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Contractual Incentive Provision and Commitment in Rent-Seeking Contests

Oliver Gürtler*

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*Oliver Gürtler, Department of Economics, BWL II, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, Germany. Tel.:+49-228-739214, Fax:+49-228-739210. oliver.guertler@uni-bonn.de

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Abstract
In this paper, we consider a symmetric rent-seeking contest, where employees lobby for a governmental contract on behalf of firms. The only verifiable information is which firm is assigned the contract. We derive the optimal wage contracts of the employees and analyze, whether commitment by determining the wage contract prior to the competitor is profitable. This is indeed the case, i.e. firms prefer to move first in the wage-setting subgame. This complements previous work on rent-seeking contests emphasizing that commitment via rent-seeking expenditures is unprofitable in symmetric contests.

Key words: Contest, First-Mover Advantage, Commitment, Wage Contract
JEL classification: D72, M52
1 Introduction


In many real-world situations, the person expending resources and the person obtaining the rent are not the same. This is oftentimes the case, when firms are competing e.g. for a contract or a license. Then, employees are usually the ones expending resources, while the firm owners receive the rent. As known from the principal-agent literature, this separation of costs and benefits of rent-seeking activities leads to motivational problems (moral hazard), as the employees are tempted to reduce expenditures to save on costs. To mitigate these problems and to align the interests of employees
with these of firm owners, firms incentivize the employees by rewarding them for good performance.

Despite the multitude of papers dealing with contests, this delegation of rent-seeking activities has not received much attention. An exception is Schoonbeek (2002), (2004), who uses an incomplete contracting approach\(^1\) to analyze delegation of rent-seeking in both, an individual and a group contest. Thereby, he focuses on the question, when delegation of rent-seeking activities is profitable. This will be the case, if (i) either the delegating party is a strongly risk-averse individual and the rent is relatively high or (ii) the delegating party is a relatively large group, which mitigates the free-rider problem by means of delegation.

In this paper, it is supposed that the single firm owners are, e.g. due to time constraints, forced to always delegate the rent seeking activities. Then, they use incentive contracts to motivate their employees. We assume contracting to be complete and maintain Schoonbeek’s assumptions about what information is verifiable to courts. We analyze whether or not contracts can be used as a commitment device. Following the literature on sequential contests, we let one firm choose the contract parameters prior to another. Under symmetric valuations for the rent, it is found that commitment via

\(^1\)Incomplete contracting here means that Schoonbeek restricts the set of feasible contracts. He assumes that a court can only distinguish between whether or not the rent has been awarded to a firm. Therefore, a contract can specify two payments, one for the case of a successful employee and one for the case of an unsuccessful employee. Schoonbeek, however, normalizes this second payment to zero and so forbids the firms to choose from a richer set of contracts.
incentive contracts is profitable. That is, a firm benefits from moving first in the wage-setting subgame. This is surprising since Leininger (1993) has shown that this will not be the case, if commitment occurs via rent-seeking expenditures. This difference in results can be explained as follows: By acting as a first-mover, one can commit to a certain behavior. In a contest, for instance, one can choose an aggressive strategy in order to show the opponent that one is extremely willing to win the contest. Committing to an aggressive behavior via rent-seeking expenditures is very costly, as it requires a high outlay choice. Therefore, it is unprofitable. In contrast, commitment via high-powered incentive contracts is only costly, if the opponent accepts the challenge and reacts by choosing high-powered incentives as well. This, however, will never be the case so that commitment is indeed profitable in the current model.

Besides its implications for the theory of rent-seeking contests, this paper contributes to the literature emphasizing the commitment role of contracts. Examples include Fershtman & Judd (1987), Sklivas (1987), Dewatripont (1988), Segendorff (1998) or Cai & Cont (2004). In the first two papers, contracts are used to commit to a more favorable behavior in an oligopoly game. In Dewatripont (1988), an incumbent signs a contract with a third party (e.g. a labor union) in order to commit to an aggressive strategy, if a potential entrant comes into the market. Finally, the commitment role of contracts in bargaining situations is analyzed in the last two papers. The current paper complements existing ideas by demonstrating that contracts can also be effective commitment devices in rent-seeking contests.
The paper is organized as follows: The next section contains the model description. The model is solved in Section 3. Finally, Section 4 concludes.

