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Short-term or long-term contracts?
- A rent-seeking perspective

Oliver Gürtler*

*Oliver Gürtler, Department of Economics, BWL II, University of Bonn,
Adenauer-allee 24-42,
D-53113 Bonn, Germany. Tel.: +49-228-739214, Fax: +49-228-739210;
oliver.guertler@uni-bonn.de

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Abstract

In this paper, firms engage in rent seeking in order to be assigned a governmental contract. We analyze how a change in the contract length affects the firms’ rent-seeking behavior. A longer contract leads to more rent seeking at a contract assignment stage, as the firms value the contract higher. On the other hand, the contract has to be assigned less often, which of course leads to less rent seeking. Finally, a longer contract makes a possible cooperation between the firms solving the rent-seeking problem more difficult to sustain.

Key words: Contract length, rent seeking, cooperation, relational contract
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**Oliver Gürtler, Department of Economics, BWL II, University of Bonn, Adenauer-allee 24-42, D-53113 Bonn, Germany. Tel.:+49-228-739214, Fax:+49-228-739210; E-mail: oliver.guertler@uni-bonn.de
1 Introduction

A situation, where several firms compete for a governmental contract or license, represents a typical example for a rent-seeking contest. During the assignment process, each firm expends time and other resources in order to convince the governmental agency that it is the right firm for the job. These resource expenditures are wasteful to a large degree and an extensive body of economic literature has analyzed the determinants of the rent-seeking problem as a basis for possible solutions.\(^1\) What has almost received no attention, however, is that a governmental contract or license is typically awarded for a fixed time period only.\(^2\) Moreover, the length of the contract or license should affect its value for the firms and, accordingly, their rent-seeking expenditures. Building on this observation, the main objective of the paper is to analyze, how long a contract or license should last in order to tackle the rent-seeking problem optimally.

We differentiate between two scenarios: In the first one, the firms are not able to enter a cooperation and so to solve the rent-seeking problem on their own. Here, the optimal length of the governmental contract is determined by a simple trade-off of two countervailing effects. On the one hand, a longer (lasting) contract is more valuable for the firms so that they increase their rent-seeking expenditures, if a contract has to be assigned. On the other hand, such a contract assignment does of course take place less often, if the


\(^{2}\)Aidt & Hillman (2006), considering a dynamic model, in which rights to a rent may be lost over time, represent a notable exception. See also McCormick et al. (1984) who analyze the costs of monopoly and briefly discuss the effects of having a monopolist being forced to continually spend resources in order to retain his status.
contract length is increased. This, in turn, makes a longer contract more attractive. Which of these effects is dominant, will be shown to depend on the specific form of the contest success-function. In particular, it depends on whether the optimal rent-seeking expenditure is a concave, linear or convex function of the contract value.\(^3\)

In a second scenario, the firms may be able to sustain a cooperation (or relational contract), under which each promises not to engage in rent seeking.\(^4\) We find that such a cooperation is easier to sustain, if the governmental contract becomes shorter. To understand this result, notice that a cooperation between the firms is sustainable, if and only if the gain from a unilateral deviation from the cooperation is overcompensated by the triggered punishment of the other firms. A firm gains from the deviation, as it becomes more likely to be assigned the present contract. The shorter this contract, the lower is its value for the firm and the less the firm gains from deviating to a positive rent-seeking expenditure. Similarly, the firm is punished for a deviation, as, starting with the next contract assignment stage, the other firms reenter the match and the situation becomes more wasteful. Here, a shorter contract yields an earlier and thus more significant punishment, as it takes less time until the next contract has to be assigned. To sum up, a shorter contract reduces the gain from a deviation, while, at the same time, leading to a stronger punishment. As both effects work into the same direction, a

\(^3\)Aidt & Hillman (2006) find similar effects in their analysis. However, they assume the rent to be always fully dissipated. Obviously, the total rent-seeking outlay then increases linearly in the rent and both effects cancel out. In this respect, the current model may be seen as a generalization of their model. Furthermore, the current model also allows the contestants to enter a cooperation, which is not the case in the model by Aidt & Hillman.

\(^4\)By introducing a possible cooperation between the firms into the analysis, the paper is related to Gürtler (2006). There, a rent-seeking theory of the firm is developed, which contains a similar cooperation as a main ingredient.
shorter contract makes the cooperation easier to sustain.

