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Advertising and Conspicuous Consumption

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Abstract

The paper formalizes the intuition that brands are consumed for image reasons and that advertising creates a brand’s image. The key idea is that advertising informs the public of brand names and creates the possibility of conspicuous consumption by rendering brands a signalling device. In a price competition framework, we show that advertising increases consumers’ willingness to pay and thus provide a foundation, based on optimization behavior, for persuasive approaches to advertising. Moreover, an incumbent might strategically overinvest in advertising to deter entry, there might be too much advertising, and competition might be socially undesirable.

Keywords: Advertising, Entry Deterrence, Brands, Conspicuous Consumption, Bertrand Competition, All-Pay Auction
JEL Classification: L12, L15, M37

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1 Introduction

The paper formalizes the intuition that consumers use brands for image reasons and that it is advertising that establishes a brand’s image.\(^1\) We consider a conspicuous consumption setup where image concerned consumers make choices so as to influence others’ (the public’s) views about themselves.\(^2\) The basic idea to capture the image-creating role of advertising is to posit that advertising informs the public of brand names.\(^3\) In doing so, advertising renders brands a potential signalling device, and a product’s image may emerge endogenously as the equilibrium outcome of a signalling game played between consumers and the public. Thus, we view advertising as a necessary requirement for establishing a product’s image: without advertising, goods would be indistinguishable and could not acquire the distinct meaning that allows conspicuous consumption.\(^4\)

Our approach yields a number of positive and normative results that are novel to the advertising literature. Most notably, the paper contributes to the discussion on the entry deterring effects of advertising. While early authors (see e.g. Braithwaite 1928, Robinson 1933) argue that an incumbent monopolist may increase his advertising to deter entry, Bagwell (2003, p. 116) concludes in his recent review that this intuition “is not strongly supported by the existing theoretical models that emphasize advertising’s possible goodwill effects.” In contrast to this literature, this paper confirms the early views and shows that an incumbent might strategically overinvest in advertising to deter entry. In addition, the paper derives welfare implications of advertising that have not been the focus of the advertising literature.

More specifically, we consider a price competition framework where advertising sellers (brands) inform a fraction of the public of their names and so make their good partially

\(^1\)This idea is suggested by a large literature in consumer research and marketing. We review this literature below.

\(^2\)The idea goes back to Veblen (1899). We use a model in the spirit of more recent accounts such as Bernheim (1995), Bagwell and Bernheim (1996), Pesendorfer (1995), Corneo and Jeanne (1997).

\(^3\)This is a key departure from the informative advertising literature in the tradition of Butters (1977) which assumes that the purpose of advertising is to inform consumers of the existence of firms.

\(^4\)The marketing literature often views advertising as a more powerful tool and seems to suggest that it can directly influence a product’s image through the style of the advertising campaign, image appeals, the form of the logo, the packaging etc. See, e.g., Park et al. (1986) or Johar and Sirgy (1991).

\(^5\)Advertising creates goodwill if it creates captive consumers who consider purchasing from the seller only from whom they received an ad.
conspicuous. Free entry ensures that the good is always available at marginal cost from a no-name seller who does not advertise. To analyze the post-advertising pricing game, we draw on well-known features from the conspicuous consumption literature: in order to create the brand’s signalling value (its image), advertising sellers raise prices so as to prevent some consumer types from purchasing their good. Thus, the market becomes endogeneously segmented: strongly image concerned consumers buy in the “premium segment” (at mark-up prices) and obtain a favourable image; and less image concerned consumers buy in the “budget segment” (at marginal cost) and obtain an unfavourable image.6 Despite Bertrand competition, premium prices do not fall, as price reductions would pool consumer types and destroy the brand’s image.7

Our main results are driven by how the pricing equilibrium affects a brand’s advertising incentives. All else equal, a premium consumer prefers a brand with higher advertising, because advertising raises the likelihood that the brand is recognized and that the consumer obtains his preferred image. Therefore, advertising is a competitive advantage in the premium segment.

Thus, the equilibrium displays three noteworthy positive features. First, advertising directly increases a consumers’ willingness to pay for a brand. Hence, the model provides a foundation, based on optimization behaviour, for persuasive approaches to advertising that simply posit that advertising shifts out (inverse) demand (see e.g. Dixit and Normann 1978). Second, the endogeneous market segmentation admits an interpretation of advertising in terms of product differentiation: because different products carry different images, consumers have heterogeneous preferences over otherwise homogeneous goods.8

Finally, the setup provides a rationale for the everyday observation that many ads do not contain useful product information. In an experience good context, the Nelson (1974) and Milgrom and Roberts (1986) tradition explains this observation by arguing that advertising is a costly signal of unobservable quality. Our story might explain why sellers of life-style

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6Throughout the formal analysis, we assume that there is exactly one ideal image that all consumers strive for (for example, to be “cool” versus to be a “bore”). If they do not obtain this image, they obtain an image loss (“stigma”). The other case, in which different consumers have different ideal images, is discussed informally.

7Bagwell and Bernheim (1996) argue that equilibrium prices above marginal cost are an artefact of the assumption that consumers consume only one unit at a time. We shall comment on this in more detail below.

8In his classic study on the US manufacturing industry, Bain (1956, p.101) concludes: “The single most important basis for product differentiation in the consumer-good industry is apparently advertising.”
products with little experience characteristics such as cigarettes, fashion clothes, or soft-drinks often appear to aim at maximum publicity through funny, shocking or otherwise eye-catching campaigns and try to associate their product with a distinct public image.\textsuperscript{9}

To study entry, we adopt Schmalensee’s (1983) seminal framework, in which brands advertise sequentially before competing in the post-entry market. In the current setup, the incumbent can deter entry, because in order to make sales, the entrant needs to “overbid” the incumbent’s advertising. Thus, by increasing her advertising, the incumbent can reduce the profitability of entry. The overinvestment result holds if brands can sustain relatively high mark-ups. In this case, overbidding tends to generate large gains in the post-entry game, and only heavy advertising by the incumbent makes entry unprofitable. As a result, entry is effectively impeded in the sense of Bain.\textsuperscript{10} Conversely, when mark-ups are relatively small, entry is already deterred when the incumbent advertises like a monopolistic brand without an entry threat, i.e. entry is blockaded in the sense of Bain.

To the best of our knowledge, this result is new to the literature in which the purpose of advertising is to increase a seller’s publicity.\textsuperscript{11} Specifically, in Schmalensee (1983) advertising informs consumers and firms compete in quantities. He establishes that the incumbent under-invests so as to commit to be more aggressive in the post-entry game. Ishigaki (2000) replaces Schmalensee’s quantity by a price competition framework and finds that entry is at most blockaded but never effectively impeded. Fudenberg and Tirole (1984) do identify conditions where the incumbent overinvests. However, the incumbent does so to accommodate, not to deter entry. When entry deterrence is optimal instead, the incumbent underinvests.

After the discussion on entry, we analyze implications for welfare and competition. Advertising exerts a negative externality on “budget consumers” since it increases the likelihood that they obtain a negative image (“stigma”). A monopolistic brand does not internalize this detrimental effect and consequently advertises too much. Moreover, in monopoly also the “premium

\textsuperscript{9}A case in point is celebrity advertising which attempts to utilize the celebrity’s publicity and to link the brand with attributes and values the celebrity stands for (see McCracken 1989).

\textsuperscript{10}In Bain’s (1956) classification, entry is blockaded when monopoly behaviour deters entry; entry is effectively impeded when the incumbent deters entry by changing her behaviour as compared to a monopolist without entry threat; and entry is easy otherwise.

\textsuperscript{11}Overinvestment however occurs in models of limit-pricing where a privately informed incumbent deters entry by overinvesting in advertising so as to signal demand or cost conditions (see Bagwell and Ramey 1988, 1990). In our model, sellers do not have private information.
consumers" do not benefit from advertising, because their image gain is entirely appropriated by the monopolist. Thus, an advertising ban would be in the interest of consumers.

Our setup also gives rise to some unconventional implications with respect to competition. When two brands advertise simultaneously, advertising takes on the form of an all-pay auction. Brands will then expend all their prospective profits in the advertising contest in an attempt to win an advantage in the post-advertising pricing game. The losing seller’s advertising does not contribute to a consumer’s image and is thus pure waste from a welfare perspective. In this sense, competition lowers welfare.\(^\text{12}\)

With regard to consumer rents, competition makes premium consumers better off, because it stimulates price competition between brands. In contrast, it makes budget consumers worse off through the increased advertising it encourages.

These results appear to confirm concerns widespread among political activists who denounce advertising as a consumer rip-off and advocate advertising bans (see e.g. Klein 2000). However, they rest critically on the assumption that all consumers share the same ideal image (see footnote 6). If this is not the case, advertising is generally beneficial, as it enables consumers to “express themselves.” We discuss the latter case informally.

