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Doping in Contest-Like Situations*

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Abstract

Individuals who compete in a contest-like situation (for example, in sports, in promotion tournaments, or in an appointment contest) may have an incentive to illegally utilize resources in order to improve their relative positions. We analyze such doping within a tournament game between two heterogeneous players. Three major effects are identified which determine a player's doping decision – a cost effect, a likelihood effect and a windfall-profit effect. Moreover, we discuss whether the favorite or the underdog is more likely to be doped, the impact of doping on overall performance, the influence of increased heterogeneity on doping, the welfare implications of doping, and possible prevention of doping.

Key words: contest, doping, drugs, fraud in research, tournament.
JEL Classification: J3, K42, M5.

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1 Introduction

In practice, there exist diverse competitive situations in which individuals illegally utilize resources in order to improve their positions. Such behavior can be characterized as *doping*. Naturally, we associate doping with professional sports where athletes sometimes take drugs to improve their performance so that their probability of winning a contest increases. Perhaps most spectacular are the cases of detected doping along with professional cycling in the last decade – in particular the disqualification of the Festina athletes during the Tour de France 1998. Moreover, there are also several well-known cases of revealed doping in connection with the Olympic Games.

However, doping as defined above can also take place in other contexts. For example, we can imagine that employees try to bribe customers or supervisors within a hierarchical contest in order to win promotion to a higher level. The first example – bribing of customers – can also be called corruption:[1] We can think, for example, of a sub-supplier's salesman who bribes an employee of another firm so that this firm orders the salesman's initial products. The bribing of supervisors has been discussed in the context of influence activities in the literature.[2] In this case, the employee transfers a monetary or non-monetary side payment to his supervisor in order to get excellent marks which improve his promotion chances. Finally, fraudulent accounting to embellish the financial status of a firm represents a further example of doping in business.[3]

Moreover, we can remember doping cases in which scientists manipulated research results in order to improve their chances of getting additional research funds or attractive positions either at universities or in industry.

[1] For the economics of corruption see, for example, Tirole (1996).
[2] See Fairburn and Malcomson (1994).
[3] See Berentsen and Lengwiler (2004).

In particular, the perils of doping may be highest in experimental research where doping can hardly be detected. The importance of scientific fraud are indicated exemplarily by the report "Fraud and Misrepresentation in Science" (Report F. I-88) of the Council on Scientific Affairs, by the joint report "Scientific Fraud and Misrepresentation" (Report I-89) of the Council on Ethical and Judical Affairs and the Council on Scientific Affairs, and by the "Framework for Institutional Policies and Procedures to Deal with Fraud in Research" of the Association of American Universities, the National Association of State Universities and Land Grant Colleges, and the Council on Graduate Schools from the years 1988 and 1989.

Although these examples are very different, they all have the common characteristics of doping as defined above: There is a tournament or contest-like situation between individuals who compete for a given winner prize (e.g. a medal, a monetary prize, research funds, promotion or appointment to an attractive position), and these individuals have the opportunity to increase their winning probabilities by using illegal activities. Of course, if such behavior is detected, the respective individual will be excluded from the competition, i.e. he will get defaulted. These heterogeneous examples should demonstrate that the doping problem is a relevant topic not only in professional sports but also in a lot of other contest-like situations.

The economic consequences of doping are meaningful. For example, if doping instead of ability and/or effort is decisive for promotion and appointment decisions, there will be a misallocation of talent and/or a decrease in incentives given that competitors observe the impact of doping. Furthermore, doping is meaningful from an economic perspective because lots of resources are spent for the implementation of drug tests in order to prevent doping in professional sports.

This paper concentrates on the doping game between two players of different ability in order to address the following questions: Are favorites or underdogs more likely to be doped? Does doping increase or decrease overall performance? Does increased heterogeneity increase or decrease the likelihood of doping? Is mutual doping welfare enhancing or decreasing? What policies should be adopted in the latter case? Are drug tests prior (ex-ante testing) or after the tournament (ex-post testing) more effective to prevent doping? Note that we focus on the game between the two players who have to decide on both doping and the use of legal inputs (i.e., effort, training or – more general – investment). In the discussion, we will point to several possibilities which can be used to prevent doping. However, we do not explicitly solve the optimization problem of a contest organizer who has to decide on the design of the contest. One important reason for this restriction is that from an economic perspective it is not clear whether the contest organizer (e.g. a private investor, a sport league or society) really wants to prevent doping.

We identify three effects which determine the use of drugs in tournaments. The first effect is called *likelihood effect* which covers the aspect that taking drugs enhances one's own likelihood of winning given that doping is not detected. The second effect is labelled *cost effect*: Recall that in the model we assume that the contestants also invest in legal inputs to win the tournament. Depending on the impact of doping on the outcome of the tournament and depending on the degree of heterogeneity, doping may or may not increase overall investment incentives and, therefore, also investment costs. Doping will increase (decrease) investment costs, if it makes the competition between the heterogeneous players less (more) uneven. The third effect is named *windfall-profit effect*: If one player is got defaulted because of doping and the other player not, the latter one will receive the winner prize for sure without having outperformed his competitor. The interplay of these three effects determines whether mutual doping or a no-doping equilibrium is more likely.[4] Typically, the likelihood effect encourages the use of drugs whereas the windfall-profit effect mitigates it, but the cost effect will be ambivalent. For a certain kind of welfare function it is shown that mutual doping increases welfare under socially optimal tournament prizes. If the organizer of the tournament wants to prevent doping, he should choose a large loser prize and a small spread between winner and loser prize. Under reasonable assumptions, the favorite is more likely to be doped than the underdog, and ex-ante testing will be more effective than ex-post testing given that doping has a sufficiently high impact on the outcome of the tournament.

There exist two strands of related literature. First, there are some papers on doping and cheating in sporting contests. Eber and Thépot (1999) consider two homogeneous athletes who have to choose between doping (D) and no-doping (ND), but do not exert legal inputs such as effort or investment. Depending on the parameter constellations, each of the four combinations (D, D), (ND, ND), (D, ND), and (ND, D) can be an equilibrium. Moreover, by doing comparative statics the two authors analyze several possibilities to prevent doping. However, they do not discuss why society should deter athletes from taking drugs. Maennig (2002) does not consider a technical model on doping. Instead he points to the parallels between doping and corruption in international sports. As a measure against doping, he suggests high financial penalties for the athletes who got defaulted due to a positive doping test. Of course, this solution will only work, if individuals do not face problems of limited liability. Haugen (2004) uses a simple game-theoretic

[4] Here, "more likely" means "exists for a larger range of parameter constellations".

model in order to discuss doping of two homogeneous athletes who do not choose efforts or investments. Under certain assumption he obtains an equilibrium in which both players takes drugs. Since doping implies a disutility when being caught, this equilibrium is a kind of prisoner's dilemma, i.e. both would be better off with not taking drugs.[5] Berentsen (2002) considers doping of heterogeneous players in a contest model. Again, input decisions on effort or investments are neglected. Berentsen shows that, for certain parameter constellations, a mixed-strategy equilibrium exists in which the favorite has a higher probability of taking drugs than the underdog. However, the favorite does not always have a higher probability of winning. Preston and Szymanski (2003) focus on different forms of cheating in contests – doping, sabotage, and match fixing. They discuss how these forms of cheating arise and how society can deal with them. Konrad (2003), contrary to the papers before but in line with this paper, discusses a tournament model in which players choose both doping and legal inputs. However, contrary to this paper the legal input and doping are complements in the players' production functions, and the probability of getting defaulted is zero.[6] Konrad shows that, given a welfare maximizing winner prize, mutual doping is always welfare enhancing. Finally, Berentsen, Bruegger and Loertscher (2004) discuss cheating and doping in an evolutionary game in order to determine those factors which enhance the dissemination of doping within society.

