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Abstract

This paper revisits the standard analysis of licensing a cost reducing innovation by an outside innovator to a Cournot oligopoly. We propose a new mechanism that combines elements of a license auction with royalty licensing by granting the losers of the auction the option to sign a royalty contract. The optimal new mechanism eliminates the losses from exclusionary licensing without reducing bidders' surplus; therefore, it is more profitable than both standard license auctions and pure royalty licensing. We also take into account that the number of licenses must be an integer, which is typically ignored in the literature.

JEL classifications: D21, D43, D44, D45.

Keywords: Patents, Licensing, Auctions, Royalty, Innovation, R&D, Mechanism Design.

1. INTRODUCTION

This paper revisits the standard analysis of licensing an outside innovator's cost reducing innovation to a Cournot oligopoly. We propose a simple new mechanism that combines elements of a fixed–fee license auction with royalty licensing in a particular way. This new mechanism is more profitable than the standard solutions evaluated in the literature such as fixed–fee license auctions, fixed–fee licensing, royalty licensing, and fixed–fee combined with royalty licensing (see Kamien and Tauman, 1984, 1986, Katz and Shapiro, 1985, 1986).

The key feature of the proposed mechanism is that it grants the losers of the license auction the option to sign a royalty contract. Like in the standard auction, the innovator auctions a restricted number of fixed–fee licenses; but, after the auction, he also grants the losers of the auction the right to sign a royalty license contract.

In equilibrium, the innovator sets the royalty rate equal to the marginal cost reduction induced by using the innovation. As a result, the royalty licensing granted in the second stage, after the auction, has no effect on equilibrium bids since losers of the auction have the same payoff functions as if no royalty option had been granted. Furthermore, in equilibrium the number of auctioned licenses is such that no loser is crowded out of the market. Thus, royalty income is collected and superiority is achieved.

Our analysis also takes into account that the number of licenses must be an integer. Recently, Sen (2005) showed that this integer constraint can make royalty contracts superior to the standard license auction, contrary to the ranking alleged in the literature. However, as we show, accounting for that integer constraint does not affect the superiority of the proposed new mechanism relative to both the standard license auction and royalty licensing.

The literature on patent licensing in oligopoly has branched out in various directions. Sen and Tauman (2005) combined a license auction with royalty licensing in the form of two-part tariffs, under complete information. Wang (1998) and Kamien and Tauman (2002) analyzed the licensing problem from the perspective of an innovator who is also an incumbent player in the downstream product market. While an outside innovator is only interested in licensing income, an "inside" innovator must also take into account how giving access to his innovation affects his downstream profit. Muto (1993), Hernández-Murillo and Llobet (2006) dealt with other market organizations such as Bertrand competition with product differentiation in lieu of the Cournot competition assumed here. And Beggs (1992), Gallini and Wright (1990), Macho-Stadler and Pérez-Castrillo (1991) examined the benefits of royalty licensing either as a screening device in the face of incomplete information concerning the users' willingness to pay for the innovation or as a signaling device if the innovator has superior information concerning the cost reductions induced by his innovation.

The licensing policy proposed in the present paper is obviously discriminatory because different buyers pay different prices for the use of the same innovation. This raises the question: is that kind of discrimination employed in industry and is it compatible with antitrust law? Unfortunately, the empirical literature on licensing practices does not provide sufficient evidence to fully address this issue. A widely cited study of 37 U.S. firms observes that “A down payment with running royalties method was used 46% of the time, while straight royalties and paid-up licenses accounted for 39% and 13%, respectively. Other forms of compensation such as periodic lump sum payments, cross licensing, stock equities and royalty free licenses, although mentioned, were used an insignificant portion of the time (2%)” (Rostoker, 1984, p.64). This finding is often interpreted as proving the predominance of royalty licensing. However, that study also reports that the same innovator often employs different licensing schemes, possibly for licensing the same innovation to different customers.

Moreover, casual evidence suggests this kind of discrimination is widely used in software licensing and in the sale of innovative products. A case in point is the “Original Equipment Manufacturer (OEM) Licensing” where PC manufacturers are sometimes given a choice between a “one-time paid-up” license, which entitles the manufacturer to unlimited distribution of the software within a specified time period, and a per copy royalty license.

