Romain Baeriswyl:
Central Bank’s Action and Communication

Munich Discussion Paper No. 2007-10

Department of Economics
University of Munich

Volkswirtschaftliche Fakultät
Ludwig-Maximilians-Universität München

Online at http://epub.ub.uni-muenchen.de/1381/
Central Bank’s Action and Communication

Romain Baeriswyl*
Munich Graduate School of Economics
e-mail: Romain.Baeriswyl@lrz.uni-muenchen.de

First version: March 2006 - This version: January 2007

Abstract

This paper contributes to the ongoing debate about the welfare effect of public information. In an environment characterized by imperfect common knowledge and strategic complementarities, Morris and Shin (2002) argue that noisy public information may be detrimental to welfare because public information is attributed too large a weight relative to its face value since it serves as a focal point. While this argument has received a great deal of attention in central banks and in the financial press, it considers communication as the sole task of a central bank and ignores that communication usually goes with a policy action. This paper accounts for the action task of a central bank and analyzes whether public disclosure is beneficial in the conduct of monetary policy when the central bank primarily tries to stabilize the economy with an instrument that is optimal with respect to its perhaps mistaken view. In this context, it turns out that transparency is particularly beneficial when central bank’s information is poorly accurate because it helps reducing the distortion associated with badly suited policies.

JEL classification: D82, E52, E58.
Keywords: differential information, monetary policy, transparency.

*I thank Camille Cornand, Frank Heinemann, and Gerhard Illing for very helpful discussions.
1 Introduction

The conduct of monetary policy has been characterized by an important switch from secrecy to transparency over the last decades. Central banks talk much more openly about their policy decisions today than they used to do in the last decades. While central bankers thought they could better achieve their target by acting in secret and taking the markets by surprise, it seems that transparency has nowadays become the new paradigm.

This trend in central banking has given rise to a growing literature about the pros and cons of higher transparency. In particular, the literature has recently raised questions about the value of having central banks provide more and better information to the public. For decisions made under uncertainty, more accurate information usually permits that decisions are better suited to the underlying fundamental. But macroeconomic environments also often entail strategic complementarities in decision making. As Keynes pointed out in his beauty contest example, decision makers face the dilemma of matching some fundamental of the economy and coordinating with the decision of others.\textsuperscript{1}

While both public information and private information play an equivalent role in guessing the fundamental, public information plays a preponderant role in guessing the decision of others because it is common to all agents and thereby better helps predicting their expectations. So, individual agents assign a higher weight to public information than justified by its informative value since it serves as a focal point. Public information is therefore extremely effective in shaping market outcomes.

In their seminal beauty contest paper, Morris and Shin (2002) (henceforth M-S) highlight that the disclosure of noisy public information may be detrimental to welfare because the overreaction to it may distort the market outcome away from the fundamental. They conclude that, if there is some upper bound in the precision of its information, the central bank may be better off withholding its information. Their argument has received a great deal of attention in central banks\textsuperscript{2} and in the financial press\textsuperscript{3} because it seems to contradict the general presumption that transparency is beneficial.

Yet, the literature in the vein of M-S analyzes the welfare effect of public information when the only task of the central bank is to communicate with the public, \textit{i.e.} to disclose or withhold its information. Typically it ignores that the primarily task of a central bank is to take action by implementing a monetary instrument. While communication is certainly a key component of monetary policy, the action implemented by a central bank must not be ignored for

\begin{itemize}
\item \textsuperscript{1}See Keynes (1936) p. 157.
\item \textsuperscript{2}See for example Kohn (2005) and Issing (2005).
\item \textsuperscript{3}See The Economist (2004).
\end{itemize}
all that. This paper argues that information policy must be thought within a framework that also considers the primarily task of the central bank, namely its action. Indeed, information disclosure – if any – rarely occurs alone but usually goes with policy implementation. More importantly, one must be aware that the action implemented by a central bank is chosen according to its perhaps noisy information. When the central bank has a mistaken view about the economic outcome (because of inevitable forecast errors) its stabilization policy may well turn out to be rather distorting. Thus, the question of transparency must account for the fact that the central bank’s action suffers from the same distortion as its disclosure. One may thus ask how a central bank should communicate with the public when the monetary instrument it implements is distorted by noisy information. Should the central bank implement its instrument in secret to avoid the private sector’s overreaction to its mistaken view? Or should it, on the contrary, bring its viewpoint to light?

This paper contributes to the ongoing debate about the welfare effect of public information when disclosure goes with action. It especially develops the idea that – as opposed to M-S – transparency reduces the distortion of monetary policy. We consider a model of monopolistic competition with imperfect common knowledge where firms’ prices are strategic complements. The economy is hit by demand shocks and firms set their price according to their own belief about the output gap and their expectations about the belief of others. Our analysis is constructed into two steps.

First, we discuss in section 3 the transparency effect in the case where information disclosure is the only purpose of the central bank. Central bank’s disclosure reduces the fundamental and strategic uncertainty. This set-up not surprisingly yields the same conclusion as M-S, namely that the central bank should withhold its information whenever it is rather noisy and when the degree of strategic complementarities is high. In this context, we introduce the concept of partial transparency. While M-S consider two extreme kinds of disclosure, transparency and opacity, we argue that some intermediate level of transparency better describes the reality and may be welfare improving. It is not necessarily true that central bank’s disclosures are common knowledge among the whole population. Indeed, central banks are known for speaking with mystique. This makes their disclosures equivocal, open to interpretation, and prevents them from becoming common knowledge. Greenspan’s testimony to the US Congress in 1987 illustrates the willingness of central bankers to speak in equivocal manners: “Since I have become a central banker, I have learned to mumble with great incoherence. If I seem unduly clear to you, you must have misunderstood what I said.” More recently (in December 2002), Mike Moskow, the
president of the Federal Reserve Bank of Chicago, claimed that “[the Fed speak] is a language in which it is possible to speak, without ever saying anything.” Imperfect or partial transparency can be well rationalized in this context. Since a central bank’s disclosure may be detrimental to welfare when it is common knowledge, introducing some uncertainty about its interpretation reduces its focal role and improves the outcome. This argument is close to that of Cornand and Heinemann (2005) who introduce the notion of partial publicity. They show that disclosing public information to a limited audience reduces the overreaction to it which can be welfare increasing. Depriving some agents of receiving public information prevents it from becoming common knowledge among the whole population. But while under partial publicity the disclosure is common knowledge among the limited audience (only), under partial transparency the disclosure is private to each firm. In this respect, partial transparency is similar to Heinemann and Illing (2002) who argue – within a game of speculative attack – that central banks should provide information to each agent in private with some idiosyncratic noise to avoid common knowledge (and yields equilibrium uniqueness).

Second, section 4 presents the case where the central bank tries to stabilize the economy by implementing a monetary instrument. As discussed below, central banks have become much more transparent about their instrument over the last decades. We show that full transparency is then preferable to partial transparency. The intuition behind this finding is as follows. Since the central bank tries to stabilize the economy based on its information, central bank’s errors influence the economic outcome even if central bank’s information remains unknown to firms. The central bank’s mistaken view distorts the economy even under opacity. The disclosure policy of the central bank however influences firms’ reaction and the price level because the monetary instrument is part of the output gap, the fundamental firms have to respond to. Under transparency, firms’ response accounts for the monetary instrument and this reduces the distorting effect of central bank’s action. For instance, if the central bank contracts the economy by mistake, prices better offset the mistaken policy action when firms’ reaction to the instrument is maximal, i.e. when the instrument is common knowledge among firms. Opacity is however optimal in this set-up for a very small and rather unrealistic range of parameter values. But interestingly, we show that the case for opacity shrinks when central bank’s information becomes less accurate: while the monetary instrument increasingly distorts the output gap, transparency, by strengthening the response of firms to central bank’s action, attenuates the distortion. Transparency is therefore particularly beneficial when the central bank has a very mistaken view of the
Section 5 compares the optimal disclosure in our two frameworks and emphasizes the benefit of transparency when the central bank tries to stabilize the economy. As a result, taking the action task of the central bank into consideration strongly contrasts with M-S according to which transparency is welfare detrimental when the central bank’s information is poorly accurate. And finally, section 6 concludes.

2 The economy

The model is derived from an economy with flexible prices, populated by a representative household, a continuum of monopolistic competitive firms, and a central bank. The economy is hit by stochastic demand shocks. Nominal aggregate demand is determined by both the demand shock and the monetary instrument set by the central bank. The baseline framework is close to Adam (2006).

2.1 Representative household

The representative household chooses its aggregate composite good \( C \) and labor supply \( H \) in order to maximize its utility subject to its budget constraint,

\[
gU(C) - V(H) \tag{1}
\]

s.t. \( WH + \Pi = PC \).