2 Description of the model and notation

Consider two firms \((i = 1, 2)\) that are in competition for a governmental contract, which is of value \(S > 0\) to the firm owners, respectively. Each firm employs a risk-neutral "rent-seeker" (who is referred to as the agent) choosing rent-seeking outlay \(x_i\) (measured in monetary terms) in order to influence the government’s decision. Firm \(i\)’s contest-success function, i.e. its probability of being selected is given by (see e.g. Tullock (1980) or, for an axiomatic approach, Skaperdas (1996))

\[
P_i = \begin{cases} \frac{x_i}{x_1 + x_2}, & \text{for } x_1 + x_2 > 0 \\ 0.5, & \text{otherwise} \end{cases}
\]  

(1)

Let \(x_i\) be unobservable. Thus, a firm cannot condition the compensation of the agent on the chosen outlay. Instead, the only verifiable information is which firm is assigned the contract. Hence, a wage contract consists of a pair \((\alpha_{0i}, \alpha_{1i})\), where \(\alpha_{0i}\) denotes a fixed payment from firm to agent and \(\alpha_{1i}\) a further payment that the agent receives, if the firm is selected by the government. The agent is assumed to possess monetary resources \(\bar{w} \geq 0\) and to be unable to get further credit. Hence, the contract parameters must satisfy \(\alpha_{0i} \geq -\bar{w}\).\(^2\) Determining the contract parameters, it is assumed that

\(^2\)In particular, it must be that \(\alpha_{0i} \geq -\bar{w}\) and \(\alpha_{0i} + \alpha_{1i} \geq -\bar{w}\). Note that setting \(\alpha_{1i} < 0\) does not make sense, as this would punish the agent for performing well. Hence \(\alpha_{1i} \geq 0\). Then, the second condition is implied by the first.
the agent possesses complete bargaining power.\footnote{\textsuperscript{3}} This may be due to different reasons: For instance, the agents may simply be the better bargainers. Alternatively, there may be many firms, but only few agents in the market. As a third explanation, one could think that the agent also works in other projects for the firm. If the agent is of great importance for those other projects, he may threat to leave the firm, if his share from achieved revenue is smaller than 1. A consequence of the bargaining power assumption is that each firm is constrained to make zero expected profit in the rent-seeking game and chooses the contract parameters such that its agent’s expected utility is maximized.\footnote{\textsuperscript{4}}

The model consists of two stages: In the first stage, the firms determine the wage parameters, in the second stage, outlays are chosen. As outlays are unobservable, the agents choose their outlays in a Cournot-fashion, that is, an agent is unable to commit to a certain behavior by choosing outlay prior to the other agent. In contrast, wage contracts are supposed to be observable. Therefore, the firms are allowed to use the wage contracts as a commitment device. Following Leininger (1993), in the wage-setting subgame, the order of moves is endogenized. Each firm may choose to announce the contract parameters at two different points in time, say at $t = 1$ or $t = 2$. If both firms choose the same $t$, we have a simultaneous wage-setting subgame. Otherwise, choices are made sequentially.

\footnote{Notice that all results to be derived will remain qualitatively unchanged, if the firms possess complete bargaining power, the agents’ reservation utilities are normalized to zero and $w \geq \frac{5}{4}$.}

\footnote{See, for example, Nalebuff & Stiglitz (1983).}
3 Solution to the model

3.1 A benchmark case

As a benchmark case, we present a model similar to the one of Leininger (1993), where rent-seeking activities are not delegated. In the case of simultaneous outlay choices, firm 1 maximizes

\[ \pi_1 = \frac{x_1}{x_1 + x_2} S - x_1 \]  

(2)

This yields the following first-order condition: \(^5\)

\[ \frac{x_2}{(x_1 + x_2)^2} S - 1 = 0 \]  

(3)

Deriving a similar condition for the second firm shows that equilibrium is symmetric and given by \( x_1 = x_2 = \frac{S}{4} \). Expected profits are also the same and equal \( \pi_1 = \pi_2 = \frac{S}{4} \).

Let us now assume sequential actions, with firm 1 acting prior to firm 2. By choosing a certain outlay, firm 1 can now affect firm 2’s outlay, i.e. firm 1 does no longer take the second firm’s outlay as given, while deciding about \( x_1 \). Firm 2’s best response function follows from maximizing \( \pi_2 \) and equals \( x_2 = \sqrt{x_1 S} - x_1 \). Inserting this function into (2), leads to a profit of \( \pi_1 = \sqrt{x_1 S} - x_1 \) for firm 1. Maximizing this profit, leads to the following first-order condition: \(^6\)

\[ 0.5 \sqrt{\frac{S}{x_1}} - 1 = 0 \Leftrightarrow x_1 = \frac{S}{4} \]  

(4)

\(^5\) The second-order condition is satisfied.

\(^6\) The second-order condition is satisfied.
This implies that the solution is the same as under simultaneous actions. There is thus no first-mover advantage. Firm 1 does not gain by acting prior to firm 2. The next subsections show that this may not be true, if commitment occurs via incentive contracts.