Similarly, new products are often sold to some users for unrestricted use while others are offered a leasing contract which is effectively a royalty licensing scheme. The only difference between these arrangements and the one proposed here is that customers are typically given a free choice between these two arrangements, whereas the proposed policy assumes that the innovator limits that choice by offering a restricted number of one-time paid-up licenses.

The plan of the paper is as follows. In Section 2. we state the licensing problem as a sequential game and introduce basic assumptions. Section 3. summarizes some general properties of the equilibrium, and Section 4. examines the superiority of the proposed mechanism in a fairly general framework. In Section 5. we specialize and consider the linear model that is assumed in a large part of the literature. This allows us to give an explicit solution of the optimal mechanism and to strengthen our results. Finally, Section 6. outlines some directions for further research.

2. THE MODEL

There are $n \geq 2$ firms with the linear cost function $C_i(q_i) := cq_i$, $c > 0$, and the inverse demand function $P(Q)$ with $Q := \sum_{i=1}^{n} q_i$. They play a Cournot game.

An outside innovator owns a patented innovation that reduces the marginal cost from c to $c - \epsilon$ with $c > \epsilon > 0$. The innovator can permit the use of that innovation by auctioning fixed–fee licenses or by offering royalty license contracts.

Throughout this text we employ the usual notion of a drastic vs. non–drastic innovation. An innovation is drastic if its exclusive use by one firm propels monopolization. Every innovation induces a natural oligopoly of a certain size, denoted by K, in the sense that if K or more firms operate with the new technology (at marginal cost $c - \epsilon$), all firms with marginal cost c exit, i.e. their equilibrium output is equal to zero. In this text we assume that the innovation is non–drastic in the sense that $K > 1$.[1]

The following stage game is played: the innovator chooses a licensing mechanism; then firms play that mechanism as a noncooperative game; finally, firms play a Cournot market game under complete information, after having observed the outcome of the previous play, knowing who gained access to the innovation and how.

We introduce the modified license auction $G := (k, r)$, which is a general class of mechanisms that includes both the fixed-fee license auction, G_A, and (linear) royalty licensing, G_R, as special cases. In these mechanisms the innovator sells k fixed-fee licenses in a first-price auction, possibly with a minimum bid (which is needed if $k = n$) and gives those firms who do not acquire a license the option to sign a linear royalty contract with the royalty rate per output unit $r > 0$. The number of licenses k must be an integer (which makes a difference but is typically ignored in the literature).

Evidently, if $r > \epsilon$, no firm will exercise the royalty option. Therefore, if $r > \epsilon$, the mechanism G is equivalent to the standard license auction G_A analyzed by Kamien (1992) and others. And if $k = 0$ it is equivalent to the standard royalty licensing G_R.

Throughout our analysis, the inverse market demand function P satisfies the following assumptions:[2]

Assumption 1 *The market demand function $Q(p)$ is strictly decreasing and continuously differentiable for $p > 0$, and its price elasticity, $\eta(p)$, is non-decreasing in p. Moreover, $P(Q)Q$ is strictly concave in Q and $P(0) > c$, and $P(Q) = 0$ for all $Q \geq \bar{Q} > 0$ (satiation point).*

In the following we refer to a fixed–fee license as a "license" and to those firms who obtain a license as "licensees". Depending upon the context, a non–licensee may either have a royalty contract or no access to the innovation.

3. Basic properties of the game

The equilibrium concept is that of a subgame perfect Nash equilibrium which is found by backward induction.

[1] The notation is borrowed from Kamien (1992). The case of drastic innovation, $K \leq 1$, is trivial. There, the innovation induces a natural monopoly where issuing one fixed–fee license is optimal.

[2] These assumptions are similar to those employed in Kamien, Oren, and Tauman (1992).

COURNOT SUBGAMES The Cournot subgame is played between licensees (L) and non–licensees (N). All non–licensees have been offered a royalty contract. We look at the particular subgames where all k fixed–fee licenses have been bought and all $n - k$ non–licensees have accepted the royalty contract if $r \leq \epsilon$ and no royalty contract has been signed if $r > \epsilon$. Signing a royalty contract changes a non–licensee's unit cost from c to $c - \epsilon + r$.