The parameter \( g \) is a stochastic demand shock, that induces variations in the efficient level of output. The utility function has the following usual properties: \( U' > 0, U'' < 0, \lim_{C \to \infty} U'(C) = 0, V' > 0, V'' < 0, \) and \( V'(0) < U'(0) \). \( C \) is the composite good defined by the Dixit-Stiglitz aggregator

\[
C = \left[ \int_0^1 (C_i)^{\frac{1}{1-\theta}} \, di \right]^{\frac{1}{\theta-1}} \tag{2}
\]

where \( \theta > 1 \) is the parameter of price elasticity of demand and where \( C_i \) is the good produced by firm \( i \). \( W \) denotes the competitive wage and \( \Pi \) the profits the household gets from firms. \( P \) is the appropriate price index which solves \( \int_0^1 P_i C_i \, di \) and satisfies

\[
P = \left[ \int_0^1 P_i^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}.
\]
Given the overall level of consumption, the household allocates its expenditure across goods according to

\[ C_i = \left( \frac{P_i}{P} \right)^{-\theta} C \]  

(3)

and optimizing the consumption-labor decision leads to the real wage

\[ \frac{W}{P} = \frac{V'(H)}{gU'(C)} \]  

(4)

2.2 Firms

Each firm \( i \) produces a single differentiated good \( C_i \) with one unit of labor \( H_i \) according to the simple production function

\[ H_i = C_i. \]  

(5)

The profit maximization problem of firm \( i \) is given by

\[ \max_{P_i} \mathbb{E}[P_i C_i(P_i) - W H_i(P_i) | I_i], \]  

(6)

where \( I_i \) is the information set of firm \( i \). Using (3), (4), and (5), the first order condition of (6) becomes

\[ \mathbb{E}\left[ (1 - \theta) \left( \frac{P_i}{P} \right)^{-\theta} + \theta \left( \frac{P_i}{P} \right)^{-\theta - 1} \frac{V'(C)}{gU'(C)} | I_i \right] = 0. \]  

(7)

Linearizing (7) around the steady state delivers

\[ p_i = \mathbb{E}_i[p + \xi c], \]  

(8)

where small letters indicate percentage deviation from the steady state and where

\[ \xi = -\frac{U''(\bar{C})\bar{C}}{U'(\bar{C})} + \frac{V''(C)\bar{C}}{V'(\bar{C})}. \]

\( \bar{C} \) is the real output at its steady state level.

The pricing rule (8) states that firms set their price as a function of their expectations of the overall price level \( p \) and the real output gap \( c \). The parameter \( \xi \) determines in what extent the optimal price responds to the output gap. Firms strongly respond to the output gap when it has a strong impact on the competitive real wage. This occurs when the household’s utility and disutility functions are very concave and convex, respectively, i.e. when \( \xi \) is large. In this case, the real wage required for additional production is high (since the
household derives a low utility from additional consumption while it suffers a high disutility from additional work) and firms strongly adjust their price to the output gap. We qualify as "weakly extensive" an economy with a high value of $\xi$ and as "highly extensive" an economy with a low value of $\xi$. In this context, $\xi$ captures the effectiveness of monetary policy for influencing the price level. As we assume later on, the central bank partially determines the nominal aggregate demand through its monetary instrument. In the case where the economy is highly extensive, output gap deviations have small impacts on the competitive real wage and thus on the price level. The monetary instrument is consequently weakly effective at influencing the price level. $\xi$ also determines whether prices are strategic complements or substitutes. Using the fact that the nominal aggregate demand (deviation) $y$ can be expressed as $y = c + p$, we rewrite the pricing rule (8) as

$$p_i = E_i[(1 - \xi)p + \xi y].$$

(9)

In the whole paper, we assume that prices are strategic complements, i.e. $\xi \leq 1$. This assumption seems very natural and captures the concept of beauty contest introduced by Keynes.

### 2.3 Central bank

The current paper underlines the relevance of two central bank’s tasks, namely information disclosure and policy implementation. In section 3, the central bank is supposed to influence the economy with the disclosure of its information about demand shocks exclusively. By contrast, section 4 additionally accounts for the monetary policy $I$ implemented by the central bank. The monetary instrument is then supposed to partially determined the nominal aggregate demand up to the demand shock $g$. The nominal aggregate demand $y$ is the sum of the central bank’s instrument $I$ (if any) and of the demand shock $g$, i.e. $y = I + g$. The demand shock is drawn from a uniform distribution over the real line: $g \in \mathbb{R}$.

### 2.4 Welfare

The welfare is defined as the utility of the representative household. One can show that in the economy described above, the welfare is decreasing in both the dispersion of prices across firms $\int_i (p_i - \bar{p})^2 di$ and the variability of the output gap $c = y - p$. Since there is currently no consensus about how coordination is socially valuable relative to macroeconomic distortion, we define a generic welfare function that accounts for alternative weights assigned to coordination.
So, the social loss is given by

\[ L = \int_i (p_i - p)^2 di + \lambda c^2, \]  

(10)

where \( \lambda \) is the weight assigned to the output gap variability. The welfare function used in the transparency debate of M-S is a matter of controversy since the detrimental effect of transparency is driven by the relative relevance of coordination and stabilization at the social level. The application of the M-S argument to different welfare functions may lead to different conclusions. For example, Hellwig (2005) and Woodford (2005) show that when coordination is socially highly valuable, transparency is welfare improving as it helps coordinating firms’ price setting. In their model, the potential destabilizing effect of transparency is neglected. The welfare function (10) is generic since the coefficient \( \lambda \) describes the relative importance of coordination for the society as a whole. As discussed below, the welfare function derived in the seminal beauty contest paper by M-S \(-\int_i (p_i - g)^2 di\) is captured by the loss (10) when the weight assigned to coordination is equal to that assigned to output gap distortion, that is to say when \( \lambda = 1 \). The loss (10) can also replicate the microfounded welfare that assigns a much strong weight on coordination at the social level. Adam (2006) shows that the weight assigned to the output gap distortion when the welfare is microfounded amounts to \( \lambda = \frac{\theta}{\theta - 1} \), where \( \theta > 1 \) is the degree of substitutability in the Dixit-Stiglitz aggregator.

### 3 Pure information disclosure

This section analyzes the welfare effect of public information when the central bank does not influence the economy except with its information disclosure. The aim of this section is to illustrate the much debated result by M-S where information disclosure is the only task of the central bank. Since the central bank does not implement any action, we set \( I = 0 \) and rewrite the pricing rule (9) as

\[ p_i = \mathbb{E}_i [(1 - \xi)p + \xi g]. \]  

(11)

One may worry about the fact that the central bank does not offset demand shocks in the present economy and claim that this is not optimal. However, the aim of this paper is not to address the merits of having a central bank stabilizing the economy but to compare the welfare effect of disclosure in the case where the central bank does not stabilize the economy to the case where it does. So, the present section must be seen as a benchmark case that replicates
the results by M-S and allows a better suited comparison. We describe the information structure in the next section. Then, we discuss the optimal information disclosure first when the central bank chooses between full transparency and opacity (i.e. the central bank either perfectly reveals its opinion or totally withholds it), and second when the central bank can choose its optimal degree of transparency (i.e. the central bank speaks with some ambiguity).

3.1 Information structure

To take its pricing decision, each firm receives two signals. First, each firm gets a private signal about the demand shock that may be interpreted as a private opinion. The private signal is centred on the true value of $g$ and has a normally distributed error term:

$$g_i = g + \varepsilon_i \quad \text{with} \quad \varepsilon_i \sim N(0, \sigma^2_\varepsilon),$$

where $\varepsilon_i$ are identically and independently distributed across firms.

Second, firms get a signal disclosed by the central bank. The central bank imperfectly observes the demand shock: it receives a signal on the demand shock that is centred on its true value and contains a normally distributed error term:

$$D = g + \eta \quad \text{with} \quad \eta \sim N(0, \sigma^2_\eta).$$

The central bank provides firms with its viewpoint about the demand shock. As discussed in the introduction, the central bank communicates its information $D$ with more or less ambiguity. For the sake of generality, we write the signal disclosed by the central bank and received by firm $i$ as

$$D_i = g + \eta + \phi_i \quad \text{with} \quad \phi_i \sim N(0, \sigma^2_{\phi}).$$

The dispersion of individual noises $\sigma^2_{\phi}$ determines the degree of transparency of the central bank. Under transparency, every firm gets the same univocal signal ($\sigma^2_{\phi} = 0$). Then, the central bank’s information $D$ is a public signal that is common knowledge among all firms. Under opacity, the individual signal got by each firm has an infinite idiosyncratic noise ($\sigma^2_{\phi} \to \infty$). The central bank’s information thus does not contain any valuable information. One can imagine any intermediate situation where the central bank provides firms with more or less equivocal information.

The introduction of idiosyncratic noise in central bank’s disclosure reduces its degree of common knowledge among firms. This communication strategy has
been proposed by Heinemann and Illing (2002) who address the issue of central bank disclosing information to every agent in private within a game of speculative attack. Cornand and Heinemann (2005) propose another disclosure strategy that also reduces the degree of common knowledge: the disclosure of a public signal $D$ to a fraction $S$ of firms. The disclosure $D$ thus becomes semi-public as the fraction $1 - S$ of firms does not receive it but only gets its private signal $g_i$. Appendix A shows that both disclosure strategies – i.e. limited transparency vs. limited publicity – are strictly equivalent in terms of welfare. More precisely, this appendix shows that the equivalence relationship between the degree of transparency $\sigma^2_\varphi$ and the degree of publicity $S$ is given by

$$\sigma^2_\varphi = \frac{1 - S}{S} (\sigma^2_\varepsilon + \sigma^2_\eta) \quad \text{or} \quad S = \frac{\sigma^2_\varphi + \sigma^2_\eta}{\sigma^2_\varepsilon + \sigma^2_\eta + \sigma^2_\varphi}.$$ 

In the remainder of the paper, we will however only address the question of optimal degree of transparency.