### 3.2 Outlay Choices

The model is solved by backward induction. Thus, we start by deriving the agents’ outlays for given contract parameters. The agent employed by firm 1 chooses his outlay to maximize his expected payoff. This payoff consists of the fixed payment, the variable payment in case of being selected by the government and the costs entailed by outlay. It is given by

\[
EU_1 = \alpha_{01} + \frac{x_1}{x_1 + x_2} \alpha_{11} - x_1
\]

Maximization of (5) yields the subsequent first-order condition:\(^7\)

\[
\frac{x_2}{(x_1 + x_2)^2} \alpha_{11} - 1 = 0
\]

A similar expression can be given for the second firm’s agent. It is obtained from (6) by replacing the numerator by \(x_1\) and \(\alpha_{11}\) by \(\alpha_{12}\). Simultaneous solution of the two conditions leads to outlays given by

\[
x_1 = \frac{(\alpha_{11})^2 \alpha_{12}}{(\alpha_{11} + \alpha_{12})^2}
\]

\[
x_2 = \frac{\alpha_{11} (\alpha_{12})^2}{(\alpha_{11} + \alpha_{12})^2}
\]

Rent-seeking only depends on the variable payments \(\alpha_{11}\) and \(\alpha_{12}\). It is straightforward to show that \(\frac{\partial x_i}{\partial \alpha_{1i}} > 0\). A higher reward for winning the

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\(^7\)The second-order condition is satisfied.
contest leads to higher outlays. On the other hand $\frac{\partial x_1}{\partial \alpha_{11}} > 0$ (respectively, $\frac{\partial x_2}{\partial \alpha_{11}} > 0$), only if $\alpha_{11} > \alpha_{12}$ ($\alpha_{11} < \alpha_{12}$). Intuitively, this means that an increase in the competitor’s reward for winning the contest will increase the other agent’s outlay, only if competition becomes more intense. If, for example, the second agent is more likely to win the contest (as $\alpha_{11} < \alpha_{12}$) and $\alpha_{12}$ is further increased, competition is weakened and the first agent chooses to rent-seek less.

3.3 Simultaneous determination of the wage parameters

We continue by determining the optimal wage parameters. Thereby, we have to analyze both cases, the case of simultaneous and sequential determination of the parameters. In this subsection, we consider the former case. The latter case is dealt with in the next subsection.

We start by deriving the first firm’s best response function. As mentioned before, the firm faces a zero-profit constraint, which implies that

$$\alpha_{01} = \frac{x_1}{x_1 + x_2} (S - \alpha_{11})$$

or, inserting the expressions for rent-seeking outlays, as

$$\alpha_{01} = \frac{\alpha_{11}S}{\alpha_{11} + \alpha_{12}} - \frac{(\alpha_{11})^2}{\alpha_{11} + \alpha_{12}}$$

Using conditions (7), (8) and (10), the agent’s expected utility becomes

$$EU_1 = \frac{\alpha_{11}S}{\alpha_{11} + \alpha_{12}} - \frac{(\alpha_{11})^2 \alpha_{12}}{(\alpha_{11} + \alpha_{12})^2}$$
The firm chooses the wage parameters such that the agent’s expected utility is maximized. Thereby, it has to consider the limited liability constraint. We solve the maximization problem by first considering the unconstrained maximization problem, i.e. by neglecting the limited liability constraint. Thereafter, we show that the optimal solution to this problem indeed satisfies the limited liability constraint. This approach leads to the following first-order condition (follows from differentiating (11)):\(^8\)

\[
\frac{\alpha_{12}}{(\alpha_{11} + \alpha_{12})^2} S - \frac{2\alpha_{11} (\alpha_{12})^2}{(\alpha_{11} + \alpha_{12})^3} = 0 \iff (12) \\
\alpha_{11} - \frac{\alpha_{12} S}{2\alpha_{12} - S} = 0
\]

The last condition characterizes the first firm’s best response function. It is strictly decreasing, i.e. increases in incentive strength of the competitor are followed by a decrease in the own strength of incentives. The second firm’s best response function results from (12) by switching \(\alpha_{11}\) and \(\alpha_{12}\). If the wage contracts are determined simultaneously, the solution to the model lies at the intersection of the two best-response functions. This solution is symmetric and given by \(\alpha_{11} = \alpha_{12} = S\). From (9), it follows that \(\alpha_{01} = 0\). Thus, at the solution the limited liability constraint is fulfilled. Each agent is paid according to a "sell-the-shop-contract". He is made residual claimant to his actions and "pays" an entrance fee equal to zero. Further, equilibrium outlays are given by \(x_1 = x_2 = \frac{S}{4}\). As each agent wins the contest with probability 0.5, the two agent’s expected utilities are the same and equal

