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On Delegation under Relational Contracts

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Abstract

In this paper, a principal’s decision between delegating two tasks or handling one of the two tasks herself is analyzed. We assume that the principal uses both, formal contracts and informal agreements sustained by the value of future relationships (relational contracts) as incentive device. It is found that the principal is less likely to delegate both tasks in a dynamic setting than in a static one (where formal contracts are the only feasible incentive device), as handling one task herself enables a much wider use of relational contracts.

Key words: Job design, relational contracts, formal contracts, delegation.
JEL classification: D82, J33, L23, M52, M54

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1 Introduction

In each multi-person firm, the firm manager (henceforth called the principal (denoted by she)) has to group the single tasks to be dealt with into jobs, i.e. she has to choose a job design. This grouping of tasks occurs in two steps. First, the firm manager has to decide which tasks to handle herself and which tasks to delegate. Thereafter, the delegated tasks have to be allocated to the single subordinates (henceforth called the agents (denoted by he)). An efficient grouping of tasks into jobs is essential for the firm’s success in the market. A firm with an inefficient job design is likely to produce at higher costs than its better organized competitors and so faces an important comparative disadvantage.

In this paper, we restrict attention to the first mentioned point and ask, when it is optimal for a principal to delegate a task and when it is not. The analysis builds on Itoh (1994, 2001), whose main result is the following: Assuming that the principal is exogenously forced to delegate at least one of two tasks, the decision of whether or not to delegate the second task as well is determined by a simple trade-off. With risk-averse agents, the principal seeks to do the second task herself in order to save on risk premiums. However, if she does so, the set of feasible incentive schemes will be constrained. Roughly speaking, total incentive strength will always sum up to 1, if the principal handles one task herself. Under complete delegation, this restriction is softened, and the principal can choose from a richer set of incentive schemes. Considering both effects, a simple, intuitive condition is derived: The principal handles one task herself, if either the agents are highly risk-averse or the
production process is very uncertain.

While Itoh’s results are clear and intuitive, his analysis is incomplete in that the principal solely relies on formal incentive contracts to motivate the agents. In many firms, however, incentives are not solely provided via formal, but also via relational contracts.\(^1\) Employees are often paid contingent on contractible measures (such as sales volume), but also on subjective assessments that are not verifiable by a third party. An example is Nokia, the world’s leading mobile phone supplier. While Nokia makes extensive use of formal incentives (e.g. payments based on project/program-success), every year, there is one subjective performance evaluation of all employees. Based on this evaluation, wages are increased or not.\(^2\) Hence, an important question is how (or whether) the delegation decision will change, if incentives are provided by a combination of formal and relational contracts.

This question is tried to be answered in the current paper. The model of Itoh (1994) is therefore combined with a model of Baker et al. (1994). Baker et al. consider a principal-agent relationship, where the principal uses a combination of formal and informal contracts to compensate the agent.\(^3\) This paper applies the Baker et al. model to situations, in which two tasks have to be dealt with and the principal is not able to handle all the tasks

\(^1\)Relational contracts are also referred to as informal, implicit or self-enforcing contracts. Throughout the paper, I use relational contracts and informal contracts as synonyms.

\(^2\)For further examples see Gibbons (2005), who reports on several other firms tying their employees’ compensations to subjective performance measures.

herself.

In the model, there are two performance measures, a very precise measure of individual performance, which is observable by the parties, but cannot be contracted upon and a less precise, but contractible measure of aggregate performance on the two tasks. As a benchmark case, a one-period model is analyzed, where formal contracts based on the second performance measure are the only feasible incentive device. In this model, the decision of whether or not to delegate the second task is determined by a trade-off similar to the one mentioned by Itoh. Thereafter, to make implicit agreements sustainable, the infinitely repeated version of the stage game is considered. In this supergame, the principal promises to pay each agent a bonus based on the realization of the unverifiable performance measure. It is found that partial delegation performs far better in the dynamic model than in the static one. In particular, partial delegation is dominant, unless the discount rate is very high (so that relational contracts are of minor relevance) and complete delegation is preferred in the absence of relational contracting. The reason for this result is as follows: The value of an informal contract depends on whether the principal’s promise to pay a certain bonus is credible. This is the case, whenever the monetary gain from not paying the bonus does not exceed the corresponding costs, which are given by lower profits in future periods. Obviously, the temptation to renege on the informal contract increases in the bonus size. Then, partial delegation has the simple, but important advantage that informal incentives are only provided for one task, as the principal handles the remaining task herself. Therefore, there is only one bonus to
be paid so that the reneging temptation is much lower than under complete delegation. This implies that higher bonuses can be sustained under partial delegation, which makes this job design widely preferable.

There additionally exists a recent, complementary paper by Schöttner (2005), who also discusses the benefits and costs of several kinds of job design under interplay of formal and relational contracts. While the current paper treats the question of whether or not to delegate a task, Schöttner, in a different setting, focuses on how to allocate the delegated tasks to the subordinates. That is, in her paper, the decision to delegate all tasks is exogenously given, and then the best form of complete delegation is derived.

The paper is organized as follows: Section 2 presents the optimal job design in the absence of relational agreements. Section 3 extends the analysis to a combined use of formal and relational contracts. Section 4 contains a model discussion and Section 5 concludes.

2 Job design in the absence of relational contracts

2.1 Description of the model

As mentioned before, the current model combines the models of Itoh (1994) and Baker et al. (1994). Consider a principal and two identical agents, all assumed to be risk-neutral. In the firm, two tasks have to be dealt with, tasks $a$ and $b$. The two tasks are assumed to be quite complex so that each person
can handle at most one task. The principal can therefore decide to assign task \( a \) to one agent and task \( b \) to another (complete delegation, henceforth CD), or to delegate one task and handle the remaining task herself (partial delegation, henceforth PD).\(^4\)

The person in charge of task \( i = a, b \) exerts unobservable effort \( e_i \geq 0 \) that stochastically determines an observable, but unverifiable output \( y_i.\)^{5} This output measures contribution to firm value on task \( i \) and equals either one or zero. Let the probability that output equals one be given by \( \text{Prob}\{y_i = 1|e_i\} = \min\{e_i, 1\} \). Total output is given by \( y = y_a + y_b \).