**Literature and background**

Our work is most closely related to previous work on conspicuous consumption that adopts a signalling perspective, in particular Coelho and McClure (1993), Bagwell and Bernheim (1996), Pesendorfer (1995), Corneo and Jeanne (1997). Our paper shares with these papers the basic feature of market segmentation through mark-up pricing.\(^\text{13}\) In contrast to our paper however, in most of these papers, the public can identify consumption choices by assumption. Thus, there is no role for advertising. An exception is Pesendorfer (1995) where sellers can create new designs at a fixed cost. Creating a design is similar to advertising in our setup. However, Pesendorfer’s focus is very different from ours. He looks at intertemporal price patterns and fashion cycles but does not study entry deterrence. Moreover, in contrast to his setup, where once a design is created everyone can distinguish it from an old design, we assume that advertising changes a brand’s publicity in a continuous way. This gives rise to the possibility

\(^{12}\)Somewhat ironically, from this perspective the entry deterrence result mentioned above is not too worrisome.

\(^{13}\)See also Bernheim (1995) for a treatment with exogenous prices.
that the public is partially unable to distinguish brands and it renders advertising an all-pay auction. While he also points out the detrimental effects of competition on social surplus, we have a more detailed analysis of consumer rents.

Our work is inspired by a large consumer research literature on symbolic consumption. Starting with the seminal work of Levy (1959), the theory of symbolic consumption views consumption goods as meaningful symbols that can be used to satisfy self-presentation needs (see also Belk 1988). Social psychologists distinguish between strategic and expressive self-presentation motives. The former aims at influencing others’ views of oneself, whereas the latter aims at constructing the self and an identity for oneself (see Baumeister (1998) for a review).14 In the context of consumption, the self-presentation function of goods and possessions is supported by several empirical and experimental studies (e.g. Prentice 1987, Richins 1994).15

The literature suggests that branding and advertising plays a central role in the process of attaching a specific symbolic meaning to a good. In their classic contribution, Gardner and Levy (1955) argue that possessing a particular image is the distinctive feature of a brand as opposed to a commodity. According to their view, a brand’s image is created by “advertising, merchandising, promotion, publicity, and even sheer length of existence” (p. 35).16

The most convincing empirical support for the hypothesis that brands are in fact used to satisfy self-presentation needs comes from Aaker (1999).17 Aaker asked subjects to evaluate brands in terms of the situations in which they typically use them. On the basis of the self-presentation motive, Aaker develops several hypotheses that predict brand preferences depending on subject characteristics such as image concerns and situation characteristics such as social reference group and finds strong support for the premise that brand images influence purchasing behavior.

The paper is organized as follows. Section 2 describes the setup. Section 3 analyzes the signalling game. Section 4 derives the entry deterrence effect, and section 5 studies welfare

14 The economics literature on conspicuous consumption often assumes that self-presentation is instrumental for matching purposes at a post-consumption stage. Similarly, we shall focus on the strategic motive and discuss the expressive motive informally.

15 Solomon (1983) extends the self-expression view and claims that the symbolic nature of consumption goods might also provide role scripts that prescribe behaviors in particular situations.

16 For an empirical study on the influence of advertising on a product’s image and on how consumers react to different image appeals see Snyder and DeBono (1985).

17 See also Aaker (1997).
properties. Section 6 concludes. All proofs are in the appendix.

2 The setup

There are two sellers, $S_1$ and $S_2$, who can each produce one unit of a good at 0 marginal cost. We refer to $S_1$ and $S_2$ as brands. In addition, the good is supplied by a no-name seller, $S_0$, at a fixed price $p_0 = 0$. There are two consumer types $t \in T = \{H, L\}$. A type might represent a particular life-style, views about the world, wealth, etc. The proportion of type $t$ in the consumer population is $\mu_t \geq 0$, and the population size is normalized to 1. A type is this type’s private information. In addition, there is a public of mass 1. The public is distinct from the consumers and does not consume a good.

Consumers seek to signal a particular type to the public and use brands to do so. We assume that each good carries the seller’s name. The idea is that it is the brand name that makes the good conspicuous and thus renders it a potential signalling device. For simplicity, attaching a name to the good is assumed to be costless. Ex ante, consumers know brand names, but the public does not. The purpose of advertising is thus to inform the public (and not the consumers) about brand names and thereby to increase the probability with which the brand is recognized by the public. More precisely, the game proceeds in three stages, an advertising stage, a pricing stage, and a consumption stage.

In the advertising stage, $S_1$ and $S_2$ inform the public of their names. The no-name seller has prohibitively high advertising costs and does not advertise. Specifically, brand $S_b$, $b = 1, 2$, informs a fraction $\alpha_b \in [0, 1]$ of the public of her brand name. Doing so costs $c(\alpha_b) = (1/2) c \alpha_b^2$ with $c > 0$. We consider both sequential and simultaneous advertising. In the sequential case, $S_1$ moves first, and $S_2$ can observe $\alpha_1$ before making her advertising choice.

When a member of the public receives an ad from a brand, she becomes able to distinguish this brand from all other sellers (including the no-name seller). We represent the public’s knowledge by the index $k \in K = \{\emptyset, 1, 2, (1, 2)\}$. E.g., if a member of the public receives an

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18 This is a convenient way to capture free entry. We would obtain the same results by considering a large number of sellers with endogenous pricing. Bertrand competition would then lead some of these sellers to supply the good at marginal cost.

19 This assumption is common in the literature and keeps the analysis simple.

20 Endogenizing the no-name seller’s advertising choice would yield similar results.

21 Any other standard cost function would do.
ad from $S_2$ only, but not from $S_1$, then $k = 2$, and she can distinguish brand $S_2$ from all other sellers, but not brand $S_1$ from the no-name seller. If she does not receive any ad, then $k = \emptyset$, and if she receives both ads, then $k = (1, 2)$. We denote the probability of $k$ by $\rho_k$.

In the pricing stage, after observing all moves by $S_1$ and $S_2$ in the previous stage, $S_1$ and $S_2$ simultaneously choose prices $p_1$ and $p_2$. We denote the profile of prices by $p = (p_0, p_1, p_2)$.

In the consumption stage, a consumer observes all moves of the previous stages and chooses whether and from which seller to purchase the good. For simplicity, we assume that a consumer always prefers the free no-name product to not consuming at all. So without loss of generality, the consumer’s choice set is $I = \{0, 1, 2\}$.

After his consumption choice, the consumer is randomly matched with a member of the public, referred to as the consumer’s (social) contact. The contact draws inferences about the consumer’s type on the basis of her knowledge of brand names, the consumer’s choice, and the distribution of prices. For $i \in I$ let $\gamma^k_i(i|p)$ be the contact’s belief that the consumer is of type $t$ conditional on having received advertisement $k$ and on being matched with a consumer who chose $i$. If his contact holds belief $\gamma_H = \gamma^H_H(i|p)$, consumer type $t$ obtains overall utility

$$u_t = \pi + \lambda_t \gamma_H,$$

where $\pi > 0$ is the intrinsic utility and $\lambda_t \gamma_H$ represents the image utility of the good. $\pi$ is assumed to be the same across consumer types and sellers, i.e. with regard to its intrinsic features (e.g. quality) the good is homogeneous.

The sign of $\lambda_t$ determines the ideal image of type $t$. If $\lambda_t \geq 0$, then type $t$ wants to appear as type $H$, and if $\lambda_t < 0$, then type $t$ wants to appear as type $L$. We assume that both types want to appear as type $H$, i.e. $\lambda_H, \lambda_L > 0$. For example, the image utility might reflect the social esteem transferred to $H$-types and the shame suffered by $L$-types due to the presence of a social norm that favours $H$-types (e.g. $H$-types are cool, $L$-types are bores), hence the notation $H$ for “high” and $L$ for “low”.

\footnote{For example, $\rho^{1,2} = \alpha_1 \alpha_2$.}
\footnote{This term is borrowed from Bagwell and Bernheim (1996).}
\footnote{One might question that a contact knows the price of a brand of which he did not receive an ad. However, if this assumption is relaxed, the contact would need to hold endogenous beliefs about prices. This raises technical issues that are beyond the scope of the paper.}
\footnote{This formulation of utility is similar to Bernheim (1995).}
\footnote{We shall comment on the case $\lambda_L < 0$ below.}
In what follows, the types’ relative intensity to appear as type $H$ will be important. Let
\[ \sigma = \frac{\lambda_L}{\lambda_H} > 0. \]
With this specification, consumer type $t$’s expected utility from choosing brand $i$ at price $p_i$ is
\[ u_t(i, p_i|\gamma) = \pi_t + \sum_{k \in K} \lambda_t \gamma^k_H(i|p) \rho^k - p_i. \]
We define a consumer type $t$’s incentive to choose $i$ rather than $j$ by
\[ \Delta u_t(i, j|p, \gamma) = u(i, p_i|p) - u_t(j, p_j|p) = \lambda_t \sum_{k \in K} (\gamma^k_H(i|p) - \gamma^k_H(j|p)) \rho^k - p_i + p_j. \]
(1)
Finally, all of what was said so far is common knowledge among the players.

3 Pricing and consumption

In this section, we describe the outcome of the price competition game after sellers have made their advertising choices. Without loss of generality, $\alpha_1 \geq \alpha_2$. For $\alpha_1 < \alpha_2$, the same results hold with a change of indices.