\(^8\)The second-order condition requires that \(-(\alpha_{11} + \alpha_{12})S - (\alpha_{12})^2 + 2\alpha_{11}\alpha_{12} < 0\). It holds for all equilibria to be derived.
\[ \pi_1 = \pi_2 = \frac{S}{4} \] Finally, note that aggregate rent-seeking is \( x_1 + x_2 = \frac{S}{2} \), which is the same as in the individual contest, where outlay choices are not delegated to agents. This is not surprising. Delegation of outlay choices to agents leads to a moral-hazard problem, as outlays are unobservable to the firm. As limited liability does not constrain the optimal solution, this moral hazard problem can be solved completely by means of incentive pay. Hence, rent-seeking expenditures are the same as in the case of non-delegation.

### 3.4 Sequential determination of the wage parameters

In the case of sequential choices, denote the first-moving firm as leader and the second-moving firm as follower. Further, suppose, without loss of generality, that firm 1 acts first. The difference between this case and the preceding case is that now the leader can affect the followers choice of \( \alpha_{12} \) by its own choice of \( \alpha_{11} \). In analogy to the argumentation Section 3.1, when determining \( \alpha_{11} \) the leader takes \( \alpha_{12} \) no longer as given. This entails the problem that one cannot be sure, how the follower’s best response function looks like, for this function depends on whether or not the limited liability constraint of the follower’s agent is binding. To derive the equilibrium, we therefore introduce a case distinction. In the first case, the limited liability constraint of the follower’s agent is assumed to be slack so that \( \alpha_{12} = \frac{\alpha_{11} S}{2\alpha_{11} - S} \). In the second case, it is assumed to be binding. The best response function is then derived from the second firm’s zero-profit condition and \( \alpha_{02} = -\bar{w} \). It is given by \( \alpha_{12} = \frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11} \bar{w}} \). Let us start with the first case and suppose additionally that \( \bar{w} > 0 \). Inserting \( \alpha_{12} = \frac{\alpha_{11} S}{2\alpha_{11} - S} \) into (11), the first agent’s
expected utility simplifies to

\[
EU_1 = \frac{S}{2} - \frac{S^2}{4\alpha_{11}}
\]  

(13)

It can immediately be seen that firm 1 wants to set \( \alpha_{11} \) as high as possible. In order to guarantee zero profit for firm 1, \( \alpha_{01} \) then has to become infinitely small. It follows that, as long as \( \bar{w} \) is finite, that is, as long as there is a limited liability constraint, this constraint must be binding so that \( \alpha_{01} = -\bar{w} \). The optimal \( \alpha_{11} \) is then the largest variable payment satisfying the zero-profit constraint. Formally, it is given by

\[
\alpha_{11} = \frac{1.5S + \bar{w}}{2} + \frac{1}{2} \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}
\]  

(14)

We started by assuming that the follower’s best response function is given by \( \alpha_{12} = \frac{\alpha_{12}S}{2\alpha_{11} - S} \). In order to show that we indeed consider an equilibrium, it must be demonstrated that the limited-liability constraint of the follower’s agent is really slack. Inserting (14) into the best-response function of the follower yields

\[
\alpha_{12} = \frac{1.5S^2 + \bar{w}S + S\sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}}{S + 2\bar{w} + 2\sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}}
\]  

(15)

From firm 2’s zero-profit condition, the fixed wage \( \alpha_{02} \) can be shown to equal

\[
\alpha_{02} = \frac{S^2}{\left(1.5S + \bar{w} + \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}\right) \left(0.5S + \bar{w} + \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}\right)}
\]  

(16)

The denominator in (16) is strictly positive. Hence, working with the best-response function \( \alpha_{12} = \frac{\alpha_{11}S}{2\alpha_{11} - S} \) is correct, whenever \( \left(\frac{-0.5S + \bar{w}}{2} + \frac{1}{2} \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}\right) \)
0. This is equivalent to \( \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2} > 0.5S - \bar{w} \), or \( 4\bar{w}S > 0 \), which is always fulfilled for \( \bar{w} > 0 \).

Before turning to the second case, where the follower’s best response function is \( \alpha_{12} = \frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11}\bar{w}} \), notice that, for \( \bar{w} > 0 \), \( \alpha_{11} > S \), \( EU_1 > \frac{S}{4} \) and \( EU_2 = \frac{S^2}{4\alpha_{11}} < \frac{S}{4} \). This means that, under sequential choices, the leader is better off and the follower worse off compared to the model of simultaneous choices. In other words, by setting \( \alpha_{11} = \frac{1.5S + \bar{w}}{2} + \frac{1}{2}\sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2} \), the leader can ensure its agent a payoff higher than \( \frac{S}{4} \). This is important for the second case, which is analyzed next.