Depending upon how many fixed–fee licenses have been sold, in equilibrium either all non–licensees are crowded out or coexist and produce positive outputs. The critical level of k above which all non–licensees are crowded out depends upon their effective unit cost, $c - \epsilon + r$. We denote it by $\mathcal{K}(r)$, and mention that for $r = \epsilon$ issuing $\mathcal{K}(\epsilon) = K$ licenses establishes a natural oligopoly of size K.

Using the measure $\mathcal{K}(r)$ it follows that all firms, licensees L and non–licensees N alike, will coexist in the Cournot market for all $k \in I_{LN}$ whereas only licensees play that game for all $k \in I_L$, where

$$I_{LN} := \{k \mid 1 \leq k \leq n - 1 \text{ and } k < \mathcal{K}(r)\}\,, \tag{1}$$
$$I_L := \{k \mid k \geq \mathcal{K}(r) \text{ or } k = n\}\,. \tag{2}$$

We denote the equilibrium Cournot quantities and profits of licensees (L) and non–licensees (N) by $q_L(k,r), q_N(k,r), \pi_L(k,r), \pi_N(k,r)$. Note that for $r \geq \epsilon$ all non–licensees have an effective unit cost equal to c (since a contract with $r > \epsilon$ is never accepted), as in the standard license auction game G_A, without royalty contract option, studied by Kamien (1992), Kamien, Oren, and Tauman (1992) and others.

LICENSING SUBGAMES Now consider the licensing subgames. The "value of a license", $v(k,r)$, is the difference between the operating profits of a licensee and a non–licensee. Thereby, one must distinguish between $k < n$ and $k = n$. If $k < n$ a bidder cannot unilaterally influence how many firms will be licensed; whereas if $k = n$, a firm can reduce the number of licenses by not bidding. Therefore,

$$v(k,r) = \begin{cases} \pi_L(k,r) - \pi_N(k,r) & \text{if} \quad k \leq n-1 \\ \pi_L(n,r) - \pi_N(n-1,r) & \text{if} \quad k = n. \end{cases} \tag{3}$$

Suppose $k = n$. If a bidder unilaterally abstains from bidding, he thus reduces the number of licensees to $n - 1$. This either crowds him out ($\pi_N(n-1,r) = 0$), which occurs if $n - 1 \geq \mathcal{K}(r)$, or allows him to earn a positive profit as a royalty contractor ($\pi_N(n-1,r) > 0$), if $n - 1 < \mathcal{K}(r)$. Therefore, in the following we partition the set I_L into

$$I_{L^-} := \{k \mid (k \leq n-1, k \geq \mathcal{K}(r)) \text{ or } (k = n, n-1 \geq \mathcal{K}(r))\} \tag{4}$$
$$I_{L^+} := \{k \mid k = n, n-1 < \mathcal{K}(r)\}\,. \tag{5}$$

Obviously, if $k = n$, the auction can only generate revenue if the innovator has set an appropriate minimum bid, because otherwise firms can buy a license with a zero bid. Whereas, if $k < n$, a minimum bid serves no purpose. Therefore, we assume that the innovator has set a minimum bid equal to $v(n,r)$ if $k = n$ (as in Kamien (1992)).

We stress that in the modified license auction G the bid functions are the same as in the standard license auction G_A if $r \geq \epsilon$. This follows immediately from the fact that for $r > \epsilon$ the two mechanisms are equivalent, and for $r = \epsilon$ the Cournot subgames are the same, because non–licensees' effective unit cost is equal to c in both environments, and therefore the value of a license is the same in both G and G_A.

We also mention that for the same number of licenses k and the royalty rate $r = \epsilon$ the innovator earns the same fixed–fee license income in both G and G_A. However, the innovator may earn royalty income in G but not in G_A. Therefore, under these conditions G is Pareto superior to G_A.

4. SUPERIORITY OF THE MODIFIED LICENSE AUCTION

The optimal modified license auction $G^* := (k^*, r^*)$ is defined as the maximizer of the innovator's payoff

$$\Pi(k,r) := kv(k,r) + (n-k)rq_N(k,r), \tag{6}$$

and G_R^*, G_A^* are similarly defined as maximizers subject to the constraint $k = 0$, resp. $r > \epsilon$.

