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On the Use of Information in Repeated Insurance Markets

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Abstract

We analyze the use of information in a repeated oligopolistic insurance market. To sustain collusion, insurance companies might refrain from changing their pricing schedules even if new information about risks becomes available. We therefore provide an explanation for the existence of “unused observables” – that is information which a) insurance companies collect or could collect, b) is correlated with the risk experience, but c) is not used by companies to set prices. Furthermore, the existence of bulk discounts becomes rationalizable. These results also obtain if we include communication among companies and market entry to our framework.

Keywords: Repeated Games, Insurance Markets, Oligopoly, Unused Observables

JEL classification codes: C72, G22, L13

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1 Introduction

The vast majority of theoretical models on the insurance market are one-shot models of either perfect competition or monopolistic behavior of insurance companies.\(^1\) However, empirical evidence seems to suggest that in reality, the common market structure in most insurance markets is oligopolistic. Concentration indices for the top 5 insurance companies in the non-life business in Europe in 2002 ranged from 27% in Germany to 89% in Finland (Buzzacchi and Valletti 2005). Concentration measures in the life insurance sector in most developed nations in the 1990s have been constantly high: even in the USA, the least concentrated market, concentration indices for the top 5 insurance companies have been above 25%, while they have been (high) above 50% in Australia, Canada, Japan and the Netherlands (Bikker and van Leuvensteijn 2008). Market concentration is also reflected in insurance premiums (Dafny et al. 2008).

Recently, numerous empirical studies have attempted to test the predictions of theoretical models of insurance markets regarding the distribution and use of information in insurance markets and its effects on market outcomes.\(^2\) Several empirical results are hard to reconcile with standard theoretical models, and they suggest more work should be devoted to analyzing imperfectly competitive models of insurance markets – or, in the words of Chiappori et al. (2006), “there is a crying need for such models”.

In this paper, we analyze a repeated oligopolistic insurance market. The main feature of our model is that insurance companies take into account the impact of pricing decisions on competitors’ actions. We think that analyzing repeated interaction is crucial to understand the use of information in insurance markets for the following reasons:

First, our model provides an explanation for the puzzle of “unused observables” that has been tested in several empirical papers, but that has not received interest in the theoretical literature. In theory, profit maximizing insurance companies should exploit any risk-relevant information available to them.\(^3\) However, there is evidence of unused observables in insurance markets, that is information which a) insurance companies collect or could collect, b) is correlated with the risk experience, but c) is not used by companies to

\(^1\)Some of the few exceptions are Anía et al. (2002) who re-examine the equilibrium non-existence problem of Rothschild and Stiglitz (1976) in a dynamic setting, and Buzzacchi and Valletti (2005) who provide a model of strategic price discrimination in compulsory insurance markets.


\(^3\)Under perfect competition, companies will use all information in order to charge the fair premium. A monopolist will use all information in order to maximize profits through price discrimination.
set prices. For example, according to Finkelstein and Poterba (2006), the address of the insured person is almost always collected, but seldom used in pricing insurance, although there is a correlation between geographic information and other individual attributes that affects both the demand for insurance and the risk type. They use data on annuity purchases in the UK to illustrate that the information on the annuitant’s residential location would help to predict future mortality risk, but that it does not influence the insurance premium. Gender is another example of an unused observable that is usually collected by default, but that is not used for pricing in certain insurance markets, the most prominent example being the the long-term care insurance market and the automotive insurance. In both markets, the expected costs for the insurer differ substantially for men and women.  

Further empirical evidence on unused observables is provided by Brown and Finkelstein (2007) (gender and place of residence in the U.S. long-term care insurance industry) and Ivaldi (1996) (smoking in the French automobile insurance industry). Finkelstein and Poterba (2006) conclude their article by stating that “a complete understanding of the limited use in pricing of available or collectible risk-related information on insurance buyers remains an open issue”.

Second, the information available to insurance companies and the correlation with the underlying risk of the insured is subject to constant change. The revolution in information technologies has enabled insurance companies to collect, analyze and make use of large amounts of information. An example of evolving information that has recently received a lot of attention is that of genetic testing (Hoy and Witt 2007, Hoy and Polborn 2000, Rees and Apps 2008). Correlation like the one between an insured’s address and socioeconomic status may change over time as the composition of residents in a certain area changes. It is therefore important to know how insurance companies respond to a constant change in their information about risks.

Third, in an oligopolistic insurance market, the existence of bulk discounts can also be rationalized if companies collude and therefore make positive profits. In competitive insurance markets with asymmetric information, high risk individuals will demand larger quantities of insurance than low-risk individuals. In order for an insurance company to break even, theory predicts that marginal prices should rise with quantity. However, in reality, many insurance companies offer discounts in bulk (Cawley and Philipson 1999 or Chiappori et al. 2006).

In our model, there are two types of individuals who face either high or low risk

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4See, for example, Finkelstein and Poterba (2006) or the “Gesamtverband der deutschen Versicherungswirtschaft”, (www.gdv.de), the association of German insurance companies.

5They mention a number of possible reasons for the existence of unused observables, e.g. regulation or implementation costs, but show that these cannot fully explain the puzzle.
of damage. There is a finite number of insurance companies. They can distinguish between these risks and compete for customers by setting insurance premia in each period. Companies interact strategically and preconceive the effect of their pricing decision on the prices set by their competitors in subsequent periods. If companies fear a price war after adjusting their prices, they may refrain from doing so. We show that even if insurance companies can distinguish between risk types, equilibria exist in which (1) insurance companies charge the same insurance premium to both risks, and (2) both risk types purchase positive amounts of insurance (however, low risks potentially acquire less insurance than high risks). Thus, we derive an equilibrium with unused observables. Furthermore, if companies make positive profits out of all risks, it may be rational for them to offer bulk discounts.

