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# A Theory of Ex Post Inefficient Renegotiation

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# A Theory of Ex Post Inefficient Renegotiation[†]

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Abstract: We propose a theory of ex post inefficient renegotiation that is based on loss aversion. When two parties write a long-term contract that has to be renegotiated after the realization of the state of the world, they take the initial contract as a reference point to which they compare gains and losses of the renegotiated transaction. We show that loss aversion makes the renegotiated outcome sticky and materially inefficient. The theory has important implications for the optimal design of long-term contracts. First, it explains why parties often abstain from writing a beneficial long-term contract or why some contracts specify transactions that are never ex post efficient. Second, it shows under what conditions parties should rely on the allocation of ownership rights to protect relationship-specific investments rather than writing a specific performance contract. Third, it shows that employment contracts can be strictly optimal even if parties are free to renegotiate.

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# 1 Introduction

Renegotiation plays a crucial role in the theory of incomplete contracts. This theory, going back to Grossman and Hart (1986) and Hart and Moore (1990), starts out from the observation that long-term contracts have to be written before the contracting parties know the realization of the state of the world that is relevant for the specifics of their trading relationship. Writing a complete, state-contingent contract is assumed to be impossible, so the parties have to rely on renegotiation to adapt the contract to the realization of the state of the world. The standard paradigm assumes that renegotiation is always ex post efficient. Once the parties observe the state of the world they will engage in Coasian bargaining and reach an efficient agreement on how to adapt the contract. An important implication of this approach is that all the inefficiencies of long-term contracting must arise ex ante, e.g., because of distorted investment incentives.

More recently, Hart and Moore (2008) and Hart (2009) have put this approach into question. They argue that the traditional approach is ill suited to studying the internal organization of firms. If ex post renegotiation is always efficient "it is hard to see why authority, hierarchy, delegation, or indeed anything apart from asset ownership matters" (Hart and Moore, 2008, p. 3). Coase (1937) and Williamson (1985) argued long ago that the organization of transactions within firms and by markets can be understood only if we understand the inefficiencies of adapting contracts to changes of their environment, i.e., the inefficiencies of renegotiation.

In this paper we propose a theory of ex post inefficient renegotiation. Our theory is based on loss aversion, a fundamental concept in behavioral economics and psychology (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991). There is ample experimental and field evidence showing that people evaluate outcomes not (only) in absolute terms but (also) relative to a reference point, and that losses (in comparison to this reference point) loom larger than gains of equal size. In a contracting environment it is natural to assume that the contract to which the parties agreed ex ante defines the reference point in the renegotiation game, because the initial contract determines the parties' payoffs when renegotiations break down. Suppose a buyer and seller agreed ex ante to trade some specification $\bar{x}$ of a good at price $\bar{p}$. After the realization of the state of the world they realize that it would be efficient to adjust the specification of the good. However, the buyer feels a loss if the renegotiated price $p$ is greater than the initially agreed payment $\bar{p}$. Similarly, the seller feels a loss if her cost to produce the new specification $x$ is larger than her cost to produce the initially agreed specification $\bar{x}$. These losses loom larger than equally sized gains of consuming a better quality for the buyer and receiving a larger payment for the seller. A crucial feature of our model is that monetary losses due to a difference between the renegotiated price $p$ and the price $\bar{p}$ are evaluated separately from losses due to a higher cost or a lower valuation of $x$ as compared to $\bar{x}$. This assumption is common in the literature on reference points (Kőszegi and Rabin, 2006, 2007).

We posit that the initial long-term contract shapes a salient reference point. The parties compare the renegotiated outcome to the outcome prescribed by the initial contract. Moreover, we assume for simplicity that loss aversion is linear, i.e., a loss of size $L$ reduces utility by $(1+\lambda)L$. The factor $\lambda > 0$ drives a wedge between the benefit of the buyer and the cost of the seller. This renders the renegotiation outcome materially inefficient, i.e., it does not maximize the material surplus (net of loss aversion) of the two parties. Furthermore, the kink in the utility function at the reference point may prevent renegotiation altogether. We show that if the realization of the state of the world is not too far from the "expected" state of the world on which the initial contract $(\bar{x}, \bar{p})$ was based, then the parties will not renegotiate at all but leave the initial contract unchanged. If the realized state of the world is sufficiently far away from the expected state, the parties will renegotiate, but they will insufficiently adjust the terms of trade. Thus, loss aversion makes the renegotiation outcome sticky and materially inefficient. This effect is reminiscent of the assumption of "sticky prices" in macroeconomics. While the macroeconomic literature attributes price stickiness to exogenously given menu costs, sticky prices can arise endogenously in our model.[1]

In this paper we restrict attention to *contracts* as reference points. Parties may have additional reference points to which they compare the renegotiation outcome. For example, they may compare this outcome to their rational expectations as in Kőszegi and Rabin (2006) or to the status quo as in Tversky and Kahneman (1991). We focus on the losses that are triggered by the comparison to the initial contract and do not analyze the effects of loss aversion on renegotiation in general. Put differently, we posit that the comparison to the

[1]That "sticky prices" can be explained by loss aversion is also shown for models with price setting firms by Heidhues and Kőszegi (2005, 2008).

initial contract is salient and therefore likely to have an important effect, but we are agnostic about the possibility that there are additional effects as well.[2]

The friction due to loss aversion is quite different from other bargaining frictions, such as asymmetric information, the risk of bargaining breakdown or other transaction costs. The difference is that loss aversion arises *only* because of the initial contract. Without an initial contract, the parties do not experience a loss. In contrast, if the parties are asymmetrically informed about the realization of the state of the world, this information asymmetry will be there no matter whether there is an ex ante contract or not. If anything, the initial contract can be used to mitigate the informational problem by setting up a sophisticated mechanism that induces the parties to reveal their private information truthfully. Thus, with asymmetric information the initial contract can only reduce the cost of contracting, but it can never be harmful, while with loss aversion there is a cost of writing the initial contract that arises endogenously.[3]

Our theory of renegotiation has several interesting and important implications for contract theory. If the parties understand that a contract sets a reference points that triggers potentially unfavorable comparisons and that gives rise to disutility from loss aversion and to materially inefficient renegotiation outcomes, then they have an incentive to design contracts so as to minimize these frictions. A first implication of our model is that it may be optimal not to write any long-term contract ex ante, even if there is some benefit to it (such as supply assurance, insurance, or the protection of relationship-specific investments). The reason is that without an initial contract there is no reference point to which gains and losses are compared, so loss aversion is not an issue. Furthermore, if the parties do write a long-term contract, it can be optimal to contract on a specification of the good that is never materially efficient ex post, but that minimizes the cost to renegotiate away from it.

Second, the theory offers a fresh view on the hold-up problem and the property rights theory. It shows under what circumstances the parties should rely on the allocation of asset ownership to protect their relationship-specific investments, and when they should rather write

[2]The assumption that contracts form a strong reference point is in line with experimental evidence obtained by Bartling and Schmidt (2012) as is explained in more detail below.

[3]The same argument applies to the risk of bargaining breakdown and other transaction costs. If anything, the risk of bargaining breakdown or the transaction costs of renegotiation are reduced by the initial contract.

a long-term specific performance contract. We show that these two instruments are mutually exclusive. A long-term specific performance contract should be used to protect relationship-specific investments if there is little uncertainty and if the cost of renegotiation due to loss aversion is small. The parties should rely on the allocation of ownership rights instead if there is a lot of uncertainty and if the investment is not too relationship-specific.

Third, our theory offers a rationale for the existence of "employment" contracts. According to Coase (1937) and Simon (1951) a key feature of an employment contract is that it fixes the price (the wage) and gives the buyer (the employer) authority to order the seller (the employee) which specification of the good (the service) to deliver. Simon (1951) compares an employment contract to a specific performance contract which fixes the specific service to be delivered. The advantage of the employment contract is that it is flexible, but it has the drawback that it leaves room for abuse. Which type of contract is optimal depends on whether the expected cost of rigidity or of abuse is more important. However, Simon's argument is incomplete. If costless renegotiation is possible the parties will always reach the efficient outcome and the difference between the two contracts disappears. If the parties suffer from loss aversion, on the other hand, then renegotiation is painful and therefore costly. The cost of renegotiation differs between a specific performance and an employment contract. Our model allows for renegotiation and confirms and extends Simon's original insight. It confirms that an employment contract strictly outperforms a specific performance contract if the scope for inefficient abuse is small as compared to the cost of rigidity. In addition, it shows that the specific performance contract outperforms the employment contract if there is little uncertainty in the sense that the costs and benefits of the ex post efficient specification $\hat{x}(\theta)$ do not differ too much from the costs and benefits of the specification $\bar{x}$ in the initial contract.

There is some recent experimental evidence that is consistent with our theory. Bartling and Schmidt (2012) conduct a laboratory experiment on (re)negotiation. They compare a situation in which a buyer and a seller renegotiate an initial contract to a situation in which they negotiate in the absence of an initial contract. In all other respects the two situations are completely identical. They find that with an initial contract prices are sticky and react much less to the realization of the state of the world as in the situation without an initial contract. This is exactly what our theory predicts for this experiment. Moreover, the experiment shows that the existence of the initial contract is causal for the stickiness of prices because the material and strategic situation is exactly the same in both treatments.

Our paper is closely related and complementary to Hart and Moore (2008) who were the first to point out that contracts may serve as reference points. They assume that a contract determines parties' feelings of entitlement if the contract was written under competitive or fair conditions. The parties do not feel entitled to outcomes that are outside the contract, but each party feels entitled to the best possible outcome that is consistent with the contract. Thus, when interpreting the contract parties have mutually inconsistent expectations with a self-serving bias. When a party does not get what it feels entitled to, it feels aggrieved and shades in non-contractible ways. Shading reduces the payoff of the other party, but is costless for the shader, i.e., it is a form of costless punishment. Under these assumptions the benefit of flexibility of a contract is that it can be better adjusted to the realization of the state of the world, but the cost is that it leads to aggrievement and shading, so there is an optimal degree of flexibility. Hart (2009), Hart and Holmstrom (2010) and Hart (2011) use this approach to develop theories of asset ownership and firm boundaries.[4] There are several important differences between the Hart-Moore approach and our approach. First, in Hart and Moore the ex post inefficiency is due to self-serving biases while our approach is based on loss aversion. Second, Hart and Moore's approach is well suited to discuss the costs and benefits of the flexibility of a contract, but it is less well suited to deal with the costs and benefits of renegotiation. Third, Hart and Moore consider "at will" contracts, i.e., each party can walk away from the contract without any penalties, while we look at specific performance contracts that are enforced by the courts if at least one party insists on it. Thus, the two approaches are quite different, but they address similar questions and complement each other.

The rest of the paper is organized as follows. The next section sets up the model. In Section 3 we take the initial contract as given and characterize the renegotiation outcome after the state of the world has materialized. We show that ex post it may be impossible to achieve the materially efficient allocation through voluntary renegotiation. Even if the materially efficient outcome is in the renegotiation set, the parties will typically not achieve

[4]Fehr, Zehnder, and Hart (2009); Fehr, Hart, and Zehnder (2011a,b) run several experiments on the Hart-Moore model. They find support for the hypothesis that people shade more when the contract is more flexible if the contract was written under competitive conditions, but not if one party had monopoly power and could dictate the terms of the contract.

it. The inefficiency of renegotiation makes writing the initial contract costly, and this cost is increasing in the degree of loss aversion and in the uncertainty of the environment. In Section 4 we look at optimal initial contracts. First, we show that it can be optimal not to write a long-term contract at all, even if this long-term contract comes with a benefit. Furthermore, we show that it may be optimal to specify a good in the initial contract that is never the materially efficient good ex post, but that minimizes the cost of renegotiation. Then we consider a hold-up problem where one party has to make a relationship specific investment and discuss under what circumstances the parties should rely either on a long-term contract or on the allocation of ownership rights to protect the relationship specific investment. Moreover, we compare an employment contract to a specific performance contract and show under which conditions which contract is optimal. Finally, we discuss the potential benefits of indexation and show that price indexation alone does not enhance ex post efficiency. All proofs of propositions, lemmas, and corollaries that are not outlined in the main text are relegated to the Appendix.

## 2 The Model

We consider two risk-neutral parties, a buyer B (he) and a seller S (she), who are engaged in a long-term relationship. The two parties can write a contract at date 0 that governs trade at date 3. The seller can deliver different specifications of a good $x \in X$, where $X$ is a compact space, that can differ in multiple dimensions (quantity, quality, time and location of delivery, etc.). The buyer's valuation $v = v(x, \theta)$ and the seller's cost $c = c(x, \theta)$ depend on the specification $x$ of the good and on the realization of the state of world $\theta \in \Theta$. The exact shapes of the cost and valuation functions become commonly known at date 1, when the state of the world $\theta \in \Theta$ is realized. The state $\theta$ reflects exogenous uncertainty that is relevant for the optimal specification of the good to be traded. We assume that there is a unique materially efficient specification $x^*(\theta) \in X$ for each possible state of the world,

$$x^*(\theta) \quad = \quad \arg\max_{x \in X} \{v(x, \theta) - c(x, \theta)\} \tag{1}$$

that maximizes the material gains from trade.

At date 0, i.e. at the contracting stage, the two parties do not know the realization of the state of the world $\theta$, which is drawn from a compact space $\Theta$ according to a commonly known cumulative distribution distribution function $F(\theta)$. At date 1, i.e. before trade takes place, the state of the world is realized and observed by both parties. We assume that the realized state cannot be verified by a court or another third party. A court can verify only payments and which if any of the goods $x \in X$ is delivered. Thus, in this setting a contract cannot specify state contingent specifications and prices.

The sequence of events is as follows:

$t = 0$ *Initial Contracting*: The buyer and the seller negotiate the initial contract $(\bar{x}, \bar{p})$.

$t = 1$ *Realization of the State of the World*: Nature draws $\theta$ which is observed by $B$ and $S$. The contract in combination with the realized state determines the default options for both parties, $\underline{U}^B = v(\bar{x}, \theta) - \bar{p}$ and $\underline{U}^S = \bar{p} - c(\bar{x}, \theta)$.