Since the standard fixed–fee license auction and royalty licensing are special cases of $G = (k,r)$ one can immediately rank G relative to G_A and G_R, as follows:

PROPOSITION 1 *The optimal modified license auction is weakly more profitable than both optimal royalty licensing and the optimal license auction:*[3] $\Pi^* \geq \max\left\{\Pi^{A^*}, \Pi^{R^*}\right\}$.

The remaining task is to examine whether the ranking of the innovator's profit can be strengthened especially if one accounts for the fact that the number of licenses is an integer.

A key result of the literature is that for an outside innovator the optimal license auction, G_A^*, is more profitable than royalty licensing, G_R. Recently, Sen (2005) qualified this result by showing that G_R can be more profitable than G_A^* if one takes into account that k is integer constrained. However, as we now show:

PROPOSITION 2 *The optimal modified license auction is strictly more profitable than optimal royalty licensing:* $\Pi^* > \Pi^{R^*}$.

[3]Note: Fixed-fee licensing is a special case of a license auction (obtained by setting $k = n$). Therefore, the stated mechanisms also dominate fixed–fee licensing.

PROOF Consider royalty licensing at the rate $r \in (0, \epsilon]$ (royalty rates greater than ϵ are never accepted). We prove the assertion by showing that the particular modified license auction $(1, r)$ that issues one license and offers the royalty rate r is more profitable for the innovator.

Denote firms' equilibrium outputs under royalty licensing and the modified license auction by q_R resp. (q_L, q_N), the associated aggregate outputs by $Q_R := nq_R$, $Q_M := q_L + (n-1)q_N$, and the equilibrium prices by p_R, p_M. Then, the innovator's profit is

$$\begin{aligned}\Pi(1,r) &= (p_M - c + \epsilon)\, q_L - (p_M - c + \epsilon - r)\, q_N + r(n-1)q_N \\ &= (p_M - c + \epsilon - r)\,(q_L - q_N) + rQ_M \\ &> rQ_M > rQ_R = \Pi(0,r).\end{aligned}$$

The first inequality follows from three facts: 1) the innovation is non–drastic and therefore the one licensee cannot crowd out other firms which assures that the Cournot equilibrium price p_M remains above the marginal cost c, $p_M > c$; hence, royalty income is generated; 2) $\epsilon \geq r$; 3) $q_L > q_N$ because the licensee has lower marginal cost. To understand the second inequality, note that both regimes induce an n-firms oligopoly, where one firm has lower marginal cost in the modified license auction, which gives rise to a higher aggregate output, as we show in detail in the Appendix. □

While the above result is unaffected by the integer constraint concerning k, the latter may upset the strict superiority of G^* relative to G_A^*.

PROPOSITION 3 *The optimal modified license auction, G^*, is strictly more profitable than $G_A^* := (k_A^*, r_A^*)$, with $r_A^* > \epsilon$, if 1) k is not integer constrained and if 2) k is integer constrained and $k_A^* < K$.*

PROOF 1) Consider the mechanism (k_A^*, ϵ) for which obviously $\Pi(k_A^*, \epsilon) \geq \Pi^{A^*}$, because switching from (k, r) with $r > \epsilon$ (which is the mechanism G_A) to (k, ϵ) does not affect the license income, $k(\pi_L - \pi_N)$. We show that it can be improved by reducing k below k_A^*, so that $\Pi^* > \Pi^{A^*}$.

Compute the left partial derivative of the innovator's profit with respect to k, evaluated at $k = k_A^*$, and one finds for $r = \epsilon$

$$\begin{aligned}\left.\frac{\partial \Pi}{\partial k}\right|_{k=k_A^*} &= \left.\frac{\partial}{\partial k}\left(k(\pi_L - \pi_N)\right)\right|_{k=k_A^*} + \left.\frac{\partial}{\partial k}\left(\epsilon(n-k)q_N\right)\right|_{k=k_A^*} \\ &= \left.\frac{\partial \Pi^A}{\partial k}\right|_{k=k_A^*} + \left.\frac{\partial}{\partial k}\left(\epsilon(n-k)q_N\right)\right|_{k=k_A^*} < 0.\end{aligned}$$

By definition of k_A^*, the first part of the RHS of the last equation is equal to zero, and the second part is negative since $(n-k)q_N$ is obviously decreasing in k. This proves the inequality, and it follows immediately that $\Pi^* > \Pi^{A^*}$.

