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Centralizing Information in Networks

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Abstract

In the dynamic game we analyze, players are the members of a fixed network. Everyone is initially endowed with an information item that he is the only player to hold. Players are offered a finite number of periods to centralize the initially dispersed items in the hands of any one member of the network. In every period, each agent strategically chooses whether or not to transmit the items he holds to his neighbors in the network. The sooner all the items are gathered by any individual, the better it is for the group of players as a whole. Besides, the agent who first centralizes all the items is offered an additional reward that he keeps for himself. In this framework where information transmission is strategic and physically restricted, we provide a necessary and sufficient condition for a group to pool information items in every equilibrium. This condition is independent of the network structure. The architecture of links however affects the time needed before items are centralized in equilibrium.

Keywords : communication network; communication dilemma; dynamic network game; strategic communication; war of attrition.

JEL Classification : D83 , C72, L22.

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1 Introduction

Bonacich (1990) reports an experiment in which success of a given group depends on an effective flow of information among its members. Precisely, subjects were initially given non-overlapping subsets of letters from a quotation that the group of participants had to identify. Only once an individual had identified the quote and independently of who did so, the group received a *collective reward*, equally shared between its members. This collective reward was reduced by a penalty that increased with the time needed to reach the common goal. To gather letters, subjects were offered several communication rounds, each being an opportunity for agents to transmit their letters along some given communication links. Indeed, participants were arranged in a fixed network, whose connections were the only possible channels letters could flow through. In addition to be physically restricted, the transmission of letters had a strategic aspect: the participant who first identified the quotation in the name of the whole group, i.e., the winner, was offered an *additional reward* that he kept for himself. Therefore, individuals had a collective interest to share their letters rapidly as well as an individual motivation to hoard them while waiting for other players' ones to arrive. Bonacich's experiment was run for different network structures and whether a subject communicated extensively or withheld letters appeared to depend on its network position. At a global level, Bonacich's experimental results support the following hypothesis: the outcome of the experimental game is affected by the architecture of the network players belong to. The present work proposes a model in which this hypothesis can be made precise and given theoretical support.

Bonacich's experiment is representative of a large class of situations in which the problem of communication between information holders arises, communication being physically restricted as well as limited by strategic retention of information. In organizations, the nature of a team's decision is often such that it requires the aggregation of some pieces of information privately obtained.[1] In this paper, we consider that the group's collective task is to put together *all* the information items that are initially dispersed. As teams often exist as a part of larger organizations, they seldom have the freedom to make adjustments of the stated patterns of communication used to pool information. Consequently, we examine the transmission of items along the links of a *fixed* communication structure.[2] We further consider that the agent who first centralizes information in the interest of his team individually benefits from this achievement. His additional gain can take the form of a monetary reward, a promotion or gratitude form other members.

In this framework, we investigate how the structure of the communication network affects the group's ability to centralize information items in equilibrium. We address the question of whether it may be that among several communication patterns, all physically adequate for the successful completion of the common task, one results in a significantly "better" equilibrium outcome than an other. As there is not a unique definition of what "better" means in this context, we examine the effect of the network structure on the group performance in two ways. First, the performance of a

[1]For instance, in Jehiel (1999), an organization is in charge of a decision and each operating unit of the firm holds a partial and crucial information on the decision to be taken.

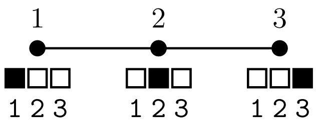
[2]A view of a firm's internal organization as a communication network can be found in Bolton and Dewatripont (1994) or Radner (1993).

team is evaluated regarding whether there is *failure or success in pooling information in equilibrium.* Next, we examine the impact of the structure on the *time the group needs to succeed in equilibrium.* If communication within the given network had no strategic aspect, the smaller the distance between a team member and every other member would be, the sooner the collective goal of items centralization could be reached. Communication networks could then be ranked regarding this distance only. However, we study items transmission that results from deliberate choices by individuals who do not act in the interest of their organization only. In that sense, we differ from the large literature on Team Theory, starting with Marschak and Radner (1972) that also aims at relating the performance of a group to its inner communication structure.

Formally, a dynamic game is introduced whose players are arranged in a fixed network. Everyone is initially endowed with an information item that he is the only player to hold. Players are next offered a finite number of periods to centralize the initially dispersed items. In every period, each player strategically chooses whether or not to transmit the items he holds to his neighbors in the network. Actions are perfectly observed in every period. Our analysis yields two main insights, which are obtained by working exclusively on graphical properties of the networks of players. In particular, our results do not rely on any further assumptions on payoffs than the two following natural ones: at a given period, every agent strictly prefers to win than to lose, and every player strictly prefers to lose sooner than later.

First, we provide a necessary and sufficient condition for a group to centralize items at some position in the network in *every* (subgame perfect) equilibrium. Interestingly, this condition is independent of the network structure. Precisely, we show that a group of n players never fails to pool information in equilibrium if and only if the number of periods offered to do so is at least equal to $n-1$, no matter the network players are arranged in. Next, we claim that network structure however affects the time needed for the dispersed items to be gathered in equilibrium. Even in the case in which every player transmits his items to all his neighbors in every period, every player needs a minimal number of periods to win that depends on his position. This minimal number of periods physically required corresponds to a graph-theoretical measure called a player's eccentricity. We prove that, for every player, there exists an equilibrium in which he is the unique winner at a date that equals his eccentricity. Said differently, we relate a person's position in the network to his ability to win and to the highest possible speed of his win. It follows that there always exists an equilibrium in which the game ends at the earliest date physically possible for the group. This date is given by the minimal eccentricity in the network, called its radius. Finally, we show that, for a particular class of network structures, there exists an upper bound on the duration before success in equilibrium.

The game we analyze contributes to the economic literature studying non-cooperative games played *on* networks extensively surveyed in Goyal (2007) and Jackson (2008). Galeotti et al. (2008) present a very general framework for *static* network games. The authors assume that a player's payoff depends on his own action as well as on the actions taken by his direct neighbors in the graph. The same assumption is made in computer sciences models of *Graphical Games* introduced by Kearns et al. (2001). Graphical games literature focuses on finding algorithms to compute equilibria in *one-stage* games played on large-scale networks. In the present work, the game played by network members is *dynamic.* Players' payoffs directly depend on the actions taken by *every* member of the network in *every* period of play and on the precise order of these actions. Indeed, in the game we build, information is pooled not only if every player transmits the items he holds, but also if it happens in a particular order that depends on the network structure. To understand this idea, consider the following figure[3] showing the initial situation in a game involving three players arranged in the network g_{line}:



To get the three dispersed information items held by distinct players at the beginning of the game, player 1 not only needs players 2 and 3 to Pass On but he also needs player 3 to Pass On *before* player 2 does so. In the network g_{line}, player 2 is an intermediary for the transmission of information from agent 3 to 1.

The paper is organized as follows. In the next section, we present the model. The necessary and sufficient condition to ensure information centralization in every equilibrium is provided in Section 3. The focus of Section 4 is on the time needed to pool information items in equilibrium. Section 5concludes. Proofs are mainly relegated to the Appendix.

2 The Model

2.1 Set-Up

Players, Actions and Networks: The set of agents is $N = \{1, \dots, n\}$. Agents are arranged in a *connected* network[4] represented by a graph g from the set $G(n)$ of all the possible connected graphs involving n nodes. We let ij be in g if player i is linked to player j. We assume that communication links are undirected so that $ij \in g$ implies $ji \in g$, meaning that information items can flow in both ways. For a given network g, the *geodesic distance* $d_{ij}(g)$ between agents i and j is the length of the shortest path[5] between them. Let $N_i(g)$ be i's *neighborhood* in g: $N_i(g) = \{ j \in N \backslash \{i\} : ij \in g\}$. We denote by $g|_S$ the *subnetwork* of g with the set of agents $S \subseteq N$ and all the links that exist between these agents in g. The subnetwork of g with the set of agents $N\backslash\{i\}$ is denoted $g\backslash\{i\}$.

The game is played over discrete time periods $t = 0, ..., T$ with a finite deadline $T \geq 1$. At each date $t \geq 1$, every player i chooses an action a_i^t from the set $A = \{P, H\}$: $a_i^t = P$ means that player i *Passes On* all the information items he holds at time t to every agents in his neighborhood $N_i(g)$

[3]Players' labels correspond to numbers written above the line. There are three different information items and a box numbered j near a player i is filled in black if player i holds the item j

[4]A network is *connected* if there exists a path between any pair of distinct agents.

[5]A *path* is a sequence of consecutive nodes and links in a graph. The length of the path is the number of links traversed.

and $a_i^t = H$ means that player i *Hides* all his information items to every player. The way pieces of information are transmitted is exposed in more detail below.

An action profile at time t is a vector $a^t = (a_i^t)_{i\in N} \in A^n$. A history h^t of the game at time t is the observed past sequence of profiles of actions $(a^1, ..., a^{t-1})$, which is an element of the set of histories at date t denoted $\mathcal{H}^t = (A^n)^{t-1}$. At date t, every player perfectly observes the history h^t.

Information Items: We assume that there are n different information items, numbered from 1 to n. Initially, every player is given a unique item, which he is the only player to hold. Player i is given the item numbered i. The state of players' information at date t is given by a matrix $V^t \in \{0,1\}^{n\times n}$ with the component v_{ij}^t of V^t equal to 1 if player i holds the item j at date t and 0 otherwise. Initially, the matrix of players' information is the identity matrix: $V^0 = Id_n$.