Taking as given $\alpha_1$ and $\alpha_2$, the players’ strategies are as follows. A seller’s strategy is a price choice $p_i \in \mathbb{R}^+$, and a consumer type $t$’s strategy $d_t(i, p)$ denotes the probability that type $t$ chooses $i$, given $p$. In addition, the public’s belief is given by the belief function $\gamma^k_t(i|p)$ introduced in the previous section.

A perfect Bayesian Nash equilibrium is a collection $p, d_H, d_L, \gamma$ such that a player’s strategy maximizes his utility given the other players’ strategies and the public’s beliefs; and the public’s beliefs are, whenever possible, Bayesian consistent with sellers’ and consumers’ strategies as well as its knowledge of brand names. As usual in signalling games, there are multiple equilibria in the consumption game following sellers’ price choices. We now describe how we select equilibria. We focus on (fully) separating equilibria, where different types make different choices, and pooling equilibrium where all types choose the same seller.\(^{27}\) The following observation is useful.

\(^{27}\)We do not consider semi-separating equilibria in which a type mixes between a brand and the no-name seller. This is for simplicity only and not substantial.
Lemma 1 In any separating equilibrium, type L consumes at price 0.

In a separating equilibrium, an L-type is identified as type L indeed. Thus, he obtains the worst possible image. Thus, he cannot lose from purchasing the no-name good at price 0.

In light of Lemma 1, we consider separating equilibria only in which an L-type chooses the no-name good. In such an equilibrium, two incentive constraints per type have to hold. Type H has to purchase from one of the two brands, and type L must not purchase from a brand. Formally, (1) implies the following incentive compatibility conditions:

\[
\begin{align*}
\Delta u_H (1, 0 | \gamma) & \geq 0 \quad \text{or} \quad \Delta u_H (2, 0 | \gamma) \geq 0, \\
\Delta u_L (1, 0 | \gamma) & \leq 0 \quad \text{and} \quad \Delta u_L (2, 0 | \gamma) \leq 0,
\end{align*}
\]

where \( \gamma \) is a belief consistent with separation.

In addition, we employ two further selection criteria. First, if there are two separating equilibria, we shall select the equilibrium in which the H-type is better off. This captures the intuitive idea that brands compete for the H-type. Second, we require that in any separating equilibrium the H-type must not obtain less than in the pooling equilibrium in which both types choose the no-name seller. This reflects the intuitive idea that brands face competition from the no-name seller. The two criteria can be formally merged as follows.

Suppose there are two equilibria \( e \) and \( \tilde{e} \) such that the L-type chooses the no-name seller in \( e \) and \( \tilde{e} \). Let \( u_H (e) \) be the H-type’s utility in equilibrium \( e \). Then:

\[ u_H (e) > u_H (\tilde{e}) \Rightarrow e \text{ is selected}. \]
To break ties, we assume that if the $H$-type is indifferent between two equilibria, then the equilibrium is selected, in which the $H$-type consumes from the brand with the higher advertising.\footnote{This includes the case when the $H$-type is indifferent between the pooling and a separating equilibrium. Then the latter is selected.} If both brand’s advertising is the same in both equilibria, then each equilibrium is selected with probability $1/2$.

In the next Lemma, we derive constraints on $\sigma$ from the IC and SC constraints.

**Lemma 2** Let $e$ be a separating equilibrium satisfying $(IC_H)$ and $(IC_L)$. Then it holds:

(i) $\sigma \leq 1$.

(ii) If $\sigma \in (\mu_L, 1)$, then the $H$-type is worse off in $e$ than in the pooling equilibrium in which both types choose the no-name seller, thus (SC) selects the pooling equilibrium.

The intuition is straightforward. (i) is a common sorting condition. If $\sigma > 1$, then at any price at which an $H$-type is willing to consume a brand in exchange for its more desirable image, an $L$-type is even more happy to do so. Hence, an $L$-type cannot be deterred from mimicking the $H$-type, and incentive compatibility breaks down. As for (ii), if $\sigma$ is large, brands need to quote high prices to prevent the $L$-type from purchasing the brand. This price outweighs the $H$-type’s image gain from separation in comparison to pooling. In light of Lemma 2, we assume from now on that $\sigma \leq \mu_L$.

We are now in the position to derive the pricing-consumption equilibrium. Intuitively, if $S_1$ advertises more than $S_2$, an $H$-type is willing to pay more for $S_1$ than for $S_2$, because when purchasing brand $S_1$ it is more likely that he meets a contact who recognizes brand $S_1$ and awards him the respective status. Consequently, $S_1$ will use this advantage and attract all $H$-types. If both brands’ advertising is the same, this advantage is absent. $S_1$ and $S_2$ will then reduce prices until a further reduction would attract $L$-types. Proposition 1 characterizes the equilibrium formally.
Proposition 1 Let $\alpha_1 \geq \alpha_2$. Then for all prices the selection criterion (SC) selects a signalling equilibrium that is unique up to out-of-equilibrium beliefs. Furthermore, there is a unique pricing equilibrium such that:

(i) $S_2$ sets price $p_2 = \lambda_L \alpha_2$ and does not make sales if $\alpha_1 > \alpha_2$;
(ii) $S_1$ sets a price such that an $H$-type is just indifferent between $S_1$ and $S_2$, i.e.

$$p_1 = \lambda_L \alpha_2 + \lambda_H \mu_L (\alpha_1 - \alpha_2),$$

and $S_1$ makes sales to all $H$-types, if $\alpha_1 > \alpha_2$; and

(iii) all $L$-types purchase the no-name product.

To illustrate the intuition, suppose $\alpha_1 > \alpha_2$. Bertrand competition implies that the non-selling brand, $S_2$, chooses the smallest price such that the $L$-type, conditional on the $H$-type choosing brand $S_2(!)$, would still choose the no-name seller. $S_2$ thereby offers the same signalling value as brand $S_1$. Despite making no sales and pricing above marginal cost, it is not profitable for $S_2$ to reduce her price, because this would attract $L$-types and worsen $S_2$’s signalling value.

Moreover, the selling brand, $S_1$, needs to quote a price such that the $H$-type, conditional on choosing $S_1$ is (weakly) better off than in the continuation game that would follow if he chose $S_2$. $S_1$’s higher advertising allows her to do so at a price higher than $p_2$. Profit maximization implies that $S_1$ leaves an $H$-type with the same utility that he would obtain if he chose $S_2$.

The proposition illuminates the main positive features mentioned in the Introduction. First, given the rival brand’s advertising, a brand’s price increases directly in her own advertising. Second, advertising leads to market segmentation and allows the selling brand to quote a price above marginal cost. Thus, advertising can be seen as product differentiation and as a means to avoid competition with no-name sellers.

In what follows, two boundary cases will be important. Fix $\alpha_1$. Then, if $\alpha_2 = 0$, $S_1$ acts as a monopolist in the “premium segment” and can charge the monopoly price

$$p^m = \lambda_H \mu_L \alpha_1.$$ 

$p^m$ is just low enough that an $H$-type is still willing to consume brand $S_1$ instead of the no-name good. If $\alpha_2 = \alpha_1$, there is perfect competition between brands, and they charge the competitive price

$$p^c = \lambda_L \alpha_1.$$
$p^c$ is just high enough to prevent an $L$-type from consuming a brand.

Bagwell and Bernheim (1996) argue that equilibrium prices above marginal cost are an artefact of consumers being restricted to consume one unit of a fixed quality at a time. This raises concerns against the use of the type of equilibria described in Proposition 1. However, the assumption of unit consumption might be appropriate in some circumstances. Moreover, Bagwell and Bernheim have a second result which says that under a tangency condition, mark-up pricing can be sustained in equilibrium even if quantity and quality can be varied. Since our main objective is to derive implications for advertising in a situation in which consumers’ image concerns allow sellers to charge mark-ups, we choose the simplest model that generates this feature. This is justified by Bagwell and Bernheim’s second result that shows that a non-trivial class of models exhibits mark-up pricing.

A final remark concerns the case when both types want to signal their types, i.e. $\lambda_L > 0$. Similar to the equilibrium described by Proposition 1, there is an equilibrium in which one type, say type $L$, consumes the no-name good at price 0, and type $R$ consumes from brand $S_1$. The key difference to the case with $\lambda_L < 0$ is that now type $L$ does not want to mimic type $R$. Hence, any positive price $p_1$ prevents the $L$-type from purchasing brand $S_1$. Thus, the competitive price is 0, the equilibrium price of the selling brand is $p_1 = \lambda_H \mu_L (\alpha_1 - \alpha_2)$, and that of the non-selling brand is $p_2 = 0$.

4 Advertising and entry

This section studies the brands’ advertising choices when advertising is sequential and $S_1$ (the incumbent) moves first and $S_2$ (the entrant) moves second. This order of moves is the same as in Schmalensee’s (1983) seminal entry deterrence model. We begin with the monopoly case as

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33 The tangency condition says that consumption-wealth indifference curves have one point in common and are tangent at this point. In our setup with quasi-linear utility, indifference curves are straight lines, and so the tangency condition cannot hold.

