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Abstract

The existing literature on the comparison of tournaments and piece rates as alternative incentive schemes has focused on the case of unlimited liability. However, in practice real workers' wealth is typically restricted. Therefore, this paper compares both schemes under the assumption of limited liability. The results show that if the cost function is sufficiently convex, first-best effort will be more likely implemented under piece rates than under tournaments. Moreover, if first-best implementation is not achieved and workers earn positive rents, efforts and profits will be larger for piece rates than for tournaments given sufficiently convex costs. While tournaments offer a partial insurance due to their fixed prizes, piece rates may not work any longer if potential losses become prohibitively high. Finally, if risk is sufficiently high, piece rates will dominate tournaments despite the partial insurance effect of tournament compensation. Since effort costs and risk may depend on an individual worker's characteristics, on the characteristics of his job and on his hierarchical position, these findings have important implications for the choice of incentive schemes and the allocation of workers in firms.

Key words: incentives, limited liability, piece rates, rank-order tournaments.
JEL Classification: J31, J33, M5.

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1 Introduction

Since the seminal paper by Lazear and Rosen (1981) there has been a wide discussion of tournaments versus piece rates as alternative incentive schemes. In a tournament, at least two workers compete against each other for given winner and loser prizes. Under a piece-rate scheme, a worker's payment consists of a fixed payment and a certain percentage – the piece rate – of the worker's realized output in monetary terms.

There exist many examples for either incentive scheme in practice. Tournaments can be observed in sports (e.g., Ehrenberg and Bognanno 1990, Becker and Huselid 1992), in broiler production (Knoeber and Thurman 1994) and also in firms when people compete for job promotion (e.g., Baker, Gibbs and Holmstrom 1994a, 1994b, Eriksson 1999, Bognanno 2001). Basically, corporate tournaments will always be created if relative performance evaluation is linked to monetary consequences for the employees. Hence, forced-ranking or forced-distribution systems, in which supervisors have to rate their subordinates according to a given number of different grades, also belong to the class of tournament incentive schemes (see, for example, Murphy 1992 on forced ranking at Merck). Boyle (2001) reports that about 25 per cent of the so-called Fortune 500 companies utilize forced-ranking systems to tie pay to performance (e.g., Cisco Systems, Intel, General Electric). Another way of combining relative performance evaluation and tournament incentives has been suggested recently by the German CEO Ulrich Schumacher of Infineon. He has proposed to dismiss the 5 per cent least successful employees of the workforce each year. Of course, there are also lots of examples for piece-rate schemes in practice (see e.g. – among many others – Lazear 2000 on the introduction of piece rates at the Safelite Glass Corporations, and Freeman and Kleiner 1998 on the decline of piece-rate systems in the American shoe industry).

Lazear and Rosen have shown that both incentive schemes lead to first-best efforts given homogeneous and risk neutral workers with unlimited liability. However, tournaments can dominate piece rates, since tournaments only require an ordinal performance measure, whereas piece rates are exclusively based on cardinal measures, and a cardinal scale usually leads to higher measurement costs than an ordinal scale. Considering risk averse workers, Lazear and Rosen (1981) show that there is no clear ranking between the two incentive schemes, as the comparison crucially depends on the shape of the workers' utility functions and the magnitude of the risk. On the one hand, if the magnitude is large, tournaments will provide a crude form of insurance, since each agent receives at least the given loser prize and at most the given winner prize. On the other hand, tournaments have the drawback that in symmetric equilibrium the probability mass is distributed equally on the winner and the loser prize. In addition, Lazear and Rosen demonstrate that tournaments will be problematic with heterogeneous contestants, if the employer is not able to choose appropriate handicaps so that the contest becomes even.

Green and Stokey (1983) emphasize that tournaments will dominate piece rates if filtering of common noise is of major interest. In tournaments, common noise cancels out because of the relative comparison of the workers' performance. Piece rates use an absolute performance measure and, therefore, cannot serve as a risk filter in a static context.

Malcomson (1984, 1986) points to an important advantage of tournaments compared to piece rates. Since winner and loser prizes are fixed in advance (i.e., the employer commits himself to certain labor costs before the tournament starts), tournaments can create incentives even if the workers' performance measure is non-contractible. However, an employer always needs a contractible performance measure for utilizing piece rates as an incentive scheme.

Demougin and Fluet (2003) compare tournaments and bonus schemes under inequity aversion. The bonus scheme is defined as an individualistic incentive scheme that gives a worker a certain base wage and, in addition, a bonus if the worker has met a fixed standard. Since under the tournament scheme wages are always unequal, tournaments turn out to be disadvantageous given a binding participation constraint. However, if workers earn positive rents and the probability of meeting the standard is sufficiently large, tournaments may dominate bonus schemes

Based on the theoretical results, several experimental papers have compared the workers' behavior in tournaments and piece rates (e.g., Bull, Schotter and Weigelt 1987, von Dijk, Sonnemans and von Winden 2001). The experimental results show that tournaments induce higher efforts than piece-rates on average, but efforts vary more in tournaments.

Although workers' liability is often limited in practice, the existing comparison of tournaments and piece rates has been restricted to the case of unlimited liability. By this assumption, loser prizes in tournaments and fixed payments in piece-rate schemes are allowed to be negative. Hence, not surprisingly given risk neutral workers first-best efforts are implemented under either incentive scheme. In tournaments, the optimal spread between winner and loser prize can always be chosen to induce first-best incentives, whereas the – possibly negative – loser prize is used by the employer to make the workers' participation constraint bind. In piece-rate schemes, optimal incentives are created by choosing a piece rate of 100% and "selling the firm" to the worker.

However, under limited liability this central result of Lazear and Rosen (1981) does not necessarily hold any longer. This paper compares both incentive schemes under limited liability and highlights the main differences between tournaments and piece rates.[1] In particular, it can be shown that the convexity of the workers' cost function plays an important role: The more convex the cost function the less likely first-best effort is implemented under the tournament scheme compared to piece rates. Moreover, if workers receive positive rents because of limited liability, efforts as well as profits will be larger under piece rates than under tournaments given a sufficiently convex cost function. Finally, if risk is sufficiently high, piece rates will dominate tournaments.

Note that the workers' cost functions and the given risk can be mainly determined either by the workers' individual characteristics or by the tasks which have been delegated to them. In the first case, effort costs and risk describe the workers' types. In the latter case, they are implied by the type of work organization chosen by the employer. On the one hand, the employer may prefer specialization of the workers so that one individual worker mostly performs either difficult (risky) or easy (non-risky) tasks. On the other hand, the employer may want to avoid too monotonous work for his employees so that he delegates different tasks to them. Note also that effort costs typically increase with the hierarchical level of a worker. In addition,

[1] For a discussion of incentive problems under limited liability see Innes (1990), Park (1995), Kim (1997) and Pitchford (1998). However, they choose a completely different approach, as they look for the optimal contract under limited liability, whereas in this paper two incentive schemes are compared that are frequently used in practice.

the limited-liability problem is less severe the higher the employee's position in the corporate hierarchy (see also Kim 1997). For example, the wealth of a manager is typically greater than that of a worker belonging to a lower hierarchical level. Altogether, given the findings of this paper, the employer carefully has to choose the optimal incentive scheme depending on the type of workers, the type of work organization and the hierarchical level of the workers.

The paper is organized as follows. The next section introduces the model. The results of the model are presented in Section 3 and discussed in Section 4. Section 5 concludes.

2 Model

To compare tournaments with piece rates, a model with one employer and two workers is considered. All players are assumed to be risk neutral. When choosing his effort e_i worker i $(i = A, B)$ can either have success (with probability $p(e_i)$) or failure (with probability $1 - p(e_i)$). The function $p(e_i)$ is assumed to be concave, i.e. $p'(e_i) > 0$ and $p''(e_i) \leq 0$, with $p(e_i) \in [0, 1]$. In addition we assume $p'''(e_i) \leq 0$. The failure case is described by a (continuous lottery or a) random output $y_L \in [-\bar{y}_L, \bar{y}_L)$ with $\bar{y}_L > 0$ and mean $E[y_L] = 0$. However, the case of success is characterized by a lottery which can only take strictly positive output levels $y_H \in [\bar{y}_L, \bar{y}_H]$ with mean $E[y_H] = \hat{Y}_H > 0$, i.e. $\hat{Y}_H$ denotes the conditional mean given the case of success.[2] In other words, output is determined by a two-stage lottery. In the first stage, a worker can either succeed or fail, in the second stage given success (failure) output is then realized according to the random variable y_H (y_L). We assume that output is contractible, whereas the employer does not observe e_i. Worker i's effort costs are described by the convex function $c(e_i)$ with $c(0) = 0$, $c'(e_i) > 0$, $c''(e_i) > 0$ and $c'''(\cdot) \geq 0$. Each worker is assumed to have a reservation value $\bar{u} \geq 0$, and, in any given case, the employer wants to hire the two workers (e.g., because of their human capital).[3] The employer maximizes expected total output minus labor costs (i.e., wages), whereas each worker maximizes expected wages minus effort costs.

If the employer organizes a *tournament* between the two workers, at the first stage of the game he will choose a winner prize w_1 and a loser prize w_2 prior to the tournament to induce incentives. Let $\Delta w = w_1 - w_2$ denote the prize spread. Then, for given tournament prizes, the two workers choose their optimal efforts at the second stage. To model limited liability, the loser and the winner prize are not allowed to become negative $(w_1, w_2 \geq 0)$. However,

[2] The two lotteries y_L and y_H are introduced instead of two deterministic values to have a sufficiently rich structure for the following analysis. However, Section 4 will show that the main results are robust with respect to the assumed production technology.