The state of players' information evolves as players *Pass On* or *Hide*. We assume that, once received, an item is never lost, even if Passed On later in the game. Formally, for every $i, j \in N$, the component v_{ij}^t evolves in the following way:

$$v_{ij}^t = max_{\{k\in N_i(g):a_k^t=P\}}\{v_{ij}^{t-1}, v_{kj}^{t-1}\}. \tag{1}$$

Payoffs, Winners and Losers: The payoff structure has common features with the one considered in Bonacich's experimental study. If there is no player who manages to gather the n items before the deadline T is reached, then players earn nothing. On the contrary, if there is at least one player who centralizes the n items in the time offered to do so, then all the players are rewarded. In this case, we denote τ the first period in which the n items are held by an agent. The game ends up at τ. At this date, a collective reward of value n is equally shared between all the players. Besides, the players who have managed to pool information items, called the *winners*,[6] receive an additional reward of value $R > 0$.[7] It follows that, at any given date, *every agent strictly prefers to win than to lose*. Payoffs are discounted according to some common discount factor $\delta \in (0,1)$ ensuring, in particular, that *every player strictly prefers to lose sooner than later*.

For a given g, each final history h^{T+1} uniquely defines a sequence of matrices representing players' information $(V^0, V^1, ..., V^T)$. Denote ι_n the vector with n components equal to 1. Formally, the present value of player i's payoff is given by:

$$u_i(V^0...V^T) = \begin{cases} 0 & \text{if } V_j^T \neq \iota_n,\ \forall\, j \in N, \\ \delta^{t-1} & \text{if } V_i^t \neq \iota_n \ \text{ and }\ \exists\, j \neq i,\ j \in N : V_j^t = \iota_n \\ & \text{and } \forall\, k \in N\ ,\ V_k^{t-1} \neq \iota_n, \\ \delta^{t-1}(1+\frac{R}{l}) & \text{if } V_i^t = \iota_n \\ & \text{and } \forall\, k \in N\ ,\ V_k^{t-1} \neq \iota_n\ \text{, with } l = \#\{k \in N : V_k^t = \iota_n\}. \end{cases}$$

A game with players in a set N arranged in a network g and lasting T periods is denoted $\Gamma(N, g, T)$.

Strategies: We restrict our attention to pure strategies. A pure strategy of player i is a profile $s_i = (s_i^1, ..., s_i^T)$ with s_i^t: $\mathcal{H}^t \to A$ for every $t = 0, \dots, T$. A strategy profile is denoted $s = (s_i)_{i\in N}$.

[6] Players who have not centralized the items at τ are called the *losers*

[7] In case there are several winners, the additional reward is equally shared between them.

Example: As an example, consider the one-shot duel $\Gamma(\{1,2\},g,1)$ where g is the complete network. Initially, players' states of information V^0 is Id_2. Since $N_1(g)=N_2(g)=N$, if player i Passes On the item he holds initially to player $j\neq i$, then $v^1_{ji}=v^1_{ii}=1$. It is easy to get the matrix of utilities of $\Gamma(\{1,2\},g,1)$ which is the well known *Chicken Game*:

	P	H
P	$1+\frac{R}{2},1+\frac{R}{2}$	$1,1+R$
H	$1+R,1$	$0,0$

The one-shot duel has two Nash Equilibria in pure strategies: $(a_1,a_2)=(P,H)$ and $(a'_1,a'_2)=(H,P)$. Note that every equilibrium outcome is such that the game ends with a winner.

2.2 Equilibrium Concept

The game $\Gamma(N,g,T)$ has a multiplicity of Nash Equilibria (NE) and we do not attempt to provide a complete characterization of these. To narrow down the set of NE, the solution concept we use is the subgame perfect Nash Equilibrium (SPNE).[8] Since we investigate the way information is pooled in a decentralized way by the members of a fixed network, we find it reasonable to assume that players do not commit themselves to the dates at which they plan to Pass On. Incorporating subgame perfection therefore makes sense. For every game $\Gamma(N,g,T)$, the set of (SP)NE is denoted $S_{(SP)NE}$.

The way subgame perfection eliminates non-credible threats in the game we propose appears in the following example. Consider $\Gamma(\{1,2,3\},g_{line},2)$ with g_{line} the three-player network presented in the Introduction. The strategy profile that consists in "every player Hiding in every period, whatever the history" is a NE. Indeed, as long as two players out of three Hide in every period, every player receives 0, whatever his strategy. Next, consider the subgame of $\Gamma(\{1,2,3\},g_{line},2)$ that starts at time $t=2$ after player 1 has Passed On at date $t=1$ while players 2 and 3 have Hidden. In this subgame, if player 3 Passes On instead of Hiding, he receives δ instead of 0 as player 2 finally holds the three information items. It follows that "players 2 and 3 Hiding in the second period of play, whatever the history" is not credible.

2.3 Graphical Objects

We define some graph-theoretical concepts that are used below, and start with a centrality measure:

Definition 1 Player i's *eccentricity* in the network g, denoted $e_i(g)$, is the distance from agent i to the agent furthest away from him: $e_i(g)=\max_{j\in N}\{d_{ij}(g)\}$.

In the game $\Gamma(N,g,T)$, player i's eccentricity is equal to the minimal number of periods required for player i to centralize the n items in case every other player Passed On in every period. In a

[8]Each finite game $\Gamma(N,g,T)$ contains subgames that are uniquely defined by each history h^t and denoted $\Gamma(N,g,T)|h^t$. The strategy profile $s\in S$ is a SPNE if, for every $h^t\in H^t$, the continuation strategy profile denoted $s|h^t$ is a NE of $\Gamma(N,g,T)|h^t$.

network g, the minimal eccentricity is called the radius $r(g)$ and the maximal eccentricity is called the diameter $d(g)$. Obviously, a player i cannot win in a game $\Gamma(N, g, T)$ that lasts strictly less than $e_i(g)$ periods:

Definition 2 For every game $\Gamma(N, g, T)$, the *set of potential winners* is given by $W(g, T) = \{ i \in N : e_i(g) \leq T \}$.

Games of interest are games $\Gamma(n, g, T)$ such that $W(g, T) \neq \emptyset$ or equivalently such that $T \geq r(g)$. We restrict our attention to such games in the present work. Note that every player can potentially win, i.e., $W(g, T) = N$, if and only if $T \geq d(g)$.

We now focus on networks' connectedness:

Definition 3 In a connected network g, an agent i is *critical* (respectively *non-critical*) if $g \backslash \{i\}$ is disconnected (resp. connected).

In other words, a critical agent is crucial in maintaining the connectedness of a network. [9] By definition, a critical agent in g is on every path between at least one pair of agents in g.

A *complete network*, denoted $g_{complete}$, is a particular architecture in which every agent is linked to every other one. Since a link exists between every pair of distinct agents, every agent is non-critical in $g_{complete}$. A *tree network*, denoted g_{tree}, is such that there is a unique path between every pair of distinct agents. It follows that there is at least one critical agent in every tree involving three players or more. Note that a connected network involving two players is both a complete and a tree network. More generally, the following theorem deals with the existence of non-critical agents in connected networks:

Theorem 1 *[Kelly and Merriell (1958)] In a connected network with $n \geq 2$ agents, there are at least two non-critical agents.*

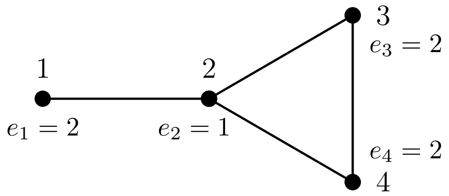
Finally, a particular type of network structure is defined with respect to the existence of a critical agent:

Definition 4 A connected network in which there is at least one critical agent is *separable.* A connected network in which every agent is non-critical is *non-separable.*

A separable network can be disconnected by removing one agent. Tree networks involving more than three players are separable whereas complete networks are not.

To illustrate the previous definitions, we consider the following network g_{kite}:

[9] The term "critical agent" refers to the term "critical link", defined by Jackson and Wolinsky (1996) as crucial to maintain the connectedness of a graph.



Players' eccentricities appear near players' labels. We have $r(g_{kite}) = 1$ and $d(g_{kite}) = 2$. The sets of potential winners are $W(g_{kite}, 1) = \{2\}$ and $W(g_{kite}, T) = N$ for every $T \geq 2$. The network g_{kite} is separable with player 2 being critical and players 3, 4 and 5 being non-critical.

3 Success or Failure in Equilibrium

Our objective is to compare network structures with respect to their efficiency in encouraging information centralization when its transmission is strategic. As already mentioned, if information transmission were not strategic but were automatic in every period, network structures could be trivially ranked as $r(g)$ would be the number of periods required to centralize the n dispersed items in a network g.

The first measure of group performance that we consider is the *achievement of the collective goal in every equilibrium.* For every game $\Gamma(N, g, T)$, the set of strategy profiles S is split into two disjoint subsets. Let $S_W \subseteq S$ be the set of strategy profiles such that the game $\Gamma(N, g, T)$ ends with at least one winner, or equivalently, such that the collective goal is reached at a time $t \leq T$. Let $S_L = S \backslash S_W$ be the set of strategy profiles such that the game ends with no winner, or equivalently, such that players have failed to centralize information at some position in the network before the deadline is reached.