We then show that the same equilibrium outcome is possible if insurance companies cannot distinguish between high and low risks, i.e. if there is asymmetric information. This renders possible the following explanation for the existence of unused observables: initially, there exists a collusive equilibrium in an insurance market with asymmetric information. Then, after analyzing their data, insurance companies learn how to distinguish between high and low risks. However, they maintain their pricing schedules in order to sustain collusion.

In the next step, we analyze the robustness of our model with regard to two extensions. First, we allow for market entry.\textsuperscript{6} Outside firms can enter the market incurring some entry costs and become incumbent firms for the rest of the game. Second, we allow for explicit collusion between firms, i.e. they can agree on charging the profit-maximizing insurance premia for low and high risks. If they can negotiate with each other, companies are likely to exploit their information. We show that if entry costs are neither too high nor too low, there exist equilibria with unused observables in which incumbent companies cannot gain by explicit collusion. The intuition for this result is as follows: if incumbent companies decide to increase their period profits by charging different premia for low and high risks, outside companies can enter the market profitably by making a one-shot gain. If on the other side one incumbent company undercuts the insurance premium of its competitors, it triggers a price war, which wipes out all gains of this deviation. We therefore show that equilibria with unused observables can be robust to explicit collusion and to the threat of market entry. In these equilibria, it does not pay off to use the information about risks as the maximal level of per-period profits can already be attained without this information.

\textsuperscript{6}Bikker and van Leuvenstelijn (2008) show empirically that market entry is indeed a relevant phenomenon in insurance markets in the countries analyzed in their study (Canada, Germany, Japan, the Netherlands, and the UK).
At a technical level, our paper also contributes to the literature of third-degree price discrimination. In the case where insurance companies are able to distinguish between high- and low-risk costumers, they have to decide whether to charge the same or different prices to two groups of costumers who differ in their willingness to pay. This literature has introduced the possibility of firms’ competition, but it has not analyzed equilibrium strategies in the repeated game.

The rest of the paper is organized as follows: The next section outlines the basic model and derives an equilibrium with unused observables. We include explicit collusion and market entry into the model in section 3. In section 4, we provide further examples of the evolution of the use of information in insurance industries, and discuss welfare and policy implications. The last section concludes.

2 A repeated Insurance Market

2.1 Framework

Time is discrete and denoted by \( t \in \{0, 1, \ldots\} \). The stage game is the simplest version of an insurance market. In each period, there is a continuum of customers of mass 1. These can be the same customers or different ones in each period. Each customer has wealth \( W \) in each period, and faces the risk of losing an amount of \( d < W \). She may have either a high-risk probability of \( \pi_H \) or a low-risk probability \( \pi_L < \pi_H \). Let \( \lambda \) be the fraction of high-risk individuals. All customers have the same von Neuman-Morgenstern utility function \( U(W) \). We assume that \( U(W) \) is twice continuously differentiable with \( U'(W) > 0 \) and \( U''(W) < 0 \).

There are \( N > 1 \) long-lived risk-neutral insurance companies in the market. Let \( I = \{1, \ldots, N\} \) be the set of insurance companies. First, we will assume that these companies can distinguish between high- and low-risk customers. At a later stage, we will turn to the case of asymmetric information. In each period, each company \( i \in I \) offers any

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7See Tirole (1988), chapter 3.2 for a summary of this issue.
8As formalized, for example, in Rees and Wambach (2008).
9There may also be a certain in- and outflow of individuals in each period. As long as not all customers are locked into a specific contract for all periods, the results of the model do not change.
10This only simplifies the exposition of the model. All of our results would also hold if customer are heterogeneous in their risk aversion (the only thing we need is that customers are risk averse to some extent). Note that in standard models of the insurance market, the assumption of a uniform utility function is not innocuous, see Smart (2000).
11An alternative interpretation would be that insurance companies cannot perfectly distinguish risks, but have imperfect information (variables which are imperfectly correlated with risk types) which can be used to categorize risks, as in Hoy (1982).
positive amount of insurance. Let $\alpha_{i,t}^H (\alpha_{i,t}^L)$ be the insurance premium for high-risk (low-risk) individuals offered by company $i$ in period $t$. If an individual of risk $j \in \{L, H\}$ purchases an insurance cover $D_j \geq 0$ in period $t$ from company $i$, she pays $D_j \alpha_{i,t}^j$ to the company in this period, regardless of whether damage occurs or not. If the damage occurs, she gets $D_j$ from the company, i.e. $D_j = d$ implies full coverage. We say that company $i$ uses the information about risks in period $t$ if $\alpha_{i,t}^H \neq \alpha_{i,t}^L$.

Customers are not modelled as strategic players: in each period, they purchase the utility maximizing insurance cover from the company that offers at the cheapest premium for their risk. If more than one company has the lowest insurance premium, each customer randomizes with equal probability from which company she buys insurance. The sequence of events in each period $t$ is as follows:

1. Insurance companies announce the insurance premia $\{(\alpha_{i,t}^L, \alpha_{i,t}^H)\}_{i \in I}$.
2. Customers purchase insurance $D_L$ and $D_H$.
3. Nature decides about the occurrence of damage and payoffs are realized.