### 3.2 Equilibrium

This section derives the perfect Bayesian equilibrium behaviour of firms. We recall the optimal pricing rule (11) for convenience and substitute successively the average price level with higher-order expectations about the cost-push shock and the monetary instrument

$$p_i = \mathbb{E}_i \left[(1 - \xi)p + \xi g\right] = \mathbb{E}_i \left[\xi g + (1 - \xi) \left[\mathbb{E}[\xi g + (1 - \xi) [\mathbb{E}[\xi g + \ldots]]]\right]\right].$$

We denote by $\mathbb{E}_i(.)$ the expectation operator of firm $i$ conditional on its information and by $\bar{\mathbb{E}}(.)$ the average expectation operator such that $\bar{\mathbb{E}}(.) = \int E_i(.)di$. With heterogeneous information, the law of iterated expectations fails and expectations of higher-order do not collapse to the average expectation of degree one. Thus, we rewrite the pricing rule as

$$p_i = \xi \sum_{k=0}^{\infty} (1 - \xi)^k \mathbb{E}_i \left[\mathbb{E}^{(k)}(g)\right],$$

and averaging over firms yields

\[ p = \xi \sum_{k=0}^{\infty} (1 - \xi)^k \left[ E^{(k+1)}(g) \right]. \]  

(12)

where \( E^{(k)} \) stands for the higher-order expectation of degree \( k \). We use the following notation of higher-order expectations: \( E^{(0)}(x) = x \) is the expected variable \( x \) itself, \( E^{(1)}(x) = E(x) \) is the average expectation of \( x \), \( E^{(2)}(x) = E E(x) \) is the average expectation of the average expectation of \( x \), and so on.

To determine the optimal price rule (12), we build the first and higher-order expectations of firm \( i \) about the demand shock \( g \) conditional on its information. The expectation of degree one about the demand shock \( E_i(g) \) yields

\[ E(g|g_i, D_i) = \sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2 g_i + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2} D_i = \Omega_{11} g_i + \Omega_{12} D_i. \]  

(13)

The best estimate of the demand shock by firm \( i \) is an average of its both signals whose weighting depends upon their relative precision. To compute the higher-order expectations of firm \( i \), one needs also to know the expectation of degree one of the central bank’s average disclosure \( E_i(D) \). This delivers

\[ E(D|g_i, D_i) = \sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2 g_i + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2} D_i = \Omega_{21} g_i + \Omega_{22} D_i. \]  

(14)

Note that under transparency (when \( \sigma_\phi^2 = 0 \)), the central bank’s disclosure is univocal and \( \Omega_{21} = 0 \) which means that the private signal \( g_i \) does not help guessing \( D \). Under opacity, when the idiosyncratic noise is infinite (\( \sigma_\phi^2 \to \infty \)), the central bank’s disclosure is of no use to estimate the demand shock \( g \) and the best estimate is the private signal \( g_i \) itself (\( \Omega_{11} = 1 \)).

Using these results, we can express the higher-order expectations of degree \( k \) as

\[ E^{(k)} \begin{pmatrix} g \\ D \end{pmatrix} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}^k \begin{pmatrix} g \\ D \end{pmatrix}. \]

Plugging this into the price rule (12), we get

\[ p = \left( \xi \quad 0 \right) \sum_{k=0}^{\infty} (1 - \xi)^k \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}^{k+1} \begin{pmatrix} g \\ D \end{pmatrix}. \]  

(15)

The price rule is a linear combination of the demand shock and the central
bank’s disclosure. Appendix B shows that the price rule is given by

\[ p = \gamma_1 g + \gamma_2 D \quad \text{with} \]

\[ \gamma_1 = \frac{\Omega_{11} \xi + (1 - \xi)\Omega_{21}}{1 - (1 - \xi)(\Omega_{11} - \Omega_{21})} = \frac{\xi \sigma_{\eta}^2 + \sigma_{\phi}^2}{\sigma_\epsilon^2 + \xi \sigma_{\eta}^2 + \sigma_{\phi}^2} \]

\[ \gamma_2 = \frac{\xi \Omega_{12} + (1 - \xi)\Omega_{12}\Omega_{21}}{\xi - (1 - \xi)(\Omega_{11} \xi - (1 + \xi)\Omega_{21} - (1 - \xi)(\Omega_{21} - \Omega_{11})\Omega_{11})} = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \xi \sigma_{\eta}^2 + \sigma_{\phi}^2}. \]

\( \gamma_1 \) and \( \gamma_2 \) sum up to 1. The equilibrium firms’ action can be interpreted as a weighted average of the fundamental \( g \) and the average disclosure \( D \). Note however that the weight assigned to the central bank’s disclosure is larger in the equilibrium action (\( \gamma_2 \)) than in the best estimate of \( g \) given in (13): \( \gamma_2 > \Omega_{12} \).

This discrepancy arises because of the coordination motive in the pricing rule. While \( \epsilon_i \) and \( \phi_i \) are idiosyncratic noises, the central bank’s error term \( \eta \) is commonly observed by all firms through the disclosure \( D_i \). The weight assigned to the central bank’s error (and thereby to \( D_i \)) increases as the coordination motive strengthens: strategic complementarities raise the incentive of firms to coordinate their action around the central bank’s disclosure. When the degree of strategic complementarities \( 1 - \xi \) increases, the weight assigned to the private signal \( g \), declines (\( \frac{\partial \gamma_1}{\partial \sigma_\epsilon^2} > 0 \)) while the weight assigned to central bank’s disclosure increases (\( \frac{\partial \gamma_2}{\partial \sigma_\epsilon^2} < 0 \)). When the degree of transparency increases (\( \sigma_\phi^2 \) falls), the weight put on the central bank’s disclosure \( D_i \) increases since its interpretation becomes less ambiguous and better conducive to guess the action of others (\( \frac{\partial \gamma_1}{\partial \sigma_\eta^2} > 0 \) and \( \frac{\partial \gamma_2}{\partial \sigma_\eta^2} < 0 \)). Signals are also given a higher weight when their precision increases: \( \frac{\partial \gamma_1}{\partial \sigma_\epsilon^2} < 0 \) and \( \frac{\partial \gamma_2}{\partial \sigma_\eta^2} < 0 \).

### 3.3 Welfare

We now examine the welfare given by (10) in the current informational context. On the one hand, the equilibrium firms’ behaviour (16) implies that the unconditional expected price dispersion across firms satisfies

\[ E \left( \int (p_i - p)^2 \, dt \right) = E \left( \int (\gamma_1 g_i + \gamma_2 D_i - \gamma_1 g - \gamma_2 D)^2 \, dt \right) = \gamma_1^2 \sigma_\epsilon^2 + \gamma_2^2 \sigma_\phi^2. \]

On the other hand, the unconditional output gap expectation is

\[ E(\epsilon^2) = E(g - p)^2 = E(g - \gamma_1 g - \gamma_2 D)^2 = \gamma_2^2 \sigma_\eta^2. \]

So, the unconditional expected social loss can be written as

\[ E(L) = \gamma_1^2 \sigma_\epsilon^2 + \gamma_2^2 \sigma_\phi^2 + \lambda \gamma_2^2 \sigma_\eta^2 \quad (17) \]
$\sigma^2 (\lambda \sigma^2 + \sigma^2) + (\xi \sigma^2 + \sigma^2)^2 \sigma^2.

Let us now discuss the welfare considered in M-S and given by $-\int_i (p_i - g)^2 di$. We write the corresponding loss as

$$E(L_{MS}) = E\left( \int_i (p_i - g)^2 di \right)$$

$$= E\left( \int_i (\gamma_1 (g + \varepsilon_i) + \gamma_2 (g + \eta + \phi_i) - g)^2 di \right)$$

$$= \gamma_1^2 \sigma^2 + \gamma_2^2 \sigma^2 + \gamma_2^2 \sigma^2.$$ 

This implies that the welfare in M-S is a particular case of our general formulation (17) where $\lambda = 1$. This means that the model of M-S equally weights coordination and stabilization at the social level.

The welfare effect of the central bank’s disclosure is analyzed in the next sections. We first restrict the discussion to the binary case of transparency vs. opacity. This is the perspective of M-S where the central bank either discloses a public signal (that is common knowledge) or withholds its information. Then, we allow for intermediate level of transparency and derive the optimal degree of transparency.

3.4 Transparency versus opacity

Opacity The welfare is calculated when the central bank withholds its information, i.e. $\sigma^2 \rightarrow \infty$. Under opacity, firms set their price equal to their private signal $g_i$, i.e. $\gamma_1 = 1$ and $\gamma_2 = 0$. The resulting expected loss is

$$E(L_O) = E\left( \int_i (\gamma_1 (g + \varepsilon_i) - \gamma_1 g)^2 di + \lambda (g - \gamma_1 g)^2 \right) = \sigma^2.$$ 

The overall price level $p$ is equal to the fundamental $g$. The price dispersion across firms is given by the variance of the idiosyncratic noise $\varepsilon_i$.

Transparency Under transparency, disclosure of the central bank is common knowledge ($\sigma^2 = 0$) and the pricing rule of firms becomes

$$p = \frac{\xi \sigma^2}{\sigma^2 + \xi \sigma^2} g + \frac{\sigma^2}{\sigma^2 + \xi \sigma^2} D.$$ 

The resulting expected loss is

$$E(L_T) = \left( \frac{\xi \sigma^2}{\sigma^2 + \xi \sigma^2} \right)^2 \sigma^2 + \lambda \left( \frac{\sigma^2}{\sigma^2 + \xi \sigma^2} \right)^2 \sigma^2.$$
Transparency is welfare improving when the loss under opacity $L_O$ is larger than the loss under transparency $L_T$. The welfare analysis of transparency yields the following proposition:

**Proposition 1:** When the central bank’s unique task is information disclosure, full transparency is preferable to opacity when

$$\lambda - 2\xi < \frac{\sigma^2}{\sigma^2_\eta}.$$  

(18)

This finding is in line with M-S: transparency is welfare detrimental whenever public information is too noisy relative to private information ($\sigma^2_\eta$ small), when the degree of strategic complementarities is rather high ($\xi$ small), and when coordination is socially not too valuable ($\lambda$ large). When complementarities are sufficiently low such that $\lambda - 2\xi < 0$, transparency is always beneficial since variances of error terms are positive by definition ($\sigma^2 \geq 0$).

