Discussion Paper No. 287
Wages and Productivity Growth in A Dynamic Oligopoly

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November 2009

Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.
WAGES AND PRODUCTIVITY GROWTH IN A DYNAMIC OLIGOPOLY

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November 2009

Abstract

This paper studies the innovation dynamics of an oligopolistic industry. The firms compete not only in the output market but also by engaging in productivity enhancing innovations to reduce labor costs. Rent sharing may generate productivity dependent wage differentials. Productivity growth creates intertemporal spill–over effects, which affect the incentives for innovation at subsequent dates. Over time the industry equilibrium approaches a steady state. The paper characterizes the evolution of the industry’s innovation behavior and its market structure on the adjustment path.

Keywords: innovation; labor productivity; oligopoly; wage differentials; productivity growth; industry dynamics

JEL Classification: D24; D42; D92; J31

*The first author is grateful for financial support by the German Science Foundation (DFG) through SFB/TR 15.
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1 Introduction

The relationship between wages and productivity growth has attracted a lot of attention in economic theory. According to the traditional view in growth theory, the causality runs from productivity growth to wage growth, with higher productivity leading to higher wages. This relation is based on the argument that “the marginal productivity equation determines the time path of the real wage” (Solow (1956), p. 68).

In this paper, we reverse the causality between wages and productivity growth and examine the impact of wages on firms’ productivity enhancing innovation investments in an oligopolistic industry. In particular, this paper studies the short- and the long-run evolution of productivity growth in an oligopolistic industry in which firms produce a homogeneous good, entry and exit are free and the time horizon is infinite. In each period, firms enter the market, they invest in capacity and in labor productivity enhancing innovation, and they compete in quantities in the following period. The competitive wage in the economy is exogenous. Yet, each firm’s specific wage is determined through bargaining with its employees. This allows us to investigate the effect of unionization on the industry’s equilibrium path. Firms have free access to the last period’s best production technology and their current innovation investments affect their labor cost at the subsequent date, and thus, the future innovation incentives. This process generates the industry’s dynamics.

We demonstrate that in the short-run, the higher is the industry’s competitive wage, and thus, the higher is the labor cost, the higher are firms’ investments in labor productivity enhancing innovation. Intuitively, when labor is costly, firms have stronger incentives to substitute against it, i.e., to use less labor by increasing the productivity of labor. In the long-run, there is a unique steady state. In the steady state, firm’s unit labor costs are constant over time and firm’s investments in productivity enhancing innovation are equal to the growth rate of the industry’s competitive wage. In the steady state also, the number of firms that enter in the market in each period, the output and the unit labor cost of each active firm depend only on the growth rate of the industry’s competitive wage and not on the level of the competitive wage. But the level of wages is important for the industry’s adjustment path towards the steady state. On this path, the number and
size of firms and their innovation activities depend on the level of their labor cost. An increase in the employees’ bargaining power reduces the innovation rate, and thus, slows down the speed of adjustment towards the steady state. In contrast, the impact of unionization on the number and size of firms is ambiguous outside the steady state.

This paper complements the analysis of Bester and Petrakis (2003, 2004) who examine the relation between wages and productivity growth in a perfectly competitive and a monopolistic industry, respectively. It extends their models to an imperfectly competitive market structure where the firms interact strategically in their capacity and innovation decisions. In contrast with the case of perfect competition, in this paper the firms’ wage rate is not necessarily identical to the competitive economy-wide wage. Instead, it depends on how unionization and wage bargaining affect the sharing of surplus between firms and their employees. As a result, unionization can have an impact on the endogenous variables of the industry both on the adjustment path and in the steady state. In contrast with the monopoly case, the number of active firms is endogenous in this paper, because there is free entry and exit. This also implies that the rate of innovation and the competitiveness of the industry are simultaneously determined on the equilibrium path. Indeed, free entry and exit have a profound impact on the firms’ innovation decisions: Whereas in Bester and Petrakis (2004) the monopolist has the highest innovation incentive for some intermediate range of unit labor cost, the present model leads to a monotone relation between these variables.

As a variation of our analysis of a homogenous market, in an appendix of this paper we adopt a demand specification based on Dixit and Stiglitz (1977) that reflects a preference for product variety. This allows us to confirm the robustness of our main findings on the industry’s long-run dynamics, as well as to examine the role of product differentiation. Regarding the latter, we find that industries characterized by stronger product differentiation tend to have a larger number of smaller and less efficient firms than industries with less differentiated products.

A number of empirical studies support our argument that labor market conditions affect productivity growth. In a recent paper, Dew-Becker and Gordon (2008) have demonstrated that changes in labour market policies, and thus, in the labour market conditions can explain the behavior of the
EU’s productivity growth after 1995, as well as the differences in the productivity growth’s trends in the EU and the US. Moreover, Gordon (1987, 2000) has found that the behavior of the ratio of wages to labor productivity plays a crucial role in explaining the trends of macroeconomic productivity growth in the US, Japan and Europe. Similar findings at the industry level are presented in Flaig and Stadler (1994), Doms et al. (1997), and Chennells and Van Reenen (1997).

Examining the interaction between unionization and firms’ innovation activities, we find that wage bargaining reduces firms’ short–run incentives to invest in productivity enhancing innovation. Intuitively, rent sharing between the employees and the firms leads to the standard hold up problem in labor markets. This observation is in line with the findings of Baldwin (1983), Grout (1984) and van der Ploeg (1987) who demonstrate that due to the hold up problem, firms’ investments decrease with the employees’ bargaining power.\(^1\) Interestingly, things change in the long–run. In particular, wage bargaining does not affect the growth rate of the industry’s competitive wage. Given that in the long–run firms’ investments are equal to the latter, it follows that unionization does not influence firms’ long–run innovation incentives and productivity growth.\(^2\) Nevertheless, higher union bargaining power means fewer firms and higher output per firm in the steady state, i.e. a more concentrated market with less efficient firms.