Now fix

$$\alpha_{i,-1}^L = \alpha_{i,-1}^H = 0$$

for all $i \in I$. For $t \in \{0, 1, \ldots\}$, we denote by $h_t$ the history of all insurance premia that were charged by all insurance companies up to period $t$:

$$h_t = \left(\left\{\left(\alpha_{i,-1}^L, \alpha_{i,-1}^H\right)\right\}_{i \in I}, \left\{\left(\alpha_{i,0}^L, \alpha_{i,0}^H\right)\right\}_{i \in I}, \ldots, \left\{\left(\alpha_{i,t-1}^L, \alpha_{i,t-1}^H\right)\right\}_{i \in I}\right).$$

The set of all possible histories at date $t$ will be denoted by $H_t$. A strategy of company $i$ is an infinite sequence of action functions $\alpha^{i,t}$ for every $t \in \{0, 1, \ldots\}$, where $\alpha^{i,t}$ determines $\alpha_{i,t}^L$ and $\alpha_{i,t}^H$ as a function of the history $h_t$:

$$\alpha^{i,t} : H_t \rightarrow \mathbb{R}^2.$$

Without loss of generality we concentrate on pure strategies. The strategies of companies determine the sequence of insurance premia

$$\left\{\left\{\left(\alpha_{i,t}^L, \alpha_{i,t}^H\right)\right\}_{i \in I}\right\}_{t=0}^{\infty}.$$ 

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12We thereby assume non-exclusive provision of insurance which is different from most insurance market models, where companies offer price-quantity combinations (such as Rothschild and Stiglitz 1976). The assumption of non-exclusivity is not crucial for some markets such as the life insurance market (see Polborn et al. 2006). However, for property-liability insurance exclusivity it is more natural (we are indebted to Michael Hoy for pointing out this fact to us). In terms of our model, non-exclusivity is not needed for the results on the insurance market with symmetric information in sections 2.2 and 3. For the results on the insurance market with asymmetric information in section 2.3 it is needed for our characterization of equilibrium outcomes, but not for the existence of collusive equilibria.
From this sequence, we can derive the profit $G^{i,t}$ of company $i$ in period $t$. Insurance companies discount future gains by $\delta$. The sum of normalized discounted profits of company $i$ is then given by

$$G^i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t G^{i,t}. \quad (5)$$

The insurance market is in equilibrium if no company $i$ can increase its profit $G^i$ by choosing unilaterally another strategy.

### 2.2 Equilibria in an oligopolistic insurance market with symmetric information

We start by analyzing the demand for insurance. Assume for a moment that company $i$ offers the lowest insurance premium to individuals with risk $j \in \{L, H\}$ in period $t$. A customer with risk $j$ purchases the utility maximizing amount of insurance:

$$\tilde{D}_j(\alpha^{i,t}_j) = \arg \max_{D_j} \pi_j U(W - d + D_j(1 - \alpha^{i,t}_j)) + (1 - \pi_j) U(W - D_j\alpha^{i,t}_j). \quad (6)$$

This demand is implicitly given by

$$\frac{U''(W - d + \tilde{D}_j(\alpha^{i,t}_j)(1 - \alpha^{i,t}_j))}{U''(W - \tilde{D}_j(\alpha^{i,t}_j)\alpha^{i,t}_j)} = \frac{\alpha^{i,t}_j (1 - \pi_j)}{(1 - \alpha^{i,t}_j) \pi_j}. \quad (7)$$

As $U''$ is continuous, $\tilde{D}_j(\alpha^{i,t}_j)$ must also be continuous. The fair insurance premium under which the customer purchases full coverage is given by

$$\alpha^f_j = \pi_j, \quad (8)$$

while the highest insurance premium such that the customer is indifferent between purchasing a marginal unit of insurance cover or not is uniquely defined by

$$\alpha^{\max}_j = \frac{\pi_j}{(1 - \pi_j) \frac{U''(W)}{U''(W - d)} + \pi_j}. \quad (9)$$

For insurance premia $\alpha^{i,t}_j \in (\alpha^f_j, \alpha^{\max}_j)$ demand is positive and company $i$ earns a positive profit from contracts with individuals of risk $\pi_j$. For higher insurance premia, profits are 0, for lower insurance premia, profits are negative. Note that if $\pi_L$ is sufficiently close to $\pi_H$, we have $\alpha^f_H < \alpha^{\max}_L$. Define

$$\hat{\alpha}_j \in \arg \max_{\alpha_j \in \mathbb{R}} (\alpha_j - \pi_j) \tilde{D}_j(\alpha_j). \quad (10)$$

This is well-defined, as $(\alpha_j - \pi_j) \tilde{D}_j(\alpha_j)$ is continuous on the interval $[\alpha^f_j, \alpha^{\max}_j]$ and therefore attains its maximum.
Assume now that only company \( i \) offers the lowest premium for customers of both types, but does not use the information about the insured risk, i.e. \( \alpha_{i,t}^{L} = \alpha_{i,t}^{H} = \alpha_{i,t}^{P} \), where \( \alpha_{i,t}^{P} \) is called the “pooling premium”. We then have

\[
G_{i,t} = \lambda \tilde{D}_{H}(\alpha_{i,t}^{P}) \left( (\alpha_{i,t}^{P} - \pi_{H}) + (1 - \lambda) \tilde{D}_{L}(\alpha_{i,t}^{P}) \right) (\alpha_{i,t}^{P} - \pi_{L}).
\]

(11)

Denote by \( \alpha_{P0} \) the pooling premium at which the right-hand-side of (??) is equal to 0 such that \( \alpha_{i,t}^{P} \in (\alpha_{P0}, \alpha_{H}^{\text{max}}) \) implies positive demand for insurance by at least the high-risk individuals and positive profits from the pooling contract. Note that \( \alpha_{L}^{f} < \alpha_{L}^{\text{max}} \) implies \( \alpha_{L}^{f} < \alpha_{P0} < \alpha_{L}^{H} \). We then can state our first main result:

**Proposition 1** (a) For each \( \alpha \in (\alpha_{P0}, \alpha_{H}^{\text{max}}) \) there is a \( \delta(\alpha) < 1 \) such that there exists a subgame-perfect equilibrium in which \( \alpha_{i,t}^{P} = \alpha \) for all companies \( i \in I \) and in all periods \( t \) if \( \delta \geq \delta(\alpha) \). (b) If \( \alpha \notin (\alpha_{P0}, \alpha_{H}^{\text{max}}) \), then no such equilibrium exists. (c) We have that \( \lim_{\alpha \to \alpha_{P0}} \delta(\alpha) = 1 \).