The general framework developed in this paper shows in what extent the welfare effect of transparency is related to the social value of coordination. In the case of M-S, as $\lambda = 1$, private information must be more accurate than public information for transparency to be detrimental. The left-hand side of inequality (18) is always smaller than one. For the right-hand side to be smaller than the left-hand one, the central bank’s noise $\sigma^2_\eta$ must be larger than the private noise $\sigma^2_\varepsilon$. Since information of public institutions (like central banks) is typically more accurate than information privately available, Svensson (2006) argues that the detrimental effect of transparency emphasized in the beauty contest framework of M-S arises under unrealistic conditions. But if the social value of coordination is smaller than in M-S ($\lambda > 1$), opacity may be superior even when public information is more accurate than private information (this arises when $\lambda - 2\xi > 1$). This means that the pertinence of the critique of Svensson strongly depends on the coordination value at the social level.

### 3.5 Optimal degree of transparency

In the former section, the central bank could either disclose its noisy information with perfect precision or withhold it. In reality, however, central bankers are known for mumbling with ambiguity. This makes central bank’s disclosures open to interpretation. The more a central bank speaks in an equivocal

---

6For instance, in an empirical analysis on US data, Romer and Romer (2000) show that the Fed better forecasts the output and inflation than any single private commercial bank.
manner, the higher the uncertainty about the interpretation of the disclosure (fundamental uncertainty) and the higher the uncertainty about its interpretation by others (strategic uncertainty). When full transparency is detrimental to welfare relative to opacity, reducing transparency may improve welfare. But even when full transparency is preferable to opacity, partial transparency may yield a superior outcome. What is the optimal degree of transparency for a central bank to disclose its information?

To determine the optimal degree of transparency \( \sigma^2_\phi \), we minimize the loss (17) with respect to \( \sigma^2_\phi \) and set it equal to zero:

\[
\frac{\partial E(L)}{\partial \sigma^2_\phi} = 2\gamma_1 \frac{\partial \gamma_1}{\partial \sigma^2_\phi} \sigma^2_\phi + \gamma_2 \frac{\partial \gamma_2}{\partial \sigma^2_\phi} \sigma^2_\phi + 2\lambda \gamma_2 \frac{\partial \gamma_2}{\partial \sigma^2_\phi} \sigma^2_\phi \\
= \frac{(\sigma^2_\phi + (3\xi - 2\lambda)\sigma^2_\eta + \sigma^2_\epsilon) \sigma^4_\epsilon}{(\sigma^2_\phi + \xi \sigma^2_\eta + \sigma^2_\epsilon)^3} \\
= 0 \quad \Leftrightarrow \quad \sigma^2_\phi = (2\lambda - 3\xi)\sigma^2_\eta - \sigma^2_\epsilon.
\]

(19)

Deriving the optimal degree of transparency in the framework described above, we get the following proposition:

**Proposition 2:** When the central bank’s unique task is information disclosure, the optimal degree of transparency is given by

\[
\sigma^2_\phi^* = \max[0, (2\lambda - 3\xi)\sigma^2_\eta - \sigma^2_\epsilon].
\]

(20)

This analysis calls for partial transparency when coordination is not very valuable at the social level (\( \lambda \) large), when the degree of strategic complementarities is high (\( \xi \) small), and/or when the central bank’s information is rather noisy (\( \sigma^2_\eta \) large).

Implementing the optimal degree of transparency (20) yields the following expected welfare:

\[
\mathbb{E}(L^*) = \min \left[ \frac{(\lambda \sigma^2_\epsilon + \xi \sigma^2_\eta) \sigma^2_\epsilon \sigma^2_\eta}{(\sigma^2_\epsilon + \xi \sigma^2_\eta)^2}, \frac{4\sigma^2_\epsilon^2(\xi - \lambda)}{4\sigma^2_\epsilon^2(\xi - \lambda) - \sigma^2_\epsilon} \right].
\]

The first panel of figure 1 illustrates the unconditional expected loss under transparency (dotted line), under opacity (dashed line), and under optimal degree of transparency (solid line). The parameter values are \( \sigma^2_\eta = 0.25 \), \( \xi = 0.1 \), and \( \lambda = 1 \). As (18) shows, full opacity is superior to full transparency when \( \sigma^2_\epsilon < (\lambda - 2\xi)\sigma^2_\eta = 0.2 \). The optimal degree of transparency is represented in the second plot below. As (20) states it, reducing the degree of transparency is
optimal when $\sigma^2 < (2\lambda - 3\xi)\sigma^2_\eta = 0.425$. Interestingly, for $0.2 < \sigma^2 < 0.425$, reducing the degree of transparency is optimal even if full transparency is superior to full opacity.

4 Action and information disclosure

We now deal with the main aim of this paper. We analyze the optimal disclosure policy when the central bank’s primarily task is to stabilize the economy. The economy is hit by demand shocks $g$ and the central bank tries to offset them by implementing its monetary instrument $I$. The nominal aggregate demand is composed of the demand shock and the monetary instrument, i.e. $y = g + I$. Thus firms set their price according to their first and higher-order expectations about both the demand shock and the monetary instrument. The central bank’s action is part of the “fundamental” firms respond to. We rewrite the pricing rule (9) for convenience:

$$p_i = \mathbb{E}_t [(1 - \xi)p + \xi g + \xi I].$$

(21)

We describe the information structure and derive the equilibrium. We discuss then the optimal information disclosure when the central bank chooses between full transparency and opacity, and then whether partial transparency is optimal.
4.1 Information structure

Each firm sets its price according to its own belief about both the demand shock $g$ and the central bank’s instrument $I$, and its belief about others’ belief about them. Again, the demand shock is drawn from the real line: $g \in \mathbb{R}$. Each firm receives a private signal $g_i = g + \varepsilon_i$ about the demand shock that has the same properties as in the former section.

Based on its own information $D = g + \eta$, the central bank sets its instrument to offset demand shocks: $I = -g - \eta$.

The central bank then provides firms with information about its instrument (or economic assessment). When the central bank is transparent, its instrument is a public signal (common knowledge among firms). Conversely, when it is opaque, firms’ observation of the instrument does not contain any valuable information at all. In intermediate situations, the central bank provides firms with more or less ambiguous information about its instrument. For the sake of generality, we write the signal disclosed by the central bank and received by firm $i$ as

$$I_i = I + \phi_i = -g - \eta + \phi_i \quad \text{with } \phi_i \sim N(0, \sigma^2_\phi).$$

As in the former section, the individual noise $\phi_i$ captures the degree of transparency of the central bank. Full transparency is reached when $\sigma^2_\phi = 0$ and full opacity when the central bank withholds information about its instrument ($\sigma^2_\phi \to \infty$).

4.2 Equilibrium

To determine the equilibrium behaviour of firms, we proceed as before. Substituting successively the average price level with higher-order expectations about the demand shock and the monetary instrument into (21) yields

$$p_i = \xi \sum_{k=0}^{\infty} (1 - \xi)^k \mathbb{E}_i \left[ \mathbb{E}^{(k)}(g + I) \right],$$

and averaging over firms, we get

$$p = \xi \sum_{k=0}^{\infty} (1 - \xi)^k \mathbb{E}^{(k+1)}(g + I). \quad (22)$$

The optimal pricing rule of firm $i$ is a weighted average of its first and higher-order expectations about the demand shock $g$ and the central bank’s instru-

---

7Since demand shock $g$ has an improper distribution, it is optimal for the central bank to fully offset its expected shock.
ment \( I \) conditional on its information. Its first-order expectations are

\[
E(g|g_i, I_i) = \frac{\sigma_n^2 + \sigma_\phi^2}{\sigma_e^2 + \sigma_n^2 + \sigma_\phi^2} g_i - \frac{\sigma_\phi^2}{\sigma_e^2 + \sigma_n^2 + \sigma_\phi^2} I_i = \Theta_{11} g_i + \Theta_{12} I_i
\]

\[
E(I|g_i, I_i) = -\frac{\sigma_\phi^2}{\sigma_e^2 + \sigma_n^2 + \sigma_\phi^2} g_i + \frac{\sigma_n^2 + \sigma_\phi^2}{\sigma_e^2 + \sigma_n^2 + \sigma_\phi^2} I_i = \Theta_{21} g_i + \Theta_{22} I_i.
\]

Plugging this result into (22), we have

\[
p = \left( \xi \quad \xi \right) \sum_{k=0}^{\infty} (1-\xi)^k \begin{pmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{pmatrix}^{k+1} \begin{pmatrix} g \\ I \end{pmatrix}
\]

and rewriting in a linear form leads to

\[
p = \gamma_1 g + \gamma_2 I \quad \text{with} \quad \gamma_1 = \frac{\xi \sigma_n^2}{1 - (1-\xi)(\Theta_{11} + \Theta_{21})} = \frac{\xi \sigma_n^2}{\sigma_e^2 + \xi \sigma_n^2 + \sigma_\phi^2} = \gamma_2.
\]

The derivation of this equilibrium pricing rule is given in appendix C. When the central bank stabilizes the economy with its monetary instrument, firms equally weight their private signal \( g_i \) and the central bank’s disclosure \( I_i \) into their pricing decision (\( \gamma_1 = \gamma_2 \)). This arises because firms respond to the nominal aggregate demand that is composed of both the demand shock and the monetary instrument.

Since the central bank tries to stabilize the economy, it is common knowledge among firms (even under opacity) that the nominal aggregate demand expected by the central bank is equal to zero. In the particular case where the central bank has perfect information about demand shocks (\( \sigma_n^2 = 0 \)), the monetary instrument perfectly offsets demand shocks and firms set their price equal to zero. For the more realistic case where central bank’s information is noisy, the demand shock is less likely to be precisely offset by the central bank and thereby the nominal aggregate demand to be zero. Firms then rely more strongly on their private information \( g_i \) and disclosure \( I_i \) to set their optimal price (\( \frac{\partial \gamma_1}{\partial \sigma_n^2} > 0 \)).