34 In addition, there is an equilibrium where no one consumes the no-name brand and each type consumes from a separate premium brand at a positive price. Such an equilibrium might be sustained by out-of-equilibrium beliefs that assign the a priori image to the no-name seller. Accordingly, the market is split in two premium segments, and to capture competition between brands, one would need to consider at least four brands, two for each premium segment. In this case, it is still true that in each premium segment the competitive price and thus the price quoted by the non-selling brand is 0.
a benchmark and then turn to the entry and entry deterrence decision.

4.1 Monopoly

Suppose that \( S_2 \) cannot advertise, i.e. \( \alpha_2 = 0 \). Then \( S_1 \) sets the monopoly price \( p^m = \mu_L \lambda_H \alpha_1 \). Hence, the monopoly profit is \( \pi^m(\alpha_1) = \mu_H p^m - (1/2) c \alpha_1^2 \), and monopoly advertising is given by

\[
\alpha^m = \frac{\lambda_H \mu_L \mu_H}{c} \land 1.
\]

4.2 Optimal entry strategy

Suppose now that \( S_2 \) can advertise and potentially enter the premium segment. If \( S_2 \) advertises less than \( S_1 \), she does not make sales in the post-entry game. Because \( S_2 \)'s profit is discontinuous at \( S_1 \)'s advertising \( \alpha_1 \), matching \( \alpha_1 \) is always dominated by slightly “overbidding” \( \alpha_1 \). If \( S_2 \) just overbids \( S_1 \)'s advertising by an \( \varepsilon \), she charges the competitive price \( p^c = \lambda_L (\alpha_1 + \varepsilon) \). If \( \alpha_1 \) becomes large, overbidding becomes less profitable, as advertising costs increase. The advertising level \( \alpha^c \) is defined as the largest advertising level such that \( S_2 \) breaks even by just overbidding, i.e. \( \lim_{\varepsilon \to 0} \pi_2(\alpha_2 = \alpha^c + \varepsilon, \alpha_1 = \alpha^c) = 0 \). We refer to \( \alpha^c \) as the competitive advertising level. It holds that \( \pi_2(\alpha_2 = \alpha^c + \varepsilon, \alpha_1 = \alpha^c) = \mu_H \lambda_L (\alpha_1 + \varepsilon) - (1/2) c (\alpha_1 + \varepsilon)^2 \).

Hence,

\[
\alpha^c = \frac{2 \lambda_L \mu_H}{c} \land 1.
\]

To avoid rather uninteresting corner cases, we assume from now on that \( c \) is large enough such that \( \alpha^c \lor \alpha^m < 1 \). In this case, straightforward algebra yields: \( \alpha^c > \alpha^m \) if and only if \( \sigma > \mu_L/2 \). Lemma 3 describes the optimal entry strategy in terms of \( \alpha^c \) and \( \alpha^m \).

Lemma 3 (i) \( S_2 \)'s optimal entry strategy \( \alpha^*_2(\alpha_1) \) against \( \alpha_1 < 1 \) is given as follows:

(A) If \( \alpha_1 < \alpha^m \), then

\[
\alpha^*_2(\alpha_1) = \begin{cases} 
\alpha^m & \text{if } \alpha_1 < \left( \mu_H \mu_L^2 \lambda_H^2 / (2c(\lambda_H \mu_L - \lambda_L)) \right) \\
0 & \text{otherwise.}
\end{cases}
\]

(B) If \( \alpha_1 \geq \alpha^m \), then

\[
\alpha^*_2(\alpha_1) = \begin{cases} 
\lim_{\varepsilon \to 0} \alpha_1 + \varepsilon & \text{if } \alpha_1 < \alpha^c \\
0 & \text{if } \alpha_1 \geq \alpha^c.
\end{cases}
\]
(ii) $S_2$’s optimal entry strategy against $\alpha_1 = 1$ is $\alpha_2^*(1) = 0$.

If $\alpha_1 < \alpha^m$, then monopoly advertising is the optimal entry strategy unless $\alpha_1$ is too large to allow $S_2$ to quote a post-entry price that covers advertising costs. Similarly, for $\alpha_1 \geq \alpha^m$, $S_2$ optimally advertises just a bit more than $S_1$ unless $\alpha_1$ is too large for $S_2$ to break even. Hence, for $\alpha_1 > \alpha^c$, $S_2$ stays out of the market. Finally, if $\alpha_1 = 1$, then $S_2$ serves half of the $H$-types if he, too, sets $\alpha_2 = 1$. However, given the abovementioned cost restriction, this does not cover costs, and $S_2$ stays out.

4.3 Entry deterrence

We are now in the position to state our central entry deterrence result.

**Proposition 2** In equilibrium, the incumbent chooses $\alpha_1^* = \alpha^c \lor \alpha^m$, and the entrant chooses $\alpha_2^* = 0$. More precisely:

(i) If $\sigma \leq \mu L/2$, then $\alpha_1^* = \alpha^m$, and entry is blockaded in the sense of Bain.

(ii) If $\sigma > \mu L/2$, then $\alpha_1^* = \alpha^c$, and entry is effectively impeded in the sense of Bain.

The incumbent can deter entry, because her advertising reduces $S_2$’s profits if $S_2$ were to exceed $S_1$’s advertising. Whether the incumbent strategically overinvests or advertises like a monopolist depends on $\sigma$. If $\sigma$ is low, the competitive price is not large enough for $S_2$ to make profits given $S_1$ chooses monopoly advertising. Hence, monopoly advertising is sufficient to keep $S_2$ out of the market.

Conversely, if $\sigma$ is low, the competitive price is high and allows $S_2$ to make profits even if $S_1$ chooses monopoly advertising. Hence, $S_1$ needs to advertise beyond the monopoly level to keep $S_2$ out, resulting in overinvestment.

We close the entry section by commenting on the case $\lambda_L < 0$. Recall from the end of the previous section that in this case the competitive price is 0 when the entrant just overbids the incumbent. Thus, this case corresponds to the case with $\sigma = 0$, and entry will always be blockaded.
5 Advertising and welfare

The objective of this section is to examine the welfare properties of the equilibrium. For simplicity, we take a utilitarian perspective and take total surplus as welfare measure. We first provide a general expression for consumer rent and total surplus and then determine the socially optimal level of advertising. We then compare this to both monopoly and duopoly with sequential advertising, and finally compare monopoly and duopoly when advertising is simultaneous.

5.1 Consumer rent and total surplus

To compute consumer and total surplus when types separate, suppose that \( \alpha_1 \) and \( \alpha_2 \) are given. Suppose further that type \( H \) chooses brand \( b \in \{1, 2\} \) at price \( p_b \), that type \( L \) chooses the no-name seller at \( p_0 = 0 \), and no one chooses \( e_b \in \{1, 2\} \), \( e_b \neq b \). Note that because no one chooses \( \tilde{b} \), a member of the public cannot better identify a type even if she is informed about \( \tilde{b} \). Thus, with probability \( \alpha_b \), type \( t \) is identified, in which case he obtains image utility \( \lambda_t \tilde{\gamma}_t \) where we define \( \tilde{\gamma}_H = 1 \) and \( \tilde{\gamma}_L = 0 \). And with probability \( 1 - \alpha_b \), he receives the a priori image utility \( \lambda_t \mu_H \), i.e. type \( t \)'s utility is given by

\[
\begin{align*}
\forall_t &= \pi + \lambda_t \tilde{\gamma}_t + \lambda_t \mu_H (1 - \alpha_b) - p_t \\
&= v_t + \lambda_t (\tilde{\gamma}_t - \mu_H) \alpha_b - p_t,
\end{align*}
\]

where we define \( v_t = \pi + \lambda_t \mu_H \). Utility is composed of three parts. \( v_t \) is the utility if there is no advertising. The term \( \lambda_t (\tilde{\gamma}_t - \mu_H) \alpha_b \) is a type’s image change through advertising, and \( p_t \) is the price paid by type \( t \).

Total surplus is then simply the sum over consumers’ utilities and sellers’ profits. Since \( S_2 \) does not make sales, and since \( S_1 \)'s profit equals the monetary transfer from an \( H \)-type minus advertising cost, total surplus is given by

\[
TS(\alpha_1, \alpha_2) = v + (\lambda_H - \lambda_L) \mu_H \mu_L \alpha_1 - \frac{1}{2} c \alpha_1^2 - \frac{1}{2} c \alpha_2^2,
\]

where \( v \) is defined as \( v_H \mu_H + v_L \mu_L \).

\[A question arises as to what the utility of a consumer’s contact is. To keep things simple, we do not introduce an additional utility function for contacts, but assume that a contact’s utility is included in \( \lambda_t \). This seems reasonable e.g. in a matching context.\]
When types pool, we suppose they choose the no-name seller. Hence, they receive their a priori image utility with probability 1 at price 0. Total surplus is then $TS_{\text{pool}} = v$.