[3] Technically, we can assume that the employer's reservation value is sufficiently negative. By this assumption, the first-best solution can only be reached within the employment relationship.

since positive incentives require $w_1 > w_2$, the limited-liability constraint actually reduces to $w_2 \geq 0$. Under a *piece-rate scheme*, at the first stage of the game the employer uses a linear incentive formula $w_i = \alpha + \beta y_i$ $(i = A, B)$ with y_i as worker i's realized output, α as a fixed payment and β as the piece rate. Again, the limited-liability assumption for the workers requires wages w_i to be non-negative $(w_i \geq 0)$. At the second stage, each worker chooses his effort e_i for a given pair (α, β).[4]

3 Results

As a benchmark result, the first-best effort e^{FB} can be calculated. This effort maximizes

$$E[y_i] - c(e_i) = p(e_i)\hat{Y}_H - c(e_i),$$

which yields

$$\hat{Y}_H = \frac{c'\left(e^{FB}\right)}{p'\left(e^{FB}\right)} =: h\left(e^{FB}\right). \tag{1}$$

Note that due to the convexity of $c(\cdot)$ and the concavity of $p(\cdot)$, the function $h(\cdot)$ is monotonically increasing. Hence, the higher the expected output in case of success, the higher first-best effort e^{FB}.

Under the *tournament scheme*, at the second stage of the game, worker i

[4] The wage parameters are decribed without a subscript because the two workers are homogeneous.

maximizes

$$\begin{aligned} EU_i(e_i) &= w_1 p(e_i)(1-p(e_j)) + \frac{w_1+w_2}{2}\left[p(e_i)p(e_j)+(1-p(e_i))(1-p(e_j))\right] \\ &\quad + w_2(1-p(e_i))p(e_j) - c(e_i). \end{aligned}$$

With probability $p(e_i)(1-p(e_j))$ worker i becomes the winner of the tournament and receives the winner prize w_1. He gets the loser prize w_2 with probability $(1-p(e_i))p(e_j)$. If the two workers produce identical outputs, the winner of the tournament will be randomly chosen by the employer using a fair coin. This event happens with probability $p(e_i)p(e_j)+(1-p(e_i))(1-p(e_j))$. The first-order condition for optimal effort yields a unique and symmetric equilibrium[5] with each worker choosing effort $e_T^*(\Delta w)$, implicitly defined by

$$\frac{\Delta w}{2} = h(e_T^*). \tag{2}$$

At the first stage, the employer chooses w_1 and w_2 to maximize $2p(e_T^*(\Delta w))\hat{Y}_H - w_1 - w_2$ subject to the workers' incentive constraint (2) and their participation constraint

$$\frac{w_1+w_2}{2} - c(e_T^*(\Delta w)) \geq \bar{u}. \tag{3}$$

It is straightforward to show that the employer can implement e^{FB} and make

[5]Since the workers' objective functions are strictly concave, the second-order condition always holds.

the participation constraint bind by choosing

$$w_1^{FB} = c(e^{FB}) + \bar{u} + \hat{Y}_H \quad \text{and} \quad w_2^{FB} = c\left(e^{FB}\right) + \bar{u} - \hat{Y}_H.$$

Note, however, that due to the limited-liability assumption $w_2 \geq 0$ this solution will only be feasible if

$$c\left(e^{FB}\right) + \bar{u} \geq h\left(e^{FB}\right) \Leftrightarrow c\left(e^{FB}\right) + \bar{u} \geq \hat{Y}_H. \tag{4}$$

Under the *piece-rate scheme*, the workers want to maximize

$$EU_i\left(e_i\right) = \alpha + \beta E\left[y_i\right] - c\left(e_i\right) = \alpha + \beta p\left(e_i\right) \hat{Y}_H - c\left(e_i\right).$$

Their incentive constraint is given by

$$\beta \hat{Y}_H = h\left(e_{PR}^*\right) \tag{5}$$

and their participation constraint by

$$\alpha + \beta p\left(e_{PR}^*\right) \hat{Y}_H - c\left(e_{PR}^*\right) \geq \bar{u}. \tag{6}$$

Therefore, the employer can implement e^{FB} and make the participation constraint bind by setting

$$\beta^{FB} = 1 \quad \text{and} \quad \alpha^{FB} = c\left(e^{FB}\right) + \bar{u} - p\left(e^{FB}\right)\hat{Y}_H.$$

Note that in the worst case each worker's wage becomes $w_i = \alpha + \beta\left(-\bar{y}_L\right)$. Hence, because of the limited-liability assumption, the first-best contract $\left(\alpha^{FB}, \beta^{FB}\right)$ is only feasible for

$$c\left(e^{FB}\right) + \bar{u} \geq p\left(e^{FB}\right)\hat{Y}_H + \bar{y}_L. \tag{7}$$

Comparing (4) with (7) leads to the following results:

Proposition 1 *(i) The higher the workers' reservation value, $\bar{u}$, the more likely e^{FB} is implemented under either incentive scheme. (ii) If $\dfrac{\bar{y}_L}{1 - p\left(e^{FB}\right)} < (>)\, \hat{Y}_H$, implementation of e^{FB} will be more likely (less likely) under a piece-rate than under a tournament scheme.*

The intuition for result (i) comes from the fact that workers can be given stronger incentives the higher their wealth. If workers have high reservation values, the employer will have to compensate the workers for these foregone values by a large lump-sum payment when they sign the contract. By this, the workers' wealth increases significantly so that it is more likely that the employer wants to create sufficiently high incentives which lead to first-best effort.[6] Hence in this context, large reservation values of the workers are strictly welfare enhancing.

Result (ii) can also be explained intuitively: Under the piece-rate scheme, the higher the possible loss $|-\bar{y}_L|$ in case of a failure, the higher the fixed payment α must be, which is necessary for compensating the worker in the worst case so that $w_i \geq 0$. Hence, the higher $|-\bar{y}_L|$, the less likely α^{FB} is sufficiently large to compensate the worker. Of course, if $\bar{y}_L = 0$, there will be no limited-liability problems when using piece rates. In the tournament case, the optimal prize spread for implementing e^{FB} is given by $\Delta w^{FB} = w_1^{FB} - w_2^{FB} = 2\hat{Y}_H$. The higher this prize spread, the higher the likelihood that the loser prize that is needed for first-best implementation becomes negative, which is not allowed under limited liability.

The inequality of Proposition 1(ii) also shows that the higher $p\left(e^{FB}\right)$ and, therefore, first-best effort e^{FB}, the less likely this effort is implemented under a piece-rate than under a tournament scheme. This result stems from the fact that the employer "sells the firm" to the workers by choosing $\beta^{FB} = 1$. Hence, the workers receive the total expected output $p\left(e^{FB}\right) E\left[y_H\right] + \left(1 - p\left(e^{FB}\right)\right) E\left[y_L\right] = p\left(e^{FB}\right) \hat{Y}_H$ which decreases α^{FB}. The higher expected output, the higher the likelihood that the workers indeed have to pay a price (i.e., $\alpha^{FB} < 0$), which is not allowed under limited liability. Note that

[6] See also Park (1995), pp. 488–489, Kim (1997), p. 910.

according to (1), the magnitude of e^{FB} directly corresponds to the shape of the cost function $c(\cdot)$. The flatter the marginal cost function $c'(\cdot)$ or the less convex the cost function $c(\cdot)$, the higher will be e^{FB} and, therefore, the less likely first-best effort is implemented under a piece-rate than under a tournament scheme.

Next, the employer's complete optimization problem under limited liability at the first stage of the game is considered. When organizing a *tournament* the employer maximizes

$$\pi_T = 2p\left(e_T^*\left(w_1 - w_2\right)\right)\hat{Y}_H - w_1 - w_2$$

subject to the participation constraint (3) and the limited-liability constraint $w_2 \geq 0$, where $e_T^*\left(w_1 - w_2\right)$ is described by the incentive constraint (2) with $\partial e_T^*/\partial w_1 = 1/\left(2h'\left(e_T^*\right)\right)$ and $\partial e_T^*/\partial w_2 = -1/\left(2h'\left(e_T^*\right)\right)$. From the Lagrangian

$$\begin{aligned} L_T\left(w_1, w_2\right) &= 2p\left(e_T^*\left(w_1 - w_2\right)\right)\hat{Y}_H - w_1 - w_2 \\ &\quad + \lambda_1\left[\frac{w_1 + w_2}{2} - c\left(e_T^*\left(w_1 - w_2\right)\right) - \bar{u}\right] + \lambda_2 w_2 \end{aligned}$$

we obtain the following optimality conditions for w_1 and w_2:[7]

$$\frac{p'\left(e_T^*\right)\hat{Y}_H}{h'\left(e_T^*\right)} - 1 + \frac{\lambda_1}{2} - \frac{\lambda_1 c'\left(e_T^*\right)}{2h'\left(e_T^*\right)} = 0 \tag{8}$$

$$-\frac{p'\left(e_T^*\right)\hat{Y}_H}{h'\left(e_T^*\right)} - 1 + \frac{\lambda_1}{2} + \frac{\lambda_1 c'\left(e_T^*\right)}{2h'\left(e_T^*\right)} + \lambda_2 = 0 \tag{9}$$

$$\lambda_1 \cdot \left[\frac{w_1 + w_2}{2} - c\left(e_T^*\right) - \bar{u}\right] = 0$$

$$\lambda_2 \cdot w_2 = 0$$

$$\lambda_1 \geq 0 \quad \text{and} \quad \lambda_2 \geq 0.$$

From (8) and (9) we get $\lambda_1 + \lambda_2 = 2$. Hence, at least one constraint must be binding in equilibrium.