The second strand of literature deals with sabotage in tournaments or contests. Similar to doping, a player gains a relative competitive advantage by choosing sabotage. However, this relative advantage arises from decreasing a competitor's output and not by illegally increasing one's own output.

[5] Bird and Wagner (1997) argue in a similar way that doping leads to a prisoner's-dilemma like situation.

[6] Instead of this, each player faces a convex cost function for consuming drugs.

Papers that deal with this subject are Lazear (1989), Konrad (2000), Chen (2003) and Kräkel (2005). Contrary to the doping literature, sabotaging players never get defaulted but have to bear costs of exerting sabotage.

The paper is organized as follows: In the next section, a tournament model with doping is introduced. This model is analyzed in Section 3 which also contains the main results. Additional results are offered in Section 4, in which ex-ante testing is compared to ex-post testing, and both endogenous tournament prizes and the implications of doping on welfare are discussed. The paper concludes in Section 5.

2 The Model

We consider a rank-order tournament between two risk neutral players or athletes A and B.[7] The output or performance of player i ($i = A, B$) can be described by the function

$$q_i = \mu_i + a_i + \varepsilon_i + d_i. \tag{1}$$

μ_i denotes the legal input of player i which is endogenously chosen by him for improving his performance. This input may be effort or training, for example. According to Lazear and Rosen (1981, p. 842) we will refer to this variable more generally as investment. There are many examples in practice which fit well with such additive performance function: Consider, for example, the case where μ_i indicates how seriously player i trains for the forthcoming tournament. In the case of bribing a supervisor, μ_i denotes those merit points that are due to real effort. Considering the case of a research

[7] Most of the assumptions follow the seminal tournament paper by Lazear and Rosen (1981).

contest, we can interpret μ_i as those research papers of scientist i which are based on correct research.[8]

The ability or talent of player i is given by $a_i > 0$. Let, in the subsequent sections, $\Delta a := a_i - a_j > 0$ $(i, j = A, B; i \neq j)$ be the ability difference. Hence, in the following the subscript "i" indicates the favorite and the subscript "j" the underdog in the tournament. ε_i is an exogenously given random term. It stands for luck, noise or measurement error. ε_i and ε_j $(i, j = A, B; i \neq j)$ are assumed to be identically and independently distributed (i.i.d.) Let $F(\cdot)$ denote the cumulative distribution function (cdf) of the composed random variable $\varepsilon_j - \varepsilon_i$ and $f(\cdot)$ the corresponding density. We assume that $f(\cdot)$ has a unique mode at zero.[9] Note that, due to the i.i.d.-assumption, the density $f(\cdot)$ is symmetric around zero. Finally, the variable d_i describes the doping decision of player i. Each player can only choose between two values $d_i = d > 0$ (doping) and $d_i = 0$ (no-doping).[10] While abilities a_i and a_j are assumed to be common knowledge, each player cannot observe the doping decision of his opponent. Hence, we consider a game of imperfect information.

It is assumed that investment μ_i entails costs on player i which are described by $c(\mu_i)$ with $c(0) = 0$, $c'(\mu_i) > 0$ and $c''(\mu_i) > 0$ for $\mu_i > 0$. Depending on the meaning of μ_i, costs may be the disutility of effort in

[8]Technically, an additively separable production function for a tournament between heterogeneous agents allows the derivation of an explicit solution for the equilibrium investments. If heterogeneity or the impact of doping were introduced via different cost functions or different labor productivities the model would not be analytically tractable any longer.

[9]For example, ε_j and ε_i may be normally distributed with mean m and variance σ^2. Then the convolution $f(\cdot)$ again describes a normal distribution with $\varepsilon_j - \varepsilon_i \sim N(0, 2\sigma^2)$. If ε_j and ε_i are uniformly distributed, the distribution of $\varepsilon_j - \varepsilon_i$ will be triangular with mean zero. In addition, the assumption is not unusual in the tournament literature; see, e.g., Drago, Garvey and Turnbull (1996), Chen (2003).

[10]The simple choice between doping and no-doping sketches the idea that, if a player decides to take drugs, he will choose the optimal degree of doping that solves the trade-off between effectiveness and the probability of being caught.

monetary terms or the opportunity costs for time-consuming training, for example. The two players compete for tournament prizes w_1 and w_2 with $w_1 > w_2 > 0$. The prizes are exogenously given. Although this assumption is not critical here since we focus on the game between the two players without a third party as principal or contest designer, the assumption will be discussed in Section 4. If no player gets defaulted and $q_i > q_j$, player i will be declared the contest winner and receives the high winner prize w_1, whereas player j only receives the loser prize. If no player gets defaulted and $q_i < q_j$, player j will receive w_1 and player i the loser prize w_2. If one player is accused for being doped and gets defaulted, he will receive no prize[11] whereas the other player gets the winner prize w_1. Let the probability of getting defaulted be $\theta_i = \theta > 0$ if the player has chosen $d_i = d$, and $\theta_i = 0$ if he has chosen $d_i = 0$. In other words, only if a player has decided to take drugs, there will be a positive probability of being accused and getting defaulted.[12]

The timing of the game is the following: At the first stage, both players simultaneously decide on d_i $(i = A, B)$. At the second stage, each player i only knows his own doping decision and chooses his input variable μ_i $(i = A, B)$. After that, nature chooses ε_i and ε_j so that the two players' outputs q_i and q_j can be compared. Finally, a doping test takes place so that player i gets defaulted with probability θ_i depending on his choice of d_i $(i = A, B)$.

[11] Assuming a fine for the detected player would not alter the qualitative results of the paper.

[12] Berentsen (2002) considers the possible case of a wrong test result for a player who has decided not to take drugs. This case may happen if, for example, the doping test is imperfect or the player has consumed an illegal drug unintentionally. Note that our results will remain qualitatively the same, if we assume a positive probability of getting defaulted when not being doped.