### 5.2 First best advertising

To determine the first best, we think of a social planner who acts as a benevolent monopolist and chooses advertising and price so as to maximize total surplus. The planner’s information, consumers’ and the public’s behavior, and the selection criterion is the same as in the case with profit-maximizing sellers. In principle, the planner could advertise both brands. However, given our indifference rule, only one brand makes sales in a separating equilibrium such that the advertising of the non-selling brand would be pure waste. So without loss of generality, $\alpha_2 = 0$. In this case, (3) obviously dominates $TS_{\text{pool}}$, and optimizing (3) with respect to $\alpha_1$ gives the first best advertising level

$$
\alpha^{FB} = \frac{(\lambda_H - \lambda_L) \mu_H \mu_L}{c}.
$$

### 5.3 Monopoly and sequential advertising

#### 5.3.1 Monopoly

The comparison of $\alpha^{FB}$ with monopoly advertising $\alpha^m = \lambda_H \mu_H \mu_L / c$ implies the following observation:

**Lemma 4** There is too much advertising in monopoly (i.e. $\alpha^m > \alpha^{FB}$).

The reason is straightforward. Advertising affects surplus in two ways. It improves the image of an $H$-type and worsens the image of an $L$-type. The monopolist does not take into account the detrimental effect of her higher visibility on an $L$-type and hence advertises too much.

The planner can achieve first best by increasing the costs of advertising, e.g. by imposing an advertising tax. A more crude policy measure is to ban advertising altogether. An advertising ban is often advocated by political activists in the name of consumer protection (see Klein 2000). We shall now briefly discuss the impact of an advertising ban on consumer rents and on total surplus.

---

$^{36}$By our assumption on $c$, $\alpha^{FB}$ is always less than 1.
5.3.2 Monopoly and advertising ban

When advertising is prohibited, each consumer type receives the a priori image utility. Thus, type $t$ receives the utility $u^\text{ban}_t = v_t$. In monopoly, by (2), type $H$ obtains $u^m_H = v_H + \lambda_H \mu_L \alpha^m - p^m$. Because $p^m = \Delta \lambda_H \mu_L \alpha^m$, we get that $u^m_H = v_H = u^\text{ban}_H$. That is, the $H$-type’s image gain is entirely appropriated by the monopolist. Moreover, an $L$-type obtains $u^m_L = v_L - \lambda_L \mu_H \alpha^m$. Therefore, since $\lambda_L > 0$, a consumer type $L$ benefits from an advertising ban, because he can avoid the stigma of being identified as an $L$-type. We summarize this observation in Lemma 5.

**Lemma 5** In monopoly, an advertising ban makes consumers (weakly) better off.

An advertising ban forgoes the producer surplus created by advertising. By (3), the difference between total surplus under an advertising ban and in monopoly is given by

$$TS^\text{ban} - TS^m = - (\lambda_H - \lambda_L) \mu_H \mu_L \alpha^m + \frac{1}{2} c (\alpha^m)^2.$$ 

A little bit of algebra yields that this is positive if $\sigma < 1/2$, i.e. an advertising ban improves total surplus when the monopolist’s profit (which is proportional to $\lambda_H$) is not too large relative to an $L$-type’s loss (which is proportional to $\lambda_L$) from advertising.

5.3.3 Sequential advertising

Proposition 2 implies that when an incumbent faces an entrant, there is never less advertising than in monopoly. Thus, the conclusion of Lemma 4 is reinforced when entry is possible:

**Lemma 6** When advertising is sequential, there is too much advertising. Moreover, social surplus in the case with potential entry is (weakly) smaller than in monopoly when no entry is possible.

The presence of the entrant reduces surplus because it might induce the incumbent to engage in inefficient deterrence advertising. Also the consumers do not benefit from the presence of a potential entrant, as it does not reduce the incumbent’s price relative to a monopolist’s. In this sense, the threat of competition reduces social surplus because it triggers wasteful rent-seeking activities. This pattern also prevails when advertising is simultaneous.
5.4 Simultaneous advertising and competition

5.4.1 Equilibrium

When sellers choose their advertising levels simultaneously, the advertising game is an all-pay auction with quadratic bid costs where the loser prize is 0, and the winner prize is the revenue from making sales to the $H$-consumers. Because a seller’s profit function is discontinuous in advertising, there is generally no equilibrium in pure strategies. We have the following result.

**Proposition 3** When advertising is simultaneous, then it holds:

(i) If $\alpha^m \geq \alpha^c$, then there are two pure advertising equilibria $(\alpha_1, \alpha_2) = (\alpha^m, 0)$ and $(\alpha_1, \alpha_2) = (0, \alpha^m)$.

(ii) If $\alpha^m < \alpha^c$, then there is no pure advertising equilibrium. There is a mixed strategy equilibrium where a seller’s advertising strategy is given by the uniform distribution on $[0, \bar{\alpha}]$ with

$$\bar{\alpha} = \frac{(\lambda_H \mu_L + \lambda_L) \mu_H}{c}.$$ \(^{37}\)

In this case, both sellers’ expected equilibrium profit is 0.

An equilibrium in pure strategies exists only when it is unprofitable to overbid the monopoly advertising level ($\alpha^m \geq \alpha^c$). In the other case, a pure equilibrium fails to exist. To see why the equilibrium strategy is uniform, notice first that in a mixed equilibrium, a seller’s profit needs to be constant in advertising levels. Second, by Proposition 1, the winner prize increases linearly in one’s own advertising, and advertising costs are quadratic. Therefore, if the probability of winning is linear in one’s own advertising, a seller’s expected benefit from advertising is quadratic and “cancels” with advertising costs, leading to constant profits.

Under (ii), sellers expend all their prospective profits in the advertising contest in an attempt to win an advantage in the post-advertising pricing game and thus end up with 0 overall profits. This is a standard result in all-pay auctions.

5.4.2 Surplus

The objective of the rest of this section is to compare welfare in monopoly and duopoly. We first compare total surplus and then turn to consumer rents. We assume that $\alpha^m < \alpha^c$. Recall

\(^{37}\)Assume that $c$ is large enough such that $\pi < 1$. 

19
that this is equivalent to $\sigma > \mu_L / 2$.

Let $\alpha^d = E[\max\{\alpha_1, \alpha_2\}]$ be the expected equilibrium advertising level of the selling brand. Likewise, let $c^d = E[(1/2) \cdot c \alpha_1^2 + (1/2) \cdot c \alpha_2^2]$ be expected equilibrium advertising costs. A little bit of algebra yields $\alpha^d = (2/3) \pi$ and $c^d = (1/2) (3/2) c (\alpha^d)^2$. Using this in (3) gives total duopoly surplus

$$TS^d = V + (\lambda_H - \lambda_L) \mu_H \mu_L \alpha^d - \frac{1}{2} c (\alpha^d)^2.$$

Recall that total monopoly surplus is

$$TS^m = V + (\lambda_H - \lambda_L) \mu_H \mu_L \alpha^m - \frac{1}{2} c (\alpha^m)^2.$$

Two effects determine the comparison between $TS^d$ and $TS^m$. On the one hand, there is a cost effect. In duopoly, since only the winning seller’s advertising matters for a consumer’s image, the loser’s advertising is useless and pure waste from a welfare perspective. $\alpha^d$ is therefore supplied at an inefficiently high cost. (This is reflected by the higher cost coefficient in $TS^d$.) On the other hand, duopoly advertising is higher than monopoly advertising.\(^{38}\) This is so, because a seller’s advertising is stimulated by the prospect of achieving positive profits in the premium segment. Accordingly, advertising in duopoly is even further away from first best than in monopoly, hence the following result.\(^ {39}\)

**Lemma 7** When advertising is simultaneous, then total surplus in duopoly is less than total surplus in monopoly

While in this sense, competition is detrimental to welfare, a more differentiated picture emerges when consumer rents are considered, because in this case also prices matter. By Proposition 1, the expected equilibrium duopoly price $p^d$ in the premium market is

$$p^d = E[\lambda_H \mu_L \max\{\alpha_1, \alpha_2\} - (\lambda_H \mu_L - \lambda_L) \min\{\alpha_1, \alpha_2\}].$$

\(^{38}\)Notice that $\alpha^m < \alpha^c$ is equivalent to $\alpha^m < \alpha^d$, hence monopoly advertising is always lower than duopoly advertising.

\(^{39}\)The result is similar to a result in Pesendorfer (1995). In his setting, an increase in sellers leads to more designs being available and this increase in variety is wasteful because the availability of more than one design does not improve the sorting of consumers but only increases the costs of supplying designs. The same logic applies in our advertising framework.
A little bit of straightforward algebra gives that \( p^d = \frac{1}{2} (\Delta \lambda_H \mu_L + \lambda_L) \alpha^d \). By (2), consumers’ utilities are thus

\[
\begin{align*}
    v_H^d &= v_H + \lambda_H \mu_L \alpha^d - p^d = v_H + \frac{1}{2} (\lambda_H \mu_L - \lambda_L) \alpha^d, \\
    v_L^d &= v_L - \lambda_L \mu_H \alpha^d.
\end{align*}
\]  

(4) (5)

An \( L \)-type is not affected by price, because he consumes the free good anyway. So he only cares for whether there is more advertising in duopoly or monopoly. As mentioned above, \( \alpha^d > \alpha^m \).

