavioral choices to find $b, b' \in \mathbb{B}$. *Third,* choose conditional damages d, $d' \in \mathbb{D}$. *Fourth*, pick a culprit $a \in c$ in equiprobable manner and likewise a victim $a' \in c'$.

[4] There is no problem in letting $\mathbb{D}$ be an interval. However, by requiring $\mathbb{D}$ to be finite we avoid some purely technical issues concerning measurability and integrability.

[5] By hypothesis, the concerned parties do not exaggerate or underestimate various damages. In particular, they agree on what harms are too ephemeral, speculative or remote to merit compensation.

[6] Under a *pure no fault system* the driver would be barred from ever suing another driver for damages. That system expressly intends to reduce the legal and administrative fees associated with insurance claims.

Thus the state set Ω is endowed with an objective probability measure, defining an expectation operator E.[7] Note that accident risk is construed as endogenous and almost idiosyncratic. Systematic or exogenous risks - caused by say, the weather - are presumed unimportant or constant, hence ignored.

Let $\mathbb{X} := \mathbb{R}^\Omega$ denote the linear space of contingent claims or liabilities. Any member $X \in \mathbb{X}$ is a random variable $\omega \mapsto X(\omega)$. An automobile owner $a \in c$ who collides with $a' \in c'$ faces state-dependent liability

$$L_a(\omega) := \begin{cases} I(b,c,d;b',c',d') & \text{when } \omega = (a,b,c,d;a',b',c',d'), \\ i(b,c,d;b',c',d') & \text{when } \omega = (a',b',c',d';a,b,c,d), \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

Plainly, L_a belongs to $\mathbb{X}$, and it depends on who caused the collision. But it does not depend on the names $a \in c$, $a' \in c'$ of the colliding parties; only their behaviors, characteristics and damages come into consideration.[8] In particular, as long as $a \in c$, we may write L_c instead of L_a. Note that, unlike Shavell's (1982) setting, care is bilateral here, and both parties to a collision may suffer direct liabilities.

Part of the rationale for liability is to deter reckless or criminal behavior. When b or b' qualify as such, L_a typically contains an element of punishment. Liability may also reflect on various psychological phenomena, be it diminished rationality, weakness of will, or lack of attention.

In (4) the functions $I, i : (\mathbb{B} \times \mathbb{C} \times \mathbb{D})^2 \to \mathbb{R}$ prescribe the expenses of the injurer and the injured respectively. Typically, the forms of these indemnity functions $I(\cdot)$ and $i(\cdot)$ are influenced by liability law and insurance institutions. Together the two functions characterize and define the regime at hand.

Example 4.1 (*Standard versus a no fault regime*) Consider two disparate liability regimes that are widely used. In the first, the culprit is responsible for the entire damage $d + d'$:

$$L_a(\omega) := \begin{cases} d + d' & \text{if } \omega = (a,b,c,d;a'b',c',d'), \\ 0 & \text{otherwise.} \end{cases} \tag{standard}$$

The second regime holds each party responsible merely for the damage on the own car:

$$\hat{L}_a(\omega) := \begin{cases} d & \text{if } \omega = (a,b,c,d;a',b',c',d'), \\ d & \text{if } \omega = (a',b',c',d';a,b,c,d), \\ 0 & \text{otherwise.} \end{cases} \tag{no fault}$$

[7] Empirical estimates of that measure can be constructed from extensive data bases; see Chiappori and Salanié (2000).

[8] Put differently: although the insurance policy is written for its particular holder, by assumption, when executed, it only invokes general structure, observable characteristics, and non-disputable contingencies.