3 Results

In this section we solve the two-stage game described above by backwards induction. First, we look at the tournament stage where the two players simultaneously choose their inputs $\mu_i^*(d_i, d_j)$ and $\mu_j^*(d_i, d_j)$ for given pairs (d_i, d_j). Then we consider the first stage where the players decide on d_i^* and d_j^* given the anticipated best responses $\mu_i^*(d_i, d_j)$ and $\mu_j^*(d_i, d_j)$ for the next stage. By assumption, each player cannot observe the doping decision of his opponent. However, in equilibrium at stage two each player can infer from the first-stage equilibrium conditions whether the other player is doped or not.

3.1 The Tournament Stage

At the second stage of the game, the two players choose μ_i and μ_j, respectively, in order to maximize their expected utilities for given values d_i and d_j. Player i's probability of winning the tournament can be written as

$$\begin{aligned}\text{prob}\{q_i > q_j\} &= \text{prob}\left\{\mu_i + a_i + \varepsilon_i + d_i > \mu_j + a_j + \varepsilon_j + d_j\right\} \\ &= F\left(\mu_i - \mu_j + \Delta a + d_i - d_j\right)\end{aligned}$$

with $\Delta a > 0$. Let $\Delta w := w_1 - w_2$ denote the spread between winner and loser prize. Then, the favorite's expected utility is given by

$$\begin{aligned}EU_i(\mu_i; d_i, d_j) &= \left(w_2 + \Delta w F\left(\mu_i - \mu_j + \Delta a + d_i - d_j\right)\right)(1-\theta_i)(1-\theta_j) \\ &\quad + w_1(1-\theta_i)\theta_j - c(\mu_i), \qquad (2)\end{aligned}$$

whereas the underdog wants to maximize

$$EU_j\left(\mu_j; d_i, d_j\right) = \left(w_2 + \Delta w\left[1 - F\left(\mu_i - \mu_j + \Delta a + d_i - d_j\right)\right]\right)(1-\theta_i)(1-\theta_j) + w_1(1-\theta_j)\theta_i - c\left(\mu_j\right). \quad (3)$$

With probability $(1-\theta_i)(1-\theta_j)$, the outcome of the tournament is not annulled, because no player gets defaulted. In this case, each player receives w_2 for sure – either directly as loser prize or as part of w_1 in case of winning – and the additional prize spread Δw with probability $F\left(\mu_i - \mu_j + \Delta a + d_i - d_j\right)$ or $1 - F\left(\mu_i - \mu_j + \Delta a + d_i - d_j\right)$, respectively. With probability $(1-\theta_i)\theta_j$, only the underdog gets defaulted so that the favorite is declared the winner of the tournament. With probability $(1-\theta_j)\theta_i$ the opposite happens. In any case, players i and j have to bear their investment costs $c(\mu_i)$ and $c\left(\mu_j\right)$.

Note that doping itself creates a natural trade-off for each player: On the one hand, taking drugs enhances a player's performance and, therefore, also his winning probability. On the other hand, doping implies a positive probability of getting defaulted. As we will see below, it will be crucial whether the influence of d or the influence of θ dominates. Of course, if the probability of being detected, θ, is sufficiently large, neither player will prefer to take drugs and we will have no real doping problem at all. Hence, the scenario in which the *influence of* d *dominates the influence of* θ will be considered as the more relevant one in the remaining part of the paper.[13]

The first-order conditions for μ_i^* and μ_j^* show that, if an equilibrium in pure strategies exists at the tournament stage,[14] it will be symmetric and

[13] Technically, we can think of $\theta \to 0$ in this case. In addition, according to Haugen (2004) the probability of being caught is rather low in practice.

[14] It is well-known in the tournament literature that the existence of pure-strategy equilibria cannot be guaranteed in general; see, e.g., Lazear and Rosen (1981), p. 845, fn. 2; Nalebuff and Stiglitz (1983). Hence, we assume that Δw is sufficiently small, $f(\cdot)$ sufficiently flat and $c(\cdot)$ sufficiently steep so that existence is no problem.

described by

$$\mu_i^* = \mu_j^* = \mu^*(d_i, d_j) = c^{'-1}(\Delta w f(\Delta a + d_i - d_j)(1-\theta_i)(1-\theta_j)). \qquad (4)$$

In Eq. (4), $c^{'-1}(\cdot)$ denotes the inverse of the marginal cost function which is monotonically increasing due to the convexity of $c(\cdot)$. Hence, equilibrium investments μ_i^* and μ_j^* increase in the prize spread Δw and the marginal probability of winning $f(\cdot)$, but decreases in each player's probability to get defaulted. If θ_i and θ_j tend to 1, equilibrium investments will be nearly zero as each player anticipates that the tournament outcome will be almost surely annulled. Given $d_i = d_j$, equilibrium investment $\mu^*(d_i, d_j)$ decreases in the ability difference Δa. Since the density $f(\cdot)$ has a unique mode at zero, it is monotonically decreasing for positive values. In other words, increasing Δa leads to values of $f(\cdot)$ at its right tail which become smaller and smaller. Intuitively, increasing the ability difference means that the competition becomes highly uneven which destroys both players' incentives.

The influence of doping on equilibrium investment is ambivalent. On the one hand, $d_i = d$ instead of $d_i = 0$ further leads to the right-hand tail of the density and, therefore, to smaller values of μ^*. On the other hand, the influence of d_j depends on the magnitude of Δa and the favorite's doping decision. If Δa is sufficiently large, $d_j = d$ leads back to the mode of $f(\cdot)$ so that μ^* increases. The same holds for arbitrarily positive values of Δa if $d_i = d$. In both cases, the underdog gets back into the race by choosing $d_j = d$. Since now the competition becomes more even, both players choose higher investment levels in the tournament. If, however, d is rather large relative to the ability difference and the favorite decides not to take drugs, $d_j = d$ will deteriorate competition. In this situation, again the contest becomes highly uneven, but now the players have switched their roles. Since

d outbalances the initial ability difference Δa, player j becomes the new favorite and player i the new underdog in the tournament. However, note that doping does not only influence μ^* via the marginal winning probability $f(\cdot)$, but also via θ_i and θ_j. As the choice of d implies θ whereas choosing no-doping implies a zero probability of getting defaulted, doping by any player always lessens equilibrium investment via θ_i or θ_j. The findings can be summarized in the following proposition:

Proposition 1 *(i)* $\mu^*(d, d_j) < \mu^*(0, d_j)$ *for* $d_j \in \{0, d\}$.
(ii) If the impact of θ *on* μ^* *dominates the impact of* d*, then* $\mu^*(d_i, d) < \mu^*(d_i, 0)$ *for* $d_i \in \{0, d\}$.
(iii) If the impact of d *on* μ^* *dominates the impact of* θ*, the following results will hold:*
If $d < 2\Delta a$*, then* $\mu^*(d_i, d) > \mu^*(d_i, 0)$ *for* $d_i \in \{0, d\}$*. If* $d > 2\Delta a$*, then* $\mu^*(d, d) > \mu^*(d, 0)$*, but* $\mu^*(0, d) < \mu^*(0, 0)$.[15]

Proposition 1 shows that doping by the favorite will always be detrimental if high investment levels are desirable. First, $d_i = d$ makes the uneven contest more uneven which weakens overall incentives. Second, by the favorite's doping a nullification of the tournament becomes more likely which further lowers incentives. However, doping by the underdog may increase incentives if it counterbalances the favorite's higher ability and may, therefore, be desirable. Note that although the doping variable and investments are substitutes in the players' production functions so that each player can save investment costs by substituting d for μ, a higher doping level may lead to increased exertion of μ^*. The results of Proposition 1 also point out that doping will

[15] Note that the last result $\mu^*(0, d) < \mu^*(0, 0)$ also holds for arbitrarily values of θ_i and θ_j.

generally decrease equilibrium investment, if doping tests are highly reliable in the sense of θ being close to 1.

3.2 The Doping Stage

At the first stage of the game, both players simultaneously have to decide on d_i and d_j while knowing that both will choose $\mu^*(d_i, d_j)$ according to (4) in the subsequent tournament stage. We will focus on two possible equilibria – the doping equilibrium $\left(d_i^*, d_j^*\right) = (d, d)$ and the no-doping equilibrium $\left(d_i^*, d_j^*\right) = (0, 0)$.[16]

We start by analyzing the doping equilibrium (d, d). Both players will choose doping as best responses if

$$\begin{aligned} EU_i\left(\mu^*(d,d); d, d\right) &> EU_i\left(\mu^*(0,d); 0, d\right) \quad \text{and} \\ EU_j\left(\mu^*(d,d); d, d\right) &> EU_j\left(\mu^*(d,0); d, 0\right) \Leftrightarrow \end{aligned}$$

$$\begin{aligned} &(w_2 + \Delta w F(\Delta a))(1-\theta)^2 + w_1(1-\theta)\theta - c(\mu^*(d,d)) > \\ &(w_2 + \Delta w F(\Delta a - d))(1-\theta) + w_1\theta - c(\mu^*(0,d)) \end{aligned} \tag{5}$$

and

$$\begin{aligned} (w_2 + \Delta w[1 - F(\Delta a)])(1-\theta)^2 + w_1(1-\theta)\theta - c(\mu^*(d,d)) &> \\ (w_2 + \Delta w[1 - F(\Delta a + d)])(1-\theta) + w_1\theta - c(\mu^*(d,0)). \end{aligned} \tag{6}$$

[16] The analysis of the two possible equilibria $\left(d_i^*, d_j^*\right) = (d, 0)$ and $\left(d_i^*, d_j^*\right) = (0, d)$ does not offer new insights since the same effects as in the two symmetric equilibria will work. When comparing the two asymmetric equilibria it is not clear which conditions are more restrictive.

By rearranging these inequalities we obtain

$$\begin{aligned}
&\Delta w\,(1-\theta)\,[F\,(\Delta a)\,(1-\theta)-F\,(\Delta a-d)] > \\
&\theta\,(w_1\theta+w_2\,(1-\theta))+c\,(\mu^*\,(d,d))-c\,(\mu^*\,(0,d))
\end{aligned} \tag{5'}$$

and

$$\begin{aligned}
&\Delta w\,(1-\theta)\,([1-F(\Delta a)]\,(1-\theta)-[1-F\,(\Delta a+d)]) > \\
&\theta\,(w_1\theta+w_2\,(1-\theta))+c\,(\mu^*\,(d,d))-c\,(\mu^*\,(d,0))\,.
\end{aligned} \tag{6'}$$

Note that the right-hand side of inequality (5') is always smaller than the right-hand side of (6') since $c\,(\mu^*\,(0,d)) > c\,(\mu^*\,(d,0))$. Hence, inequality (6') will always be more restrictive than (5') if[17]

$$\begin{aligned}
&F\,(\Delta a)\,(1-\theta)-F\,(\Delta a-d) > \\
&[1-F\,(\Delta a)]\,(1-\theta)-[1-F\,(\Delta a+d)] \Leftrightarrow
\end{aligned}$$

$$[F\,(\Delta a+d)+F\,(\Delta a-d)-1] < [2F\,(\Delta a)-1]\,(1-\theta)\,.$$

If θ is sufficiently large, the inequality will never hold. However, given the more interesting case in which the influence of d dominates the influence of θ, the inequality will hold, if

$$\begin{aligned}
F\,(\Delta a+d)+F\,(\Delta a-d) &< 2F\,(\Delta a) \Leftrightarrow \\
F\,(\Delta a+d)-F\,(\Delta a) &< F\,(\Delta a)-F\,(\Delta a-d)\,. \qquad (7)
\end{aligned}$$

[17] Note that, because of the symmetry of $f\,(\cdot)$, $2F\,(\Delta a) > 1$ since $\Delta a > 0$, and $F\,(\Delta a+d)+F\,(\Delta a-d)-1 = F\,(\Delta a+d)-F\,(-\Delta a+d) > 0$ since each cdf is monotonically increasing.

Since $f(\cdot)$ has a unique mode at zero, the corresponding cdf $F(\cdot)$ is strictly convex for negative values and strictly concave for positive values, which implies that (7) holds as $\Delta a > 0$. In this case, the underdog's condition for a doping equilibrium is more restrictive than the favorite's equilibrium condition. In other words, the favorite prefers mutual doping under more parameter constellations than the underdog. To sum up, we can rearrange condition (6') in order to obtain the following result:

Proposition 2 *Let the influence of d dominate the influence of θ. A doping equilibrium $\left(d_i^*, d_j^*\right) = (d, d)$ will exist if and only if*

$$\Delta w (1-\theta)\left[F(\Delta a + d) - F(\Delta a)(1-\theta)\right] >$$
$$\theta w_1 + c\left(\mu^*(d,d)\right) - c\left(\mu^*(d,0)\right). \qquad (8)$$

Proposition 2 highlights three effects which determine the doping decision of the two players:[18] The left-hand side of (8) can be referred to as **likelihood effect**. As $F(\cdot)$ is monotonically increasing, the choice of $d_j = d$ leads to a higher probability of winning for the underdog. Hence, the likelihood effect supports condition (8). However, note that the left-hand side of (8) only shows a reduced form of the likelihood effect. According to (6') the original version of the likelihood effect is characterized by

$$(1-\theta)\left(\left[1-F(\Delta a)\right](1-\theta) - \left[1-F(\Delta a + d)\right]\right),$$

and this difference will only be positive, if the influence of d dominates the one of θ. Since this expression describes the additional probability mass of winning when taking drugs given that the other player is doped, it should be

[18] Condition (8) only captures the underdog's preferences. However, similar considerations also hold for the favorite.

indeed positive, if the doping problem should be of any relevance. Otherwise, it will not be plausible at all for players to take drugs.