Suppose now that the limited liability constraint of the follower’s agent is binding and the best-response function is given by \( \alpha_{12} = \frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11}\bar{w}} \). It is extremely messy to derive the solution in analogy to the approach in the first case. Instead, we derive the following Lemma, which states that the leader always prefers to make the limited liability constraint of the follower’s agent slack rather than binding. It follows that, in equilibrium, the limited liability constraint of the follower’s agent is never binding.

**Lemma 1** For \( \bar{w} > 0 \), the leader’s agent is better off, if the limited liability constraint of the follower’s agent is slack than if it is binding. Thus, the first

\(^9\)Note that this is somewhat critical, if we assume the agents’ bargaining powers to stem from the relative scarcity of rent-seekers. In this case, the follower’s agent is likely to leave the firm. On the other hand, the two remaining reasons for having agents possessing complete bargaining power do not imply this strong result. An agent being a better bargainer than the firm he works in does not necessarily leave the firm, if agents in other firms get a higher payoff. Further, notice that the first-mover advantage and, accordingly, our results would even be enforced, if the follower would lose its agent.
firm always chooses \( \alpha_{11} = \frac{1.5S + \bar{w}}{2} + \frac{1}{2} \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}. \)

**Proof.** See Appendix. ■

From Lemma 1, the following proposition immediately follows:

**Proposition 2** If \( \bar{w} > 0 \), the leader sets

\[
\alpha_{11} = \frac{1.5S + \bar{w}}{2} + \frac{1}{2} \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}
\]

and the follower reacts by setting

\[
\alpha_{12} = \left( \frac{1.5S^2 + \bar{w}S + S \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}}{S + 2\bar{w} + 2\sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}} \right).
\]

The leader’s agent is better off and the follower’s agent is worse off than in the contest with simultaneous actions.

We see that, under sequential contract announcements, the leader’s agent is better off and the follower’s agent worse off compared to the case of simultaneous actions. This complements the findings of Leininger (1993) who demonstrates that contestants are indifferent between moving sequentially or simultaneously, when valuations for the prize are symmetric. Naturally, the question arises, why the results differ. In general, as a first-mover, one can commit to a certain behavior. In a contest, for instance, one can choose an aggressive strategy in order to show the opponent that one is extremely willing to win the contest. Now, compare the two instruments available for committing purposes. In Leininger (1993), the instrument is the chosen outlay. However, committing to an aggressive behavior is then very costly, as it requires a high outlay choice. Therefore, it is unprofitable. In contrast, in the current model, commitment is via high-powered incentive contracts. This will only be costly, if the opponent accepts the challenge and reacts by choosing high-powered incentives as well.\(^{10}\) But as the best-response functions are

\(^{10}\)Recall that \( \frac{\partial \alpha_{11}}{\partial \alpha_{12}} > 0 \iff \alpha_{11} > \alpha_{12}. \)
downward-sloping, the opponent will never do so. Incentive contracts are thus profitable incentive devices so that becoming leader is beneficial.

We conclude this subsection by briefly commenting on the case, where \( \bar{w} = 0 \). In this case, the follower always sets \( \alpha_{12} = S \),\(^{11}\) which implies that \( \alpha_{11} = S \) as well. Hence, the solution is the same as in the case of simultaneous actions. Here, the first-mover advantage of the leader disappears.

### 3.5 The timing of events

As mentioned in Section 2, the order of moves is endogenized in that each firm may choose to announce the contract parameters either at date \( t = 1 \) or \( t = 2 \). The following matrix depicts the agents’ expected utilities for each possible scenario.

<table>
<thead>
<tr>
<th>Firm 1/Firm 2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{S}{T}, \frac{S}{T} )</td>
<td>( Y, Z )</td>
</tr>
<tr>
<td>2</td>
<td>( Z, Y )</td>
<td>( \frac{S}{T}, \frac{S}{T} )</td>
</tr>
</tbody>
</table>

with \( Y = \frac{S}{2} - \frac{S^2}{4\left(1.5\bar{w} + \frac{1}{2}\sqrt{\frac{S^2}{2} + 3\bar{w}S + \bar{w}^2}\right)} \) and \( Z = \frac{S^2}{4\left(1.5\bar{w} + \frac{1}{2}\sqrt{\frac{S^2}{2} + 3\bar{w}S + \bar{w}^2}\right)} \).