When the degree of strategic complementarities increases, firms respond less strongly to their private signal \( g_i \) and to the instrument disclosure \( I_i \), and assign a higher weight to the nominal aggregate demand expected by the central bank (that is to say zero) since the latter is common knowledge (\( \frac{\partial \gamma_1}{\partial \xi} > 0 \)).

When private noises increase, fundamental and strategic uncertainty increases as well. Hence, firms less strongly respond to their private signal and disclosure and higher weight the nominal demand of zero expected by the central
bank \( (\frac{\partial \gamma_1}{\partial \sigma^2_z} < 0 \) and \( \frac{\partial \gamma_1}{\partial \sigma^2_\phi} < 0 \).

### 4.3 Welfare

We now turn to the welfare analysis. First, the equilibrium firms’ behaviour (23) implies that the price dispersion across firms satisfies

\[
\mathbb{E} \left( \int_i (p_i - p)^2 di \right) = \mathbb{E} \left( \int_i (\gamma_1 g_i + \gamma_2 I_i - \gamma_1 g - \gamma_2 I)^2 di \right) = \gamma_1^2 \sigma^2_z + \gamma_2^2 \sigma^2_\phi.
\]

Second, with the central bank stabilizing the economy, the output gap is

\[
\mathbb{E}(\sigma^2_c) = \mathbb{E} \left( (g + I - p)^2 \right) = \mathbb{E} \left( (g + (-g - \eta) - \gamma_1 g - \gamma_2 (-g - \eta))^2 \right) = (\gamma_2 - 1)^2 \sigma^2_\eta.
\]

So, since \( \gamma_1 = \gamma_2 \), the unconditional expected loss can be written as

\[
\mathbb{E}(L) = \gamma_1^2 \sigma^2_z + \gamma_2^2 \sigma^2_\phi + \lambda (\gamma_1 - 1)^2 \sigma^2_\eta
\]

\[
= \frac{\lambda (\sigma^2_z + \sigma^2_\phi) + \xi^2 \sigma^2_\eta}{(\sigma^2_z + \xi \sigma^2_\eta + \sigma^2_\phi)^2 (\sigma^2_z + \sigma^2_\phi) \sigma^2_\eta}.
\]  

(24)

### 4.4 Transparency versus opacity

**Opacity** The welfare is now computed when the central bank is opaque and implements its instrument in secret, i.e. \( \sigma^2_\phi \rightarrow \infty \). Under opacity, firms set their price equal to zero since \( \gamma_1 = 0 \) and \( \gamma_2 = 0 \). In so far as firms know that the central bank stabilizes the economy but have no information about the instrument, their private information \( g_i \) does not help them guessing the nominal aggregate demand. Their best nominal aggregate demand estimation is therefore zero and the resulting unconditional expected loss is

\[
\mathbb{E}(L_O) = \lambda \sigma^2_\eta.
\]

**Transparency** When the central bank is transparent, its monetary instrument is common knowledge: \( \sigma^2_\phi = 0 \). Under transparency, the pricing rule of firms becomes

\[
p_i = \frac{\xi \sigma^2_\eta}{\sigma^2_z + \xi \sigma^2_\eta} g_i + \frac{\xi \sigma^2_\eta}{\sigma^2_z + \xi \sigma^2_\eta} I_i.
\]

and the resulting unconditional expected loss yields

\[
\mathbb{E}(L_T) = \left( \frac{\xi \sigma^2_\eta}{\sigma^2_z + \xi \sigma^2_\eta} \right)^2 \sigma^2_z + \lambda \left( \frac{\sigma^2_\phi}{\sigma^2_z + \xi \sigma^2_\eta} \right)^2 \sigma^2_\eta.
\]
Transparency is welfare improving when the loss under opacity $L_O$ is larger than the loss under transparency $L_T$. Comparing both expected losses, we get the following proposition.

**Proposition 3:** When the central bank tries to offset demand shocks with its monetary instrument, full transparency is preferable to opacity when

$$\lambda > \frac{\xi \sigma^2_{\gamma}}{2\sigma^2_{\epsilon} + \xi \sigma^2_{\eta}}.$$  

Whether transparency is beneficial depends on the value of four parameters, the relevance of output gap stabilization at the social level $\lambda$, the noise of central bank’s information $\sigma^2_{\eta}$, the noise of firms’ private information $\sigma^2_{\epsilon}$, and the degree of strategic complementarities $1 - \xi$. Note first that transparency is particularly welfare improving when the weight assigned to the output gap stabilization $\lambda$ is large. When the central bank actively shapes the nominal aggregate demand with its monetary instrument, transparency reduces the potential detrimental effect of the policy owing firms to account for it in their price setting.

Second, transparency improves welfare when central bank’s information is rather noisy (the derivative of the right-hand side (RHS) of inequation (25) is negative with respect to central bank’s noise $\sigma^2_{\eta}$). When the monetary instrument implemented by the central bank is very likely not to precisely offset the demand shock, transparency helps reducing the possible distortion generated by the policy.

Third, switching from opacity to transparency increases the price dispersion since prices are all homogeneous under opacity ($\gamma_1 = 0$).\(^8\) The loss linked to the rise in dispersion depends on the precision of firms’ private information. High precision of firms’ private information reduces the cross section price dispersion. Hence, transparency is welfare improving when firms’ private information is rather precise (the derivative of the RHS of inequation (25) is positive with respect to firms noise $\sigma^2_{\epsilon}$).

Fourth, transparency is beneficial when strategic complementarities are strong (\(\xi\) small) because strong complementarities reduces the weight assigned to private signals and thereby the cross sectional price dispersion (the derivative of the RHS of inequation (25) is positive with respect to $\xi$).

It is worth underlining here that welfare effects of transparency fundamen-

\(^8\)This mechanism is similar to that of Kondor (2004). He shows that when the fundamental is split into two parts (as it is the case in this section) more information increases the disagreement between agents.
tally depends on whether the central bank tries to offset demand shocks with its monetary instrument or not. As discussed in section 3.4, when the central bank does not influence the nominal aggregate demand, transparency is welfare increasing (compared to opacity) when (i) the output gap stabilization is socially not very valuable ($\lambda$ small), (ii) the central bank’s information is quite accurate ($\sigma^2$ small), (iii) the firms’ private information is rather noisy ($\sigma^2_\varepsilon$ large), and (iv) strategic complementarities are strong ($\xi$ small). The conditions for transparency to be welfare improving in an economy where the central bank does not influence the nominal aggregate demand are simply the opposite to that derived in an economy where the central bank partially determines the nominal aggregate demand with its monetary instrument.

4.5 Optimal degree of transparency

In this section, we allow the central bank to disclose more or less equivocal information about its instrument and derive the optimal degree of transparency. The recent development of the US Federal Reserve disclosure about its monetary policy provides a good illustration of various degrees of transparency. Before 1994, the Federal Reserve did not publicly announce the federal funds rate it was targeting. The private sector had to observe the market operations implemented by the trading desk of the Fed to guess the policy decisions of the Federal Open Market Committee. This lack of transparency was a source of fundamental uncertainty about the rate targeted by the Fed and of strategic uncertainty about the beliefs of others about this target. Since February 1994, the Fed has been publishing the new target after each meeting of the FOMC. While such a publication reduces uncertainty about the numerical target, uncertainty still remains about how restrictive or expansive the Fed considers its policy decision to be. Hence, from 1998 on, the FOMC has decided to indicate after each meeting its current bias with respect to possible changes in the future policy. And even more recently, the FOMC has made the release of the minutes of its deliberations available to the public.\textsuperscript{9} This process clearly increases the degree of common knowledge about the impact of monetary policy on the aggregate nominal demand among firms. While the previous subsection has compared the welfare under both extreme cases of full transparency and opacity, we focus now on intermediate level of transparency and determine the optimal degree of transparency.

To determine the optimal degree of transparency $\sigma^2_{\phi^*}$, we set the first derivative

\textsuperscript{9}See Poole (2005).
of the loss (24) with respect to $\sigma^2_\phi$ equal to zero:

$$\frac{\partial E(L)}{\partial \sigma^2_\phi} = 2\gamma_1 \frac{\partial}{\partial \sigma^2_\phi} \sigma^2_\epsilon + \gamma_1^2 + 2\gamma_1 \frac{\partial}{\partial \sigma^2_\phi} \sigma^2_\eta - 2\lambda \frac{\partial}{\partial \sigma^2_\phi} \sigma^2_\eta + 2\lambda \gamma_1 \frac{\partial}{\partial \sigma^2_\phi} \sigma^2_\eta$$

$$= \frac{[(2\lambda - \xi)\sigma^2_\epsilon + (2\lambda - \xi)\sigma^2_\eta][\xi \sigma^4_\eta]}{(\sigma^2_\epsilon + \xi \sigma^2_\eta + \sigma^2_\phi)^3}$$

$$= 0 \iff \sigma^2_\phi = \frac{\xi^2}{\xi - 2\lambda} \sigma^2_\eta - \sigma^2_\epsilon. \quad (26)$$

To check whether extrema lead to minimum expected losses, the second derivative of the loss with respect to $\sigma^2_\phi$ yields

$$\frac{\partial^2 E(L)}{\partial (\sigma^2_\phi)^2} = \frac{2[(\xi - 2\lambda)\sigma^2_\epsilon + \xi(\lambda - 2\xi)\sigma^2_\eta + (\xi - 2\lambda)\sigma^2_\phi][\xi \sigma^4_\eta]}{(\sigma^2_\epsilon + \xi \sigma^2_\eta + \sigma^2_\phi)^4}$$

$$> 0 \iff (\xi - 2\lambda)\sigma^2_\phi > (2\lambda - \xi)\sigma^2_\epsilon + \xi(2\xi - \lambda)\sigma^2_\eta. \quad (27)$$

We show that limiting the degree of transparency is never optimal. Substituting equation (26) into (27), we observe that inequation (27) is satisfied when $\lambda > \xi$. That is to say that implementing the degree of transparency given by the RHS of (26) yields a minimum expected loss only if $\lambda > \xi$. But this condition implies that the RHS of (26) is negative. In other words, the extrema described as in equation (26) are maximum expected losses. As a result, the optimal disclosure strategy consists of choosing between full transparency and full opacity according to Proposition 3.