Under the *piece-rate scheme* with limited liability, the employer wants to maximize

$$\pi_{PR} = 2\left(1 - \beta\right) p\left(e_{PR}^*\left(\beta\right)\right)\hat{Y}_H - 2\alpha$$

subject to the participation constraint (6) and the limited-liability constraint $\alpha - \beta\bar{y}_L \geq 0$, with $e_{PR}^*\left(\beta\right)$ being implicitly defined by the incentive constraint (5) with $de_{PR}^*/d\beta = \hat{Y}_H/h'\left(e_{PR}^*\right)$. Hence, the corresponding Lagrangian is

[7] To simplify notation $e_T^*\left(w_1 - w_2\right)$ is written as e_T^*.

given by

$$\begin{aligned} L_{PR}(\alpha,\beta) &= 2\left(1-\beta\right)p\left(e^*_{PR}\left(\beta\right)\right)\hat{Y}_H - 2\alpha \\ &\quad +\lambda_1\left[\alpha+\beta p\left(e^*_{PR}\left(\beta\right)\right)\hat{Y}_H - c\left(e^*_{PR}\left(\beta\right)\right)-\bar{u}\right]+\lambda_2\left[\alpha-\beta\bar{y}_L\right] \end{aligned}$$

and the respective optimality conditions for β and α yield:[8]

$$\left(\lambda_1-2\right)p\left(e^*_{PR}\right)\hat{Y}_H+\left(2\left(1-\beta\right)+\lambda_1\beta\right)\frac{p'\left(e^*_{PR}\right)\hat{Y}_H^2}{h'\left(e^*_{PR}\right)}-\frac{\lambda_1 c'\left(e^*_{PR}\right)\hat{Y}_H}{h'\left(e^*_{PR}\right)}-\lambda_2\bar{y}_L=0 \tag{10}$$

$$\lambda_1+\lambda_2=2 \tag{11}$$

$$\lambda_1\cdot\left[\alpha+\beta p\left(e^*_{PR}\right)\hat{Y}_H-c\left(e^*_{PR}\right)-\bar{u}\right]=0$$

$$\lambda_2\cdot\left[\alpha-\beta\bar{y}_L\right]=0$$

$$\lambda_1\geq 0 \quad \text{and} \quad \lambda_2\geq 0.$$

Eq. (11) shows that – analogously to the tournament case – at least one constraint must be binding in equilibrium.

Let e^*_T (e^*_{PR}) denote the equilibrium effort under the tournament (piece-rate) scheme. We obtain the following results:

Proposition 2 *Let $c'''\left(e_i\right)>0$. In equilibrium, three cases have to be dis-*

[8]To simplify notation $e^*_{PR}\left(\beta\right)$ is replaced with e^*_{PR}.

tinguished: (i) If the participation constraint is binding but not the limited-liability constraint, we get $e_T^* = e_{PR}^* = e^{FB}$. *(ii) If both constraints are binding, equilibrium efforts are described by*

$$h\left(e_T^*\right) = \bar{u} + c\left(e_T^*\right) \quad \text{and} \quad h\left(e_{PR}^*\right)\left[\frac{\bar{y}_L}{\hat{Y}_H} + p\left(e_{PR}^*\right)\right] = \bar{u} + c\left(e_{PR}^*\right). \quad (12)$$

(iii) If only the limited-liability constraint is binding, equilibrium efforts are given by

$$\frac{p'\left(e_T^*\right)\hat{Y}_H}{h'\left(e_T^*\right)} = 1 \quad \text{and} \quad \frac{p'\left(e_{PR}^*\right)\hat{Y}_H}{h'\left(e_{PR}^*\right)} = \frac{\bar{y}_L + p\left(e_{PR}^*\right)\hat{Y}_H}{\hat{Y}_H - h\left(e_{PR}^*\right)}. \quad (13)$$

Proof. See appendix. ■

Result (i) is not surprising. As we know from Lazear and Rosen (1981), if there are no limited-liability problems (i.e., the limited-liability constraint is not binding), first-best effort is implemented under either incentive scheme. However, if the limited-liability constraint is binding, we will either have an interior solution given that $\bar{u}$ is not too large, so that the participation constraint does not become binding (result (iii)) or a corner solution otherwise (result (ii)).

In case (ii) with both constraints being binding, we have

$$\begin{aligned}
\frac{\partial e_{PR}^*}{\partial\left(\bar{y}_L/\hat{Y}_H\right)} &= -\frac{h\left(e_{PR}^*\right)}{h'\left(e_{PR}^*\right)\left[\frac{\bar{y}_L}{\hat{Y}_H}+p\left(e_{PR}^*\right)\right]+h\left(e_{PR}^*\right)p'\left(e_{PR}^*\right)-c'\left(e_{PR}^*\right)} \\
&= -\frac{h\left(e_{PR}^*\right)}{h'\left(e_{PR}^*\right)\left[\frac{\bar{y}_L}{\hat{Y}_H}+p\left(e_{PR}^*\right)\right]}<0,
\end{aligned}$$

since $h\left(\cdot\right) := c'\left(\cdot\right)/p'\left(\cdot\right)$. Hence, e_{PR}^* decreases in $\bar{y}_L$ and increases in $\hat{Y}_H$, whereas e_T^* is independent of both parameters. Therefore, given scenario (ii), e_{PR}^* will be more (less) likely to exceed e_T^* if $\bar{y}_L$ is small (large) and $\hat{Y}_H$ is large (small). The intuition for the influence of $\hat{Y}_H$ immediately comes from the incentive constraint (5): The higher the expected output in case of success, the higher are the workers incentives induced by the piece rate. The intuition for the impact of $\bar{y}_L$ can be obtained from the limited-liability constraint. The constraint is relaxed when decreasing $\bar{y}_L$, which leads to higher incentives: The binding limited-liability constraint $\alpha = \beta\bar{y}_L$ can be rewritten as $\beta = \alpha/\bar{y}_L$, which – by using (5) – yields

$$h\left(e_{PR}^*\right) = \frac{\alpha\hat{Y}_H}{\bar{y}_L}.$$

Case (iii) describes the typical scenario in which the two workers receive a positive rent due to limited liability. For this case, (13) indicates, which incentive scheme generates a larger effort. Again, e_T^* is independent of $\bar{y}_L$, but now increases in $\hat{Y}_H$, because $h'(\cdot)$ is an increasing function so that $p'(\cdot)/h'(\cdot)$ is decreasing. $h'(\cdot)$ is monotonically increasing since $h(e_i) := c'(e_i)/p'(e_i)$ and we have

$$\begin{aligned} h'(e_i) &= \frac{c''(e_i)\,p'(e_i) - c'(e_i)\,p''(e_i)}{[p'(e_i)]^2} \quad \text{and} \\ h''(e_i) &= \frac{[c'''(e_i)\,p'(e_i) - c'(e_i)\,p'''(e_i)]\,p'(e_i) - [c''(e_i)\,p'(e_i) - c'(e_i)\,p''(e_i)]\,2p''(e_i)}{[p'(e_i)]^3} > 0. \end{aligned}$$

The intuition for $\partial e_T^*/\partial\hat{Y}_H > 0$ can be seen from the employer's objective function. The employer wants to maximize expected profits. Hence, the higher the expected outcome in case of success, the larger the effort level he wants to implement. Rewriting condition (13) for e_{PR}^* and using the definition for $h(\cdot)$ gives

$$\hat{Y}_H p'(e_{PR}^*) - c'(e_{PR}^*) - h'(e_{PR}^*)\frac{\bar{y}_L}{\hat{Y}_H} - h'(e_{PR}^*)\,p(e_{PR}^*) = 0$$

so that

$$\frac{\partial e_{PR}^*}{\partial \bar{y}_L} = -\frac{-h'(e_{PR}^*)/\hat{Y}_H}{\hat{Y}_H p''(e_{PR}^*) - c''(e_{PR}^*) - h''(e_{PR}^*)\left(\frac{\bar{y}_L}{\hat{Y}_H} + p(e_{PR}^*)\right) - h'(e_{PR}^*)\,p'(e_{PR}^*)} < 0$$

and

$$\frac{\partial e_{PR}^*}{\partial \hat{Y}_H} = -\frac{p'\left(e_{PR}^*\right) + h'\left(e_{PR}^*\right)\frac{\bar{y}_L}{\hat{Y}_H^2}}{\hat{Y}_H p''\left(e_{PR}^*\right) - c''\left(e_{PR}^*\right) - h''\left(e_{PR}^*\right)\left(\frac{\bar{y}_L}{\hat{Y}_H} + p\left(e_{PR}^*\right)\right) - h'\left(e_{PR}^*\right)p'\left(e_{PR}^*\right)} > 0.$$

Therefore, the smaller (larger) $\bar{y}_L$ and the larger (smaller) $\hat{Y}_H$, the more (less) likely will the optimal effort under the piece-rate scheme exceed the optimal effort under the tournament scheme. The intuition for this result is the same one as discussed under case (ii) above, but note that in this scenario the participation constraint is not binding, i.e. both workers receive a positive rent. Hence, additional incentives created by a decrease of $\bar{y}_L$ or an increase of $\hat{Y}_H$ are free for the employer – they only reduce the workers' positive rents. The parameter $\bar{y}_L$ characterizes the maximum loss/gain in case of a failure and, therefore, the risk of the failure lottery (see Section 2). If the employer were able to control this risk to some extent, we would have a trade-off between the costs of risk reduction and additional incentives under the piece rate scheme although both workers are risk neutral.