Recall that if a game ends with at least a winner, every player earns a strictly positive payoff whereas if the game ends with no winner, every player earns 0. It follows that failure in performing the collective task is an outcome that is Pareto dominated by any outcome in which success is ensured. The following proposition provides a necessary and sufficient condition for success to be ensured in *every* SPNE outcome of $\Gamma(N, g, T)$:

Proposition 1 *$S_{SPNE} \subseteq S_W$ if and only if the game $\Gamma(N, g, T)$ is such that $T \geq n - 1$.*

That is, *every* equilibrium yields at least one winner if and only if the game lasts sufficiently many periods. On the contrary, if the deadline is $T \leq n - 2$, there exists equilibria that lead to failure in the collective task. In such cases, there may also be equilibrium outcomes such that there is a winner.

Interestingly, the condition $T \geq n-1$ is independent of the structure of the connected network g and, in particular, depends neither on its radius nor on its diameter. Given a deadline T and a fixed number of players n, all communication networks are therefore equally efficient with respect to the efficiency criterion considered in this section, namely collective success in equilibrium. Precisely, a complete network happens to be as efficient as any connected structure that minimizes the number of links such as tree networks do. In settings in which building communication links is costly but neither the identity of the winner nor the time needed to succeed matters, a tree network can be chosen rather than any other structure.

The proof of the fact that $T \geq n-1$ is a sufficient condition to get $S_{SPNE} \subseteq S_W$ is done by induction and is quite constructive. It mainly relies on the following two-player result: *every NE of a dynamic duel* $\Gamma(\{1,2\}, g, T)$ *yields at least one winner.* Indeed, for a duel to end up with a winner, it is sufficient to have one of the two players Pass On before the deadline is reached. It follows that both players loosing cannot be an equilibrium outcome since every one can unilaterally prevent such an outcome. Next, to see how this fact about duel is used, consider the one-shot game $\Gamma(\{1,2,3\}, g_{line}, 1)$, with g_{line} represented in the Introduction. In this game, if two or more players Hide, the game ends with no winner and this is a SPNE since no player can unilaterally prevent this outcome.

Proposition 1 states that adding a period to $\Gamma(\{1,2,3\}, g_{line}, 1)$ is sufficient to rule out such an equilibrium outcome. This is due to the fact that, in $\Gamma(\{1,2,3\}, g_{line}, 2)$, player 1 or player 3 have the ability to unilaterally make the game evolve into a duel between the two other players that would last at least one period. This happens to be a general feature of non-critical agents whose existence relies on Theorem 1. More precisely, if a non-critical player, say player 1, Passes On at date $t = 1$ while the other players Hide (which means they behave in the worst way regarding items centralization), then the subgame that starts at time $t = 2$ is strategically equivalent to the one-shot duel: players 2 and 3 are directly linked to each other and each player is holding some items that, if transmitted, make the other player win immediately. This is represented as *Situation A* in Figure 1.[10] Next, as stated earlier, once a duel is reached, every equilibrium yields at least a winner. The same reasoning can be applied to the game $\Gamma(\{1,2,3\}, g_{complete}, 2)$ as it is illustrated by *Situation B* on the following figure.

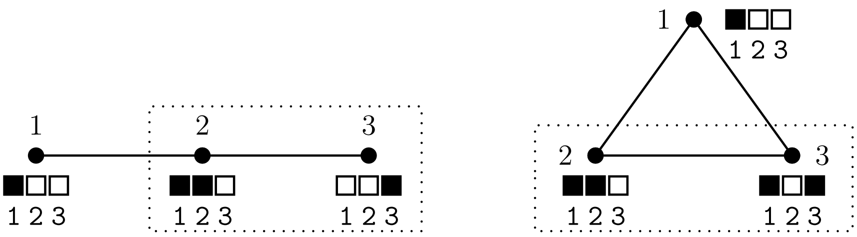


Figure 1: Informational situations once player 1 has Passed On and players 2 and 3 have Hidden.

From the previous paragraph, we have that every equilibrium outcome of $\Gamma(\{1,2,3\}, g, 2)$ is such that there is at least a winner. One can then get the same result for games $\Gamma(\{1,2,3,4\}, g, 3)$ by noting that there always exists a non-critical agent in g who can, by Passing on at time $t = 1$,

[10]On Figure 1, players' labels correspond to numbers written above the line. Every player i's informational situation is represented by three boxes numbered 1,2 and 3 and situated near player i: box numbered j near player i is filled in black if player i holds the item j and is empty otherwise.

make the subgame that starts at time $t = 2$ be such that the other three players are in a situation strategically equivalent either to $\Gamma(\{1,2,3\}, g_{line}, 2)$ or to $\Gamma(\{1,2,3\}, g_{complete}, 2)$. This inductive reasoning enables to state that the minimal number of periods sufficient to get a winner in every equilibrium of $\Gamma(N, g, T)$ is $n - 1$.

The intuition behind the number $n-1$ is found in the following observation: any n-player connected network can be transformed into a 2-player connected network by a number $n-2$ of successive removals of non-critical agents. Said differently, $n-2$ is to the number of periods required to reduce any n-player network to a 2-player network by dropping out one non-critical agent at a time. In our game, $n-2$ is therefore the number of periods required to reduce a n-player game to a duel through successive items transmissions by non-critical agents. $n-1$ corresponds to that number $n-2$ of periods to reach a duel plus one period which is the unique one required for the duel to solve with a winner.

4 Duration in Equilibrium

Among equilibria that yield success, aggregate payoffs are lower when information is centralized at time t than at any earlier date $t' < t$.[11] This section focuses on *the time needed for the group of players to succeed in equilibrium.* From a global point of view, the minimal duration before success in equilibrium gives the *best* equilibrium outcome. Once success is ensured in every equilibrium, maximal duration before success then gives the *worst* equilibrium outcome.

4.1 Minimal Duration

In a game $\Gamma(N, g, T)$, duration before success has a lower bound that depends on the architecture of the network g and corresponds to the radius $r(g)$. The first question we ask is therefore whether the end of the game at time $t = r(g)$ is a SPNE outcome. The next proposition claims that it is the case, meaning that, with respect to the *best* equilibrium outcomes, networks could be ranked according to their radii. More generally, this proposition relates a person's position in the network to his ability to win and the highest possible speed of his win. It states that, for every potential winner, there exists an equilibrium in which this player is effectively the unique winner after a number of periods just equal to the time he "physically" requires to be so. Every of these equilibria is Pareto efficient. We show that there are no strategic matters which prevent players from reaching them:

Proposition 2 *For every player $i \in W(g, T)$, there exists a $SPNE$ of $\Gamma(N, g, T)$ such that player i is the unique winner at time $t = e_i(g)$.*

Complete networks are particular in that every member's eccentricity is equal to the radius $r(g) = 1$. Indeed, in $g_{complete}$, an information item which is Passed On is immediately held by every player. As a consequence, a member of a complete network is the unique winner if and only if he

[11] Recall that if a game ends up with at least a winner at time $t \leq T$, then aggregate payoffs equal $\delta^{t-1}(n + R)$.

is the last player to Pass On. Said differently, as soon as $n-1$ players have "conceded", the game ends with the player who has not conceded yet holding the n items and winning. To that extent, the game $\Gamma(N, g_{complete}, T)$ can be viewed as a *war of attrition* of complete information in which n symmetric players compete for one prize in discrete and finite time. As in usual such games, every player strictly prefers to win than to lose but prefers to lose sooner than later.

Interestingly, the previous proposition applied to games $\Gamma(N, g_{complete}, T)$ corresponds to a well-known result of the *war of attrition* literature. In Kornhauser et al. (1988), a concession game with complete information is played in discrete time by two players 1 and 2 with different discount factors. The authors state that there is an infinity of SPNE outcomes: one of these outcomes is for player 1 to concede immediately, another is for player 1 to wait and for player 2 to concede immediately.[12] Proposition 2 corresponds to the generalization of that observation to the case of n agents competing for one prize. We get that, for every individual in the complete network, there is a SPNE outcome in which the $n-1$ other players concede immediately. This statement is further generalized to any network structure by showing that, for every individual, there is a SPNE outcome in which only that individual wins at a date that is equal to his eccentricity. Indeed, if the network is not complete, a number $n-1$ of immediate concessions do not always enable the player who has not conceded to win immediately. The time needed by the latter player depends on the network structure.

4.2 Maximal Duration

In the previous section, we have shown that, for every $\Gamma(N, g, T)$, there is a lower bound on the equilibrium duration of the game. The present section focuses on the existence of an upper bound on that duration. We recall that, for games $\Gamma(N, g, T)$ such that $T < n-1$, Proposition 1 states that the worst equilibrium outcome corresponds to the case in which the game reaches the deadline with no winner. On the contrary, success is ensured in every equilibrium for games $\Gamma(N, g, T)$ such that $T \geq n-1$. Our goal is now to examine whether there is an upper bound on the time needed for that success to happen. Let $S_{end \leq n-1} \subseteq S_W$ be the set of winning strategy profiles S_W that are such that the game ends with at least a winner at a date $t \leq n-1$.

4.2.1 A graphical sufficient condition

For every given n, we consider a particular set of networks $K(n) \subseteq G(n)$. Let a n-player network g belong to $K(n)$ if and only if every of its connected subnetwork $g|_S$ with $S \subseteq N$ is either a separable or a complete network. In particular, for every n, we have $g_{complete} \in K(n)$ since every subnetwork of a complete network is complete. For every n, the set $K(n)$ also encompasses all the n-player trees, since tree networks have every of their subnetworks separable or complete.[13] Networks in

[12]Departing from that statement, their paper then develops a criterion for selecting an intuitive equilibrium outcome, namely one in which the weaker player, i.e., with the lower discount factor, concedes immediately.