This yields the following proposition:

**Proposition 4:** When the central bank tries to offset demand shocks with its monetary instrument, partial transparency is never optimal.

In sharp contrast to the economy where the central bank does not stabilize the nominal aggregate demand, reducing the degree of transparency does not improve welfare when the central bank actively influences the nominal aggregate demand with its policy. As we bring it up in the next section, the framework where the central bank stabilizes the economy with its instrument calls for full transparency under realistic parameter conditions.

## 5 Discussion

This section compares the optimal information disclosure when the only task of the central bank is to disclose information with the case where it also stabilizes the economy. The optimal disclosure in both situations is a function of the
degree of strategic complementarities $1 - \xi$, the weight assigned to output gap variability $\lambda$, and the relative precision of firms’ private information $\frac{\sigma^2}{\sigma_0^2}$.

Figure 2 illustrates the optimal disclosure when firms’ private information is as precise as central bank’s information, i.e. $\frac{\sigma^2}{\sigma_0^2} = 1$. Figure 3 considers the more realistic case where firms’ private information is less accurate than central bank’s information, i.e. $\frac{\sigma^2}{\sigma_0^2} = 2$.

The optimal disclosure derived in section 3 where the unique central bank’s task is to disclose is as follows. The dotted line in both figures is given by $\lambda = (\frac{\sigma^2}{\sigma_0^2} + 3\xi)/2$ (see Proposition 2). As discussed in section 3.5, full transparency is optimal when $\lambda < (\frac{\sigma^2}{\sigma_0^2} + 3\xi)/2$ while partial transparency is optimal otherwise. The optimal disclosure is partial transparency for parameter combinations of $\lambda$ and $\xi$ given by the area called A in both figures. Full transparency
is optimal for parameter combinations in areas B and C. Partial transparency is beneficial when the degree of strategic complementarities $1 - \xi$ is high and when firms’ private information is relatively accurate. As shown above, opacity is never optimal.\textsuperscript{10} This arises because of the coordination motive: public information (or more information) allows private agents to better coordinate. The optimal degree of transparency (20) indicates that full opacity is optimal only in the extent that coordination does not matter at all at the social level. Withholding central bank’s information is optimal when there is no concern for coordination.

We now turn to the case described in section 4 where the central bank stabilizes the economy with its instrument. The dashed line is given by $\lambda = \frac{\xi \sigma^2}{2\sigma^2 + \xi \sigma^2}$ (see Proposition 3). As discussed in the previous section, full transparency is optimal for values of $\lambda$ larger than the dashed line. So, full transparency is optimal for parameter combinations in areas A and B, while opacity is optimal for area C. The framework of section 4 that accounts for the stabilization purpose of the central bank makes a case for full transparency in almost all parameter configurations unless price dispersion is assigned a much higher weight than output gap stabilization. There is no price dispersion when the central bank withholds its information since every firm sets a price of zero under opacity ($\gamma_1 = \gamma_2 = 0$). But opacity creates however higher cost in terms of output gap variability.

Yet, the case for opacity is extremely unlikely. For instance, when firms’ private information is as accurate as central bank’s information and $\xi = 0.25$, opacity would be optimal if the weight assigned to price dispersion would be more than 9 times higher than that assigned to output gap variability (from Proposition 3, we obtain $\lambda < \frac{\xi}{2\xi + \xi} = 0.11$). It is interesting to emphasize that when central bank’s information becomes less accurate (the relative precision $\sigma^2 / \xi$ decreases) transparency becomes beneficial for a larger range of parameter combinations. Since the central bank’s instrument is part of the fundamental firms respond to, an increase in central bank’s uncertainty makes transparency more beneficial.

We now briefly discuss the case of microfounded welfare function. As shown by Adam (2006), the microfounded welfare function is given by equation (10) with $\lambda = \frac{\xi}{\theta}$ where $\theta > 1$ is the degree of substitutability in the Dixit-Stiglitz
aggregator. Plugging the microfounded weight $\lambda$ into (20), we get

\[ \sigma^2_\phi^* = \max\{0, (2\xi_\theta^2 - 3\xi)\sigma^2_n - \sigma^2_n \} < 0 \]

When the central bank does not stabilize the economy with its instrument, full transparency is always optimal for the microfounded welfare function. This has been underlined by Hellwig (2005): when coordination is socially highly valuable transparency is welfare improving since it helps coordinating. By contrast, when the central bank tries to stabilize the nominal aggregate demand with its instrument, the microfounded welfare function can lead to full opacity. As Proposition 3 indicates, opacity is superior to transparency when coordination is socially highly valuable ($\lambda$ small). For large value of $\theta$, the weight $\lambda$ becomes arbitrarily small and may call for opacity (area C in figures 2 and 3).

6 Conclusion

Can a central bank speak too much? This question has been the subject of a very controversial literature over the last years. While transparency has been an important point of central banks’ agenda, the argument by Morris and Shin (2002) has received a great deal of attention because it seems to contradict the general presumption that transparency is always beneficial. According to their analysis, the disclosure of central bank’s noisy information can be welfare detrimental and destabilizing since it serves as a focal point in a context of strategic complementarities. The current paper contributes to this ongoing debate by highlighting the dual tasks of monetary policy: action and communication. While the literature in the vein of M-S considers the case where the sole task of the central bank is to provide the private sector with information, we also account for the action task of the central bank and draw opposite conclusions: when central bank’s information is poorly accurate, transparency reduces the distorting effect of the monetary instrument. This finding challenges the stabilizing role of public disclosure under imperfect information. Our analysis highlights the beneficial effects of transparency when the stabilization policy of the central bank is implemented on the base of imprecise information. Yet, in monetary policy, decisions under imperfect information are rather the rule than the exception. Indeed, since monetary policy affects the economy with a substantial delay, central banks must act in advance and take their decisions according to their forecasts. The Inflation Reports of the Bank of England provide a good example of the information accuracy a central
bank bases its decision on. As an inflation targeter, the Bank of England mainly conducts its policy in compliance with its expected inflation and output growth that are published in its *Inflation Report*. The uncertainty surrounding central bank’s forecasts is surprisingly high. As pointed out by Morris and Shin (2005) for the August 2005 Report, the “fan chart” for output growth looks rather like a “hammer” than a “fan”. Under these circumstances, the instrument set by the central bank may well be proved inadequate for the actual state of the economy.

Our analysis addresses the question of central bank’s communication when the conduct of monetary policy suffers from inaccurate information and shows that transparency helps reducing the distortion associated with poorly suited policies. This result supports the recent development in central banking towards more transparency with respect to policy implementations.
References


Kondor, P. (2004). The more we know, the less we agree: public announcements and higher-order expectations. *LSE FMG Discussion Paper*, 532.


Limited publicity versus limited transparency

As Morris and Shin (2002) show, firms overreact to the public signal because it is common knowledge among them. Consequently, limiting the degree of common knowledge reduces the overreaction and may improve welfare. Which disclosure strategies can reduce the degree of common knowledge? This appendix compares two strategies and shows that they are strictly equivalent for a large class of coordination games.

First, the central bank can reduce the degree of transparency by disclosing its information with idiosyncratic noise to each firm. This strategy has been proposed by Heinemann and Illing (2002) and is discussed in section 3. The disclosure received by firm $i$ is given by

$$I_i = I + \phi_i = g + \eta + \phi_i \quad \text{with } \phi_i \sim N(0, \sigma^2_\phi).$$

Each firm receives the central bank’s disclosure in private. This disclosure strategy captures the so-called mystique of central banks’ speech, i.e. the ambiguity surrounding the interpretation of central banks’ message. Indeed, central banks are known for speaking with some ambiguity that gives rise to fundamental and strategic uncertainty about the interpretation of their speeches.

Second, the central bank can reduce the degree of publicity by disclosing its information with perfect precision but not to all agents. This strategy has been proposed by Cornand and Heinemann (2005). In this set-up, a fraction $S$ of agents receives the semi-public signal $D = g + \eta$ in addition to its private signal $g_i = g + \varepsilon_i$ while the other fraction $1 - S$ only gets its private signal.

A.1 Information structure

We allow now the central bank to dispose of both disclosure strategies simultaneously. So, we have

- a fraction $S$ of firms who gets a private signal and a central bank’s disclosure
  - $g_i = g + \varepsilon_i$ \quad with $\varepsilon_i \sim N(0, \sigma^2_\varepsilon)$
  - $D_i = g + \eta + \phi_i$ \quad with $\eta \sim N(0, \sigma^2_\eta)$ and $\phi_i \sim N(0, \sigma^2_\phi)$.

$\sigma^2_\eta$ captures the degree of transparency of the central bank’s disclosure and drives the degree of common knowledge among the fraction $S$ of firms that gets the disclosure.

- a fraction $1 - S$ of firms who only get a private signal
  - $g_i = g + \varepsilon_i$. 