Proposition 3 describes the equilibrium order of moves:

**Proposition 3** If \( \bar{w} > 0 \), both firms announce their contract parameters at \( t = 1 \). Thus, the contract announcements occur simultaneously. If \( \bar{w} = 0 \),

\[ \alpha_{12} = \frac{S + \bar{w}}{S + 2\bar{w} + 2\sqrt{\frac{S^2}{2} + 3\bar{w}S + \bar{w}^2}} \] or \( \alpha_{12} = \frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11}\bar{w}} \).

---

\(^{11}\)This follows from inserting \( \bar{w} = 0 \) into the two best response functions \( \alpha_{12} = \frac{S + \bar{w}}{S + 2\bar{w} + 2\sqrt{\frac{S^2}{2} + 3\bar{w}S + \bar{w}^2}} \) or \( \alpha_{12} = \frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11}\bar{w}} \).
the firms are indifferent between announcing the contract parameters at \( t = 1 \) or \( t = 2 \). Hence, there may be either simultaneous or sequential play.

Proof. Obvious from the payoff matrix and therefore omitted.

For \( \bar{w} > 0 \), there is a first mover advantage. Each firm prefers to act as a leader to acting simultaneously and the latter to acting as a follower. It is thus a dominant strategy for the firms to announce their contract parameters at date \( t = 1 \). This necessarily leads to simultaneous play. Note that this is exactly the logic that Leininger (1993) has shown to be incorrect, if outlay is the only commitment device. Further, for \( \bar{w} = 0 \), the first mover advantage disappears and the firms do no longer care about the order of moves.

4 Concluding remarks

In this paper, a rent-seeking contest was considered, where agents spend resources on behalf of firms in order to influence the government’s decision, which firm to assign a contract. The agents are rewarded for success, that is, for attracting the contract. The main focus of the paper was on, whether commitment by determining the wage contract prior to the competitor is profitable. It was found that this is indeed the case. As a result, firms have an interest to move first, and competition for the first-mover advantage leads to simultaneous choices. This complements the findings by Leininger (1993), who shows that, in symmetric contests, commitment via rent-seeking expenditures is unprofitable.

Finally, it should be emphasized that there are many real-world situations,
where the person expending resources is not the same as the person obtaining the rent. In these cases, the rent-seekers have to be offered incentive contracts to engage in rent-seeking. In this paper, we tried to join the two fields of contest and contract theory by introducing a moral hazard problem into a contest model. This, however, was only a first step. Many exciting problems such as e.g. the screening of certain types of "rent-seekers" await.

Appendix

In this Appendix, we prove Lemma 1. It says that, for all \( \alpha_{11} \geq 0 \) and \( \bar{w} > 0 \), \( EU_{1}^{nb} > EU_{1}^{b} \), where

\[
EU_{1}^{nb} = \frac{\alpha_{11} S}{\alpha_{11} + \alpha_{12}} - \frac{(\alpha_{11})^{2} \alpha_{12}}{(\alpha_{11} + \alpha_{12})^{2}} \quad \text{with} \quad \alpha_{12} = \frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11} \bar{w}}.
\]

This is equivalent to \( Z(\bar{w}, \alpha_{11}) > 0 \), with \( Z(\bar{w}, \alpha_{11}) = EU_{1}^{nb} - EU_{1}^{b} \). \( EU_{1}^{nb} \) is clearly increasing in \( \bar{w} \). Thus, if \( EU_{1}^{b} \) is (weakly) decreasing in \( \bar{w} \), \( Z(\bar{w}, \alpha_{11}) \) is increasing in \( \bar{w} \). Consider the derivative

\[
\frac{\partial EU_{1}^{b}}{\partial \bar{w}} = -\frac{\alpha_{11} S \frac{\partial \alpha_{12}}{\partial \bar{w}}}{(\alpha_{11} + \alpha_{12})^{2}} - \frac{(\alpha_{11})^{3} \frac{\partial \alpha_{12}}{\partial \bar{w}} - (\alpha_{11})^{2} \alpha_{12} \frac{\partial \alpha_{12}}{\partial \bar{w}}}{(\alpha_{11} + \alpha_{12})^{3}}
\]

As \( \frac{\partial \alpha_{12}}{\partial \bar{w}} > 0 \), this derivative is (weakly) negative, if \( (\alpha_{11})^{2} - \alpha_{11} \alpha_{12} + S(\alpha_{11} + \alpha_{12}) \geq 0 \), or \( (\alpha_{11} - S)(\alpha_{11} - \alpha_{12}) + 2 \alpha_{11} S \geq 0 \). Suppose, for the moment, that \( \alpha_{11} \leq S \) and \( \alpha_{11} \leq \alpha_{12} \). Then, \( Z(\bar{w}, \alpha_{11}) \) is increasing in \( \bar{w} \). Note that \( Z(0, \alpha_{11}) = \frac{S}{4} - \frac{\alpha_{11} S^{2}}{(\alpha_{11} + S)^{2}} \). If \( Z(0, \alpha_{11}) \geq 0 \), then \( Z(\bar{w}, \alpha_{11}) > 0 \), for all \( \bar{w} > 0 \). \( Z(0, \alpha_{11}) \geq 0 \) is equivalent to \( (\alpha_{11} + S)^{2} \geq 4 \alpha_{11} S \), which, using the second binomial, can be shown to always hold.