2) Now we assess what is changed due to the integer constraint. Note that generically K is not an integer. Therefore, one has either $k_A^* < K$ or $k_A^* > K$.

If $k_A^* < K$ the mechanism (k_A^*, ϵ) generates the same license income as G_A^* yet adds positive royalty income; therefore, $\Pi^* > \Pi^{A*}$.

If $k_A^* > K$, the innovator's profit can only be higher under G if $k < k_A^*$, because otherwise there is no royalty income. However, one cannot reduce by less than 1 unit, which may be too much to be profitable. □

In the following we specialize and assume the linear model that is typically employed in the license auction literature (see Kamien, 1992, Kamien and Tauman, 1984, 1986). In that framework the optimal modified license auction can be solved explicitly, and Proposition 3 can be strengthened to the strict superiority of G^*.

5. THE MODIFIED LICENSE AUCTION IN THE STANDARD LINEAR MODEL

The literature on patent licensing typically assumes linear market demand $P(Q) := a - Q$ with $a > c > 0$. We now solve G^* for that linear model and show that Proposition 3 can be strengthened.

COURNOT SUBGAME For $r \le \epsilon$ the equilibrium outputs, size of the natural oligopoly $\mathcal{K}(r)$, and operating profits, π_L, π_N, are

$$q_L(k,r) = \begin{cases} \frac{(K+1)\epsilon + r(n-k)}{n+1} & \text{if} \quad k \in I_{LN} \\ \frac{(K+1)\epsilon}{k+1} & \text{if} \quad k \in I_L \end{cases} \tag{7}$$

$$q_N(k,r) = \begin{cases} \frac{(K+1)\epsilon - r(k+1)}{n+1} & \text{if} \quad k \in I_{LN} \quad \text{or} \quad k = 0 \\ 0 & \text{if} \quad k \in I_L \end{cases} \tag{8}$$

$$\mathcal{K}(r) = \frac{(K+1)\epsilon - r}{r}, \quad K := \frac{a-c}{\epsilon} = \mathcal{K}(\epsilon) \tag{9}$$

$$\pi_i(k,r) = q_i(k,r)^2, \qquad i \in \{L, N\}. \tag{10}$$

And for $r > \epsilon$ one has $q_i(k,r) = q_i(k,\epsilon)$, $i \in \{L,N\}$, $\mathcal{K}(r) = K$. (Of course, if $k = 0$, all firms are non–licensees.)

We do not explicitly solve the other subgames in which either not all licenses were sold or some losers failed to sign a royalty contract. Evidently, being a licensee is more profitable than being a non–licensee for all k. Similarly, non–licensees are never worse–off if they sign the royalty contract. Therefore, these subgames are not encountered by rational players.[4]

[4] The case $r = \epsilon$ is special. There, the licensing subgame has several equilibria, one where all non–licensees sign the royalty contract and others were some do not sign. However, only the equilibrium where all non–licensees sign are part of the overall equilibrium of the game that includes the innovator's choice of mechanism.

LICENSING SUBGAME The unique equilibrium strategy is to bid the value of a license: $b(k,r) = v(k,r)$. If $k < n$, then $\pi_N(k,r)$ is equal to zero for $k \geq \mathcal{K}(r)$ and positive for $k < \mathcal{K}(r)$. If $k = n$, $\pi_N(k-1,r)$ is equal to zero for $n-1 \geq \mathcal{K}(r)$ and positive otherwise. Using these facts, one can easily compute $v(k,r)$ and hence the asserted equilibrium strategy, using the equilibrium profits of the Cournot subgame (10) together with the value of the innovation defined in (3). Therefore, for $r \leq \epsilon$ one obtains the following equilibrium bid function

$$b(k,r) = \begin{cases} \frac{r^2(n-2k-1)+2r\epsilon(K+1)}{n+1} & \text{if} \quad k \in I_{LN} \\ \left(\frac{(K+1)\epsilon}{k+1}\right)^2 & \text{if} \quad k \in I_{L^-} \\ \frac{nr(2\epsilon(K+1)-nr)}{(n+1)^2} & \text{if} \quad k \in I_{L^+}. \end{cases} \tag{11}$$

For $r > \epsilon$ one has $b(k,r) = b(k,\epsilon)$.