The remainder of the paper is organized as follows. Section 2 describes our model. In Section 3, we derive the equilibrium for a given state of the environment. The firms’ innovation decisions change the state of the environment over time. The steady state of this process is studied in Section 4. In Section 5, we show that the industry monotonically approaches the steady state and describe the industry’s dynamics on its adjustment path. We conclude in Section 6. The proofs of all formal results are relegated to Appendix A. In Appendix B we extend our steady state analysis to an

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\(^1\)See Malcomson (1997) for an overview. Tauman and Weiss (1987) and Ulph and Ulph (1994, 1998, 2001) consider different environments, with asymmetric firms and a patent race respectively; they find that unionization can lead to overinvestment in innovation.

\(^2\)Note that the empirical evidence on the relation between unionization and innovation is mixed (see e.g., Hirsh and Link (1984), Connolly et al. (1986), Acs and Audretsch (1987a&b), Machin and Wadhwnani (1991), Menezes-Filho et al. (1998)). For a review of the empirical literature see Flanagan (1999).
alternative specification of demand by considering Dixit–Stiglitz preferences.

2 The Model

We consider an oligopolistic industry in which firms produce a homogeneous product and entry and exit are free. The market demand is given by:

\[ p = d - X, \]

where \( p \) is the product’s price, \( X \) is the aggregate output of all firms. For simplicity, we assume that the demand function (1) is stationary over time. Accordingly, demand does not change with the growth of incomes.\(^3\) We further assume that \( d \), which captures the size of demand, is large enough so that entering the market is always profitable for a positive number of firms.

Time is discrete, it is denoted by \( t \), with \( t = 0, 1, 2, \ldots \), and the horizon is infinite. At date \( t \), all firms have access to the best available current technology, which is described by its level of labor productivity, \( a_t \). This implies that all firms that enter in the market at date \( t \) are identical. However, to produce output \( x_{it} \) at date \( t + 1 \), each firm \( i \), with \( i = 1, 2, \ldots, n_t \), must invest in capacity \( kx_{it} \) at date \( t \); that is, the unit cost of capacity investments is \( k > 0 \). Further, at date \( t \), each firm \( i \) can invest in process innovation, \( q_{it} \), in order to increase its labor productivity from \( a_t \) to \( a_t(1 + q_{it}) \) at date \( t + 1 \). The cost of the process innovation investments is given by \( K(q) \), with \( K(0) = K'(0) = K''(0) = 0 \), and \( K'(q) > 0, K''(q) > 0 \) for all \( q > 0 \). We also assume that \( K(\cdot) \) satisfies the following inequality:

\[ K''(q) \geq \frac{K'(q)^2}{2K(q)}. \]  

This condition requires that the innovation cost \( K(\cdot) \) is sufficiently convex. It is satisfied, for instance, when \( K(q) = \mu q^m \), with \( m \geq 2 \) and \( \mu > 0 \). As a consequence, at date \( t + 1 \), each firm \( i \) produces its output \( x_{it} \) by hiring \( x_{it}/[a_t(1 + q_{it})] \) units of labor. It is important to note that the industry dynamics are generated by the firms’ innovation behavior. This determines

\(^3\)This could be justified by assuming that the demand function is derived from a standard quasi-linear utility function in which wealth effects are absent.
the best available in the industry technology and the incentives for further innovation in the subsequent period.

We assume that the competitive wage rate of labor is exogenously given as \( \bar{w}_t \) at date \( t \). We also assume that at the following date, date \( t + 1 \), the competitive wage becomes:

\[
\bar{w}_{t+1} = \bar{w}_t (1 + \gamma), \quad \gamma > 0,
\]

where \( \gamma \) is the growth rate of the competitive wage. One could think of \( \gamma \) as the rate of average productivity growth and wage growth in the entire economy. This means that the industry under consideration constitutes a tiny part of the whole economy, and thus, its impact on the growth of \( \bar{w}_t \) is negligible. In what follows we define the competitive wage per efficiency unit of labor at date \( t \),

\[
c_t \equiv \bar{w}_t \frac{a_t}{c_t},
\]

and consider it as the industry’s state variable at the beginning of date \( t \). From (3), (4) and the result of the process innovation investments, it follows that the competitive wage per efficiency unit of labor that firm \( i \) faces at date \( t + 1 \) is:

\[
\frac{\bar{w}_{t+1}}{a_t(1 + q_{it})} = \frac{\bar{w}_t(1 + \gamma)}{a_t(1 + q_{it})} = \frac{1 + \gamma}{1 + q_{it}} c_t. \tag{5}
\]

Firm \( i \)’s specific wage rate at date \( t + 1 \) may differ from the competitive wage as it may be positively related to the firm’s labor productivity enhancement due to the firm’s innovation activities at date \( t \). The latter activities generate quasi-rents over which the employees of the firm have a ‘stake’ - this is the well-known hold-up problem. Such productivity dependent wage differentials reflect the employees’ bargaining power within the firm. The firm \( i \)’s specific wage is determined through bargaining between the firm and its employees at the beginning of date \( t + 1 \), i.e. just before production. In particular, since at the previous date \( t \) firm \( i \)’s output and process innovation investments have been determined, the employment level is fixed during the wage negotiations. Therefore, the only variable at stake during the negotiations is the surplus per unit of firm \( i \)’s output given by:

\[
\rho_{t+1} - k - \frac{\bar{w}_{t+1}}{a_t(1 + q_{it})}. \tag{6}
\]
Clearly, firm $i$ would prefer to pay the minimum possible wage, i.e. the competitive wage $\bar{w}_{t+1}$, and retain for itself the whole surplus. On the other hand, firm $i$’s employees would prefer to set the wage equal to $(p_{t+1} - k)a_t(1 + q_{it})$ so that they are the ones who capture the whole surplus. As a consequence, the firm $i$’s specific wage is expected to be a weighted average of the two bargaining parties most preferred wages, with weights equal to their respective bargaining powers. Assuming that the employees’ bargaining power is given by $r$, with $0 \leq r < 1$, it follows that firm $i$’s specific wage rate at date $t+1$ is:

$$(1 - r)\bar{w}_{t+1} + r(p_{t+1} - k)a_t(1 + q_{it}).$$

(7)

Therefore, by (3) and (5), firm $i$’s labor cost per unit of output is

$$(1 - r)c_t\frac{1 + \gamma}{1 + q_{it}} + r(p_{t+1} - k).$$

(8)

Firm $i$’s profits upon entry at date $t$ are thus given by:

$$(1 - r)\left[d - x_{it} - \sum_{j \neq i} x_{jt} - k - c_t\frac{1 + \gamma}{1 + q_{it}}\right]x_{it} - K(q_{it}).$$

(9)

## 3 Static Equilibrium

In this section, we obtain the static equilibrium when the state of the industry at date $t$ is given by $c_t$.