[13]Precisely, all the connected subnetworks of a tree are separable, except two-player connected subnetworks which are complete.

$K(n)$ which are neither complete nor trees can be pictured as "made up" of both structures. The network $g_{kite} \in K(4)$ presented in Section 2.3 is an example of such structures.

The following proposition establishes that $g \in K(n)$ is a sufficient condition for the existence of an upper bound equal on the equilibrium duration of every game $\Gamma(N, g, T)$ such that $T \geq n-1$. This upper bound is equal to $n-1$:

Proposition 3 *In every $\Gamma(N, g, T)$ with $T \geq n-1$, we have: if $g \in K(n)$, then $S_{SPNE} \subseteq S_{end \leq n-1}$.*

In Section 3, Proposition 1 stated that every equilibrium yields a winner if the game lasts at least $n-1$ periods. Proposition 3 now gives conditions under which *every* of these winning equilibria is such that the game ends up at the latest at date $n-1$, while the number of periods offered to the players can be strictly larger than $n-1$. In other words, it establishes that if the network g belongs to the particular class of structures $K(n)$, then the available periods from date $t = n$ to $t = T$ are never used by the group of agents to succeed in equilibrium. In the following section, we will examine a four-players game $\Gamma(\{1, 2, 3, 4\}, g, T \geq 3)$ played on a network $g \notin K(4)$ and show that the game can end up at a date $T > 3$ in equilibrium.

The proof of Proposition 3 is done by induction and the reasoning is similar to the one developed in Section 3 to establish one part of Proposition 1. However, two additional ingredients are needed. First, to show that the statement is true for $n+1$ players using the assumption that it is true for n, we use the fact that a network $g \in K(n+1)$ has every of its n-player connected subnetworks in $K(n)$.[14] Second, we use the following result about equilibrium sets of winners:

Proposition 4 *i) In every $\Gamma(N, g, T)$ with g separable, every $s \in S_W$ is such that there is at least one non-critical agent who loses. ii) In every $\Gamma(N, g, T)$ with g complete, every $s \in S_{NE} \cap S_W$ is such that the winner is unique, implying that there is at least one non-critical agent who loses.*[15]

To get an intuition of how Proposition 4*i)* works, let's consider g_{kite} presented in Section 2.3. Assume that non-critical players 1, 3 and 4 win together at time t. To get such an outcome, player 1 must hold, at date t, the items initially held by players 3 and 4 and vice versa. In g_{kite}, player 2 is the intermediary for items transmission between player 1 and players 3 and 4. It follows that players 1, 3 and 4 winning together at time t implies that player 2 already held the four items at a date $t' \leq t-1$, which contradicts the initial assumption. While part *i)* relies on the graphical properties of separability, part *ii)* is established for equilibrium strategy profiles. Observe that Proposition 4 is independent of $W(g, T)$. It is not because some non-critical players are not potentially able to win, that there always exists (at least) one loser. Even in case in which $W(g, T) = N$, it is the structure that prevents some players from winning together in equilibrium.

Now, let's come back to Proposition 3 and first establish it for two players: *every equilibrium of a duel is such that the game ends up in the first period of play.* Indeed, a single period is sufficient for every agent to make a duel end. Since a two-player connected network belongs to $K(2)$, we get

[14] It is the case because every network $g \in K(n)$ is precisely defined by the structure of each of its subnetworks. Every subnetwork itself contains subnetworks with the same structure.

[15] In a complete network, every agent is non-critical.

from Proposition 4 that the winner is unique in every equilibrium of $\Gamma(\{1,2\}, g, T)$. It follows that an equilibrium strategy profile of the duel cannot be such that it lasts strictly more than one period as the loser would have a profitable deviation to a strategy that makes him lose in the first period of play.

Second, let's consider a game $\Gamma(\{1,2,3\}, g, T)$ with $T \geq 2$ and an equilibrium strategy profile of that game such that it ends up with a winner at a date $t \geq 3$. Since every three-player connected network belongs to $K(3)$[16], we have from Proposition 4, that this profile is then such that a non-critical agent loses at t. By Passing on in the first period of play, this non-critical agent can make the game evolve into a duel that ends up at time $t = 2$. Since every player prefers to lose sooner than later, the strategy profile considered cannot be an equilibrium. Finally, we get: *in a three-player game lasting at least two periods, every equilibrium yields a winner in one or two periods.*

Finally, let's consider a game $\Gamma(\{1,2,3,4\}, g, T)$ with $T \geq 3$. To establish the fact that, if $g \in K(4)$, then $S_{SPNE} \subseteq S_{end \leq 3}$, we extend the inductive reasoning detailed above. Note however that contrary to the cases of networks involving two or three players, four-player networks do not all belong to $K(4)$.[17] The ones in $K(4)$ are such that, by Passing On, every non-critical player i can make the game evolve into a subgame played in a connected three-player network $g \backslash \{i\} \in K(3)$. Again, since that subgame lasts at most two periods, a strategy profile such that $\Gamma(\{1,2,3,4\}, g, T)$ ends at $t \geq 4$ with a non-critical player i loosing cannot be an equilibrium. This inductive reasoning generalizes easily to n-player networks in $K(n)$.

The strength of Proposition 3 lies in that the existence of an upper bound only relies on the graphical properties of the networks in $K(n)$. In particular, it does not depend on any further assumption on payoffs than the two natural ones we made: first, at any given date, one always strictly prefers to win than to lose, i.e., $R > 0$, and second, one strictly prefers to lose sooner than later, i.e., $\delta \in (0,1)$. It is Proposition 4 which is crucial to establish Proposition 3. It shows, for the set of separable and complete networks which encompasses the set $K(n)$, that there is a non-critical agent who loses in *every* equilibrium. Without such a result on the effect of the structure on the set of winners in equilibrium, the existence of an upper bound on duration in every equilibrium depends on further assumptions about payoffs. The example developed in the following section aims at making this point clear.

4.2.2 The square

We will consider a game played on a network g which is neither separable nor complete, and therefore do not belong to $K(n)$, and first show that it has an equilibrium in which *all* the players win together.[18] We will then demonstrate how this enables to construct an equilibrium in which the game ends up at the very last date $T > n - 1$.

Let's take the game $\Gamma(\{1,2,3,4\}, g_{square}, 2)$ with $g_{square} \notin K(4)$. We start by analyzing the

[16] A three-player connected network is either a complete network (the triangle) or a separable network (the tree).

[17] Precisely, among the six different connected structures that four-player networks can have, two structures do not belong to $K(4)$. The four-player square with no diagonal and the four-player square with one diagonal are neither separable nor complete.

[18] However, there does not exist such an equilibrium in every game played on incomplete and non-separable networks.

subgame that would result from a first period in which every player Passes On. The informational situation of such a subgame is illustrated by *Situation C* on Figure 2. In *Situation C*, if every player Passes On at T, then the game ends with four winners. At date $t = 2$, by Hiding while the other players Pass On, no player can prevent the other three players from winning. Consequently, starting from *Situation C*, "every player Passing On" is a NE yielding four simultaneous winners.

Next, we analyze the subgame that result from a first period of play in which one player, say 1, Hides and the other three players Pass On. *Situation D* on Figure 2 shows the resulting informational situation. In *Situation D*, if player 1 Passes On while the other three players Hide, the game ends up with players 2 and 4 receiving $\delta(1+\frac{R}{2})$ whereas players 1 and 3 earn δ. At date $t = 2$, by Hiding instead of Passing On, player 1 makes the game end with no winner. Given *Situation D*, player 3's action has no impact on the outcome of the game. Finally, if player 2 or 4 deviates from Hiding, it makes three players win $(1, 2$ and $4)$ instead of two (2 and 4). Therefore, starting from *Situation D*, "player 1 Passing On while the other players Hide" is a SPNE yielding two simultaneous winners, players 2 and 4.

Comparing *Situations C and D*, player 1 has no interest in deviating from Passing On in the first period of play when the other three players Pass On. The same is true for every player as their positions are symmetric. We conclude that there is a SPNE of $\Gamma(\{1,2,3,4\}, g_{square}, 2)$ such that ends with four winner at $t = 2$.

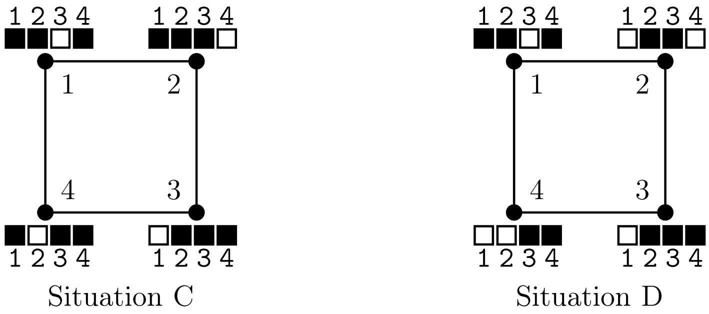


Figure 2: Two informational situations at time $t = 2$.

