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Regional integration and economic development: A theoretical approach
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Abstract
We use a model of combined endogenous growth and economic geography to study the impact of regional economic integration on the member and non-member countries of a regional union. Regional integration affects growth through interregional technology diffusion symbolized by knowledge spillovers generated at home and spreading to the partner countries. Spillovers flow from the leader to the follower. Following integration, the lagging country has access to a bigger stock of knowledge that fosters an increase in its rate of growth and extends the diversity of its products. Trade in goods or in FDI and flows of ideas are two faces of the same coin. We show that the progressive decrease in transaction costs through the phasing out of barriers to trade together with product imitation can foster growth and convergence in the member countries. However, in order to avoid eventual trade and investment diversions, the non-member should envisage to join the integrated zone.

Keywords: regional economic integration, endogenous growth, economic geography

JEL classification: F12, F15, F43, O18, O30, O41, R11, R12, R13

1 Introduction
This paper intends to draw a framework capable of answering several questions relative to the regional economic integration between dissimilar countries. Our objective is to analyze the impact of the integration process on the growth and localization of the economic activities in each of these countries. Economic activity is not homogeneously distributed across space. This is a source of inequality in revenues between countries due to agglomeration effects. Why do some locations attract economic activities while others stagnate or disappear

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is an important issue in development economics with an obvious geographic dimension.

The model is broadly based on the new trade, new growth and new geography theories. Fortunately, the two theories focusing on growth and geography, are based on similar assumptions and discuss similar issues (see Dion, 2004a). Their combination provides then strong results and allows us to answer analytically numerous questions related to the implications in terms of growth and location following global or regional economic integration.

Regional economic integration concerns three different actors: the two insiders and the one outsider. Each actor having its specificities, we henceforth consider that each differs from the two others in terms of its stock of human capital. In that case, its stock of knowledge will also be different as its output growth. However, by trading and opening their economy to the exchange of goods, services and ideas, union members can increase their stock of knowledge by benefiting from the stock of their trading partners (contained in their partners’ exports). The lowering of transaction costs (in the sense of barriers to trade and to commercial transactions) is one of the most efficient way of acquiring knowledge both private and public either through relocation of activity or increase in trade volumes.

We build as a departure a system where each country is specialized when trade costs are originally high: the North being specialized in modern goods, the South in traditional goods and the East either in one or the other whether we consider it advanced or laggard. Laggard countries dispose of a lower wage rate and smaller market for modern goods. However, with the phasing out of trade barriers, the inconvenience of having a small home market diminishes. Taking advantage of their lower cost level (cheaper labor), laggard countries may capture market shares by underpricing their richer rivals of the more advanced economy in imitating their goods.

Imitation means the copy of product-specific features. Although the technology for imitation is similar as the one for innovation, it requires less input requirement and is thus less costly. Imitation is profitable as soon as both the innovator and the imitator can obtain profits. We base our specification for imitation along the lines of the leader-follower model of Grossman and Helpman (1991, chapter 11) and Barro and Sala-i-Martin (1995, Chapter 8) while assuming limit pricing competition between the innovator and its imitator to allow the imitator to gain market shares.\footnote{Imitation differs here from counterfeiting in the sense that the former is an authorized copy whereas the latter is a pirated copy.}

Imitation may occur if the genuine invention is not fully protected (the patent or copyrights are not strict enough to prevent imitation) and thus if the imitator can benefit in the marketplace thanks to its imitation. Imitation thus plays the role of transfer of technology. Imitation eventually combines both the public-general and private-specific features of knowledge. More fundamentally, imitation may be a non-zero sum game where innovating countries specialize in advanced goods (and quit modern), whereas less innovating countries specialize

\footnote{Imitation differs here from counterfeiting in the sense that the former is an authorized copy whereas the latter is a pirated copy.}
in modern goods (and quit traditional). If countries are symmetric, one needs a reallocation of labor towards R&D to raise productivity permanently. However, we will see that the innovator still has incentives in pursuing research despite the risk of being imitated.

Whereas the models of local spillovers conclude that R&D is forced to be concentrated in one single location, we show that it still can be spread and even evenly in a model allowing imitation (no pure secrecy, there is still room for copying). We find in this context no reason why a poorer country should stay permanently lagging after joining a regional union. Indeed, as soon as it keeps its wages lower and maintain an advanced sector, it should be able to capitalize on its cost advantages (lower wages and softer competition) and build on its available stock of knowledge. Conversely, a country staying outside of the liberalization process (no access to goods and thus ideas) might impoverish. Transaction costs, acting as an exogenous force, shifts the balance between centripetal or concentration forces (economies of scale and local spillovers) and centrifugal or dispersion forces (wage differentials and global spillovers).

The remainder of this chapter is organized as follows. The chapter is in six sections. In section 2 we draw the broad lines of the model by presenting the countries, sectors and factors. In section 3 we present the behavior of the representative consumer and producer. In section 4 we propose the market equilibrium conditions. In section 5 we determine the static and dynamic equilibria. Section 6 concludes and suggests directions for future research.

2 The setting

The model puts together an endogenous growth framework (Romer, 1990; Rivera-Batiz and Romer, 1991a and 1991b; Grossman and Helpman, 1991) and an economic geography framework (Krugman, 1991 and Fujita et al., 1999) to propose a synthesis of the two theories along the lines of Walz (1995, 1996 and 1998), Martin and Ottaviano (1999) and Manzocchi and Ottaviano (2001). Whereas the models of economic geography are static in nature, we develop a dynamic framework by introducing features of economic geography in a dynamic growth model.

We consider a world economy with a three country-setting (North - N, South - S and East - E), three factors (Knowledge capital - K, Low-skilled labor - L and Human capital or High-skilled labor - H) and three sectors (Traditional - T, Modern - M, and Advanced - A).

2.1 Countries

The three countries are identical except for their initial level of human capital. Thus, they are symmetric in terms of preferences and type of technology but differ in the (absolute among countries and relative among sectors) proportions of their labor endowments between skilled and unskilled workers. Since - by assumption - the North is relatively human capital abundant, it has a compara-
tive advantage in the production of the modern and advanced activities, which are relatively intensive in human capital (HOS theorem). So that the North is a net exporter of the modern goods, while the two countries are net exporters of the traditional good (with free mobility of capital, trade in goods does not have to be balanced).

Rather than allowing households to be mobile and choose their location according to the higher nominal wage rate, we have already stressed that the proportion of mobile households is indeed very small and thus non-significant. However, firms can be located in any location.

We assume that the North has relatively more increasing returns to scale (IRS) firms than the other countries. Hence, it also benefits from a higher income, level of expenditures and market share in the modern sector. Thanks to its higher stock of human capital, the North is also more specialized in modern and advanced activities than the two other countries. Moreover, since the North owns the largest home market in modern and advanced activities, it exports modern goods and offers to its workers in these two sectors a higher real wage (high nominal wage and low price index) than the other countries. Since firms in the North have a good access to the largest market they can afford to pay higher wages (backward linkage).

And likewise, thanks to its relatively large modern sector, the North can propose a broader variety of differentiated goods, permitting lower production costs for advanced goods (forward linkages). The combination of these linkages allows specialization and concentration of the modern and advanced sectors in the North. However, R&D is not fully concentrated in the North since there exists in each of the two other countries an advanced sector capable of copying the new blueprints (if need be). Obviously, since the North has the highest share of human capital, it is responsible for most of the innovation.

2.2 Sectors

We consider a three sector economy. The three sectors are distinguished by the opportunities that they afford for technological progress and by the intensity with which they employ the two primary inputs. To simplify we affect one single type of labor to each sector.\footnote{Results would be similar if we were instead considering that sectors differ in terms of relative factor intensities.}

**Traditional sector** Competitive firms produce the traditional good according to a constant returns to scale (CRS) production function. The technology used is similar in each location. Since the traditional good is assumed to be produced in each location, we have:

\[
p_T = c_T(w^c_L)
\]

where \( w^c_L = w_L \) (because there is free-trade so that \( \tau = 1 \) in that sector) is the reward to unskilled labor in each country \( c \) and \( c_T \) is the cost of producing
a unit of good \( T \). Choosing its products as the numeraire, we set \( p_T = 1 \), and the pricing condition becomes:

\[
1 = c_T(w) \tag{2}
\]

It uses unskilled labor \((L)\) as its only input. By choice of units, one unit of unskilled labor is required to produce one unit of the traditional good (one unit of \( T \) is made with one unit of \( L \))\(^3\).

**Modern sector** Since our approach deals with the models of varieties in consumer goods we consider that the consumer-goods sector manufactures previously invented consumer goods and sells them to the consumers. These goods are horizontally differentiated goods with increasing returns to scale (IRS) in a monopolistic competition framework. Each of the \( n \) monopolistic firms manufactures one variety or brand of the set of differentiated products. Every variety is produced with the same IRS production function. The number of firms in that industry is equal to \( n \), with each firm producing a different variety and charging the same price \( p \) for its products. In a symmetrical equilibrium, the quantity \( x \) is equal for all \( n \).

\( M \) uses both skilled labor \((H)\) and knowledge capital \((K)\) as inputs. By choice of units, one unit of knowledge capital is required to produce one variety of the modern good, but the scale of production is determined by the labor input, so that we get IRS in the production of each variety. Variable cost in \( x \)-production is the labor cost for skilled labor. So, marginal cost \( c_x \) in \( x \)-production is given by:

\[
c_x = u(w) \tag{3}
\]

Its cost function includes fixed costs paid in terms of knowledge capital with a unit input requirement set to one so that the number of firms in a given country is equal to the knowledge capital endowment.