Unfortunately, without specifying $c\left(e_i\right)$ and $p\left(e_i\right)$ no direct comparison of the workers' efforts and the employer's expected profits is possible. Hence, let $c\left(e_i\right) = \frac{\kappa}{3}e_i^3$ (with $\kappa > 0$), $p\left(e_i\right) = \gamma e_i$ (with $\gamma > 0$) and $\bar{u} = 0$. Furthermore, let

$$\gamma^3 \hat{Y}_H < \kappa \tag{14}$$

so that $p\left(e^{FB}\right) < 1$.[9] For this example the following results can be obtained:

Proposition 3 *(i) Workers receive a positive rent under either incentive scheme. (ii) Under the tournament scheme, the employer implements* $e_T^* = \frac{\gamma^2 \hat{Y}_H}{2\kappa}$ *and receives expected profits* $\pi_T^* = \frac{\gamma^3 \hat{Y}_H^2}{2\kappa}$. *(iii) Under the piece-rate scheme, the employer implements* $e_{PR}^* = \frac{\sqrt{\bar{y}_L^2 \kappa + 3\gamma^3 \hat{Y}_H^3} - \bar{y}_L \sqrt{\kappa}}{3\gamma \hat{Y}_H \sqrt{\kappa}}$ *and gets* $\pi_{PR}^* = \frac{2}{27}\left(\sqrt{\bar{y}_L^2 \kappa + 3\gamma^3 \hat{Y}_H^3} - \bar{y}_L \sqrt{\kappa}\right) \frac{6\gamma^3 \hat{Y}_H^3 \kappa + \bar{y}_L^2 \kappa^2 - \bar{y}_L \kappa^{\frac{3}{2}} \sqrt{\bar{y}_L^2 \kappa + 3\gamma^3 \hat{Y}_H^3}}{\gamma^3 \hat{Y}_H^3 \kappa^{\frac{3}{2}}} > 0$. *(iv)* $\bar{y}_L \to \hat{Y}_H$ *leads to* $e_T^* > e_{PR}^*$ *and* $\pi_T^* > \pi_{PR}^*$, *whereas* $\bar{y}_L \to 0$ *implies* $e_T^* < e_{PR}^*$ *and* $\pi_T^* < \pi_{PR}^*$. *Moreover, there exists a cut-off value* $\bar{\kappa} = \frac{3\gamma^3 \hat{Y}_H^2}{4\left(\hat{Y}_H - \bar{y}_L\right)}$ *so that* $e_T^* > (<)\, e_{PR}^*$ *if* $\kappa < (>)\, \bar{\kappa}$.

Proof. See appendix. ■

The parametric example considered in Proposition 3 belongs to case (iii) of Proposition 2. The reservation value $\bar{u}$ is sufficiently small so that an interior solution is achieved in which only the limited-liability constraint is binding and both workers earn positive rents. The impact of $\bar{y}_L$ and $\hat{Y}_H$ on workers' efforts as claimed in the discussion of Proposition 2(iii) is supported by Proposition 3, which can be seen from the parametric expressions for e_T^* and e_{PR}^*. Moreover, for large (small) values of $\bar{y}_L$ workers exert more (less) effort in the tournament than under the piece-rate scheme. $\bar{y}_L$ influences the

[9] We have $\hat{Y}_H = c'\left(e^{FB}\right)/p'\left(e^{FB}\right)$ which implies $e^{FB} = \sqrt{\frac{\gamma \hat{Y}_H}{\kappa}}$. Hence, $p\left(\sqrt{\frac{\gamma \hat{Y}_H}{\kappa}}\right) < 1$ yields $\gamma^3 \hat{Y}_H < \kappa$.

employer's expected profits in the same way. If $\bar{y}_L$ is large (small), expected profits will be higher under the tournament (piece-rate) scheme. Finally, the results of Proposition 3 indicate that the workers' cost function influences the effort choices under the two incentive schemes differently. Of course, the more convex the cost function (i.e., the higher κ), the lower workers' effort under either incentive scheme, but effort declines more rapidly under the tournament than under the piece-rate scheme.

Proposition 2 as well as the example of Proposition 3 assume that the third derivative of the cost function is positive. Now we consider the special case in which this assumption does not hold, i.e. we assume $c'''(e_i) = 0$. Note that this assumption corresponds to a quadratic cost function. Let this cost function be described by $c(e_i) = \frac{c}{2}e_i^2$ with $c > 0$. If in this situation $p''(e_i) < 0$, the results of Proposition 2 will still go through, since $h(e_i)$ is convex. However, if $p''(e_i) = 0$, the first equation of (13) given in Proposition 2 becomes problematic, because now $p(e_i)$ and $h(e_i)$ are linear. Let in this scenario $p(e_i) = \gamma e_i$ (with $\gamma > 0$) denote the linear probability function. Together with quadratic costs $c(e_i) = \frac{c}{2}e_i^2$ we obtain $h(e_i) = \frac{ce_i}{\gamma}$ and, therefore, first-best effort

$$e^{FB} = \frac{\gamma \hat{Y}_H}{c}.$$

Let again $p\left(e^{FB}\right) < 1$ to have a reasonable result, i.e.

$$\gamma^2 \hat{Y}_H < c \tag{15}$$

is assumed. Furthermore, we assume that, given first-best effort, the unconditional mean of the composed output lottery exceeds the minimum output in case of success:

$$\begin{aligned} & p\left(e^{FB}\right) E\left[y_H\right] + \left(1 - p\left(e^{FB}\right)\right) E\left[y_L\right] > \bar{y}_L \\ \Leftrightarrow \quad & p\left(e^{FB}\right) \hat{Y}_H > \bar{y}_L \Leftrightarrow \frac{\gamma^2 \hat{Y}_H^2}{c} > \bar{y}_L. \end{aligned} \tag{16}$$

Finally, define four cut-off values for the reservation utility:[10]

$$\begin{aligned} \hat{u}_T & \equiv \hat{Y}_H \left(1 - \frac{\gamma^2 \hat{Y}_H}{2c}\right) \\ \hat{u}_{PR}^1 & \equiv \frac{\gamma^2 \hat{Y}_H^2}{2c} + \bar{y}_L \\ \hat{u}_{PR}^2 & \equiv \frac{\gamma^4 \hat{Y}_H^4 - c^2 \bar{y}_L^2}{2c\gamma^2 \hat{Y}_H^2} \\ \hat{u}_{PR}^3 & \equiv \left(\gamma^2 \hat{Y}_H^2 - \bar{y}_L c\right) \frac{3\bar{y}_L c + \gamma^2 \hat{Y}_H^2}{8\gamma^2 \hat{Y}_H^2 c}. \end{aligned}$$

Note that $\hat{u}_{PR}^1 > \hat{u}_{PR}^2 > \hat{u}_{PR}^3$. Now we obtain the following proposition:

Proposition 4 *Let $c'''(e_i) = 0$. (1) If $p''(e_i) < 0$, the results of Proposition*

[10] (16) ensures that $\hat{u}_{PR}^2 > 0$ and $\hat{u}_{PR}^3 > 0$.

2 still hold. (2) If $p''(e_i) = 0$, the following results can be derived: (i) Under the tournament scheme, e^{FB} will be implemented if $\bar{u} \geq \hat{u}_T$; otherwise $e_T^* = 0$. Dropping the assumption that in any given case the employer wants to hire the two workers, yields $w_1^* = w_2^* = e_T^* = \pi_T^* = 0$. (ii) Under the piece-rate scheme, if $\bar{u} \geq \hat{u}_{PR}^1$, first-best effort e^{FB} will be implemented; if $\hat{u}_{PR}^1 > \bar{u} > \hat{u}_{PR}^3$, then $e_{PR}^* = \frac{\sqrt{\bar{y}_L^2 c + 2\gamma^2 \hat{Y}_H^2 \bar{u}} - \bar{y}_L \sqrt{c}}{\gamma \hat{Y}_H \sqrt{c}}$, and if $\hat{u}_{PR}^3 \geq \bar{u}$, then $e_{PR}^* = \frac{\gamma^2 \hat{Y}_H^2 - \bar{y}_L c}{2\gamma \hat{Y}_H c}$. Dropping the assumption that in any given case the employer wants to hire the two workers, leads to the following results:

$$
\begin{aligned}
e_{PR}^* &= 0 \textit{ and } \pi_{PR}^* = 0 \textit{ if } \bar{u} > \hat{u}_{PR}^2 \\
e_{PR}^* &= \frac{\sqrt{\bar{y}_L^2 c + 2\gamma^2 \hat{Y}_H^2 \bar{u}} - \bar{y}_L \sqrt{c}}{\gamma \hat{Y}_H \sqrt{c}} \textit{ and} \\
\pi_{PR}^* &= 2\left(\sqrt{\bar{y}_L^2 c^2 + 2c\gamma^2 \hat{Y}_H^2 \bar{u}} - \bar{y}_L c\right) \frac{\gamma^2 \hat{Y}_H^2 - \sqrt{\bar{y}_L^2 c^2 + 2c\gamma^2 \hat{Y}_H^2 \bar{u}}}{\gamma^2 \hat{Y}_H^2 c} > 0 \\
& \textit{if } \hat{u}_{PR}^2 \geq \bar{u} > \hat{u}_{PR}^3 \\
e_{PR}^* &= \frac{\gamma^2 \hat{Y}_H^2 - \bar{y}_L c}{2\gamma \hat{Y}_H c} \textit{ and } \pi_{PR}^* = \frac{\left(\gamma^2 \hat{Y}_H^2 - \bar{y}_L c\right)^2}{2\gamma^2 \hat{Y}_H^2 c} > 0 \textit{ if } \hat{u}_{PR}^3 \geq \bar{u}.
\end{aligned}
$$

Proof. See appendix. ■

The results of Proposition 4 show that, under quadratic costs and a linear probability function, tournament incentives will completely break down, if the employer is able to choose between hiring and no-hiring to guarantee non-negative profits. This result can be explained by the fact that effort costs are too high for generating positive incentives under the tournament scheme. According to the proof of Proposition 4 in the appendix, the employer's profit function is given by

$$\pi_T = 2\gamma \underbrace{\frac{\Delta w \gamma}{2c}}_{e_T^*(\Delta w)} \hat{Y}_H - \Delta w - 2w_2.$$

Hence, inducing higher incentives by marginally increasing the prize spread Δw leads to marginal gains $\frac{\hat{Y}_H \gamma^2}{c}$, but due to (15) marginal net gains $\frac{\hat{Y}_H \gamma^2}{c} - 1$ are negative. Here the cost parameter c prevents the employer from generating any positive incentives. However, we have a completely different result for the piece-rate scheme. Consider again the scenario in which the employer does not unambiguously hire the two workers. Of course, if the workers' reservation value is too large ($\bar{u} > \hat{u}_{PR}^2$), it will not pay for the employer to hire the two workers, but otherwise strictly positive effort levels are implemented: The incentive constraint $e_{PR}^* = \frac{\beta \gamma \hat{Y}_H}{c}$ shows that – analogously to the tournament case – a large cost parameter c also decreases incentives under piece rates, but for $\bar{u} \leq \hat{u}_{PR}^2$ efforts are always positive.