**Proof.** (a) Consider the following simple grim-trigger strategy which is played by all companies \( i \in I \): charge \( \alpha_{i,t}^{0} = \alpha \). In period \( t > 0 \), charge \( \alpha_{i,t}^{P} = \alpha \) if and only if \( \alpha_{i,t}^{\tau} = \alpha_{L}^{H} = \alpha, \lambda \in I \), in all periods \( \tau \in \{0, ..., t - 1\} \). Otherwise, charge \( \alpha_{i,t}^{P} = \alpha_{L}^{f} \) and \( \alpha_{i,t}^{H} = \alpha_{L}^{H} \). We employ the one stage deviation principle in order to show that this can be an equilibrium. If at least one company charges the fair insurance premia \( \alpha_{L}^{f} \) and \( \alpha_{H}^{f} \), no other company can make positive profits. Thus, the maximal normalized discounted profit from a deviation of company \( i \) in period \( t \) is given by

\[
G_{i,t}^{d} = (1 - \delta) \left[ \lambda(\tilde{\alpha}_{H} - \pi_{H})\tilde{D}_{H}(\tilde{\alpha}_{H}) + (1 - \lambda)(\tilde{\alpha}_{L} - \pi_{L})\tilde{D}_{L}(\tilde{\alpha}_{L}) \right].
\]

(12)

The normalized discounted profit from compliance is given by

\[
G_{i,t}^{c} = \frac{1}{N} \left[ \lambda(\alpha - \pi_{H})\tilde{D}_{H}(\alpha) + (1 - \lambda)(\alpha - \pi_{L})\tilde{D}_{L}(\alpha) \right].
\]

(13)

As \( \alpha \in (\alpha_{P0}, \alpha_{H}^{\text{max}}) \), this term is positive. Thus, if \( \delta \) is sufficiently close to unity, we have \( G_{i,t}^{c} \geq G_{i,t}^{d} \). (b) If \( \alpha_{L}^{f} < \alpha \leq \alpha_{P0} \) and \( \alpha_{i,t}^{P} = \alpha \) for all \( i \in I \) and in all periods \( t \), a single company \( l \) could increase its normalized discounted profit by charging \( \alpha_{i,t}^{L0} = \alpha, \alpha_{i,t}^{L0} = \alpha_{L}^{f} \) and the fair premia thereafter. If \( \alpha \leq \alpha_{L}^{f} \) and \( \alpha_{i,t}^{P} = \alpha \) for all \( i \in I \) and in all periods \( t \), a single company \( l \) could increase its normalized discounted profit by charging \( \alpha_{i,t}^{L0} = \alpha_{L}^{f} \) and \( \alpha_{i,t}^{H0} = \alpha_{L}^{H} \) in all periods \( t \). If \( \alpha_{H}^{\text{max}} \leq \alpha \) and \( \alpha_{i,t}^{P} = \alpha \) for all \( i \in I \) and in all periods \( t \), a single company \( l \) could increase its normalized discounted profit by charging \( \alpha_{i,t}^{L0} = \tilde{\alpha}_{H}, \alpha_{i,t}^{L0} = \tilde{\alpha}_{L} \) and the fair premia thereafter. (c) Assume that an equilibrium exists in which \( \alpha_{i,t}^{P} = \alpha \) for all \( i \in I \) and in all periods \( t \). Then the normalized discounted profit for each firm is equal to the term in (??). Note that this term converges to 0 as \( \alpha \) approaches \( \alpha_{P0} \) from above. The maximal normalized discounted profit of a deviating firm can be at least

\[
(1 - \delta)(1 - \lambda)(\alpha_{P0} - \pi_{L})\tilde{D}_{L}(\alpha_{P0}) > 0.
\]

(14)
Thus, we must have \( \lim_{\alpha \to \alpha_{P0}} \delta(\alpha) = 1. \)

In the equilibria of proposition 1, insurance companies fear a price war if they change their insurance premia. Thus, they maintain a pooling premium, which guarantees them positive profits. This situation exhibits the following features:

- Although companies have more detailed information about risks, they do not use it. Thus, we have an equilibrium with unused observables.
- Given that \( \pi_L \) is sufficiently close to \( \pi_H \), both low- and high-risk individuals purchase positive amounts of insurance. However, there is adverse selection: as we can derive from equation (??), low-risk customers purchase less insurance than high-risk customers.