Upon entry at date $t$, firm $i$ chooses $x_{it}$ and $q_{it}$ in order to maximize its profits (9). In a symmetric Nash equilibrium, all active firms produce the same quantity $x^*_t$ and have the same innovation rate $q^*_t$. Keeping this in mind, it follows from the first order conditions of firms $i$’s maximization problem that $x^*_t$ and $q^*_t$ are determined by:

$$x^*_t = \frac{1}{1 + n_t}\left[d - k - c_t\frac{1 + \gamma}{1 + q^*_t}\right],$$

(10)

---

*Note that we assume that firms do not discount future profits. This assumption is without loss of generality. If the firm discounts its future profits by a factor $0 < \delta < 1$, the analysis goes through by simply redefining $k_\delta \equiv k/\delta$ and $K_\delta(q) \equiv K(q)/\delta$. 

*One can show that the first order conditions are sufficient if $[K(q)(1 + q)]'' \geq (1 - r)[c_t(1 + \gamma)]^2/2$. 

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\begin{align*}
(1 - r)c_t(1 + \gamma) \left[ d - k - c_t \frac{1 + \gamma}{1 + q_t^*} \right] &= K'(q_t^*)(1 + q_t^*)^2. \quad (11)
\end{align*}
Substituting (10) into (9), firm \( i \)'s profits at date \( t \) can be rewritten as:
\begin{align*}
\frac{(1 - r)}{(1 + n_t)^2} \left[ d - k - c_t \frac{1 + \gamma}{1 + q_t^*} \right]^2 - K(q_t^*). \quad (12)
\end{align*}

Due to free entry and exit, firm \( i \)'s profits (12) have to be zero.\(^6\) Using (11) and setting (12) equal to zero, we can determine the equilibrium values of \( n_t^* \) and \( q_t^* \) for a given \( c_t \). More specifically, in order to derive the equilibrium innovation rate \( q_t^* \), we define the following function:
\begin{align*}
\varphi_I(q) &\equiv \frac{K'(q)^2 (1 + q)^4}{K(q)}. \quad (13)
\end{align*}
By Lemma 1 in the Appendix, \( \varphi_I(q) \) is strictly increasing in \( q \). Moreover, our assumptions on \( K(\cdot) \) ensure that \( \lim_{q \to 0} \varphi_I(q) = 0 \) and \( \lim_{q \to \infty} \varphi_I(q) = \infty \). This is so, because by L' Hospital rule \( \lim_{q \to 0}[K'(q)^2/K(q)] = \lim_{q \to 0}[2K''(q)] = 0 \) and similarly for \( q \to \infty \).

Combining (11) with the zero profit condition resulting from (12), we get:
\begin{align*}
(1 - r)[c_t(1 + \gamma)]^2 &= \varphi_I(q_t^*). \quad (14)
\end{align*}
The properties of \( \varphi_I(\cdot) \) imply the following results.

**Proposition 1** For a given value of the state variable \( c_t \), there is a unique equilibrium innovation rate \( q_t^* \). Moreover, \( q_t^* \) is:

(i) strictly increasing in \( c_t \) and \( \gamma \),

(ii) strictly decreasing in \( r \),

(iii) independent of \( d - k \).

According to Proposition 1(i), the competitive wage growth \( \gamma \) stimulates firm’s productivity enhancing innovation. As can be seen from (11), each firm chooses its innovation rate so that its marginal benefit from the higher labor productivity tomorrow equals the marginal cost of its innovation investments today. A higher \( \gamma \) means higher unit labor cost for the firm (see (8)). This also holds when the competitive wage per efficiency unit of labor at date \( t \), \( c_t \),

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\(^6\)As usual, we ignore the problem that \( n_t \) should be an integer number.
becomes higher. Under these circumstances, a firm has stronger incentives to use less labor and it does so by enhancing its labor productivity, i.e. by investing more in process innovation.

Proposition 1(ii) asserts that firm’s productivity enhancing investments are negatively related to the employees’ bargaining power \( r \). This is an immediate consequence of the hold up problem. Clearly, an increase in the employees’ bargaining power leads to an increase in the employees’ share of the quasi rents generated by innovation and a decrease in the respective firm’s share. As the firm enjoys a smaller share of the outcome of its investments, it has weaker incentives to invest.

According to Proposition 1(iii), the market size, as captured by \( d - k \), has no impact on firm’s innovation investments. This is so because the marginal benefit of the innovation investments is proportional to the equilibrium output of each firm. As we will see later on, the latter is independent of the market size, and thus, the equilibrium innovation investments are also independent of the market size.

Having determined the relation between \( c_t \) and \( q^*_t \), we use the zero profit condition to derive the number \( n^*_t \) of firms that are active in the market in state \( c_t \).

**Proposition 2** For a given value of the state variable \( c_t \), there is a unique equilibrium number \( n^*_t \) of active firms. Moreover, \( n^*_t \) is:

(i) strictly decreasing in \( c_t \) and \( \gamma \),

(ii) strictly increasing in \( d - k \).

Proposition 2 states that there is a negative relationship between the number of firms that enter in the market in equilibrium \( n^*_t \) and the competitive wage per efficiency unit of labor \( c_t \). As mentioned above, higher \( c_t \) means higher unit labor cost. The latter translates into lower efficiency for the firm, and thus, into a lower profit margin. Since the profit margin is low, fewer firms are willing to enter in the market. A similar reasoning applies for an increase in the competitive wage growth \( \gamma \).