The differentiated good can be freely tradable across countries, although it incurs an international trade cost when crossing the border modelled according to Samuelson (1954) iceberg costs. Previous to the marketing of a modern good, it must be invented through research implemented in the advanced sector.

**Advanced sector** The advanced sector invents the new varieties. Each country has an advanced sector (e.g. the innovative or R&D sector of the economy) although of a different size than its partners. In a situation of autarky, the fastest growing economy is the one owing the highest share of human capital

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\(^3\)Factor prices will be equalized if it is possible to reproduce through trade the production of a hypothetical integrated world economy in which factors of production are costless tradable. We suppress transaction costs of international trade in that sector as usually done in the models of economic geography. This simplification does not alter our results. The \( T \) good is costless tradable (all countries still produce it after trade). So that the wage rate will be the same in all countries for that sector. In that case, only the relative size of markets for the differentiated product is the determinant of the trade pattern.
devoted to the advanced sector. The research technology is deterministic and works as follows. The advanced sector $A$ produces knowledge or new ideas in a perfectly competitive fashion for the $M$ sector. It uses skilled labor ($H_A$ for the human capital engaged in the advanced sector $A$) as its only input.

The flows of new differentiated final goods ($\dot{n}$) depend on the aggregate level of employment in the advanced sector ($H_A$) responsible for the innovation that generates knowledge and ultimately growth. It also depends on a productivity factor ($\eta$) and on the improvements or advancements realized in the research field measured by the stock of knowledge ($K$). In effect, these new developments in research decrease the cost (or the labor requirements) for creating new products. The production function for innovation is then the following:

$$\dot{n} = \frac{H_A K}{\eta}$$

(4)

The dots denote the time derivatives. Then, the number of goods on offer becomes an endogenous variable thanks to innovation. Moreover, in order not to have to choose between increasing or decreasing returns to scale (IRS or DRS) in the production of knowledge, we follow the traditional literature and impose the following restriction:

$$K = n$$

(5)

where the global stock of knowledge is equal to the global number of products. The key assumption is the linearity in $n$. This reflects the public good features of the existing stock of knowledge $K$.

We also assume that each known variety of the differentiated product is manufactured by a single, atomistic firm. To support this assumption one may think that the government grants infinitely lived patents to the inventor. However, per country, the stock of knowledge is: $K^c = n^c$. Superscript $c$ designates one of the three countries. For instance, in the North, the stock of knowledge is: $K^N = n^N$. And for the two other countries, we obtain: $K^S = n^S$ and $K^E = n^E$ with - by assumption - $K^E = n^E \leq K^S = n^S < K^N = n^N$. By summing up the three stocks, we obtain: $n^E + n^S + n^N = n$. The number of researchers in each country is then proportional to the number of designs invented in each country.

This sector produces both private knowledge (in the form of patented blueprints according to a profit-maximizing and forward-looking approach) and public knowledge in the form of knowledge spillovers that automatically increase the stock of knowledge and thus productivity in the advanced sector. Romer (1990) assumes that the cost of innovation, $\eta$, decreases with the discovery of new ideas, represented by $n$, the number of products. Current research has a positive impact on future research. Finally, researchers make use of patents to protect their ideas. In that way, they can appropriate the returns to product-specific infor-

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$^4$ $\eta$ is the parameter reflecting the productivity of human capital in product development. The smaller $\eta$, the higher the productivity.
mation which enables them to create new products, but not the social return to general information which serves as an input in inventive activity 5.

2.3 Factors

We assume that knowledge stocks and factor rewards differ between locations. Indeed, $L$ and $H$ are imperfect substitutes since they perform very different tasks. Human capital gathers a broad range of skills obtained through time devoted to education. The number of varieties (and hence of firms) depends on the number of ideas and $H$ workers.

Because each country produces some quantities of good $M$, but employs different levels of technologies, equilibrium wage rates may differ across countries. The three locations are inhabited by $L + H$ identical households who perform the tasks of consumers (of the modern and traditional final goods $M$ and $T$), workers and researchers. In each period the representative household is endowed with one unit of labor. Initially, each household forms expectations about the sequences of all prices which he takes as given. Each household supplies inelastically one unit of labor either skilled or unskilled for a wage rate equals to either $w_L$ or $w_H$ with $w_L < w_H$.

Labor is inter-sectorial mobile but internationally imperfectly mobile (almost nationally fixed). In fact, we consider that the incentives for firms to relocate due to wage differentials (and corrected for transaction costs) are higher than for the workers who are facing natural and cultural barriers that impede their ability to migrate. That is: a wage differential between North and South will incite, all things equal, the Northern firms to relocate in the cheaper South whereas only few workers will be able to migrate. So few that this impact is negligible and is not a force of agglomeration. In that sense, the incomes of households are also geographically fixed contrary to the firms that can migrate. Hence, agglomeration and disagglomeration forces cannot occur through the movements of labor but will have to be generated through other dynamics, namely transactions costs, wages (or costs) differentials and economies of scale.

The Northerners own $H^N$ units of human capital, the Southerners $H^S$ and the Easterners $H^E$. The total number of varieties and firms depends on the world stock of human capital. The firms can be located in any of the three countries and their location is given by $n^N$, $n^S$ and $n^E$ (or in proportionate terms: $l^c = n^c/n$). We assume that the North is initially richer in human capital than its two partners.

Knowledge capital $K$ is produced by the advanced sector $A$. There is one unit of $K$ per variety and the growth rate of the number of varieties is therefore: $\dot{K}/K = g$. However, the marginal cost of producing new $K$ declines (i.e. $\eta$ falls) as the sector’s cumulative output rises. This is due to the specific public good attributes of knowledge such as non-rivalry and non-excludability 6.

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5 By assumption and for simplification, there is no depreciation in that sector since knowledge does not depreciate in the same way as physical capital would.

6 Financial capital is freely mobile across countries so that we should expect the equalization of interest rates.
3 The model

We present in that section the respective consumer and producer behaviors.

3.1 Consumer behavior

The household sector is structured as in the Ramsey-Cass-Koopmans model. Households allocate spending over time so as to maximize an intertemporal utility function. Households in the three countries are consumers of the modern and traditional final goods $M$ and $T$ and determine labor supply and the demand of assets held in the economy.

3.1.1 Utility function

Households share identical preferences and have a taste for diversity in consumption, which firms can only supply by bearing fixed costs. The utility of a representative household is:

$$U^c(t) = \int_{t=0}^{\infty} e^{-\rho t} \ln Q^c dt$$

(6)

with:

$$Q^c = (D^c)^{\alpha} (T^c)^{1-\alpha}, \quad 0 < \alpha \leq 1,$$

(7)

We assume constant expenditure shares. The intertemporal elasticity of substitution has been chosen at unity for simplicity. The use of the natural logarithm aims at measuring instantaneous utility at a moment in time. $\rho$ is the rate of time preference also called the subjective discount rate and $\alpha$ is the share of consumption devoted to $D$. It is similar in the other countries. Instantaneous utility $Q$ is a composite good of the homogenous good $T$ and the differentiated good $D$.

3.1.2 Consumption index

$T$ is the traditional and numeraire good whereas $D$ is a composite or aggregator good made up of a number of differentiated products. Indeed, $D$ expresses household’s tastes for diversity in consumption that will create demands for differentiated goods. Thus, $D$ is a quantity index that follows the Dixit and Stiglitz framework (1977) and thus allows a C.E.S. between any pair of goods. Consumers’ preferences cover an infinite set of goods indexed by $i$ in the Northern case, $j$ in the Southern case and $k$ in the Eastern case. At any given moment in time $t$, only a subset $n$ of these varieties is available. Thus, $n(t)$ is the amount of available goods or varieties invented before time $t$ so that $n$ is a function of time and in equilibrium, $n$ grows at a constant rate.

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7 Population stays constant.
8 Innovation is thus used to augment the number of available varieties.
where \( \varepsilon \) is the elasticity of substitution between any two varieties and the elasticity of demand for each variety of the modern good. \( x(i) \) is the consumption of the \( i \)-th variety in the Northern case. We have the same structure in the \( j \) (South) and \( k \) (East) cases. Indeed, commodities supplied by different producers are imperfect substitutes with \( \varepsilon \) as their constant elasticity of substitution. To simplify the writing, we define the elasticity of substitution between any two products as:

\[
\varepsilon = 1/(1 - \sigma) > 1 \implies \sigma = (\varepsilon - 1)/\varepsilon
\]

with \( 0 < \sigma < 1 \). It measures the heterogeneity of modern goods, the intensity of the preference for variety and the degree of scale economies. Hence, a smaller \( \sigma \) implies a greater preference for variety and conversely a larger \( \sigma \) close to 1 reflects the fact that differentiated products are almost perfect substitutes.

Basically, this framework shows that the more differentiation, the more varieties, the more intense the specialization into a set of varieties, the more increasing returns to scale.

\[ n_N, n_S \text{ and } n_E \text{ are the respective numbers of differentiated goods produced in the three countries with } n = n_N + n_S + n_E. \] 

is therefore the total number of differentiated goods produced in the North, South and East together.