### 2.3 Equilibria in an oligopolistic insurance market with asymmetric information

We now turn to the case where insurance companies cannot distinguish between high- and low-risk individuals and show that a result similar to proposition 1 holds. Because of asymmetric information, each firm \( i \in I \) only charges a pooling premium \( \tilde{\alpha}_{i,t} \) in period \( t \) and customers purchase the amount of insurance which maximizes their expected utility from the firm that charges the lowest insurance premium. Fix

\[
\tilde{\alpha}_{i,-1} = 0
\]

for all \( i \in I \). For \( t \in \{0, 1, \ldots\} \), we denote by \( \tilde{h}_t \) the history of all insurance premia that were charged by all insurance companies up to period \( t \) :

\[
\tilde{h}_t = \left( \{\tilde{\alpha}_{i,-1}^{i,t}\}_{i \in I}, \{\tilde{\alpha}_{i,0}^{i,t}\}_{i \in I}, \ldots, \{\tilde{\alpha}_{i,t-1}^{i,t}\}_{i \in I} \right).
\]

The set of all possible histories at date \( t \) will be denoted by \( \tilde{H}_t \). A strategy of company \( i \) is an infinite sequence of action functions \( \tilde{\alpha}^{i,t} \) for every \( t \in \{0, 1, \ldots\} \), where \( \tilde{\alpha}^{i,t} \) determines \( \tilde{\alpha}_{i,t} \) as a function of the history \( \tilde{h}_t \):

\[
\tilde{\alpha}^{i,t} : \tilde{H}_t \to \mathbb{R}.
\]

Again, we concentrate on pure strategies. The strategies of companies determine the sequence of insurance premia

\[
\left\{ \{\tilde{\alpha}_{i,t}^{i,t}\}_{i \in I} \right\}_{t=0}^{\infty}
\]

from which the sum of normalized discounted profits \( \tilde{G}^i \) can be calculated as in the last subsection. The rest of the model remains unchanged. Define

\[
A_P = \left\{ \tilde{\alpha}_P \mid \tilde{\alpha}_P \in \arg \max_{\alpha \in \mathbb{R}} \lambda(\alpha - \pi_H)\tilde{D}_H(\alpha) + (1 - \lambda)(\alpha - \pi_L)\tilde{D}_L(\alpha) \right\},
\]

where \( \lambda \) is the discount factor and \( \pi_L \) and \( \pi_H \) are the expected losses for low- and high-risk individuals, respectively.
For each \( \alpha \)

Proposition 2 Assume that companies cannot distinguish between high- and low risks. For each \( \alpha \in [\alpha_{P_0}, \alpha_{H}^{\text{max}}] \) there is a \( \hat{\delta}(\alpha) < 1 \) such that there exists a subgame-perfect equilibrium in which \( \hat{\alpha}_{P}^{i,t} = \alpha \) for all companies \( i \in I \) and in all periods \( t \) if \( \hat{\delta} \geq \hat{\delta}(\alpha) \). (b) If \( \alpha \not\in [\alpha_{P_0}, \alpha_{H}^{\text{max}}] \), then no such equilibrium exists. (c) If \( \alpha > \alpha_{P_0} \) is sufficiently small, then \( \hat{\delta}(\alpha) = 1 - \frac{1}{N} \).

\[ \hat{\alpha}_{P}^{i} = \min \{ \hat{\alpha}_{P} \in A_{P} \}, \]

which is the smallest element in this set. We then can derive:

\[ G^{i,d} = (1 - \delta) \left[ \lambda(\hat{\alpha}_{P}^{i} - \pi_{H})\hat{D}_{H}(\hat{\alpha}_{P}^{i}) + (1 - \lambda)(\hat{\alpha}_{P}^{i} - \pi_{L})\hat{D}_{L}(\hat{\alpha}_{P}^{i}) \right]. \]

The normalized discounted profit from compliance is given by

\[ G^{i,c} = \frac{1}{N} \left[ \lambda(\alpha - \pi_{H})\hat{D}_{H}(\alpha) + (1 - \lambda)(\alpha - \pi_{L})\hat{D}_{L}(\alpha) \right]. \]

As \( \alpha \in (\alpha_{P_0}, \alpha_{H}^{\text{max}}) \), this term is positive. Thus, if \( \hat{\delta} \) is sufficiently close to unity, we have \( G^{i,c} \geq G^{i,d} \). (b) If \( \alpha < \alpha_{P_0} \) and \( \hat{\alpha}_{P}^{i,t} = \alpha \) for all \( i \in I \) and in all periods \( t \), a single company \( l \) could increase its normalized discounted profit by charging \( \hat{\alpha}_{L}^{l,t} = \alpha_{P_0} \) in all periods \( t \). If \( \alpha_{H}^{\text{max}} \leq \alpha \) and \( \hat{\alpha}_{P}^{i,t} = \alpha \) for all \( i \in I \) and in all periods \( t \), a single company \( l \) could increase its normalized discounted profit by charging \( \hat{\alpha}_{L}^{l,0} = \hat{\alpha}_{P}^{i} \) and \( \hat{\alpha}_{L}^{l,\tau} = \alpha_{P_0} \) in all periods \( \tau \) thereafter. (c) Define

\[ G(\alpha) = \lambda(\alpha - \pi_{H})\hat{D}_{H}(\alpha) + (1 - \lambda)(\alpha - \pi_{L})\hat{D}_{L}(\alpha) \]