Proposition 2 also states that when the market size increases, there are stronger entry incentives. The intuition is straightforward. The bigger is the size of the market, the more space there is in the market for firms to enter.

Equation (10) allows us to determine each firm’s equilibrium output \( x^*_t \).
Proposition 3 For a given value of the state variable $c_t$, there is a unique equilibrium output $x^*_t$ for each firm. Moreover, $x^*_t$ is:

(i) strictly increasing in $c_t$ and $\gamma$,
(ii) independent of $d - k$.

According to Proposition 3(i), the higher is the competitive wage per efficiency unit of labor $c_t$, the higher is each firm’s output $x^*_t$. The intuition is as follows. We know from Proposition 1(i) that higher $c_t$ leads to higher $q^*_t$. We also know that firm’s output and innovation investments are complements. This holds because when output increases the marginal benefit of innovation also increases ("output effect"). As a consequence, since $q^*_t$ increases with $c_t$, $x^*_t$ also increases with $c_t$.

As we saw in Proposition 2(ii), when the market size $d - k$ increases, the equilibrium number of entering firms $n^*_t$ increases, and thus, each firm tends to be smaller. Yet, an increase in the market size, increases each firm’s profit margin which tends to increase its equilibrium output. These two effects cancel out each other. As a consequence, firm’s equilibrium output turns out to be independent of the market size (Proposition 3(ii)).

We know from Proposition 1(ii) that an increase in the employees’ bargaining power has a negative impact on firm’s investments in labor productivity enhancing innovation. Similarly, one might wonder about the impact of the employees’ bargaining power on the equilibrium number of entering firms, as well as on the equilibrium output of each firm. An increase in the employees’ power $r$ has two opposite effects on firms’ entry incentives. First, it leads to a decrease in firm’s innovation (Proposition 1(i)), and thus, to a decrease in the “entry costs” $K(q^*_t)$. Second, it leads to an increase in the firm’s unit labor costs. The latter, together with the decrease in the share of the quasi rents $1 - r$ that a firm enjoys, translate into lower firm’s gross profits and they lead, in turn, to a decrease in firm’s entry incentives. As a consequence, the equilibrium number of firms might increase or decrease with $r$.

Setting $K(q) = q^2$ and using numerical simulations we find that when the competitive wage per efficiency unit of labor $c_t$ is low, the stronger is the employees’ bargaining power, the more firms enter into the industry. Instead, when $c_t$ is sufficiently high, an increase in the employees’ bargaining power can discourage firms’ entry. Regarding the impact of the employees’
bargaining power \( r \) on firm’s equilibrium output, our numerical simulations indicate that an increase in \( r \) discourages firm’s production when \( c_t \) is low, while it can encourage it when \( c_t \) is high and \( r \) is sufficiently low. The respective impact of an increase in \( r \) on the aggregate output \( n^*_t x^*_t \) is instead always negative.

4 Steady State Equilibrium

We now turn to the study of the long-run dynamics of the industry. In particular, in this section we study the existence, the uniqueness and the properties of the steady state. In the subsequent section, we investigate the industry’s adjustment path towards the steady state.

A firm that at date \( t \) enters the market and invests in process innovation, has a one-period monopoly over its productivity improvement in the following date \( t + 1 \). A firm instead that enters at date \( t + 1 \), has access to the most advanced technology that has been developed at the previous date \( t \), and it can further improve upon this technology by investing in innovation that it will use in order to produce its output at date \( t + 2 \). Clearly, this means that current innovations generate spillover effects on the starting point of future innovations. This process determines the evolution of the industry’s state variable \( c_t \) and, therefore, also the intertemporal equilibrium path of the variables \( n^*_t, q^*_t \) and \( x^*_t \) that, as we saw in the static equilibrium, depend on \( c_t \).

We infer from (4), (5) and the symmetry of the static equilibrium that at date \( t + 1 \), the industry’s state \( c_{t+1} \) depends on the state of the previous date \( c_t \) according to:

\[
c_{t+1} = \frac{1 + \gamma}{1 + q^*_t} c_t.
\]

(15)

Since \( q^*_t \) is determined by \( c_t \), it follows that equation (15) describes the evolution of \( c_t \) over time for any given initial value \( c_0 \). What happens in a steady state? In a steady state, the variable \( c_t \) remains constant at some value over time, \( c_t = \hat{c} \). Accordingly, the variables \( n^*_t, q^*_t \) and \( x^*_t \) also remain constant over time, \( n^*_t = \hat{n}, q^*_t = \hat{q} \) and \( x^*_t = \hat{x} \).

From \( c_{t+1} = c_t = \hat{c} \) and (15) follows immediately that the state variable
is in a steady state \( \hat{c} > 0 \) if and only if

\[
q_t^* = \hat{q} = \gamma \tag{16}
\]

for all \( t \). This means that in a steady state, the industry’s rate of productivity growth \( q_t^* \) equals the growth rate of the competitive wage \( \gamma \). Note that if, according to our previous discussion, the competitive wage reflects average productivity growth in the rest of the economy, then in turn the condition for the existence of a steady state (16) means that the industry’s innovation performance is identical to the average performance of all other industries.

Note also that in a steady state the firm–specific wage, as specified in (7), increases at the same rate as the competitive wage. This means that the relative wage differential remains constant over time. Further, by (8), the firms’ unit labor cost is stationary in a steady state, and it is given by \( \hat{c} + r(d - \hat{n}\hat{x} - k - \hat{c}) \). In other words, the firm’s unit labor cost exceeds the steady state competitive wage per efficiency unit of labor \( \hat{c} \) by an amount which is proportional to its employees’ power and to the industry’s steady state profits margin. Using (16), the equilibrium conditions (11) and (12) in a steady state are:

\[
(1 - r)\hat{c}^2(1 + \hat{n}) = \frac{K'(q)(1 + \gamma)}{K(q)}, \tag{17}
\]

and

\[
\frac{(1 - r)}{(1 + \hat{n})^2} [d - k - \hat{c}]^2 = K(\gamma). \tag{18}
\]

Conditions (17) and (18) determine the steady state values \( \hat{c} \) and \( \hat{n} \). The output of each firm \( \hat{x} \) in the steady state can be derived from equation (10) by using \( \hat{c}, \hat{n}, \) and (16).