### 3.1.3 Budget Constraint

**Expenditure function** \( E^c \) represents aggregate spending of the households on the composite good in country \( c \) so that \( E = E^N + E^S + E^E \). The value of total expenditure is:

\[
E^N = \left[ \begin{array}{c}
\int_{i=0}^{n_N} \tau^N p^N(i)x(i)di + \\
\int_{j=n_N}^{n_N+n_S} \tau^S p^S(j)x(j)dj + \\
\int_{k=n_N+n_S}^{n_N+n_S+n_E} \tau^E p^E(k)x(k)dk + T_i
\end{array} \right]
\]

\[
E^S = \left[ \begin{array}{c}
\int_{i=0}^{n_N} \tau^N p^N(i)x(i)di + \\
\int_{j=n_N}^{n_N+n_S} \tau^S p^S(j)x(j)dj + \\
\int_{k=n_N+n_S}^{n_N+n_S+n_E} \tau^E p^E(k)x(k)dk + T_j
\end{array} \right]
\]

\[
E^E = \left[ \begin{array}{c}
\int_{i=0}^{n_N} \tau^N p^N(i)x(i)di + \\
\int_{j=n_N}^{n_N+n_S} \tau^S p^S(j)x(j)dj + \\
\int_{k=n_N+n_S}^{n_N+n_S+n_E} \tau^E p^E(k)x(k)dk + T_k
\end{array} \right]
\]

\[ \text{These spending shares also depend on the free mobility of capital, since net asset positions are part of national wealth.} \]
where $p_N$, $p_S$ and $p_E$ are respectively the producer prices of the differentiated good in the North, South and East. As in Samuelson (1954) and in the models of economic geography, international transaction costs on the differentiated goods are in the form of iceberg costs. $\tau$ is more than one which means that only a fraction of the good purchased is actually consumed or said differently: to sell a unit of the differentiated good from one country to another, more than one unit has to be sent. It then measures the barriers to trade.

**Demand function** The optimal choice of the $D$ goods can be found by solving the following subutility maximization problem:

$$\max u(D) \text{ such that } pD = E_D$$

(13)

This means that as long as we know the expenditure on the $D$ goods, $pD = E_D$, we can solve the subutility maximization problem to determine the optimal choice of the $D$ goods.

$$D_i = \left[ \frac{p_i^{-\varepsilon}}{\int_{j=0}^{m} p_j^{1-\varepsilon} dj} \right] \alpha E_i$$

(14)

$$= \left[ \frac{n_N (p_N)^{1-\varepsilon} + n_S (p_S)^{1-\varepsilon} + n_E (p_E)^{1-\varepsilon}}{n_N (p_N)^{1-\varepsilon} + n_S (p_S)^{1-\varepsilon} + n_E (p_E)^{1-\varepsilon}} \right] \alpha E_i$$

(15)

**Spending function** Under the specified properties of the utility function, the maximization of $U$ subject to an intertemporal budget constraint necessitates that expenditures evolve in a specific way. Expenditures $E$ indeed must grow at an instantaneous rate equal to the difference between the interest rate $r$ on a safe asset and the subjective discount rate $\rho$. We assume free financial capital movements between the three locations, so this implies that the same condition applies in all three countries. That allows us to obtain equal rates of return so that $r = r^c$ where $r^c$ is the interest rate on bonds issued by firms in country $c$:

$$\begin{align*}
\frac{\dot{E}_N}{E_N} &= \frac{\dot{E}_S}{E_S} = \frac{\dot{E}_E}{E_E} = r - \rho
\end{align*}$$

(16)

where as usual the dot refers to the change in the variable. This is the standard Ramsey formula describing the optimal path of spending as the difference between the real rate of interest and the pure rate of time preference (originally multiplied by the elasticity of intertemporal substitution that we have normalized to 1). We normalize prices so that spending is constant through time and thus equal to one. Indeed, it turns out that in equilibrium expenditures and the spending shares are constant so that interest rates converge to $\rho$:

$$E(t) = (E_N + E_S + E_E) = 1 \implies r^c(t) = \rho$$

(17)
3.2 Producer behavior

Producers are researchers inventing new varieties of differentiated goods and workers producing goods previously invented. Each variety of a differentiated good is produced by a single firm. Although the original innovators can protect their invention thanks to a patent, we allow for imitation.

3.2.1 Price function

The choice of the price $p_M$ in the modern sector (the domestic price of any variety of the differentiated good) that maximizes profits follows the standard rule in monopolistic competition. It will be priced according to the standard Chamberlin mark-up rule over marginal costs. $p$ is thus higher than the marginal cost of the differentiated product and this discrepancy is due to the degree of differentiation between products measured by the constant mark-up $\sigma$. The inverse elasticity rule provides the free-on-board (f.o.b.) price of the monopolist:

**In the North**

$$p^N = \frac{\beta^N w^N}{\sigma}$$

where $w^c$ is the wage rate in country $c$. Each variety of the differentiated good has a linear cost function where variable costs are paid in terms of labor with a unit input requirement of $\beta$.

**In the South** We have two cases:
- **if autarky:**

$$p^S = \frac{\beta^S w^S}{\sigma}$$

(19)

- **if free-trade:**

$$p^S \leq \tau \beta^N w^N$$

(20)

for an imitator in the southern market because of limit pricing behavior. Indeed, we recall that wages in the South are lower than in the North allowing the South to underprice the North\(^{10}\). So that we obtain:

$$p^S \leq p^N \implies \tau w^N < \frac{w^N}{\sigma} \iff \tau < \frac{1}{\sigma}$$

(21)

We introduce the possibility of imitation (the copy of product-specific information) in our framework. Imitation is profitable if a successful imitator is able to earn positive profits in competition with the original inventor of a new differentiated good. We assume that the enforcement of patents is not strict

\(^{10}\) A firm in the South experiences a transport cost disadvantage in supplying the North so that it can not afford to pay a wage as high as in the North. See also Appendix 2 for a detailed presentation.
enough to make imitation too expensive. Or said differently, the legislation on patents tolerates imitation. Imitators in the South can capture market shares and quasi-rents by underpricing rivals in the technologically advanced North. Imitation encourages the spread of the public and private features of goods and allows the poor countries to reallocate their relatively low-skilled labor towards the modern sector and their high-skilled labor towards the advanced sector\textsuperscript{11}.

Moreover, in freer-trade, for the South to sell its imitated products, their price has to be either equal or lower than the one in the North. We thus consider that the Southern country enjoys a cost advantage over its Northern partner and that it sets its prices at the level of its partner’s costs (inclusive of trade costs). The South can do so thanks on the one hand to the lower cost of imitation in comparison with the cost of innovation and on the other hand to the wage differential.

Due to the excessive costs caused by a failure to compete, we count one potential imitator for one innovator. As soon as the imitator has acquired the technology, it can expect to earn oligopoly profits. We could thus envisage that South and East produce world-wide demand for the traditional good and that $w_E^H = w_S^H < w_N^H$. However, as soon as $\tau = 1$ between North and South, we will have $w_E^H = w_S^H = w_N^H$ since location between the two partners will become irrelevant. All active producers of differentiated goods in a given country charge the same price.

**In the East**

$$p^E = \frac{\beta^E w^E}{\sigma} \quad (22)$$

The wage rates might only be equalized over the long run in a situation of free-trade. This implies that the cost insurance freight (c.i.f.) price of an imported good is $\tau$ times higher than its f.o.b. price\textsuperscript{12}.

### 3.2.2 Profit function

Confronted with the demand function previously derived, the monopolistic producer of variety $i$ maximizes operating profits according to the following function where the operating profits of each firm in the IRS sector $M$ are equal to the difference between revenues and labor costs. $s^c$ is the share of world spending on modern goods that is devoted to the brands of country $c$. The demand functions for the individual varieties can be used to compute the share $s^c$ of world spending on modern goods that is devoted to the brands of country $c$:

$$s^c_i = \frac{n_i^c (p_i^c)^{1-\varepsilon}}{\sum_j n_j (p_j^c)^{1-\varepsilon}}, \quad c = N, S, E \quad (23)$$

\textsuperscript{11}See also Appendix 3 for a detailed presentation.

\textsuperscript{12}So that the price of the traditional good $T$ is also equalized in each country over the long run in case of free-trade. If we assume that the demand for the traditional good is high enough, then it must be produced in each country and allow for international wage equalization in that sector.
We thus have:

$$\pi_c = p_c x_c - \beta_c w_c x_c$$

(24)

$$= \frac{(1 - \sigma) \beta_c \alpha s_c}{n_c}$$

(25)

Profits per variety are then a decreasing function of the number of competitors and an increasing function of the amount of aggregate spending (equal to final output since producers of modern goods earn zero profits) spent on modern goods. In each country, producers of differentiated products earn profits that are a fraction ($1 - \sigma$) of their revenues. We see that a rise in $\sigma$ lowers profit as the margin decreases if varieties are better substitutes and that profit also falls in the number of varieties because new competitors increase competition. The profit functions for each country can thus be derived in a similar fashion:

In the North

$$\pi^N = p^N x^N - \beta^N w^N x^N$$

(26)

$$= \frac{(1 - \sigma) \beta^N \alpha s^N}{n^N}$$

(27)

In the South We have two cases:

- if autarky:

$$\pi^S = p^S x^S - \beta^S w^S x^S$$

(28)

$$= \frac{(1 - \sigma) \beta^S \alpha s^S}{n^S}$$

(29)

- if free-trade:

$$\pi^S = p^S x^S - \beta^S w^S x^S$$

(30)

$$= \frac{(1 - \beta^S w^S \tau^S) \alpha s^S}{\tau^S n^S}$$

(31)

The changes between the autarky and the free-trade cases come from the modifications in the price function (the limit pricing strategy introduced by the South in case of free-trade) that were derived in the previous section.