Before we proceed, note that a governmental agency assigning a temporary contract or license is not the only example to which the model applies. As a second example, one may think of an employer using subjective performance evaluations in order to motivate and reward his employees. Here, an employee may engage in costly influence activities in order to shift his superior’s judgment in a favorable direction. Following the arguments made before, the amount of influence activities may clearly depend on how often the employees are evaluated and rewarded. Alternatively, the model may also be thought of as to describe a vertically organized production process, where a downstream firm governs the supply of an input with an upstream firm contractually. Then, the upstream firms may engage in rent seeking in order to be assigned the contract. Again, the contract length may crucially affect the rent-seeking decision.

The paper is organized as follows: In the next section, the basic model is described. Section 3 solves the model in a situation, where the firms are not able to enter a cooperation. In Section 4, such a cooperation is introduced into the analysis. Finally, Section 5 concludes.

2 Description of the model and notation

Consider a game with an open time-horizon (i.e. $t = 1, 2, ...$), where all players - a governmental agency and $N$ firms - share a common discount factor $\delta \in (0, 1)$. In each period, one of the firms provides a service for the agency and receives a net payoff of $S > 0$. This service is governed by a

\footnote{Influence activities are a special form of rent seeking. They are of particular relevance, if superiors in a firm have some discretion concerning their subordinates’ compensations. See, for instance, Milgrom (1988) or Kräkel (2006).}
contract of length \( T \in \{1, 2, \ldots, \tilde{T}\} \), where \( T \) is determined by the agency at the beginning of the game and after the old contract has expired, respectively. We assume that the contract must not be longer than \( \tilde{T} \), which could be justified by other efficiency considerations (besides rent seeking) that are not explicitly modeled.

If a new contract has to be assigned, the firms compete for this contract and choose rent-seeking expenditures \( x_i \geq 0, \ i = 1, \ldots, N \). Accordingly, firm \( i \) is awarded the contract with probability\(^6\)

\[
P_i = \begin{cases} \frac{f(x_i)}{\sum_{j=1}^{N} f(x_j)}, & \text{if } \sum_{j=1}^{N} f(x_j) > 0 \\ \frac{1}{N}, & \text{otherwise} \end{cases}
\]

(1)

Here, \( f(\cdot) \) is a strictly increasing function satisfying \( f(0) \geq 0 \) and not being too convex. Specifically, we assume \( f''(x_i) \sum_{j=1}^{N} f(x_j) - 2 (f'(x_i))^2 < 0 \ \forall x_i, x_j \geq 0 \), which guarantees that the firms’ objective functions are strictly concave.

The agency is interested in maximizing social welfare. Therefore, it determines \( T \) such that the present value of the total rent-seeking outlays and, accordingly, the social waste is minimized.\(^7\) The firms choose their rent-seeking expenditures in order to maximize the present value of profits.

\(^6\)See e.g. Skaperdas (1996).

\(^7\)One may also think that the agency benefits from rent seeking and so tries to maximize it. In this case, its optimal decision as determined in Propositions 1 and 2 is simply reversed.
3 Model solution, if cooperation is not possible

In this section, we assume the parameter constellations to be such that no cooperation between the firms is sustainable. Bearing this in mind, we consider the beginning of the game and suppose that $T$ has already been determined. Furthermore, we denote by $S_T$ the total value of the contract for each of the firms. Then, firm $i$ chooses its rent-seeking outlay such that

$$\pi_i = \frac{f(x_i)}{\sum_{j=1}^{N} f(x_j)} S_T - x_i$$

is maximized. We assume that an interior solution to this problem exists, which requires $S$ to be finite, but sufficiently high. If this is the case, the solution is characterized by the following first-order condition (the second-order condition is satisfied):

$$\frac{f'(x_i) \sum_{j \neq i} f(x_j)}{\left(\sum_{j=1}^{N} f(x_j)\right)^2 S_T} - 1 = 0$$

For any other firm $j \neq i$, we obtain the first-order condition analogously. Comparing all first-order conditions, one can directly see that there is a symmetric equilibrium (i.e. $x_1 = \ldots = x_N =: x^*$), which is implicitly given by

$$\frac{(N-1)f'(x^*)}{N^2 f(x^*)} S_T - 1 = 0,$$

or, defining $h(y) := \left(\frac{f'}{f}\right)^{-1}(y)$, by

$$x^* = h\left(\frac{(N-1)}{N^2} S_T\right)$$

Let us now consider the agency’s optimization problem. This can be
stated as

\[ \text{Min } PV_T = Nh \left( \frac{(N-1)}{N^2} S_T \right) (1 + \delta^T + \delta^{2T} + ...) \]  