Efforts \( e_a \) and \( e_b \) additionally affect a second performance measure \( p \) that is contractible and so may be the basis of an enforceable contract. \( p \) is an imperfect measure of joint contribution to firm value on the two tasks and also equals either one or zero. The probability of a measure realization of one is given by \( \text{Prob}\{p = 1|e_a, e_b\} = \min\{\mu_a e_a + \mu_b e_b, 1\} \). The realization of each parameter \( \mu_i \) is unknown, when the principal determines the job design

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\(^4\)It is implicitly assumed that task sharing is impossible. One reason for this assumption could be that each task requires the use of a machine that cannot be operated by two people at the same time.

\(^5\)As pointed out by Malcomson (1984), a rank-order tournament between agents could be arranged, even if output is unverifiable by a third party. With the assumptions made in this paper (in particular, risk neutrality and unlimited liability of the agents) such a tournament would always yield the first-best solution, in the static as well as in the dynamic case. However, a tournament scheme may also lead to serious problems such as collusion between the agents (see e.g. Dye (1984)) or sabotage (see e.g. Lazear (1989), Konrad (2000) or Chen (2003)). Throughout the paper it is assumed that these problems are so severe that the tournament scheme is never desired.
and when the agents are offered a wage contract. Thereafter, it is revealed to the respective person in charge, that is, the person in charge of task \( a \) (\( b \)) privately learns the realization of \( \mu_a \) (\( \mu_b \)). The parameter \( \mu_i \) characterizes the actual difference between the effect of \( e_i \) on \( y \) and its effect on \( p \). Following Baker et al. \( \mu_i \) can be interpreted as follows: There are days (i.e. values of \( \mu_i \)), where high effort spent on task \( i \) leads to similar increases in \( y \) and \( p \) (\( \mu_i \) around one), days, where high effort increases \( y \) but not \( p \) (\( \mu_i \) near zero) and days, where small effort increases \( p \) but not \( y \) (\( \mu_i \) much larger than one). It is further assumed that the mean of \( \mu_i \) equals one so that, in expectation, \( p \) is an unbiased measure of total output \( y \). This assumption allows to characterize the expected difference of \( p \) from \( y \) by a single measure, namely the variance \( \text{Var} [\mu_i] = E_{\mu_i} [(\mu_i)^2] - 1 \). The parameters \( \mu_a \) and \( \mu_b \) are independently, identically distributed (i.i.d. assumption), i.e. \( \text{Var} [\mu_a] = \text{Var} [\mu_b] =: \text{Var} [\mu] \). Their distribution is common knowledge.

Effort entails costs, which, to derive several closed-form solutions, are assumed to be quadratic and given by \( C (e_i) = \frac{c}{2} (e_i)^2 \), with \( c > 0 \). Throughout the paper, it is assumed that the parameter \( c \) and the support of \( \mu_i \) are such that, in equilibrium, \( e_i < 1 \) and \( \mu_a e_a + \mu_b e_b < 1 \). In negotiations, the principal is assumed to possess the complete bargaining power. Therefore, she only has to make sure that each agent receives expected utility weakly exceeding his reservation utility \( \bar{U} \), which is normalized to zero.

The timing of the model is as follows: At stage 1, the principal decides on which tasks to delegate. At stage 2, she offers a wage contract to one or two agents, respectively. At stage 3, the agent(s) accept(s) or reject(s) the
offer. An agent rejecting the offer as well as an agent not being offered a wage contract receives his reservation utility. If all wage offers are rejected, the principal handles one task herself, whereas the other task is not handled at all. At stage 4, the person in charge of task $i$ learns the realization of $\mu_i$, while efforts are chosen at stage 5. At stage 6, $p$ and $y$ are realized and payments are made.

2.2 Model solution

Before the model is solved, we consider the first-best solution, in which efforts are assumed to be contractible. In the first-best solution, the principal could hire two agents and determine their efforts to maximize

$$S = e_a + e_b - \frac{\varepsilon}{2} (e_a)^2 - \frac{\varepsilon}{2} (e_b)^2.$$  

First-best efforts are therefore given by $e^{fb}_a = e^{fb}_b = \frac{1}{c}$ and the corresponding surplus by $S^{fb} = \frac{1}{c}$.

Let us now solve the model. As we are in a static scenario, relational contracts are not feasible and the principal solely relies on formal contracts to motivate the agents. Hence, the wage contracts are given by $w^{CD,f}_i = \alpha^{CD,f}_0 + \alpha^{CD,f}_1 p$ and $w^{PD,f}_i = \alpha^{PD,f}_0 + \alpha^{PD,f}_1 p$, where the $f$ indicates the isolated consideration of formal contracts. While the agents always receive a fixed wage of $\alpha_0$, they will receive the variable component $\alpha_1$, only if the joint performance measure $p$ equals one.

The model is solved by backward induction. I start with the CD case. After observing the realization of $\mu_a$, the agent working on task $a$ chooses his effort to maximize expected utility. This expected utility is given by (1).
It consists of the expected wage payment minus costs entailed by effort.

\[ EU_{a, e, p}^C = \alpha_{0a}^C + \alpha_{1a}^C (e_a \mu_a + E_{\mu_b} [e_b \mu_b]) - \frac{c}{2} (e_a)^2 \tag{1} \]

\( E_{\mu_i} [\cdot] \) denotes the expectation operator with respect to \( \mu_i \) and \( e_p \) stands for ex post since (1) denotes the expected utility after observing the parameter \( \mu_a \).