THE OPTIMAL MECHANISM

PROPOSITION 4 *In the linear model $G^* = (k^*, r^*)$ is unique, at least one and at most $n-1$ firms are awarded a fixed-fee license, all others a royalty contract, and no firm is crowded out. Specifically, $r^* = \epsilon$ and*

$$k^* = \begin{cases} Round\left(\frac{K+1}{2}\right) & 1 < K < 2n-3 \\ n-1 & K > 2n-3, \end{cases} \tag{12}$$

where "Round" means rounding to the nearest integer.

The proof is in the Appendix.

Since G^* always generates positive royalty income (which is absent in G_A^*), G^* is unique, and G_A is a special case of G, we conclude immediately:

COROLLARY 1 *In the linear model, the optimal modified license auction is strictly more profitable than the optimal license auction, $\Pi^* > \Pi^{A^*}$.*

6. DISCUSSION

We close with a sketch of some interesting extensions for further research. The purpose of these extensions is to assess whether the proposed mechanism can be expected to perform well in a variety of circumstances.

The literature has suggested that the use of pure royalty licensing can be justified by uncertainty concerning the success of the innovation. This is due to the fact that royalty licensing entails a sharing of that risk between innovator and licensees. In this regard, the proposed mechanism could perform even better than pure royalty licensing. If firms have different degrees of risk aversion, the more risk averse firms would tend to lose the auction and then exercise the royalty licensing option. And the less risk averse firms would tend to win one of the fixed–fee licenses in the auction. In this way, the proposed mechanism would allow the innovator to gain from price discrimination between firms with different degrees of risk aversion.

In a recent paper, Sen and Tauman (2005) combined a license auction with royalty licensing by assuming that the innovator employs a two-part tariff, under complete information. It seems that adding royalty licensing to the losers of the auction is even better. This suggests that one should combine the auctioning of a limited number of royalty licenses, with a given royalty rate, with the pure royalty licensing option proposed in the present paper.

Aoki and Tauman (2001) have explored how spillovers affect the optimal license auction. Spillovers reduce the royalty dividends collected by the innovator, since part of the cost reduction due to the innovation is already available without licensing. This suggests that spillovers make the option to sign a royalty licensing contract less valuable. But it should not eliminate that benefit altogether, unless the complete cost reduction spills over.

Finally, it should be interesting to evaluate our proposal in the context of other market rules, such as under price competition in differentiated goods markets.

7. APPENDIX

7.1. *Supplement to the Proof of Proposition 2*

We compare royalty licensing with the royalty rate r with the particular modified license auction $(1, r)$ and prove that $Q_M > Q_R$.

Under royalty licensing the aggregate equilibrium output Q_R solves the condition

$$p_R\,(n - p_R/\eta_R) = n(c - \epsilon + r). \tag{13}$$

Similarly, under the modified license auction $(1, r)$, one has

$$p_M\,(n - p_M/\eta_M) = n(c - \epsilon + r) - r. \tag{14}$$

By assumption, η is non-decreasing in p. Since the right-hand-side of (13) is greater than that of (14) it follows that $p_R > p_M$ and therefore $Q_M > Q_R$.

7.2. *Proof of Proposition 4*

PROOF In the proof of Proposition 2 we have already shown that $k^* \geq 1$. Therefore, in the following we ignore the case $k = 0$.