The difference $c(\mu^*(d,d)) - c(\mu^*(d,0))$ implies a **cost effect**. If the impact of d on μ^* dominates the impact of θ, this difference will be positive. In this case, the underdog's doping increases equilibrium investments and investment costs which makes condition (8) less likely to hold. The term θw_1 characterizes a **windfall-profit effect**. If the underdog chooses $d_j = d$ instead of $d_j = 0$, he will benefit with a lower probability from the favorite getting defaulted (so that the underdog receives the winner prize). This effect also makes condition (8) less likely to hold. The interplay of all three effects determines whether both players mutually prefer to take drugs or not. Of course, the importance of each effect depends on the specific sport under consideration and on the concrete meaning of the diverse variables. In particular, if μ_i and μ_j only capture the actual effort exerted in the contest so that $c(\cdot)$ describes the disutility of effort, the cost effect can be neglected in most sports. In this case, each athlete will typically do his best without thinking about the corresponding exertions. If, however, μ_i and μ_j describe time consuming training, the cost effect may become very important.

Now we can do some comparative statics concerning condition (8). As we have seen in Proposition 1, the effect of doping on investment and, hence, investment costs is ambiguous. For this reason we ignore the cost effect for a moment.[19] For $\theta \to 0$ or $d \to \infty$ we obtain the trivial result that mutual doping is always an equilibrium. Similarly, if $\theta \to 1$, the combination (d,d) will never be an equilibrium. Next, we can analyze the influence of heterogeneity on condition (8). The ability difference Δa can serve as a measure

[19] For example, we can think of μ_i and μ_j only describing effort as argued above. However, the influence of the cost effect can easily be added by considering Eq. (4) and the discussion of Proposition 1, but such a complete analysis would be less concise.

of heterogeneity. Differentiating the term in brackets at the left-hand side of (8) with respect to Δa yields $f(\Delta a + d) - f(\Delta a)(1-\theta)$ so that we obtain two results: 1. This term tends to be *negative*, if θ is rather low, so that d is decisive. 2. The term will become *positive*, if d is sufficiently small, so that θ becomes crucial. Therefore, higher heterogeneity will *not support* a doping equilibrium if the doping test is highly unreliable, whereas it will *support* mutual doping if doping has only a small impact on the outcome of the tournament. An intuition for the first finding comes from the concavity of $F(\cdot)$ in the positive range. If Δa is quite large, the underdog's additional probability of winning by choosing $d_j = d$ instead of of $d_j = 0$, i.e. $[1 - F(\Delta a)] - [1 - F(\Delta a + d)] = F(\Delta a + d) - F(\Delta a)$, is very small because the favorite will almost surely win the tournament due to the large ability difference. In this situation the likelihood effect has only a low impact. The second result can be explained by the fact that, in the end, the relevant additional probability of winning is $F(\Delta a + d) - F(\Delta a)(1-\theta)$, i.e. the extra probability conditional on whether the doping underdog will not get defaulted. Of course, if θ is large and hence becomes crucial, the likelihood effect will be of great importance which supports condition (8). As we have assumed in Proposition 2 that the influence of d dominates the one of θ, we can conclude that in this situation more heterogeneity would rather work against a doping equilibrium.

Finally, we can analyze the influence of the tournament prizes w_1 and w_2 on (8). At first sight, one might expect that high winner prizes provoke more doping and are therefore detrimental. However, condition (8) shows that both the likelihood effect and the windfall-profit effect increase in w_1 so that the influence of the winner prize is not clear at all. The left-hand side of (8) points out that the prize spread Δw is a crucial parameter. Standard tournament results show that investment incentives do not depend on the absolute values of w_1 and w_2 but on the spread Δw (see also Eq. (4)). The likelihood effect demonstrates that the incentives to be doped also increase in the prize spread. Hence, a tournament organizer can try to prevent doping by setting only a moderate prize spread.[20] Note that this decision would also lessen the perils of other forms of cheating like sabotage (see Lazear 1989), but comes at the cost that productive incentives would also decrease. Since $\Delta w = w_1 - w_2$, mutual doping can be prevented by a high loser prize w_2. This policy would have two effects: First, the prize spread and hence the expected gains from doping would decrease. Second, by inspection of (6) we can see that each player earns an expected base salary $w_2(1-\theta_i)(1-\theta_j)$. If w_2 is high, the players might prefer not to take drugs in order to receive the base salary w_2 with higher probability.

In the next step, we can analyze under which conditions a no-doping equilibrium $\left(d_i^*, d_j^*\right) = (0,0)$ will exist. By rearranging the equilibrium conditions $EU_i\left(\mu^*(0,0);0,0\right) > EU_i\left(\mu^*(d,0);d,0\right)$ and $EU_j\left(\mu^*(0,0);0,0\right) > EU_j\left(\mu^*(0,d);0,d\right)$ we get the following proposition:

Proposition 3 *A no-doping equilibrium* $\left(d_i^*, d_j^*\right) = (0,0)$ *will exist if and only if*

$$\begin{aligned}\Delta w\left[F(\Delta a + d)(1-\theta) - F(\Delta a)\right] &< \\ \theta w_2 + c\left(\mu^*(d,0)\right) - c\left(\mu^*(0,0)\right)&\end{aligned} \tag{9}$$

[20] See also Eber and Thépot (1999), 441-442. Up to now, it is not clear whether a tournament organizer really wants to prevent doping. This point will be discussed in Section 4.

and

$$\Delta w\left(\left[1-F\left(\Delta a-d\right)\right]\left(1-\theta\right)-\left[1-F\left(\Delta a\right)\right]\right) \;<\; \theta w_2+c\left(\mu^*\left(0,d\right)\right)-c\left(\mu^*\left(0,0\right)\right). \tag{10}$$

Contrary to the doping equilibrium it is not clear whether the favorite's or the underdog's equilibrium condition is more restrictive. The windfall-profit effect is identical for both players. Since $\mu^*(d,0) < \mu^*(0,d)$ the right-hand side of (9) is smaller than the right-hand side of (10). Hence, according to the cost effect the favorite's equilibrium condition seems to be more restrictive. However, comparing the left-hand sides of (9) and (10) we can see from condition (7) (i.e. from the concavity of the cdf in the positive range) that – for all values of θ – the likelihood effect is always stronger for the underdog. Comparative statics give insights similar to the results in the discussion of Proposition 2. In particular, the higher the loser prize w_2 and the lower the prize spread Δw the more likely both players will prefer not to be doped.