We have shown that, for \( \alpha_{11} \leq S \) and \( \alpha_{11} \leq \alpha_{12} \), \( EU_{1}^{nb} \) strictly exceeds \( EU_{1}^{b} \). To complete the proof of Lemma 1, we need to show that, in equilibrium, it will never be the case that \( \alpha_{11} > S \) or \( \alpha_{11} > \alpha_{12} \). We start with
\( \alpha_{11} > \alpha_{12} \). Divide the case, where the limited liability constraint of the follower’s agent binds into two subcases. In the first subcase, the limited liability constraint of the leader’s agent binds as well. Then, it must be the case that \( \alpha_{11}S - (\alpha_{11})^2 = \alpha_{12}S - (\alpha_{12})^2 \). From this condition, it follows that either \( \alpha_{11} = \alpha_{12} \) contradicting \( \alpha_{11} > \alpha_{12} \), or \( \alpha_{11} \neq \alpha_{12} \) and \( \alpha_{11} + \alpha_{12} = S \). Using \( \alpha_{12} = \frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11} \bar{w}} \), \( \alpha_{11} > \alpha_{12} \) can be rewritten as \( \alpha_{11} > S + 2\bar{w} \). Therefore, it can never be the case that \( \alpha_{11} > \alpha_{12} \) and \( \alpha_{11} + \alpha_{12} = S \) together hold. In the second subcase, the limited liability constraint of the leader’s agent is slack, hence \( \alpha_{01} > -\bar{w} \). This implies that the following two conditions are met:

\[
\begin{align*}
\frac{\alpha_{11}}{\alpha_{11} + \alpha_{12}}S - \frac{(\alpha_{11})^2}{\alpha_{11} + \alpha_{12}} &> -\bar{w} \\
\frac{\alpha_{12}}{\alpha_{11} + \alpha_{12}}S - \frac{(\alpha_{12})^2}{\alpha_{11} + \alpha_{12}} &= -\bar{w}
\end{align*}
\]

Combining these conditions offers the set of parameters, for which the first limited liability constraint is slack and the second is binding. This set is given by

\[ A = \{\alpha_{11}, \alpha_{12}| \alpha_{11} > \alpha_{12} \wedge \alpha_{11} + \alpha_{12} < S \vee \alpha_{11} < \alpha_{12} \wedge \alpha_{11} + \alpha_{12} > S\} \]

Analogously to the first subcase, it can never be that \( \alpha_{11} > \alpha_{12} \) and \( \alpha_{11} + \alpha_{12} < S \) together hold, which proves that \( \alpha_{11} \leq \alpha_{12} \).

It remains to demonstrate that it is never optimal to set \( \alpha_{11} > S \). First, we show that \( x_1 = \frac{(\alpha_{11})^2\alpha_{12}}{(\alpha_{11} + \alpha_{12})^2} \) is strictly increasing in \( \alpha_{11} \). Differentiating \( \frac{(\alpha_{11})^2\alpha_{12}}{(\alpha_{11} + \alpha_{12})^2} \) with respect to \( \alpha_{11} \) yields