$t = 2$ *Renegotiation*: The buyer and the seller can renegotiate the initial contract to a new contract $(\hat{x}, \hat{p})$ that must be feasible and individually rational for both parties. If the parties do not agree upon a new contract, then the initial contract $(\bar{x}, \bar{p})$ remains in place.

$t = 3$ *Trade*: Trade is carried out according to the (renegotiated) contract.

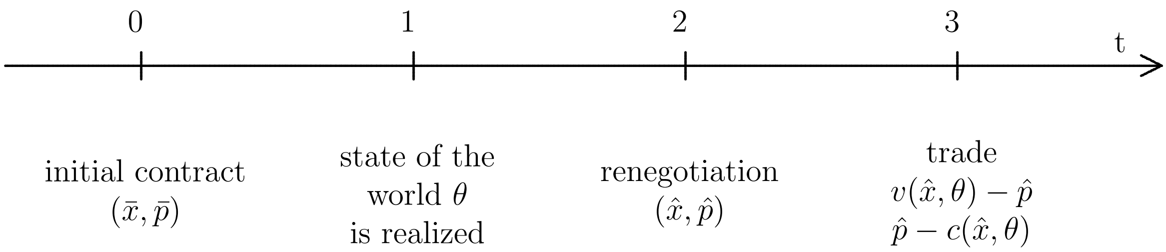


Figure 1: Time structure

Note that the initial contract $(\bar{x}, \bar{p})$ is a "specific performance contract" that can be enforced by each party. Other forms of initial contracts are discussed in Section 4.

So far our model of renegotiation is completely standard. We now depart from the existing literature by assuming that the initial contract creates a reference point that determines how the parties evaluate the new contract. The parties compare the new contract $(\hat{x}, \hat{p})$ to what they would have received under the old contract in the realized state $\theta$.[5] This evaluation is distorted from standard neoclassical preferences by *loss aversion*: The buyer feels a loss if the renegotiated price $\hat{p}$ is greater than the initially agreed price $\bar{p}$. Furthermore, he also feels a loss if his valuation for the renegotiated good $\hat{x}$ is smaller than his valuation for the good $\bar{x}$ given the realized state of nature. Similarly, the seller feels a loss if the renegotiated price $\hat{p}$ falls short of the initially agreed price $\bar{p}$ and if her cost for the renegotiated good $\hat{x}$ is greater than her cost for the good $\bar{x}$ in the realized state $\theta$. Put differently, we posit that the default option—determined by the initial contract and the realized state of nature—shapes a reference point for the two parties.[6]

Thus, the utility functions of the two parties at the renegotiation stage are given by

$$U^B = v(\hat{x},\theta) - \hat{p} - \lambda\,[\hat{p} - \bar{p}]^+ - \lambda\,[v(\bar{x},\theta) - v(\hat{x},\theta)]^+ \tag{2}$$

$$U^S = \hat{p} - c(\hat{x},\theta) - \lambda\,[\bar{p} - \hat{p}]^+ - \lambda\,[c(\hat{x},\theta) - c(\bar{x},\theta)]^+ \tag{3}$$

with $\lambda > 0$ and $[z]^+ \equiv \max\{z, 0\}$.

Loss aversion is a behavioral regularity that is well documented in the experimental and psychological literature. Note that monetary losses due to a difference between $\hat{p}$ and $\bar{p}$ are evaluated separately from losses due to a lower valuation or a higher cost. This assumption is imposed in most of the literature on reference points (Kőszegi and Rabin, 2006, 2007) and is crucial for reference dependence to capture many well-documented deviations from standard theory.[7]

[5] This reference point formation can be interpreted as a forward looking status quo. The reference point depends on the contract and is adjusted to the state of nature. The reference point is what the parties get in case renegotiation fails. Note, however, that the reference point is not completely forward looking, because it does not depend on whether the parties expect renegotiation to take place.

[6] If the initial contract is not a specific performance contract but rather a menu of prices and services from which one party can choose ex post, then the reference point is the payoff that results if renegotiation do not take place—the service and the price preferred by the party that is allowed to select from the menu for the realized state of nature.

[7] The separability of losses across dimensions is needed, for instance, to capture the endowment effect. See Kahneman, Knetsch, and Thaler (1990).

# 3 Renegotiation

In this section we take the initial contract $(\bar{x}, \bar{p})$ as exogenously given and analyze the renegotiation game at date 2. We first characterize the renegotiation set, i.e., the set of specifications $\hat{x}$ that are feasible and individually rational given the initial contract $(\bar{x}, \bar{p})$. Then, we impose some structure on how the parties renegotiate the initial contract and characterize the renegotiation outcome. We will show that the renegotiation outcome is sticky and materially inefficient, i.e., often the specification is not adjusted to the state of nature and if it is adjusted it often does not react strongly enough to changes in the state of nature. Put differently, typically the parties do not agree ex post on trading the materially efficient $x^*(\theta)$ which maximizes $v(\cdot) - c(\cdot)$. Finally, we characterize the likelihood and the cost of renegotiation.

## 3.1 The Renegotiation Set

Suppose that an initial contract $(\bar{x}, \bar{p})$ is in place and that the state of the world $\theta$ has materialized. Thus, if the initial contract is not renegotiated the parties will trade $\bar{x}$ at price $\bar{p}$ which yields the outside option utilities $\underline{U}^B = v(\bar{x}, \theta) - \bar{p}$ and $\underline{U}^S = \bar{p} - c(\bar{x}, \theta)$.

The buyer prefers a new contract $(\hat{x}, \hat{p})$ to the initial contract if and only if his utility under the new contract is greater than his utility from the initial contract. This is the case if and only if

$$\begin{aligned} v(\hat{x}, \theta) - \hat{p} - \lambda[v(\bar{x}, \theta) - v(\hat{x}, \theta)]^+ - \lambda[\hat{p} - \bar{p}]^+ &\geq v(\bar{x}, \theta) - \bar{p} \\ \iff \quad v(\hat{x}, \theta) - v(\bar{x}, \theta) - \lambda[v(\bar{x}, \theta) - v(\hat{x}, \theta)]^+ &\geq \hat{p} - \bar{p} + \lambda[\hat{p} - \bar{p}]^+ . \end{aligned} \tag{4}$$

The seller prefers the new contract $(\hat{x}, \hat{p})$ to the original contract if and only if

$$\begin{aligned} \hat{p} - c(\hat{x}, \theta) - \lambda[c(\hat{x}, \theta) - c(\bar{x}, \theta)]^+ - \lambda[\bar{p} - \hat{p}]^+ &\geq \bar{p} - c(\bar{x}, \theta) \\ \iff \quad c(\hat{x}, \theta) - c(\bar{x}, \theta) + \lambda[c(\hat{x}, \theta) - c(\bar{x}, \theta)]^+ &\leq \hat{p} - \bar{p} - \lambda[\bar{p} - \hat{p}]^+ . \end{aligned} \tag{5}$$

Contracts $(\hat{x}, \hat{p})$ satisfying (4) and (5) are called individually rational. The *renegotiation set* is the set of goods $\hat{x}$ to which the parties could voluntarily renegotiate to, i.e., the set of $\hat{x} \in X$ for which there exists a price $\hat{p}$ such that $(\hat{x}, \hat{p})$ is individually rational for both parties.

We have to distinguish whether $\hat{x}$ leads to higher or lower benefits for the buyer and higher or lower costs for the seller as compared to $\bar{x}$. Obviously, if $\hat{x}$ leads to higher costs and lower benefits than $\bar{x}$, then there does not exist any price $\hat{p}$ such that $(\hat{x}, \hat{p})$ is preferred by both parties to $(\bar{x}, \bar{p})$. This leaves us with the following three cases:

(i) $v(\hat{x}, \theta) \geq v(\bar{x}, \theta)$ and $c(\hat{x}, \theta) \leq c(\bar{x}, \theta)$. In this case the buyer's valuation of $\hat{x}$ is (weakly) greater than his valuation of $\bar{x}$ while the seller's cost of producing $\hat{x}$ is (weakly) lower than producing $\bar{x}$. Clearly, there always exists a set of prices $\hat{p}$ such that both parties prefer $(\hat{x}, \hat{p})$ to $(\bar{x}, \bar{p})$. In particular $\hat{p} = \bar{p}$ is an element of this set. Thus, these $\hat{x}$ always belong to the renegotiation set.

(ii) $v(\hat{x}, \theta) > v(\bar{x}, \theta)$ and $c(\hat{x}, \theta) \geq c(\bar{x}, \theta)$. In this case moving from $\bar{x}$ to $\hat{x}$ increases the buyers valuation, but it also (weakly) increases the seller's cost. The buyer is willing to accept an increase in price if and only if

$$\hat{p} \leq \bar{p} + \frac{v(\hat{x}, \theta) - v(\bar{x}, \theta)}{1 + \lambda}. \tag{6}$$

The seller is willing to incur the higher production cost if and only if she is compensated by a higher price $\hat{p}$ where

$$\hat{p} \geq \bar{p} + (1 + \lambda)[c(\hat{x}, \theta) - c(\bar{x}, \theta)]. \tag{7}$$

Combining the two inequalities above reveals that there exists a price $p > \bar{p}$ for $\hat{x}$ that is acceptable to both parties if and only if

$$v(\hat{x}, \theta) - v(\bar{x}, \theta) \geq (1 + \lambda)^2 [c(\hat{x}, \theta) - c(\bar{x}, \theta)]. \tag{8}$$

(iii) $v(\hat{x}, \theta) \leq v(\bar{x}, \theta)$ and $c(\hat{x}, \theta) < c(\bar{x}, \theta)$, i.e., the buyer's valuation for $\hat{x}$ is (weakly) smaller than his valuation of $\bar{x}$, but the seller's cost is also smaller. The buyer prefers the new contract $(\hat{x}, \hat{p})$ to the initial contract if and only if

$$\hat{p} \leq \bar{p} - (1 + \lambda)[v(\bar{x}, \theta) - v(\hat{x}, \theta)]. \tag{9}$$

The seller is prepared to accept a lower price because she incurs lower costs of production. She prefers the new contract $(\hat{x}, \hat{p})$ to the initial contract if and only if

$$\hat{p} \geq \bar{p} - \frac{c(\bar{x}, \theta) - c(\hat{x}, \theta)}{1 + \lambda}. \tag{10}$$

Combining (9) and (10) shows that there exists a price $\hat{p} < \bar{p}$ for $\hat{x}$ that is acceptable to both parties if and only if

$$c(\bar{x},\theta) - c(\hat{x},\theta) \geq (1+\lambda)^2[v(\bar{x},\theta) - v(\hat{x},\theta)]. \tag{11}$$

The following proposition summarizes these results.

**Proposition 1.** *Consider an initial contract $(\bar{x}, \bar{p})$ and suppose that state $\theta \in \Theta$ is realized. The renegotiation set, i.e. the set of all $\hat{x} \in X$ to which the parties may voluntarily renegotiate to, is characterized as follows:*

*(i) If $\hat{x} \in X$ yields (weakly) higher benefits for the buyer and (weakly) lower costs for the seller as compared to $\bar{x}$, then it can always be reached by renegotiation.*

*(ii) If $\hat{x} \in X$ yields higher benefits for the buyer but is more costly to produce for the seller as compared to $\bar{x}$, then it can be reached by renegotiation if and only if*

$$v(\hat{x},\theta) - v(\bar{x},\theta) \geq (1+\lambda)^2[c(\hat{x},\theta) - c(\bar{x},\theta)]. \tag{12}$$

*(iii) If $\hat{x} \in X$ is less costly to produce for the seller but also less beneficial to the buyer as compared to $\bar{x}$, then it can be reached by renegotiation if and only if*

$$c(\bar{x},\theta) - c(\hat{x},\theta) \geq (1+\lambda)^2[v(\bar{x},\theta) - v(\hat{x},\theta)]. \tag{13}$$

Obviously, if both parties are loss neutral ($\lambda = 0$) the efficient good can always be reached by renegotiation. Moreover, independent of the parties' degree of loss aversion, goods that are unambiguously preferred can always be reached by renegotiation. The interesting cases arise when there is a tradeoff, i.e., either the buyer or the seller suffers if the new good is implemented and the price is not adjusted. For instance, the parties will agree to a new specification $\hat{x}$ that benefits the buyer only if the increase in valuation of the buyer exceeds the increase in cost of the seller by at least the factor $(1+\lambda)^2$. Similarly, they will agree to a new specification $\hat{x}$ that reduces the cost of the seller only if the seller's cost reduction exceeds the reduction of the buyer's valuation again by at least the factor $(1+\lambda)^2$. Thus, the renegotiation set becomes smaller when $\lambda$ increases. The renegotiation set for $\lambda = 0$ and $\lambda > 0$ is depicted in Figure 2. If the parties are not loss averse, all goods that are north-east of the straight line can be reached by renegotiation. If the parties are loss averse, only goods that are located north-east of the

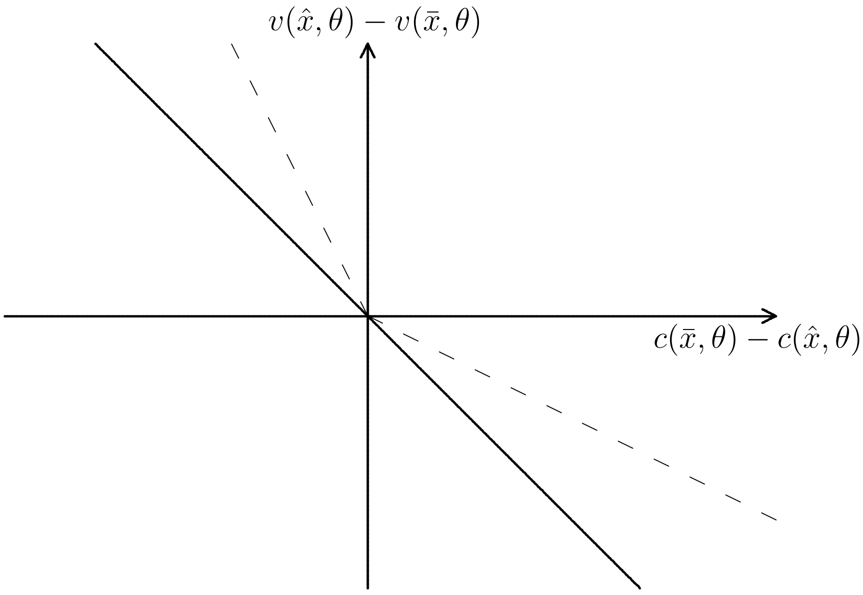


Figure 2: The renegotiation set

dotted lines can be reached by renegotiation. In the extreme case when $\lambda \to \infty$, i.e. when both parties are not willing to accept a good that leads to lower benefits or higher costs, only goods in the positive orthant can be implemented by renegotiation. Note finally, that even if $x^*(\theta)$ is not in the renegotiation set, it is possible that some other $x \neq \bar{x}$ can be reached by renegotiation.