Finally, we can infer from the results of Propositions 1 to 3, which type of player more likely tends to be doped.[21] As can be seen from conditions (5') and (6') as well as from (9) and (10), the windfall-profit effect influences both players in the same way. Hence, this effect can be neglected in our comparison. Concerning the cost effect, doping will be more attractive for the favorite than for the underdog: According to Proposition 1 doping by the favorite always decreases the players' investment costs, whereas the impact of the underdog's doping on costs is not clear. Whether the likelihood effect stronger supports doping by the favorite than by the underdog or vice versa,

[21] In the model by Berentsen (2002) for certain parameter constellations there exists a mixed-strategy equilibrium where the favorite will be doped with a higher probability than the underdog if doping is sufficiently effective, the costs of doping are sufficiently small and the winner prize is sufficiently large.

depends on the given situation. Recall that the cdf is convex in the negative range and concave in the positive range. Consider, for example, the situation of the favorite.[22] If the underdog decides to take drugs, the favorite's original lead Δa will be reduced by $-d$. In this situation, by also choosing d the favorite increases his winning probability by the amount $F(\Delta a) - F(\Delta a - d)$ (see (5')). If, however, the underdog chooses no-doping, the favorite's additional winning probability from doping will only be $F(\Delta a + d) - F(\Delta a)$ (see (9)). When discussing the relative importance of the likelihood effect for both players we also have to take into account that basically the favorite already has a lead Δa so that the additional winning probability from doping should be lower compared to an underdog who takes drugs. Of course, the higher the lead Δa the lower will be the probability gains from taking drugs for the favorite. To sum up, – when abstracting from θ_i and θ_j – the likelihood and the cost effect always work into the same direction for the favorite, whereas this is not clear for the underdog: By choosing $d_i = d$ the favorite does not only increase his winning probability but also lowers investment costs (see Proposition 1(i)). However, choosing $d_j = d$ instead of $d_j = 0$ increases the underdog's winning probability, too, but will increase investment costs if the players significantly differ in ability or the favorite also chooses doping (see Proposition 1(iii)).

4 Discussion

Ex-ante versus ex-post testing

As we have seen in Section 3, the organizer of a tournament can decrease the players' incentives to take drugs by choosing a high loser prize or a low

[22] Of course, similar considerations also hold for the underdog.

prize spread. However, up to now it is not clear whether the organizer would really be interested in preventing the consumption of drugs. On the one hand, doping and the disqualification of players who have consumed drugs harms the reputation of a specific sport which may imply fewer spectators or lower revenues from the selling of broadcasting rights. On the other hand, the organizer may be interested in high performances q_i and high investments μ^*. Note that q_i always increases in d_i, and that μ^* may increase under certain conditions if the underdog takes drugs (see Proposition 1 (iii)). For the moment, let us assume that the organizer is interested in preventing the consumption of drugs and that he can choose between ex-post testing after the tournament (as in Section 3) and ex-ante testing before the tournament starts. In the case of ex-ante testing, the doping test takes place between the two stages of the game which have been discussed in the previous section (i.e., after the doping decision but before the players choose investment).[23] In the following, we will discuss the question, whether the organizer of the tournament should prefer ex-ante or ex-post training in order to combat doping.

In the case of ex-ante testing, the objective function of the favorite at the tournament stage is given by

$$\begin{aligned} EU_i\left(\mu_i; d_i, d_j\right) &= \left(w_2 + \Delta w F\left(\mu_i - \mu_j + d_i - d_j + \Delta a\right) - c\left(\mu_i\right)\right)\left(1-\theta_i\right)\left(1-\theta_j\right) \\ &\quad + w_1\left(1-\theta_i\right)\theta_j \end{aligned} \tag{11}$$

[23] In practice, if μ_i and μ_j denote efforts ex-ante testing will mean a test directly before the tournament. If the investments stand for (final) training, ex-ante testing will take place in an early training period.

and that of the underdog by

$$EU_j\left(\mu_j; d_i, d_j\right) = \left(w_2 + \Delta w[1 - F(\mu_i - \mu_j + d_i - d_j + \Delta a)] - c\left(\mu_j\right)\right)(1-\theta_i)(1-\theta_j) + w_1(1-\theta_j)\theta_i. \quad (12)$$

The important difference of (11) and (12) compared to the objective functions (2) and (3) is given by the fact that if using ex-ante testing the tournament will not take place in any case. The two players will only compete (by choosing investments) with probability $(1-\theta_i)(1-\theta_j)$. With probability $1-(1-\theta_i)(1-\theta_j)$ there will be no tournament, no investments and no investment costs because of the disqualification of at least one contestant.[24]

In analogy to Section 3, we obtain a symmetric equilibrium described by

$$\hat{\mu}_i^* = \hat{\mu}_j^* = \hat{\mu}^*(d_i, d_j) = c'^{-1}\left(\Delta w f\left(d_i - d_j + \Delta a\right)\right). \quad (13)$$

By comparing (13) and (4) we can see that, for a given pair (d_i, d_j), equilibrium investments will always be larger under ex-ante than under ex-post testing. The intuition for this result is straightforward. In the case of ex-post testing, players do not know whether the tournament will be annulled afterwards so that their investments are lost. This leads to lower incentives which decrease in θ_i and θ_j according to (4).

Now we can turn to the doping stage where the two players have to decide on d_i and d_j, respectively. Again, we can consider a possible doping equilibrium (d, d). Note that the corresponding equilibrium conditions will be identical to (5') and (6') with the exception that $c(\mu^*(d,d))$ has to be

[24] Note that if one player gets defaulted under ex-ante testing, the other player will receive the winner prize although no tournament takes place. This special situation is due to the fact that we consider a two-person tournament. However, for $n > 2$ contestants we would have a similar effect. In that case, the remaining contestants' winning probability would increase by the disqualification of a player.

replaced with $c(\hat{\mu}^*(d,d))(1-\theta)^2$, $c(\mu^*(0,d))$ with $c(\hat{\mu}^*(0,d))(1-\theta)$, and $c(\mu^*(d,0))$ with $c(\hat{\mu}^*(d,0))(1-\theta)$. Since $c(\hat{\mu}^*(0,d)) > c(\hat{\mu}^*(d,0))$, again because of (7) the underdog's condition for a doping equilibrium is more restrictive than the favorite's condition, given that the influence of d dominates the one of θ. Hence, under ex-ante testing and dominance of d, we will have an equilibrium $\left(d_i^*, d_j^*\right) = (d,d)$ if and only if

$$\begin{aligned} \Delta w (1-\theta)\left[F(\Delta a + d) - F(\Delta a)(1-\theta)\right] &> \\ \theta w_1 + c(\hat{\mu}^*(d,d))(1-\theta)^2 - c(\hat{\mu}^*(d,0))(1-\theta). & \end{aligned} \tag{14}$$