Since $\sum_a L_a = \sum_a \hat{L}_a$, we get $\sum_a \pi_a = \sum_a \hat{\pi}_a$ and $P = \hat{P}$. Suppose utilities independent of state and vehicle - and identical so that $u_a = u$. Then, agents opt to retain the same risk $X_a = \bar{L} := \sum_a L_a/n$. The price density becomes $P = u'(-\bar{L})$, giving uniform gross payoffs

$$\Pi_a = \hat{\Pi}_a = Eu(-\bar{L}) + E\left[u'(-\bar{L}) \cdot \bar{L}\right] \text{ for each } a.$$

However, the difference between net payoffs can become large. To see this, let $\mathbb{A}$ comprise merely two agents: a, a', owning vehicles v, v', and suppose damages d, d' are deterministic. Ex ante expectation is then taken over only three exclusive events, namely: either *one* agent a or a' causes an accident, presumed to happen with probability $p > 0$, or there is no mishap. Suppose d is significantly smaller than d'. Posit $\bar{d} = (d + d')/2$ to get

$$\pi_{av} - \hat{\pi}_{av} = E\left[P \cdot (\hat{L}_a - L_a)\right] = 2pu'(-\bar{d})\left[d_a - \bar{d}\right],$$

and quite similarly for the other agent a'. Hence $\pi_{av} < \hat{\pi}_{av}$ and $\pi_{a'v'} > \hat{\pi}_{a'v'}$. That is, to no surprise, the owner of the more expensive car prefers the standard liability regime over no fault.

As said, a problem with either regime is its negligence of car-driver characteristics. A standard regime favors an expensive Mercedes Benz - and particularly so when its driver is infallible. In contrast, no fault liability benefits the SUV - and especially so when driven with little care. Either combination externalizes accident costs caused by particular vehicles or drivers - and thus induces an undesirable profile of cars.

To indicate how one might mitigate these defects let v denote the value (or weight) of the vehicle. Then, with $a \in c = (t, v)$, $a' \in c' = (t', v')$, a hybrid regime

$$\tilde{L}_a(\omega) := \begin{cases} d + \frac{v}{v+v'}d' & \text{if } \omega = (a, b, c, d; a', b', c', d'), \\ \frac{v'}{v+v'}d' & \text{if } \omega = (a', b', c', d'; a, b, c, d), \\ 0 & \text{otherwise,} \end{cases}$$

which covers portions of inflicted and suffered damages, appears better at internalizing some accident costs caused by expensive or heavy vehicles. ♢

Real regimes differ in strictness and negligence standards. On the one hand, strictness is embodied in (4). On the other, (b, b') presumably determines the parties' degree of negligence. As is fairly well known, strictness and standards, alongside insurance, affect drivers' behavior; see Cohen and Dehejia (2004). We do not consider the strength or nature of such mechanisms.[9] Our focus is rather on the link between liability rules and choice of vehicle. That link passes via insurance.

[9]Conversely though, the model accounts for the fact that behavior affects the frequency and severity of accidents.

Liability insurance.[10] There are, of course, several ways to organize provision and purchase of coverage. We choose to model the arrangement as a mutual company. Operation of that company largely depends on the vehicle profile (v_a) in place. Accordingly, *for the remainder of this section fix* (v_a) *and thereby the distribution* δ.

For sake of greater generality and realism let utility now depend on the state. So, $u_{av}(\omega, r)$ denotes the utility of agent a in state ω, when driving vehicle v and getting revenue r added to his wealth.[11] If agent a opts to retain $X_a \in \mathbb{X}$ of his liability risk, he gets expected utility $Eu_{av}(-X_a) := Eu_{av}(\omega, -X_a(\omega))$ - the expectation being taken with respect to ω.[12]

Since the vehicle choice of each agent is fixed in this section, we write only u_a instead of u_{av_a}. Further, to simplify arguments, in the main, *suppose that utility be transferable* - whence referred to as *payoff.* Accordingly, for any state $\omega \in \Omega$ and *coalition* $A \subseteq \mathbb{A}$ let

$$r \in \mathbb{R} \mapsto u_A(\omega, r) := \sup \left\{ \sum_{a \in A} u_a(\omega, r_a) : \sum_{a \in A} r_a \leq r \right\}$$

denote the state-dependent, aggregate payoff function, often assigned to a fictitious agent declared *representative.*[13] Our chief interest is with the *grand coalition* $A = \mathbb{A}$ and the corresponding function $u_{\mathbb{A}}(\omega, \cdot)$, evaluated at aggregate liability $L(\omega) := \sum_{a \in \mathbb{A}} L_a(\omega)$.[14]