## 3.2 The Renegotiation Outcome

So far we characterized the set of renegotiation outcomes that are feasible and individually rational. In order to characterize the renegotiation outcome that will actually obtain we employ the Generalized Nash Bargaining Solution (GNBS). The GNBS is the only bargaining solution that is Pareto efficient, invariant to equivalent utility representations and independent of irrelevant alternatives. Furthermore, it reflects the relative bargaining power of the two parties.[8] The GNBS is the contract $(\hat{x}(\theta), \hat{p}(\theta))$ that maximizes the Generalized Nash Product

[8]See Roth (1979) for a discussion of the Generalized Nash Bargaining Solution and other axiomatic models of bargaining. Binmore, Rubinstein, and Wolinsky (1986) derive the GNBS as a non-cooperative equilibrium of an alternating offer game between one seller and one buyer.

(GNP), i.e.,

$$(\hat{x}(\theta), \hat{p}(\theta)) \equiv \arg\max_{x,p} \left\{ \left(U^B(x,p|\theta) - \underline{U}^B\right)^\alpha \cdot \left(U^S(x,p|\theta) - \underline{U}^S\right)^{1-\alpha} \right\}, \tag{14}$$

where $\underline{U}^B$ and $\underline{U}^S$ are the outside option utilities of the buyer and the seller, respectively—i.e., the utilities they achieve if no agreement is reached and the initial contract is carried out. The share of the surplus going to the buyer increases with $\alpha$, and $\alpha$ is commonly interpreted as a measure of the buyer's relative bargaining skill/power.

Because of the very general structure of $X$ which may be a discrete or multi-dimensional space, it is not possible to characterize $\hat{x}(\theta)$ without imposing additional structure on the renegotiation problem. We will do this in the next subsections. However, for a given renegotiated $\hat{x}(\theta)$ we can characterize the renegotiated price $\hat{p}(\theta)$ in general.

**Proposition 2.** *Let $\Delta_v := [v(\hat{x},\theta) - v(\bar{x},\theta)]$ and $\Delta_c := [c(\hat{x},\theta) - c(\bar{x},\theta)]$. The Generalized Nash Bargaining Solution implies that for a given $\hat{x}(\theta)$ the renegotiated price $\hat{p}(\theta)$ is given by*

$$\hat{p}(\theta) = \begin{cases} \bar{p} + (1-\alpha)\frac{1+\lambda_1}{1+\lambda}\Delta_v + \alpha(1+\lambda_2)\Delta_c & \text{if } (1-\alpha)\frac{1+\lambda_1}{1+\lambda}\Delta_v + \alpha(1+\lambda_2)\Delta_c \geq 0 \\ \bar{p} & \text{otherwise} \\ \bar{p} + (1-\alpha)(1+\lambda_1)\Delta_v + \alpha\frac{1+\lambda_2}{1+\lambda}\Delta_c & \text{if } (1-\alpha)(1+\lambda_1)\Delta_v + \alpha\frac{1+\lambda_2}{1+\lambda}\Delta_c \leq 0 \end{cases} \tag{15}$$

*with*

$$\lambda_1 = \begin{cases} \lambda & \text{if } v(\bar{x},\theta) - v(\hat{x},\theta) > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \lambda_2 = \begin{cases} \lambda & \text{if } c(\hat{x},\theta) - c(\bar{x},\theta) > 0 \\ 0 & \text{otherwise} \end{cases}$$

Note first that that there is a range where $\hat{p} = \bar{p}$, i.e., the initial price is left unchanged even though the parties agree to trade a new specification $\hat{x} \neq \bar{x}$. This requires that $\Delta_v > 0$ and $\Delta_c < 0$, i.e., both parties have to benefit from changing $\bar{x}$ to $\hat{x}$. To interpret equation (15) in case the price changes suppose that $\hat{x}$ is such that the buyer's valuation and the seller's cost go up as compared to $\bar{x}$, so $\Delta_v > 0$ and $\Delta_c > 0$ which implies $\lambda_1 = 0$ and $\lambda_2 = \lambda$. In this case the price must go up, too. If the buyer has all the bargaining power ($\alpha = 1$), the price increases by $(1+\lambda)[c(\hat{x},\theta) - c(\bar{x},\theta)]$, just enough to compensate the seller for her increase in cost and her feeling of a loss because of this cost increase. If the seller has all the bargaining power ($\alpha = 0$), the price increases by $\frac{v(\hat{x},\theta)-v(\bar{x},\theta)}{1+\lambda}$, so the price increase multiplied by $(1+\lambda)$ just equals the increase of the buyer's valuation (because the buyer feels a loss due to the price increase). Interestingly, the increase in price, $m = \hat{p} - \bar{p}$, is decreasing in $\lambda$ if both parties have the same bargaining power in the renegotiation game—i.e., prices are sticky. The result that prices are sticky if both parties have the same bargaining power holds also true if the parties agree upon a lower prices, which is in particular the case for $\Delta_v < 0$ and $\Delta_c < 0$. Here, the markup is negative, $m < 0$, and increasing in $\lambda$, i.e. prices adjust less the more loss averse the parties are.[9]

## 3.3 The Stickiness of the Initial Contract

In this subsection we assume that the specification of the good $x$ is one-dimensional and can be changed continuously, i.e. $X \equiv \mathbb{R}_0^+$. We will call $x$ the quantity of the good, but it could also be some continuous dimension of quality. Furthermore, we assume that the state of the world is drawn from a one-dimensional continuous space $\Theta \subset \mathbb{R}$.

**Assumption 1.** *For any state $\theta \in \Theta \subset \mathbb{R}$ and any quantity $x \in X \equiv \mathbb{R}_0^+$ the buyer's valuation and the seller's cost function are twice continuously differentiable and satisfy the following (Inada) conditions: $\forall\ x > 0$*

*(a)* $v(0,\theta) = 0,\ \frac{\partial v(x,\theta)}{\partial x} > 0,\ \frac{\partial^2 v(x,\theta)}{\partial x^2} < 0,\ \frac{\partial^2 v(x,\theta)}{\partial x \partial \theta} > 0,$

*(b)* $c(0,\theta) = 0,\ \frac{\partial c(x,\theta)}{\partial x} > 0,\ \frac{\partial^2 c(x,\theta)}{\partial x^2} \geq 0,\ \frac{\partial^2 c(x,\theta)}{\partial x \partial \theta} \leq 0,$

*(c)* $\lim_{x\to 0} \frac{\partial v(x,\theta)}{\partial x} > \lim_{x\to 0} \frac{\partial c(x,\theta)}{\partial x} = 0,\ \lim_{x\to\infty} \frac{\partial v(x,\theta)}{\partial x} < \lim_{x\to\infty} \frac{\partial c(x,\theta)}{\partial x}.$

Assumption 1 guarantees that there exists a unique materially efficient quantity $x^*(\theta) > 0$ that is fully characterized by the first-order condition. Furthermore, it implies that an increase in $\theta$ increases marginal benefits and reduces marginal costs. Thus, the higher the state, the higher is the materially efficient quantity, i.e., $x^*(\theta)$ is increasing in $\theta$.

[9] Proposition 2 is consistent with the experimental evidence in Bartling and Schmidt (2012). In this experiment the seller can make a take-it-or-leave-it renegotiation offer, so $\alpha = 0$, and the buyer always benefits from renegotiation, i.e., $\Delta_v > 0$. This also implies sticky prices. Bartling and Schmidt find that sellers often deliver the ex post efficient specification of the good without charging any markup if $x^*(\theta)$ is less costly to produce than $\bar{x}$. Moreover, they find that if the seller demands a higher price, which typically happens if $x^*(\theta)$ is more costly to produce, then the demanded markup is lower with an initial contract than in an equivalent situation without an initial contract.

The parties start out from the initial contract $(\bar{x}, \bar{p})$. Let $\bar{\theta}$ denote the state of the world in which $\bar{x}$ is the materially efficient quantity, i.e. $\bar{x} = x^*(\bar{\theta})$. Two cases have to be distinguished. First, if $\theta > \bar{\theta}$ the parties want to increase $x$ which increases the buyer's valuation and the seller's cost. In this case the price $p$ has to go up. Second, if $\theta < \bar{\theta}$ the parties want to reduce the quantity which reduces the buyer's valuation and the seller's cost. In this case the price $p$ has to go down. The following proposition fully characterizes the renegotiation outcome for both cases.

**Proposition 3.** *Suppose that Assumption 1 holds. Consider any initial contract $(\bar{x}, \bar{p})$ with $\bar{x} > 0$ and any realized state of the world $\theta \in \Theta$. The GNBS implies that the parties will renegotiate to*

$$(\hat{x}(\theta), \hat{p}(\theta)) = \begin{cases} \left(\hat{x}^L(\theta), \hat{p}^L(\theta)\right) & if \quad \theta < \theta^L \\ (\overline{x}, \overline{p}) & if \quad \theta^L \leq \theta \leq \theta^H \\ \left(\hat{x}^H(\theta), \hat{p}^H(\theta)\right) & if \quad \theta^H < \theta \end{cases} \tag{16}$$

*where $\hat{x}^i$ and $\hat{p}^i$, $i \in \{L, H\}$ are given by:*

$$\frac{\partial v\left(\hat{x}^L(\theta), \theta\right)}{\partial x} = \frac{1}{(1+\lambda)^2} \frac{\partial c\left(\hat{x}^L(\theta), \theta\right)}{\partial x} \tag{17}$$

$$\frac{\partial v\left(\hat{x}^H(\theta), \theta\right)}{\partial x} = (1+\lambda)^2 \frac{\partial c\left(\hat{x}^H(\theta), \theta\right)}{\partial x} \tag{18}$$

$$\hat{p}^L(\theta) = \overline{p} + (1-\alpha)(1+\lambda)\left[v\left(\hat{x}^L(\theta), \theta\right) - v(\overline{x}, \theta)\right] + \frac{\alpha}{1+\lambda}\left[c\left(\hat{x}^L(\theta), \theta\right) - c(\overline{x}, \theta)\right] \tag{19}$$

$$\hat{p}^H(\theta) = \overline{p} + \frac{1-\alpha}{1+\lambda}\left[v\left(\hat{x}^H(\theta), \theta\right) - v(\overline{x}, \theta)\right] + \alpha(1+\lambda)\left[c\left(\hat{x}^H(\theta), \theta\right) - c(\overline{x}, \theta)\right] \tag{20}$$

*and $\theta^L < \bar{\theta}$ and $\theta^H > \bar{\theta}$ are the unique solutions to $\hat{x}^L(\theta^L) = \overline{x}$ and $\hat{x}^H(\theta^H) = \overline{x}$ if these solutions exist; otherwise, $\theta_L$ and $\theta_H$ coincide with* $\inf\{\Theta\}$ *and* $\sup\{\Theta\}$, *respectively.*

This result is remarkable. First of all, the relative bargaining power $\alpha$ of the parties does not affect the renegotiated quantity $\hat{x}(\theta)$, it only affects the price that has to be paid. This is reminiscent of the Coase Theorem. However, the Coase Theorem assumes efficient bargaining and transferable utility, while in our model it is costly to transfer utility and therefore renegotiation yields a materially inefficient outcome. Second, there is a range of states of the world $[\theta^L, \theta^H]$ around state $\bar{\theta}$ in which the parties do not renegotiate but stick to the initial contract, even though this is inefficient in the absence of loss aversion. However, if a state materializes that is far enough away from $\bar{\theta}$ the parties will renegotiate, but the contract is sticky. The quantity change falls short of the quantity change that would be necessary to achieve the materially efficient $x^*(\theta)$. The price change can be smaller as well as larger than the price change that would occur without loss aversion depending on the relative bargaining power and the direction of renegotiation.

The stickiness of the initial contract is due to the well established psychological fact that losses loom larger than gains. For example, if $\theta > \bar{\theta}$ a small increase in $x$ benefits the buyer more than it costs the seller. However, the loss that the seller incurs is inflated by the factor $(1 + \lambda)$. Thus, the buyer has to reimburse at least $(1 + \lambda)$ times the seller's cost increase in order to get the seller to agree to the increase of $x$. But paying more than $\bar{p}$ is a loss to the buyer. Thus, the cost to the buyer of paying $(1 + \lambda)$ to the seller is $(1 + \lambda)^2$. It is this wedge between the actual and perceived cost of the seller and the actual and perceived price paid by the buyer that gives rise to the inefficiency of renegotiation.