1) We show that $r^* = \epsilon$ is optimal for each k. The choice of k is restricted to integers. Consider $k \in I_{LN}$. There, the innovator's profit, $\Pi_{LN}(k, r) := \Pi(k, r)$ for $k \in I_{LN}$, is equal to

$$\Pi_{LN}(k, r) = \frac{r\epsilon(K + 1)(k + n) - r^2(k^2 + n)}{n + 1}. \tag{15}$$

Using the fact that $k < \mathcal{K}(r)$, which is equivalent to $\epsilon(K+1) > r(k+1)$, one has

$$\begin{aligned}\frac{\partial}{\partial r}\Pi_{LN}(k,r) &= \frac{\epsilon(K+1)(k+n) - 2r(k^2+n)}{n+1} \\ &> \frac{r(n-k)(k-1)}{n+1} \geq 0.\end{aligned} \tag{16}$$

Therefore, for $k \in I_{LN}$ it is optimal to set the highest possible royalty rate, $r = \epsilon$.

If $k \in I_{L^-}$, one has $q_N(k,r) = 0$. Therefore, the innovator's profit is equal to $b(k,r)k$, which in turn is independent of r (see (11)). Therefore, all royalty rates, including $r = \epsilon$, are equally profitable.

Similarly, for the parameter set I_{L^+}, where $q_N(n,r) = 0$, one obtains (using the fact that $n-1 < \mathcal{K}(r)$ is equivalent to $(K+1)\epsilon - nr > 0$)

$$\frac{\partial}{\partial r}\Pi_{L^+}(n,r) = \frac{2n^2((K+1)\epsilon - nr)}{(n+1)^2} > 0. \tag{17}$$

This completes the proof that $r = \epsilon$ is optimal, regardless of k.

Since $r^* = \epsilon$, the equilibrium bid function simplifies to:

$$b(k) = \begin{cases} \frac{(2(K-k)+n+1)\epsilon^2}{n+1} & \text{if} \quad k \in I_{LN} \\ \left(\frac{(K+1)\epsilon}{k+1}\right)^2 & \text{if} \quad k \in I_{L^-} \\ \frac{(2K+2-n)\epsilon^2 n}{(n+1)^2} & \text{if} \quad k \in I_{L^+} \end{cases} \tag{18}$$

2) We now compute k^*, given that $r^* = \epsilon$. Note that $\mathcal{K}(\epsilon) = K$. We proceed as follows: First, we compute the profit maximizing k that would be obtained if k were restricted to the subsets I_{LN}, I_{L^-}, and I_{L^+}, respectively. These restricted maximizers are denoted by k^*_{LN}, $k^*_{L^-}$, $k^*_{L^+}$. Then, we examine which of these is the global maximizer, depending upon the parameter K, taking into account the integer constraint concerning k. Thereby we use the fact that K is generically not an integer.

2a) The innovator's equilibrium profit over the subset I_{LN} is

$$\Pi_{LN}(k,\epsilon) = \frac{(-k^2 + k(K+1) + Kn)\epsilon^2}{n+1}. \tag{19}$$

We compute the profit maximizing k over this subset I_{LN}, at first ignoring the integer constraint concerning k. Note that this profit function is quadratic in k and strictly concave.

The maximizer is the interior solution $k_{LN} = \frac{K+1}{2}$ if $\frac{K+1}{2} \in I_{LN}$, i.e. if $1 \leq \frac{K+1}{2} \leq n-1$ and $\frac{K+1}{2} < K$. Since $K > 1$ (by assumption) and since K is not an integer, these conditions are equivalent to $K < 2n-3$. In turn, if $K > 2n-3$ one obtains the corner solution $k_{LN} = n-1$. Therefore, ignoring the integer constraint one has

$$k^*_{LN} = \begin{cases} \frac{K+1}{2} & \text{if } K < 2n-3 \\ n-1 & \text{if } K > 2n-3. \end{cases} \tag{20}$$

Now we take into account that k must be an integer. If $K > 2n-3$, one has $k_{LN} = n-1$ which is an integer. Whereas if $K < 2n-3$, $k_{LN} = \frac{K+1}{2}$ is never an integer. Recall that the equilibrium profit is quadratic and therefore symmetric around $\frac{K+1}{2}$. Therefore, the true maximizer is the nearest integer within I_{LN}. We now show that it can be found by simply rounding to the nearest integer since that number is always in I_{LN}.