The various cases for the piece-rate result can be best explained by the employer's profit function

$$\pi_{PR}(\beta) = 2\beta \frac{(1-\beta)\gamma^2 \hat{Y}_H^2 - \bar{y}_L c}{c}$$

and the workers' participation constraint

$$\beta \bar{y}_L + \frac{\beta^2 \gamma^2 \hat{Y}_H^2}{2c} \geq \bar{u}.$$

The function $\pi_{PR}(\beta)$ describes a parabola open to the bottom, whereas the left-hand side of the participation constraint describes the ascending part of a parabola open to the top which goes through the origin. The profit function has a unique maximum and the employer wants to implement the corresponding effort by choosing the optimal piece rate β^*. If the reservation value is sufficiently small ($\bar{u} \leq \hat{u}_{PR}^3$), this effort implementation will not contradict the participation constraint. However, if this optimal piece rate is too low to satisfy the participation constraint, the employer will choose a higher β that leads to a binding participation constraint but lower profits. As long as profits remain positive (i.e., as long as $\bar{u}$ is not too large), the employer will choose this corner solution ($\hat{u}_{PR}^2 \geq \bar{u} > \hat{u}_{PR}^3$). If $\bar{u}$ is very large ($\bar{u} > \hat{u}_{PR}^2$), all piece rates that satisfy the participation constraint yield negative profits. Consequently, the employer prefers not to hire the two workers.

To sum up, on the one hand the previous results show that a small $\bar{y}_L$ favors piece rates since the limited-liability constraint of the piece-rate scheme is relaxed. On the other hand, a small $\hat{Y}_H$ makes tournaments relatively attractive compared to piece rates, because the optimal prize spread is small, which relaxes the limited-liability constraint of the tournament scheme, and because piece-rate incentives are directly decreased. Moreover, the findings indicate that tournaments will become more problematic than piece rates if the workers' cost function is very steep. In the next section, the robustness of these results will be discussed.

4 Discussion

The findings above may be criticized by the fact that we have not considered the standard tournament model by Lazear and Rosen (1981). Hence, the derived results may not hold in general. However, there is a good reason for looking at a different model: Contrary to our model, in the Lazear-Rosen framework pure-strategy equilibria will not always exist. Existence is only guaranteed, if there is sufficient luck in the tournament and the cost function is sufficiently convex.[11] Moreover, the endogenously derived tournament prizes also enter the existence condition. Finally, doing comparative statics may be problematic, since the existence condition can be violated.

In this section, we abstract from all these problems and assume existence (as, among others, Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983)

[11] See Lazear and Rosen (1981), p. 845, Nalebuff and Stiglitz (1983), p. 27.

do, for example) to check how the results above will change within the Lazear-Rosen framework, which assumes a different production technology: Now let worker i's ($i = A, B$) output be described by the production function $q_i = e_i + \varepsilon_i$. e_i denotes i's effort choice and ε_i exogenous noise which is distributed over $[-\bar{\varepsilon}_L, \bar{\varepsilon}_H]$ with mean $\hat{\varepsilon}$ and $\bar{\varepsilon}_L, \bar{\varepsilon}_H > 0$. As in Lazear and Rosen (1981), ε_A and ε_B are assumed to be identically and independently distributed (i.i.d.). Let $G(\cdot)$ denote the cumulative distribution function and $g(\cdot)$ the density of the composed random term $\varepsilon_j - \varepsilon_i$ ($i, j = A, B$; $i \neq j$). All the other assumptions of Section 2 are retained.

In this framework, the first-best effort e^{FB} maximizes $E[q_i] - c(e_i)$, which yields

$$c'\left(e^{FB}\right) = 1 \qquad (i = A, B).$$

Under the *tournament scheme*, at the second stage of the game, worker i maximizes

$$EU_i(e_i) = w_2 + \Delta w \cdot p_i(e_i, e_j) - c(e_i)$$

for given w_1 and w_2 with $p_i(e_i, e_j)$ denoting i's probability of winning ($i, j = A, B$; $i \neq j$). Since i will win, if $q_i > q_j$, we have $p_i(e_i, e_j) = G(e_i - e_j)$ and $p_j(e_i, e_j) = 1 - G(e_i - e_j)$. Hence, $\partial p_i / \partial e_i = \partial p_j / \partial e_j = g(e_i - e_j)$, and the equilibrium will be unique and symmetric with each agent choosing effort

$e_T^*(\Delta w)$ implicitly defined by

$$\Delta w g(0) = c'(e_T^*) .$$

At the first stage, the employer chooses w_1 and w_2 to maximize $2e_T^*(\Delta w) - w_1 - w_2$ subject to the workers' incentive constraint $\Delta w g(0) = c'(e_T^*)$ and their participation constraint

$$\frac{w_1 + w_2}{2} - c(e_T^*(\Delta w)) \geq \bar{u}.$$

The employer can implement e^{FB} and make the participation constraint bind by choosing

$$w_1^{FB} = c(e^{FB}) + \bar{u} + \frac{c'\left(e^{FB}\right)}{2g(0)} \text{ and } w_2^{FB} = c\left(e^{FB}\right) + \bar{u} - \frac{c'\left(e^{FB}\right)}{2g(0)}.$$

Because of the limited-liability constraint $w_2 \geq 0$, this solution will only be feasible if

$$c\left(e^{FB}\right) + \bar{u} \geq \frac{c'\left(e^{FB}\right)}{2g(0)} \Leftrightarrow c\left(e^{FB}\right) + \bar{u} \geq \frac{1}{2g(0)}. \tag{17}$$

Under the *piece-rate scheme*, the workers' incentive constraint is given by $\beta = c'(e_{PR}^*)$, which implicitly defines optimal effort $e_{PR}^*(\beta)$, and their participation constraint by $\alpha + \beta(e_{PR}^*(\beta) + \hat{\varepsilon}) - c(e_{PR}^*(\beta)) \geq \bar{u}$. Therefore, the employer can implement e^{FB} and make the participation constraint bind by choosing

$$\beta^{FB} = 1 \quad \text{and} \quad \alpha^{FB} = c\left(e^{FB}\right) + \bar{u} - e^{FB} - \hat{\varepsilon}.$$

Due to limited liability, the workers' wages still have to be non-negative in the worst case, i.e. $w_i = \alpha + \beta\left(e^*_{PR}(\beta) - \bar{\varepsilon}_L\right) \geq 0$. Hence, first-best implementation will be feasible if

$$c\left(e^{FB}\right) + \bar{u} \geq \hat{\varepsilon} + \bar{\varepsilon}_L. \tag{18}$$

The employer's complete optimization problems are described by the two Lagrangians

$$\begin{aligned} L_T(w_1, w_2) &= 2e^*_T(\Delta w) - w_1 - w_2 + \lambda_1\left[\frac{w_1 + w_2}{2} - c\left(e^*_T(\Delta w)\right) - \bar{u}\right] + \lambda_2 w_2 &(19)\\ L_{PR}(\alpha, \beta) &= 2(1-\beta)\left(e^*_{PR}(\beta) + \hat{\varepsilon}\right) - 2\alpha &(20)\\ &\quad +\lambda_1\left[\alpha + \beta\left(e^*_{PR}(\beta) + \hat{\varepsilon}\right) - c\left(e^*_{PR}(\beta)\right) - \bar{u}\right] + \lambda_2\left[\alpha + \beta\left(e^*_{PR}(\beta) - \bar{\varepsilon}_L\right)\right]. \end{aligned}$$

and we obtain the following results:

Proposition 5 *(1) If* $\frac{1}{2g(0)} > (<)\, \bar{\varepsilon}_L + \hat{\varepsilon}$*, implementation of* e^{FB} *will be more (less) likely under a piece-rate than under a tournament scheme.*

(2) Let $c'''(\cdot) > 0$*. In the employer's optimization problems at least one*

constraint is binding: (i) If only the participation constraint is binding, we will have $e^*_T = e^*_{PR} = e^{FB}$. (ii) If both constraints are binding, equilibrium efforts will be described by

$$\frac{c'(e^*_T)}{2g(0)} = \bar{u} + c(e^*_T) \qquad \text{and} \qquad c'(e^*_{PR}) \cdot \bar{\varepsilon}_L = \bar{u} + c(e^*_{PR}).$$

(iii) If only the limited-liability constraint is binding, equilibrium efforts will be characterized by

$$2g(0) = c''(e^*_T) \qquad \text{and} \qquad \frac{1}{\bar{\varepsilon}_L} = c''(e^*_{PR}).$$

(3) Let $c(e_i) = \eta e_i^\delta$ with $\delta > 2$ and $\eta > 0$. Given $\bar{u} = 0$ and the limited-liability constraint is binding, if $\frac{1}{2g(0)} > (<) \, \bar{\varepsilon}_L + \hat{\varepsilon}$, the workers will more (less) likely receive a positive rent under the tournament than under the piece-rate scheme.

