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Abstract

It is well-known that, in static models, minimum wages generate positive worker rents and, consequently, inefficiently low effort. We show that this result does not necessarily extend to a dynamic context. The reason is that, in repeated employment relationships, firms may exploit workers' future rents to induce excessively high effort.

Key Words: bonuses; limited liability; minimum wages

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1 Introduction

As is well-known in economics, minimum wages are inefficient for at least two reasons: On an aggregate level, they may prevent labor market clearing, and on a disaggregate level, they can imply inefficiently low effort. The latter problem has been highlighted by contract-theoretic models analyzing moral-hazard problems under limited liability (see, among many others, Laffont and Martimort, 2002, chapter 4; Schmitz 2005): When agents are protected by minimum wages or limited liability, they usually earn positive rents under an incentive contract. These rents raise the principal's costs of eliciting effort. Consequently, he optimally induces less than first-best effort.

We show that this conclusion may no longer hold in a dynamic setting. To do so, we also consider a moral-hazard problem under minimum wages, which leads to positive rents and inefficiently low effort in a static model. However, in a two-period model, the principal optimally uses second-period rents to generate extra incentives for the agent in the first period. This is achieved by combining a bonus contract with an extension clause that allows the agent to sign a second-period contract only if he was successful in the first period. When the expected second-period rent is large, the principal uses the extra incentives to induce more than first-best effort.

In practice, this "reversed" inefficiency problem of minimum wages (i.e., *excessively large* efforts) should typically apply to low-skilled blue-collar workers. The introduction of minimum wages that are enforced by law (or collective agreements) usually compels firms to increase wages for unskilled labor, whereas wages for high-skilled employees are unaffected because they already earn more than the minimum wage. Translating this to our model means that, for low-skilled workers, the minimum wage constraint is binding. Hence, blue-collar workers are likely to earn rents in a one-shot game. In a dynamic environment, firms then optimally respond by exploiting these rents, which may result in inefficiently high effort.

The paper is related to the contract-theoretic literature on moral hazard and limited liability, which usually considers a static relationship.[1] Two ex-

[1] Note, however, that there is a rich literature on repeated moral hazard with risk averse agents. Contrary to our paper, the focus of these models is on consumption smoothing

ceptions are Ohlendorf and Schmitz (2008) and Kräkel and Schöttner (2008), who also analyze a two-period principal-agent relationship and show that the principal employs future rents to generate extra incentives. However, in those papers, the agent's effort remains below first-best. We obtain a contrary result by considering a situation where the firm can replace the worker after the first period.

2 The Model

A firm needs one worker to carry out a task in each of two periods. In each period, the firm can randomly hire a worker from a pool of homogeneous agents available on the labor market. Alternatively, in period 2, the firm may again employ the worker hired in period 1. However, only one-period contracts are feasible because the firm cannot commit not to renegotiate contractual terms referring to period 2 at the beginning of the second period.

All players are risk neutral. The monetary output of the worker hired in period t ($t = 1, 2$) is $v_t Y_t$ with $Y_t \in \{0, 1\}$ and $\Pr[Y_t = 1|e_t] = p(e_t)$. The variable e_t denotes the worker's effort in period t, and $p(e_t)$ is a concave probability function with $p'(e_t) > 0$ and $p''(e_t) < 0$. At the beginning of $t = 1$, the firm knows v_1. However, due to uncertainty about the future, v_2 is still unknown and considered to be the realization of a non-negative random variable v with commonly known cdf $F(\cdot)$. The firm learns v_2 at the beginning of $t = 2$.

Effort is not observable, but output Y_t is verifiable. Hence, at the beginning of period t, the firm offers a worker a bonus contract (b_{Lt}, b_{Ht}) contingent on output Y_t, where the low bonus b_{Lt} is paid to the worker if $Y_t = 0$, and the high bonus b_{Ht} if $Y_t = 1$. The firm's payment to the worker must be at least as high as the minimum wage, which is normalized to zero in both periods. Thus, $b_{Ht}, b_{Lt} \geq 0$. To supplement the period-1 bonus contract, the firm announces a probability $q \in [0, 1]$ of hiring the period-1 worker again in period 2, provided that the worker achieved a high output in period 1, i.e., if $Y_1 = 1$.[2]

and the renegotiation of long-term contracts.