Naturally, each $u_a(\omega, \cdot)$ hence $u_{\mathbb{A}}(\omega, \cdot)$ is increasing, but neither need be differentiable. To handle such objects, and to account for implicit constraints, if any, the following generalized notion of differentiability comes expedient. Given a function f from a vector space $\mathbb{Y}$ into $\mathbb{R} \cup \{-\infty\}$, a linear $g : \mathbb{Y} \to \mathbb{R}$ is declared a *supergradient* of f at $y \in \mathbb{Y}$, written $g \in \partial f(y)$, iff $f(y)$ is finite and

$$f(\cdot) \leq f(y) + g(\cdot - y).$$

[10] Liability - and associated insurance - often creates incentive problems and legal disputes. So, the concept has a somewhat negative ring to it. Many difficultes stem from requiring mainly one party to pay attention and take precautions; see papers by Cooter, Priest, Shapiro and Winter (1991). Traffic accidents stand apart because care had better be bilateral.

[11] Agents need, however, not rank welfare according to final wealth. They might instead consider appropriate reference levels and treat gains different from losses; see Safra and Segal (2008) or references therein.

[12] As said, personal injuries are separate whence not considered. To ccount for them would require inclusion of more states - and state-dependent utilities - that reflect on after-accident health, work ability, or physical pain. Admittedly, the notion of pecuniary compensation for personal injuries is rather problematic.

[13] For a nice presentation, see Magill and Quinzii (1996).

[14] Clearly, for this construction to make sense, liabilities must be perfectly divisible and - like utilities - be transferable at no extra cost.

In these terms, we call $P \in \mathbb{X}$ a *shadow price density* iff $P(\omega) \in \partial_r u_{\mathbb{A}}(\omega, -L(\omega))$ for each $\omega \in \Omega$, where ∂_r denotes the partial subdifferential. Note that when $\mathbb{X} = \mathbb{R}^{\Omega}$ has inner product $\langle x, \hat{x} \rangle = E(x \cdot \hat{x})$, it holds $P \in \partial E u_{\mathbb{A}}(-L) = [\partial_r u_{\mathbb{A}}(\omega, -L(\omega))]_{\omega \in \Omega}$.

As already indicated in Example 4.1, a price density P structures risk-sharing. For a more comprehensive and precise statement, let now $L_A := \sum_{a \in A} L_a$ denote the aggregate liability for any coalition $A \subseteq \mathbb{A}$.

Theorem 4.1 (On transferable utility, risk sharing, shadow price, and core solution, Evstigneev and Flåm (2001))
• *Given a shadow price density P, the payoff profile $a \mapsto \pi_a$ defined by* (1) *constitutes a **core allocation** in the transferable-utility, cooperative game where coalition $A \subseteq \mathbb{A}$ can secure itself expected payoff $Eu_A(-L_A)$.*
• *A shadow price density exists when $u_{\mathbb{A}}(\omega, \cdot)$ is globally concave, and finite-valued at $-L(\omega)$ for each $\omega \in \Omega$. If moreover, $u_{\mathbb{A}}(\omega, \cdot)$ is differentiable at every $-L(\omega)$, then the density is unique.*
• *For any shadow price density P, the allocation (X_a) of the aggregate liability L is optimal iff Borch's rule holds in the generalized, state-dependent form:*

$$P(\omega) \in \partial_r u_a(\omega, -X_a(\omega)) \quad \textit{for each} \quad a \in \mathbb{A} \text{ and } \omega \in \Omega.$$

• *Conversely, if these inclusions hold, and $\sum_{a \in \mathbb{A}} X_a = L$, then P is a shadow price density.*
• *Any shadow price density P decentralizes the choice of retained liability across agents and events:*

$$X_a(\omega) \in \arg\max_{r \in \mathbb{R}} \{u_{av_a}(\omega, -r) + P(\omega) \cdot r\} \quad \textit{for each} \quad a \in \mathbb{A} \text{ and } \omega \in \Omega. \quad \square$$

Indirect utility, a chief object in our analysis, now emerges forthwith:

Corollary 4.1 (Indirect utility) *The joint distribution of L_a and the shadow price density P is determined by the underlying distribution δ. Thus, any agent a, upon choosing vehicle v, foresees an indirect expected utility*

$$\pi_{av}(\delta) := \sup_{X_a} E\left[u_{av}(-X_a) - P \cdot (L_a - X_a)\right] \tag{5}$$

that depends only on δ. $\square$

By hypothesis: *each $a \in \mathbb{A}$ regards δ as unaffected by himself.* The optimal choice of $X_a(\omega)$ in (5) can be settled state by state. That is,

$$\pi_{av}(\delta) := E\left[\sup_{r \in \mathbb{R}} \{u_{av}(\omega, -r) + P(\omega) \cdot r\}\right] - E\left[P \cdot L_a\right].$$

The upshot is that core imputation (5) is the reduced payoff function in (3).[15] Note that insurance policies and premiums are endogenous. Also note that the contingent price $P(\omega)$ depends on car-driver characteristics.

On market-based insurance. It is appropriate to digress briefly and sketch another construction of $\pi_{av}(\cdot)$ - one that does not presume transferable utility. A retained liability profile $a \in \mathbb{A} \mapsto X_a \in \mathbb{X}$, together with a price density $P \in \mathbb{X}$, constitutes a *competitive equilibrium* iff $\sum_a X_a = L$, where X_a will

$$\max Eu_{av_a}(-X_a) \text{ subject to } E\left[P \cdot X_a\right] \geq E\left[P \cdot L_a\right]. \tag{6}$$

(6) models the insurance choice of agent a in a pure exchange economy where $P(\omega)$ is the Arrow-Debreu price for a claim to one unit of account in state ω. For existence and properties of such equilibrium see [1] and [8]. As above, to block arbitrage, pricing $E\left[P\cdot\right]$ must be linear on the space $\mathbb{X}$ of contingent claims. Clearly, the net insurance premium $E\left[P \cdot (L_a - X_a)\right]$ paid ex ante depends on manifold features of the vehicle and its driver. Everything else equal, the distribution of P is determined by δ. So, competitive equilibrium again yields indirect expect payoff as in (5).

5. Equilibrium Choice of Car

The preceding sort of insurance is marked by anonymity. Likewise, the choice of car defines a large game in which interaction works only via the distribution of choices. This section considers existence of Nash equilibrium (3) in that game. We follow Schmeidler (1973) and Rath (1992).[16]

Recall that any distribution $\delta := (\delta_c)$ - alongside a liability regime - generates a profile $a \mapsto L_a$ defined by (4). In turn, that profile gives rise to at least one shadow price density P. Further, taking L_a and P as given, agent a chooses a vehicle

$$v_a \in \arg\max_v \pi_{av}(\delta) = \arg\max_{v \in V} \left\{ \sup_{X_a \in \mathbb{X}} E\left[u_{av}(-X_a) + P \cdot (X_a - L_a)\right] \right\}.$$

Finally, the resulting vehicle profile (v_a) defines a unique distribution δ across $\mathbb{C}$. This string of connections, just outlined, is summarized next:

$$\text{distribution } \delta \rightsquigarrow \text{ liability profile } (L_a) \rightsquigarrow \text{ price density } P \rightsquigarrow$$

$$\text{a set of vehicle profiles } (v_a) \rightsquigarrow \text{ a set } D(\delta) \text{ of distributions.}$$

δ is called an *equilibrium distribution* iff $\delta \in D(\delta)$. Any such object belongs to the standard simplex

$$\Delta(\mathbb{C}) := \left\{ \delta \in \mathbb{R}_+^{\mathbb{C}} : \sum_{c \in \mathbb{C}} \delta_c = 1 \right\}.$$

[15] This arrangement differs from the mutuals studied by Henriet and Rochet (1987) where fees are flat and modified by lump sum transfers.

[16] For material on anonoymous games see Bergin (2005) and references therein.

Note that D is a correspondence from $\Delta(\mathbb{C})$ into itself. For a precise definition of D, denote by $\mathbf{1}(c)$ the unit vector in $\mathbb{R}^{\mathbb{C}}$ that has 1 in component c, and zero elsewhere. In these terms posit

$$D(\delta) := \frac{1}{n} \sum_{a \in \mathbb{A}} conv \left\{ \mathbf{1}(t_a, v_a) : v_a \in \arg\max \pi_{av}(\delta) \right\}. \tag{7}$$

Here $n := \#\mathbb{A}$ is the number of agents; t_a is the type of agent a; and *conv* denotes the operation of taking convex hull. That operation allows and reflects randomized choice of vehicle.