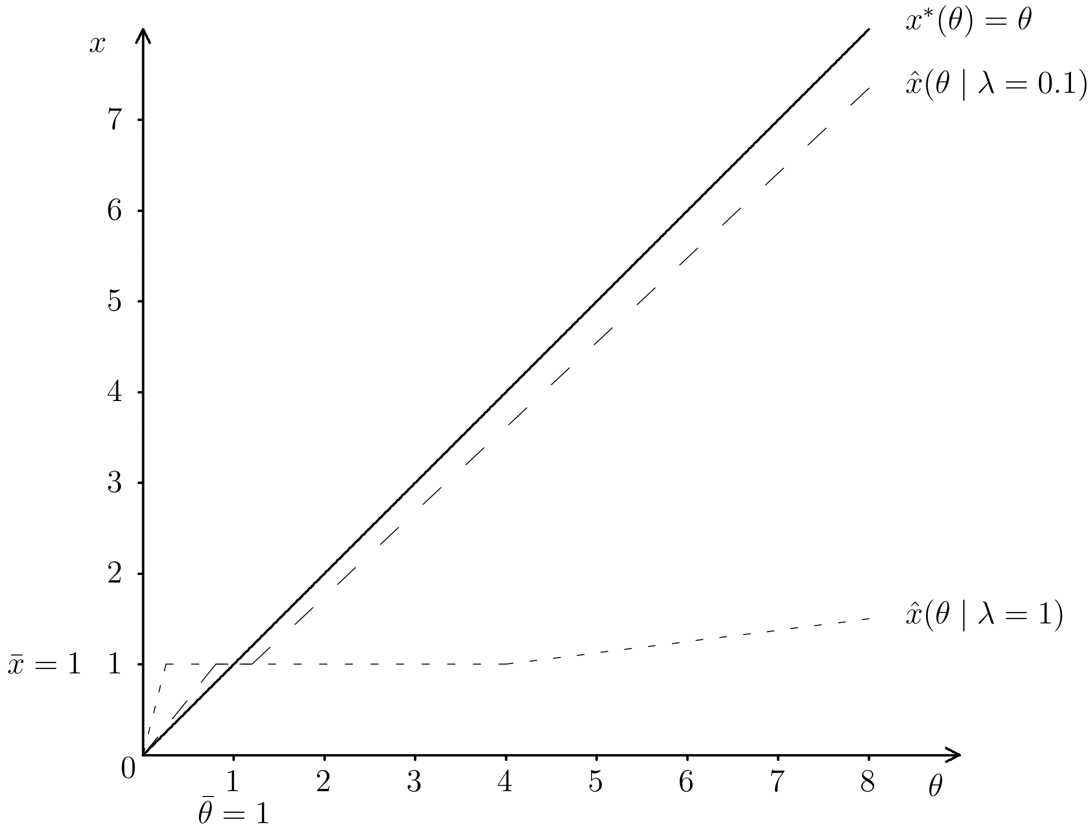


Figure 3: Ex post implemented service as function of $\theta$.

Figure 3 illustrates the renegotiation outcome for a simple example with $v(x,\theta) = \theta x$, $c(x,\theta) = \frac{1}{2}x^2$, and $X = \Theta = [0, 10]$. In this example the ex post efficient quantity is $x^*(\theta) = \theta$. The initial contract has $\bar{x} = 1$ which implies $\bar{\theta} = 1$. The figure shows the renegotiated quantity $\hat{x}(\theta)$ for $\lambda = 1$ and $\lambda = 0.1$. Many experimental studies found that losses are valued about twice as much as equally sized gains, which corresponds to $\lambda = 1$. If $\lambda = 1$ (dashed line), there is very little renegotiation. Only in extreme states of the world ($\theta < 0.25$ and $\theta > 4$) do the parties renegotiate. On the other hand, the experimental evidence also suggests that experienced "traders" (i.e. people who frequently trade goods not to own them but in order to make money) are much less attached to the goods they trade and suffer much less from loss aversion.[10] But even if $\lambda = 0.1$ (dotted line) there is a significant effect. There is no renegotiation for $\theta \in [0.87, 1.21]$. If there is renegotiation the renegotiated quantity is sticky and does not fully adjust to $x^*(\theta)$. In this example the relative distortion, $\left|\frac{x^*(\theta)-\hat{x}(\theta)}{x^*(\theta)}\right|$, increases when $\theta$ moves away from $\bar{\theta}$ until it reaches $\theta^H$ ($\theta^L$, respectively). From then on the relative distortion is constant.

## 3.4 The Likelihood and Cost of Renegotiation

From Proposition 3 it seems intuitive that an initial sales contract $(\bar{x}, \bar{p})$ is more likely to be renegotiated if the environment is more uncertain. In a more uncertain environment, it turns out more often that the initially contracted specification $\bar{x}$ is far from optimal and thus will not be delivered ex post, even though the parties are loss averse and dislike renegotiations. In order to formalize this intuition, assume that $\theta$ is distributed according to some cumulative distribution function $F(\theta)$. The initial contract will be renegotiated for $\theta < \theta^L$ and $\theta > \theta^H$, where $\theta^L$ and $\theta^H$ are characterized by Proposition 3. Note that $\theta^L$ and $\theta^H$ are independent of the cumulative distribution function $F(\cdot)$. We denote the ex-ante probability of renegotiation by $\rho(F) = F(\theta^L) + 1 - F(\theta^H)$ which depends on the distribution function and the initial contract. The following result shows that our conjecture is correct.

**Corollary 1.** *Suppose Assumption 1 holds. If $F_1(\theta)$ crosses $F_2(\theta)$ once from below at $\tilde{\theta} \in$*

[10]One explanation that has been put forth in the literature in order to explain the different behavior of traders and non-traders is that traders expect to sell their items while non-traders expect to keep the item. People who expect not keep an item are less attached to that item and in turn suffer less from loss aversion when loosing the item.

$(\theta^L, \theta^H) \subset \Theta$*, then the initial contract* $(\bar{x}, \bar{p})$ *is more likely to be renegotiated if* $\theta$ *is drawn from* $F_2$ *than from* $F_1$*, i.e.,* $\rho(F_1) < \rho(F_2)$*.*

The condition that $F_1(\theta)$ crosses $F_2(\theta)$ once from below at $\tilde{\theta} \in (\theta^L, \theta^H)$ means that $F_2(\theta)$ has more "weight in the tails" than $F_1(\theta)$ and is "more risky" in this sense.[11]

Another direct implication of Proposition 3 is that renegotiation becomes less likely the higher $\lambda$. An increase of $\lambda$ shifts $\theta^L$ to the left and and $\theta^H$ to the right and thereby reduces the set of states of the world in which renegotiation takes place.

**Corollary 2.** *Suppose Assumption 1 holds and that* $(\theta^L, \theta^H) \subset \Theta$*. The probability that the initial contract is renegotiated is decreasing in* $\lambda$*.*

We now turn to the cost of renegotiation. If the parties do not write a long-term contract but rely on spot contracting instead they will achieve the materially efficient social surplus $S^*(\theta) = v(x^*(\theta), \theta) - c(x^*(\theta), \theta)$. Recall that without an initial contract there is no $(\bar{x}, \bar{p})$ to which the parties can compare the actual exchange to, so there is no scope for loss aversion. With an initial contract $(\bar{x}, \bar{p})$, on the other hand, they only achieve the social surplus

$$\begin{aligned} S(\theta \mid \lambda, \bar{x}, \bar{p}) \quad &= \quad v(\hat{x}(\theta), \theta) - c(\hat{x}(\theta), \theta) \\ &\quad -\lambda \left[v(\bar{x}, \theta) - v(\hat{x}(\theta), \theta)\right]^+ - \lambda \left[c(\hat{x}(\theta), \theta) - c(\bar{x}, \theta)\right]^+ - \lambda|\hat{p} - \bar{p}| \qquad (21) \end{aligned}$$

The cost of renegotiation is the difference between the materially efficient social surplus and the social surplus that the parties actually achieve through renegotiation.

Thus, the cost of writing an initial contract that later may have to be renegotiated is the expected cost of renegotiation

$$C(\lambda, \bar{x}, \bar{p}) = E_\theta \left[S^*(\theta) - S(\theta \mid \lambda, \bar{x}, \bar{p})\right] \qquad (22)$$

**Proposition 4.** *Suppose Assumption 1 holds. The cost of writing an ex ante contract* $C(\lambda, \bar{x}, \bar{p})$ *is independent of* $\bar{p}$ *and increasing in* $\lambda$*. It is strictly increasing in* $\lambda$ *at* $\lambda = 0$*.*

[11] This definition of "more risky" differs from Second Order Stochastic Dominance (SOSD) which is in turn equivalent to a mean preserving spread. On the one hand, our definition is more general than SOSD because it does not require that $F_1$ and $F_2$ have the same mean. On the other hand, it is possible to construct $F_2$ by adding a mean preserving spread to $F_1$ in such a way that $F_2$ has less weight in the tails than $F_1$ (see Levy (1992, p. 563) for an example). In this case, the likelihood of renegotiation is smaller under $F_2$ than under $F_1$. Thus, SOSD is not sufficient for Corollary 1.

The initial price $\hat{p}$ does not affect the renegotiated good $\hat{x}$ nor the adjustment in price $|\hat{p}-\bar{p}|$ (in which $\bar{p}$ cancels out). Thus, the cost of writing a long-term contract is independent of the initial price. Furthermore, an increase in the degree of loss aversion has two effects on the costs of writing a long-term contract. First, keeping the good $\hat{x}$ fixed, increasing $\lambda$ increases the costs of writing a long-term contract because the social surplus decreases. Second, the renegotiated good $\hat{x}$ also depends on $\lambda$. If $\lambda$ increases, $\hat{x}$ reacts less strongly to changes in the state of nature. This decreases the material surplus $v(\cdot) - c(\cdot)$.

# 4 Optimal Contracts

## 4.1 Materially Inefficient Contracts or No Contracts Can Be Optimal

The cost of writing an ex ante contract makes it less attractive to write such a contract. However, there are many situations in which a long-term contract brings important benefits. A long-term contract may be useful for supply assurance, it may be desirable to allocate risk, or it may be beneficial to protect relationship specific investments. In the following we will assume that these benefits $B > 0$ are exogenously given and unaffected by renegotiation. Clearly, if $B$ is so large that the parties prefer a long-term contract that cannot be renegotiated to spot contracting, then it must be optimal to write this long-term contract. However, if $B$ is smaller then this amount, the parties may prefer not to write the ex-ante contract even if this contract can be renegotiated. We will show that this is the case if the costs of renegotiation are high, i.e., the parties are sufficiently loss averse.

To fix ideas, we consider a simple example with only two states $\Theta = \{\theta_1, \theta_2\}$ which are equally likely to materialize ex post. The seller can deliver three goods $X = \{x_1, x_2, x_3\}$. Good $x_1$ is the materially efficient good if state $\theta_1$ is realized, while $x_2$ is materially efficient if $\theta_2$ is realized. Good $x_3$ is in no case materially efficient, but it always leads to intermediate costs and benefits. The costs and benefits of the three goods in the two states are given in the following table:

| $x \backslash \theta$ | $\theta_1$ | $\theta_2$ |
|---|---|---|
| $x_1$ | $v = v^*$ <br> $c = c^*$ | $v = 0$ <br> $c = 0$ |
| $x_2$ | $v = 0$ <br> $c = 0$ | $v = v^*$ <br> $c = c^*$ |
| $x_3$ | $v = \beta v^*$ <br> $c = \beta c^*$ | $v = \beta v^*$ <br> $c = \beta c^*$ |

with $v^* > c^* > 0$ and $\beta \in (0, 1)$.

The parties either write no long-term contract, or a long-term contract specifying one of the three goods to be delivered. Without an initial contract, the parties rely on spot contracting, i.e., they do not obtain the benefit $B$ of a long-term contract but the materially efficient good is always traded. If the parties write a long-term contract, then renegotiation may be necessary to implement the materially efficient good ex post. Irrespective of which long-term contract $(\bar{x}_i, \bar{p})$, $i \in \{1, 2, 3\}$, is written ex ante, by Proposition 1 renegotiation takes place if and only if $\theta \neq \theta_i$ and

$$\lambda \leq \sqrt{\frac{v^*}{c^*}} - 1 =: \bar{\lambda}. \tag{23}$$

**Proposition 5.**

(i) *For* $0 < B < \min\{1 - \beta, \frac{1}{2}\}(v^* - c^*)$ *there exists a critical value* $\tilde{\lambda}(B) \in (0, \bar{\lambda})$ *such that it is optimal not to write a long-term contract if and only if* $\lambda > \tilde{\lambda}(B)$.

(ii) *A long-term contract specifying the compromise good* $x_3$ *dominates all other specific performance contracts if and only if* $\beta \geq \frac{1}{2}$. *For* $\lambda > \bar{\lambda}$ *renegotiation does not take place and the compromise good is traded. For* $\lambda \leq \bar{\lambda}$ *renegotiation takes place in all states and the materially efficient good is implemented ex post.*

Proposition 5 shows that it can be optimal for the parties not to write a long-term contract if they are sufficiently loss averse, even in situations where a long-term contract involves substantial benefits $B > 0$. Furthermore, no matter how large $B$ it may be optimal to contract on a specification of the good that is never materially efficient ex post. On the one hand, this good $x_3$ makes renegotiation less painful because smaller adjustments in prices and costs are necessary to get to the materially efficient good. This is the case for low degrees of loss aversion. On the other hand, if $\lambda$ is large and renegotiation costs are prohibitive, good $x_3$ is an attractive compromise that yields an intermediate surplus in both states of the world which is preferable to getting the full surplus in one state and nothing in the other state if $\beta > 1/2$. Thus, even though the compromise good is never materially efficient it minimizes renegotiation costs.

## 4.2 Asset Ownership, Long-term Contracts, and the Hold-up Problem

In the previous subsection, the benefit of writing a long-term contract was exogenously given. Now we endogenize both the cost and the benefit of writing a long-term contract by assuming that the buyer can make a non contractible relationship-specific investment. If a complete, state-contingent contract could be written, there would be no problem to induce the buyer to invest efficiently. However, in a complex environment with many different states of the world it may be prohibitively costly to write a long-term contract that specifies the rights and obligations of both parties in all possible contingencies. The incomplete contracts literature (Grossman and Hart, 1986; Hart and Moore, 1990) goes one step further and assumes that it is impossible to write any long-term contract on trade. The only contracts that can be written to protect relationship-specific investments are contracts on the allocation of ownership rights. If a party owns an asset that is required for production this party has a stronger bargaining position when the terms of trade are negotiated. Thus, the party will get a larger share of the surplus which increases his or her investment incentives.