[2] Note that such an extension clause is renegotiation-proof: At the beginning of period

In each period, workers have a reservation value $\bar{u} \geq 0$. Exerting effort e_t entails cost $c(e_t)$ with $c(0) = c'(0) = c''(0) = 0$ and $c'(e_t), c''(e_t) > 0$ for all $e_t > 0$. To guarantee that the firm is interested in hiring a worker and implementing efficient effort, we assume that $v_t p(e_t^{FB}) - c(e_t^{FB}) > \bar{u}$,$\forall v_t$, with first-best effort e_t^{FB} being defined by $v_t p'(e_t^{FB}) = c'(e_t^{FB})$. Concavity of the firm's objective function in the second-best case is ensured by the technical assumptions $c'''(e_t) > 0$ and $p'''(e_t) \leq 0$.

The timeline for each period t is the following: First, the firm observes v_t. Then the firm offers a bonus contract (b_{Lt}, b_{Ht}) supplemented by an extension probability q if $t = 1$. The worker accepts or rejects the contract. In case of acceptance, the worker chooses effort e_t. Finally, output is realized and payoffs are made.

3 Solution to the Model

We solve the problem by first considering $t = 2$. If the worker has accepted the contract (b_{L2}, b_{H2}), his expected utility is

$$EU_2(e_2) = b_{L2} + (b_{H2} - b_{L2})\, p(e_2) - c(e_2). \tag{1}$$

Hence, the worker optimally chooses effort e_2 given by

$$(b_{H2} - b_{L2}) = c'(e_2)/p'(e_2). \tag{2}$$

Therefore, the worker's expected utility is

$$EU_2(e_2) = b_{L2} + G(e_2) \quad \text{with} \quad G(e_2) := \frac{c'(e_2)}{p'(e_2)} p(e_2) - c(e_2). \tag{3}$$

The function $G(e_2)$ denotes the worker's expected gain from exerting effort e_2, i.e., the resulting expected wage increase net of effort cost. $G(e_2)$ is strictly increasing in e_2.

The firm maximizes $v_2 p(e_2) - b_{L2} - (b_{H2} - b_{L2})\, p(e_2)$, taking into account the worker's participation constraint (PC) $EU_2(e_2) \geq \bar{u}$ and the minimum-

2, the firm is indifferent between employing the period-1 worker for another period or hiring a new worker.

wage condition (MWC) $b_{L2} \geq 0$. Thus, using (3), the firm's Lagrangian reads as follows:[3]

$$L_2(b_{L2}, b_{H2}) = v_2 p(e_2) - b_{L2} - G(e_2) - c(e_2) + \lambda_1 [b_{L2} + G(e_2) - \bar{u}] + \lambda_2 b_{L2}.$$

Maximization leads to our first result:[4]

Proposition 1 *(i) If $\bar{u} < G(e_2^*)$ with e_2^* being implicitly defined by $v_2 p'(e_2^*) - c'(e_2^*) - G'(e_2^*) = 0$, only the MWC will be binding and the firm induces e_2^*. (ii) If $\bar{u} \in \left[G(e_2^*), G\left(e_2^{FB}\right)\right]$, both MWC and PC will be binding and the firm implements e_2^{**} with $G(e_2^{**}) = \bar{u}$. (iii) If $\bar{u} > G\left(e_2^{FB}\right)$, then only the PC will be binding and the firm implements e_2^{FB}. (iv) We have $e_2^* < e_2^{**} < e_2^{FB}$.*

The proposition shows that an increasing reservation value $\bar{u}$ relaxes the MWC, thus leading to higher implemented effort. The worker earns a positive rent if and only if case (i) applies. Because e_2^* and, consequently, $G(e_2^*)$ is increasing in v_2, case (i) occurs if v_2 is sufficiently large. More precisely, v_2 needs to exceed the threshold $\hat{v}$ implicitly defined by $G(e_2^*(\hat{v})) = \bar{u}$. Since we are interested in situations where workers may earn rents, we assume that $v_2 > \hat{v}$ occurs with positive probability. Thus, before uncertainty about v_2 is resolved, the expected rent of the period-2 worker is $\bar{R} := (1 - F(\hat{v}))(E[EU_2(e_2^*) | v > \hat{v}] - \bar{u}) > 0$.