The optimal effort satisfies \( e_a = \frac{\alpha_{1a}^C \mu_a}{c} \). Similarly, the agent working on task \( b \) exerts effort \( e_b = \frac{\alpha_{1b}^C \mu_b}{c} \). As the principal is assumed to possess the complete bargaining power, the agents’ participation constraints are always binding. This implies that the fixed wages are set such that the agents’ ex ante expected utilities, that is, their expected utilities before observing the parameters \( \mu_a \) and \( \mu_b \), respectively, equal zero. Therefore, the principal maximizes expected surplus, which is given by

\[ E\pi_{CD,f} = E \left[ e_a + e_b - \frac{c}{2} (e_a)^2 - \frac{c}{2} (e_b)^2 \right] \tag{2} \]

\[ = \frac{\alpha_{1a}^C + \alpha_{1b}^C}{c} - \left( \frac{\alpha_{1a}^C}{2} \right)^2 E [\mu_a]^2 + \left( \frac{\alpha_{1b}^C}{2} \right)^2 E [\mu_b]^2 \]

In equation (2) as well as in the following, notation is simplified by writing

\[ E_{\mu_i} [(\mu_i)^2] = E [(\mu_i)^2]. \]

Maximizing (2) yields the solution \( \alpha_{1a}^C = \frac{1}{E[(\mu_a)^2]} \) and \( \alpha_{1b}^C = \frac{1}{E[(\mu_b)^2]} \), which, using the i.i.d. assumption, becomes \( \alpha_{1a}^C = \alpha_{1b}^C = \frac{1}{c E[\mu^2]} \). The principal’s expected profit is then given by \( E\pi_{CD,f} = \frac{1}{c E[\mu^2]} \).

The optimal formal contract under PD can be derived analogously. Let us suppose in this case, without loss of generality, that the principal delegates the second task and handles the first task herself. The principal’s and
the agent’s optimal effort are then $e_a = \frac{1 - \alpha_{1b}^{PD,f} \mu_a}{c}$ and $e_b = \frac{\alpha_{1b}^{PD,f} \mu_b}{c}$. The optimal incentive parameter satisfies $\alpha_{1b}^{PD,f} = \frac{1}{E[(\mu_a)^2] + E[(\mu_b)^2]}$, or with the i.i.d. assumption, $\alpha_{1b}^{PD,f} = \frac{1}{2E[\mu^2]}$. The principal achieves an expected profit of $E\pi_{PD,f} = \frac{2E[\mu^2] + 1}{4cE[\mu^2]}$.

A comparison of $E\pi_{CD,f}$ and $E\pi_{PD,f}$ immediately yields the following proposition:

**Proposition 1** The principal chooses CD (PD, is indifferent between both job designs), if and only if $E[\mu^2] < (>,\equiv)1.5$.

Let me explain the intuition behind Proposition 1. Compared to CD, PD has one important advantage and one important disadvantage. The disadvantage stems from a restriction of the set of incentive contracts. Under both kinds of job design, the agents are compensated contingent on the realization of some aggregate performance measure. Hence, they receive only part of their marginal product, but bear the complete effort costs and so choose inefficiently low efforts. This problem can be mitigated effectively under CD by installing high-powered incentives, i.e. by increasing both variable components. Under PD, on the other hand, providing the principal and the agent with high-powered incentives is impossible. The joint performance measure is positively correlated to total output. Thus, if the principal provides the agent with high incentives, she will automatically decrease her marginal payoff from exerting effort. That is, installing high incentives for the agent leads to low incentives for the principal and vice versa.\(^6\)

\(^6\)As shown by Holmström (1982), this problem might be solved by introducing a third
However, as seen in Proposition 1, PD may be the preferred choice of job design, too. There also exists a relative advantage of PD. As mentioned before, the measure $p$ is only an imperfect measure of total contribution to firm value. Due to this imperfection, an agent’s behavior shows distortions with respect to desired behavior. This distortion depends on the realization of $\mu_i$. For $\mu_i < 1$, the agent responsible for task $i$ exerts undesirably low effort. On the contrary, for $\mu_i > 1$, the actual effort is undesirably high. Since the principal must compensate the agents for their effort costs such distortions from desired effort are costly. Under PD, this distorting behavior is less serious. There is only one agent behaving inefficiently. The principal focuses on the realization of output and therefore chooses a more desired effort.

The cut-off in Proposition 1 results from the interaction of these two effects. As is clear from the preceding argumentation, PD will be preferable, if the distortion in an agent’s effort with respect to desired effort is very high. Since, in expectation, $p$ is an unbiased measure of $y$, the agents’ efforts will be highly distorted, if $\text{Var}[\mu]$ is high. This variance can be rewritten as $\text{Var}[\mu] = E[\mu^2] - 1$. Hence, this variance as well as the relative advantage of PD compared to CD is strictly increasing in $E[\mu^2]$. Consequently, there exists a cut-off value for $E[\mu^2]$, where the optimal job design changes.

party being able to “break the budget”. Since such a solution entails new complications (see e.g. Eswaran and Kotwal (1984)), it is not considered in this paper.
3 Job design under a combination of formal and relational contracts

In the one-period model in Section 2, no relational agreement could be sustained since every such agreement would be reneged on. In order to make relational contracts sustainable and to analyze the interaction of formal and relational contracts, I consider the infinitely-repeated version of the model from Section 2.\(^7\)

A relational contract specifies a bonus payment from principal to agent contingent on the agent’s contribution to firm value. As this contribution is non-verifiable, it must be in the parties’ interests to honor the agreement, i.e. the agreement must be self-enforcing. This implies that the bonus has to be non-negative. To see this, recall the assumption that the principal has the complete bargaining power. Therefore, each agent always receives his reservation utility and has no incentive to pay a bonus to the principal.

To determine the principal’s incentives to honor the relational contract, we have to specify two things: First, we have to specify, how the principal values future profits in relation to present profit. Second, we must clarify the consequences of a breach of the relational contract. Concerning the first point, we assume that the principal discounts future profits at a discount rate \(r\), i.e. a one-unit profit in the next period is worth \(\frac{1}{1+r}\) units in the present.