In the East

$$\pi^E = p^E x^E - \beta^E w^E x^E$$

(32)

$$= \frac{(1 - \sigma) \beta^E \alpha s^E}{n^E}$$

(33)
The owners of firms receive profits as described above (for instance in the form of dividends if we consider that profit corresponds to the annual dividend paid by a monopolist). Moreover, they can incur capital gains (or losses) on their shares. In order to measure these sums, we need to build a valuation function.

3.2.3 Valuation function

We assume free-entry in the access to R&D if the cost of R&D, $\eta w/n$, is paid in order to receive the actualized value $v(t)$ such that $\eta w/n = v(t)$ \footnote{See also the free-entry condition in Section 4.1 below for further details.}. Indeed, the value of a firm that manufactures modern goods shall be at least equal to innovation costs in order to allow an entrepreneur to invest in inventing new goods. Firms issue equity to finance their R&D investment. Indeed, the purpose of the monopolist in the general equilibrium is to generate monopoly rents that allow through issuing shares for the financing of the R&D investment. $v$ is the highest amount that a monopolist is ready to pay for the right to manufacture one differentiated good from period $t$ onwards. He can also raise $v$ by issuing new shares.

Households can borrow against the present value of their income, hold net claims against other households or buy shares with their savings and receive a claim on future dividends from the firms. The dividends are paid out of the profits made by successful innovators. The stock market value of a firm at time $t$ is represented by the total profit of inventing a new idea that is the present discounted value of all future operating profits of the firm minus the cost of invention. With free capital flows, these operating profits are equalized. Then, the value of any firm is:

$$v^c(t) = \int_{s=t}^\infty e^{-R[s-t]} c^c(s)d(s)$$

$$= \int_{s=t}^\infty e^{-R[s-t]} \frac{\beta^c w^c x^c}{1 - e^{-1}} d(s)$$

where $R(t)$ represents the cumulative discount factor applicable to profits earned at time $t$. In each country, free-entry ensures a value of the representative high-tech firm that is no higher than the cost of product development, and equal to it when R&D actually is taking place. So, in equilibrium, the present value of the whole stream of profits that the firm can receive must be equal to the cost of purchasing the patent for producing the specific durable. Modern firms cover the fixed costs of R&D thanks to operating profits.

4 Market equilibrium conditions

We draw here the free-entry, capital market and labor market conditions.
4.1 Free-entry condition

R&D firms enter freely into the advanced sector so that - thanks to the assumed continuous entry of new firms and the CRS condition in that sector - the profits in the advanced sector are driven to zero. Moreover, the financing of product development is done by issuing equities. R&D firms are competitive and charge the price $v$ per new invention when it is sold to a manufacturing monopolist. Then $vdn = vH_A n / \eta dt$ is the value for the entrepreneur of paying a wage bill $w H_A dt$. Value maximization in the R&D sector implies that $H_A$ will be chosen as large as possible when $\frac{v}{\eta} n - w > 0$ and will equal zero when $\frac{v}{\eta} n - w < 0$. The former case cannot arise in a general equilibrium as it implies an unbounded demand for labor by research enterprises. The latter case corresponds to a momentary equilibrium without investment in R&D. Hence, $v \leq \frac{w}{\eta}$ with $" = \"$, if $\dot{n} > 0$. In equilibrium, free-entry condition requires that the market value of a new design equals its cost. Since the cost of a blueprint is $w \eta / K = w \eta / n$, the free-entry condition giving the value of each firm is then:

In the North

$$v^N = \frac{w^N \eta^N}{n^N} = \frac{w^N \eta^N}{n}$$ (36)

where $n^N = n$ when the North is responsible for the whole innovation.\(^{14}\)

In the South

We have two cases:
- if autarky:

$$v^S = \frac{w^S \eta^S}{n^S} = \frac{w^S \eta^S}{nl^S}$$ (37)

- if free-trade:

$$v^S = \frac{w^S \eta^S}{n^S, N} = \frac{w^S \eta^S}{nl^S, N}$$ (38)

where $n^S \leq n^S, N \leq n^N$ and we recall that $l^S = \frac{n^N}{n}$. Note that $n^S, N$ represents the amount of varieties available in the intermediate case when the South has access to more varieties than in autarky but still less than in the North. It means that both local and global stock of knowledge contribute to the imitation process. The number of products that the South can access and eventually copy is then higher than in the case of autarky $n^S$ but still lower than the number of products in the North $n^N$ due to the presence of transaction costs still not completely phased out.

In the East

$$v^E = \frac{w^E \eta^E}{n^E} = \frac{w^E \eta^E}{nl^E}$$ (39)

Thanks to free-entry and exit, the number of firms is determined endogenously by a process which drives net profits to zero. Since the model allows for

---

\(^{14}\)As we will see later in the case of free-trade where the North becomes the innovation leader and the South becomes the imitator or the so-called follower.
perfect-foresight, there is no uncertainty and stocks and bonds are perfect substitutes and yield at equilibrium equal rates of return. This absence of profitable arbitrage condition gives us the dynamic capital market equilibrium.

4.2 Capital market equilibrium condition

The equilibrium in the capital market requires that the returns which a shareholder can expect be equal to the return of a riskless loan. So, investors buy shares if expected return on investment is equal to the risk-free interest rate. By investing through shares, they help the creation of new knowledge for producing new good. This raises productivity in production lines, hence revenue, hence the knowledge base in R&D activities and it brings a capital gain (or loss in case of failure). Differentiating the valuation function with respect to time, we get the arbitrage condition on capital markets:

$$\pi^c + \dot{v}^c = rv^c \iff \dot{v}^c = \rho v^c - \pi^c$$

which says that the returns $rv dt$ on the different riskless assets must be equalized. On an investment of value $v$ in a firm, the return is equal to the operating profits (or the dividends paid to the shareholders) plus the change in the value of the firm (the capital gains or losses). The arbitrage condition that equates the total return on equity claims to the discount rate can also be written in a more convenient way:

$$\frac{\pi^c}{v^c} + \frac{\dot{v}^c}{v^c} = \rho \iff \frac{(1 - \sigma)\beta c \alpha s^c}{n v^c} + \frac{\dot{v}^c}{v^c} = \rho$$

where the left hand side represents the rate of return from owning the innovated product (profit rate plus capital gain - or loss) and the right hand side represents the rate of return from selling the product.

In full equilibrium, profits are driven to zero, which results in a relation between growth and the number of firms. This is the zero-profit condition. It determines, together with the short-run equilibrium, the long-run equilibrium solution for $n$ and $g$. We define $g$ as the rate of innovation or the rate at which new products are created. At the steady state, $g = \frac{n}{n} = \dot{n}$. In equilibrium $v$ decreases at the same rate as $n$ increases. Since we know that the value of shares or the value of a typical firm $v$ decreases at rate $g$, the rate of innovation or of product development is:

$$\dot{v} = \frac{\dot{v}}{v} = -\frac{\dot{n}}{n} = -g$$

we can thus obtain from the two previous equations:

$$\frac{\dot{v}}{v} = \rho - \frac{\pi}{v} \implies v = \frac{(1 - \sigma)\beta \alpha s}{n(\rho + g)}$$

The no-arbitrage condition for each country can be derived in a similar way by replacing $\pi$ by its value from the profit function and by replacing $v$ from the free-entry condition. We then obtain the following no-arbitrage conditions:
In the North  We have two cases:
- if autarky:
  \[
  \frac{\pi^N}{v^N} + \frac{\dot{v}^N}{v^N} = \rho \iff \frac{(1 - \sigma)\beta^N \alpha s^N}{n^N v^N} = \rho + g^N
  \]  (44)
  However, following the integration process and the imitation that occurs, we obtain a slightly different result in case of free-trade:
  - if free-trade:
  After the integration occurs, the profits in the North will be reduced due to the imitation of some of its products by its Southern partner in the union. The probability of imitation depends on the number of Northern goods imitated by the South, \( \dot{n}^S \), relatively to the number of all Northern goods in place, \( n^N \). The Northern firm, whose good has been imitated, experiences then a decrease in profit of \( v^N \). The no-arbitrage condition in case of free-trade with imitation will thus take the following form:
  \[
  \frac{\pi^N}{v^N} + \frac{\dot{v}^N}{v^N} - \frac{\dot{n}^S}{n^N} = \rho \iff \frac{(1 - \sigma)\beta^N \alpha s^N}{n^N v^N} = \rho + g^N + \gamma
  \]  (45)
  where \( \gamma = \frac{\dot{n}^S}{n^N} \) measures the rate of imitation by the South of the products previously invented in the North. The gap between the two partners will be nil as soon as the barriers will have been completely dismantled, since at that point, the South will be able to fully profit from the whole range of products to imitate or even begin to innovate. In the Southern and Eastern cases, we obtain in a similar fashion:

In the South  We have two cases:
- if autarky:
  \[
  \frac{\pi^S}{v^S} + \frac{\dot{v}^S}{v^S} = \rho \iff \frac{(1 - \sigma)\beta^S \alpha s^S}{n^S v^S} = \rho + g^S
  \]  (46)
  - if free-trade:
  \[
  \frac{\pi^S}{v^S} + \frac{\dot{v}^S}{v^S} = \rho \iff \frac{(\tau \beta^N w^N - \beta^S w^S) \alpha s^S}{v^S} = \rho + g^S
  \]  (47)

In the East
\[
\frac{\pi^E}{v^E} + \frac{\dot{v}^E}{v^E} = \rho \iff \frac{(1 - \sigma)\beta^E \alpha s^E}{n^E v^E} = \rho + g^E
\]  (48)

4.3 Labor market equilibrium condition
In equilibrium, the world market must clear and the free-entry condition requires that the marginal product of labor is the same for manufacturing and R&D activities. Households provide \( L + H \) units of labor (skilled and unskilled) at any moment in time. We know that workers will either be working in the R&D
sector or in the two other sectors. For a flow of new products equal to \( \dot{n} \), then total employment in R&D is equal to \( \eta^A \) of them.