We now turn to the optimal contract for $t = 1$. The period-1 worker earns $\bar{R}$ in the second period if $Y_1 = 1$ and he is hired again in period 2 (which happens with probability q). Thus, a worker's expected utility from accepting a contract in $t = 1$ is given by

$$EU_1(e_1) = b_{L1} + \bar{u} + \left(b_{H1} - b_{L1} + \bar{R}q\right) p(e_1) - c(e_1).$$

Hence, the worker chooses effort according to $b_{H1} - b_{L1} + \bar{R}q = c'(e_1)/p'(e_1)$. For $t = 1$, the PC is $EU_1(e_1) \geq 2\bar{u}$ and the MWC is $b_{L1}, b_{H1} \geq 0$. In order to rule out extreme cases where, in the absence of a minimum wage in period

[3] Note that $b_{L2} \geq 0$ together with $(b_{H2} - b_{L2}) = c'(e_2)/p'(e_2)$ ensures that $b_{H2} \geq 0$. Recall that e_2 is a function of (b_{L2}, b_{H2}).

[4] All proofs are relegated to the appendix.

1, the firm would like to punish the worker for success (i.e., $b_{H1} < 0$), let $\bar{R} < \bar{u}$. We obtain the following result:

Proposition 2 *Assume that $\bar{R} < \bar{u}$. In the optimal contract, the firm sets $q = 1$. There exists a cut-off value $\hat{u} < G(e_1^{FB})$ such that the firm implements more than first-best effort, e_1^{FB}, if and only if $\bar{u} > \hat{u}$.*

According to Proposition 2, the firm optimally combines a bonus contract with an extension clause that guarantees a period-1 worker another contract in $t = 2$ in case of success (i.e., $Y_1 = 1$). Thereby, the firm can use the entire expected second-period rent $\bar{R}$ to generate extra incentives in $t = 1$. Formally, this means that the firm's cost of inducing a given effort level e_1 decreases by $p(e_1)\bar{R}$ relative to a situation without extension clause (i.e., $q = 0$). Consequently, the optimal period-1 effort is higher than under $q = 0$ or, equivalently, in a static employment setting. Moreover, if period-1 effort would already be quite large without extension clause (i.e., $\bar{u} > \hat{u}$), extra incentives due to contract extension lead to more than efficient effort in period 1.

Appendix

Proof of Proposition 1: We have

$$\frac{\partial L_2}{\partial b_{L2}} = v_2 p'(e_2)\frac{\partial e_2}{\partial b_{L2}} - 1 - [G'(e_2) + c'(e_2)]\frac{\partial e_2}{\partial b_{L2}} + \lambda_1\left[1 + G'(e_2)\frac{\partial e_2}{\partial b_{L2}}\right] + \lambda_2 = 0 \tag{4}$$

and

$$\frac{\partial L_2}{\partial b_{H2}} = v_2 p'(e_2)\frac{\partial e_2}{\partial b_{H2}} - [G'(e_2) + c'(e_2)]\frac{\partial e_2}{\partial b_{H2}} + \lambda_1 G'(e_2)\frac{\partial e_2}{\partial b_{H2}} = 0. \tag{5}$$

Adding both optimality conditions, using that, by (2),

$$\frac{\partial e_2}{\partial b_{H2}} = -\frac{p'(e_2)}{(b_{H2} - b_{L2})\, p''(e_2) - c''(e_2)} = -\frac{\partial e_2}{\partial b_{L2}} > 0, \tag{6}$$

yields $\lambda_1 + \lambda_2 = 1$. Hence, either (i) only the PC is binding, or (ii) only the MWC, or (iii) both. In case (i) we have $\lambda_2 = 0$ and $\lambda_1 = 1$. Inserting into (5) shows that the firm implements first-best effort: $v_2 p'\left(e_2^{FB}\right) = c'\left(e_2^{FB}\right)$. From the binding PC and the non-binding MWC $b_{L2} > 0$ we obtain $G\left(e_2^{FB}\right) < \bar{u}$. In case (ii), $\lambda_1 = 0$, $\lambda_2 = 1$ and $b_{L2} = 0$. Inserting into (5) gives

$$v_2 p'(e_2) - c'(e_2) - G'(e_2) = 0. \tag{7}$$

Obviously, the solution to (7), e_2^*, satisfies $e_2^* < e_2^{FB}$. The non-binding PC yields $G(e_2^*) > \bar{u}$. Finally, in case (iii), $\lambda_1, \lambda_2 > 0$ and $b_{L2} = 0$. From the binding PC optimal effort in this scenario, e_2^{**}, is characterized by $G(e_2^{**}) = \bar{u}$. Solving (5) for the multiplier λ_1 yields

$$\lambda_1 = 1 - \frac{v_2 p'(e_2^{**}) - c'(e_2^{**})}{G'(e_2^{**})}. \tag{8}$$

$\lambda_1 < 1$ implies that $v_2 p'(e_2^{**}) - c'(e_2^{**}) > 0$ and, hence, $e_2^{**} < e_2^{FB}$. From (8) and $\lambda_1 > 0$ we obtain

$$v_2 p'(e_2^{**}) - c'(e_2^{**}) - G'(e_2^{**}) < 0. \tag{9}$$

Using that $p''' < 0$ and $c''' > 0$, it is straightforward to verify that $G(\cdot)$ is a convex function. Thus $v_2 p(\cdot) - c(\cdot) - G(\cdot)$ is concave in effort. Consequently, comparison of (7) and (9) gives $e_2^{**} > e_2^*$.