\(^7\)Note that the infinite-horizon approach is not the only possible approach to model relational contracts. There also exists a finite-horizon approach (see e.g. Hart & Holmström (1987) or Gürtler (2006)). I make use of the first approach, because it is mainly used in the literature on relational contracting and it is much more tractable.
one. The discount rate $r$ could, for example, represent the interest rate, to which the principal could lend or borrow money.\footnote{Similarly, the agents are assumed to discount future utility at a rate $r_a$. This rate, however, is unimportant for the analysis. Discounting only affects a party’s temptation to renege on the relational contract. As the bonus accrues to the agents and their efforts are utility-maximizing, they are never interested in breaking this contract.}

Concerning the second point, we assume that the principal as well as the agents follow a modified grim trigger strategy. This means, they start by cooperating (that is, by honoring the relational agreement) and continue cooperation unless one player defects, in which case they refuse to cooperate forever after. Hence, if the relational agreement is reneged on once\footnote{Under CD, it is assumed that both agents lose trust in the principal, even if she reneges on the relational contract of only one agent.}, the parties will rely on the formal contracts derived in Section 2.\footnote{One problem with these strategies is that they are not renegotiation-proof. To see this, note that the game remaining after one party defects coincides with the game as a whole. As a consequence, equilibria being available in the game as a whole should also be available after the relational agreement was broken. Hence, the parties should be able to renegotiate from punishment to a different equilibrium with higher payoffs. This problem, however, can be easily solved (see e.g. the textbook by Bolton & Dewatripont (2005), p. 467): Instead of playing the stage-game Nash equilibrium in the punishment phase, the parties could play jointly efficient punishments, but change the division of the surplus after a deviation. In particular, the division should be changed such that the deviating party receives exactly the same payoff as in the Nash-equilibrium of the stage game. Note that all results to be derived remain the same, if we assume the parties to follow this second type of strategy.} Moreover, we have to say something about what happens with the job design, if the
relational contract is reneged on. Two possibilities arise: First, one could assume that a change from a certain job design to another entails prohibitively high costs so that the principal always maintains the job design she initially has chosen. This assumption seems to map practice very well, for firms seem to change their organization of work very rarely. Further, this assumption makes the model much more tractable and is therefore adopted in this section. However, this assumption is not crucial to the results to be derived. This will be demonstrated in Section 4. There, it is assumed that a change in job design is costless so that, after reneging on the relational contract, the principal always switches to the preferred job design.

In order to derive the optimal combination of formal and relational contracts, I search for a subgame perfect equilibrium of the game. I consider only stationary contracts, under which the principal in every period offers the same wage contract and the agents choose the same efforts on the equilibrium path. This is, as shown by Levin (2003), without loss of generality. The wage payment to the agent dealing with task $i$ is in each period given by

$$w_{i}^{CD,r} = \alpha_{0i}^{CD,r} + \alpha_{1i}^{CD,r} p + \beta_{i}^{CD,r} y_{i}$$

or

$$w_{i}^{PD,r} = \alpha_{0i}^{PD,r} + \alpha_{1i}^{PD,r} p + \beta_{i}^{PD,r} y_{i},$$

where $r$ indicates the combined use of formal and relational contracts. The term $\beta_{i}y_{i}$ corresponds to an informal promise of the principal to pay the agent a bonus depending on the realization of unverifiable output. Since such an informal promise cannot be enforced by court, it must be self-enforcing.

\[\gamma_{i}y_{j}\]

where $\gamma_{i}$ is a payment the agent will receive, if the contribution of the person in charge of the other task equals one. With the restriction $\gamma_{i} \geq 0$, the principal will always set $\gamma_{i}$ equal to zero. The element $\gamma_{i}y_{j}$ is therefore not considered in the wage contract.
Consider first the CD case. The incentives provided by a relational contract depend on whether or not the agents believe that the principal will honor the contract. If, in a given period, they trust the principal, the agents will choose their efforts, after observing $\mu_j$, to maximize ex post expected utilities given by (3) and (4), respectively:

$$EU_{a,CD,r,ep}^{CD,r} = \alpha_{0a}^{CD,r} + \alpha_{1a}^{CD,r}(e_a \mu_a + E[\mu_b | e_b \mu_b]) + \beta_a^{CD,r}e_a - \frac{c}{2}(e_a)^2 \quad (3)$$

$$EU_{b,CD,r,ep}^{CD,r} = \alpha_{0b}^{CD,r} + \alpha_{1b}^{CD,r}(e_b \mu_b + E[\mu_a | e_a \mu_a]) + \beta_b^{CD,r}e_b - \frac{c}{2}(e_b)^2 \quad (4)$$

The optimal efforts therefore satisfy $e_a = \frac{\alpha_{1a}^{CD,r} + \beta_a^{CD,r}}{c}$ and $e_b = \frac{\alpha_{1b}^{CD,r} + \beta_b^{CD,r}}{c}$.

Again, the principal determines the fixed wages such that the agents’ ex ante expected utilities become zero. Then, her expected profit equals the expected surplus and is given by

$$E\pi_{CD,r} = \frac{\alpha_{1a}^{CD,r} + \alpha_{1b}^{CD,r} + \beta_a^{CD,r} + \beta_b^{CD,r}}{c}$$

$$- \left( \frac{\alpha_{1a}^{CD,r}}{c} \right)^2 E[(\mu_a)^2] - \left( \frac{\alpha_{1b}^{CD,r}}{c} \right)^2 E[(\mu_b)^2] + \frac{\beta_a^{CD,r}}{c} - \frac{\beta_b^{CD,r}}{c} \quad (5)$$

Note that the principal will honor the relational contract, only if the discounted additional future profits arising from the combined use of formal and relational agreements exceed the present gain from not paying the two relational bonuses. Otherwise, she would renege on the relational contract and the agents, anticipating this, would not trust her. Therefore, a non-reneging constraint has to be fulfilled, which is given by

$$\left( E\pi_{CD,r} - E\pi_{CD,f} \right) \frac{1}{r} \geq \beta_a^{CD,r} + \beta_b^{CD,r} \quad (6)$$
It can easily be seen that this constraint is more likely to be satisfied, the higher the additional profit from relying on relational agreements, the lower the discount rate $r$, and the lower the relational bonuses to be paid. This is intuitive. If the principal does gain very much from the use of relational contracts and if she is rather patient (that is, future profits are hardly discounted), the benefit from not paying the relational bonuses will probably be outweighed by the loss in future profits. On the other hand, the gain from not paying the bonuses and, hence, the reneging temptation certainly increases in the size of the bonuses.