To study the industry’s steady state equilibrium, we define the function

\[
\varphi_{II}(q) = \frac{K'(q)^2(1 + q)^2}{K(q)}. \tag{19}
\]

By Lemma 1 in the Appendix, \( \varphi_{II}(q) \) is strictly increasing in \( q \). Moreover, our assumptions on \( K(\cdot) \) ensure that \( \lim_{q \to 0} \varphi_{II}(q) = 0 \) and \( \lim_{q \to \infty} \varphi_{II}(q) = \infty \), for the same reasons as for the case of \( \varphi_I(q) \).

The combination of (17) and (18) shows that \( \hat{c} \) is the solution of the equation

\[
(1 - r)\hat{c}^2 = \varphi_{II}(\gamma). \tag{20}
\]
In the remainder of this section we show that the steady state equilibrium is unique and discuss its properties.\textsuperscript{7}

**Proposition 4** For a given value of $\gamma$, the steady state value $\hat{c}$ of the industry’s state variable is unique. Moreover, $\hat{c}$ is:

(i) strictly increasing in $\gamma$,
(ii) strictly increasing in $r$,
(iii) independent of $d-k$.

As stated in Proposition 4(i), a higher competitive wage growth leads to a higher competitive wage per efficiency unit of labor at the steady state. Intuitively, since in the steady state $\hat{q} = \gamma$, an increase in the rate of the competitive wage growth is countervailed by an increase in each firm’s innovation investments. For the latter to occur though, the competitive wage per efficiency unit of labor should be higher in the steady state (see Proposition 1(i)). In other words, a higher competitive wage growth rate can be supported in the steady state only if higher unit labor costs force firms to increase their innovation investments.

Regarding the impact of the employees’ bargaining power on the competitive wage per efficiency unit of labor in the steady state, Proposition 4(ii) tells us that it is positive. The intuition is as follows. By Proposition 1(ii) we know that the higher is the employees’ power, the lower are the firms’ innovation investments. However, in the steady state firms’ investments are constant over time. Therefore, for an increase in the employees’ power not to lead to a decrease in the innovation investments, there must be an opposite force in action. This is, in fact, an increase in the competitive wage per efficiency unit of labor that, in contrast to the employees’ power, reinforces firms’ innovation incentives (Proposition 1(i)). Interestingly, an increase in the employees’ power has no impact on the firms’ investment incentives in the steady state. The latter are determined exclusively by the exogenous competitive wage growth. This implies that there is no hold-up problem in the industry’s steady state. Nevertheless, the higher employees’ power implies that the active firms face less favorable production conditions, i.e. their unit labor cost is higher.

\textsuperscript{7}Note that a necessary and sufficient condition for the existence of a steady state equilibrium is that $\gamma$ is not too large. In particular, $\gamma$ should be such that $\phi_{II}(\gamma) < (1-r)(d-k)^2$. Otherwise, the equilibrium quantity becomes negative.
Finally, since we know from Proposition 1(iii) that the firms’ innovation incentives are independent of the market size, it follows that, as stated in Proposition 4(iii), the competitive wage per efficiency unit of labor is also independent of the market size.

The solution $\hat{c}$ of (20) allows us to derive the number $\hat{n}$ of active firms.

**Proposition 5** For a given value of $\gamma$, there is a unique steady state number $\hat{n}$ of active firms in the industry. Moreover, $\hat{n}$ is:

(i) strictly decreasing in $\gamma$,
(ii) strictly decreasing in $r$,
(iii) strictly increasing in $d - k$.

We know from Proposition 2(i) that when the competitive wage per efficiency unit of labor increases, the equilibrium number of entering firms decreases. We also know from Proposition 4(i) that the competitive wage growth rate is positively related to the competitive wage per efficiency unit of labor in the steady state. Combining these two, it follows that, as stated in Proposition 5(i), an increase in the competitive wage growth rate leads to a decrease in the steady state number of firms.

How does the employees’ power influence the number of active firms in the steady state? We saw in Proposition 4(ii) that an increase in the employees’ power has a positive impact on the competitive wage per efficiency unit of labor. We also saw in Proposition 2(i) that the latter has a negative impact on the number of entering firms. As a consequence, when the employees’ bargaining power is increased, firms have weaker incentives to enter the market. Finally, and as expected, an increase in the market size offers stronger market entry incentives (Proposition 5(iii)).

The solution $\hat{c}$ of (20) together with (18) and (10) determines the output $\hat{x}$ of each firm.

**Proposition 6** For a given value of $\gamma$, there is a unique steady state output $\hat{x}$ for each firm. Moreover, $\hat{x}$ is:

(i) strictly increasing in $\gamma$,
(ii) strictly increasing in $r$,
(iii) independent of $d - k$.  

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According to Proposition 6(i), an increase in the competitive wage growth rate has a positive impact on each firm’s equilibrium output. Intuitively, an increase in the competitive wage growth rate leads to an increase in the competitive wage per efficiency unit of labor (Proposition 4(i)). An increase though in the latter, as we know from Proposition 3(i), leads to an increase in the equilibrium output of each firm. Thus, $\gamma$ and $\hat{x}$ move in the same direction. Similarly, as stated in Proposition 6(ii), $r$ and $\hat{x}$ move in the same direction. The intuition for the latter result is a straightforward implication of Propositions 4(ii) and 3(i). Finally, as in the static equilibrium, in the steady state too, each firm’s output is independent of the market size $d - k$.

Combining Propositions 5 and 6, we end up with the following implications. First, a higher competitive wage growth rate is expected to lead to industries with a smaller number of larger firms. Second, industries in which employees have strong power are expected to be more concentrated, i.e. have fewer and larger firms, than industries with weak employees’ power.