and observe that \( G(\alpha) \) is continuous on the interval \( (\alpha_{P_0}, \alpha_{H}^{\text{max}}) \) with \( G(\alpha_{P_0}) = 0 \). Thus, there must exist an \( \hat{\alpha} \) with \( \alpha_{P_0} < \hat{\alpha} \leq \hat{\alpha}_{P}^{i} \), such that \( G(\alpha) \) strictly increases in the interval \( (\alpha_{P_0}, \hat{\alpha}) \). Consider the same strategy as in part (a) for \( \alpha \in (\alpha_{P_0}, \hat{\alpha}) \). As before, the normalized discounted profit from compliance is then \( \frac{1}{N} G(\alpha) \), while the normalized discounted profit from a deviation is \( (1 - \delta)G(\alpha) \). Consequently, the critical discount factor equals \( 1 - \frac{1}{N} \). \( \blacksquare \)
The results of proposition 1 and proposition 2 enable the following interpretation: the arrival of new information about risks does not necessarily change the equilibrium outcome. Suppose that the market is in an equilibrium with asymmetric information in which all insurance companies charge a premium of \( \bar{\alpha} \in (\alpha_{P0}, \alpha_{H}^{\text{max}}) \) in each period (proposition 2 says that this is possible). If in that situation new information about risks arrives (as a result of data-collection, for example), then, by proposition 1, the market can enter an equilibrium with symmetric information and with exactly the same equilibrium outcome (given that \( \delta \) is sufficiently high). This is especially relevant if collusion must be tacit and companies avoid explicit negotiations. In particular, we have shown that almost every equilibrium outcome under asymmetric information is also an equilibrium outcome under symmetric information.

Whether an equilibrium outcome survives the arrival of new information about risks or not, might depend on the extent of collusion: observe from proposition 1 (c) and proposition 2 (c) that if profits are low under asymmetric information (i.e. if \( \bar{\alpha}_{i,t}^{\text{lim}} \) is close to \( \alpha_{P0} \) for all \( i \)), the critical discount factor is just \( 1 - \frac{1}{N} \), while it is very close to 1 under symmetric information. Thus, if profits are low, then the arrival of new information potentially triggers a price war and a change of the equilibrium outcome. Compared to that, the critical discount factor equals \( 1 - \frac{1}{N} \) under both asymmetric and symmetric information if \( \bar{\alpha}_{i,t}^{\text{lim}} = \bar{\alpha}_{i}^{*} \) for all \( i \) (which can be shown by going through the same steps as in the proof of proposition 2 (c)), i.e. if profits are relatively high.

Whenever insurance companies can make expected profits out of all contracts, it is not difficult to imagine a situation when they do so by granting bulk discounts (instead of linear pricing) to customers, as discovered by Cawley and Philipson (1999). This especially makes sense when firms face administrative fixed costs per contract, such that selling more insurance to some risks increases the expected profit per unit of insurance.

3 Explicit Collusion and Market Entry

The equilibria in the last section had a number of attributes that are inconsistent with the results of one-shot models of the insurance market, but consistent with empirical results. However, there remain two important issues:

- If an industry makes profits, we would expect market entry.
- If companies are able to sustain collusion, they should be able to increase their profits even further by using the information about risks, i.e. they may coordinate
on an equilibrium in which all firms discriminate between risks and charge $\alpha_{i,t}^H = \tilde{\alpha}_H$ and $\alpha_{i,t}^L = \tilde{\alpha}_L$ in all periods $t$.\footnote{Here we implicitly assume that $\tilde{\alpha}_H \neq \tilde{\alpha}_L$ which is true for most standard utility-functions.}

We will deal with both questions in this section and show that the equilibria of proposition 1 still can be robust against market entry and explicit collusion. In all what follows, we will again consider a scenario with symmetric information. Note that explicit collusion is illegal in most legislations and tacit collusion (i.e. collusion without communication between firms) hard to detect.\footnote{For a discussion about the difference between explicit and tacit collusion, see Rees (1993).} We will not rely on this, but assume that firms can negotiate without being exposed to the danger of punishment.

Denote the set of incumbent companies in each period by $I_t$, where

$$I_0 = \{1, \ldots, N\}.$$ In each period $t$, there is an infinite number of outside firms $k \in \mathbb{N} \setminus I_t$ which can enter the market at cost $c > 0$.\footnote{This also could be insurance companies which offer the same insurance contracts, but at substantially higher rates, such that only a small fraction of uninformed consumers (who do not compare insurance premia, but randomly choose some contract) purchases those contracts.} These entry costs can be interpreted as installation costs, for example, the costs of acquiring the necessary distribution channels. If an outside company enters the market, it belongs to the set of incumbents in all future periods and can distinguish between high and low risks.\footnote{This particular model of market entry was introduced by Harrington (1989).} As tie-breaking rule we define that a company only enters the market if it can make strictly positive profits. Furthermore, we define:

**Definition** An equilibrium is robust against explicit collusion if there is no other weakly pareto superior equilibrium for incumbent companies.

If an equilibrium in which companies $i \in I_0$ do not use the information about risks is robust against explicit communication, any agreement on adjusting insurance premia to increase profits must result in a decrease of profits for at least one incumbent company, and therefore would not be accepted by this company.\footnote{Note that robustness against explicit collusion is weaker than (weak) renegotiation proofness. For details about renegotiation proofness, see Mailath and Samuelson (2006), pages 134 - 143. One also could construct weak renegotiation proof equilibria in our setting, however, their structure is not interesting for our purpose.}
or not. Thus, incumbent companies are Stackelberg leaders and market entry is endoge-
nous as in Etro (2008). The sequence of events now is as follows:

1. Insurance companies announce the insurance premia \( \{(\alpha_{i,t}^L, \alpha_{i,t}^H)\}_{i \in I} \).

2. Outside companies decide whether to enter the market at cost \( c \) or not. If a company \( k \in \mathbb{N} \setminus I_t \) enters the market, it subsequently sets its insurance premia \( (\alpha_{k,t}^L, \alpha_{k,t}^H) \).