The assumption that it is impossible to write any long-term contract on trade is very strong. Surely, the parties could write an unconditional contract that specifies some good $\bar{x}$ to be traded at some price $\bar{p}$ at some date in the future. This contract is not state-contingent and likely to be suboptimal after the realization of the state of the world in which case the parties may want to renegotiate it. Nevertheless it may offer some protection against hold-up.

It is important to note that writing a specific performance contract and allocating ownership rights on assets are mutually exclusive instruments to encourage relationship-specific investments. Ownership of an asset improves the bargaining position of the owner only if he can threaten to trade with some outside party and take the asset with him. A specific performance contract precludes this possibility. With a specific performance contract each party can insist that good $\bar{x}$ is traded at price $\bar{p}$. Thus, the parties have to take a decision: Either they write a specific performance contract or they rely on the allocation of ownership rights to give investment incentives. In this section we discuss under what circumstances which of these two mechanisms the parties prefer. We assume that allocating asset ownership does not provide a reference point because it does not specify $(\bar{x}, \bar{p})$ to which the actual trade $(x, p)$ can be compared.

We augment the model introduced in Section 2 in order to allow for one-sided relationship-specific investments and asset ownership in the spirit of Grossman and Hart (1986). At date 0 the two parties can write a contract. At date 1 the buyer can make a relationship-specific investment $I \in \mathbb{R}_0^+$ that increases his benefit from trade at cost $\psi(I) = (1/2)I^2$. The investment is beneficial only if the buyer has access to an asset $A$. At date 2 the state of the world, $\theta \in \Theta$, materializes that affects the valuation of the buyer, $v(x, \theta, I)$, and the cost of the seller, $c(x, \theta)$. At date 3 parties can (re)negotiate the specification of the good $x \in X$ to be traded and the price $p \in \mathbb{R}$ to be paid, the good is exchanged, and payoffs are made.

In order to keep the analysis tractable consider a simple model with $n \geq 2$ states of the world, $\theta \in \{\theta_1, \dots, \theta_n\} \equiv \Theta$, and $n$ relevant specifications of the good that can be produced at stage 3, $x \in \{x_1, \dots, n_n\} \equiv X$. Each good is ex-post efficient in exactly one state of nature, i.e.,

$$x_i = x^*(\theta_i) = \arg\max_x \{v(x, I, \theta_i) - c(x, \theta_i)\}. \tag{24}$$

Let $Prob(\theta = \theta_i) = \pi_i$ with $\sum_{i=1}^n \pi_i = 1$, and assume w.l.o.g. that $\pi_1 \geq \pi_i$ for all $i \geq 2$. For simplicity, we assume that only two configurations of costs and benefits can arise ex post depending on whether or not the efficient good is traded. Given $\theta_i$ has materialized, the buyer's and the seller's ex-post utility is

$$U^B = \begin{cases} v^* + I - p - \frac{1}{2}I^2 & \text{if } \ x = x_i \\ \underline{v} - p - \frac{1}{2}I^2 & \text{if } \ x \neq x_i \end{cases}, \tag{25}$$

and

$$U^S = \begin{cases} p - c^* & \text{if } \ x = x_i \\ p - \underline{c} & \text{if } \ x \neq x_i \end{cases}, \tag{26}$$

respectively, with $v^* - c^* > \underline{v} - \underline{c} > 0$ and $c^* > \underline{c}$. Note that the investment pays off only if the efficient good is traded. Efficiency requires that the parties trade $x_i$ in state $\theta_i$ and that the buyer invests

$$I^* = \arg\max_I \{v^* - c^* + I - \frac{1}{2}I^2\} = 1 \ . \tag{27}$$

**Asset Ownership and Spot Contracting.** Suppose the parties do not write an ex ante contract on trade at date 0. In this case, when they negotiate the terms of trade on the spot market at date 3, each party is free to opt out and trade on the outside market. Leaving the relationship is inefficient, but it determines the threatpoints of the bargaining game. If negotiations fail and the parties turn to the outside market they get

$$\tilde{U}^B = \begin{cases} -\frac{1}{2}I^2 & \text{if the seller owns } A \\ \beta I - \frac{1}{2}I^2 & \text{if the buyer owns } A \end{cases}, \tag{28}$$

and

$$\tilde{U}^S = \begin{cases} 0 & \text{if the seller owns } A \\ 0 & \text{if the buyer owns } A \end{cases}, \tag{29}$$

The parameter $\beta \in [0,1]$ measures the specificity of the buyer's investment. The smaller $\beta$ the more specific is his investment to the relationship with the seller.

Because there is no initial contract to which the parties compare the outcome of the bargaining game they will always reach an efficient agreement and split the surplus in proportion $(\alpha, 1-\alpha)$. Thus, the final payoff of the buyer is $U^B = \beta I - \frac{1}{2}I^2 + \alpha[v^* + I - c^* - \beta I]$ if he owns the asset and $U^B = -\frac{1}{2}I^2 + \alpha[v^* + I - c^*]$ if he does not own it. Clearly, to improve the buyer's investment incentives he should own the asset. In this case he invests

$$I^A = \beta + \alpha(1-\beta) < 1, \tag{30}$$

where $I^A$ stands for the investment level if the parties rely on asset ownership. Note that the buyer invests too little as compared to the first best. His investment increases with $\alpha$ (i.e. the larger his bargaining power) and with $\beta$ (i.e. the less relationship specific his investment is).

**Long-term Specific Performance Contracts.** Suppose now that the parties write a specific performance contract $(\bar{x}, \bar{p})$ at stage 0. Obviously, it is optimal to specify good $x_1$ in this contract, because this is the good which is most likely to be materially efficient ex post. If state $\theta = \theta_1$ materializes, the sales contract is materially efficient and will be executed. If some other state $\theta \neq \theta_1$ materializes trading $x_1$ is materially inefficient. In this case, the contract will be renegotiated if and only if

$$\lambda \leq \sqrt{\frac{v^* - \underline{v} + I}{c^* - \underline{c}}} - 1 \equiv \bar{\lambda}(I). \tag{31}$$

If renegotiation takes place $B$'s expected utility is

$$E[U^B] = v^* + I - \frac{1}{2}I^2 - \bar{p} - (1 - \pi_1)\left[(1-\alpha)(v^* + I - \underline{v}) + \alpha(1+\lambda)^2(c^* - \underline{c})\right]$$

and he will choose

$$I^{CR} = \pi_1 + \alpha(1 - \pi_1) < 1, \tag{32}$$

where $I^{CR}$ stands for the investment level that is optimal with a specific performance contract which is renegotiated. Again, the buyer invests too little as compared to the first best. His investment increases the more likely it is that state 1 materializes and the higher his bargaining power in the renegotiation game. The investment induced by a long-term contract with renegotiation is larger than the investment induced by asset ownership if and only if $\pi_1 > \beta$. Note that neither $\alpha$ nor $\lambda$ affect this comparison.

If the parties are sufficiently loss averse so that renegotiation does not take place ($\lambda > \bar{\lambda}(I)$), then $B$'s expected utility is

$$E[U^B] = \pi_1[v^* + I] + (1 - \pi_1)\underline{v} - \bar{p} - \frac{1}{2}I^2. \tag{33}$$

and he will invest

$$I^{CN} = \pi_1 \ < I^{CR} \tag{34}$$

Thus, without renegotiation the buyer will always invest less than with renegotiation.

**Which Contractual Arrangement Is Optimal Ex Ante?** Comparing the two contractual arrangements it is not enough to compare the investment levels. Even if the investment with a long-term contract is more efficient than the investment without a long-term contract, the parties may prefer not to write a long-term contract because renegotiation is costly: It may either fail if $\lambda$ is too large or cause a utility loss due to loss aversion.

The following proposition completely characterizes the conditions under which relying on the allocation of asset ownership is jointly preferred by the two parties to writing a long-term specific performance contract.

**Proposition 6.** *There exists a unique cutoff* $\bar{\lambda} = \sqrt{\frac{v^* - \underline{v} + \pi_1 + \frac{\alpha}{2}(1-\pi_1)}{c^* - \underline{c}}} - 1 > 0$ *such that a specific performance contract will be renegotiated if and only if* $\lambda \leq \bar{\lambda}$.

*(1) Suppose that* $\lambda \leq \bar{\lambda}$. *Then the parties jointly prefer to rely on allocating asset ownership to B rather than on a long-term specific performance contract if and only if*

$$[\beta + \alpha(1-\beta)] - \frac{1}{2}[\beta + \alpha(1-\beta)]^2 \geq [\pi_1 + \alpha(1-\pi_1)] - \frac{1}{2}[\pi_1 + \alpha(1-\pi_1)]^2 - \Phi(\lambda) \quad (35)$$

*where* $\Phi(\lambda) = \lambda(1-\pi_1)\left[\frac{1-\alpha}{1+\lambda}[v^* - c^* + \pi_1 + \alpha(1-\pi_1)] + (\alpha(1+\lambda)+1)[c^* - \underline{c}]\right] > 0$ *is the utility loss due to loss aversion in case of renegotiation which is increasing in* $\lambda$.

*(2) Suppose that* $\lambda > \bar{\lambda}$. *Then the parties jointly prefer to rely on asset ownership rather than on a long-term specific performance contract if and only if*

$$[\beta + \alpha(1-\beta)] - \frac{1}{2}[\beta + \alpha(1-\beta)]^2 \geq \pi_1^2 - \frac{1}{2}\pi_1^2 - (1-\pi_1)[v^* - c^* - \underline{v} + \underline{c}] \ . \quad (36)$$

To interpret Proposition 6 suppose that there is no loss aversion ($\lambda = 0$). In this case the cost to renegotiation $\Phi(\lambda)$ disappears and the parties prefer relying on asset ownership if and only if this induces a more efficient investment level. This is the case if and only if $\beta \geq \pi_1$. Asset ownership provides better investment incentives the less relationship-specific the investment is, i.e. the larger $\beta$. The specific performance contract provides better investment incentives the higher the probability that the contract need not be renegotiated, i.e. the higher $\pi_1$. Note that the buyer's bargaining power $\alpha$ does not affect this comparison, because it affects investment incentives under both contracts in exactly the same way.

Suppose now that $\lambda > 0$. In this case the specific performance contract becomes more costly. If $\lambda \leq \bar{\lambda}$ the parties do renegotiate, but they suffer a utility loss $\Phi(\lambda)$ from loss aversion. If $\lambda > \bar{\lambda}$ the parties do not renegotiate and lose $(1-\pi_1)[v^* - c^* + I^{CN} - \underline{v} + \underline{c}]$ because of inefficient trade if $\theta \neq \theta_1$. The costs of renegotiation are increasing in $\lambda$, decreasing in $\pi_1$ and increasing in the loss due to inefficient trade $[v^* - c^* - \underline{v} + \underline{c}]$.

We conclude from this discussion that the parties should rely on asset ownership if asset specificity is low and if the future is highly uncertain in the sense that a contract would have to be renegotiated with a high probability, while a long-term contract is optimal if asset specificity is high and if there is not too much uncertainty. Moreover, asset ownership is more likely to be the optimal contractual arrangement ex ante if the parties are loss averse.

## 4.3 Authority Contracts and the Employment Relation

Instead of writing an ex-ante contract that specifies a particular good $\bar{x}$ to be traded, the parties could also write an "authority contract" that gives one party the right to choose $x$ out of some admissible set $\mathcal{A} \subseteq X$. For example, the buyer could have the right to "order" the seller to deliver any good or service $x \in \mathcal{A}$. According to Simon (1951) this is the nature of the employment relation. An employment contract does not specify a specific service to be delivered by the employee (the seller), it rather gives the employer (the buyer) the right to tell the employee which service to provide (within the limits specified by the employment contract). Simon compares an authority contract to a specific performance contract and argues that there is a tradeoff. The authority contract has the advantage of flexibility, i.e., the employer can easily adjust the service to be provided to the realization of the state of the world. However, the authority contract is also prone to abuse. The employer has an incentive to choose $\tilde{x}(\theta) = \arg\max_{x \in \mathcal{A}} v(x, \theta)$ which maximizes his own utility rather than the materially efficient service $x^*(\theta) = \arg\max_{x \in \mathcal{A}}[v(x, \theta) - c(x, \theta)]$. The employee anticipates this and has to be compensated ex ante for her expected cost $E_\theta[c(\tilde{x}(\theta), \theta)]$. Thus, the efficiency loss will be borne by the employer. A specific performance contract, on the other hand, leaves no scope for abuse. But, this advantage comes at the cost of rigidity. The employee will provide $\bar{x}$ in all states of the world. Thus, according to Simon, whether an authority contract or a specific performance contract is optimal depends on whether the cost of abuse exceeds the cost of rigidity.

The problem with Simon's argument is that the specific performance contract need not be rigid because the parties are free to renegotiate. If the parties write a contract $(\bar{x}, \bar{p})$ they can later renegotiate it to $(x^*(\theta), \hat{p})$. The specific performance contract protects the employee against abuse (he must always get at least $\bar{p} - c(\bar{x}, \theta)$), while renegotiation makes the contract flexible. With a specific performance contract the employer has to "bribe" the employee to provide $x^*(\theta)$ rather than $\bar{x}$. The authority contract can also be renegotiated to prevent that the employer orders the inefficient good $\tilde{x}(\theta)$. With an authority contract the employee has to "bribe" the employer to choose $x^*(\theta)$ rather than $\tilde{x}(\theta)$. If renegotiation is costless the final outcome will always be materially efficient and the expected payments will be the same under both contracts.

If renegotiation is imperfect due to loss aversion, however, the two contracts are no longer equivalent. In the following, we show that loss aversion affects an authority contract differently than a specific performance contract. Thus an authority contract will be strictly optimal in some situations, while a specific performance contract will dominate in others.