(6)

Note that we can rewrite \( S_T \) as \( S_T^{1-\delta^T} - 1 \) and \( 1 + \delta^T + \delta^{2T} + ... \) as \( \frac{1}{1-\delta^T} \). The present value of the rent-seeking outlays can thus be rewritten as

\[ PV_T = Nh \left( \frac{(N-1)}{N^2} S^{1-\delta^T} - 1 \right) \frac{1}{1-\delta^T} \]  

(7)

This latter expression nicely reflects the trade-off that the agency faces. On the one hand, a longer contract is more valuable for the firms so that each expends more resources, if a contract has to be assigned. On the other hand, a longer contract is assigned less often, which obviously mitigates the rent-seeking problem.

From (7), one can see that the solution to the agency’s minimization problem depends on the curvature of \( h(\cdot) \). The following proposition makes this argument more precise:

**Proposition 1** Let \( f(0) = 0 \). Then, the agency chooses \( T = 1 \) (\( T = \bar{T} \)), if \( h(\cdot) \) is strictly convex (concave). If \( h(\cdot) \) is linear, the agency is indifferent between all \( T \in \{1, 2, ..., \bar{T}\} \).

**Proof.** First recall that \( h(y) \) is defined as \( \left( \frac{f}{f'} \right)^{-1}(y) \). From \( f(0) = 0 \), it follows that \( \frac{f}{f'}(0) = 0 \) since \( f'(\cdot) > 0 \). As the graph of \( h(y) \) is the reflection about the line \( y = x \) of the graph of \( \frac{f}{f'(x)} \), we have \( h(0) = 0 \), too. The easiest case to handle is the case of a linear function \( h(y) \), which means that \( h(y) = ky \), with \( k = \text{const} \). Here, \( PV_T \) can be written as

\[ PV_T = Nk \frac{(N-1)}{N^2} S \frac{1-\delta^T}{1-\delta} \frac{1}{1-\delta^T} = k \frac{(N-1)}{N} S \frac{1}{1-\delta} \]

\( ^8 \)Note that the agency’s problem does not change over time. Therefore, we can assume w.l.o.g. that it sets the same \( T \) for every contract to be assigned.
which is clearly independent of $T$. Therefore, the agency is indifferent between all possible contract durations.

Assume now that $h(\cdot)$ is strictly convex. In this case, we show that $PV_1 < PV_2$. The proof that $PV_1 < PV_t$, for $t = 3, 4, \ldots, \bar{T}$, is completely analogous. Define $\Delta PV$ as $PV_2 - PV_1$. Hence, we have

$$\Delta PV = N \left[ h \left( \frac{(N-1)}{N^2} S \frac{1-\delta^2}{1-\delta} \right) \frac{1}{1-\delta^2} - h \left( \frac{(N-1)}{N^2} S \frac{1}{1-\delta} \right) \frac{1}{1-\delta} \right]$$

Notice that $h(\cdot)$ being strictly convex means that

$$h \left( tu + (1-t) v \right) < th \left( u \right) + (1-t) h \left( v \right)$$

for any $t \in (0,1)$ and $u \neq v$. Now let $t = \frac{1-\delta}{1-\delta^2}$, $u = \frac{(N-1)}{N^2} S \frac{1-\delta^2}{1-\delta}$ and $v = 0$.

Then, the convexity condition implies

$$h \left( \frac{1-\delta}{1-\delta^2} \frac{(N-1)}{N^2} S \frac{1-\delta^2}{1-\delta} \right) + \left( 1 - \frac{1-\delta}{1-\delta^2} \right) h \left( 0 \right)$$

$$< \frac{1-\delta}{1-\delta^2} h \left( \frac{(N-1)}{N^2} S \frac{1}{1-\delta} \right) + \left( 1 - \frac{1-\delta}{1-\delta^2} \right) h \left( 0 \right),$$

or, equivalently,

$$h \left( \frac{(N-1)}{N^2} S \right) \frac{1}{1-\delta} < h \left( \frac{(N-1)}{N^2} S \frac{1-\delta^2}{1-\delta} \right) \frac{1}{1-\delta^2}$$

From the last condition, we see that $\Delta PV$ is strictly positive and the agency prefers $T = 1$ to $T = 2$. As mentioned before, the same argumentation applies, if we compare the situation $T = 1$ to a situation, where the contract lasts for three or more periods. Hence, if $h(\cdot)$ is strictly convex, the agency sets $T = 1$.