Theorem 5.1 (Existence of equilibrium) *Suppose each mapping* $\delta \mapsto \pi_{av}(\delta)$ *is continuous. Then a Nash equilibrium exists in pure strategies if* $\arg\max_v \pi_{av}(\delta)$ *always is single-valued.*

Proof. The correspondence $\delta \rightsquigarrow D(\delta)$, as defined here above, has closed graph and convex, non-empty values. Hence it admits a fixed point alias an equilibrium distribution $\delta \in D(\delta)$. Since $v_a := \arg\max_v \pi_{av}(\delta)$ is always a singleton, we get a pure Nash equilibrium, and $\delta = D(\delta) = \sum_{a \in \mathbb{A}} \mathbf{1}(t_a, v_a)/n$. $\Box$

It suits the generality of argument, and the size of the player population, to accommodate *infinite* agent sets. Moreover, in doing so, we shall dispense with the assumption that vehicle choice be unique.

Regard $\mathbb{A}$ now as a measure space, endowed with sigma-algebra $\mathcal{A}$ and probability measure da. By a *vehicle profile* is meant a mapping $a \in \mathbb{A} \mapsto v_a \in V$ such that $a \mapsto \mathbf{1}(t_a, v_a)$ is measurable. It follows forthwith from Schmeidler [21]:

Theorem 5.2 (More on existence of equilibrium) *Suppose each mapping* $\delta \mapsto \pi_{av}(\delta)$ *is continuous. Now let* $(\mathbb{A}, \mathcal{A}, da)$ *be a non-atomic probability space. Assume*

$$\{a \in \mathbb{A} : t_a = t\} \in \mathcal{A}, \quad \text{and} \quad \{a \in \mathbb{A} : \pi_{av}(\delta) > \pi_{av'}(\delta)\} \in \mathcal{A}$$

when $t \in T$, $v, v' \in V$, *and* $\delta \in \Delta(\mathbb{C})$. *Posit*

$$D(\delta) := \int_{\mathbb{A}} \{\mathbf{1}(t_a, v_a) : v_a \in \arg\max \pi_{av}(\delta)\}\, da.$$

Then a Nash equilibrium exists in pure strategies. $\Box$

On drivers' behavior. For simplicity, we assumed that a driver's behavior is determined by his characteristics. Hence it has not been viewed as a decision variable. It is conceivable though, that his utility depends directly on own behavior. If so, behavioral choice becomes part of his strategy. This feature complicates matters. Nonetheless, it could be incorporated rather easily along the following lines.

Suppose all agents $a \in c$ have the same utility function u_{bc}, depending on own behavior b and characteristic c. Suppose agents with characteristic c randomize their behavior independently but identically, each using the same behavioral mix

$$\beta \in \Delta(\mathbb{B}) := \left\{ \beta \in \mathbb{R}^{\mathbb{B}}_{+} : \sum_{b \in \mathbb{B}} \beta_b = 1 \right\}.$$

Then, for any agent $a \in c = (t, v)$, replace (5) by

$$\pi_{av}(\delta) := \max_{\beta} \max_{X_a} E\left[u_{bc}(-X_a) - P \cdot (L_a - X_a)\right]. \tag{8}$$

Since $L_a = L_c$ for all $a \in c$, it follows that the optimal $X_a = X_c$. That is, those with equal characteristics retain equal risk and pay the same premium. Using (8) instead of the objective in (2), and

$$D(\delta) := \frac{1}{n} \sum_{c \in \mathbb{C}} \sum_{a \in c} conv \left\{ \mathbf{1}(t_a, v_a) : v_a \in \arg\max \pi_{av}(\delta) \right\}$$

instead of (7), the fixed point argument in Theorem 5.1 still holds verbatim.

6. Some Properties of Equilibrium

Singled out in this section are some straightforward features of risk sharing and acquisition of automobiles.