Proof of Proposition 2: To make the firm's problems for the two periods easily comparable and to be able to apply Proposition 1, we state the optimization program in a general form that incorporates both periods $t = 1, 2$. In period t, the firm's problem is:

$$\begin{gathered}
\max_{b_{Ht}, b_{Lt}, e_t, q} \; v_t p(e_t) - b_{Lt} - (b_{Ht} - b_{Lt})\, p(e_t) \\
\text{s.t. } b_{Ht} - b_{Lt} = \frac{c'(e_t)}{p'(e_t)} - \bar{R}q \\
b_{Lt} + \left(b_{Ht} - b_{Lt} + \bar{R}q\right) p(e_t) - c(e_t) \geq \bar{u} \\
b_{Lt}, b_{Ht} \geq 0 \\
q = 0 \;\text{ if }\; t = 2.
\end{gathered}$$

Inserting for $b_{Ht} - b_{Lt}$ and using the definition for $G(\cdot)$ from (3), this problem simplifies to

$$\begin{aligned} &\max_{b_{Lt}, e_t, q} \left[v_t + \bar{R}q\right] p(e_t) - b_{Lt} - \left[G(e_t) + c(e_t)\right] \\ &\text{s.t. } b_{Lt} = \max\left\{\bar{u} - G(e_t), \bar{R}q - \frac{c'(e_t)}{p'(e_t)}, 0\right\} \\ &\qquad\qquad q = 0 \text{ if } t = 2, \end{aligned}$$

where the expression for b_{Lt} follows from the PC and the MWC for b_{Ht} and b_{Lt}, respectively. Note that

$$\bar{u} - G(e_t) > \bar{R}q - \frac{c'(e_t)}{p'(e_t)} \Leftrightarrow \bar{u} - \bar{R}q > \left[p(e_t) - 1\right] \frac{c'(e_t)}{p'(e_t)} - c(e_t)$$

is true since the right-hand side of the last inequality is negative and $\bar{u} > \bar{R}$ by assumption. Hence, $b_{Ht} \geq 0$ is satisfied and $b_{Lt} = \max\{\bar{u} - G(e_t), 0\}$. The firm's problem can then be transformed to:

$$\max_{e_t, q} \begin{cases} \left[v_t + \bar{R}q\right] p(e_t) - \left[G(e_t) + c(e_t)\right] & \text{if } G(e_t) \geq \bar{u} \\ \left[v_t + \bar{R}q\right] p(e_t) - \left[\bar{u} + c(e_t)\right] & \text{otherwise} \end{cases}$$

s.t. $q = 0$ if $t = 2$. Thus, the firm sets $q = 1$ in $t = 1$. To see that the firm may induce $e_1 > e_1^{FB}$, assume for a moment that $\bar{R} = 0$. Then, the optimal period-1 contract is equivalent to the optimal period-2 contract. Hence, by Proposition 1, the firm would implement $e_1 = e_1^{FB}$ if $\bar{u} > G(e_1^{FB})$. Consequently, because $\bar{R} > 0$, there is a critical value $\hat{u}$ such that $e_1 > e_1^{FB}$ for all $\bar{u} > \hat{u}$. Moreover, $\hat{u} < G(e_1^{FB})$.

References

Kräkel, M. and A. Schöttner (2008): Relative Performance Pay, Bonuses and Job-Promotion Tournaments. Bonn ECON Papers No. 16/2008.

Laffont, J.-J. and D. Martimort (2002): *The Theory of Incentives.* Princeton University Press.

Ohlendorf, S. and P.W. Schmitz (2008): Repeated Moral Hazard, Limited Liability, and Renegotiation. CEPR Discussion paper No. 6725.

Schmitz, P.W. (2005): Workplace Surveillance, Privacy Protection, and Efficiency Wages, *Labour Economics* 12, 727–738.