28
A.2 Equilibrium action

The average equilibrium action of the fraction $1 - S$ receiving only a private signal is given by

$$p_{1-S} = g$$

since private signals $g_i$ are centred on the true value $g$.

The average equilibrium action of the fraction $S$ receiving both a private signal and a central bank’s disclosure is given by

$$p_S = \gamma_1 g + \gamma_2 D = \frac{(1 - (1 - \xi)S)\sigma^2_{\varepsilon} + \sigma^2_\phi}{\sigma^2_\varepsilon + (1 - (1 - \xi)S)\sigma^2_{\varepsilon} + \sigma^2_\phi} g + \frac{\sigma^2_\varepsilon}{\sigma^2_\varepsilon + (1 - (1 - \xi)S)\sigma^2_{\varepsilon} + \sigma^2_\phi} D.$$

The overall average equilibrium action (over both fractions of firms with and without central bank’s disclosure) can be written as

$$p = \Gamma_1 g + \Gamma_2 D = S \cdot p_S + (1 - S) \cdot p_{1-S} = S(\gamma_1 g + \gamma_2 D) + (1 - S)g = (S\gamma_1 + 1 - S)g + S\gamma_2 D = \frac{(1 - S)\sigma^2_{\varepsilon} + (1 - (1 - \xi)S)\sigma^2_\phi + \sigma^2_\phi}{\sigma^2_\varepsilon + (1 - (1 - \xi)S)\sigma^2_{\varepsilon} + \sigma^2_\phi} g + \frac{S\sigma^2_\varepsilon}{\sigma^2_\varepsilon + (1 - (1 - \xi)S)\sigma^2_{\varepsilon} + \sigma^2_\phi} D.$$

A.3 Welfare

We consider the general form of social loss

$$L = \int_i (p_i - p)^2 di + \lambda(g - p)^2, \quad (28)$$

where $\lambda$ describes to what extent coordination is socially valuable. Using equilibrium actions of our set-up, we express the unconditional expected loss as

$$E(L) = E\left[ S \int_S (\gamma_1 g_i + \gamma_2 D_i - \Gamma_1 g - \Gamma_2 D)^2 di + (1 - S) \int_{(1-S)} (g_i - \Gamma_1 g - \Gamma_2 D)^2 di \right] + \lambda(g - \Gamma_1 g - \Gamma_2 D)^2$$

$$= S[\gamma^2_1 \sigma^2_{\varepsilon} + (1 - S)^2 \gamma^2_2 \sigma^2_\phi + \gamma^2_2 \sigma^2_\phi] + (1 - S)[\sigma^2_\varepsilon + \gamma^2_2 \sigma^2_\phi + \lambda \gamma^2_2 \sigma^2_\phi]$$

$$= S[\gamma^2_1 \sigma^2_{\varepsilon} + (1 - S + \lambda S)\gamma^2_2 \sigma^2_\phi + \gamma^2_2 \sigma^2_\phi] + (1 - S)\sigma^2_\varepsilon. \quad (29)$$

As discussed in section 3.3, the welfare in M-S given by $-\int_i (p_i - g)^2 di$ is a
particular case of (28) where $\lambda = 1$. The corresponding unconditional expected loss with full publicity (i.e. $S = 1$) and full transparency (i.e. $\sigma_\phi^2 = 0$) is

$$
E(L_{MS}) = \gamma_1^2 \sigma_\epsilon^2 + \gamma_2^2 \sigma_\eta^2,
$$

$$
= \frac{\sigma_\epsilon^2 \sigma_\eta^2 (\sigma_\epsilon^2 + \xi^2 \sigma_\eta^2)}{(\sigma_\epsilon^2 + \xi \sigma_\eta^2)^2}.
$$

A.4 Optimal transparency

A.4.1 Transparency versus opacity

We address the question whether full transparency ($\sigma_\phi^2 = 0$) is superior to full opacity ($\sigma_\phi^2 \rightarrow \infty$) in terms of welfare (29). It is straightforward to show that transparency is superior to opacity if

$$
\frac{S^2 \lambda - 2 + 3S - S^2 - 2S \xi}{2 - S} < \frac{\sigma_\epsilon^2}{\sigma_\eta^2}.
$$

(30)

In the particular case where the degree of publicity is maximal ($S=1$), we get condition (18) in the text.

A.4.2 Optimal degree of transparency

General case We derive the optimal degree of transparency $\sigma_\phi^2$. The degree of publicity $S$ is considered as given. The first derivative of the unconditional expected loss (29) with respect to $\sigma_\phi^2$ is

$$
\frac{\partial E(L)}{\partial \sigma_\phi^2} = \frac{(\sigma_\epsilon^2 + (1 - S - 2S \lambda + 3S) \sigma_\eta^2 + \sigma_\phi^2) S \sigma_\epsilon^4}{(\sigma_\epsilon^2 + (1 - S - 1 - \xi) \sigma_\eta^2 + \sigma_\phi^2)^3}
$$

$$
= 0 \quad \Leftrightarrow \quad \sigma_\phi^2 = (S - 1 + 2S \lambda - 3S \xi) \sigma_\eta^2 - \sigma_\epsilon^2.
$$

(31)

We ensure that extrema yield minimum losses. The second derivative of the expected loss with respect to $\sigma_\phi^2$ leads to

$$
\frac{\partial^2 E(L)}{\partial (\sigma_\phi^2)^2} = \frac{-2S \sigma_\epsilon^4 (\sigma_\epsilon^2 + (1 - S - 3S \lambda + 4S \xi) \sigma_\eta^2 + \sigma_\phi^2)}{(\sigma_\epsilon^2 + (1 - (1 - \xi) S \sigma_\eta^2 + \sigma_\phi^2)^4}
$$

$$
> 0 \quad \Leftrightarrow \quad \sigma_\phi^2 < (S - 1 + 3S \lambda - 4S \xi) \sigma_\eta^2 - \sigma_\epsilon^2.
$$

(32)

To show that reducing the degree of transparency according to (31) always leads to a minimum expected loss, we plug (31) into (32). The second derivative of the expected loss is then positive only if $\xi < \lambda$, which turns to be a necessary condition for the optimal variance $\sigma_\phi^2$ of (31) to be positive (the expression $(S - 1 + 2S \lambda - 3S \xi)$ is larger than zero only if $\xi < \lambda$). This means that when (31)
calls for increasing $\sigma^2$ (i.e. reducing transparency), the resulting expected loss is a minimum.

One can show that when the right hand side (RHS) of (31) is negative, condition (30) is always satisfied. So, full transparency is always superior to opacity when the (RHS) of equation (31) is negative.

For the sake of generality, the optimal degree of transparency is given by

$$\sigma^2 = \max[0, (S - 1 + 2S\lambda - 3S\xi)\sigma^2 - \sigma^2].$$  \hspace{1cm} (33)

Reducing the degree of transparency is optimal improving when the precision of central bank’s information $1/\sigma^2$ is low, when the weight $\lambda$ assigned to economic stabilization is large, when complementarities are strong ($\xi$ small), and when the degree of publicity is large.

$$\sigma^2 > 0 \Leftrightarrow S(1 + 2\lambda - 3\xi) > \frac{\sigma^2 + \sigma^2}{\sigma^2}. \hspace{1cm} (34)$$

**Full publicity** For the particular case of full publicity (i.e. $S = 1$) discussed in section 3 we have:

$$\frac{\partial E(L)}{\partial \sigma^2} = 0 \Leftrightarrow \sigma^2 = (2\lambda - 3\xi)\sigma^2 - \sigma^2 \hspace{1cm} (34)$$

$$\frac{\partial^2 E(L)}{\partial (\sigma^2)^2} > 0 \Leftrightarrow \sigma^2 < (3\lambda - 4\xi)\sigma^2 - \sigma^2 \hspace{1cm} (35)$$

To show that reducing the degree of transparency according to (34) always leads to a minimum expected loss, we plug (34) into (35). The second derivative of the expected loss is then positive only if $\xi < \lambda$, which turns to be a necessary condition for the optimal variance $\sigma^2$ of (34) to be positive (the expression $(2\lambda - 3\xi)$ is larger than zero only if $\xi < \lambda$). This means that when (34) calls for increasing $\sigma^2$ (i.e. reducing transparency), the resulting expected loss is a minimum.

We now check whether transparency is superior to opacity when the RHS of equation (34) is negative. We distinguish two cases. First, when $\xi < \lambda$, the condition $(2\lambda - 3\xi) < \frac{\sigma^2}{\sigma^2}$ (for negative RHS of (34)) implies $(\lambda - 2\xi) < \frac{\sigma^2}{\sigma^2}$, which calls for full transparency according to (18). Second, when $\xi > \lambda$, condition (18) is always satisfied and full transparency optimal. As a result, full transparency is always superior to opacity when the RHS of equation (34) is negative.

The optimal degree of transparency is given by

$$\sigma^2 = \max[0, (2\lambda - 3\xi)\sigma^2 - \sigma^2].$$  \hspace{1cm} (36)
This is equation (20) in the text.

### A.5 Optimal publicity

#### A.5.1 Full versus zero publicity

Again, we address the question whether full publicity ($S = 1$) is superior to zero publicity ($S = 0$). One can show that full publicity is superior to zero publicity in terms of welfare (29) if

$$(\lambda - 2\xi) < \frac{\sigma^2 + \sigma_\phi^2}{\sigma_\eta^2}. \quad (37)$$

In the particular case where the central bank’s disclosure is fully transparent ($\sigma_\phi^2 = 0$), the condition for full publicity is identical to the condition for full transparency (under full publicity) (18) in the text. In other words, the condition for full publicity under full transparency is identical to the condition for full transparency under full publicity.