Hence, at the regional level, labor market equilibrium requires:

\[
H_M + H_A = H
\]  

(49)

where the aggregate labor demand in manufacturing and research must be equal to the households’ labor supply.

Labor market equilibrium requires the full employment of unskilled and skilled labor. Since unskilled labor is nil in both the advanced and modern sectors, there is no need to investigate the sectorial allocation of unskilled labor. So, here we restrict our analysis to the full allocation of skilled labor (human capital) between the modern and advanced sectors. Although wage rates may differ across countries they are the same within the country whether human capital is allocated in the modern or the advanced sector. In equilibrium, total expenditure is equal to total factor permanent income. At the steady state, we have:

**In the North**

\[
\hat{n}^N \eta^N + \beta^N x^N n^N = H^N
\]  

(50)

where \( \beta^N x^N n^N \) are the units of labor demanded by the manufacturing firms of the M sector and \( \hat{n}^N \eta^N \) are the units of labor demanded by the R&D firms of the A sector. Since \( g = \hat{n}/n \), we can simplify and obtain:

\[
\eta^N g^N + \beta^N n^N x^N = H^N \iff g^N = \frac{H^N - \hat{n}^N \beta^N x^N}{\eta^N}
\]  

(51)

**In the South**  We have two cases:

- **if autarky:**

\[
\hat{n}^S \eta^S + \beta^S x^S n^S = H^S
\]  

(52)

or after simplification:

\[
\eta^S g^S + \beta^S n^S x^S = H^S \iff g^S = \frac{H^S - \hat{n}^S \beta^S x^S}{\eta^S}
\]  

(53)

- **if free-trade:**

\[
\hat{n}^{S,N} \eta^S + \beta^S x^S n^{S,N} = H^S
\]  

(54)

or, after simplification:

\[
\eta^S g^{S,N} + \beta^S n^S x^S = H^S \iff g^{S,N} = \frac{H^S - \hat{n}^{S,N} \beta^S x^S}{\eta^S}
\]  

(55)
The difference between autarky and free-trade stems from the notation and the superscript \( n,s \) that symbolizes the intermediary situation of partial free-trade in which the South is evolving: i.e. between complete isolation and complete free-trade.

In the East

\[
\eta^E \frac{n^E}{n^E} + \beta^E x^E n^E = H^E
\]

(56)
or, after simplification:

\[
\eta^E g^E + \beta^E n^E x^E = H^E \iff g^E = \frac{H^E - n^E \beta^E x^E}{\eta^E}
\]

(57)

5 Static and dynamic equilibria

We now derive the static (or short-run) equilibrium and dynamic (or long-run) equilibrium.

5.1 Static or short-run equilibrium

With freedom in the movements of goods and capital, we obtain the following equilibrium conditions determining firms’ size and location.

5.1.1 Size function

The demands for differentiated modern goods being produced in each country (inclusive of transport costs) must equal supplies at home and abroad. In equilibrium the supply of each variety \( X \) must equal its demand \( n^X \). For each member of the union, the gross output of the differentiated industry is given by our previously derived demand functions \( (D^N, D^S, D^E) \).

In the North

\[
x^N = \frac{\alpha}{n} \left[ \frac{(p^N)^{1-\epsilon^N} x^N}{(p^N)^{1-\epsilon^N} + (p^S)^{1-\epsilon^S} + (p^E)^{1-\epsilon^E}} \right]
\]

(58)

where the three terms inside the brackets describe respectively insider’s (North and South) and outsider’s (East) demands.

In the South

\[
x^S = \frac{\alpha}{n} \left[ \frac{(p^S)^{1-\epsilon^S} x^S}{(p^S)^{1-\epsilon^S} + (p^N)^{1-\epsilon^N} + (p^E)^{1-\epsilon^E}} \right]
\]

(59)
In the East

\[ x^E = \frac{\alpha}{n} \left[ \frac{(p^E)^{1-\varepsilon} E^E}{(p^E)^{1-\varepsilon} \delta^S + (p^E)^{1-\varepsilon} \delta^N + (p^E)^{1-\varepsilon} \delta^S N + (p^E)^{1-\varepsilon} \delta^N + (p^E)^{1-\varepsilon} \delta^S N} + \frac{(p^S)^{1-\varepsilon} E^S}{(p^S)^{1-\varepsilon} \delta^N + (p^S)^{1-\varepsilon} \delta^S} \right] \]

5.1.2 Location function

The determination of the results Our objective is to find an expression for \( l^c \): the variable that measures the localization of modern firms in country \( c \). In order to proceed, we note that the firms can be located in any of the three countries without incurring any relocation costs. Since we have assumed free financial capital movements, the operating profits of a firm situated in the South but owned by a Northern researcher will be repatriated. The Northern researcher does indeed possess a patent on the idea that he developed. With the free mobility of capital we obtain an equalization of the operating profits so that in equilibrium:

\[ \pi^N = \pi^S = \pi^E \implies x^N = x^S = x^E \]

Due to our assumption of free mobility of capital, the share of firms in one country is endogenous since it is determined by an arbitrage condition that says that location of firms is in equilibrium when profits are equalized in the three countries. However, this does not imply that capital mobility is a stabilizing factor preventing a permanent core-periphery setting. Indeed, ownership does not mean localization. In order to obtain an expression for \( l^N \) and \( l^S \), we solve for \( x^N = x^S \) so that we get:

\[ \left[ \frac{(p^N)^{1-\varepsilon} E^N}{(p^N)^{1-\varepsilon} \delta^S + (p^N)^{1-\varepsilon} \delta^N + (p^N)^{1-\varepsilon} \delta^S N} \right] = \left[ \frac{(p^S)^{1-\varepsilon} E^S}{(p^S)^{1-\varepsilon} \delta^N + (p^S)^{1-\varepsilon} \delta^S} \right] \]

and obtain:

In the North

\[ l^N = - \left( p^S \right)^{1-\varepsilon} l^S - \frac{2 \left( p^S \right)^{1-\varepsilon} E^S \delta (1 - \delta) + (p^N)^{1-\varepsilon} E^N (\delta^2 - 1)}{\left( p^N \right)^{1-\varepsilon} \left( (p^N)^{1-\varepsilon} E^S (1 - \delta) + (p^N)^{1-\varepsilon} E^N (\delta - 1) \right)} \]

with

\[ \frac{\partial l^N}{\partial \delta} < 0; \frac{\partial l^N}{\partial \varepsilon} < 0 \]

Localization of firms in the North depends on the respective expenses, the elasticity of substitution, the prices (and thus wage rates) in the North and the
South and the transaction costs. We note that a decrease in transport costs (an increase in $\delta$) between the North and the South has a negative impact on the localization of modern firms in the North. That is, it decreases the number of modern firms in the North. Or, said differently: the decrease in transport costs provokes the relocation of firms from the North towards the South.

In the South

$$l^S = - (p^N)^{1-\varepsilon} l^N \frac{\left(p^S\right)^{-\varepsilon} E^S (1 - \delta) + (p^N)^{-\varepsilon} E^N \delta (\delta - 1)}{\left(2 \left(p^S\right)^{-\varepsilon} E^S \delta (1 - \delta) + (p^N)^{-\varepsilon} E^N (\delta^2 - 1)\right)}$$

with

$$\frac{\partial l^S}{\partial \delta} > 0; \frac{\partial l^S}{\partial \varepsilon} > 0$$

(65)

Conversely to the Northern case, we observe that in the Southern case, the decrease in transport costs fosters the relocation of firms to the South. \footnote{The location function in the East is the same as in the South as long as we consider that both East and South are similar.}

The interpretation of the results It thus appears that the country with the largest market size (the highest $n^*$) or the highest expenditure (the highest $E^*$) will get the highest number of firms. Because of transaction costs and increasing returns, firms want to be situated close to the largest markets. The exogenous factor that may foster a relocation is the level of transaction costs measured by $\tau$ (or $\delta$). Further regional trade integration implies a lowering of the trade barriers and of the transaction costs ($\tau$ decreases or $\delta$ increases). This decrease in $\tau$ affects the location of firms and thus the growth rate.

We detail now the impact of a change in transaction costs. With transaction costs rapidly decreasing towards zero (low $\tau$ or large $\delta$), firms have less incentive to locate close to the largest markets and export from that base towards the other locations. Indeed, with very low trade barriers, they can afford to be located close to their targeted markets. Furthermore, they also want to profit from the lower costs occurring in the less-advanced location (lower wage rates) and are thus concerned in arbitraging between two incentives: a bigger market (demand incentive through expenditures) or a cheaper market (cost incentive through wages and less competition). These are the two forces: centripetal in the former case and centrifugal in the latter that concurrently foster and dampen agglomeration.

We see these two opposite forces in action in the location functions. Following integration (an increase in $\delta$) between North and South, localization of firms in the North decreases as long as there is a price (i.e. wage) differential. Conversely, the South, benefiting from its cost advantage, experiences an
increase in the number of firms operating in the country. The South also attracts firms thanks to the weaker competition (small knowledge capital stock and hence small amount of modern firms) in the country. Profiting from relocation, the South is eventually in a position to build its own vertical linkages among its modern industries and between modern and advanced sectors. It also profits from the possibility of acquiring more goods at a cheaper price.