If $h(\cdot)$ is strictly concave, we have

$$h \left( tu + (1-t) v \right) > th \left( u \right) + (1-t) h \left( v \right)$$

for any $t \in (0,1)$ and $u \neq v$. From the previous analysis, it directly follows that $T = \bar{T}$ is optimal for the agency. □
Proposition 1 is very intuitive. It states that the optimal contract duration depends on whether a firm's rent-seeking expenditure is a concave, linear or convex function of the contract value. If it is concave, the (marginal) effect of a higher contract valuation on the rent-seeking outlay is decreasing. Accordingly, the disadvantage of a longer contract in form of higher rent seeking at a contract assignment stage loses some of its impact. This implies that the longest possible contract is optimal. A similar argumentation holds, if \( h(\cdot) \) is linear or convex. In the former case, the contract length does not affect the present value of rent-seeking outlays, as rent seeking increases linearly in the contract valuation. In the latter case, a higher contract value has an increasing (marginal) effect on the rent-seeking expenditures. Consequently, a long-term contract leads to excessively high social waste and a short-term contract with \( T = 1 \) is optimal.

Remark 1 If \( f(0) > 0 \), we have \( h(0) < 0 \) implying that the results from the proposition are shifted in the direction of a shorter contract. In particular, a contract with \( T = 1 \) is strictly optimal, even if \( h(\cdot) \) is linear.

We conclude this section with three examples that help to provide a better understanding of the results derived before.

**Example 1** In the first example, we consider a widely-used contest-success function, where \( f_1(x_i) = x_i^\gamma, \gamma \in (0, 1] \). It is straightforward to see that \( \frac{h_1}{f_1}(x_i) = \frac{x_i}{\gamma} \) and, accordingly, \( h_1(y) = \gamma y \). Hence, \( h_1(\cdot) \) is linear so that Proposition 1 implies the governmental agency to be indifferent between all \( T \in \{1, ..., \bar{T}\} \).

**Example 2** In the second example, \( f_2(x_i) = x_i + k, k > 0 \). In analogy to the first example, we can easily show that \( \frac{h_2}{f_2}(x_i) = x_i + k \). Consequently,
$h_2(\cdot)$ is given by the linear function $h_2(y) = y - k$. Applying Remark 1, we can see that the agency chooses $T = 1$.

**Example 3** In Example 3, we have $f_3(x_i) = ax_i - x_i^2$, for $x_i < 0.5a$ ($a > 0$) and $f_3(x_i) = \frac{a^2}{4}$, otherwise.\(^9\) Assuming $x_i < 0.5a$, it follows that $\frac{f_3}{f_3'}(x_i) = \frac{ax_i-x_i^2}{a-2x_i}$. After a few calculations, one obtains\(^10\) $h_3(y) = \frac{2y+a}{2} - \sqrt{\frac{4y^2+a^2}{4}}$, which is strictly concave, as $h_3'' < 0$. Consequently, Proposition 3 tells us that $T = \bar{T}$ is the optimal policy for the agency.

## 4 Model solution, if cooperation is possible

In the previous section, each firm chose a positive rent-seeking expenditure and was assigned the contract with probability $\frac{1}{N}$. Obviously, the firms would do better, if each did not engage in rent seeking, as the winning-probabilities did not change, but the rent-seeking expenditures were saved. In the following, we will analyze, whether such a cooperation is sustainable. To do so, we assume that firm $i$ can observe, whether firm $j \neq i$ chooses $x_j > 0$ or $x_j = 0$, i.e. whether or not firm $j$ engages in rent seeking. This means that the firms can enter a cooperation, under which each promises to choose $x_i = 0$.

The firms use a Grim-Trigger strategy to sustain the cooperation. Roughly speaking, they start by cooperating and continue to do so unless one party defects. In the latter case, the firms refuse to cooperate forever after and switch to the solution described in Section 2. The agency observes, whether

\(^9\)Note that $f_3(\cdot)$ is only weakly, but not strictly increasing. This, however, is not problematic, as $f_3'(0) > 0$ and the optimal solution always lies at the strictly increasing part of $f_3(\cdot)$.