3. Costumers purchase insurance \( D_L \) and \( D_H \).

4. Nature decides about the occurrence of damage and payoffs are realized.

5. If a company \( k \in \mathbb{N} \setminus I_t \) has entered the market, then \( I_{t+1} = I_t \cup \{k\} \).

Clearly, as entry costs are positive, incumbents can price outside companies out of the
market. However, if entry costs are small, then per-period profits also must be small. If
these profits are generated by charging a pooling premium such that low risks subsidize
high risks, then it can be profitable for an outside firm to enter the market and to make
a one-shot gain by offering contracts only to low risks. We therefore get:

**Proposition 3** If \( c \) is sufficiently small, then in equilibrium all companies that make
positive profits in period \( t \) use the information about risks in this period.

**Proof.** Assume that this is not the case and an incumbent company \( i \in I_t \) charges
\( \alpha_{i,t}^P \in (\alpha_{P0}, \alpha_{P}^{\text{max}}) \) and makes a positive profit in period \( t \). If follows that \( \alpha_{j,t}^L \geq \alpha_{i,t}^P \) for all
\( l \in I_t \) and \( j \in \{L, H\} \). If an outside company \( k \in \mathbb{N} \setminus I_t \) enters the market, then it earns at least
\[
-c + (1 - \lambda)(\alpha_{P0} - \pi_L)\hat{D}_L(\alpha_{P0}),
\]
by charging \( \alpha_{k,t}^L = \alpha_{P0} \) and \( \alpha_{k,t}^H = \alpha_{H}^L \), given that there is no other outside company
which enters the market. The term in (??) is positive if \( c \) is sufficiently low. Therefore,
the situation outlined above cannot be an equilibrium outcome if \( c \) is sufficiently low. \( \blacksquare \)

Thus, the equilibria of proposition 1 are not robust against market entry, if entry
costs are sufficiently small. However, we do not expect entry barriers to be negligible for
insurance markets. If \( c \) is sufficiently high, the existence of equilibria with pooling premia
might be restored. Define \( \tilde{\alpha}_{P}^* \) as in the last section and denote

\[
G^{\text{high}} = \lambda(\tilde{\alpha}_{P}^* - \pi_H)\hat{D}_H(\tilde{\alpha}_{P}^*) + (1 - \lambda)(\tilde{\alpha}_{P}^* - \pi_L)\hat{D}_L(\tilde{\alpha}_{P}^*),
\]
\[G^{\text{low}} = \max_{\alpha \in [\tilde{\alpha}_{P}^*, \tilde{\alpha}_{H}^*]} (1 - \lambda)(\alpha - \pi_L)\hat{D}_L(\alpha).
\]

\( G^{\text{high}} \) is the highest period profit from a pooling contract, \( G^{\text{low}} \) is the highest period profit
that can be made by selling contracts only to low risks and by charging a premium in the
interval $[\alpha_f^L, \alpha_f^H]$. For $\pi_H \to \pi_L$, we have $\alpha_f^H \to \alpha_f^L$, such that $G^{low} \to 0$. Thus, if $\pi_H$ is sufficiently close to $\pi_L$, then $G^{high} > G^{low}$. We then can show:

**Proposition 4** Assume that $\delta > 1 - \frac{1}{N}$ and $G^{high} > G^{low}$. If $c \in (G^{low}, G^{high})$, then there is a subgame-perfect equilibrium which is robust against explicit collusion and in which $\alpha_{p, l} = \alpha, \alpha \in (\alpha_f^H, \tilde{\alpha}_p)$, for all incumbent companies $i \in I_0$ in all periods $t$, while outside firms do not enter the insurance market.

**Proof.** Define for $G \in (G^{low}, G^{high})$

$$\alpha^G = \min_{\alpha} \left\{ \alpha \in (\alpha_f^H, \tilde{\alpha}_p) \mid \alpha \in (\alpha_f^H, \tilde{\alpha}_p) \right\}.$$

Fix a value $G^* \in (G^{low}, G^{high})$. Assume that in each period, incumbent companies play a grim-trigger strategy that also deters entry: Charge $\alpha_{p, l} = \alpha^*_{\pi} = \alpha^*_{\pi}$ if and only if $\alpha^{l, \tau}_l = \alpha^{l, \tau}_H = \alpha^*, l \in I_0$, and $I_{\tau} = I_0$ in all periods $\tau \in \{0, ..., t - 1\}$. Otherwise, charge $\alpha^{l, t}_l = \alpha^F_l$ and $\alpha^{l, t}_H = \alpha^F_H$. We show that this strategy can support an equilibrium. If a company $i \in I_0$ undercuts $\alpha^*_{\pi}$ in period $t$, then the definition of $G$, the continuity of $\alpha \widehat{D}_j(\alpha)$ and the fact that $G^* > G^{low}$ ensure that $G_{i, l} < G^*$. Given that no outside company ever enters the market, an incumbent company complies to this strategy if

$$\frac{1}{N} G^* > (1 - \delta) G^*, \quad (28)$$

which is equivalent to

$$\delta > 1 - \frac{1}{N}. \quad (29)$$

The tie-breaking rule implies that an outside company will not enter if and only if

$$G^* \leq c. \quad (30)$$

Thus, if $c \in (G^{low}, G^{high})$ and (29) holds, then an equilibrium with no market entry, $G^* = c, \alpha_{p, l} = \alpha_{\pi}$ for all $i \in I_0$ and all $t$ exists and is robust against explicit collusion. ■

The logic of these equilibria is again simple. As incumbent companies play a grim-trigger strategy, they refrain from changing their pricing schedule. The punishment is also triggered if an outside company enters the market. Therefore, the period profit is limited to entry costs, otherwise it would pay off for an outside company to enter the market and make a one-shot gain. Therefore, incumbent companies cannot coordinate on insurance premia, such that they earn strictly higher profits.