First, since the game is large, and risks are almost independent, one may, to good approximation, regard aggregate liability as fairly constant. Then, in extremis, car owners can fully protect themselves:[17]

Proposition 6.1 (No aggregate liability risk) *Suppose aggregate liability L is constant. Also suppose agents have strictly concave, state-independent utility functions. Then, no agent carries any risk. That is, the optimal allocation (X_a) of retained liability risk makes each X_a constant. Moreover, the price density $\omega \mapsto P(\omega)$ is constant as well. These assertions hold even if utilities are not transferable.*

Proof. By assumption there exists welfare weights $\lambda_a > 0$ such that the equilibrium allocation of liability risk will

$$\text{maximize} \sum_{a \in \mathbb{A}} \lambda_a E u_{av_a}(-X_a) \text{ subject to } \sum_{a \in \mathbb{A}} X_a \geq L.$$

Clearly, since L is constant, for any feasible allocation (X_a) the corresponding mean-value profile (EX_a) remains feasible. Moreover, by risk aversion, $u_{av_a}(-EX_a) \geq$

[17] This result goes back to Malinvaud (1972-3). For an extension see Cass et al. (1996).

$Eu_{av_a}(-X_a)$. In fact, if some X_a is non-degenerate, that last inequality is strict. The constancy of P follows from Borch's rule. □

Proposition 6.2 (No aggregate risk makes a driver care for his expected liability) *Let aggregate liability L be constant. Suppose the utility of agent a is not directly affected by his vehicle choice. Then he prefers the regime that yields smallest expected liability EL_a.*

Proof. Since the aggregate loss is constant, so is P as well. Upon comparing one liability instance L_a with another $\hat{L}_a$, agent a sees a difference

$$\pi_{av} - \hat{\pi}_{av} = E\left[P \cdot (\hat{L}_a - L_a)\right] = P \cdot \left[E\hat{L}_a - EL_a\right]. \ \square$$

Since insurance is purchased at anonymous prices, one naturally expects that equilibrium treats equal agents equally:

Proposition 6.3 (Equal treatment) *Suppose two agents a, a' have identical utility functions: $u_{av}(\cdot) = u_{a'v}(\cdot)$ and are of the same type. Then, presuming they select the same sort of vehicle, they carry the same liability: $X_a = X_{a'}$ and pay the same gross premium: $E[P \cdot L_a] = E[P \cdot L_{a'}]$, hence $\pi_a = \pi_{a'}$.* □

The introduction alluded to dynamic aspects - and the instability of some situations. Invoking standard evolutionary arguments, we may state forthwith:

Proposition 6.4 (Vanishing inferior vehicles) *Suppose all agents of type t have the same utility function $u_{t,v}$. Let $\delta(v\,|t)$ denote the frequency with which vehicle v is chosen among agents of type t. Out of equilibrium, suppose the rate of change $\dot{\delta}(v\,|t)$ in that frequency satisfies*

$$\dot{\delta}(v\,|t) = f_{t,v}(\delta, \pi_{t,v} - \sum_{v' \in V} \delta(v'\,|t)\pi_{t,v'}),$$

featuring continuous functions $f_{t,v}$ which satisfy $\sum_v f_{t,v} \equiv 0$, $f_{t,v}(\delta, r) = 0 \Leftrightarrow r = 0$, and $sign\,[f_{t,v}(\delta, r)] = sign\,[r]$. Then, in equilibrium, expected payoff $\pi_{t,v}$ is constant across vehicles v for which $\delta(v\,|t) > 0$. In particular, if two categories $c = (t, v)$, $c = (t, v')$ give $u_c = u_{c'}$, but liabilities $E[P \cdot L_c] < E[P \cdot L_{c'}]$ respectively, then c' will disappear from the pool. □

7. Concluding Remarks

Automobile use is a major source of negative externalities - hence heavily regulated and taxed; see Parry et al.(2007). Plainly, putting another vehicle on the road increases those externalities. Our concern has been with a less recognized spill-over. It stems from replacing vehicles, already on the road, by others, more expensive to collide with. Such replacement heightens other motorists' liability for property damages.

Upon dealing with such damages, liability law aims at compensating victims, deterring injurers, and setting standards of behavior. Legal practice has, however, not fully accounted for externalities associated with vehicle choice. Economic theory, in contrast, ceaselessly advocates that costs better be internalized, and it recommends arrangements that facilitate voluntary exchange. Faithful to those maxims, economists would advice that liability rules and insurance policies should reflect the choice of vehicles. Characteristics $c = (t, v)$, more prone to inflict or suffer large damages, should face less favorable liabilities, indemnities or premiums. Otherwise, the share of vehicle brand v in the pool will be undesirably large.