Finally, let's take a game $\Gamma(\{1,2,3,4\}, g_{square}, T)$ with $T \geq 4$ and a strategy profile s resulting in "every player Hiding in every period $t \in [1, T-2]$ and Passing On in periods $T-1$ and T". Following s from time $t = 1$ to time $T-2$, the subgame that starts at time $T-1$ is equivalent to $\Gamma(\{1,2,3,4\}, g_{square}, 2)$ examined above and has a SPNE yielding four winners at time T. It is always possible to construct the profile s such that, if any player, say i, deviates from Hiding in a period $t \in [1, T-2]$, then, in the subgame that starts, players play a NE such that player i loses in period $t+1$.[19] Given a discount factor δ, one can then easily find an addition reward $R > 0$ such that the strategy profile s yielding four winners at time T is a SPNE. Indeed, it is the case for R such that $\delta^{T-1}(1+\frac{R}{4}) > \delta$. For the network g_{square}, Proposition 4 does not apply. It follows

[19]For instance, if player 1 Passes On in the first period of play, then, in the subgame that starts, a SPNE is played in which players 2 and 4 always Pass On while 3 always Hides and therefore win at time $t = 2$.

that, for some couple (δ, R), we cannot find any agent who has a strictly profitable deviation from a strategy profile such that the game ends at $t \geq n$.

5 Conclusion

In the dynamic game we analyze, the members of a fixed network face a *communication dilemma* in the sense that they have a collective interest to transmit information items via communication links as well as an individual interest to withhold them. We show that, a group of n players centralizes all the initially dispersed items in every subgame perfect equilibrium if and only if the game lasts sufficiently many periods, precisely more than $n-1$ periods. It follows that *whether or not the collective task is performed in every equilibrium is independent of the network structure*, as long as it is physically adequate for the successful completion of this task, i.e., connected. On the contrary, *the architecture of the communication links affects the time needed before information items are pooled in equilibrium.* For every network, we show that the minimal time needed in equilibrium is given by the radius of the network. Finally, once success is ensured in every equilibrium, we provide a sufficient condition on network structure to get an upper bound on the duration of the game in equilibrium. The number $n-1$ corresponds to that the maximal number of periods used by players for items centralization.

We have proposed to evaluate the ability of a group to solve communication dilemmas regarding whether it fails or succeeds in every equilibrium, and the time it needs to do so. According to these criteria, a network structure that appears adapted to the task of centralizing dispersed items is the *star network.* Indeed, it exhibits a radius equal to one, meaning that the *best* equilibrium outcome of the game played in the star is the best the group can get in any connected network. Besides, the star exhibits the graphical properties that are sufficient to provide an upper bound on the time needed to pool information in equilibrium. If information transmission in the network were automatic, suggesting the star network for centralization purposes would be quite straightforward. The interesting point here is that this structure remains adapted to players' task despite the conflicting interests - collective and individual - which characterize our communication dilemma.

Information items that are considered in this paper are not private information in the usual sense of incomplete information games. For instance, one could imagine that the items transmitted by players are some keys that have to be centralized by the agents for them to open a box. Note that it would then be as if each player Passed On copies of the keys he possessed. To answer the question about the structure that is the most appropriate for the pooling of these keys, we introduce graphical notions and results from graph theory that are used in some areas of operations research.[20] A building block of our analysis is a graphical result stating that, in every connected network, there exists at least two non-critical agents. Since such agents can be dropped from a network without disconnecting the resulting subnetwork, proofs can be done by induction within networks.

[20]For instance, Buckley (1986) uses the eccentricity measure to define and find the center of a tree network. More generally, see *network location theory* that addresses the question of the optimal location of a single-point facility in a graph.

Even if Bonacich (1990)'s experimental results stated that the outcome of such communication dilemmas is affected by the network structure, his study rather examined the influence of an agent's position on his individual behavior. For instance, it seemed that agents with peripheral positions behaved more cooperatively than central agents. In the present work, for agents who are not in the set of potential winners because they are peripheral in the sense that their eccentricities are too large, Passing On in every period of play is a weakly dominating strategy. That is, the effective chances of victory determined by network positions clearly affect one's communication behavior. Our focus was on the effect of the players' positions and network structure on equilibrium outcomes. We leave for further research the impact positions have on individual behaviors.

6 Appendix

For every proposition presented in a previous section, the proof is given in a subsection of the Appendix entitled as the section. We denote $\Gamma(N,g,T)|h^t$ the subgame of $\Gamma(N,g,T)$ that starts at time $t \leq T$ after history h^t. Player i's continuation strategy after history h^t is denoted $s_i|h^t$.

6.1 Success or Failure in Equilibrium

6.1.1 Proposition 1: Sufficient Condition

Lemma 1 *If the game $\Gamma(N,g,T)$ is such that $T \geq n-1$, then $S_{SPNE} \subseteq S_W$.*

Proof of Lemma 1 is by induction: assume it is true for n players and show it stays true for $n+1$. To do so, fix n and consider three kinds of games. First, games $\Gamma(N,g,T)$ with $|N| = n$. Next, *augmented* games $\Gamma(N',g',T)$ with $|N'| = n+1$. Without loss of generality, let player $(n+1)$ be in $N_n(g')$ and be non-critical in g'. Finally, *augmented modified* games $\tilde{\Gamma}(N',g',T)$ that differ from *augmented* games only in that the initial matrix of information $\tilde{V}'^0 \neq Id_{n+1}$ is such that, for every $i \in N'$, we have $\tilde{v}'^0_{ii} = 1$ and such that $\tilde{v}'^0_{n\,n+1} = 1$ meaning that player n initially holds the item numbered $(n+1)$.

Let two games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$ form ***a pair*** if the two connected networks g and g' are such that $g = g' \backslash \{n+1\}$. Given either $\Gamma(N,g,T)$ or $\tilde{\Gamma}(N',g',T)$ only, one can always construct a pair. Indeed, a connected g' can be constructed from a connected g by linking agent $(n+1)$ only to agent n. Since agent $(n+1)$ has a unique neighbor in g', he is non-critical in g'. A connected g can be constrcuted from a connected g' by removing the non-critical agent $(n+1)$ and all its links.

The sets of (SP)NE of games $\Gamma(N',g',T)$ and $\tilde{\Gamma}(N',g',T)$ are denoted $S'_{(SP)NE}$ and $\tilde{S}'_{(SP)NE}$ respectively. The sets of strategy profiles such that games $\Gamma(N',g',T)$ and $\tilde{\Gamma}(N',g',T)$ end with at least one winner (no winner, resp.) are denoted S'_W and $\tilde{S}'_W$ respectively (S'_L and $\tilde{S}'_L$, resp.). Before proving Lemma 1, we show:

Lemma 2 *For every pair of games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$, we have: if $S_{SPNE} \subseteq S_W$, then $\tilde{S}'_{SPNE} \subseteq \tilde{S}'_W$.*

Proof: Take a pair of games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$. We show that if there exists a strategy profile $\tilde{s}'$ in $\tilde{S}'_{SPNE} \cap \tilde{S}'_L$, then there exists a strategy profile s in $S_{SPNE} \cap S_L$. In $\tilde{\Gamma}(N',g',T)$, consider a profile $\tilde{s}' \in \tilde{S}'$ such that player $(n+1)$ Hides in every period whatever the history and such that, for every player $i \in N$, player i's action at time t is independent of player $(n+1)$'s actions at dates $t' \in [1, t-1]$. Next, in $\Gamma(N,g,T)$, consider a profile $s \in S$ such that s and $\tilde{s}'$ describe, for every player $i \in N$ and every date $t \le T$, the same action profile in games $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$ respectively.

Considering the process of items' transmission given by (1), it is easy to show that the sequences $(V^0,...V^T)$ and $(\tilde{V}'^0,...,\tilde{V}'^T)$ determined by s and $\tilde{s}'$ in $\Gamma(N,g,T)$ and $\tilde{\Gamma}(N',g',T)$ respectively are such that, for every $i \in N$ and every $t \le T$, we have *(A)*: for each item $j \in N$, $\tilde{v}'^t_{ij} \ge v^t_{ij}$.[21] Next, since $\tilde{s}' \in \tilde{S}'_L$, there exists, for every $i \in N$, at least one item $k \in N'$ such that $\tilde{v}'^T_{ik} = 0$. Given that items n and $(n+1)$ are transmitted together in $\tilde{\Gamma}(N',g',T)$ as $\tilde{v}'^0_{n\,n+1} = 1$, we get that $\tilde{s}' \in \tilde{S}'_L$ implies that there exists, for every $i \in N$, an item $k \in N$ such that $\tilde{v}'^T_{ik} = 0$. Using *(A)*, we have that $\tilde{s}' \in \tilde{S}'_L$ implies $s \in S_L$.

Finally, if $\tilde{s}' \in \tilde{S}'_{SPNE}$, the profile of continuation strategy $\tilde{s}'|h'^t$ is a NE of the subgame $\tilde{\Gamma}(N',g',T)|h'^t$ for every $h'^t \in H'^t$. Since $\tilde{s}'$ is such that, for every $i \in N$, player i's action in every period is independent of player $(n+1)$'s past actions and player $(n+1)$'s actions are independent of the history, we directly get: if $\tilde{s}'|h'^t$ is a NE of $\tilde{\Gamma}(N',g',T)|h'^t$, then $s|h^t$ is a NE of $\tilde{\Gamma}(N,g,T)|h^t$ with h'^t and h^t describing the same action profile for every $i \in N$ and every date $t \le T$. It follows that $\tilde{s}' \in \tilde{S}'_{SPNE}$ implies $s \in S_{SPNE}$ which completes the proof. $\square$

Proof of Lemma 1: As stated in Section 3, every NE of a dynamic duel $\Gamma(\{1,2\},g,T)$ yields at least one winner, which implies that Lemma 1 is true for $n=2$. We assume that Lemma 1 is true for n agents and prove that it stays true for $n+1$ agents: if the game $\Gamma(N',g',T+1)$ is such that $T+1 \ge n$, then $S'_{SPNE} \cap S'_L = \emptyset$.