To determine the optimal incentive parameters, we maximize $E \pi_{CD,r}$ subject to the non-reneging constraint. Using the i.i.d. assumption, the Lagrangian to the maximization-problem is given by

$$L = \frac{1 + \lambda}{c} \left[ \alpha_{1a}^{CD,r} + \alpha_{1b}^{CD,r} - 0.5 \left( \left( \alpha_{1a}^{CD,r} \right)^2 + \left( \alpha_{1b}^{CD,r} \right)^2 \right) E [\mu^2] \right]$$

$$+ \beta_a^{CD,r} + \beta_b^{CD,r} - 0.5 \left( \left( \beta_a^{CD,r} \right)^2 + \left( \beta_b^{CD,r} \right)^2 \right) - \alpha_{1a}^{CD,r} \beta_a^{CD,r}$$

$$- \alpha_{1b}^{CD,r} \beta_b^{CD,r} - \lambda \left[ \frac{1}{cE[\mu^2]} + r \left( \beta_a^{CD,r} + \beta_b^{CD,r} \right) \right]$$

The first-order conditions to this maximization-problem are

$$\frac{\partial L}{\partial \alpha_{1a}^{CD,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_a^{CD,r} - \alpha_{1a}^{CD,r} E [\mu^2] \right) = 0$$

$$\frac{\partial L}{\partial \alpha_{1b}^{CD,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_b^{CD,r} - \alpha_{1b}^{CD,r} E [\mu^2] \right) = 0$$

$$\frac{\partial L}{\partial \beta_a^{CD,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_a^{CD,r} - \alpha_{1a}^{CD,r} \right) - \lambda r = 0$$

$$\frac{\partial L}{\partial \beta_b^{CD,r}} = \frac{1 + \lambda}{c} \left( 1 - \beta_b^{CD,r} - \alpha_{1b}^{CD,r} \right) - \lambda r = 0$$

These conditions lead to a symmetric solution with $\alpha_{1a}^{CD,r} = \alpha_{1b}^{CD,r} = \alpha_1^{CD,r}$, $\beta_a^{CD,r} = \beta_b^{CD,r} = \beta^{CD,r}$. Using this symmetry, the first-order conditions
simplify to
\[ \frac{1 + \lambda}{c} \left( 1 - \beta^{CD,r} - \alpha_1^{CD,r} E[\mu^2] \right) = 0 \]  
\[ \frac{1 + \lambda}{c} \left( 1 - \beta^{CD,r} - \alpha_1^{CD,r} \right) - \lambda r = 0 \]  
(12)  
(13)

If, in the optimum, the non-reneging constraint is slack (i.e. \( \lambda = 0 \)), the solution is \( \alpha_1^{CD,r} = 0 \) and \( \beta^{CD,r} = 1 \). That is, if the principal is sufficiently patient, a relational contract will be installed leading to the first-best solution. Each agent bases his effort decision solely on the realization of output and, as a consequence, no distorting behavior will arise.

Of more interest is the case, in which the principal is less patient so that the non-reneging constraint binds in the optimum. From (12) and the binding condition (6), the second-best relational bonus and the second-best expected profit can be derived. The possible values of relational bonus and expected profit are given by (14) and (15), respectively:

\[ \beta^{CDS,r} = \begin{cases} 
1 & \text{for } r \leq \hat{r}^c \\
2 - \frac{2rCE[\mu^2]}{E[\mu^2]-1} & \text{for } \hat{r}^c > r > \hat{r}^c \\
0 & \text{for } r \geq \hat{r}^c 
\end{cases} \]  
(14)

\[ E\pi_{CDS,r} = \begin{cases} 
\frac{1}{c} & \text{for } r \leq \hat{r}^c \\
\frac{(E[\mu^2]-1)(1+4rE[\mu^2]) - 4r^2c^2(E[\mu^2])^2}{E[\mu^2](E[\mu^2]-1)c} & \text{for } \hat{r}^c > r > \hat{r}^c \\
\frac{1}{cE[\mu^2]} & \text{for } r \geq \hat{r}^c 
\end{cases} \]  
(15)

with \( \hat{r}^c = \frac{E[\mu^2]-1}{2cE[\mu^2]} \) and \( \hat{r}^c = \frac{E[\mu^2]-1}{cE[\mu^2]} \).

The derivation of the optimal combination of formal and relational contract in the PD case is analogous. The optimal relational bonus in this case
is given by (16), the optimal expected profit by (17).

\[
\beta_{PD,r} = \begin{cases} 
1 & \text{, for } r \leq \hat{r}^p \\
2 - \frac{4rcE[\mu^2]}{2E[\mu^2]-1} & \text{, for } \hat{r}^p > r > \hat{r}^c \\
0 & \text{, for } r \geq \hat{r}^c 
\end{cases}
\]  

(16)

\[
E\pi_{PD,r} = \begin{cases} 
\frac{1}{c} & \text{, for } r \leq \hat{r}^p \\
\frac{4(E[\mu^2])^2 - 1 + 8rcE[\mu^2](2E[\mu^2](1-rc) - 1)}{4E[\mu^2](2E[\mu^2]-1)c} & \text{, for } \hat{r}^c < r < \hat{r}^p \\
\frac{2E[\mu^2] + 1}{4cE[\mu^2]} & \text{, for } r \geq \hat{r}^p 
\end{cases}
\]  

(17)

with \(\hat{r}^c = \frac{2E[\mu^2] - 1}{4cE[\mu^2]}\) and \(\hat{r}^p = \frac{2E[\mu^2] - 1}{2cE[\mu^2]}\).

We now compare the expected profits to see, which kind of job design the principal prefers. When comparing the profits, it is convenient to distinguish between the cases \(E[\mu^2] < 1.5\), \(E[\mu^2] > 1.5\) and \(E[\mu^2] = 1.5\). Proposition 2 describes the optimal job design in the first case (note that, with \(E[\mu^2] < 1.5\), \(\hat{r}^c < \hat{r}^c < \hat{r}^p < \hat{r}^p\) holds).