The role played by the transaction costs is thus twofold. On the one hand, they trigger the disagglomeration forces when they decrease. On the other hand, they also provide a measure of the degree of localization of the knowledge spillovers (to the extent that they foster trade or relocation). Indeed, without any transaction costs, distance by definition does not matter. On the contrary, as long as transaction costs are present, they foster the local dimension of spillovers.

We see that clearly in our localization function. If the transaction costs tend to zero, then $\tau$ tends to 1 in which case, the localization of companies is indifferent and firms are evenly spread across the three locations. In that case, spillovers are also spread across the three locations without any friction. Obviously, since transaction costs will never be entirely eliminated, they still have an impact on the localization of firms. The lowering of the transaction costs diminishes the agglomeration forces and thus increases the global reach of the spillovers.

However, the East still faces the same transaction costs, but following their decrease within the union, the products of the two union countries are now cheaper. This competitive advantage, in terms of higher productivity in the modern sector, leads to a better return to capital within the zone than outside of it. This leads to the migration of capital from the outsider to the insiders implying both trade-investment creation in the insiders’ market and trade-investment diversions in the outsider market.

Since we began from a situation where wages in the South and the East were equal, we see that following integration, the South benefits from integration relatively more than the East that does not participate in the union. Moreover, the East will have to import more varieties from the integrated center due to the relocation of some of its own former firms into the integrated center (trade diversion). Finally, the relocation of its firms towards the bigger market will push up its cost of innovation and encourage the relocation of its advanced sector also. This is a case of divergence between the integrated core and the non-integrated periphery.

\[16\] We often observe in the real economy that the industries the most dependent on labor costs (labor intensive) are the first to relocate followed by others seeking the benefits of linkages.
5.2 Dynamic or long-run equilibrium

At the steady-state\textsuperscript{17}, we solve for an equilibrium with a specific allocation of labor in each country. We recall that $l^c = n^c/n$ is the share of country $c$ in the total number of differentiated products and measures the degree of concentration of production in each country\textsuperscript{18}. We first pursue a growth analysis and then study the trade implications.

5.2.1 Growth analysis

The determination of the results By using both the labor market equilibrium condition and the capital market equilibrium condition, we get the constant growth rate of $n$. That is, we reintroduce, for each country, the value of $x$ obtained thanks to the labor market condition into the capital market condition in order to obtain the following results\textsuperscript{19}:

In the North We have two cases:

- if autarky:

$$\frac{(1 - \sigma)}{\sigma \eta^N} (H^N - \eta^N g^N) = \rho + g^N$$

So that, after simplification, we obtain the growth rate for the North in autarky:

$$g^N = \frac{(1 - \sigma) H^N}{\eta^N} - \sigma \rho$$

- if free-trade:

The results are slightly different due to the presence of imitation:

$$\frac{(1 - \sigma)}{\sigma \eta^N} (H^N - \eta^N g^N) \frac{g^S + \gamma}{g^S} = \rho + g^N + \gamma$$

or after simplification:

$$\frac{(1 - \sigma)}{\sigma \eta^N} (H^N - \eta^N g^N) \frac{1}{g^S} = \rho + g^N + \gamma$$

where:

\textsuperscript{17}The steady state is a situation at which all the quantities grow at a constant rate. To study these steady states, we need to assume that population growth is nil. If population was consistently increasing, then this would imply that labor keeps on increasing as well as growth per capita.

\textsuperscript{18}Whereas Grossman and Helpman (1991, p. 289) assume convergence of the $l^c$ in the long run to some constant, the literature on economic geography has proved that there could be stable equilibria without convergence and that the situation with equal shares of production is not unique and not even the most probable. Indeed, in the long-run, the growth rates might be the same in each location without the $l^c$ equalizing.

\textsuperscript{19}For the following transformations, recall that $\alpha s = xnp$. 

23
\[
\frac{g^S}{l^N + \gamma} = l^N \leq 1
\] (71)

so that:

\[
g^N = \frac{l^N}{(\sigma l^N + 1 - \sigma)} \left[ \frac{(1 - \sigma)}{l^N} \left( \frac{H^N}{\eta^N} \right) - \sigma (\rho + \gamma) \right]
\] (72)

where:

\[
\frac{\delta g^N}{\delta \sigma} < 0; \quad \frac{\delta g^N}{\delta H^N} > 0; \quad \frac{\delta g^N}{\delta \eta^N} < 0; \quad \frac{\delta g^N}{\delta \rho} < 0
\] (73)

However, we shall not run partial derivatives in the case of \(l\) and \(\gamma\) since there exists a functional relationship between the two so that their variation might affect each other. Due to the interdependence between these two variables, it is complex to find a clear-cut answer on the combined impact of the variables. For instance, we know that an increase in human capital or in the degree of localization translates in an increase in the number of varieties invented (and potentially imitated) and thus a lower cost of innovation that can foster growth.

We display below several functional relationships linking growth, localization and imitation in order to better acknowledge their interdependence.

\[
g^N = f_1 (\gamma, l^N) = \frac{\gamma}{l^N} - \gamma
\] (74)

\[
g^S = f_2 (\gamma, l^S) = \frac{\gamma}{l^S} - \gamma
\] (75)

We obtain these functions by recalling that:

\[
g^c = \frac{\dot{\gamma}}{\dot{n}}; \quad l^c = \frac{\dot{n}}{\dot{m}}; \quad \gamma = \frac{\dot{g}^S}{\dot{m}} \text{ with } l^c = h (\gamma, g^c).
\]

We are interested in 2 * 2 total derivatives:

\[
\frac{dg^N}{dl^N} = \frac{g^S (1 - l^N) + \gamma l^N}{(l^N)^4} > 0
\] (76)

\[
\frac{dg^N}{d\gamma} = \frac{1 - l^N}{l^N} - \gamma (\gamma - 1) \left( \frac{H^N}{\eta^N} \right) > 0
\] (77)

We see here that both the concentration of modern firms and the imitation of modern goods foster growth in the North. We can also determine whether the growth rate in the North in case of autarky is higher than in the case of free-trade in the following way:

\[
g^N_{\text{free-trade}} > g^N_{\text{autarky}}
\] (78)

\[
\iff \frac{1}{(\sigma l^N + 1 - \sigma)} \left[ (1 - \sigma) \left( \frac{H^N}{\eta^N} \right) - (\sigma l^N) (\rho + \gamma) \right] > \frac{(1 - \sigma)H^N}{\eta^N} - \sigma \rho
\] (79)

\[
> \frac{(1 - \sigma)H^N}{\eta^N} - \sigma \rho
\] (80)
We see that in the case of free-trade with imitation (where $\gamma > 0$), the North can grow faster than in a case of autarky (where $\gamma = 0$) provided that imitation reinforces diversity (specific case due to our CES specification) or that the new stock of knowledge in free-trade is also higher for the North (otherwise it grows at the same rate as in autarky). Nonetheless, because of imitation, Northern innovators experience a shortening of the expected duration of their monopoly rents.

However, the non-imitated Northern producers experience an increase in their profits when Southern imitators get rid off their former rivals. These firms can increase their mark-up and sell more to customers whose taste for diversity is central. The surviving firms can hire the laid-off workers and increase their business by selling more goods and receiving more profits. Indeed, the positive effect of imitation on the profit rate is higher than the negative effect on the cost of capital. Moreover, laid-off workers may also join the advanced sector, encouraging hence the invention of even more goods (and thus more growth).

In the South We have two cases:
- if autarky:

$$\frac{(1 - \sigma)}{\sigma \eta^S}(H^S - \eta^S g^S) = \rho + g^S$$

so that:

$$g^S = \frac{(1 - \sigma)H^S}{\eta^S} - \sigma \rho$$

- if free-trade:

The results are slightly different because of the limit pricing strategy:

$$\frac{(\tau \beta^N w^N - \beta^S w^S)}{\sigma \eta^{S,N}}(H^S - \eta^{S,N} g^{S,N}) = \rho + g^{S,N}$$

so that:

$$g^{S,N} = \frac{1}{\sigma + (\tau \beta^N w^N - \beta^S w^S)} \left[ \frac{\tau \beta^N w^N - \beta^S w^S}{\eta^{S,N}} H^S - \sigma \rho \right]$$

where:

$$\frac{\delta g^{S,N}}{\delta \sigma} < 0; \frac{\delta g^{S,N}}{\delta \tau} < 0; \frac{\delta g^{S,N}}{\delta H^S} > 0; \frac{\delta g^{S,N}}{\delta \eta^{S,N}} < 0; \frac{\delta g^{S,N}}{\delta \rho} < 0$$

The derivative of particular interest is the one for the transaction costs. Here, the transaction cost is directly connected to the wage rate in the North. So as long as the transaction cost stays high, the wage differential is substantial and the South can take advantage of it to underprice the North and gain market shares. The lowering of barriers to trade reduces the margin of the underpricing and thus negatively affects profits for the Southern economy. However, we remember that the lowering of transaction costs had a positive impact on the
number of firms located in the South. This is the paradox of transaction costs. Finally, the smaller the wage differential, the slower the growth rate in the South.