First, in $\Gamma(N',g',T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that $a'^1_{n+1} = P$ and we show that $T+1 \ge n$ implies $s' \in S'_W$. By definition of SPNE, the profile of continuation strategy $(s'_i|h'^2)_{i \in N'}$ is a SPNE of the subgame $\Gamma(N',g',T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i\in N}, P)$. This subgame is equivalent to the *augmented modified* game $\tilde{\Gamma}(N',g',T)$. More precisely, games $\Gamma(N',g',T+1)|h'^2$ and $\tilde{\Gamma}(N',g',T)$ have the same set of players N', the same network g', the same number of periods of play T and the same matrix of players' information: at the beginning of $\Gamma(N',g',T+1)|h'^2$ the matrix V'^1 is such that, for every $i \in N'$, we have $v'^1_{ii} = 1$ and such that $v'^1_{n\,n+1} = 1$ since $n \in N_{n+1}(g')$ and $a'^1_{n+1} = P$. By assumption, if $\Gamma(N,g,T)$ is such that $T \ge n-1$, then $S_{SPNE} \subseteq S_W$. Given $\tilde{\Gamma}(N',g',T)$, we can find a game $\Gamma(N,g,T)$ to get a pair and then deduce from Lemma 2 that $\tilde{S}'_{SPNE} \subseteq \tilde{S}'_W$. Therefore, $(s'_i|h'^2)_{i\in N'} \in \tilde{S}'_{SPNE}$ implies $(s'_i|h'^2)_{i\in N'} \in \tilde{S}'_W$ which implies that $s' = (s'^1_i(h'^1), s'_i|h'^2)_{i\in N'} \in S'_W$.

Next, in $\Gamma(N',g',T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that $a'^1_{n+1} = H$ and we show that $T+1 \ge n$ implies $s' \notin S_L$. By definition of SPNE, the profile of continuation strategy

[21] Note that we have $\tilde{v}'^t_{ij} \ge v^t_{ij}$ and not $\tilde{v}'^t_{ij} = v^t_{ij}$ because we do not exclude that the initial matrix $\tilde{V}'^0$ of players' information in $\tilde{\Gamma}(N',g',T)$ is such that there exists a pair of players $i,j \in N$, $i \ne j$ such that $\tilde{v}'^0_{ij} = 1$ whereas this is excluded for the initial matrix of players' information $V^0 = Id_n$ of $\Gamma(N,g,T)$.

$(s'_i|h'^2)_{i\in N'}$ is a SPNE of $\Gamma(N', g', T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i\in N}, H)$. As shown in the previous paragraph, if $T \geq n-1$, then every SPNE played in a subgame $\Gamma(N', g', T+1)|h'^2$ that starts after a history $h'^2 = ((a'^1_i)_{i\in N}, P)$ is such that the game $\Gamma(N', g', T+1)$ ends up with a winner. Therefore, if we assume that $s' \in S'_L$, then $T \geq n-1$ implies that player $(n+1)$ has an interest in deviating from s'_{n+1} such that $a'^1_{n+1} = H$ to a strategy s''_{n+1} such that $a''^1_{n+1} = P$. This profitable deviation in the first period of play contradicts $s' \in S'_{SPNE}$ which is why $s' \notin S'_L$.

Proof is completed by noting that every $s' \in S'_{SPNE}$ is either such that $a'^1_{n+1} = H$ or such that $a'^1_{n+1} = P$. $\square$

6.1.2 Proposition 1: Necessary Condition

Lemma 3 *If the game $\Gamma(N, g, T)$ is such that $T \leq n-2$, then $S_{SPNE} \cap S_L \neq \emptyset$.*

We prove Lemma 3 for complete networks only and use the the following lemma to get it for any connected network.

Lemma 4 *If $S_{SPNE} \cap S_L \neq \emptyset$ in $\Gamma(N, g_{complete}, T)$, then $S_{SPNE} \cap S_L \neq \emptyset$ in $\Gamma(N, g, T)$.*

Proof: In $\Gamma(N, g_{complete}, T)$, consider a strategy profile $s^c \in S_L$. Next, in $\Gamma(N, g, T)$, consider a strategy profile s such that s and s^c describe the same action profiles for every $i \in N$ and every $t \leq T$ in $\Gamma(N, g_{complete}, T)$ and $\Gamma(N, g, T)$ respectively. Since $N_i(g) \subseteq N_i(g_{complete}) = N\backslash\{i\}$ for every $i \in N$, we directly have that $s^c \in S_L$ implies $s \in S_L$. Equivalently, we get that $s \in S_W$ implies $s^c \in S_W$. Therefore, if there exists a player i with a strictly profitable deviation from the profile $s \in S_L$ for a history h^t in $\Gamma(N, g, T)$, then the same deviation from $s^c \in S_L$ is strictly profitable in $\Gamma(N, g_{complete}, T)$. We conclude that if the strategy profile $s \in S_L$ is not in S_{SPNE}, then the strategy profile $s^c \in S_L$ is not in S_{SPNE}. $\square$

Lemma 5 *If the game $\Gamma(N, g_{complete}, T)$ is such that $T \leq n-2$, then $S_{SPNE} \cap S_L \neq \emptyset$.*

Proof: We show that if $T \leq n-2$, then there exists a strategy profile $s \in S_{SPNE} \cap S_L$. For every h^t, denote $M(h^t)$ the set $\{i \in N : \forall j \in N\backslash\{i\}, v^{t-1}_{ji} = 0\}$ and let $m(h^t) = |M(h^t)|$. Players in $M(h^t)$ have Hidden in every period $t' \in [1, t-1]$. Note that as soon as a history h^t is such that $M(h^t)$ is a singleton, say $M(h^t) = \{l\}$, the game ends at $t-1$ with player l being the unique winner. Consider the profile s such that, for every $i \in N$, we have:

- $s^t_i(h^t) = H$ if $i \notin M(h^t)$
- $s^t_i(h^t) = H$ if $i \in M(h^t)$ and $T - t + 1 \leq m(h^t) - 2$
- $s^t_i(h^t) = H$ if $i \in M(h^t)$ and $T - t + 1 > m(h^t) - 2$ and $i = min_{j\in M(h^t)} j$
- $s^t_i(h^t) = P$ if $i \in M(h^t)$ and $T - t + 1 > m(h^t) - 2$ and $i \neq min_{j\in M(h^t)} j$.

First, let's show $s \in S_L$. Since for every $i, j \in N$, $i \neq j$, $v^0_{ij} = 0$, we have that $M(h^1) = N$ and $m(h^1) = n$. If $T - 1 + 1 \leq n - 2$, then, following s, V^1 remains equal to Id_n. Repeating the reasoning, we get that everybody Hides in every period, which establishes $s \in S_L$.

Next, let's show $s \in S_{SPNE}$ by showing that s satisfies the one-stage deviation principle. We distinguish two kinds of histories h^t and check that, conditional on h^t reached, no player $i \in N$

has an strict interest in unilaterally deviating from the continuation strategy $s_i|h^t$ at date t and conforming to $s_i|h^t$ thereafter.[22]

First, consider a subgame $\Gamma(N, g_{complete}, T)|h^t$ with h^t such that $T - t + 1 > m(h^t) - 2$. Let $l = min_{j \in M(h^t)} j$. Following $(s_i|h^t)_{i \in N}$, the action profile $(a_i^t)_{i \in N}$ is such that *(1)* for every $i \notin M(h^t)$, $a_i^t = H$, *(2)* for every $i \in M(h^t) \backslash \{l\}$, $a_i^t = P$ and *(3)* $a_l^t = H$. Therefore, we get $M(h^{t+1}) = \{l\}$.[23] As a consequence, following $(s_i|h^t)_{i \in N}$ in $\Gamma(N, g_{complete}, T)|h^t$, the game $\Gamma(N, g_{complete}, T)$ ends at t with l being the unique winner. Obviously, player l has no interest in unilaterally deviating from $s_l|h^t$ at time t. In addition, in $g_{complete}$, the action of every $i \notin M(h^t)$ has no effect in $\Gamma(N, g_{complete}, T)|h^t$ [24], so there is no strict interest for these players in deviating from $s_i|h^t$ at t. Finally, consider a deviation of a player $j \in M(h^t) \backslash \{l\}$. A strategy $s_j'|h^t$ that agrees with $s_j|h^t$ except at date t consists in Hiding at t instead of Passing On. If period $t = T$, then player j has no interest in such a deviation as the game would end at T with no winner instead of ending at T with player l winning. If period $t < T$, then at time $t + 1$ after player j's deviation, we have $M(h^{t+1}) = \{j, l\}$ and $m(h^{t+1}) = 2$ which implies that $m(h^{t+1}) - 2 = 0$. Since $t < T$, we have $T - t = T - (t + 1) + 1 > 0 = m(h^{t+1}) - 2$. As a consequence, following $(s_i|h^t)_{i \in N}$ in the subgame that starts at $t + 1$ after player j's deviation, every agent $i \neq j$ Hides and player j Passes On. It follows that player l is still the unique winner but at time $t + 1$ instead of t: if player j deviates, he then receives δ^t instead of δ^{t-1}. Conditional on h^t reached, we conclude that no player $i \in N$ has a strict interest in unilaterally deviating from $s_i|h^t$ at time t only.