In emphasizing this feature we have worked with reduced payoffs and ignored personal injuries. Further, for simplicity, we assumed that, after a collision, there is perfect information, complete solvency, and costless resolution of disputes.

Format (4) is general enough to account for manifold concerns. It can incorporate deductibles and feature caps on punitive damages. Most important, it can reflect car-driver characteristics. In designing a liability rule (4) it is prudent to pursue at least two goals: *compensation* and *deterrence* - twin goals often in conflict. The regime could and should, however, also incite drivers to choose vehicles so as to reduce the frequency and severity of accidents. As customary, to that end it is expedient that cost be internalized.

References

[1] K. K. Aase, Perspectives on risk sharing, *Scandinavian Actuarial J.* 2, 73-128 (2002)

[2] J. Bergin, *Microeconomic Theory*, Oxford University Press (2005)

[3] K. H. Borch, Equilibrium in a reinsurance market, *Econometrica* 424-44 (1962)

[4] D. Cass, G. Chichilnisky and H.-M. Wu, Individual risk and mutual insurance, *Econometrica* 64, 2, 333-341 (1996)

[5] P-A. Chiappori and B. Salanié, Testing for asymmetric information in insurance markets, *J. Political Economy* 108. 45-78 (2000)

[6] A. Cohen and R. Dehejia, The effect of automobile insurance and accident liability laws on traffic fatalities, *J. Law and Economics* 47, 2, 357-393 (2004)

[7] R. D. Cooter, Economic theories of legal liability, *J. Economic Perspectives* 5,3, 11-30 (1991)

[8] R.-A. Dana, Existence and uniqueness of equilibria when preferences are additively separable, *Econometrica* 61,2, 953-958 (1993)

[9] I. Evstigeneev and S. D. Flåm, Sharing nonconvex cost, *J. Global Optimization* 20, 3-4, 257-71 (2001)

[10] E. Helpman and J-J. Laffont, On moral hazard in general equilibrium theory, *J. Economic Theory* 15, 8-23 (1975)

[11] D. Henriet and J-C. Rochet, Some reflections on insurance pricing, *European Economic Review* 31, 863-885 (1987)

[12] M. Magill and M. Quinzii, *Theory of Incomplete markets*, MIT Press (1996)

[13] E. Malinvaud, The allocation of individual risks in large markets, *J. Economic Theory* 4, 312-328 (1972)

[14] E. Malinvaud, Markets for an exchange economy with individual risks, *Econometrica* 41, 383-410 (1973)

[15] A. Mas-Colell, On a theorem of Schmeidler, *J. Mathematical Economics* 13, 201-206 (1984)

[16] I. W. H. Parry, M. Walls and W. Harrington, Automobile externalities and policies, *J. Economic Literature* XLV, 373-399 (2007)

[17] G. L. Priest, The modern expansion of tort liability: its sources, its effects, and its reform, *J. Economic Perspectives* 5, 3, 31-50 (1991)

[18] K. P. Rath, A direct proof of the existence of pure strategy equilibria in games with a continuum of players, *Economic Theory* 2 (3),426-433 (1992)

[19] R. T. Rockafellar, *Convex Analysis*, Princeton University Press (1970)

[20] Z. Safra and U. Segal, Calibration results for non-expected utility theories, *Econometrica* 76, 5, 1143-1166 (2008)

[21] D. Schmeidler, Equilibrium points of nonatomic games, *J. Statistical Physics* 7, 4, 295-300 (1973)

[22] C. Shapiro, Symposium on the economics of liability, *J. Economic Perspectives* 5,3, 3-10 (1991)

[23] S. Shavell, On liability and insurance, *The Bell J. Economics* 13, 1, 120-132 (1982)

[24] R. Wilson, The theory of syndicates, *Econometrica* 36, 119-32 (1968)

[25] R. A. Winter, The liability insurance market, *J. Economic Perspectives* 5, 3, 115-136 (1991)