Thus, an \( L \)-type is always worse off in duopoly. An \( H \)-type, however, is affected by price. In monopoly, his image gain is just taken away by the monopolist. In duopoly, advertising makes brands more similar. Thus prices fall and make an \( H \)-type better off than in monopoly.

**Lemma 8** When advertising is simultaneous, then an \( H \)-type is better off and an \( L \)-type is worse off in duopoly than in monopoly.

We close this section with two remarks. First, note that our welfare results hinge on the assumption that the good is intrinsically homogeneous and all consumers purchase a good. This implies that all inefficiencies result from inefficiencies in advertising but not in trade. If consumer types’ willingness to pay is heterogeneous, monopoly pricing might exclude some consumers, and this might improve the welfare properties of duopoly relative to monopoly.

Second, our welfare results depend critically on the assumption that \( \lambda_L > 0 \). If \( \lambda_L \leq 0 \), then each type benefits from separation. In particular, consider the equilibrium informally discussed at the end of section 3. Since the competitive price is 0, it follows that the competitive advertising level \( \alpha^c \) is 0 and thus always smaller than \( \alpha^m \). In this case, under both sequential and simultaneous advertising, the equilibrium outcome will be that exactly one brand advertises like a monopolist and the other brand does not advertise. But since \( \lambda_L \leq 0 \), advertising exerts a positive externality on the \( L \)-type. Hence, there is generally too little advertising in equilibrium, and an advertising ban can never be beneficial.

6 Conclusion

The paper studies the role of advertising in the process of conspicuous consumption. The key idea is that advertising informs the public about brand names and thus creates the possibility
of conspicuous consumption by rendering brands a signalling device. By linking advertising and conspicuous consumption in this way, we provide a foundation, based on optimization behavior, for persuasive approaches to advertising. We derive an overinvestment entry deterrence result that is novel to the formal advertising literature and also derive some unconventional welfare implications.

While in this paper consumers care only about their image conveyed to others, psychological research suggests that individuals also engage in activities that allow them to hold favorable views about themselves (see e.g. Bem 1972). Thus, brands could also be seen as self-signalling devices when consumers have imperfect self-knowledge. The issues that arise from such a perspective are left for future research.

Appendix

Proof of Lemma 1: Towards a contradiction, suppose the $L$-type chooses brand $b_L \in \{1, 2\}$ in a separating equilibrium at price $p_L > 0$. Suppose first that $b_L = 1$. Hence, the $H$-type chooses either $S_2$ or $S_0$, and consistency of beliefs implies

$$\gamma_{H}^{12}(b_L) = 0, \gamma_{H}^{1}(b_L) = 0, \gamma_{H}^{2}(b_L) = \frac{d_H (0, p) \mu_H}{d_H (0, p) \mu_H + \mu_L}, \gamma_{H}^{0}(b_L) = \mu_H,$$

and

$$\gamma_{H}^{12}(0) \geq 0, \gamma_{H}^{1}(0) \geq 0, \gamma_{H}^{2}(0) = \frac{d_H (0, p) \mu_H}{d_H (0, p) \mu_H + \mu_L}, \gamma_{H}^{0}(0) = \mu_H.$$

Hence, the public’s belief of facing type is $H$ upon observing the no-name product is never smaller than its belief of facing type is $H$ upon observing $b_L$. Thus, because $\lambda_L > 0$, $L$’s image utility can only rise when he consumes the no-name product instead of consuming $b_L$. Moreover $p_L > 0$, while the no-name product is free. Thus, type $L$ would benefit by deviating from $b_L$ to the no-name seller which is in contradiction to the assumption that $b_L$ was his equilibrium choice. The same argument works if $b_L = 2$. $\square$

Proof of Lemma 2: As for (i). Suppose a separating equilibrium exists. Then at least one seller, say $S_1$, needs to advertise (otherwise, a contact cannot distinguish any brand names and there cannot be separation). So suppose $\alpha_1 > 0$. Consider the case in which an $H$-type consumes from $S_1$. (The case in which the $H$-type consumes from $S_2$ can be treated similarly.)
By assumption, an $L$-type consumes from the no-name seller. Consistency of beliefs therefore implies
\[ \gamma^1_H(1) = 1, \quad \gamma^1_H(0) = 1, \quad \gamma^2_H(0) = \mu_H, \quad \gamma^0_H(1) = \mu_H, \]
and
\[ \gamma^1_H(0) = 0, \quad \gamma^1_H(0) = 0, \quad \gamma^2_H(0) = \mu_H, \quad \gamma^0_H(0) = \mu_H. \]

Using these beliefs in (1) gives type $L$’s incentive to choose $S_1$ rather than $S_0$ as
\[
\Delta u_L(1,0) = \lambda_L \left[(1 - 0) \alpha_1 \alpha_2 + (1 - 0) \alpha_1 (1 - \alpha_2)
\right.
\left. + (\mu_H - \mu_H)(1 - \alpha_1) \alpha_2 + (\mu_H - \mu_H)(1 - \alpha_1)(1 - \alpha_2)\right] - p_1,
\]
which simplifies to
\[ \Delta u_L(1,0) = \lambda_L \alpha_1 - p_1. \]  \hfill (6)

Similarly, it follows that
\[ \Delta u_H(1,0) = \lambda_H \alpha_1 - p_1. \]  \hfill (7)

Incentive compatibility requires $\Delta u_L(1,0) \leq 0$ and $\Delta u_H(1,0) \geq 0$. By (6) and (7) this can be true only if $\lambda_H \geq \lambda_L$, that is, only if $\sigma \leq 1$. This proves part (i).

As for (ii), let $\sigma \in (\mu_L,1)$. Consider first the pooling equilibrium, say $\bar{e}$, in which both types consume from the no-name seller at price 0. In $\bar{e}$, an $H$-type receives the a priori image utility with probability 1. Hence, $u_H(\bar{e}) = \bar{\pi} + \lambda_H \mu_H$.

Let now $e$ be a separating equilibrium and consider the same case as in (i). We want to show that $u_H(e) < u_H(\bar{e})$. To see this, notice first that in $e$, by (6), $p_1 \geq \lambda_L \alpha_1$.

Second, using the public’s beliefs, an $H$-type’s expected equilibrium utility in $e$ is given by
\[ u_H(e) = \bar{\pi} + \lambda_H \alpha_1 + \lambda_H \mu_H (1 - \alpha_1) - p_1, \]
which can also be written as $u_H(\bar{e}) + \lambda_H \mu_L \alpha_1 - p_1$. Because $p_1 \geq \lambda_L \alpha_1$, we obtain that
\[ u_H(e) \leq u_H(\bar{e}) + (\lambda_H \mu_L - \lambda_L) \alpha_1. \]

Finally, because $\sigma \in (\mu_L,1)$, we have that $\lambda_H \mu_L - \lambda_L < 0$. Thus, $u_H(e) < u_H(\bar{e})$, and this completes the proof. $\square$

**Proof of Proposition 1:** In what follows, we denote the conjectured equilibrium price stated
in Proposition 1 by \( p_1^* \) and \( p_2^* \) and shall use plain \( p_1 \) and \( p_2 \) to denote generic prices. We first consider the case \( \alpha_1 > \alpha_2 \). We begin by describing the candidate signalling equilibria for given \((p_1, p_2)\): an \( L \)-type always chooses \( S_0 \). An \( H \)-type’s choice is illustrated in Figure 1. Here, \( p_b^* = \lambda_L a_b \) and \( p_b^\mu = \lambda_H \mu_L a_b \), \( b = 1, 2 \). Notice that \( p_2^* = p_2^\mu \). A letter \( i \in I = \{0, 1, 2\} \) indicates the \( H \)-type’s choice in this area. The little arrows point towards the \( H \)-type’s choice on the boundary of areas with different choices. The circle indicates the candidate price equilibrium \((p_1^*, p_2^*)\).

![Figure 1: signalling equilibria for \( \alpha_1 > \alpha_2 \)](image)

We denote by \( e_1 \) the candidate signalling equilibrium in which the \( L \)-type chooses \( S_0 \) and the \( H \)-type chooses \( i \in I \). Table 1 specifies a contact’s belief in \( e_1 \) that she faces type \( H \), given her knowledge \( k \), a consumer choice \( i \), and prices \( p \) (i.e. \( \gamma^k_H (i|p) \)).

\[
\begin{array}{c|cccc}
  i = 1 & 1 & 1 & \mu_H & \mu_H \\
  i = 2 & \zeta & 0 & \zeta & \mu_H \\
  i = 0 & 0 & 0 & \mu_H & \mu_H \\
\end{array}
\]

Table 1

Notice that the second line specifies out-of-equilibrium beliefs: conditional on observing and recognizing 2, the contact assigns an arbitrary but fixed probability \( \zeta \in [0, 1] \) to the event that she faces type \( H \). However, not recognizing 2 \((k = 1 \text{ or } k = \emptyset)\) are not out-of-equilibrium events. In particular, if the contact encounters brand 1 but recognizes 2 only, then she deduces that she faces type \( L \). This is because no one chooses 2 in \( e_1 \), so she would recognize the choice as 1 if she faced type \( H \).