Finally, consider a subgame $\Gamma(N, g_{complete}, T)|h^t$ with h^t such that $T - t + 1 \leq m(h^t) - 2$. Following $(s_i|h^t)_{i \in N}$, the action profile $(a_i^t)_{i \in N}$ is such that, for every $i \in N$, $a_i^t = H$. Therefore, we get $m(h^{t+1}) = m(h^t)$. Since $T - t + 1 \leq m(h^t) - 2$, we have $T - (t + 1) + 1 \leq m(h^{t+1})$. As a consequence, following $(s_i|h^t)_{i \in N}$ in $\Gamma(N, g_{complete}, T)|h^t$, we have that for every $i \in N$, $a_i^{t+1} = H$ yielding $m(h^{t+2}) = m(h^{t+1})$. The same reasoning applies for every $t' \in [t + 2, T]$ meaning that following $(s_i|h^t)_{i \in N}$, the game $\Gamma(N, g_{complete}, T)|h^t$ ends at T with no winner. As mentioned in the previous paragraph, in $\Gamma(N, g_{complete}, T)|h^t$, players $i \notin M(h^t)$ have no strict interest in deviating from $s_i|h^t$ at t. For a player $i \in M(h^t)$, a strategy $s_i'|h^t$ that agrees with $s_i|h^t$ except at date t consists in Passing On at t instead of Hiding. If a player i Passes On at time t, we get $m(h^{t+1}) = m(h^t) - 1$. Since h^t is such that $T - t + 1 \leq m(h^t) - 2$, we have that $T - (t + 1) + 1 \leq m(h^t) - 1 - 2 = m(h^{t+1}) - 2$. Therefore, following $(s_i|h^t)_{i \in N}$ in the subgame of $\Gamma(N, g_{complete}, T)|h^t$ starting at $t + 1$ after history h^{t+1}, we get for every $i \in N$, $a_i^{t+1} = H$. Repeating the reasoning, we get for every $i \in N$, $a_i^{t+2} = H$ and so until date T. Conditional on h^t reached, we conclude that no player $i \in N$ has a strict interest in deviating from $s_i|h^t$ at time t only. $\square$

Proof of Lemma 3: Directly from Lemmas 5 and 4. $\square$

Proof of Proposition 1 : Directly from Lemmas 1 and 3. $\square$

[22] See one-stage deviation principle for finite horizon games in Fudenberg and Tirole (1991)[pp 108-110].

[23] If a player $i \in M(h^t)$ Passes On at time t, then $i \notin M(h^{t+1})$.

[24] In $g_{complete}$, a Passed On item immediately reaches every player, i.e., no one further transmits previously received items.

6.2 Duration in Equilibrium

6.2.1 Minimal Duration

Let $W(g,T)|h^t$ denote the set of *potential winners* in $\Gamma(N,g,T)|h^t$. Given h^t, every player in $W(g,T)|h^t$ is such that there exists a strategy profile $s|h^t$ in $\Gamma(N,g,T)|h^t$ that enables him to hold the n items at a date $t \leq T$.

Lemma 6 *Consider a history h^t of $\Gamma(N,g,T)$ such that there exists a player $i \in W(g,T)|h^t$ who has Hidden in every period $t' \in [1,t-1]$. There exists a continuation strategy profile $s|h^t$ that is a NE of $\Gamma(N,g,T)|h^t$ and such that player i is the unique winner of $\Gamma(N,g,T)$.*

Proof: Consider the profile $s|h^t$ such that i Hides in every period whatever the history and such that every $j \neq i$ Passes On in every period whatever the history. $\square$

Proof of Proposition 2 : Pick a player $i \in W(g,T)$. Consider a strategy profile $s \in S$ that results in a final history $h^{e_i(g)+1} = (a^1, \ldots, a^{e_i(g)})$ such that :
- for all $t \leq e_i(g)$, $a_i^t = H$
- every player $j \neq i$ starts to Pass On in every period at a specific date $t_j = e_i(g) - d_{ij}(g) + 1$ (that depends on his distance to player j) : $a_j^t = H$ for all $t < t_j$ and $a_j^t = P$ for all $t \geq t_j$.

The history $h^{e_i(g)+1}$ is such that player i Hides in every period and the further a player j is from player i, the earlier he starts Passing On in every subsequent period. At every date t, the set of players $N\backslash\{i\}$ can be divided into two sets : the set $\{j \ : t_j \geq t\}$ of players who are "t-close" to i, that is who have not Passed on in any period $t' < t$, and the set $\{j \ : t_j < t\}$ of players who are "t-distant" from i, that is who have started to Pass On at a date $t' < t$. According to $h^{e_i(g)+1}$, in every period t, every piece of information initially held by a "t-distant" player is held by at least one "t-close" player. Indeed, every player $j \neq i$ who Passes On at time $t = 1$, i.e is such that $d_{ij}(g) = e_i(g)$, is linked to at least one player $k \neq i$ who Passes On at time $t = 2$, i.e., who is such that $d_{ik}(g) = e_i(g) - 1$, and so on. At date $t = e_i(g)$, player i wins the game because he receives the $n-1$ items that he did not hold initially from his direct neighbors. He is the unique winner as every player lacks at least player i's information item.

Let's prove that $s \in S_{SPNE}$ by showing that s satisfies the one-stage deviation principle. We consider three kinds of histories h^t and check that, conditional on h^t reached, no player has a strict interest in unilaterally deviating from the continuation strategy $s_i|h^t$ at date t and conforming to $s_i|h^t$ thereafter.

(i) First, consider a history $h^t = (a^1, \ldots, a^{t-1})$ with $t - 1 < e_i(g)$ that describes the same actions as $h^{e_i(g)+1}$ for every $i \in N$ and every date $t' \leq t-1$. Let's examine each kind of player :
- *Player i :* In $\Gamma(N,g,T)|h^t$, player i has no strict interest in unilaterally deviating from the continuation profile $(s_i|h^t)_{i\in N}$ as being the unique winner at date $e_i(g)$ is his best possible outcome.
- *Players in $\{j \neq i : t_j < t\}$:* Conditional on h^t, a "t-distant" player's deviation from $s_j|h^t$ at date t consists in Hiding. Then, the subgame $\Gamma(N,g,T)|h^{t+1}$ that starts at time $t+1$ is such that

player i is still able to win the game[25], i.e., belongs to $W(g,T)|h^{t+1}$, and has Hidden in every period $t' \in [1,t]$. It follows from Lemma 6 that s can be constructed so that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ such that player i is the unique winner at a date $t \geq e_i(g)$. Therefore, "t-distant" players have no strict interest in deviating unilaterally from $s_j|h^t$ at the single date t.

• *Players in* $\{j \neq i : t_j > t\}$ *:* Conditional on h^t, such a "t-close" player's deviation from $s_j|h^t$ at date t consists in Passing On. Then, the subgame $\Gamma(N,g,T)|h^{t+1}$ that starts at time $t+1$ is such that player i is still able to win the game[26], i.e., belongs to $W(g,T)|h^{t+1}$, and has Hidden in every period $t' \in [1,t]$. It follows from Lemma 6 that s can be constructed so that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ such that player i is the unique winner at a date $t \geq e_i(g)$. Therefore, such "t-close" players have no strict interest in deviating unilaterally from $s_j|h^t$ at the single date t.

• *Player in* $\{j \neq i : t_j = t\}$ *:* Conditional on h^t, such a "t-close" player's deviation from $s_j|h^t$ at date t consists in Hiding, which can have two different effects :

1st case, $i \in W(g,T)|h^{t+1}$ *:* it follows from Lemma 6 that s can be constructed so that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ such that i is the unique winner at a date $t \geq e_i(g)$. Therefore, such "t-close" players have no strict interest in deviating unilaterally from $s_j|h^t$ at the single date t.

2nd case, $i \notin W(g,T)|h^{t+1}$ *:* this case corresponds to a situation in which player i is excluded from potential winners of $\Gamma(N,g,T)|h^{t+1}$ by the fact that player j Hides at t. If this single deviation prevents i from winning, it must be that $d_{ij}(g) > T-t \Leftrightarrow e_i(g) = T$, which means that the number of remaining periods after date t is too small to enable player i to get player j's information item before the end of the game. Conditional on h^{t+1} reached, player i has never Passed On in $t' \in [1,t]$. It follows from $d_{ij}(g) > T-t$ that j cannot hold the item i at a date $t \in [t+1,T]$ either. Therefore, player j cannot win. Finally, we have to check that the outcome of the game is such that player j has no interest in deviating because the game would end with a player different from i winning at a date $t \leq e_i(g)$.

First, let players i and j be such that $d_{ij}(g) = 1$. In that case, it follows from $d_{ij}(g) > T-t$ that player j's deviation at date $t_j = t = T$ yields to the game ending up with no winner - player j has never Passed On at a date $t' < T$ and does not Pass On at time T. Therefore, player j has no strict interest in deviating from $s_j|h^t$ at the single date t.