**Proposition 2** Suppose that \(E[\mu^2] < 1.5\). (i) For \(r \leq \hat{r}^c\), both job designs lead to the first-best solution. In this case, the principal is indifferent between the two job designs. (ii) For \(\hat{r}^c < r \leq \hat{r}^p\), PD yields the first-best solution, whereas CD does not. PD is thus preferred. (iii) For \(\hat{r}^p < r\), there exists a cut-off \(\bar{r}\) with \(\hat{r}^p > \bar{r} < \hat{r}^p\) such that PD is preferred, only if \(r \in [\bar{r}, \hat{r}]\).

**Proof.** See the Appendix.  

Note first that \(r > \hat{r}^p\) corresponds to the case, where the principal is so impatient or the interest rate is so high that any informal contract would be reneged on. Then, we obtain the same result as in Section 2, which states that, for \(r > \hat{r}^p\) and \(E[\mu^2] < 1.5\), CD is preferred.
CD, however, will no longer be necessarily preferred, if relational contracts become feasible, i.e. if \( r < \tilde{r}_p \). On the contrary, if the discount rate \( r \) is not too large (in particular, if \( r \leq \tilde{r} \)), PD will even be the (weakly) dominant job design. In other words, the results derived in the less general model in Section 2 are not robust to an introduction of relational agreements. The different parts of Proposition 2 indicate that, for different values of the discount rate, the form of compensation used under the two job designs may differ. For \( r \leq \hat{r}_c \), the discount rate is so low that the principal only uses relational contracts as incentive device. Consequently, there is no distorting behavior by the agents and both job designs lead to the first-best solution. For \( \hat{r}_c < r \leq \hat{r}_p \), this is still true under PD, but not under CD. Under CD, the principal must decrease the bonus in order to credibly commit not to renege on it. Then, she makes use of formal contracts, too, which leads to a deviation from the first-best solution. Therefore, PD is preferred. Further, for \( \hat{r}_p < r \leq \tilde{r}_p \), the principal uses a combination of formal and relational compensation under PD, while, under CD, only formal contracts are feasible. Here, the profit under CD is independent of \( r \). In contrast, the profit under PD decreases, as \( r \) increases, for the relational bonus to be sustained becomes smaller. Hence, PD performs relatively worse, as \( r \) increases. Since CD is optimal in the absence of relational agreements, there exists a clear cut-off \( \tilde{r} \), where the optimal job design changes.

Summarizing these findings, one can say that PD allows a much wider use of relational contracts and is therefore oftentimes preferred in the dynamic scenario. The next paragraphs explain, why this is the case.
For the principal, it is always better to rely on informal contracts rather than on formal ones since, in this way, distortions in the agents’ efforts are mitigated. However, she may not be able to credibly commit not to renege on a relatively high relational bonus. In this spirit, a job design might be preferred, if it leads to a higher sustainable bonus. To determine the respective reneging temptations under the two job designs (and, accordingly, the respective size of the sustainable bonus), let us compare condition (6) with the following non-reneging constraint from the PD case:

\[ (E\pi_{PD,r} - E\pi_{PD,f}) \frac{1}{r} \geq \beta^{PD,r} \tag{18} \]

Comparing the two conditions, we can isolate two effects that determine, which job design leads to higher sustainable bonuses. First, PD has the simple, but important advantage that only one bonus has to be paid so that the gain from reneging on the relational contract is much lower. This effect leads to a higher sustainable bonus under PD. Second, the relative reneging temptation depends on the relative profit increases, if formal incentives are replaced by relational ones. Without a thorough analysis, the sign of this effect cannot be assessed. On the one hand, a relational contract seems to be more beneficial under PD. From (12), we see that formal and relational incentives are substitutes under CD. This is also true under PD. That is, the introduction of relational contracts leads to lower remuneration based on the realization of the contractible measure \( p \). Under PD, this mitigates problems connected with the restriction of the set of possible incentive contracts since, as explained in Section 2, lower formal incentives for the agent yield higher incentives for the principal inducing the latter to choose effort closer to the
first-best. This advantage is absent under CD. CD, on the other hand, especially benefits from the introduction of relational agreements, as, under that job design, distortions in effort behavior of two agents are mitigated.

To determine, which job design leads to a higher profit increase, we need to compare $E\pi_{CD,r} - E\pi_{CD,f} := \Delta CD$ and $E\pi_{PD,r} - E\pi_{PD,f} := \Delta PD$ for a fixed relational bonus $\beta$. Using (5), the symmetry of the solution and (12), one can show that $\Delta CD = \frac{1}{\sigma E[\mu^2]} (-2\beta + \beta^2 + 2E[\mu^2] \beta - E[\mu^2] \beta^2)$. Similarly, $\Delta PD$ can be shown to equal $\frac{1}{\sigma E[\mu^2]} (-2\beta + \beta^2 + 4E[\mu^2] \beta - 2E[\mu^2] \beta^2)$. It is then straightforward to show that $\Delta CD > \Delta PD \iff E[\mu^2] > 1.5$. The condition says that the job design performing relatively worse in the absence of relational contracts benefits more strongly from their introduction. In case $E[\mu^2] < 1.5$ it follows that, under PD, the principal benefits more strongly from the introduction of relational agreements.\footnote{Note that this result is partly driven by our assumption that renegotiation of the job design is impossible. Therefore, PD leads to a lower profit off the equilibrium path and, accordingly, to a higher profit increase. Nevertheless, I show in Section 4 that this assumption is not crucial for the model results. Even if renegotiation of the job design is possible, PD leads to higher sustainable bonuses.} Hence, both effects are enforcing and PD always leads to higher relational bonuses than CD. This can be easily confirmed comparing (14) and (16). To sum up, PD allows a much wider use of relational contracts and is therefore preferred for many values of the discount rate $r$.