\[
\frac{\partial}{\partial \alpha_{11}} \left( \frac{(\alpha_{11})^2\alpha_{12}}{(\alpha_{11} + \alpha_{12})^2} \right) = \frac{2\alpha_{11}(\alpha_{12})^2 + (\alpha_{11})^3 \frac{\partial \alpha_{12}}{\partial \alpha_{12}} - (\alpha_{11})^2 \alpha_{12} \frac{\partial \alpha_{12}}{\partial \alpha_{11}}}{(\alpha_{11} + \alpha_{12})^3}
\]
This derivative is positive, if \(2(\alpha_{12})^2 + (\alpha_{11})^2 \frac{\partial \alpha_{12}}{\partial \alpha_{11}} > \alpha_{11} \alpha_{12} \frac{\partial \alpha_{11}}{\partial \alpha_{11}}\). Recall that 
\[
\alpha_{12} = \frac{s + \bar{w}}{2} + \sqrt{\left(\frac{s + \bar{w}}{2}\right)^2 + \alpha_{11} \bar{w}},
\]
and, hence, 
\[
\frac{\partial \alpha_{12}}{\partial \alpha_{11}} = \frac{\bar{w}}{2\sqrt{(\frac{s + \bar{w}}{2})^2 + \alpha_{11} \bar{w}}}.\]
Then, the inequality changes to 
\[
2\left(\frac{s + \bar{w}}{2} + \sqrt{\left(\frac{s + \bar{w}}{2}\right)^2 + \alpha_{11} \bar{w}}\right)^2 + (\alpha_{11})^2 \frac{\bar{w}}{2\sqrt{(\frac{s + \bar{w}}{2})^2 + \alpha_{11} \bar{w}}}
\]
\[
> \alpha_{11} \left(\frac{s + \bar{w}}{2} + \sqrt{\left(\frac{s + \bar{w}}{2}\right)^2 + \alpha_{11} \bar{w}}\right) \frac{\bar{w}}{2\sqrt{(\frac{s + \bar{w}}{2})^2 + \alpha_{11} \bar{w}}},
\]
After some calculations, the inequality becomes 
\[
((S + \bar{w})^2 + 1.5\alpha_{11} \bar{w}) \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + \alpha_{11} \bar{w}} + (S + \bar{w})^3
\]
\[
> -1.75\alpha_{11} (S + \bar{w}) \bar{w} - \frac{(\alpha_{11})^2 \bar{w}}{2}
\]
which is clearly fulfilled. Hence, 
\[
\frac{\partial (\alpha_{11}^2 \alpha_{12})}{\partial \alpha_{11}} > 0.
\]
It follows that, with \(\alpha_{11} > S\), \(EU^b_1 \leq \frac{S}{2} - \frac{s^2 \left(\frac{s + \bar{w}}{2} + \sqrt{\left(\frac{s + \bar{w}}{2}\right)^2 + \bar{w}}\right)}{\left(\frac{3s + \bar{w}}{2} + \sqrt{\left(\frac{3s + \bar{w}}{2}\right)^2 + \bar{w}}\right)}\). Thus, 
if \(EU^{nb}_1 > \frac{S}{2} - \frac{s^2 \left(\frac{s + \bar{w}}{2} + \sqrt{\left(\frac{s + \bar{w}}{2}\right)^2 + \bar{w}}\right)}{\left(\frac{3s + \bar{w}}{2} + \sqrt{\left(\frac{3s + \bar{w}}{2}\right)^2 + \bar{w}}\right)}\), \(EU^{nb}_1\) always exceeds \(EU^b_1\). \(EU^{nb}_1 > \frac{S}{2} - \frac{s^2 \left(\frac{s + \bar{w}}{2} + \sqrt{\left(\frac{s + \bar{w}}{2}\right)^2 + \bar{w}}\right)}{\left(\frac{3s + \bar{w}}{2} + \sqrt{\left(\frac{3s + \bar{w}}{2}\right)^2 + \bar{w}}\right)}\) is equivalent to 
\[
\left(3S + 2\bar{w} + 2\sqrt{\frac{s^2}{4} + 3\bar{w}S + \bar{w}^2}\right) \left(\frac{S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + S\bar{w}}\right)
\]
\[
> \left(\frac{3S + \bar{w}}{2} + \sqrt{\left(\frac{S + \bar{w}}{2}\right)^2 + S\bar{w}}\right)^2
\]
Define \(X := \frac{s^2}{4} + 3\bar{w}S + \bar{w}^2\) and \(Y := (\frac{S + \bar{w}}{2})^2 + S\bar{w}\) and note that \(X > Y\). The inequality can be rewritten as 
\[
S\bar{w} + 0.75 \bar{w}^2 + \bar{w}\sqrt{Y} + \sqrt{X(S + \bar{w}) + 2\sqrt{XY}} > 0.75S^2 + Y
\]
Notice that $\sqrt{XY} > Y$. Further, $\sqrt{X} (S + \bar{w}) + \sqrt{XY} > 0.75S^2$. Hence, even for $\alpha_{11} > S$, $EU_{1}^{ub}$ always exceeds $EU_{1}^{b}$. Therefore, firm 1 never prefers to make the limited liability constraint of the second firm’s agent binding. As it can guarantee an equilibrium, where this constraint is slack, by choosing $\alpha_{11} = \frac{1.5S + \bar{w}}{2} + \frac{1}{2} \sqrt{\frac{S^2}{4} + 3\bar{w}S + \bar{w}^2}$, it will always do so. Q.E.D.

References