5 Equilibrium Dynamics

We now show that the steady state, studied in the previous section, indeed describes the long–run industry equilibrium, i.e. we show that for any initial value $c_0$, the industry’s state variable monotonically approaches the steady state value $\hat{c}$ over time. Obviously, this implies that the equilibrium variables $(n^*_t, q^*_t, x^*_t)$ tend towards $(\hat{n}, \hat{q}, \hat{x})$ in the limit as $t \to \infty$.

The evolution of $c_t$ is given by equation (15), where $q^*_t$ is the industry’s equilibrium innovation rate at date $t$ in state $c_t$. Thus, (15) represents a first–order difference equation. Its solution has the following property:

**Proposition 7** The state variable $c_t$ monotonically approaches $\hat{c}$ over time. That is, $\{c_t\}_{t=0}^\infty$ is a monotone sequence with $\lim_{t \to \infty} c_t = \hat{c}$.

Proposition 7 implies that, since the state variable $c_t$ approaches its steady state value $\hat{c}$ monotonically, it increases over time if the initial state $c_0$ lies below $\hat{c}$; while it decreases over time if $c_0 > \hat{c}$. In other words, the industry’s competitive wage per efficiency unit of labor is decreasing over time when, for given $w_0$, the level of labor productivity in the industry is initially low ($a_0$ low). The opposite is true when $a_0$ is high, in which case we expect to observe
$c_t$ to be increasing over time. Furthermore, the industry’s labor cost per unit of output is expected to exhibit a similar behavior to $c_t$. That is, the industry becomes increasingly more efficient when its initial labor productivity is low; and vice versa.

Propositions 1, 2 and 3, therefore, allow us to characterize the industry’s behavior on its adjustment path:

**Proposition 8** If $c_0 < \hat{c}$, then on the equilibrium path $q_t^*$ and $x_t^*$ are increasing over time while $n_t^*$ is decreasing over time. Moreover, total industry output $X_t^* = n_t^*x_t^*$ is decreasing over time. Conversely, $q_t^*$ and $x_t^*$ are decreasing while $n_t^*$ and $X_t^*$ are increasing over time if $c_0 > \hat{c}$.

According to Proposition 8, depending on the initial state of the industry, labor productivity growth either increases, or decreases continuously over time. In particular, when the industry’s initial labor productivity $a_0$ is high (for given $w_0$), firms invest increasingly more over time in innovation, and thus, there is acceleration in productivity growth. When instead the industry’s initial productivity is low, the opposite occurs. Moreover, Proposition 8 states that changes in the labor productivity growth are positively related to changes in the size of firms and negatively related to changes in the number of firms and the aggregate industry output. Therefore, when initial labor productivity is low, the industry becomes increasingly less concentrated through entry of new firms and a decrease in the size of the existing firms; moreover, the aggregate industry output expands over time. In contrast, when the initial labor productivity is high, the industry becomes increasingly more concentrated through the exit of firms and the increase in the size of the active firms, while its aggregate output shrinks over time.

The above results have a number of empirically testable implications for the industry’s adjustment following a change in exogenous parameters. First, an increase in the employees’ bargaining power is expected to lead to a pattern of exit of firms and an increased concentration in an industry that has already reached its steady state. Second, a similar pattern is expected to occur when the growth rate of the competitive wage becomes higher. In contrast, a decrease in the employees’ power or the competitive wage growth rate should be followed by entry of new firms in the industry and a downside of the size of the existing firms. Finally, a change in the market size is not
expected to have a significant impact on industry dynamics, since it should be accommodated by the entry of a number of new firms of similar size to the existing ones.

6 Concluding Remarks

We have shown that the labor market characteristics can play a crucial role in our understanding of the innovative performance, as well as of the productivity of different industries and countries.

We have considered an imperfectly competitive market in a partial equilibrium model which is based on a cost-push argument of innovation. Firms react by innovating to increases in the exogenous economy-wide wage rate. But also wage bargaining at the firm level influences their innovation decision. We have shown that unionization lowers the incentives for innovation in the short-run. But, perhaps surprisingly, long-run productivity growth in the steady state is independent of wage bargaining.

Our findings give rise to a number of interesting empirically testable implications. An increase in the growth rate of the competitive wage is expected to lead to a more concentrated industry, i.e. fewer and larger firms, and to a pattern of exit of firms in an industry that has already reached its steady state. A similar pattern is more likely to be observed in industries in which employees have strong bargaining power than in industries with weak employees’ power.

An interesting extension of this paper would be to embed the analysis in a general equilibrium model, in which the economy-wide wage rate is endogenous. In such a model, aggregate productivity growth in all industries of the economy would determine the path of real wages. Hellwig and Irmen (2001) present a model of this type; but they assume all industries to be perfectly competitive. By extending their model along the lines of this paper, one might address the question of how imperfect competition and wage bargaining affect productivity growth not only in a single industry but also in the entire economy.
7 Appendix A: Proofs of Propositions 1–8

This appendix contains the proofs of Propositions 1–8. We begin with the following auxiliary Lemma.

Lemma 1 By condition (2), $\varphi_I(q)$ in (13) and $\varphi_{II}(q)$ in (19) are strictly increasing in $q$.

Proof: The functions $\varphi_I(\cdot)$ and $\varphi_{II}(\cdot)$ are certainly strictly increasing in $q$ if

$$\frac{K'(q)^2}{K(q)}$$

is non-decreasing in $q$. This is the case if and only if

$$2K''(q)K(q) - K'(q)^2 \geq 0.$$  \hspace{1cm} (22)

By condition (2), $2K''(q)K(q) \geq K'(q)^2$. This implies that (22) is satisfied.

Q.E.D.

Proof of Proposition 1: Results (i)–(iii) immediately follow from (14) and the properties of $\varphi_I(\cdot)$ stated in Lemma 1. Q.E.D.