Suppose that $X \subset \mathbb{R}^N$ and $\Theta \subset \mathbb{R}^S$ are some continuous subsets of Euclidean spaces and that $\theta$ is drawn by nature according to the density function $f(\theta)$ out of set $\Theta$. Let $x^*(\theta) : \Theta \to X$ be a bijective function, i.e., for any $x \in X$ there exists at most one $\theta \in \Theta$ in which $x$ is efficient. Similarly, let $\tilde{x}(\theta) : \Theta \to X$ be also a bijective function, i.e., for any $x \in X$ there is at most one $\theta \in \Theta$ in which $x$ is profit maximizing for the buyer. Furthermore, we assume that $x^*(\theta) \neq \tilde{x}(\theta)$ for all $\theta \in \Theta$. These assumptions imply that without renegotiation the specific performance contract and the authority contract implement the efficient outcome with probability zero.[12]

**Assumption 2.** *For all $\theta \in \Theta$ we have that $v(x^*(\theta), \theta) = v^*$, $c(x^*(\theta), \theta) = c^*$, $v(\tilde{x}(\theta), \theta) = \tilde{v}$, $c(\tilde{x}(\theta), \theta) = \tilde{c}$, $v(x, \theta) = \underline{v}$, and $c(x, \theta) = \underline{c}$ for all $x \in X \setminus \{x^*(\theta), \tilde{x}(\theta)\}$. Furthermore, $\tilde{v} > v^* > \underline{v}$, $\tilde{c} > c^* > \underline{c}$, $v^* - c^* > \tilde{v} - \tilde{c} > 0$, and $v^* - c^* > \underline{v} - \underline{c} > 0$.*

Assumption 2 simplifies the problem considerably by assuming that there are only three different outcomes and two relevant services in each state of the world at the renegotiation stage. The relevant two services are the materially efficient service $x^*(\theta)$ and the service $\tilde{x}(\theta)$ that maximizes the buyer's benefit.

The following Lemma fully characterizes the social surplus that is generated by the two types of contracts.

[12] These assumptions are useful to avoid cumbersome case distinctions, but they are not crucial for any of the following results. It is straightforward to set up a similar model with a discrete state space and without the bijectivity assumptions.

**Lemma 1.** *If the parties write a specific performance contract the contract will be renegotiated if and only if* $\lambda \leq \sqrt{\frac{v^*-\underline{v}}{c^*-\underline{c}}} - 1 \equiv \bar{\lambda}^S$. *The total surplus that is generated by this contract is given by*

$$S^S(\lambda) = \begin{cases} v^* - c^* - \lambda(1+\alpha(1+\lambda))[c^* - \underline{c}] - \frac{\lambda(1-\alpha)}{1+\lambda}[v^* - \underline{v}] & \text{if } \lambda \leq \bar{\lambda}^S \\ \underline{v} - \underline{c} & \text{if } \lambda > \bar{\lambda}^S \end{cases} \tag{37}$$

*If the parties write an authority contract the contract will be renegotiated if and only if* $\lambda \leq \sqrt{\frac{\tilde{c}-c^*}{\tilde{v}-v^*}} - 1 \equiv \bar{\lambda}^A$. *The total surplus that is generated by this contract is given by*

$$S^A(\lambda) = \begin{cases} v^* - c^* - \lambda(1+(1-\alpha)(1+\lambda))[\tilde{v} - v^*] - \frac{\lambda\alpha}{1+\lambda}[\tilde{c} - c^*] & \text{if } \lambda \leq \bar{\lambda}^A \\ \tilde{v} - \tilde{c} & \text{if } \lambda > \bar{\lambda}^A \end{cases} \tag{38}$$

Note that renegotiation of a specific performance contract increases the benefit of the buyer and (to a lesser degree) the cost of the seller, while renegotiation of an authority contract reduces the cost of the seller and (to a lesser degree) the benefit of the buyer.

**Proposition 7.** *If* $\lambda = 0$ *the authority contract and the specific performance contract are equivalent. Both implement* $x^*(\theta)$ *with probability one and yield* $S^A(0) = S^S(0) = v^* - c^*$.
*If* $\lambda > 0$, *generically, either the authority contract or the specific performance contract is uniquely optimal, i.e.* $S^A(\lambda) \neq S^S(\lambda)$. *The authority contract is more likely to be optimal*

- *the smaller the efficiency loss of abuse if there is no renegotiation, i.e. the smaller* $(v^* - c^*) - (\tilde{v} - \tilde{c})$,
- *the less costs and benefits have to be shifted to reach* $x^*(\theta)$ *from* $\tilde{x}(\theta)$, *i.e. the smaller* $(\tilde{v} - v^*)$ *and* $(\tilde{c} - c^*)$.

*The specific performance contract is more likely to be optimal*

- *the smaller the efficiency loss of rigidity if there is no renegotiation, i.e. the smaller* $(v^* - c^*) - (\underline{v} - \underline{c})$,
- *the less costs and benefits have to be shifted to reach* $x^*(\theta)$ *from any* $x \notin \{x^*(\theta), \tilde{x}(\theta)\}$, *i.e. the smaller* $(v^* - \underline{v})$ *and* $(c^* - \underline{c})$.

This result confirms and extends the original insights of Simon. If $\lambda > \max\{\bar{\lambda}^A, \bar{\lambda}^S\}$ so that neither the authority contract nor the sales contract are renegotiated the comparison boils down to Simon's comparison of whether $\underline{v} - \underline{c}$ is greater than $\tilde{v} - \tilde{c}$, i.e., whether rigidity or abuse is more efficient. However, if $\lambda < \max\{\bar{\lambda}^A, \bar{\lambda}^S\}$, it is also important by how much costs and benefits have to be shifted to reach efficiency. To see this compare two situations in which specific performance contracts (without renegotiation) yield $(\underline{v}, \underline{c})$ and $(\underline{v} - \Delta, \underline{c} - \Delta)$, respectively, while the materially efficient good always yields $(v^*, c^*)$. Even though the specific performance contracts are equally inefficient in the two situations, it is less costly to renegotiate in the first situation than in the second. This is because $v^* - \underline{v} < v^* - (\underline{v} - \Delta)$ and $c^* - \underline{c} < c^* - (\underline{c} - \Delta)$, i.e., loss aversion kicks in more strongly.

## 4.4 Price Indexation

Suppose that there is a verifiable signal $\sigma$ that is correlated with the state of the world $\theta$. Is it possible to improve efficiency by making the payment in the initial contract conditional on this signal? One of the two main results in Hart (2009) is that indexation can be very useful. By making the price $\bar{p}$ conditional on $\sigma$ it becomes more likely that $c(\cdot, \theta) < \bar{p}(\sigma) < v(\cdot, \theta)$, so that parties are willing to trade voluntarily and costly renegotiation can be avoided. Perhaps surprisingly, this is not the case in our set-up. To show this we return to the case of Section 3.3 where $x$ is a continuous function of $\theta$.

**Proposition 8.** *Suppose that Assumption 1 holds and that there exists a verifiable signal $\sigma$ that is correlated with the state of the world $\theta$. Making the initially agreed upon price $\bar{p}$ a function of $\sigma$ has no effect on the renegotiation outcome and on the efficiency of the initial contract.*

The intuition for this result is simple. In our model the only role of the initial price $\bar{p}$ is to share the available surplus ex ante. Renegotiation is only about $\hat{x}$ and the markup $\hat{p} - \bar{p}$ in which $\bar{p}$ cancels out. Once the state of the world has materialized and some $\bar{p}(\sigma)$ is in place, this $\bar{p}(\sigma)$ defines the reference point. Only deviations from the reference point matter, but not the reference point itself.

What explains the striking difference to Hart (2009)? Hart considers at will contracts in which each party can freely walk away from the contract. Thus, the default point is not the initial contract, but the no trade situation. If e.g. $p < c < v$ the seller will refuse to trade even though trade is efficient. Similarly, if $c < v < p$ the buyer will refuse to trade and walk away. By making the price contingent on $\sigma$ these inefficient outcomes become less likely. In contrast, we consider specific performance contracts. In our set up the parties cannot simply walk away but are obliged to honor the contract even if this turns out to result in a loss ex post. Hence, preventing the parties to leave is not an issue.

Of course, if it was possible to make the specification of the good contingent on $\sigma$ this would improve efficiency if it reduces $\bar{x}(\sigma) - x^*(\theta)$ as compared to $\bar{x} - x^*(\theta)$ and thereby the welfare loss due to renegotiation in expectation. However, indexation of the specification of the good is much more difficult than price indexation and rarely observed in practice.

What is frequently observed is a contract that makes the price per unit of output conditional on an index (say inflation, the exchange rate, or the price of oil) and gives the buyer the right to choose the quantity of trade ex post. Such a contract is similar to the employment contract because one party can tell the other party what to deliver, but it is more complex because the total payment depends on the quantity chosen by the buyer.

The following example shows that such a contract with price indexation can be very beneficial and may implement the first best if the signal is sufficiently informative. Let $x \in \mathbb{R}_0^+$ denote the quantity of trade and let $\theta = (\sigma, \tau)$ be a two-dimensional state of the world, where $\sigma$ is publicly observable and verifiable. Assume that $c(x, \theta) = c(\sigma) \cdot x$. In this case a contract with price indexation that gives the buyer the right to decide on the quantity $x$ is very useful. If the contract stipulates that the price per unit is $w(\sigma) = c(\sigma)$, i.e $p = w(\sigma) \cdot x$, then the buyer will choose[13]

$$\tilde{x} = \arg\max_x \{v(x, \theta) - w(\sigma) \cdot x\} \;=\; \arg\max_x \{v(x, \theta) - c(\sigma) \cdot x\} \;=\; x^*(\theta)\ . \tag{39}$$

Thus, this contract implements the first best without renegotiation. Note, however, that for price indexation to be beneficial it is crucial that it not only affects the total payment but

[13]Here, we stick to our assumption that the buyer feels a loss if total payment after renegotiation is higher than total payment under the initial contract, which depends on the verifiable signal and the buyer's action. With the contract specifying a unit price, it might also be sensible to assume that the buyer feels a loss if the renegotiated unit price is higher than the initially specified unit price. In the case with indexation this would not change the example, because the initial contract is not renegotiated if the contract is properly indexed.

also implicitly the quantity (or specification) $x$ that is traded if the initial contract is not renegotiated.

## 5 Conclusions

This paper explores the implications of one important behavioral phenomenon, loss aversion, for optimal (incomplete) contracting and renegotiation. It shows that loss aversion makes the initial contract sticky and prevents parties to adjust the contract to the ex post (materially) efficient allocation. This ex post inefficiency of renegotiation has important implications for the optimal design of contracts. In particular, it can explain why people often abstain from writing (beneficial) long-term contracts or why they write long-term contracts that are obviously inefficient, it can explain under what conditions the allocation of ownership rights should be used to promote investment incentives rather than specific performance contracts, and it predicts under what conditions employment contracts strictly outperform specific performance contracts. Moreover, the model we propose is simple and tractable and thus can easily be applied to other contracting problems as well.

We assume that the contracting parties are aware that they are loss averse when they write the initial contract. Nevertheless, they do not manage to get rid of this distortion of their preferences and continue to weigh gains and losses differently. This is of course irrational behavior. The behavior of the contracting parties in our model is akin to the behavior of a house owner who is reluctant to sell his house at a price that is below the price he bought it for, even though he understands that the historic price at which he bought is bygone and should not affect his decision to sell.[14] It would be interesting to extend our model to the case of contracting parties who are less sophisticated and do not anticipate that loss aversion will distort renegotiation in the future. If the parties are "naïve" and believe that all future renegotiations will be materially efficient then they will write contracts that are suboptimal not only ex post but also ex ante.

[14]Empirically investigating the Boston condominium market in the 1980's, Genesove and Mayer (2001) provide evidence that the original purchase price has a significant effect on seller behavior in line with nominal loss aversion. Moreover, they show that not only owner-occupants but also professional investors behave in a loss averse fashion.

Finally, it would be interesting to study the interaction of loss aversion with other behavioral biases such as concerns for fairness, self-serving biases, and overconfidence. Parties could also be affected by additional reference points such as their (rational) expectations, social norms, or the status quo. To model the interaction of these effects and their impact on contracting is a fascinating topic for future research.

# A Appendix

*Proof of Proposition 2.* The generalized Nash product can be written as follows

$$
\begin{aligned}
GNP(p) &= \big[v(\hat{x},\theta) - p - \lambda_1[v(\bar{x},\theta) - v(\hat{x},\theta)] - \lambda_3[p-\bar{p}] - v(\bar{x},\theta) + \bar{p}\big]^{\alpha} \\
&\quad \times \big[p - c(\hat{x},\theta) - \lambda_4[\bar{p}-p] - \lambda_2[c(\hat{x},\theta) - c(\bar{x},\theta)] - \bar{p} + c(\bar{x},\theta)\big]^{1-\alpha} \qquad \text{(A.1)}
\end{aligned}
$$

where

$$
\lambda_3 = \begin{cases} \lambda & \text{if } \; p - \bar{p} > 0 \\ 0 & \text{otherwise} \end{cases} \qquad \lambda_4 = \begin{cases} \lambda & \text{if } \; \bar{p} - p > 0 \\ 0 & \text{otherwise} \end{cases}
$$

Note that $GNP(p)$ is strictly concave and differentiable for all $p$ but $p = \bar{p}$. Because we consider a given $\hat{x}(\theta)$ it is clear whether or not $\lambda_1 = 0$ and/or $\lambda_2 = 0$. For $\Delta_v \geq 0$ and $\Delta_c \geq 0$ only prices $p \geq \bar{p}$ can lead to $U^S(\hat{x}(\theta), p|\theta) \geq \underline{U}^S$. Thus, in this case $GNP(p)$ is differentiable for all prices in the relevant range. Moreover, for $\Delta_v \leq 0$ and $\Delta_c \leq 0$ only prices $p \leq \bar{p}$ can lead to $U^B(\hat{x}(\theta), p|\theta) \geq \underline{U}^B$, and thus $GNP(p)$ is differentiable for all prices in the relevant range. Only for $\Delta_v > 0$ and $\Delta_c < 0$ we need to consider prices $p$ that are higher as well as lower than $\bar{p}$. (For $\Delta_v \leq 0$ and $\Delta_c \geq 0$ with at least one inequality being strict renegotiation does not take place.)