However, South grows faster in free-trade than in autarky since its growth rate in free-trade benefits from its access to Northern technologies (this reduces the cost of knowledge: \( \eta^S \) decreases to \( \eta^{S,N} \)). It leads to more rapid innovation and thus faster growth. It appears that \( g^{S,N} > g^S \) and \( g^{S,N} > g^N \) are possible if:

\[
g^{S,N} > g^S \quad \Longleftrightarrow \quad \frac{1}{\sigma + (\tau \beta^N w^N - \beta^S w^S)} \left[ \left( \tau \beta^N w^N - \beta^S w^S \right) \frac{H^S}{\eta^{S,N}} - \sigma \rho \right] > \frac{(1 - \sigma)H^S}{\eta^S} - \sigma \rho
\]

That is true for:

\( \beta^S w^S < \tau \beta^N w^N \) and \( \eta^{S,N} < \eta^S \) \hspace{1cm} (89)

and:

\[
g^{S,N} > g^N \quad \Longleftrightarrow \quad \frac{1}{\sigma + (\tau \beta^N w^N - \beta^S w^S)} \left[ \left( \tau \beta^N w^N - \beta^S w^S \right) \frac{H^S}{\eta^{S,N}} - \sigma \rho \right] > \frac{1}{(\sigma l^N + 1 - \sigma)} \left[ (1 - \sigma) \left( \frac{H^N}{\eta^N} \right) - (\sigma l^N) (\rho + \gamma) \right]
\]

That is true for:

\( \beta^S w^S < \tau \beta^N w^N \) and \( \eta^{S,N} < \eta^N \) \hspace{1cm} (93)

It means that when the wage differential decreases, the growth rate in the South decreases as expected. But, as long as the differential exists, the South grows faster than the North, until it catches up and its wage rate equals the one in the North. In that sense, the dynamic spillover effect depends on the wage differential that reflects the technology gap between the two countries.

Another interpretation of this result is based on the idea that thanks to a higher number of firms in the South and its access to a higher stock of knowledge, the cost of R&D in the South diminishes even farther. With a relocation of firms from the North to the South, the emergence of new ideas in the South may allow the South to grow faster than its Northern partner. This leads to a decrease in the wage gap and reduces the monopoly power of existing firms. This interpretation can be shown thanks to the following total derivatives (built in the same way as in the Northern case) that confirm that relocation of domestic
firms towards a foreign location impedes domestic growth and that imitation fosters imitator’s growth:

\[
\frac{dg^S}{dN} = \frac{-g^N l^N - \gamma (1 - l^N)}{(1 - l^N)^4} < 0 \quad (94)
\]

\[
\frac{dg^S}{d\gamma} = \frac{1 - l^S}{l^S} + \frac{1}{(l^S (g^N + \gamma))^2} > 0 \quad (95)
\]

Whereas the leader discovers the new goods, the follower imitates the goods invented by the leader. The cost of imitation is assumed to be lower than the cost of innovation. Inventors do not receive indemnities from their imitators. In the follower country (the South), we have \(n^S\), the number of goods available such that, \(n^S < n^N\) \(^{20}\). The imitator of a good \(j\) invented in country \(N\) becomes the sole producer and seller of that copied good in country \(S\). We assume that \(\eta^S < \eta^N\) in the sense that the cost of imitation is lower than the cost of innovation. We recall that the factors may differ between the two countries and express the differences in endowments and types of economic policies. More fundamentally, the spread between \(H^N\) and \(H^S\) reflects the differences in scale between the two economies.

If \(n^S\) grows faster than \(n^N\), then country \(S\) may eliminate the innovative gap: the rate of return \((\rho + g^S)\) in country \(S\) will then decrease. Growth will decelerate in \(S\) but still stay above \(g^N\) as long as there is no complete catch-up. Only if the date, when the reserve of innovation will be nil, is far away in time, will the rate of return in \(S\) stays more or less constant. As long as \(\eta^S < \eta^N\) we shall have \(g^S > g^N\). Then, at the end \(n^S = n^N\). From that point on, either both countries grow at the same rate or one takes the lead after a positive shock and the other one follows either immediately or with a lag.

To summarize the Southern case, we observe that the South accumulates public knowledge thanks to spillovers. Its knowledge stock varies with cumulative experience in imitation, measured by the number of technologies that the South has acquired. The productivity of Southern imitators also partly depends upon the number of products developed in the North, since these contribute to a global pool. There is convergence in the sense that countries initially backward grow faster. This is due to the diffusion of technology itself thanks to a lower cost of imitation relatively to innovation.

However, empirically, it appears that very few countries have been able to catch up and also become inventors. It may be that the barriers to imitate and to trade ought to be reduced to accelerate the diffusion of knowledge while at the same time allowing a high enough protection for innovation to keep incentives to innovate. The presence of such barriers reinforces the dominant positions. In any case the convergence process stays conditional since it also depends on the economic policies implemented in the country.

\(^{20}\)For mathematical tractability, we had considered that \(n^S\) is a sub-sample of \(n^N\). Thus, we do not consider the case where the follower innovates and the leader imitates.
In the East

\[
\frac{(1 - \sigma)}{\sigma \eta^E} (H^E - \eta^E g^E) = \rho + g^E
\]  

(96)

and after simplification:

\[
g^E = \left(1 - \frac{\sigma}{\eta^E}\right) H^E - \sigma \rho
\]  

(97)

The East, as long as it stays outside the union, will keep its cost of knowledge at a higher level than the one prevailing in the integrated zone. Moreover, the return on knowledge capital in the East will be lower than in the Union but its cost higher, lowering its growth rate. If the South and the East were initially similar, the newly integrated South benefits from its access to a bigger stock of knowledge and grows faster than the autarkic East. The latter might even see its growth rate temporarily decreasing as long as its cost of capital stays higher than in the integrated union.

The interpretation of the results

Due to the local spillovers, the concentration of industries in a country measured by \( l^c \) has a positive impact on its growth rate. The equilibrium growth rate depends positively on the level of concentration \( l \), the stock of skilled labor \( H \), the expenditure share of differentiated good \( \alpha \), and the degree of increasing returns to scale. A greater taste for diversity - a low \( \sigma \) - implies a less elastic demand for each product thus a larger opportunity for monopoly profits and a higher return to R&D. Increasing product variety has an impact on knowledge spillovers that diminishes the cost of innovation and hence fosters growth. Conversely, a high \( \sigma \) implies smaller differentiation between products. Likewise growth depends negatively on the cost of innovation (\( \eta \) : a smaller \( \eta \) implies a higher productivity in the resources devoted to the advanced sector) and the rate of time preference (\( \rho \)). A higher desire to save or more patience - \( \rho \) smaller - increases the rate of growth.

Moreover, a decrease in the cost of innovation (or an increase in the productivity of the labor force in the advanced sector), \( \eta \), increases the rate of return, \( r \), and thus the rate of growth, \( g \). There is also a scale effect, since an increase in \( H \) increases \( g \). The bigger the economy (the bigger \( H \)), the smaller the cost of innovation for each \( H \), since the cost of unit for R&D is \( \eta/H \). However, any restriction in terms of knowledge diffusion limits the impact of the scale effect. Finally, we see that when we allow for trade, innovation - and therefore growth - accelerates in each member country. Indeed, genuine research in each country combines and thus contributes to the creation of a global stock of knowledge bigger than the respective national stocks\(^{21}\).

Because of local spillovers, agglomeration of firms in the most advanced country fosters global growth. But we have seen that concentration depends on the wage or cost differential across countries and on the expenditure differential. More importantly, location of firms depends on location of human capital in the

\(^{21}\)Only in the improbable case of perfect redundancy would the global stock equals the national ones.
R&D sector. In that framework, a country still lagging after integration has obviously been unable to shift its human capital from the modern sector to the advanced sector (and from unskilled to skilled labor).

Economic integration conducts to the expansion of market size (and its related higher aggregate demand combining the domestic and foreign ones) and the intensification of monopolistic competition (and its related relative decrease in market share following the rivalry between domestic and foreign producers). By leading to static gains, they permit level effects. For dynamic gains or growth effects to occur, we need an increase in the rate of creation of new products. Through the direct effect of trade, households can purchase a bigger set of goods rather than the restricted local set measured by \( n' \). Because the number of varieties has increased, the union experiences a permanent increase in the level of utility but not in its rate. The increase in the rate of utility is due to the exchange of ideas. Indeed, even without international knowledge spillovers, productivity in the modern sector rises due to a better access to foreign-made differentiated goods. When we allow for international spillovers, then researchers across the whole world can draw on a global common stock of knowledge.

These benefit each trade partner since national research accumulates in proportion to global R&D activity (and not only local). This is due to the IRS features of R&D at the external level\(^{22}\). Because of the relocation of firms from the outsider country to the insiders one, we should also expect the relocation of the R&D sector from the East towards the North and the South. Households will rather invest in the North-South center than in the Eastern periphery. This leads to a decrease in the cost of R&D in these two countries and thus a higher growth. Nevertheless, the East can still profit from its lower wage rate if it decides to join the union.

The demand effect reflected by the size of the expenditures lead to a center-periphery solution (a corner solution, i.e. Walz, 1995) where the larger market receives the larger share of firms. That will incite innovators to also locate in the main market and hence increase the number of newly invented goods in that location. This is a typical case of circular cumulative causation through vertical linkages both forward and backward.

Conversely, there is a competition effect pushing in the adverse direction towards an interior solution. It relates to the fact, that due to barriers to trade, average sales prices are higher in the lagging country endowed with less modern sector goods. Hence, competition is less fierce, enabling higher sales and thus bigger profits in that location. Added to lower costs (lower wages), firms are incited to invest and produce in the lagging country. In our model, it appears that the competition and cost effects dominate the demand effect\(^ {23}\).

\(^{22}\)R&D is CRS at the internal level of the firm.

\(^{23}\)Profits are indeed affected in two opposite ways by liberalization. They increase through the rise in exports of local goods towards the partner market while they diminish because of the increased competition established by lower trade barriers and thus lower prices.
5.2.2 Trade analysis

For the insiders  At every moment in time, the pattern of trade is determined by the number of blueprints in the hands of each country’s firms. Over time, the trade pattern evolves in accordance with the number of new discoveries made by entrepreneurs in each country. This in turn depends upon the R&D investments that take place in each location.