\(^10\)Note that $\frac{2y+a}{2} - \sqrt{\frac{4y^2+a^2}{4}}$ is indeed smaller than $0.5a$. 

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the firms cooperate or not. Accordingly, if it observes that cooperation breaks down, it changes the contract duration to its optimal value as determined in the previous section. This value is denoted by $\bar{T}$, while the corresponding rent-seeking expenditure by each of the firms is $\bar{x}$.

To solve the model, suppose the agency to have chosen a contract of length $T$. Notice that cooperation is only sustainable, if each firm $i$’s gain from deviating is overcompensated by the triggered costs. A firm gains from deviating to a positive rent-seeking outlay $\hat{x}_{iT} > 0$ by becoming more likely to be assigned the present contract. Formally, the gain can thus be written as

$$G_T = S_T \left( \frac{f(\hat{x}_{iT})}{f(\hat{x}_{iT}) + (N-1)f(0)} - \frac{1}{N} \right) - \hat{x}_{iT}$$

(8)

The following lemma describes the relation between $G_T$ and $T$:

**Lemma 1** $G_T$ is strictly increasing in $T$.

**Proof.** From the definition of $S_T$, $S_T = S_{1-T}$, it can directly be seen that $S_T$ increases in $T$. Hence, if $T$ increases and the deviating firm does not change $\hat{x}_{iT}$, $G_T$ becomes higher. Moreover, as the firm changes $\hat{x}_{iT}$, only if this change pays off (in the sense that $G_T$ must further increase), it directly follows that $G_T$ must necessarily get higher, if $T$ is increased. ■

The lemma states that the gain from the deviation is increasing in the contract length. This is intuitive. If a firm deviates, it becomes more likely to be assigned the current governmental contract. If this contract is longer, its value for the firm is higher so that the firm gains relatively stronger from the deviation.

Let us now consider the costs that a deviation from the cooperative agreement entails. If a firm deviates from the agreement, cooperation between the firms break down. This implies that, starting with the next contract, the
firms switch to the solution from the previous section, which, as mentioned before, is characterized by a contract duration $\tilde{T}$ and rent-seeking outlay $\tilde{x}$. Hence, the costs of a deviation from the cooperation are given by

$$C_T = \delta^T \left[ \frac{1}{N} S_T \left( 1 + \delta^T + \delta^{2T} + \ldots \right) - \left( \frac{1}{N} S_T - \tilde{x} \right) \left( 1 + \delta^T + \delta^{2T} + \ldots \right) \right]$$

(9)

From the expression in the lower line of (9), we directly obtain the second lemma:

**Lemma 2** The costs of deviating from the cooperation, $C_T$, are strictly decreasing in $T$.

Lemma 2 is very intuitive, too. If a firm deviates from the cooperation, it is punished by the other firms from the next contract assignment stage on. If the current contract is rather long, it takes considerable time until the next contract has to be assigned. In other words, the punishment occurs later. Therefore, the punishment is less strong (or more heavily discounted) so that the costs of a deviation are lower.\(^{11}\)

Summarizing, we have seen that a relatively longer contract increases a firm’s direct gain from reneging on the cooperative agreement, while the corresponding costs become lower. It directly follows that a shorter contract facilitates the sustainability of the cooperation. This is formalized in the following proposition:

**Proposition 2** Let the minimal discount factor, which allows to sustain the cooperation under a contract of duration $T$, be denoted by $\hat{\delta}_T$. Then, $\hat{\delta}_1 < \ldots$\(^{11}\)Sasaki & Strausz (2006) find a similar effect analyzing the sustainability of implicit cartels.
\[ \hat{\delta}_2 < \hat{\delta}_3 < \ldots \] Hence, the cooperation is most easily sustainable, if \( T = 1 \). For \( \delta \geq \hat{\delta}_1 \), \( T = 1 \) is also the (weakly) optimal contract length. Otherwise, a cooperation is not feasible and the results from Proposition 1 apply.

5 Conclusion

In this paper, it was analyzed, whether an agency could mitigate rent seeking for a governmental contract by changing the contract length. Such a change was shown to entail three effects on the rent-seeking behavior. First, a longer contract leads to more rent seeking at a contract assignment stage, as the firms value the contract higher. Second, however, the contract has to be assigned less often so that rent seeking occurs less often, too. Finally, a longer contract makes a cooperative agreement solving the rent-seeking problem more difficult to sustain. Building on these effects, we determined the optimal contract length that the agency should choose.

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