Comparing (14) and (8) yields the following result:

Proposition 4 *Let the influence of d dominate the influence of θ. A doping equilibrium $\left(d_i^*, d_j^*\right) = (d,d)$ will be more likely under ex-ante than under ex-post testing if and only if*

$$\begin{aligned} c(\mu^*(d,d)) - c(\mu^*(d,0)) &> \\ c(\hat{\mu}^*(d,d))(1-\theta)^2 - c(\hat{\mu}^*(d,0))(1-\theta) & . \end{aligned} \tag{15}$$

Here, "more likely" means that the equilibrium condition (14) is less restrictive than condition (8). The proposition points out that only the cost effect is decisive whether mutual doping is more likely under ex-ante or ex-post testing. If the influence of d dominates the influence of θ (i.e., if d is sufficiently large relative to θ) both the left-hand and the right-hand side of (15) will be positive. We obtain the following finding:

Corollary 1 *If $c'''(\mu) < \left[c''(\mu)\right]^2 / c'(\mu)$, there will exist a threshold value $\bar{d}$ so that for $d > \bar{d}$ a doping equilibrium is more likely under ex-post testing.*

Proof. See appendix. ■

First note that the condition given in the corollary is not restrictive at all, since it holds for most of the cost functions (e.g. for the family of cost functions $c(\mu) = \frac{\mu^\delta}{\delta}$ with $\delta > 1$). According to Corollary 1, given a sufficiently large impact of doping, the organizer of the tournament will always prefer ex-ante testing if he wants to prevent mutual doping by the two players. Note that if d goes to infinity, investments $\mu^*(d, 0)$ and $\hat{\mu}^*(d, 0)$ – and hence the corresponding investment costs – will tend to zero. In this case, the cost effect is solely determined by the (expected) investment costs for the pair (d, d). The lower these costs, the more attractive will be mutual doping for the players. Since for $d > \bar{d}$ we have $c(\hat{\mu}^*(d, d))(1-\theta)^2 > c(\mu^*(d, d))$ (i.e. expected costs are higher under ex-ante testing), the players more likely take drugs under ex-post testing in this situation.

Endogenous tournament prizes

Until now, tournament prizes have been assumed to be exogenous in order to focus on the doping game between the two heterogeneous players. Discussing endogenous prizes seems to be problematic in this context. First, we have to specify the objective function of the organizer of the tournament. In the economic literature on sport contests, several possible objective functions have been discussed.[25] The organizer may be interested in competitive balance to guarantee an attractive competition and, therefore, high revenues from the selling of broadcasting rights. Alternatively, the organizer may want to maximize total expected performance minus tournament prizes, $E[q_i + q_j] - w_1 - w_2$. As another alternative, the organizer's revenues may increase in the realization of top performances (e.g., beating records) so that the organizer may want to maximize $max\{q_i, q_j\} - w_1 - w_2$. To sum up, it

[25] See, for example, Fort and Quirk (1995), Szymanski (2003), Szymanski and Kesenne (2004), Falconieri, Palomino and Sákovics (2004), Kesenne (2005).

is not quite clear how the correct objective function should look like.

Second, given a certain objective function, it is not obvious whether the organizer wants to prevent doping or not. For example, the organizer may want to implement doping by the underdog but prevent doping by the favorite in order to increase competitive balance.[26] As can be seen from (1), (4) and (13), the impact of such increased competitive balance on both expected total performance and maximum individual performance is ambiguous. Besides the basic ambiguity resulting from the trade-off concerning d_i (or d_j) and θ_i (or θ_j), on the one hand equilibrium investments will increase in more balanced competition via $(d_i, d_j) = (0, d)$ if the influence of d dominates the one of θ. This in turn increases total expected performance as well as maximum individual performance. On the other hand, the pair $(d_i, d_j) = (0, d)$ may lead to low values of both $E[q_i + q_j]$ and $max\{q_i, q_j\}$, since q_i directly increases in d_i so that no-doping by the favorite may be unfavorable for the organizer in the end. Finally, if the reputation of a certain sport is fundamentally harmed by detected doping, preventing doping may be the organizer's primary aim in any case.

When calculating optimal prizes, the organizer would choose w_1 and w_2 in order to maximize his objective function subject to the players' incentive constraint (4), the two participation constraints $EU_i\left(\mu^*\left(d_i^*, d_j^*; d_i^*, d_j^*\right)\right) \geq \bar{u}$ and $EU_j\left(\mu^*\left(d_i^*, d_j^*; d_i^*, d_j^*\right)\right) \geq \bar{u}$ with $\bar{u}$ denoting the players' reservation utility, and two constraints implementing a favored pair $\left(d_i^*, d_j^*\right)$. If, for example, the organizer wants to implement a doping equilibrium (d, d) and the players are not wealth-restricted (i.e., there is no limited liability), the organizer will choose the lowest possible loser prize w_2^* which makes the participation constraint of the player with the lower expected utility just

[26] Here, we have to assume that $d < 2\Delta a$ is satisfied, because otherwise competition would become more uneven.

bind. This loser prize both guarantees that the organizer's labor costs become as low as possible and supports the doping condition (8). In this case, we have $d_i^* = d_j^* = d$ so that the underdog's participation constraint is binding in the optimum and the favorite, who starts with a lead Δa in the tournament, receives a positive rent $EU_i - \bar{u} > 0$. Furthermore, a rather large prize spread Δw^* is chosen by the organizer in order to support condition (8) and to implement considerable investment levels μ^* according to (4). Note that despite risk neutral players and unlimited liability, the organizer will not implement first-best effort because of the players' heterogeneity.

Welfare analysis

For a welfare analysis, similar problems arise as in the discussion of optimal tournament prizes since first we have to define welfare in the given context. Following the analysis of Konrad (2003) we can, for example, define welfare as the difference of expected total output minus total costs. Hence, in our model welfare would be

$$W = \sum_{k \epsilon \{i,j\}} \left(\mu_k + a_k + d_k + E[\varepsilon_k] - c\left(\mu_k\right)\right) \tag{16}$$

Konrad (2003) considers a model in which the doping and the investment inputs are complements in the production function. Furthermore, both inputs have separable convex cost functions.[27] There is only one prize in the Konrad model which is given to the tournament winner. Konrad shows that if the tournament prize that maximizes welfare is chosen, mutual doping will be welfare improving. In our model, both inputs are substitutes in the production function. However, we find the same curious result:

[27] In the Konrad model, doping does not imply the possibility to get defaulted.