Differentiating the generalized Nash product with respect to $p$ yields the following first-order condition:

$$
\begin{aligned}
\frac{\partial GNP(p)}{\partial p} = 0 \quad \Leftrightarrow \quad & \alpha(1+\lambda_3)\left[U^B(\hat{x}, p \mid \theta) - \underline{U}^B\right]^{\alpha-1}\left[U^S(\hat{x}, p \mid \theta) - \underline{U}^S\right]^{1-\alpha} \qquad \text{(A.2)} \\
& + (1-\alpha)(1+\lambda_4)\left[U^S(\hat{x}, p \mid \theta) - \underline{U}^S\right]^{-\alpha}\left[U^B(\hat{x}, p \mid \theta) - \underline{U}^B\right]^{\alpha} \;=\; 0,
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
&\alpha(1+\lambda_3)\Big[(1+\lambda_4)[p-\bar{p}] - (1+\lambda_3)[c(\hat{x},\theta) - c(\bar{x},\theta)]\Big] \\
&\quad = (1-\alpha)(1+\lambda_4)\Big[(1+\lambda_1)[v(\hat{x},\theta) - v(\bar{x},\theta)] - (1+\lambda_3)[p-\bar{p}]\Big]. \qquad \text{(A.3)}
\end{aligned}
$$

Solving for $p$ yields the expressions for $\hat{p}(\theta)$, given by equation (15) for the cases $(1-\alpha)\frac{1+\lambda_1}{1+\lambda}\Delta_v + \alpha(1+\lambda_2)\Delta_c \geq 0$ and $(1-\alpha)(1+\lambda_1)\Delta_v + \alpha\frac{1+\lambda_2}{1+\lambda}\Delta_c \leq 0$. Note that the two price formulas coincide for $\Delta_v = 0$ and $\Delta_c = 0$ the unique case where both conditions are satisfied with equality. Recall that it is impossible that $\Delta_v \leq 0$ and $\Delta_c \geq 0$ because in this case no renegotiation takes place.

Therefore, $(1-\alpha)\frac{1+\lambda_1}{1+\lambda}\Delta_v + \alpha(1+\lambda_2)\Delta_c > 0$ implies that $(1-\alpha)(1+\lambda_1)\Delta_v + \alpha\frac{1+\lambda_2}{1+\lambda}\Delta_c > 0$ and $(1-\alpha)(1+\lambda_1)\Delta_v + \alpha\frac{1+\lambda_2}{1+\lambda}\Delta_c < 0$ implies that $(1-\alpha)\frac{1+\lambda_1}{1+\lambda}\Delta_v + \alpha(1+\lambda_2)\Delta_c < 0$. Hence, the two cases are disjunct.

It remains to analyze the case where $(1-\alpha)\frac{1+\lambda_1}{1+\lambda}\Delta_v + \alpha(1+\lambda_2)\Delta_c < 0 < (1-\alpha)(1+\lambda_1)\Delta_v + \alpha\frac{1+\lambda_2}{1+\lambda}\Delta_c$. This case can occur only if $\Delta_v > 0$ and $\Delta_c < 0$, i.e., if $\hat{x}(\theta)$ is unambiguously better than $\bar{x}$. By the concavity of $GNP(p)$ it can readily be shown that

$$\left.\frac{\partial GNP(p)}{\partial p}\right|_{p\nearrow\bar{p}} > 0 \quad \text{and} \quad \left.\frac{\partial GNP(p)}{\partial p}\right|_{p\searrow\bar{p}} < 0. \tag{A.4}$$

Thus, in this case the renegotiated price is $\bar{p}$, which completes the proof. □

*Proof of Proposition 3.* The proof is decomposed into two steps. First, we analyze the case $\theta > \bar{\theta}$, with $\bar{\theta}$ implicitly defined by $\partial v(\bar{x},\bar{\theta})/\partial x = \partial c(\bar{x},\bar{\theta})/\partial x$. Thereafter, the renegotiation outcome for states $\theta < \bar{\theta}$ is solved.

*Case 1:* Suppose $\theta > \bar{\theta}$. First, observe that the parties will never agree upon implementing a good $x < \bar{x}$ ex post. For $x < \bar{x}$ the buyer feels a loss in the good dimension and thus demands a price reduction. The necessary price reduction making the buyer accepting the contract is higher than the seller's reduction in costs, because $\theta > \bar{\theta}$. Hence, the parties either renegotiate to a $x > \bar{x}$ or agree on performing the initially specified service $\bar{x}$.

If the parties agree on a $x > \bar{x}$ then the price necessarily needs to increase, since the seller incurs higher costs for the new good. This implies that if renegotiation is successful, i.e. $x > \bar{x}$, then the buyer feels a loss in the money dimension and the seller feels a loss in the good dimension. The $GNP(x,p)$ in this case is given by:

$$GNP(x,p) = \Big\{v(x,\theta) - v(\bar{x},\theta) - (\lambda+1)(p-\bar{p})\Big\}^{\alpha} \times \Big\{p - \bar{p} - (\lambda+1)[c(x,\theta) - c(\bar{x},\theta)\Big\}^{1-\alpha} \tag{A.5}$$

If there is an interior solution, then the interior solution is characterized by the following first-order conditions:

$$\frac{\partial GNP}{\partial p} = 0 \iff$$
$$-\alpha\left[U^B - \underline{U}^B\right]^{\alpha-1}\left[U^S - \underline{U}^S\right]^{1-\alpha}(\lambda+1) + \left[U^B - \underline{U}^B\right]^{\alpha}\left[U^S - \underline{U}^S\right]^{-\alpha}(1-\alpha) = 0, \tag{A.6}$$

and

$$\frac{\partial GNP}{\partial x} = 0 \iff \alpha \left[U^B - \underline{U}^B\right]^{\alpha-1} \left[U^S - \underline{U}^S\right]^{1-\alpha} \frac{\partial v(x,\theta)}{\partial x} + \left[U^B - \underline{U}^B\right]^{\alpha} \left[U^S - \underline{U}^S\right]^{-\alpha} (1-\alpha)(\lambda+1)\frac{\partial c(x,\theta)}{\partial x} = 0. \quad \text{(A.7)}$$

Rearranging (A.6) yields

$$\frac{U^B - \underline{U}^B}{U^S - \underline{U}^S} = (\lambda+1)\frac{\alpha}{1+\alpha}. \quad \text{(A.8)}$$

Similarly, (A.7) can be written as

$$\frac{U^B - \underline{U}^B}{U^S - \underline{U}^S} = \frac{1}{\lambda+1}\,\frac{\alpha}{1+\alpha}\,\frac{\partial v(x,\theta)/\partial x}{\partial c(x,\theta)/\partial x}. \quad \text{(A.9)}$$

The first-order conditions (A.8) and (A.9) together imply that

$$R(x,\theta) \equiv \frac{\partial v(x,\theta)/\partial x}{\partial c(x,\theta)/\partial x} = (\lambda+1)^2. \quad \text{(A.10)}$$

Note that $R(x,\theta)$ is strictly decreasing in $x$ by Assumption 1. Moreover, $\lim_{x\to\infty} R(x,\theta) < 1$. Hence, if there are $x > \bar{x}$ such that $R(x,\theta) > (1+\lambda)^2$ then there is a unique $x$ at which $R(x,\theta) = (1+\lambda)^2$. We denote this solution by $\hat{x}^H(\theta)$. If $R(x,\theta) \leq (1+\lambda)^2$ for all $x > \bar{x}$ then it does not pay off for the parties to renegotiate the original contract, because this would lead to losses for the parties that are higher than the net benefit in intrinsic utilities $v - c$.

For which realizations of the state $\theta$ does the optimality condition (A.10) characterize a $\hat{x}^H > \bar{x}$? Put differently, when do goods $x > \bar{x}$ exist such that $R(x,\theta) > (1+\lambda)^2$. This is the case if the realized state is sufficiently high, i.e., if $\theta > \theta^H$, with $\theta^H$ being implicitly defined by $R(\bar{x},\theta^H) = (\lambda+1)^2$. Note that $\hat{x}^H(\theta^H) = \bar{x}$ by definition.

We conclude the first step by noting that the parties are indeed better of when $\hat{x}^H(\theta) > \bar{x}$ is implemented ex post for $\theta > \theta^H$. By Proposition 1 $x$ can be implemented ex post iff

$$\begin{aligned} v(x,\theta) - v(\bar{x},\theta) &\geq (1+\lambda)^2[c(x,\theta) - c(\bar{x},\theta)] && \text{(A.11)} \\ \iff \int_{\bar{x}}^{x} \frac{\partial v(z,\theta)}{\partial x} dz &\geq (1+\lambda)^2 \int_{\bar{x}}^{x} \frac{\partial c(z,\theta)}{\partial x} dz. && \text{(A.12)} \end{aligned}$$

The above condition is satisfied for $x = \hat{x}^H$ by Assumption 1. Hence, there are prices $p \geq \bar{p}$ such that both parties prefer the new contract with good $\hat{x}^H$ to the initial contract.

*Case 2:* Suppose $\theta < \bar{\theta}$. This case can be proved by similar reasonings as used in the proof of case 1. We outline only the few differences. Obviously, if renegotiation is successful, then the parties agree upon a good $x < \bar{x}$ and a price $p < \bar{p}$. The $GNP$ is given by

$$GNP(x,p) = \Big\{\bar{p} - p - (\lambda+1)[v(\bar{x},\theta) - v(x,\theta)]\Big\}^{\alpha} \times \Big\{c(\bar{x},\theta) - c(x,\theta) - (\lambda+1)(\bar{p} - p)\Big\}^{1-\alpha} \tag{A.13}$$

From the two first-order conditions we obtain the following optimality condition which is independent of the price,

$$R(x,\theta) = \frac{1}{(1+\lambda)^2}. \tag{A.14}$$

If $R(x,\theta) > 1/(1+\lambda)^2$ for all $x \in [0,\bar{x}]$, then the parties carry out the initial service $\bar{x}$. Note that $\lim_{x\to 0} R(x,\theta) = \infty$. Thus, if there are $x < \bar{x}$ such that $R(x,\theta) < 1/(1+\lambda)^2$, then there is a unique $x \in (0,\bar{x})$ at which $R(x,\theta) = 1/(1+\lambda)^2$. We denote this solution by $\hat{x}^L(\theta)$. The solution is indeed lower than the initially specified good ($\hat{x}^L < \bar{x}$) if $\theta$ is sufficiently low, i.e. if $\theta < \theta^L$, implicitly defined by $R(\bar{x},\theta^L) = 1/(1+\lambda)^2$. Noting that $\hat{x}^L(\theta^L) = \bar{x}$ by definition completes the second step.

Obviously, for $\theta = \bar{\theta}$ the parties cannot benefit from renegotiating the initial contract, which completes the proof. □

*Proof of Corollary 1.* First, note that the bounds $\theta^L$ and $\theta^H$ do not depend on the distribution. Moreover, for $\theta < \tilde{\theta}$ we have $F_1(\theta) < F_2(\theta)$ and for $\theta > \tilde{\theta}$ it holds that $F_2(\theta) < F_1(\theta)$. From the definition of $\rho(F)$ it follows immediately that $\rho(F_1) < \rho(F_2)$. □

*Proof of Corollary 2.* We have to show that $\rho(F,\lambda) = F(\theta^L(\lambda)) + 1 - F(\theta^H(\lambda))$ is decreasing in $\lambda$. We first show that $\frac{\partial\theta^L(\lambda)}{\partial\lambda} < 0$. By Proposition 3 $\theta^L$ is implicitly defined by $\hat{x}^L(\theta^L) - \bar{x} = 0$. By the implicit function theorem

$$\frac{\partial\theta^L}{\partial\lambda} = -\frac{\partial\hat{x}^L/\partial\lambda}{\partial\hat{x}^L/\partial\theta^L}. \tag{A.15}$$

$\hat{x}^L(\theta,\lambda)$ is implicitly defined by $\frac{\partial v(\hat{x}^L,\theta)}{\partial x} - \frac{1}{(1+\lambda)^2}\frac{\partial c(\hat{x}^L,\theta)}{\partial x} = 0$. Using the implicit function theorem twice again we get

$$\frac{\partial \hat{x}^L}{\partial \theta^L} = -\frac{\frac{\partial^2 v}{\partial x \partial \theta} - \frac{1}{(1+\lambda)^2}\frac{\partial^2 c}{\partial x \partial \theta}}{\frac{\partial^2 v}{\partial x^2} - \frac{1}{(1+\lambda)^2}\frac{\partial^2 c}{\partial x^2}} > 0, \tag{A.16}$$

$$\frac{\partial \hat{x}^L}{\partial \lambda} = -\frac{2(1+\lambda)^{-3}\frac{\partial c}{\partial x}}{\frac{\partial^2 v}{\partial x^2} - \frac{1}{(1+\lambda)^2}\frac{\partial^2 c}{\partial x^2}} > 0. \tag{A.17}$$

In both equations the numerator is positive by Assumption 1, while the denominators are negative by Assumption 1. Thus, $\frac{\partial \theta^L}{\partial \lambda} < 0$. By the same line of argument it is straightforward to show that $\frac{\partial \theta^H}{\partial \lambda} > 0$. Hence, we get

$$\frac{\partial \rho}{\partial \lambda} = \underbrace{\frac{\partial F}{\partial \theta}}_{>0}\underbrace{\frac{\partial \theta^L}{\partial \lambda}}_{<0} - \underbrace{\frac{\partial F}{\partial \theta}}_{>0}\underbrace{\frac{\partial \theta^H}{\partial \lambda}}_{>0} < 0 \ . \tag{A.18}$$

$\square$

*Proof of Proposition 4.* By Proposition 2 the renegotiated price $\hat{p}$ is the initial price $\bar{p}$ corrected by a term that depends only on $\lambda$, $\bar{x}$ and $\theta$. For $C(\lambda, \bar{x}, \bar{p})$ only $|\hat{p} - \bar{p}|$ plays a role. But in this term $\bar{p}$ cancels out. Thus, $C(\cdot)$ is independent of $\bar{p}$.