Proof. See appendix. ■

Proposition 5 strongly supports the qualitative results of Propositions 1 and 2. The larger $\bar{\varepsilon}_L$ and the smaller $1/g(0)$, the more advantageous tournaments will be relative to piece rates. In particular, if

$$\frac{1}{2g(0)} < \bar{\varepsilon}_L, \tag{21}$$

then first-best effort e^{FB} will be more likely implemented under the tournament scheme (result (1)), $e_T^* > e_{PR}^*$ if workers receive a positive rent (result (2)(iii)), and workers will less likely earn positive rents under the tournament scheme given the scenario of result (3). The intuition for Proposition 5(1) is the same as the one for Proposition 1(ii): Here, $\bar{\varepsilon}_L$ (instead of $\bar{y}_L$) characterizes the worst case under the piece-rate scheme, in which the workers' compensation must be still non-negative, and under the tournament scheme $w_2^{FB} = c\left(e^{FB}\right) + \bar{u} - \frac{c'\left(e^{FB}\right)}{2g(0)} = c\left(e^{FB}\right) + \bar{u} - \frac{1}{2g(0)}$, which will become negative if $\frac{1}{2g(0)}$ is too large. Note that the marginal winning probability, $g(\cdot)$, determines incentives in the tournament and, hence, optimal prizes. If $g(\cdot)$ is flat (i.e., the outcome of the tournament is mainly determined by luck) – and, therefore, $g(0)$ is small[12] – effort incentives will be rather low (see $e_T^*(\Delta w)$). In this situation, the employer has to choose a sufficiently high prize spread Δw to restore incentives. This means, however, that the loser prize w_2 has to be rather small, and that w_2^{FB} may become negative. Altogether, a small value of $\bar{\varepsilon}_L$ relaxes the limited-liability constraint under the piece-rate scheme, whereas a small $\frac{1}{2g(0)}$ relaxes the one under the tournament scheme, which drives the remaining results of Proposition 5.

On the one hand, the results of Proposition 5 show that within the Lazear-

[12] Following Lazear (1995, p. 29), we can interpret $1/g(0)$ as a measure of luck or risk in the tournament. Alternatively, we could interpret $g(0)$ as a measure for the monitoring precision in the tournament.

Rosen framework no clear statements are possible whether the convexity of the cost function has a higher impact on piece-rate incentives than on tournament incentives. For example, if workers receive a positive rent, the more convex the cost function the lower equilibrium efforts will be under either incentive scheme, but the impact of this cost effect solely depends on $\frac{1}{2g(0)}$ and $\bar{\varepsilon}_L$ (result (2)(iii)). Moreover, inequality (21) is completely independent of the cost function.

On the other hand, the Lazear-Rosen framework allows to examine whether risk harms tournament incentives more than piece-rate incentives. First, note that risk will not influence piece-rate incentives given risk neutral workers, if there is unlimited liability. However, under limited liability maximum bad luck clearly influences inequality (21): If $\bar{\varepsilon}_L$ and, therefore, risk is large, piece rates will be disadvantageous. As mentioned above, $\frac{1}{2g(0)}$ can also be used as a measure of risk. If $\frac{1}{2g(0)}$ is large, tournaments will become disadvantageous, too. Hence, we have to examine which of these two effects is dominant. Of course, for $\bar{\varepsilon}_L \to \infty$ (e.g., if the error terms are normally distributed) piece rates become prohibitively expensive for the employer, but tournaments still work. They offer workers a partial insurance, since minimum and maximum income are determined by the loser and the winner prize, respectively. When looking at less extreme cases, the comparison may lead to a different result. Assume, for example, that the i.i.d. error terms ε_i and ε_j follow a normal distribution $N(0,\sigma^2)$ that has been truncated on the left at $-\bar{\varepsilon}_L = -\bar{\varepsilon}$ and on the right at $\bar{\varepsilon}_H = \bar{\varepsilon}$. This implies that the convolution $g(\cdot)$ for $\varepsilon_j - \varepsilon_i$ is also a truncated normal distribution with mean zero. However, now the variance of the underlying normal distribution is given by $2\sigma^2$, and the composed random variable $\varepsilon_j - \varepsilon_i$ is distributed over the interval $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$. Defining $z := \varepsilon_j - \varepsilon_i$ the convolution can be written as

$$g(z) = \frac{1}{\sqrt{2\sigma^2}} \frac{\phi\left(\frac{z}{\sqrt{2\sigma^2}}\right)}{1 - 2\Phi\left(\frac{-2\bar{\varepsilon}}{\sqrt{2\sigma^2}}\right)}$$

with $\phi(\cdot)$ denoting the density and $\Phi(\cdot)$ the cumulative distribution function of the standardized normal distribution. We obtain

$$\frac{1}{2g(0)} = \frac{\sqrt{2\sigma^2}}{2} \frac{1 - 2\Phi\left(\frac{-2\bar{\varepsilon}}{\sqrt{2\sigma^2}}\right)}{\phi(0)} = \sigma\sqrt{\pi}\left(1 - 2\int_{-\infty}^{\frac{-2\bar{\varepsilon}}{\sqrt{2\sigma^2}}} \frac{1}{\sqrt{2\pi}} \exp\left\{\frac{-x^2}{2}\right\} dx\right), \tag{22}$$

which yields the following result:

Proposition 6 *Let ε_i and ε_j follow a normal distribution $N(0,\sigma^2)$ truncated at $-\bar{\varepsilon}$ and $\bar{\varepsilon}$. If $\sigma^2 < (>) \frac{\bar{\varepsilon}^2}{\ln 2}$, the left-hand side of (21) will increase less (more) rapidly in $\bar{\varepsilon}$ than the right-hand side. If $\sigma^2 \to 0$ ($\sigma^2 \to \infty$), inequality (21) will always (never) hold.*

Proof. See appendix. ■

Proposition 6 shows that for low variances of the initial normal distribution, inequality (21) is more likely to hold,[13] whereas for high values of σ^2 the opposite is true.[14] If the variance tends to zero, the inequality will always be satisfied, whereas for sufficiently high variances it will always be violated. Hence, tournaments will only dominate piece rates, if risk – i.e. the variance of ε_i and ε_j – is not too large. Following Lazear and Rosen (1981) and Lazear (1995), ε_i and ε_j can be interpreted in different ways. For example, they can (a) measure the exogenous risk of the given production technologies, (b) the individual measurement errors when workers are evaluated, or (c) the ex ante unknown abilities of the workers in case of symmetric uncertainty. This means that, given limited liability, tournaments will only be attractive for the employer compared to piece rates, if workers use quite safe production technologies, the supervisors' monitoring precision is not too low, or initial uncertainty about the workers' talents is sufficiently reduced by introducing appropriate recruiting techniques.[15]

[13] In other words, increasing risk by a mean preserving spread will be less (more) problematic for tournaments than for piece rates, if the risk of the initial normal distribution is low.

[14] Note that the results qualitatively also hold for the variance of the truncated convolution, $Var[z] = \sigma^2 \left[1 - \frac{\frac{4\bar{\varepsilon}}{\sqrt{2\sigma^2}}\phi\left(\frac{2\bar{\varepsilon}}{\sqrt{2\sigma^2}}\right)}{1-2\Phi\left(\frac{-2\bar{\varepsilon}}{\sqrt{2\sigma^2}}\right)}\right]$, since $Var[z]$ is monotonically increasing in σ^2.

[15] However, note that tournaments may be even problematic under such conditions, because existence of pure-strategy equilibria requires a sufficiently high risk, i.e. a sufficiently flat density $g(\cdot)$ and/or a sufficiently convex cost function $c(\cdot)$.

5 Conclusion

In this paper, tournaments and piece rates have been compared under the assumption of limited liability. The comparison has shown that, on the one hand, risk or luck (i.e., the variance of the error terms) has a large impact on the profitability of both incentive schemes. If risk is sufficiently high, piece rates will dominate tournaments, because first-best implementation will be more likely under a piece-rate scheme, and because efforts will be larger under piece rates when workers earn positive rents.

On the other hand, the convexity of the workers' cost function also plays a crucial role: The more convex the cost function the less likely first-best effort is implemented under the tournament scheme relative to piece-rates. If the workers receive positive rents, efforts as well as profits will be larger under piece rates than under tournaments.

Appendix

Proof of Proposition 2:

(i) Substituting $\lambda_1 = 2$ and $\lambda_2 = 0$ into the optimality conditions immediately replicates the benchmark result of Lazear and Rosen (1981) for the case of unlimited liability. (ii) Combining the binding limited-liability constraints, the binding participation constraints, and the incentive constraints (2) and (5), leads to (12). (iii) Inserting $\lambda_1 = 0$ into (8) yields e_T^*. Using $\lambda_1 = 0$ in (10) and the incentive constraint (5) gives e_{PR}^*.

Proof of Proposition 3:

(i)–(iii) First, note that the employer does not implement e^{FB} under either incentive scheme. Under the tournament scheme, first-best implementation requires $c\left(e^{FB}\right) + \bar{u} \geq \hat{Y}_H \Leftrightarrow c\left(e^{FB}\right) \geq \hat{Y}_H \Leftrightarrow \gamma^3 \hat{Y}_H \geq 9\kappa$, which is not true because $\gamma^3 \hat{Y}_H < \kappa$ according to (14). For the implementation of $e^{FB} = \sqrt{\frac{\gamma \hat{Y}_H}{\kappa}}$ under the piece-rate scheme, we must have $c\left(e^{FB}\right) + \bar{u} \geq p\left(e^{FB}\right) \hat{Y}_H + \bar{y}_L \Leftrightarrow \frac{\kappa}{3}\left(\sqrt{\frac{\gamma \hat{Y}_H}{\kappa}}\right)^3 \geq \gamma \sqrt{\frac{\gamma \hat{Y}_H}{\kappa}} \hat{Y}_H + \bar{y}_L \Leftrightarrow -\frac{2}{3}\frac{\gamma}{\kappa}\sqrt{\gamma \hat{Y}_H \kappa}\hat{Y}_H \geq \bar{y}_L$, a contradiction. As we know from Proposition 2, in this case the limited-liability constraint is binding for both incentive schemes, i.e. we have $w_2^* = 0$ and $\alpha^* = \beta \bar{y}_L$.