Proposition 5 *If tournament prizes are chosen which maximize welfare* W*, mutual doping* $(d_i, d_j) = (d, d)$ *will be welfare enhancing.*

Proof. Obviously, the first-best investment for each player which maximizes W as given in (16), is

$$\mu^{FB} = c'^{-1}(1). \tag{17}$$

We know that players choose investments according to (4). Comparing (17) and (4) shows that the prize spread

$$\Delta w^{FB} = \frac{1}{f(\Delta a + d_i - d_j)(1 - \theta_i)(1 - \theta_j)} \tag{18}$$

implements first-best incentives for any pair (d_i, d_j). Hence, the only influence of doping on welfare remains via d_k $(k = i, j)$ in (16) so that $d_i = d_j = d$ leads to maximum welfare. ■

If welfare maximizing tournament prizes are chosen, these prizes will always be adjusted to the doping levels (d_i, d_j). In other words, incentives are not influenced by doping under optimal prizes. In this case, doping d will only increase both players' aggregate performance and therefore overall welfare. Of course, this curious result crucially depends on the definition of welfare. If overall welfare is reduced by detected consumption of drugs since the spectators' utilities decrease, mutual doping will not necessarily be welfare maximizing.[28]

[28] Note that the athletes' health costs from taking drugs typically also reduce welfare.

5 Conclusion

Although doping in contests is an important topic from an economic perspective, it is not clear whether the utilization of illegal resources should be generally prevented from the viewpoint of the contest organizer or from society's perspective. Hence, in this paper we focus on a stylized doping game between two players in order to analyze the determinants of doping and possible alternatives for preventing doping given that such behavior is welfare reducing.

Contrary to most of the existing doping models, we assume heterogeneous players who choose both doping and a legal input (e.g., effort or training) and face a positive probability of getting defaulted in case of doping. Furthermore, we assume that doping, the legal input and ability are substitutes in the players' production function. We identify three effects which determine the attractiveness for taking drugs: a likelihood effect (i.e. doping increases one's own probability of winning if not getting defaulted), a cost effect (i.e. doping influences the exertion of the legal input and hence costs), a windfall-profit effect (i.e. if one player is disqualified, the other player wins for sure). Interestingly, under reasonable assumptions the favorite is more likely to use drugs than the underdog, and mutual doping by both players may be welfare enhancing. Comparing ex-ante and ex-post drug tests shows that ex-ante testing will be more effective in preventing the consumption of drugs, if doping has a sufficiently high impact on the outcome of the tournament.

In a next step, it would be interesting to analyze the players' doping decision within a closed model where the organizer of the tournament wants to maximize a certain objective function and doping is a continuous variable. However, this step will not be a trivial one since there exist several trade-offs when discussing doping in such a context. In particular, more doping will typically lead to a higher probability of getting defaulted and to higher health costs of the players. Hence, it will be difficult to find out the optimal level of doping from both the organizer's and society's perspective. Perhaps, it will be helpful to concentrate on a specific application, for example on doping in a certain professional sport. By this it might be easier to formulate a concrete objective function for the organizer of the contest since in professional sports the selling of broadcasting rights is of major importance.

Appendix:

Proof of Corollary 1:

By using (4) and (13), condition (15) can be rewritten as

$$\begin{aligned} H\left(\Delta w f\left(\Delta a+d\right)\right)\left(1-\theta\right)-H\left(\Delta w f\left(\Delta a+d\right)\left(1-\theta\right)\right) &> \\ H\left(\Delta w f\left(\Delta a\right)\right)\left(1-\theta\right)^{2}-H\left(\Delta w f\left(\Delta a\right)\left(1-\theta\right)^{2}\right) & \qquad \text{(A1)} \end{aligned}$$

with $H\left(\cdot\right)=c\left(c^{'-1}\left(\cdot\right)\right)$. First, we will show that $H\left(\cdot\right)$ is convex given the condition in Corollary 1. In order to simplify notation, let

$$\mu\left(x\right):=c^{'-1}\left(x\right) \qquad \text{(A2)}$$

so that

$$H\left(x\right)=c\left(\mu\left(x\right)\right). \qquad \text{(A3)}$$

From (A2) we have $c'\left(\mu\left(x\right)\right)=x$ and, hence, $c''\left(\mu\left(x\right)\right)\mu'\left(x\right)=1$ which gives

$$\mu'\left(x\right)=\frac{1}{c''\left(\mu\left(x\right)\right)} \qquad \text{(A4)}$$

so that we obtain

$$\mu''\left(x\right)=-\frac{c'''\left(\mu\left(x\right)\right)\mu'\left(x\right)}{\left[c''\left(\mu\left(x\right)\right)\right]^{2}}. \qquad \text{(A5)}$$

$H''\left(x\right)>0$ ensures convexity of $H\left(\cdot\right)$. We obtain $H'\left(x\right)=c'\left(\mu\left(x\right)\right)\mu'\left(x\right)$ and, by using (A4) and (A5),

$$\begin{aligned} H''\left(x\right) &= c''\left(\mu\left(x\right)\right)\left[\mu'\left(x\right)\right]^{2}+c'\left(\mu\left(x\right)\right)\mu''\left(x\right) \\ &= \frac{c''\left(\mu\left(x\right)\right)}{\left[c''\left(\mu\left(x\right)\right)\right]^{2}}-\frac{c'''\left(\mu\left(x\right)\right)c'\left(\mu\left(x\right)\right)}{\left[c''\left(\mu\left(x\right)\right)\right]^{3}} \\ &= \frac{1}{c''\left(\mu\left(x\right)\right)}-\frac{c'''\left(\mu\left(x\right)\right)c'\left(\mu\left(x\right)\right)}{\left[c''\left(\mu\left(x\right)\right)\right]^{3}} \end{aligned}$$

which is positive, if $\left[c''\left(\mu\left(x\right)\right)\right]^2/c'\left(\mu\left(x\right)\right) > c'''\left(\mu\left(x\right)\right)$.

Next we can show that (A1) and, therefore, (15) can never hold, if d becomes sufficiently large. Note that the argument in the first $H\left(\cdot\right)$-function at the left-hand side of (A1) describes a convex combination of 0 and $\Delta w f\left(\Delta a + d\right)$, whereas the argument in the first $H\left(\cdot\right)$-function at the right-hand side of (A1) is a convex combination of 0 and $\Delta w f\left(\Delta a\right)$. Since $H\left(\cdot\right)$ is convex, both the left-hand and the right-hand side of (A1) are positive. Recall that $f\left(\cdot\right)$ has a unique mode at zero and, therefore, monotonically decreases for positive values. Hence, $d \to \infty$ leads to $f\left(\Delta a + d\right) \to 0$, and (A1) reduces to

$$0 > H\left(\Delta w f\left(\Delta a\right)\right)\left(1-\theta\right)^2 - H\left(\Delta w f\left(\Delta a\right)\left(1-\theta\right)^2\right), \tag{A6}$$

which is always false because of $H\left(\cdot\right)$'s convexity. In other words there exists a threshold $\bar{d}$ so that (15) never holds for $d > \bar{d}$.

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