Consider now the case $E[\mu^2] > 1.5$. In this case, conditions $\hat{r}^c < \hat{r}^p < \tilde{r}^c < \tilde{r}^p$ hold. Proposition 3 describes the principal’s optimal choice.

**Proposition 3** Suppose that $E[\mu^2] > 1.5$. (i) For $r \leq \hat{r}^c$, both job designs
lead to the first-best solution. The principal is in this case indifferent between
the two job designs. (ii) For \( r > \bar{r}^c \), PD is always preferred.

**Proof.** See the Appendix. ■

The intuition behind Proposition 3 is similar to the one given for Propo-
sition 2. Here, both effects on the sustainable bonus are countervailing, as
fewer bonuses have to be paid under PD, but the profit increase for a fixed
bonus is higher under CD. The first effect, however, is dominant. There-
fore, the relational bonus to be sustained is higher under PD so that this job
design is weakly dominant, independent of the discount rate \( r \).

Finally, suppose that \( E[\mu^2] = 1.5 \). In this case, we have \( \bar{r}^c < \bar{r}^p = \bar{r}^c < \bar{r}^p \).
Again, the same effects as in the first two cases determine the optimal job
design. In Proposition 4, I therefore only present the optimal job design,
without further explaining the intuition behind the results.

**Proposition 4** With \( E[\mu^2] = 1.5 \), the following results hold: (i) For \( r \leq \bar{r}^c \),
both job designs lead to the first-best solution. The principal is in this case
indifferent between the two job designs. (ii) For \( \bar{r}^c < r < \bar{r}^p \), PD is always
preferred. (iii) For \( r \geq \bar{r}^p \), the principal is indifferent between the two job
designs.

**Proof.** Obvious and omitted. ■

4 Discussion

Up to this point, we have assumed that renegotiation of the organizational
structure is impossible. One might guess that this drives some of the derived
results. In particular, the wide preferableness of PD in the case $E[\mu^2] < 1.5$ might be argued to stem from the relatively higher punishment the principal incurs under PD.\textsuperscript{13} In this section, we show that this is not true. Even if renegotiation of the organizational structure is possible, the qualitative model results do not change at all. PD is still preferred, unless the discount rate is very high and complete delegation is preferred in the absence of relational contracting.

To prove this, we start by assuming that $E[\mu^2] < 1.5$. Then, after reneging on the relational contract, the principal would switch to CD and the corresponding profit was $E\pi_{CD,f} = \frac{1}{cE[\mu^2]}$. The analysis in the CD case therefore does not change at all. Under PD, on the other hand, it does. In this case, consider the binding version of the non-reneging constraint, which is given by

$$\left(\beta^{PD,r}\right)^2 - \beta^{PD,r} \left(2 + \frac{4rcE[\mu^2]}{1 - 2E[\mu^2]}\right) - \frac{3 - 2E[\mu^2]}{1 - 2E[\mu^2]} = 0 \quad (19)$$

To find the optimal bonus, we determine the largest value for $\beta^{PD,r}$ satisfying (19). This value is given by

$$\beta^{PD,r} = 1 + \frac{2rcE[\mu^2]}{1 - 2E[\mu^2]} + \sqrt{\frac{4 - 4E[\mu^2]}{1 - 2E[\mu^2]} + \frac{4rcE[\mu^2]}{1 - 2E[\mu^2]} + \left(\frac{2rcE[\mu^2]}{1 - 2E[\mu^2]}\right)^2} \quad (20)$$

It is easy to show that the first-best solution (i.e. $\beta^{PD,r} = 1$) will be realized, whenever $r \leq \frac{E[\mu^2] - 1}{cE[\mu^2]} = \bar{r}^c$. It directly follows that PD is weakly dominant.

\textsuperscript{13}This effect is already known from the theory of the firm (see e.g. Garvey (1995) or Halonen (2002)), where it is oftentimes assumed that renegotiation of the ownership structure is prohibitively costly. This assumption drives the result that the ownership structure being dominated in the static scenario may become optimal, if relational contracts are introduced into the analysis.
for $r \leq \tilde{r}^c$. If $r \leq \hat{r}^c$, both job designs are equally good, as they both achieve the first-best solution. If, however, $\hat{r}^c < r \leq \tilde{r}^c$, under PD the first-best solution is achieved, while under CD it is not.

Suppose now that $r$ is slightly above $\tilde{r}^c$. In this case, the principal uses a combination of formal and relational contracts under PD, while, under CD, formal contracts are the only feasible incentive device. If then $r$ increases, the PD profit decreases, whereas the CD profit stays the same. Hence, CD becomes relatively more attractive. As CD is optimal in the absence of relational contracts, there will arise a cut-off value for $r$, say $\bar{r}$, at which CD becomes optimal. Further, note that the bonus to be sustained under PD is smaller than in Section 3, for the reneging temptation is higher. Therefore, PD performs worse than in Section 3 so that $\bar{r}$ has to be smaller than $\hat{r}$. The following proposition summarizes the results (proof in the text):

**Proposition 5** Suppose that $E[\mu^2] < 1.5$ and the job design can be renegotiated at no costs. (i) For $r \leq \tilde{r}^c$, both job designs lead to the first-best solution. In this case, the principal is indifferent between the two job designs. (ii) For $\hat{r}^c < r \leq \tilde{r}^c$, PD yields the first-best solution, whereas CD does not. PD is thus preferred. (iii) For $\tilde{r}^c < r$, there exists a cut-off $\bar{r} < \hat{r}$ such that PD is preferred, only if $r \in [\hat{r}^c, \bar{r}]$.

I now briefly discuss the case $E[\mu^2] > 1.5$. In this case, the principal would, after reneging on the relational contract, always switch to PD. Therefore, the analysis under PD is exactly the same as in Section 3. CD, on the other hand, performs relatively worse, as the bonus to be sustained decreases.
In Proposition 3, PD was shown to be weakly dominant. Therefore, PD is naturally still dominant, if renegotiation of the job design is possible. The only difference to the results in Proposition 3 is that the range of parameter values, for which CD leads to the first-best solution, too, becomes smaller.