Proof of Propositions 2 and 3: We first show that in equilibrium

$$c_t \frac{1 + \gamma}{1 + q_t^*}$$

is strictly increasing in $c_t$. Indeed, by (14) we have

$$(1 - r) \left[ c_t \frac{1 + \gamma}{1 + q_t^*} \right]^2 = \frac{K'(q_t^*)^2(1 + q_t^*)^2}{K(q_t^*)}.$$  \hspace{1cm} (24)

The r.h.s. of this equation is strictly increasing in $q_t^*$ because the proof of Lemma 1 shows that $K'(q)^2/K(q)$ is non-decreasing in $q$. By Proposition 1, $q_t^*$ is strictly increasing in $c_t$ and so the r.h.s. of (24) is strictly increasing in $c_t$. It thus follows from (24) that the term in (23) is strictly increasing in $c_t$.

Now consider the zero profit condition

$$\frac{(1 - r)}{(1 + n_t)^2} \left[ d - k - c_t \frac{1 + \gamma}{1 + q_t^*} \right]^2 = K(q_t^*).$$  \hspace{1cm} (25)
As we have just shown, the term in the bracket of the l.h.s. of this equation is strictly decreasing in $c_t$. By Proposition 1, the r.h.s of this equation is strictly increasing in $c_t$. This immediately implies that $n_t^*$ is strictly decreasing in $c_t$.

Because, by Proposition 1, $q_t^*$ is strictly increasing in $\gamma$, it is easy to show that (24) implies that in equilibrium the term in (23) is strictly increasing in $\gamma$. Therefore, the term in the bracket on the l.h.s. of equation (25) is strictly increasing in $\gamma$. By Proposition 1, the r.h.s of this equation is strictly increasing in $\gamma$. This immediately implies that $n_t^*$ is strictly decreasing in $\gamma$.

The term in the bracket of the l.h.s. of equation (25) is strictly increasing in $d - k$, because, by Proposition 1, $q_t^*$ is independent of $d$ and $k$. Therefore, (25) implies that $n_t^*$ is strictly decreasing in $k$ and strictly increasing in $d$.

Finally, note that by (10) and (12), the zero profit condition can be written as

$$(1 - r)[x_t^*]^2 = K(q_t^*).$$

Therefore, the comparative statics properties of $x_t^*$ and $q_t^*$ are identical, with the exception of $r$. Q.E.D.

Proof of Proposition 4: Results (i)–(iii) immediately follow from (20) and the properties of $\varphi(I(\cdot))$ stated in Lemma 1. Q.E.D.

Proof of Propositions 5 and 6: By Proposition 4, $\hat{n}$ is uniquely determined by the steady state zero profit condition

$$
\frac{(1 - r)}{(1 + \hat{n})^2} [d - k - \hat{c}]^2 = K(\gamma).
$$

As $\hat{c}$ is strictly increasing in $\gamma$, the term in the bracket on the l.h.s of this equation is strictly decreasing in $\gamma$. Since the r.h.s is strictly increasing in $\gamma$, it follows that $\hat{n}$ is strictly decreasing in $\gamma$.

By Proposition 4, $\hat{c}$ is independent of $d$ and $k$. As the term in the bracket on the l.h.s of equation (27) is strictly increasing in $d - k$, this implies that $\hat{n}$ is strictly decreasing in $k$ and strictly increasing in $d$.

By Proposition 4, $\hat{c}$ is strictly increasing in $r$. Thus the term in the bracket of the l.h.s. of (27) is strictly decreasing in $r$. This in turn implies that $\hat{n}$ is strictly decreasing in $r$.

Finally, note that by (10) and (12), the zero profit condition in the steady state can be written as

$$(1 - r)x^2 = K(\gamma).$$

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Therefore, \( \hat{x} \) is strictly increasing in \( \gamma \) and \( r \) and independent of the parameters \( k \) and \( d \).

\[ \text{Q.E.D.} \]

**Proof of Propositions 7 and 8:** We first show that \( c_t < \hat{c} \) implies \( c_t < c_{t+1} \) for all \( t \). Let \( q_t^* \) denote the equilibrium innovation rate in state \( c_t \).

By Proposition 1, \( q^*(c) \) is strictly increasing in \( c \). As \( q^*(\hat{c}) = \gamma \), \( c_t < \hat{c} \) implies \( q_t^* = q^*(c_t) < \gamma \). Therefore, (15) implies \( c_{t+1} > c_t \).

Next, we show that \( c_t < \hat{c} \) implies \( c_{t+1} \leq \hat{c} \) for all \( t \). Note that the first argument in the proof of Proposition 2 shows that

\[
\frac{c_t}{1 + q^*(c_t)} \leq \frac{c_t}{1 + q^*(c_t)} = \hat{c}.
\]

(29)

We have thus shown that \( c_t > \hat{c} \) implies \( c_t < c_{t+1} \leq \hat{c} \) for all \( t \). An analogous argument completes the proof of the proposition by showing that \( c_t > \hat{c} \) implies \( c_t > c_{t+1} \geq \hat{c} \) for all \( t \).

Finally, from (10) we have:

\[
X_t^* = x_t^* n_t^* = d - k - c_t \frac{1 + \gamma}{1 + q_t^*(c_t)} - x_t^*
\]

(31)

From (31) it is obvious that \( X_t^* \) is decreasing over time when \( c_0 < \hat{c} \) and is increasing over time otherwise. This, in turn, implies that the industry’s labor cost per unit of output,

\[
(1 - r)c_t \frac{1 + \gamma}{1 + q_t^*(c_t)} + r(d - k - X_t^*)
\]

is increasing over time when \( c_0 < \hat{c} \) and is decreasing over time otherwise.

\[ \text{Q.E.D.} \]

8 **Appendix B: Dixit-Stiglitz Preferences**

In this appendix, we extend our analysis to a differentiated industry and confirm our main findings under an alternative specification of the demand.
function which has been extensively used in the literature and originates from Dixit and Stiglitz (1977). It has the important feature to reflect a preference for product diversity. We show that the main properties of the steady state equilibrium extend to this demand specification.\footnote{Numerical simulations suggest that the industry monotonically approaches the steady state.} In addition, we investigate how the steady state depends on the preference for diversity.