In \( e_2 \), beliefs are specified alike, and in \( e_0 \) we assume for simplicity that the contact always holds the a priori belief \( \mu \).
We now show that the candidate equilibrium thus described is an equilibrium indeed and is selected by our criterion. To do so, we use the following claims that we show below. Let \( b = 1, 2 \), then it holds:

(a) \( p_b < p_b^* \Rightarrow e_b \) is not an equilibrium.
(b) \( p_b > p_b^* \Rightarrow u_H (e_0) > u_H (e_b) \).
(c) \( p_b \in [p_b^c, p_b^m] \Rightarrow e_b \) is an equilibrium and \( u_H (e_b) > u_H (e_0) \)
(d) For \( (p_1, p_2) \in [p_1^c, p_1^m] \times [p_2^c, p_2^m] \): \( u_H (e_1) \geq u_H (e_2) \Leftrightarrow p_1 - p_2 \leq \lambda_H \mu_L (\alpha_1 - \alpha_2) \).

We shall now go through the price space and deduce from (a)-(d) that the candidate equilibrium is selected. In region \([0, p_1^c) \times (0, p_2^m), \) (a) implies that \( e_0 \) is the only equilibrium and thus selected in this region. In \((p_1^m, \infty) \times (p_2^m, \infty), \) (b) implies that \( e_0 \) is selected due to SC. In \([0, p_1^c) \times (p_2^m, \infty), \) (a) implies that \( e_1 \) is not an equilibrium, and (b) implies that \( e_0 \) is selected due to SC. Likewise, in \((p_1^m, \infty) \times [0, p_2^m), \) \( e_0 \) is selected. In \([0, p_1^c) \times [p_2^c, p_2^m], \) (a) and (c) imply that \( e_2 \) and \( e_0 \) are the only equilibria and because of the second claim in (c), SC selects \( e_2 \). Likewise, in \([p_1^c, p_1^m] \times [0, p_2^m), \) \( e_1 \) is selected. Finally, in \([p_1^c, p_1^m] \times [p_2^c, p_2^m], \) (c) and (d) implies that all three equilibria exist. (c) and (d) together with SC imply that \( e_1 \) is selected if \( p_1 - p_2 < \lambda_H \mu_L (\alpha_1 - \alpha_2) \). If \( p_1 - p_2 = \lambda_H \mu_L (\alpha_1 - \alpha_2) \), then our tie-breaking rule selects \( e_1 \), because \( \alpha_1 > \alpha_2 \) by assumption.

This establishes that the candidate equilibrium is a signalling equilibrium indeed. Notice also, that the selection criterion selects exactly one equilibrium. Thus the selection is unique up to to out-of-equilibrium beliefs \( \zeta \). It remains to show that \( p_1^* \) and \( p_2^* \) are equilibrium prices. But this can be seen directly from Figure 1: if \( S_2 \) reduces or increases price, \( e_1 \) is selected, and \( S_2 \) does not make sales. If \( S_1 \) increases price, \( e_2 \) is selected, and \( S_1 \) loses all sales. If she reduces price, \( e_1 \) is still selected, but she gets a smaller price. Since at the equilibrium price, \( e_1 \) is selected, features (i)-(iii) stated in Proposition 1 hold true.

To complete the proof, we have to show claims (a) to (d). We prove the claims only for \( b = 1 \) (the proof for \( b = 2 \) is identical).

As for (a): We prove that in \( e_1 \) the constraint IC\(_L\) is violated. Using the beliefs in the first and third row in Table 1 in (1) gives an L-type’s incentive to choose 1 rather than 0:

\[
\Delta u_L (1, 0) = \lambda_L [(1 - 0) \alpha_1 \alpha_2 + (1 - 0) \alpha_1 (1 - \alpha_2) + (\mu_H - \mu_H) (1 - \alpha_1) \alpha_2 + (\mu_H - \mu_H) (1 - \alpha_1) (1 - \alpha_2)] - p_1.
\]
This simplifies to \( \Delta u_L(1, 0) = \lambda_L \alpha_1 - p_1 \). Hence, because \( p_1 < p_1^c = \lambda_L \alpha_1 \) by assumption, it follows that \( \Delta u_L(1, 0) > 0 \), a contradiction to IC\(_L\). This completes (a).

As for (b): We first compute an \( H \)-type’s utility in \( e_1 \). Using the first row in Table 1 gives:

\[
  u_H(e_1) = \bar{\pi} + \lambda_H \alpha_1 + \lambda_H \mu_H (1 - \alpha_1) - p_1 \\
  = \bar{\pi} + \lambda_H \mu_H + \lambda_H \mu_L \alpha_1 - p_1.
\]

Likewise, in \( e_0 \), an \( H \)-type’s utility is \( u_H(e_0) = \bar{\pi} + \lambda_H \mu_H \). Since, \( p_1 > p_1^m = \lambda_H \mu_L \alpha_1 \) by assumption, comparison of \( u_H(e_0) \) and \( u_H(e_1) \) yields the claim immediately. This completes (b).

As for (c): For existence, we simply check the IC constraints. Identical computations as in (a) yield that \( \Delta u_L(1, 0) \leq 0 \). Likewise, it is easy to check that \( \Delta u_H(1, 0) \geq 0 \). Moreover, the claim that \( u_H(e_1) > u_H(e_0) \) follows from identical computations as in (b). This completes (c).

As for (d): We compute the \( H \)-type’s utility in the respective equilibria. By the same calculations as in (b), we obtain for \( b = 1, 2 \):

\[
  u_H(e_b) = \bar{\pi} + \lambda_H \mu_H + \lambda_H \mu_L \alpha_b - p_b.
\]

Comparison of \( u_H(e_1) \) and \( u_H(e_2) \) yields the claim immediately. This completes (d).

Finally, if \( \alpha_1 = \alpha_2 \), the candidate equilibrium looks similar as in Figure 1 except that now the rectangular region between \( p_1^c \) and \( p_1^* \) disappears, and in region \([p_1^c, p_1^m] \times [p_2^c, p_2^m] \), if \( p_1 = p_2 \), then \( e_1 \) and \( e_2 \) are each played with probability 1/2. It follows then from identical arguments as in the case \( \alpha_1 > \alpha_2 \) that the candidate equilibrium is an equilibrium indeed and is selected by our selection criterion. This completes the proof. \( \Box \)

**Remark:** In this remark, we illustrate why the intuitive criterion in the style of Cho and Kreps (1987) does not rule out separating equilibria that appear implausible in our context. Consider the case \( \alpha_1 > \alpha_2 \). Recall from claim (c) in the proof of Proposition 1 that for \((p_1, p_2) \in [p_1^c, p_1^m] \times [p_2^c, p_2^m] \) both \( e_1 \) and \( e_2 \) are equilibria. Suppose that the price difference \( p_1 - p_2 \) is slightly larger than \( \lambda_H \mu_L (\alpha_1 - \alpha_2) \). In this case, claim (d) shows that \( e_1 \) does not survive our selection criterion. We shall now show that, by contrast, \( e_1 \) is not ruled out by the intuitive criterion. To do so, we first show that in \( e_1 \), \( S_2 \) is a dominated choice for type \( L \) for all out-of-equilibrium beliefs \( \zeta \). Indeed, by Table 1 and (1), in \( e_1 \) an \( L \)-type’s incentive to choose
A little bit of algebra shows that this condition is equivalent to

\[ \Delta u_L (2, 0) = \lambda_L [(\zeta - 0) \alpha_1 \alpha_2 + (0 - 0) \alpha_1 (1 - \alpha_2) + (\zeta - \mu_H) (1 - \alpha_1) \alpha_2 + (\mu_H - \mu_H) (1 - \alpha_1) (1 - \alpha_2)] - p_2. \]

Because \( \zeta \leq 1 \), this is smaller than \( \lambda_L [\alpha_1 \alpha_2 + (1 - \mu_H) (1 - \alpha_1) \alpha_2] - p_2 \). Hence, since \( p_2 \geq p' = \lambda_L \alpha_2 \), it follows that \( \Delta u_L (2, 0) \leq 0 \).