Under the tournament scheme, the worker's incentive constraint is given by

$$e_T^* = \sqrt{\frac{w_1 \gamma}{2\kappa}},$$

and the participation constraint by

$$\frac{w_1}{2} - \frac{\kappa}{3}\left(\frac{w_1\gamma}{2\kappa}\right)^{\frac{3}{2}} \geq 0.$$

The employer chooses w_1 to maximize

$$\pi_T = 2p\left(e_T^*\left(w_1\right)\right)\hat{Y}_H - w_1 = 2\gamma\sqrt{\frac{w_1\gamma}{2\kappa}}\hat{Y}_H - w_1.$$

The first-order condition yields

$$w_1^* = \frac{\gamma^3\hat{Y}_H^2}{2\kappa}.$$

Inserting into the participation constraint gives

$$\frac{\gamma^3\hat{Y}_H^2}{24}\frac{6\kappa - \gamma^3\hat{Y}_H}{\kappa^2} \geq 0,$$

which is always true because of (14). Hence, the workers receive a positive rent in equilibrium. Using the result for w_1^* yields

$$e_T^* = \frac{\gamma^2\hat{Y}_H}{2\kappa} \quad \text{and} \quad \pi_T^* = \frac{\gamma^3\hat{Y}_H^2}{2\kappa}.$$

Under the piece-rate scheme, the incentive constraint becomes

$$e_{PR}^{*}=\sqrt{\frac{\beta\gamma\hat{Y}_{H}}{\kappa}},$$

and the participation constraint

$$\begin{aligned}
& \alpha^{*}+\beta p\left(e_{PR}^{*}\right)\hat{Y}_{H}-c\left(e_{PR}^{*}\right)\geq 0 \\
\Leftrightarrow\quad & \beta\bar{y}_{L}+\frac{2}{3}\frac{\beta\gamma\hat{Y}_{H}}{\kappa}\sqrt{\beta\gamma\hat{Y}_{H}\kappa}\geq 0,
\end{aligned}$$

which is always satisfied, i.e. workers receive a positive rent under piece rates, too. The employer maximizes

$$\begin{aligned}
\pi_{PR} &= 2\left(1-\beta\right)p\left(e_{PR}^{*}\left(\beta\right)\right)\hat{Y}_{H}-2\beta\bar{y}_{L} \\
&= 2\gamma\hat{Y}_{H}\left(\frac{\gamma\hat{Y}_{H}}{\kappa}\right)^{\frac{1}{2}}\left(1-\beta\right)\beta^{\frac{1}{2}}-2\bar{y}_{L}\beta.
\end{aligned}$$

From the first-order condition we obtain

$$\beta^{*}=\left(\sqrt{\frac{\bar{y}_{L}^{2}\kappa}{9\gamma^{3}\hat{Y}_{H}^{3}}+\frac{1}{3}}-\frac{\bar{y}_{L}\kappa^{\frac{1}{2}}}{3\gamma^{\frac{3}{2}}\hat{Y}_{H}^{\frac{3}{2}}}\right)^{2}.$$

Inserting into the incentive constraint and the profit function leads to

$$e_{PR}^{*}=\frac{\sqrt{\bar{y}_{L}^{2}\kappa+3\gamma^{3}\hat{Y}_{H}^{3}}-\bar{y}_{L}\sqrt{\kappa}}{3\gamma\hat{Y}_{H}\sqrt{\kappa}}\qquad\text{and}$$

$$\pi_{PR}^* = \frac{2}{27}\left(\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3} - \bar{y}_L\sqrt{\kappa}\right)\frac{6\gamma^3\hat{Y}_H^3\kappa + \bar{y}_L^2\kappa^2 - \bar{y}_L\kappa^{\frac{3}{2}}\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3}}{\gamma^3\hat{Y}_H^3\kappa^{\frac{3}{2}}}.$$

(iv) The comparison of e_T^* and e_{PR}^* yields

$$\begin{aligned} e_T^* = \frac{\gamma^2\hat{Y}_H}{2\kappa} > (<) \frac{\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3} - \bar{y}_L\sqrt{\kappa}}{3\gamma\hat{Y}_H\sqrt{\kappa}} = e_{PR}^* \\ \Leftrightarrow \quad 3\gamma^3\hat{Y}_H^2 > (<)\, 4\kappa\left(\hat{Y}_H - \bar{y}_L\right). \end{aligned} \tag{23}$$

Hence, for $\bar{y}_L \to \hat{Y}_H$ we have $3\gamma^3\hat{Y}_H^2 > 0$, whereas for $\bar{y}_L \to 0$ we get $3\gamma^3\hat{Y}_H < 4\kappa$ which always holds because of (14). Solving inequality (23) for κ leads to the cut-off $\bar{\kappa} = \frac{3\gamma^3\hat{Y}_H^2}{4\left(\hat{Y}_H - \bar{y}_L\right)}$.

Comparing the two profits gives

$$\begin{aligned} \pi_T^* \quad = \quad & \frac{\gamma^3\hat{Y}_H^2}{2\kappa} > (<) \\ & \frac{2}{27}\left(\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3} - \bar{y}_L\sqrt{\kappa}\right)\frac{6\gamma^3\hat{Y}_H^3\kappa + \bar{y}_L^2\kappa^2 - \bar{y}_L\kappa^{\frac{3}{2}}\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3}}{\gamma^3\hat{Y}_H^3\kappa^{\frac{3}{2}}} = \pi_{PR}^* \\ \Leftrightarrow \quad & 27\gamma^6\hat{Y}_H^5 + 4\bar{y}_L\kappa\left(2\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3 - 2\bar{y}_L\sqrt{\kappa}\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3}\right) > (<) \\ & 24\kappa^{\frac{1}{2}}\gamma^3\hat{Y}_H^3\left(\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3} - \bar{y}_L\sqrt{\kappa}\right) \\ \Leftrightarrow \quad & 27\gamma^6\hat{Y}_H^5 + 8\kappa^2\bar{y}_L^3 + 12\bar{y}_L\kappa\gamma^3\hat{Y}_H^3 + 24\kappa\bar{y}_L\gamma^3\hat{Y}_H^3 > (<) \\ & \left(24\kappa^{\frac{1}{2}}\gamma^3\hat{Y}_H^3 + 8\kappa^{\frac{3}{2}}\bar{y}_L^2\right)\sqrt{\bar{y}_L^2\kappa + 3\gamma^3\hat{Y}_H^3} \\ \Rightarrow \quad & 27\gamma^6\hat{Y}_H^5\left(27\gamma^6\hat{Y}_H^5 + 16\kappa^2\bar{y}_L^3 + 72\bar{y}_L\kappa\gamma^3\hat{Y}_H^3 - 16\bar{y}_L^2\kappa^2\hat{Y}_H - 64\gamma^3\hat{Y}_H^4\kappa\right) > (<)\, 0 \\ \Leftrightarrow \quad & \hat{Y}_H^3\gamma^3\left(27\gamma^3\hat{Y}_H^2 + 72\kappa\bar{y}_L - 64\kappa\hat{Y}_H\right) > (<)\, 16\kappa^2\bar{y}_L^2\left(\hat{Y}_H - \bar{y}_L\right) \end{aligned}$$

For $\bar{y}_L \to \hat{Y}_H$ the inequality boils down to $\hat{Y}_H^3\gamma^3\left(27\gamma^3\hat{Y}_H^2 + 72\kappa\hat{Y}_H - 64\kappa\hat{Y}_H\right) > 0 \Leftrightarrow 27\gamma^3\hat{Y}_H + 8\kappa > 0$. However, if $\bar{y}_L \to 0$, the inequality becomes $\hat{Y}_H^3\gamma^3\left(27\gamma^3\hat{Y}_H - 64\kappa\right) < 0$ which is true because of (14).

Proof of Proposition 4:

(1) The proof directly follows from the preceding discussion.

(2)(i) First-best implementation under the *tournament scheme* requires $c\left(e^{FB}\right)+\bar{u} \geq \hat{Y}_H \Leftrightarrow \bar{u} \geq \hat{Y}_H\frac{2c-\gamma^2\hat{Y}_H}{2c} = \hat{Y}_H\left(1 - \frac{\gamma^2\hat{Y}_H}{2c}\right) =: \hat{u}_T$. If this condition is not met, the employer will implement a smaller effort level. The workers' incentive constraint is given by

$$e_T^* = \frac{(w_1 - w_2)\,\gamma}{2c}$$

and the participation constraint by

$$\frac{w_1 + w_2}{2} - \frac{c}{2}\left(\frac{(w_1 - w_2)\,\gamma}{2c}\right)^2 \geq \bar{u}.$$

The employer chooses w_1 and w_2 to maximize

$$\begin{aligned}
\pi_T &= 2p\left(e_T^*\left(w_1 - w_2\right)\right)\hat{Y}_H - w_1 - w_2 \\
&= 2\gamma\frac{(w_1 - w_2)\,\gamma}{2c}\hat{Y}_H - w_1 - w_2 \\
&= \frac{\gamma^2\hat{Y}_H - c}{c}\left(w_1 - w_2\right) - 2w_2 \overset{(15)}{<} 0.
\end{aligned}$$

Hence, the employer wants to minimize incentives and optimally chooses $w_1^* = 0$ to induce $e_T^* = 0$. By this, the participation constraint becomes

$$\frac{w_2}{2} \geq \bar{u}$$

and the employer chooses $w_2^* = 2\bar{u}$ to make the constraint just bind. Profits are negative and given by $\pi_T^* = -2\bar{u}$. The assumption that the employer unambiguously wants to keep the two workers is, of course, crucial here. Otherwise, he would prefer to offer a tournament contract with $w_1^* = w_2^* = 0$ so that the participation constraint is not met. This would lead to $e_T^* = \pi_T^* = 0$. The employer would neither want to induce first-best effort nor any other positive effort level as his losses increase in any unit of effort he wants to implement.