The upper bound on entry costs, $G^{high}$, ensures that a period profit equal to $\frac{1}{N} c$ per incumbent company can be attained by charging a pooling premium. If entry costs are much higher than $G^{high}$, incumbent companies can increase their profits by explicit
collusion and by using their information about risks. The lower bound, $G^{low}$, is needed to make sure that no incumbent company can gain by undercutting the premium for customers with small risk if the period payoff is equal to $\frac{1}{N}c$ for each incumbent company. If entry costs are lower, incumbent companies could still deter entry by charging low insurance premia, but they would have to use the information about risks in some periods, otherwise each incumbent company could gain by one-shot deviation. The measure of admissible values of $c$ can be substantial: $G^{low}$ strictly decreases in $\lambda$ and will be small if $\pi_H$ is close to $\pi_L$, while $G^{high}$ can be large if customers are very risk averse and ready to pay a high risk premium.

The result of proposition 3 remains valid if incumbents use other punishment strategies to deter market entry or deviation from pooling premia. However, the maximal period profit for incumbent companies may decrease. Consider, for example, a tit-for-tat strategy where incumbent companies again start to charge profitable pooling premia after a finite number of periods with zero-profits. Then in an equilibrium with entry-deterrence, the period profit per incumbent company must be lower than $\frac{1}{N}c$. Otherwise, an outside company could enter the market, cover its entry costs by capturing the whole market (as it earns $c$), and participate in future business profitably after the price war has been finished.

4 Discussion

4.1 Examples of the evolution of information in insurance markets

Our model interprets the presence of unused observables as a sign of collusion. This is in accordance with experience in the US automotive insurance market where, as long as companies were making extensive profits, contracts were almost not differentiated by risk class (Carter 2005). However, as profits in the market started to deteriorate in the late 1990s, one insurance company (Allstate), changed the number of pricing categories from 3 to over 1,500. As a consequence, Allstate’s return on equity almost doubled in the following two years. However, as the author points out, this strategy might not be of lasting success, as other insurance companies also start to change their pricing system, and a price war in the auto insurance market is on its way.

It seems to be the case that it is often small firms or new entrants who start using a finer risk classification (Finkelstein and Poterba 2006). Ainslie (2006) provides an example of the U.K. annuity market where new start-up companies were formed to offer impaired
annuity products to those individuals in observable poor health. Only under increased competitive pressure did the existing companies follow suit.

The evolution of the use of information in the European Union has followed the evolution of competition in the insurance industry. Before 1994, when the European Commission completed a series of directives in order to remove obstacles to competition, the insurance markets in several European countries such as Germany and Italy were tightly regulated. Considering the use of information in automotive insurance in Germany, risk categories were rather coarse and involved extensive pooling (Rees and Kessner 1999), while in Italy, companies were even restricted by law to a very limited number of parameters they could use in their pricing schemes (Buzzacchi and Valletti 2005). After deregulation, as a consequence of increased competition, premiums in automotive insurance have undergone large reductions, and at the same time companies introduced contracts with finer risk categorization, see Rees and Kessner (1999) and Buzzacchi and Valletti (2005). However, in some markets, such as the annuity market in Germany, the companies remain in an equilibrium where contracts are almost not differentiated by risks classes at all.

4.2 Welfare and Policy Implications

The sole existence of unused observables is a signal of anti-competitive behavior in the insurance industry for the regulator. Therefore, the presence of unused observables could be used as a policy tool by competition authorities. Given that costumers are aware of their individual risk, equilibria with unused observables are clearly inefficient: as long as costumers are not forced to purchase full coverage (for example, by regulation), they will buy too little insurance.

Considering the debate on whether insurance companies should be allowed to gather genetic information or not, there are cases where it might be welfare enhancing if not all information is used to set prices: if customers do not know their individual risk, genetic testing might impose ex-ante a classification risk on potential insurance buyers. However, our analysis shows that there are good reasons for insurance companies not to use genetic information in their pricing schedules: firstly, adjusting pricing schedules without coordination with other companies might trigger a price war. Secondly, using more information only makes sense for companies if profits rise. Companies might refrain from using all additional information in their pricing decisions for fear of market entry.

\footnote{For an in-depth discussion of this issue consult Polborn et. al. (2006), Strohmenger and Wambach (1999) and the papers cited there.}
5 Conclusion

Recently, several empirical findings have contradicted the predictions of the standard one-shot model of an insurance market. Our model of an oligopolistic insurance market rationalizes the occurrence of two formerly unexplained phenomena, unused observables and bulk discounts.

From the model we derived two explanations why firms may not use all risk-relevant information: first, if firms collude tacitly and available information changes over time, then they possibly refrain from using new information in order to prevent a price war. Second, if there is the threat of market entry, then per-period profits are limited. Whenever firms attain the upper limit of per-period profits, it does not pay off for incumbents to include more information. Under both scenarios, firms can make positive profits out of contracts with high and low risks and therefore may offer bulk discounts to customers.

A number of extensions to our model can be made. We used a very simple model of an insurance market, where in each period a new cohort of customers arrives. Usually, a customer is insured over a longer time horizon and the insurance company can condition premia for this customer on her history of damages. Thus, experience rating could be introduced into the model. Eventually, firms may find it optimal to skip experience rating in order to simplify collusion. Furthermore, one can analyze more complex forms of collusion like, for example, collusion on several different insurance markets as in Bernheim and Whinston (1990). Note that many insurance companies offer various types of insurance. Finally, more empirical work on contracts and collusion in insurance markets would be desirable to investigate the use of information by insurance companies.

References