Since $\frac{K+1}{2} \in [1, n-1]$, and 1 and $n-1$ are integers, $\text{Round}\left(\frac{K+1}{2}\right) \in [1, n-1]$. Therefore, in order to show that $\text{Round}\left(\frac{K+1}{2}\right) \in I_{LN}$ we only need to show that $\text{Round}\left(\frac{K+1}{2}\right) < K$. Evidently, that condition is potentially violated only if one rounds up. Assume the nearest integer is found by rounding up. Then, $\text{Round}\left(\frac{K+1}{2}\right) \leq \frac{K+1}{2} + \frac{1}{2} = \frac{K+2}{2}$. Since $\frac{K+2}{2} < K$ holds if and only if $K > 2$ we only need to worry about the case $K < 2$. However, if $K < 2$, one has $\frac{K+1}{2} < 1.5$, and thus one does not round up.

We conclude that

$$k^*_{LN} = \begin{cases} \text{Round}\left(\frac{K+1}{2}\right) & \text{if } K < 2n-3 \\ n-1 & \text{if } K > 2n-3. \end{cases} \tag{21}$$

2b) The innovator's equilibrium profit over the subset I_{L^-} is

$$\Pi_{L^-}(k, \epsilon) = \frac{(K+1)^2 k \epsilon^2}{(k+1)^2}. \tag{22}$$

We assess the profit maximizing k over this subset I_{L^-}, denoted by $k^*_{L^-}$.

First, notice that this profit function is strict monotone decreasing in k for $k \geq 1$. Therefore, the maximizer is smallest integer in I_{L^-}. That set is empty iff $K > n-1$. And if $K < n-1$ the smallest integer in I_{L^-} is the smallest integer that satisfies the condition $k > K$. Obviously, the maximum profit over I_{L^-} is smaller than the profit obtained from inserting $k = K$ into $\Pi_{L^-}(k, \epsilon)$.

2c) Finally, notice that $I_{L^+} = \emptyset$ iff $K < n-1$. And if $K > n-1$ one has $I_{L^+} = n$; therefore, in that case $k^*_{L^+} = n$.

We summarize the results 2a)-2c) in the following Table:

	$1 < K < n-1$	$n-1 < K < 2n-3$	$K > 2n-3$
k^*_{LN}	$\text{Round}\left(\frac{K+1}{2}\right)$	$\text{Round}\left(\frac{K+1}{2}\right)$	$n-1$
$k^*_{L^-}$	K	$-$	$-$
$k^*_{L^+}$	$-$	n	n

3) Finally, we find the global maximum for all values of K.

3a) If $1 < K < n-1$ one finds:

$$\begin{aligned} \Pi_{LN}(k^*_{LN}, \epsilon) - \Pi_{L^-}(k^*_{L^-}, \epsilon) >& \Pi_{LN}(k, \epsilon) - \Pi_{L^-}(K, \epsilon) \\ =& \frac{(-k^2 + k(K+1) + Kn)\epsilon^2}{n+1} - \frac{(K+1)^2 K \epsilon^2}{(K+1)^2} \\ =& \frac{(K-k)(k-1)\epsilon^2}{n+1} \geq 0 \end{aligned}$$

Therefore, $k^* = k^*_{LN}$.

3b) If $n-1 < K < 2n-3$ one obtains (notice that $k = n-1 \in I_{LN}$ in that case):

$$\begin{aligned} \Pi_{LN}(k^*_{LN}, \epsilon) - \Pi_{L^+}(n, \epsilon) \geq& \Pi_{LN}(n-1, \epsilon) - \Pi_{L^+}(n, \epsilon) \\ =& \frac{((n-1)(K+1) + Kn - (n-1)^2)\epsilon^2}{n+1} - \frac{n^2(2+2K-n)\epsilon^2}{(n+1)^2} \\ =& \frac{(n-2+K(n-1))\epsilon^2}{(n+1)^2} > 0. \end{aligned}$$

Therefore, $k^* = k^*_{LN}$ in that case.

3c) If $K > 2n-3$ one obtains $k^* = k^*_{LN}$ from the fact that $\Pi_{LN}(n-1,\epsilon) - \Pi_{L^+}(n,\epsilon) > 0$ which has already been established in 3b).

We conclude that for all parameters K one has $k^* = k^*_{LN}$. Therefore, by the definition I_{LN} both licensees and non-licensees coexist and produce positive equilibrium outputs, and hence royalty income is always generated. □

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