(ii) Implementation of e^{FB} under the *piece-rate scheme* requires $c\left(e^{FB}\right) + \bar{u} \geq p\left(e^{FB}\right)\hat{Y}_H + \bar{y}_L \Leftrightarrow \bar{u} \geq \frac{\gamma^2 \hat{Y}_H^2}{2c} + \bar{y}_L =: \hat{u}_{PR}^1$. However, if $\hat{u}_{PR}^1 > \bar{u}$, the employer will induce lower effort incentives. The incentive constraint is given by

$$e_{PR}^* = \frac{\beta\gamma\hat{Y}_H}{c}$$

and the participation constraint by

$$\alpha+\beta p\left(e_{PR}^{*}\right)\hat{Y}_{H}-c\left(e_{PR}^{*}\right)\geq\bar{u}\Leftrightarrow\alpha+\frac{\beta^{2}\gamma^{2}\hat{Y}_{H}^{2}}{2c}\geq\bar{u}.$$

From the optimality conditions (10) and (11) above we know that the limited-liability constraint will be binding, if first-best effort is not implemented. Inserting $\alpha^{*}=\beta\bar{y}_{L}$ into the participation constraint yields

$$\beta\bar{y}_{L}+\frac{\beta^{2}\gamma^{2}\hat{Y}_{H}^{2}}{2c}\geq\bar{u}.$$

The employer wants to maximize

$$\begin{aligned}\pi_{PR} &= 2\left(1-\beta\right)p\left(e_{PR}^{*}\left(\beta\right)\right)\hat{Y}_{H}-2\beta\bar{y}_{L}\\ &= 2\beta\frac{\left(1-\beta\right)\gamma^{2}\hat{Y}_{H}^{2}-\bar{y}_{L}c}{c}.\end{aligned}$$

The first-order condition yields

$$\beta^{*}=\frac{\gamma^{2}\hat{Y}_{H}^{2}-\bar{y}_{L}c}{2\gamma^{2}\hat{Y}_{H}^{2}}.$$

Inserting into the incentive constraint and the employer's objective function leads to

$$e^*_{PR} = \frac{\gamma^2\hat{Y}_H^2 - \bar{y}_L c}{2\gamma\hat{Y}_H c} \qquad \text{and} \qquad \pi^*_{PR} = \frac{\left(\gamma^2\hat{Y}_H^2 - \bar{y}_L c\right)^2}{2\gamma^2\hat{Y}_H^2 c}.$$

This solution is only feasible, if the participation constraint holds:

$$\hat{u}^3_{PR} := \left(\gamma^2\hat{Y}_H^2 - \bar{y}_L c\right)\frac{3\bar{y}_L c + \gamma^2\hat{Y}_H^2}{8\gamma^2\hat{Y}_H^2 c} \geq \bar{u} \quad \text{with} \quad \hat{u}^1_{PR} > \hat{u}^3_{PR} \geq \bar{u}.$$

However, if this condition is not met, the interior optimum cannot be achieved, because the corresponding piece rate is too low to satisfy the workers' participation constraint. Hence, the employer has to increase β so that the participation constraint holds, but now his profits decrease with any increase in β. Therefore, given $\hat{u}^1_{PR} > \bar{u} > \hat{u}^3_{PR}$ the employer chooses the lowest β that makes the participation constraint

$$\beta\bar{y}_L + \frac{\beta^2\gamma^2\hat{Y}_H^2}{2c} \geq \bar{u}$$

just bind, i.e. we have a corner solution with

$$\beta^* = \frac{\sqrt{\bar{y}_L^2 c^2 + 2\gamma^2\hat{Y}_H^2 c\bar{u}} - \bar{y}_L c}{\gamma^2\hat{Y}_H^2}$$

which implies

$$
\begin{aligned}
e^*_{PR} &= \frac{\sqrt{\bar{y}_L^2 c + 2\gamma^2 \hat{Y}_H^2 \bar{u}} - \bar{y}_L\sqrt{c}}{\gamma \hat{Y}_H \sqrt{c}} \text{ and} \\
\pi^*_{PR} &= 2\left(\sqrt{\bar{y}_L^2 c^2 + 2c\gamma^2 \hat{Y}_H^2 \bar{u}} - \bar{y}_L c\right) \frac{\gamma^2 \hat{Y}_H^2 - \sqrt{\bar{y}_L^2 c^2 + 2c\gamma^2 \hat{Y}_H^2 \bar{u}}}{\gamma^2 \hat{Y}_H^2 c}.
\end{aligned}
$$

Dropping the assumption that in any case the employer wants to hire the two workers also changes the results for the piece-rate scheme. Of course, the interior-solution result still holds, since the employer earns positive expected profits. However, the corner-solution result changes. Note that expected profits $\pi_{PR} = 2\beta \frac{(1-\beta)\gamma^2 \hat{Y}_H^2 - \bar{y}_L c}{c}$ are only positive for piece rates that meet

$$
\beta < 1 - \frac{c\bar{y}_L}{\gamma^2 \hat{Y}_H^2}.
$$

Hence, if

$$
\begin{aligned}
\beta^* &= \frac{\sqrt{\bar{y}_L^2 c^2 + 2\gamma^2 \hat{Y}_H^2 c\bar{u}} - \bar{y}_L c}{\gamma^2 \hat{Y}_H^2} > 1 - \frac{c\bar{y}_L}{\gamma^2 \hat{Y}_H^2} \Leftrightarrow \\
\bar{u} &> \frac{\left(\gamma^2 \hat{Y}_H^2\right)^2 - \bar{y}_L^2 c^2}{2\gamma^2 \hat{Y}_H^2 c} =: \hat{u}^2_{PR} \quad \text{with} \quad \hat{u}^2_{PR} > \hat{u}^3_{PR},
\end{aligned}
$$

profits will become negative and the employer prefers not to hire the workers. He offers a piece-rate contract with $\alpha^* = \beta^* = 0$ which leads to $e^*_{PR} = \pi^*_{PR} =$

0. Note that the employer never wants to implement first-best effort since $\hat{u}_{PR}^1 > \hat{u}_{PR}^2$.

Proof of Proposition 5:

(1) This result immediately follows from comparing (17) and (18).

(2) Using (19) and (20), the equilibrium efforts can be derived analogously to Proposition 2.

(3) Given a binding limited-liability constraint and $\bar{u} = 0$, workers will receive a positive rent under the tournament scheme, if $\frac{w_1}{2} - c\left(e_T^*\right) > 0$ with e_T^* being described by subcase (2)(iii) (i.e., $2g\left(0\right) = c''\left(e_T^*\right)$). Substituting for w_1 according to the incentive constraint $w_1 g\left(0\right) = c'\left(e_T^*\right)$ leads to $\frac{c'\left(e_T^*\right)}{2g(0)} - c\left(e_T^*\right) > 0 \Leftrightarrow \frac{1}{2g(0)} > \frac{c\left(e_T^*\right)}{c'\left(e_T^*\right)}$. Using the specific form of the cost function yields

$$e_T^* = c''^{-1}\left(2g\left(0\right)\right) = \left(\frac{2g\left(0\right)}{\eta\delta\left(\delta-1\right)}\right)^{\frac{1}{\delta-2}}$$

and the inequality becomes

$$\frac{\delta}{2g\left(0\right)} > \left(\frac{2g\left(0\right)}{\eta\delta\left(\delta-1\right)}\right)^{\frac{1}{\delta-2}}. \tag{24}$$

Under the piece-rate scheme, workers will get a positive rent, if $\alpha+\beta\left(e_{PR}^* + \hat{\varepsilon}\right) >$

$c\left(e^*_{PR}\left(\beta\right)\right)$ with e^*_{PR} being characterized by subcase (2)(iii), i.e.

$$e^*_{PR} = \left(\frac{1}{\eta\delta\left(\delta-1\right)\bar{\varepsilon}_L}\right)^{\frac{1}{\delta-2}}.$$

Because of the binding limited-liability constraint $\alpha + \beta\left(e^*_{PR} - \bar{\varepsilon}_L\right) = 0$ and the incentive constraint $\beta = c'\left(e^*_{PR}\right)$ the inequality can be rewritten as $\bar{\varepsilon}_L + \hat{\varepsilon} > \frac{c\left(e^*_T\right)}{c'\left(e^*_T\right)}$. By using the parametric form of the cost function and the concrete expression for the equilibrium effort e^*_{PR} we obtain

$$\left(\bar{\varepsilon}_L + \hat{\varepsilon}\right)\delta > \left(\frac{1}{\eta\delta\left(\delta-1\right)\bar{\varepsilon}_L}\right)^{\frac{1}{\delta-2}}. \tag{25}$$

Comparing (24) with (25) completes the proof.

Proof of Proposition 6:

Differentiating (22) (and therefore the left-hand side of (21)) with respect to $\bar{\varepsilon}$ gives

$$\frac{\partial \frac{1}{2g(0)}}{\partial} = 2\exp\left\{-\frac{\bar{\varepsilon}^2}{\sigma^2}\right\}.$$

Hence, the left-hand side of (21) will increase less rapidly in $\bar{\varepsilon}$ than the right-hand side, if

$$2\exp\left\{-\frac{\bar{\varepsilon}^2}{\sigma^2}\right\} < 1 \Leftrightarrow \sigma^2 < \frac{\bar{\varepsilon}^2}{\ln 2}.$$

The second result of Proposition 6 becomes obvious by inspection of (22).

For $\sigma^2 \rightarrow 0$, the upper limit of the integral tends to $-\infty$ and the whole integral tends to zero so that (22) goes to infinity. If $\sigma^2 \rightarrow \infty$, the upper limit of the interval tends to zero so that the whole interval goes to $\frac{1}{2}$ and, therefore, the term in brackets to zero. However, the expression $\sigma\sqrt{\pi}$ in front of the brackets grows more rapidly to infinity.

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