Next, let players i and j be such that $d_{ij}(g) \geq 2$, which means that there is at least one agent, say k, on the shortest path between player i and j. Since we have $d_{ik}(g) < d_{ij}(g)$, we have at least three players, namely i, j and k, who have never Passed On at the beginning of the subgame $\Gamma(N,g,T)|h^{t+1}$. It follows that s can be constructed so that $s|h^{t+1}$ is a NE of $\Gamma(N,g,T)|h^{t+1}$ in which these three players Hides in every period whatever the history in the subgame.[27] This leads to the game ending up with no winner. Therefore, player j has no strict interest in deviating from $s_j|h^t$ at the single t.

[25]Indeed, at date t, every information item initially held by a "t-distant" player is already held by a "t-close" player. It follows that if every "t-close" player j sticks to $s_j|h^{t+1}$ and Passes On in every period from time t on, player i can hold the n item at $t = e_i(g)$.

[26]Indeed, it cannot be detrimental to player i's ability to win that a player j Passes On earlier than what is imposed by $s_j|h^t$ and then sticks to $s_j|h^{t+1}$.

[27]It is straightforward to check that such a profile is a NE of the subgame : given that two players Hide in every period whatever the history, there is no strictly profitable deviation from doing the same for the third agent.

(ii) Next, consider a history $h^t = (a^1, \dots, a^{t-1})$ with $t-1 \geq e_i(g)$ that describe the same actions as $h^{e_i(g)+1}$ for every $i \in N$ and every $t' \leq t-1$: for any such history, the subgames $\Gamma(N, g, T)|h^t$ do not exist as the game ends at date $e_i(g)$.

(iii) Finally, for any other history h^t, s can be constructed so that $s|h^t$ is a NE of $\Gamma(N, g, T)|h^t$. $\square$

6.2.2 Maximal Duration

We prove Proposition 4 before Proposition 3:

Proof of Proposition 4 i): By definition of separable graphs (chapter 3 in Tutte (2001)), if g is separable, then there exists a pair (g_1, g_2) of connected subnetworks of g such that $g_1 \cup g_2 = g$ and $g_1 \cap g_2$ is a critical agent of g, say k. Letting N_1 be the agents in g_1 and N_2 the agents in g_2, we get $N_1 \cup N_2 = N$ and $N_1 \cap N_2 = \{k\}$. From Theorem 1, there exists at least one agent in $N_1 \backslash \{k\}$ who is non-critical in the connected network g_1, say i, and at least one agent in $N_2 \backslash \{k\}$ who is non-critical in the connected network g_2, say j. It follows from the fact that $g_1 \backslash \{i\}$ is connected that $(g_1 \backslash \{i\}) \cup g_2 = g_1 \cup g_2 \backslash \{i\} = g \backslash \{i\}$ is connected[28], meaning that i is non-critical in g. The same is true for agent j. Next, we show that i and j cannot win together at a date $t \leq T$. If i and j win together at t, it must be that player i has every item $l \in N_2$ and player j has every item $l \in N_1$ at date t. Since k is on every path between i and j, every item $l \in N_2$ was held by k at least one period before it was held by i and every item $l \in N_1$ was held by k at least one period before it was held by j. Since $N_1 \cup N_2 = N$, there was a period $t' \leq t-1$ in which k held the n items. This contradicts the fact that i and j win together at t. $\square$

Proof of Proposition 4 ii) First, we show that every $s \in S_W$ is either such that the winner is unique or such that there are n winners. Consider a strategy profile $s \in S_W$ such that the game ends with strictly more than one winner, say players i and j win together at t. Since i wins at t, i holds every item $l \in N \backslash \{i\}$ at that date. Since the network is complete, every agent $k \in N$ also holds every item $l \in N \backslash \{i\}$ at t. Applying the same reasoning to j, we get that players i and j both winning at t implies n players winning at that date. Second, we consider a strategy profile $s \in S_W$ such that there are n winners at t and show that $s \notin S_{NE}$. If n players win at t, every $i \in N$ has Passed On at least at one date $t' \leq t$. Nevertheless, since the n players have not won at $t-1$, at least two agents, say i and j, had not Passed On yet at time $t-1$ but both Pass On at t.[29] Given that i Passes On at t, j has a strict interest in deviating from Passing On to be the only winner at t. $\square$

Proof of Proposition 3 is by induction. As in Section 6.1.1, we fix n and consider games $\Gamma(N, g, T)$, *augmented* games $\Gamma(N', g', T)$, and *augmented modified* games $\tilde{\Gamma}(N', g', T)$.

Let two games $\Gamma(N, g, T)$ and $\tilde{\Gamma}(N', g', T)$ form ***a K-pair*** if the two connected networks g and g' are such that $g = g' \backslash \{n+1\}$ with $g \in K(n)$ and $g' \in K(n+1)$. Given either $\Gamma(N, g, T)$ with $g \in K(n)$ or $\tilde{\Gamma}(N', g', T)$ with $g' \in K(n+1)$, one can always construct a K-pair. Indeed, a $g' \in K(n+1)$ can be constructed from a $g \in K(n)$ by linking agent $(n+1)$ only to agent n.[30]

[28]The union of two connected networks is a connected network.

[29]If there is a unique i who has not Passed On yet at time $t-1$, then player i is the unique winner at time $t-1$.

[30]Then g' is separable as n is critical.

All the connected subnetworks of g' are A $g \in K(n)$ can be constructed from a $g' \in K(n+1)$ by removing the non-critical agent $(n+1)$ and all its links.

The sets of strategy profiles such that the games $\Gamma(N', g', T)$ and $\tilde{\Gamma}(N', g', T)$ end up with a winner at a date $t \leq n$ (at date $t \geq n+1$, resp.) are denoted $S'_{end\leq n}$ and $\tilde{S}'_{end\leq n}$ respectively ($S_{end\geq n+1}$ and $\tilde{S}'_{end\geq n+1}$, resp.). To prove Proposition 3, we use:

Lemma 7 *For every K-pair of games $\Gamma(N, g, T)$ and $\tilde{\Gamma}(N', g', T)$, we have: if $S_{SPNE} \subseteq S_{\leq n-1}$, then $\tilde{S}'_{SPNE} \subseteq \tilde{S}'_{end\leq n}$.*

Proof: Similar to Lemma 2. $\square$

Proof of Proposition 3: As stated in Section 4.2.1, Proposition 3 is true for $n = 2$ and $n = 3$. We assume that Proposition 3 is true for n agents and we prove that it stays true for $n+1$ agents: in every $\Gamma(N', g', T+1)$ with $T+1 \geq n$, we have that if $g' \in K(n+1)$, then $S'_{SPNE} \subseteq S'_{end\leq n}$. From Proposition 4, we have that if $g' \in K(n+1)$, every $s' \in S'_{SPNE}$ is such that there is one non-critical agent in g' who loses.

First, in $\Gamma(N', g', T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that player $(n+1)$ loses and such that $a'^1_{n+1} = P$. By definition of SPNE, the profile of continuation strategy $(s'_i|h'^2)_{i\in N'}$ is a SPNE of the subgame $\Gamma(N', g', T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i\in N}, P)$. This subgame is equivalent to the *augmented modified* game $\tilde{\Gamma}(N', g', T)$ in the same sense as in the proof of Lemma 1. By assumption, if $g \in K(n)$, then $S_{SPNE} \subseteq S_{end\leq n-1}$. Given $\tilde{\Gamma}(N', g', T)$, we can find a game $\Gamma(N, g, T)$ to get a K- pair and then deduce from Lemma 7 that $\tilde{S}'_{SPNE} \subseteq \tilde{S}'_{end\leq n-1}$. Therefore, $(s'_i|h'^2)_{i\in N'} \in \tilde{S}'_{SPNE}$ implies $(s'_i|h'^2)_{i\in N'} \in \tilde{S}'_{end\leq n-1}$ which implies $s' = ((s'^1_i(h'^1)), (s'_i|h'^2))_{i\in N'} \in S'_{end\leq n}$.

Next, in $\Gamma(N', g', T+1)$, we consider a strategy profile $s' \in S'_{SPNE}$ such that player $(n+1)$ loses and such that $a'^1_{n+1} = H$. By definition of SPNE, the profile of continuation strategy $(s'_i|h'^2)_{i\in N'}$ is a SPNE of $\Gamma(N', g', T+1)|h'^2$ with $h'^2 = ((a'^1_i)_{i\in N}, H)$. As shown in the previous paragraph, if $g \in K(n)$, then every SPNE played in a subgame $\Gamma(N', g', T+1)|h'^2$ that starts after a history $h'^2 = ((a'^1_i)_{i\in N}, P)$ is such that the game $\Gamma(N', g', T+1)$ ends up with a winner different from player $(n+1)$ at a time $t \leq n$. Therefore, if we assume that $s' \in S'_{end\geq n+1}$, then the fact that g' belong to $K(n+1)$ implies that player $(n+1)$ has an interest in deviating from s'_{n+1} such that $a'^1_{n+1} = H$ to a strategy s''_{n+1} such that $a''^1_{n+1} = P$. Such a deviation would not make player $(n+1)$ win but make him lose at $t \leq n$ instead of $t \geq n+1$. This profitable deviation in the first period of play contradicts $s' \in S'_{SPNE}$ which is why $s' \notin S'_{end\geq n+1}$. Proof is completed by noting that every $s' \in S'_{SPNE}$ is either such that $a'^1_{n+1} = H$ or such that $a'^1_{n+1} = P$. $\square$

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