Therefore, \( S_2 \) is equilibrium dominated for the \( L \)-type, and the intuitive criterion would break \( \epsilon_1 \) if the \( H \)-type wanted to deviate to \( S_2 \), given \( \zeta = 1 \). However, this is not the case. Indeed, given \( \zeta = 1 \), it follows from Table 1 and (1), that in \( \epsilon_1 \) an \( H \)-type’s incentive to choose 2 rather than 1 is

\[ \Delta u_H (2, 1) = \lambda_H [(1 - 1) \alpha_1 \alpha_2 + (0 - 1) \alpha_1 (1 - \alpha_2) + (1 - \mu_H) (1 - \alpha_1) \alpha_2 + (\mu_H - \mu_H) (1 - \alpha_1) (1 - \alpha_2)] - p_2 + p_1. \]

The key point to notice is that \( \gamma_H (2) = 0 \), because conditional on \( k = 1 \), the contact does not detect the out-of-equilibrium move of choosing \( S_2 \) (this is reflected by the second entry in the squared bracket). This reduces the \( H \)-type’s incentive to deviate to \( S_2 \). Indeed, \( \Delta u_H (2, 1) \) can be straightforwardly re-arranged to \(-\lambda_H \mu_L (\alpha_1 - \alpha_2) - \lambda_H \mu_H \alpha_1 (1 - \alpha_2) - p_2 + p_1 \). Hence, if \( p_1 - p_2 \) is slightly larger than \( \lambda_H \mu_L (\alpha_1 - \alpha_2) \), the incentive to deviate to \( S_2 \) is slightly larger than \(-\lambda_H \mu_H \alpha_1 (1 - \alpha_2) \) which is non-positive, and this is what we sought to show. \( \square \)

**Proof of Lemma 3:** Suppose first that \( \alpha_1 < 1 \). Because \( S_2 \)'s demand makes a discrete upward jump at \( \alpha_2 = \alpha_1 \) and advertising costs are continuous, matching \( S_1 \)'s advertising is dominated by advertising slightly but strictly more than \( S_1 \). For \( \alpha_1 < \alpha_2 \), \( S_2 \)'s profit is given by

\[ \pi_2 (\alpha_2, \alpha_1) = \mu_H [\lambda_H \mu_L \alpha_2 + (\lambda_L - \lambda_H \mu_L) \alpha_1] - (1/2) c \alpha_2, \]

and the first order condition is solved by monopoly advertising \( \alpha^m = \lambda_H \mu_L \mu_H / c \). If \( \alpha_1 < \alpha^m \), then \( S_2 \) optimally chooses \( \alpha_2 = \alpha^m \) unless this yields negative profits, i.e. unless \( \pi_2 (\alpha^m, \alpha_1) \leq 0 \).

A little bit of algebra shows that this condition is equivalent to \( \alpha_1 \geq (\mu_H \mu_L^2 \lambda_H^2) / (2c (\lambda_H \mu_L - \lambda_L)) \), and this establishes part (A) of the Lemma.

If \( \alpha_1 \geq \alpha_2^m \leq \), then \( S_2 \) optimally chooses \( \alpha_2 = \alpha_1 + \varepsilon \) for small \( \varepsilon \) unless this yields negative profits, i.e. unless \( \lim_{\varepsilon \to 0} \pi_2 (\alpha_1 + \varepsilon, \alpha_1) \leq 0 \). By definition of \( \alpha^c \), this condition is equivalent to \( \alpha_1 \geq \alpha^c \), and this establishes part (B) of the Lemma.
Suppose finally that \( \alpha_1 = 1 \). Then \( S_2 \) can at most match \( S_1 \)'s advertising and makes no sales otherwise. If \( S_2 \) chooses \( \alpha_2 = 1 \), he sells to all \( H \)-types at the competitive price \( p^c = \lambda_L \) with probability 1/2. Thus, her profit is \( \pi_2(1, 1) = (1/2) \mu_H \lambda_L - (1/2) c \). This is non-negative if \( c \leq \lambda_L \mu_H \). But because, by assumption, \( \alpha^c = 2\lambda_L \mu_H / c < 1 \), we have that \( c > 2\lambda_L \mu_H > \lambda_L \mu_H \). Therefore, \( \alpha_2 = 1 \) cannot be the optimal response, and this completes the proof.

\[ \square \]

**Proof of Proposition 2**: Inspection of \( \alpha^*_1 \) implies that \( S_2 \) optimally chooses \( \alpha^*_1 = \alpha^c \vee \alpha^m \) unless this yields negative profits. However, \( S_1 \) makes always non-negative profits when \( \alpha_1 = \alpha^*_1 \). To see this, notice that \( \alpha_2 = \alpha^c \) is \( S_2 \)'s break-even point given \( S_1 \) chooses \( \alpha_1 = \alpha^c \) and all \( H \)-types purchase from \( S_2 \). Moreover, the price of the seller who advertises more falls in the advertising of her rival. Hence, \( S_1 \) cannot do worse than break even when she chooses \( \alpha_1 = \alpha^c \), given \( S_2 \) chooses \( \alpha_2 = 0 \) and all \( H \)-types purchase from \( S_1 \). \[ \square \]

**Proof of Proposition 3**: As for (i): Let \( \alpha^m \geq \alpha^c \). By definition of \( \alpha^m \), the best response against \( \alpha = 0 \) is \( \alpha^m \). As for the best response against \( \alpha^m \), notice that overbidding \( \alpha^m \) leads to losses, because \( \alpha^m \) exceeds the break even advertising level \( \alpha^c \). Moreover, bidding a positive amount less than \( \alpha^m \) has costs only but no benefits. Thus, the best response against \( \alpha^m \) is \( \alpha = 0 \).

As for (ii): Let \( \alpha^m < \alpha^c \). Suppose, there is a pure strategy equilibrium \( (\alpha_1, \alpha_2) \). Suppose first \( \alpha_1 = \alpha_2 \). Then each seller serves the premium market with probability 1/2. By bidding slightly more, either seller would capture the whole market at the same price. Hence, a seller could gain a strictly positive amount at a negligible additional cost. Suppose next that, say \( \alpha_1 > \alpha_2 \). Then \( S_2 \) does not make sales, thus she sets \( \alpha_2 = 0 \), and makes 0 profits. \( S_1 \)'s best reply is thus \( \alpha_1 = \alpha^m \). Because \( \alpha^m < \alpha^c \), \( S_2 \) could make positive profits by slightly overbidding \( \alpha_1 = \alpha^m \). Thus, no equilibrium in pure strategies exist.

To establish that the proposed strategy is an equilibrium, let \( F \) be the c.d.f. of the uniform distribution on \( [0, \alpha] \). We have to show that the expected profit for, say \( S_1 \), given \( S_2 \) plays the mixed strategy \( F \), is (A) constant for all \( \alpha_1 \in [0, \alpha] \) and (B) does not increase for \( \alpha_1 > \alpha \). As for (A), \( S_1 \)'s profit is 0 for \( \alpha_1 < \alpha_2 \), the event \( \alpha_1 = \alpha_2 \) has zero probability, and he sells to all
$H$-types at the equilibrium price for $\alpha_1 > \alpha_2$. Therefore, $S_1$’s expected profit is

$$\pi_1 (\alpha_1; F) = \mu_H \int_0^{\alpha_1} \lambda_L \alpha_2 + \lambda_H \mu_L (\alpha_1 - \alpha_2) \, d\alpha_2 \frac{1}{\alpha} - \frac{1}{2} c \alpha_1^2.$$  

Solving the integral and collecting terms gives

$$\pi_1 (\alpha_1; F) = \left[ \mu_H (\lambda_H \mu_L + \lambda_L) \frac{1}{\alpha} - c \right] \times \frac{1}{2} \alpha_1^2.$$  

But by definition of $\bar{\alpha}$, the term in the squared brackets vanishes, and this establishes (A).

As for (B), note that by spending $\alpha_1 > \bar{\alpha}, S_1$ sells to all $H$-types with probability 1, and therefore obtains expected profit

$$\pi_1 (\alpha_1; F) = \mu_H \left[ \lambda_H \mu_L \alpha_1 - (\lambda_H \mu_L - \lambda_L) \int_0^{\bar{\alpha}} \alpha_2 dF (\alpha_2) \right] - \frac{1}{2} c \alpha_1^2.$$  

With $\int_0^{\bar{\alpha}} \alpha_2 dF (\alpha_2) = (1/2) \bar{\alpha}$, we get

$$\pi_1 (\alpha_1; F) = \lambda_H \mu_H \mu_L \alpha_1 - \frac{1}{2} \mu_H (\lambda_H \mu_L - \lambda_L) \bar{\alpha} - \frac{1}{2} c \alpha_1^2.$$  

Because $\alpha_1 > \bar{\alpha}$, the cost term is strictly less than $- (1/2) c \bar{\alpha}^2$. Hence, the last three terms in $\pi_1 (\alpha_1; F)$ are strictly less than $- (1/2) \bar{\alpha} [\mu_H (\lambda_H \mu_L - \lambda_L) + c \bar{\alpha}]$. Inserting $\bar{\alpha}$ into the squared brackets yields that the squared bracket can be written as $2 \lambda_H \mu_L$. Combining this with the first term in $\pi_1 (\alpha_1; F)$ yields that $\mu_H \lambda_H \mu_L (\alpha_1 - \bar{\alpha})$ is a strict upper bound on $\pi_1 (\alpha_1; F)$. Because $\alpha_1 > \bar{\alpha}$, this bound is strictly negative, and thus $S_1$ cannot gain by deviating to an advertising level larger than $\bar{\alpha}$. \hfill $\Box$

References