#### A.5.2 Optimal degree of publicity

**General case** We derive the optimal degree of publicity $S^*$. The central bank seeks to determine the optimal degree of publicity for a given degree of transparency $\sigma_\phi^2$. The first and second derivatives of the unconditional expected loss (29) are given by

$$\frac{\partial E(L)}{\partial S} = 0 \quad \Leftrightarrow \quad S = \frac{\sigma^2 + \sigma_\phi^2}{(1 + 2\lambda - 3\xi)\sigma_\eta^2} \quad \text{(38)}$$

$$\frac{\partial^2 E(L)}{\partial S^2} > 0 \quad \Leftrightarrow \quad (\lambda - 1 + S + 2S\lambda - 2(2 + \lambda)S\xi + 3S\xi^2)\sigma_\eta^2 \
+ (\lambda - 1)(\sigma^2 + \sigma_\phi^2) > 0. \quad \text{(39)}$$

Substituting (38) into (39), we see that the extrema yield a minimum expected loss if and only if $\lambda > \xi$. This is however a necessary condition for the RHS of (38) to be positive.

For the case where the RHS of (38) is negative, we see that $\lambda + 2(\lambda - 3\xi) < 0$ implies $\lambda - 2\xi < 0$, which calls for zero publicity according to (37). For the case where the RHS of (38) is greater than 1, we rewrite it as $(2(\lambda - 3\xi) < \frac{\sigma^2 + \sigma_\phi^2}{\sigma_\eta^2}$ and see that it implies the condition for full publicity (37) when $\lambda > \xi$, which turns to be a necessary condition for the RHS of (38) to be greater than 1 (or even positive).
For the sake of generality, the optimal degree of publicity is given by

\[ S^* = \min[1, \max(0, \frac{\sigma_\varepsilon^2 + \sigma_\eta^2 + \sigma_\phi^2}{(1 + 2\lambda - 3\xi)\sigma_\eta^2})]. \]  

(40)

Reducing the degree of publicity is optimal when the precision of central bank’s information \(1/\sigma_\eta^2\) is low, when the weight assigned to stabilization \(\lambda\) is large, when complementarities are strong (\(\xi\) small), and when the degree of transparency is large (\(\sigma_\phi^2\) small).

\[ S^* < 1 \iff 2\lambda - 3\xi > \frac{\sigma_\varepsilon^2 + \sigma_\phi^2}{\sigma_\eta^2}. \]

**Full transparency**  When the central bank’s disclosure is common knowledge among receivers (\(\sigma_\phi^2 = 0\)), the condition for limiting publicity becomes

\[ S^* < 1 \iff 2\lambda - 3\xi > \frac{\sigma_\varepsilon^2}{\sigma_\eta^2}. \]

(41)

Note that the RHS of (38) is negative when

\[ S^* < 0 \iff 1 + 2\lambda - 3\xi < 0, \]

what must be foreclosed because it has no economic sense. Since Cornand and Heinemann (2005) consider the case where \(\lambda = 1\) (as in M-S), the RHS of (38) is never negative in their analysis.

### A.6 Welfare under optimal degree of publicity vs. transparency

We analyze the welfare (29) when the central bank implements the optimal degree of transparency (33) or the optimal degree of publicity (40). It turns out that the loss under both disclosure strategies is strictly identical and is given by

\[ \mathbb{E}(L^*) = \sigma_\varepsilon^2 + \frac{\sigma_\phi^4}{4\sigma_\eta^2(\xi - \lambda)}. \]

### A.7 Publicity-transparency equivalence

Since implementing a limited degree of publicity or a limited degree of transparency yields the same welfare, the central bank can indifferently implement one of both disclosure strategies to reduce the degree of common knowledge about its disclosure. The relation between the degree of publicity \(S\) and the
degree of transparency $\sigma^2_\phi$ is
\[
\sigma^2_\phi = \frac{1 - S}{S} (\sigma^2_\epsilon + \sigma^2_\eta) \quad \text{or} \quad S = \frac{\sigma^2_\epsilon + \sigma^2_\eta}{\sigma^2_\phi + \sigma^2_\eta + \sigma^2_\phi}.
\]
Interestingly, while the weight $\lambda$ assigned to economic stabilization in the welfare drives the optimal degree of publicity or transparency (optimal publicity or transparency are low when coordination is given a small weight at the social level), it does not challenge the publicity-transparency equivalence result.

**B Linear pricing rule: pure information disclosure**

This appendix solves the rational expectations equilibrium for the pricing rule of firms given by equation (15).

We first postulate that the optimal price of firm $i$ is a linear combination of its two signals
\[
p_i = \gamma_1 g_i + \gamma_2 D_i.
\]  
(42)

The optimal weights $\gamma_1$ and $\gamma_2$ depend on firms’ expectations about the pricing behaviour of other firms. The conditional estimate of the average price is therefore given by
\[
E_i(p) = \gamma_1 E_i(g) + \gamma_2 E_i(D).
\]  
(43)

Plugging $E_i(p)$ in the pricing rule (11) and replacing the expectations of firm $i$ about $g$ and $D$ yields
\[
p_i = (1 - \xi)[\gamma_1 E_i(g) + \gamma_2 E_i(D)] + \xi E_i(g)
\]
\[
= (1 - \xi)[\gamma_1(\Omega_{11} g_i + \Omega_{12} D_i) + \gamma_2(\Omega_{21} g_i + \Omega_{22} D_i)] + \xi(\Omega_{11} g_i + \Omega_{12} D_i).
\]

Rearranging gives
\[
p_i = g_i[(1 - \xi)(\Omega_{11} \gamma_1 + \Omega_{21} \gamma_2) + \xi \Omega_{11}]
\]
\[
+ D_i[(1 - \xi)(\Omega_{12} \gamma_1 + \Omega_{22} \gamma_2) + \xi \Omega_{12}].
\]

Identifying the coefficients, we get
\[
\gamma_1 = \frac{(1 - \xi)\Omega_{21} \gamma_2 + \xi \Omega_{11}}{1 - (1 - \xi)\Omega_{11}},
\]
\[
\gamma_2 = \frac{(1 - \xi)\Omega_{12} \gamma_1 + \xi \Omega_{12}}{1 - (1 - \xi)\Omega_{22}}.
\]
And solving this system of equations yields
\[
\gamma_1 = \frac{\xi \Omega_{11} + (1 - \xi) \Omega_{21}}{1 - (1 - \xi)(\Omega_{11} - \Omega_{21})} = \frac{\xi \sigma_y^2 + \sigma_\phi^2}{\sigma_z^2 + \xi \sigma_y^2 + \sigma_\phi^2},
\]
\[
\gamma_2 = \frac{\xi \Omega_{12} + (1 - \xi) \Omega_{12} \Omega_{21}}{\xi - (1 - \xi)[\xi \Omega_{11} - (1 + \xi) \Omega_{21} - (1 - \xi)(\Omega_{21} - \Omega_{11}) \Omega_{11}]} = \frac{\sigma_y^2}{\sigma_z^2 + \xi \sigma_y^2 + \sigma_\phi^2}.
\]
This solution is equivalent to equations (16) in the text.

C Linear pricing rule: Action and information disclosure

This appendix solves the rational expectations equilibrium for the pricing rule of firms given by equation (21).

We first postulate that the optimal price of firm \(i\) is a linear combination of its two signals
\[
p_i = \gamma_1 g_i + \gamma_2 I_i. \tag{44}
\]
The optimal weights \(\gamma_1\) and \(\gamma_2\) depend on firms’ expectations about the pricing behaviour of other firms. The conditional estimate of the average price is therefore given by
\[
E_i(p) = \gamma_1 E_i(g) + \gamma_2 E_i(I). \tag{45}
\]
Plugging \(E_i(p)\) in the pricing rule (21) and replacing the expectations of firm \(i\) about \(g\) and \(I\) yields
\[
p_i = (1 - \xi)[\gamma_1 E_i(g) + \gamma_2 E_i(I)] + \xi E_i(g) + \xi E_i(I)
= (1 - \xi)[\gamma_1 (\Theta_{11} g_i + \Theta_{12} I_i) + \gamma_2 (\Theta_{21} g_i + \Theta_{22} I_i)]
+ \xi (\Theta_{11} g_i + \Theta_{12} I_i) + \xi (\Theta_{21} g_i + \Theta_{22} I_i).
\]
Rearranging gives
\[
p_i = g_i[(1 - \xi)(\Theta_{11}\gamma_1 + \Theta_{21}\gamma_2) + \xi(\Theta_{11} + \Theta_{21})]
+ I_i[(1 - \xi)(\Theta_{12}\gamma_1 + \Theta_{22}\gamma_2) + \xi(\Theta_{12} + \Theta_{22})].
\]
Identifying the coefficients, we get
\[
\gamma_1 = \frac{(1 - \xi)\Theta_{21}\gamma_2 + \xi(\Theta_{11} + \Theta_{21})}{1 - (1 - \xi)\Theta_{11}},
\]
\[
\gamma_2 = \frac{(1 - \xi)\Theta_{12}\gamma_1 + \xi(\Theta_{12} + \Theta_{22})}{1 - (1 - \xi)\Theta_{22}}.
\]
And solving this system of equations yields

\[
\gamma_1 = \frac{\xi (\Theta_{11} + \Theta_{21})}{1 - (1 - \xi)(\Theta_{11} + \Theta_{21})} = \frac{\xi \sigma_q^2}{\sigma^2 + \xi \sigma_q^2 + \sigma_\phi^2}
\]

\[
\gamma_2 = \frac{\xi \sigma_q^2}{\sigma^2 + \xi \sigma_q^2 + \sigma_\phi^2}.
\]

This solution is equivalent to equation (23) in the text.