Since we allow for mobility of financial capital, each country can finance its R&D with international savings. In that sense, countries are not forced to specialize in a unique type of industry either T or M and their trade account needs only to be balanced in terms of present value of their trade flows. However, the human-capital-rich country specializes relatively in the production of innovative goods. But its residents consume the same share $E^N$ of world output of every good. Therefore, country $N$ must be the one that develops a sectorial trade surplus in high technology. The labor-abundant countries $S$ and $E$, on the other hand, import differentiated products on net and export the traditional good. This pattern of trade is a direct consequence of the Heckscher-Ohlin theorem.

Factor intensity reversal comes from a change in specialization triggered by openness and the lowering of $\tau$. There are three effects of a decrease in trade costs. One negative effect due to the home market becoming less protected. Two positive effects when the access to the export markets improves and imported goods become less expensive. Economic integration indeed fosters market access. When manufacturing costs in the two countries are not the same, imitation may be a profitable activity in the low-cost country. Entrepreneurs there who develop clone products can earn positive profits in duopolistic competition with innovators in the high-cost country. If these profits are substantial enough, they may justify the costs of imitation. This makes imitation by the South more profitable and thus accelerates growth.

However, at the same time, wages increase in the South until it becomes unsustainable to underprice the North. At this stage, South might be forced to stop imitating and begins innovating while pricing its goods at a mark-up and compete on the foreign markets. The higher the degree of imitation, the higher the required rate of return $(\rho + g + \gamma)$ shall be. This also reflects that innovators lose market share to lower price competitors (imitators).

Imitation also speeds growth in the North because import of imitated goods means that a lower number of workers in the North will be necessary for each percentage point growth. Imitation forces workers out of modern sector and pushes them into advanced sector. Indeed, according to Stolper-Samuelson theorem, an increase in the price of $M$ for $T$ given raises the cost of innovation since it raises the reward to $H$ in the $M$ sector relatively to $H$ in the $A$ sector while reducing the reward to $L$.

However, according to Rybczynski theorem, if $H$ augments, the $A$ sector, since it uses $H$ most intensively, must expand to absorb the increased supply of $H$. This shall foster a reallocation of $L$ into the expanding sectors and lead to the contraction of the traditional sector. Actually, rather than a reallocation, this is rather more a transformation of $L$ into $H$ due to the new wage incentives for $L$.
to invest in education and training in order to become \( H \). Indeed, the interest of the traditional sector remains in the learning from HOS theorems. The case of the new industrialized economies reminds us of the technology transfer feature of imitation in encouraging convergence through the reallocation-transformation of labor towards sectors more intensive in human capital.

**For the outsider**  The competition between the South and the East depends on their relative factor endowment (their relative skilled labor share in the total population). Thanks to productivity improvements following the integration (access to a higher stock of knowledge), the need for unskilled labor decreases while the need for skilled labor increases shifting labor from \( L \) (low-skilled labor) to \( H \) (high-skilled labor) through the transformation of \( L \) into \( H \). While the South is going to more resemble the North, the East will follow the same pattern at condition that it engages in liberalizing its trade with the union. If not, it will combine the inconveniences of being a cheaper country (lower demand) without being able to attract foreign firms (interested in lower costs and competition) and profit from their knowledge spillovers since foreign firms will be discouraged by high transaction costs and weak productivity. The East might thus specialize in the traditional sector and its relatively lower wages.

In the case of an initially rich East, this country would simply have a growth rate equal to the one in the North in case of autarky (thus lower than the one in case of free-trade) although it will progressively face a higher cost of investment. Integrating the East increases the global growth rate and the one of each participant. The global capital stock of knowledge is possibly higher and thus innovation is cheaper encouraging the employment of more labor in R&D activities. Following integration, we experience a labor quality upgrading ladder where workers progressively quit the unskilled pool to join the skilled pool through wage incentives and knowledge spillovers.

### 6 Conclusion

Following integration, the global economy becomes less geographically concentrated since the centripetal forces (advantage of being close to your suppliers) weaken. Two results may occur: either a convergence setting involving the three countries in a common union or a core-periphery pattern featuring the integrated insiders (center) and outsider (periphery). Because of imitation we observe two major effects: 1) the wage differential between unequals diminishes until complete disappearance; 2) a reallocation-transformation of labor from the modern sector to the advanced sector fosters growth in the rich country. All partners benefit from liberalization. The incentives to innovate for the originally laggard country come from an equalization between the imitation and innovation costs that presuppose that human capital, knowledge capital and number

\[24\] The models of economic geography by missing the role of knowledge spillovers and a dynamic dimension had long been unable to deal with the relocation of economic activity towards the laggard countries thanks to knowledge spillovers.
of new goods also reach equality in both locations. We also showed that the best way for an outsider to avoid trade and investment diversion - and eventually the constitution of a center-periphery setting - is to join the union.

On the empirical side, Dion (2004b) has shown that large rich countries benefit essentially from their respective national stock of knowledge while small poor countries profit essentially from their access to foreign stocks of knowledge. The more open a lagging country towards advanced foreign countries, the higher its total factor productivity (TFP). The cases of regional economic integration that we have studied in this paper present obvious similarities with the cases of regional integration encountered in the world economy: European Union, NAFTA, MERCOSUR, ASEAN, etc... where the trade partners (members or not) are at different stages of development.

The recommendations are thus: promotion of openness to foreign ideas thanks to free-movement in goods, capital, services and labor with one essential condition: investment in human capital and in R&D in the lagging countries. Not only should countries pursue economic integration but they should also foster knowledge diffusion in order to avoid unfavorable resource reallocation effects. The access to international knowledge diffusion is a precondition for convergence. If intranational diffusion is stronger than international diffusion, there is risk of divergence.

7 References


8 Appendices

8.1 Appendix 1: Notation summary

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<tr>
<th>Variables and Parameters</th>
<th>Notation Summary</th>
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<tr>
<td>$n$</td>
<td>number of varieties</td>
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<tr>
<td>$x$</td>
<td>demand for variety</td>
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<tr>
<td>$p$</td>
<td>price</td>
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<td>taste parameter</td>
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<td>$K$</td>
<td>knowledge capital</td>
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<td>$\eta^I$</td>
<td>efficiency parameter for innovation</td>
</tr>
<tr>
<td>$\eta^S$</td>
<td>efficiency parameter for imitation</td>
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<tr>
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<td>value of firm</td>
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<td>$r$</td>
<td>rate of returns on riskless bond</td>
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### Variables and Parameters

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<th>Variables and Parameters</th>
<th>Notation Summary</th>
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<td>$g$</td>
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<td>$A$</td>
<td>advanced or R&amp;D sector</td>
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### 8.2 Appendix 2: Level of barriers and volume of trade

According to economic geography theories, the North, where the modern sector is concentrating has higher wages than the two other locations. Due to a higher labor demand generated by the modern sector and a lower cost of living index (the North does not spend so much money as the others on transaction costs for differentiated goods since it produces most of them), wages in the North are indeed originally higher. Conversely, confronted to these transaction costs, the South and the East can only grant their workers with a lower wage. Only the progressive lowering of these transaction costs can allow the reduction of the wage differential until factor price equalization for $\tau = 1$. At this level, proximity to demand and supply loses its appeal and a symmetric equilibrium is possible where labor costs are the main determinant of location.

The equilibrium may thus depend on the relationship between the level of the trade barriers and the degree of increasing returns to scale (or elasticity of substitution between products). However, for $p^N \leq p^S$ where $\tau \geq 1/\sigma$, Southern imitators would not be able to compete against Northern innovators in the Northern market and would thus be absent from that market. There is thus a clear relationship between the decrease in trade barriers and the increase in bilateral trade. Furthermore, the higher (lower) the IRS [low (high) $\sigma$], the higher (lower) the trade barriers may be to allow bilateral trade in modern goods. The more differentiated the goods, the higher the IRS, the broader the extent of the limit pricing and the sharper the eventual competition from the imitators.

This implies that due to a high degree of differentiation, the Northern products can be more easily competed with by potential imitators. Moreover, we see that the lowering of the trade barriers allows the increase in the wage rates in the South. Indeed, in order to maintain the balance between innovator and imitator’s prices, the phasing out of the trade barriers get along with the filling of
the wage gap. There is thus a compensation scheme between transaction costs and Southern wages triggered by the lowering of $\tau$ that fosters the increase in $w^S$.

8.3 Appendix 3: Innovation versus imitation

We assume that the South, as soon as it joins the union with the North, stops innovating but does imitate. The South innovates in the case of autarky although at a higher cost than in the Northern case because of its lower productivity in R&D. However, in case of free-trade - as soon as the trade barriers begin to gradually phase out between the South and the North - it has a strong incentive to stop innovating and begin imitating in order to rip off some easy profit by underpricing the products of its Northern competitors.

The R&D sector in the South is similar as in the North apart that its share of $H$ is smaller. $\gamma$ is the rate of imitation where $\gamma = K^S/K^N = n^S/n^N$. This variable represents the proportion of Northern products copied per unit of time. The idea is that the South imports goods from the North according to $\tau$ and then imitates some of these imported goods according to $\gamma$. Finally, the South will use its own imitation function (similar to the innovation function of the North except for its productivity parameter $\eta^S$ lower than the productivity parameter of the North $\eta^N$) to copy and reproduce the imitated varieties.