5 Concluding Remarks

This paper started by comparing two different job designs in a static environment. A very nice and intuitive condition was derived indicating when each job design is optimal, respectively. Thereafter, a model with infinite horizon was considered. The purpose was to allow the principal to use both, formal and informal contracts, as incentive device. It was shown that the introduction of relational contracts generally makes partial delegation more attractive.

The reason is that partial delegation leads to higher relational bonuses to be sustained. As the principal handles one task himself, fewer relational bonuses have to be paid so that the reneging temptation is lower. This result was shown to be quite robust, it holds both, in the case, where the job design can be changed and where it is prohibitively costly to do so.

An interesting extension of the model would be to consider a situation with $n > 2$ tasks to be dealt with. A guess is that the wide preferableness of PD would disappear, if $n$ becomes large enough. In this case, the difference in the reneging temptation, which results from either paying $n - 1$ (PD) or $n$ (CD) bonuses should be negligible so that the main advantage of PD
vanishes. If this is true, PD would be especially useful in small groups, where PD enables a much larger bonus to be sustained.

Appendix

Proof of Proposition 2:

The proof of parts (i) and (ii) is obvious and therefore omitted. It remains to prove part (iii). First, note that, for \( r > \hat{r}^p \), relational contracts are not feasible under either job design and CD is preferred. Suppose now that \( \hat{r}^p < r \leq \tilde{r}^p \). Then, PD leads to a mixture of formal and relational contracting, while under CD only formal contracts are available. Thus, the profits to be compared are

\[
E_{PD,r} = \frac{4\left(E[\mu^2]\right)^2 - 1 + 8rcE[\mu^2]\left((2E[\mu^2] - 1) - 2rcE[\mu^2]\right)}{4E[\mu^2](2E[\mu^2] - 1)c}
\]

and

\[
E_{CD,r} = \frac{1}{cE[\mu^2]}
\]

PD is the preferred choice of job design, if the following condition holds:

\[
4\left(E[\mu^2]\right)^2 - 1 + 8rcE[\mu^2]\left((2E[\mu^2] - 1) - 2rcE[\mu^2]\right) > 8E[\mu^2] - 4
\]

Simplifying yields

\[
z(r) := 4\left(E[\mu^2]\right)^2 - 1 + 16rc\left(E[\mu^2]\right)^2 - 8rcE[\mu^2] - 16r^2c^2\left(E[\mu^2]\right)^2 > 0.
\]

The derivative of \( z \) with respect to \( r \) will be positive, only if \( 2E[\mu^2] - 1 - 4rcE[\mu^2] > 0 \). For \( r = \hat{r}^p \), the left-hand-side of the inequality is zero. It directly follows that the derivative of \( z \) with respect to \( r \) is negative for \( r > \hat{r}^p \). Since \( z \) is positive for \( r = \hat{r}^p \) (PD achieves the first-best solution) and negative for \( r = \tilde{r}^p \) (under both job designs relational contracts are not available), there must be a cut-off \( \tilde{r} \), with \( \hat{r}^p < \tilde{r} < \tilde{r}^p \), at which the optimal job design changes. This proves part (iii) of Proposition 2.

Proof of Proposition 3:

The proof of part (i) is again obvious and so omitted. It remains to prove
part (ii). Let \( r \in (\hat{r}^c, \check{r}^p) \). Then, under PD, the first-best solution is achieved, while, under CD, it is not. PD is therefore preferred. Further, let \( r > \check{r}^c \). Then, under CD, only formal contracts are available, whereas, under PD, relational contracts may or may not be feasible. As, for \( E [\mu^2] > 1.5 \), PD will be preferred, even if it does not enable relational contracting and as the principal always benefits from the introduction of these contracts, PD must be dominant for \( r > \check{r}^c \). Finally, it must be shown that PD will also be preferred, if \( \hat{r}^p < r \leq \check{r}^c \). In this case, both job designs lead to a mixture of formal and relational contracting. Hence, condition \( E_\pi_{CD,r} > E_\pi_{PD,r} \) is equivalent to 
\[
\left( (E [\mu^2] - 1)(1 + 4rcE [\mu^2]) - 4r^2c^2 (E [\mu^2])^2 \right) 4 (2E [\mu^2] - 1) \text{ strictly exceeding} \left( \frac{(E [\mu^2])^2}{4} (2E [\mu^2] - 1) - 2rcE [\mu^2] \right) (E [\mu^2] - 1).
\]
Simplifying this condition yields \( y(r) := -U (r) - V > 0 \), with
\[
U (r) = 16r^2c^2 (E [\mu^2])^3 + r c \left( 24 (E [\mu^2])^2 - 16 (E [\mu^2])^3 - 8E [\mu^2] \right) \text{ and} \quad V = 4 (E [\mu^2])^3 - 12 (E [\mu^2])^2 + 11E [\mu^2] - 3. \]
The function \( y \) is strictly concave in \( r \). It has two nulls, \( r_1 = \frac{1}{4c(E [\mu^2])^3} \left( -3E [\mu^2] + 2 (E [\mu^2])^2 + 1 + \sqrt{-3E [\mu^2] + 2 (E [\mu^2])^2 + 1} \right) \), \( r_2 = \frac{1}{4c(E [\mu^2])^3} \left( -3E [\mu^2] + 2 (E [\mu^2])^2 + 1 + \sqrt{-3E [\mu^2] + 2 (E [\mu^2])^2 + 1} \right) \). In order to show that PD will perform better than CD, if \( \hat{r}^p < r \leq \check{r}^c \), it suffices to show that the right null \( r_2 \) is smaller than \( \hat{r}^p \). The right null will be smaller than \( \hat{r}^p \), if and only if \( -3E [\mu^2] + 2 (E [\mu^2])^2 + 1 + \sqrt{-3E [\mu^2] + 2 (E [\mu^2])^2 + 1} < 2 (E [\mu^2])^2 - E [\mu^2] \). Rearranging this condition leads to \( 2 (E [\mu^2])^2 - E [\mu^2] > 0 \), which is always fulfilled. This proves part (ii) of Proposition 3.

References

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