The (representative) consumer’s utility is given by:

$$U(x_0, x_1, \ldots, x_n) = x_0^{1-\delta} [x_1^\rho + \ldots + x_n^\rho]^{\delta/\rho}, \quad 0 < \rho < 1, \quad 0 < \delta < 1,$$

where $x_0$ is the quantity of the numeraire good, $x_i, \ i = 1, \ldots, n$, is the quantity of the differentiated good produced by firm $i$, $\delta$ is the share of the endowment income that the consumer spends on the differentiated products, and $\rho$ is the degree of product substitutability. The higher is $\rho$, the closer substitutes the products of the $n$ firms are. Denoting by $I$ the endowment income of the consumer, and maximizing (32) subject to the consumer’s budget constraint $I = p_0 x_0 + \sum_{i=1}^n p_i x_i$, with $p_0$ normalized to 1, we obtain the (inverse) demand function faced by each firm $i$:

$$p_i = \frac{\delta I}{x_i} \left( \frac{x_i}{y} \right)^\rho, \text{ where } y = [x_1^\rho + \ldots + x_n^\rho]^{1/\rho}. \quad (33)$$

Upon replacing the demand part in (9) by (33), we obtain the following maximization problem for firm $i$ when the state of the industry at date $t$ is given by $c_t$:

$$\max_{x_i, q_i} (1 - r) \left[ \frac{\delta I}{x_i} \left( \frac{x_i}{y} \right)^\rho - k - c_i \frac{1 + \gamma}{1 + q_i} \right] x_i - K(q_i). \quad (34)$$

Taking the first order conditions and assuming symmetry yields

$$(1 - r) c_i \frac{1 + \gamma}{(1 + q_i)} x_i^* = K'(q_i^*);$$  \hspace{1cm} (35)

$$\delta I \rho \frac{n_i^* - 1}{n_i^* x_i^*} = k + c_i \frac{1 + \gamma}{1 + q_i^*}. \quad (36)$$

Finally, using (34), symmetry and (36), we obtain the zero-profit condition:

$$\frac{(1 - r) \delta I}{n_i^*} [n_i^* - \rho (n_i^* - 1)] = K(q_i^*).$$  \hspace{1cm} (37)
In the steady state we have \( q_t^* = \hat{q} = \gamma \) and so \( c_t = \hat{c}, \ x_t = \hat{x}, \) and \( n_t = \hat{n}. \) Therefore, (35), (36), and (37) can be rewritten in the following way:

\[
(1 - r)\hat{c}\hat{x} = K' (\gamma) (1 + \gamma); \tag{38}
\]

\[
\delta I \rho \frac{\hat{n} - 1}{\hat{n}^2} = k + \hat{c}; \tag{39}
\]

\[
(1 - r)\delta I \frac{\hat{n} - \rho(\hat{n} - 1)}{\hat{n}^2} = K(\gamma). \tag{40}
\]

By (40) there is a unique number of active firms \( \hat{n} \) in the steady state and this number is strictly decreasing in \( \gamma \) and \( r. \) It is decreasing also in the preference parameter \( \rho. \) When the firms’ products are closer substitutes, competition becomes more intensive and so fewer firms enter the market. However, now \( \hat{n} \) is independent of the cost of the capacity investments \( k \) while with our initial specification it was decreasing in \( k. \) Also note that \( \hat{n} \) is strictly increasing in the consumer’s expenditures \( \delta I. \)

Since for each level of the competitive wage growth \( \gamma \) there is a unique \( \hat{n}, \) we reconfirm that there is also a unique \( \hat{x} \) and a unique \( \hat{c}. \) Indeed, from (39) we have:

\[
\hat{c}\hat{x} + k\hat{x} = \delta I \rho \frac{\hat{n} - 1}{\hat{n}^2}, \tag{41}
\]

and using (38), we obtain

\[
\hat{x} = \frac{1}{k} \left[ \delta I \rho \frac{\hat{n} - 1}{\hat{n}^2} \right] \tag{42}
\]

From (42) it follows that \( \hat{x} \) is strictly decreasing in \( k. \)

Finally, from (38) and (42), we get

\[
\hat{c} = \frac{K'(\gamma)(1 + \gamma) k}{(1 - r)\delta I \rho(\hat{n} - 1) / \hat{n}^2 - K'(\gamma)(1 + \gamma)}. \tag{43}
\]

From inspection of (43) we conclude that \( \hat{c} \) is strictly increasing in \( k. \)

To characterize the effect of the preference parameter \( \rho \) on \( \hat{x} \) and \( \hat{c} \) we note that \( \partial [(\hat{n} - 1) / \hat{n}^2]/\partial n < 0 \) if and only if \( \hat{n} > 2. \) The latter is a reasonable assumption in a market with monopolistic competition. We know from above that \( \hat{n} \) is strictly decreasing in \( \rho. \) This, in turn, means that if \( \hat{n} > 2, \) the term \( (\hat{n} - 1) / \hat{n}^2 \) that enters both in (42) and (43) is strictly increasing in \( \rho. \) Taking this into account, as well as the first term in the brackets of (42), it follows that \( \hat{x} \) is strictly increasing in \( \rho \) for \( \hat{n} > 2. \) Applying a similar reasoning to
(43), we can also conclude that the reverse is true for $\hat{c}$, i.e. $\hat{c}$ is strictly decreasing in $\rho$ if $\hat{n} > 2$. Thus, when products are closer substitutes there are fewer and larger firms in the industry and these firms are more efficient (i.e. lower $\hat{c}$).

Finally, in order to examine the impact of $\gamma$ and $r$ on $\hat{x}$ and $\hat{c}$, we set $K(q) = m + sq^2$, with $m, s > 0$, and perform numerical simulations. The latter indicate that an increase in the employees’ bargaining power $r$ has a positive impact on both $\hat{x}$ and $\hat{c}$ as with our initial specification. An increase in $\gamma$ continues to have a positive impact on $\hat{c}$, but it now has a negative impact on $\hat{x}$.
9 References