Showing that $C(\lambda, \bar{x}, \bar{p})$ is increasing in $\lambda$ is equivalent to showing that for any initial contract $(\bar{x}, \bar{p})$ the surplus from renegotiation $\Delta S = \Delta U^B + \Delta U^S$ is decreasing with $\lambda$. Two cases have to be distinguished:

1. Suppose that $\theta > \bar{\theta}$. In this case $\hat{x} \geq \bar{x}$ and $\hat{p} \geq \bar{p}$. Thus

$$\begin{aligned}
\Delta S &= v(\hat{x},\theta) - \hat{p} - \lambda[\hat{p} - \bar{p}] - v(\bar{x},\theta) + \bar{p} + \hat{p} - c(\hat{x},\theta) - \lambda[c(\hat{x},\theta) - c(\bar{x},\theta)] - \bar{p} + c(\bar{x},\theta) \\
&= [v(\hat{x},\theta) - v(\bar{x},\theta)] - (1+\lambda)[c(\hat{x},\theta), -c(\bar{x},\theta)] - \lambda[\hat{p} - \bar{p}] \\
&= \Delta_v - (1+\lambda)\Delta_c - \lambda[\hat{p} + \frac{1-\alpha}{1+\lambda}\Delta_v + \alpha(1+\lambda)\Delta_c - \bar{p}] \\
&= \frac{1+\alpha\lambda}{1+\lambda}[\Delta_v - (1+\lambda)^2\Delta_c] \qquad \text{(A.19)}
\end{aligned}$$

Differentiating with respect to $\lambda$ we get:

$$\begin{aligned}
\frac{\partial \Delta S}{\partial \lambda} &= \frac{\alpha(1+\lambda)-(1+\alpha\lambda)}{(1+\lambda)^2}\left[\Delta_v-(1+\lambda)^2\Delta_c\right] \\
&\quad + \left[\frac{\partial v}{\partial x}\frac{\partial \hat{x}}{\partial \lambda} - 2(1+\lambda)\Delta_c - (1+\lambda)^2\frac{\partial c}{\partial x}\frac{\partial \hat{x}}{\partial \lambda}\right]\frac{1+\alpha\lambda}{1+\lambda} \\
&= \underbrace{-\frac{1-\alpha}{(1+\lambda)^2}}_{<0}\underbrace{\left[\Delta_v-(1+\lambda)^2\Delta_c\right]}_{>0} \\
&\quad + \left[\underbrace{\left[\frac{\partial v}{\partial x}-(1+\lambda)^2\frac{\partial c}{\partial x}\right]}_{=0}\frac{\partial \hat{x}}{\partial \lambda}\underbrace{-2(1+\lambda)\Delta_c}_{\leq 0}\right]\underbrace{\frac{1+\alpha\lambda}{1+\lambda}}_{>0} \leq 0. \qquad \text{(A.20)}
\end{aligned}$$

To see that $\frac{\partial \Delta S}{\partial \lambda} < 0$ at $\lambda = 0$, note $\frac{\partial \Delta S}{\partial \lambda} = 0$ if and only if $\Delta_c = 0$. However, for every $\theta \neq \bar{\theta}$ there exists a $\bar{\lambda}$ sufficiently close to zero such that for all $\lambda < \bar{\lambda}$ we have $\Delta_c > 0$, and the strict inequality holds.

2. Suppose now that $\theta < \bar{\theta}$. In this case $\hat{x} \leq \bar{x}$ and $\hat{p} \leq \bar{p}$. Thus

$$\begin{aligned}
\Delta S &= v(\hat{x},\theta) - \hat{p} - \lambda[\hat{v}(\hat{x},\theta) - v(\hat{x},\theta)] - v(\bar{x},\theta) + \bar{p} + \hat{p} - c(\hat{x},\theta) - \lambda[\hat{p} - p] - \bar{p} + c(\bar{x},\theta) \\
&= -(1+\lambda)[v(\hat{x},\theta) - v(\bar{x},\theta)] + [c(\bar{x},\theta), -c(\hat{x},\theta)] - \lambda[\hat{p} - \bar{p}] \\
&= \Delta_c - (1+\lambda)\Delta_v - \lambda\left[\bar{p} - \bar{p} + (1-\alpha)(1+\lambda)\Delta_v + \frac{\alpha}{1+\lambda}\Delta_c\right] \\
&= \frac{1+\lambda-\alpha\lambda}{1+\lambda}[\Delta_c - (1+\lambda)^2\Delta_v]. \qquad \text{(A.21)}
\end{aligned}$$

Differentiating with respect to $\lambda$ we get:

$$\begin{aligned}
\frac{\partial \Delta S}{\partial \lambda} &= \frac{(1-\alpha)(1+\lambda)-(1+\lambda-\alpha\lambda)}{(1+\lambda)^2}\left[\Delta_c-(1+\lambda)^2\Delta_v\right] \\
&\quad + \left[\frac{\partial c}{\partial x}\frac{\partial \hat{x}}{\partial \lambda} - 2(1+\lambda)\Delta_v - (1+\lambda)^2\frac{\partial v}{\partial x}\frac{\partial \hat{x}}{\partial \lambda}\right]\frac{1+\lambda-\alpha\lambda}{1+\lambda} \\
&= \underbrace{-\frac{\alpha}{(1+\lambda)^2}}_{<0}\underbrace{\left[\Delta_c-(1+\lambda)^2\Delta_v\right]}_{>0} \\
&\quad + \left[\underbrace{\left[\frac{\partial c}{\partial x}-(1+\lambda)^2\frac{\partial v}{\partial x}\right]}_{=0}\frac{\partial \hat{x}}{\partial \lambda}\underbrace{-2(1+\lambda)\Delta_v}_{\leq 0}\right]\underbrace{\frac{1+\alpha\lambda}{1+\lambda}}_{>0} \leq 0. \qquad \text{(A.22)}
\end{aligned}$$

To see that $\frac{\partial \Delta S}{\partial \lambda} < 0$ at $\lambda = 0$, note again that $\frac{\partial \Delta S}{\partial \lambda} = 0$ if and only if $\Delta_v = 0$. However, for every $\theta \neq \bar{\theta}$ there exists a $\bar{\lambda}$ sufficiently close to zero such that for all $\lambda < \bar{\lambda}$ we have $\Delta_v > 0$, and the strict inequality holds. □

*Proof of Proposition 5.* The parties either write no long-term contract or a sales contract specifying one of the three goods.

*No contract (spot contracting):* If the parties have not written a contract ex ante, then they meet on the spot market and the materially efficient good is always traded. The generated surplus from spot contracting is $S^{nc} = v^* - c^*$.

*(Long-term) contract $(x_i, \bar{p})$ with $i = 1, 2$:* Let $\bar{x} = x_1$ w.l.o.g., the analysis for $\bar{x} = x_2$ is equivalent due to the imposed symmetry. If $\theta_1$ materializes the materially efficient good $x_1$ is traded. If, on the other hand, $\theta_2$ materializes the parties need to renegotiate the initial contract in order to implement the materially efficient good $\hat{x}(\theta_2) = x_2$. The parties will agree upon the new contract if and only if

$$v^* \geq (1+\lambda)^2 c^* \quad \Longleftrightarrow \quad \lambda \leq \sqrt{\frac{v^*}{c^*}} - 1 =: \bar{\lambda}. \tag{A.23}$$

If this is the case the good $x_2$ is traded at the renegotiated price

$$\hat{p} = \bar{p} + (1-\alpha)\frac{1}{1+\lambda}v^* + \alpha(1+\lambda)c^*. \tag{A.24}$$

The ex ante expected social surplus in this case is given by

$$S(x_1, x_2|x_1) = v^* - c^* - \frac{1}{2}\lambda\left[\frac{1-\alpha}{1+\lambda}v^* + [1+\alpha(1+\lambda)]c^*\right] + B. \tag{A.25}$$

If $\lambda > \bar{\lambda}$ renegotiation does not take place and $x_1$ is delivered in both states of nature. The expected surplus is given by $S(x_1, x_1|x_1) = \frac{1}{2}(v^* - c^*) + B$. Note that $S(x_1, x_2|x_1)$ is strictly decreasing in $\lambda$ and approaches $S(x_1, x_1|x_1)$ for $\lambda \to \bar{\lambda}$.

Instead of implementing $x_2$ after renegotiation the parties could also agree upon delivering $\hat{x}(\theta_2) = x_3$. This is feasible, again, if and only if $\lambda \leq \bar{\lambda}$. It can easily be verified, however, that the parties always prefer to implement the materially efficient good if renegotiation is feasible, i.e. if $\lambda \leq \bar{\lambda}$.

*(Long-term) contract specifying the compromise good* $(x_3, \bar{p})$*:* If the initial contract is $(x_3, \bar{p})$ the parties will renegotiate this contract such that the materially efficient good is traded in all states if and only if $\lambda \leq \bar{\lambda}$. If renegotiation is successful, then the materially efficient good is traded at the new price:

$$\hat{p} = \bar{p} + (1-\alpha)\frac{1}{1+\lambda}(1-\beta)v^* + \alpha(1+\lambda)(1-\beta)c^*. \tag{A.26}$$

For $\lambda > \bar{\lambda}$ the materially efficient good is not in the renegotiation set and the compromise good $x_3$ is traded in both states. In this case the social surplus is given by

$$S(x_3, x_3|x_3) = \beta(v^* - c^*) + B. \tag{A.27}$$

If, on the other hand, $\lambda \leq \bar{\lambda}$, the expected surplus from contracting on the compromise good is

$$S(x_1, x_2|x_3) = v^* - c^* - (1-\beta)\lambda\left[\frac{1-\alpha}{1+\lambda}v^* + [1+\alpha(1+\lambda)]c^*\right] + B. \tag{A.28}$$

Similar as above, $S(x_1, x_2|x_3)$ is strictly decreasing in $\lambda$ and approaches $S(x_3, x_3|x_3)$ for $\lambda \to \bar{\lambda}$.

*Statements of the proposition:* Now we are prepared to prove the two parts of the proposition. We start with part (ii). For $\lambda > \bar{\lambda}$ renegotiation does not take place under all long-term contracts. It is readily obtained that $S(x_3, x_3|x_3) \geq S(x_1, x_1|x_1)$ if and only if $\beta \geq 1/2$. For $\lambda \leq \bar{\lambda}$ the materially efficient good is always traded ex post. Comparing the expressions for the social surpluses reveals that $S(x_1, x_2|x_3) \geq S(x_1, x_2|x_1)$ if and only if $\beta \geq 1/2$.

Now we prove part (i). Consider $\lambda \leq \bar{\lambda}$. For $\beta > 1/2$ the maximum expected surplus generated by a sales contract is $S(x_1, x_2|x_1)$. For $\lambda \to 0$ we have $S(x_1, x_2|x_1) > S^{nc}$, because $B > 0$. For $\lambda \to \bar{\lambda}$ it holds that $S(x_1, x_2|x_1) \to \frac{1}{2}(v^* - c^*) + B = S(x_1, x_1|x_1)$. By noting that $S(x_1, x_2|x_1)$ is continuous and strictly decreasing in $\lambda$ and that $B < \frac{1}{2}(v^* - c^*)$ we can conclude that there is a unique value $\tilde{\lambda}(B) \in (0, \bar{\lambda})$ such that $S(x_1, x_2|x_1) = S^{nc}$.

For $\beta \geq 1/2$ the maximum expected surplus generated by a sales contract is $S(x_1, x_2|x_3)$. For $\lambda \to 0$ we have $S(x_1, x_2|x_3) > S^{nc}$, because $B > 0$. For $\lambda \to \bar{\lambda}$ it holds that $S(x_1, x_2|x_3) \to \beta(v^* - c^*) + B = S(x_3, x_3|x_3)$. By noting that $S(x_1, x_2|x_3)$ is continuous and strictly decreasing in $\lambda$ and that $B < (1-\beta)(v^* - c^*)$ we can conclude that there is a unique value $\tilde{\lambda}(B) \in (0, \bar{\lambda})$ such that $S(x_1, x_2|x_3) = S^{nc}$, which establishes the desired result. $\square$

*Proof of Proposition 6.* Because $I^{CN} = \pi_1 < \pi_1 + \alpha(1-\pi_1) = I^{CR}$ we have $\bar{\lambda}(I^{CN}) < \bar{\lambda}(I^{CR})$. Thus, if $\lambda \leq \bar{\lambda}(I^{CN})$ the buyer anticipates that there will be renegotiation if $\theta \neq \theta_1$, so invests $I^{CR}$. Similarly, if $\lambda > \bar{\lambda}(I^{CR})$ the buyer anticipates that there will be no renegotiation if $\theta \neq \theta_1$, so he invests $I^{CN}$. If, however, $\bar{\lambda}(I^{CN}) < \lambda \leq \bar{\lambda}(I^{CR})$ there are two candidates for the optimal strategy of the buyer. He may invest $I^{CR}$ which is the optimal investment given that with $I^{CR}$ there will be renegotiation if $\theta \neq \theta_1$, or he may invest $I^{CN}$ which is the optimal investment given that with $I^{CN}$ there will be no renegotiation if $\theta \neq \theta_1$. If he chooses the first strategy and invests $I^{CR}$ his expected utility is

$$EU^B(I^{CR}, \lambda) = v^* + I^{CR} - \frac{1}{2}(I^{CR})^2 - \bar{p} - (1-\pi_1)\left[(1-\alpha)(v^* + I^{CR} - \underline{v}) + \alpha(1+\lambda)^2(c^* - \underline{c})\right] \tag{A.29}$$

If he follows the second strategy his expected utility is

$$\begin{aligned} EU^B(I^{CN}, \lambda) &= \pi_1(v^* + I^{CN}) + (1-\pi_1)\underline{v} - \frac{1}{2}(I^{CN})^2 - \bar{p} \\ &= v^* + I^{CN} - \frac{1}{2}I^{CN2} - \bar{p} - (1-\pi_1)\left[v^* + I^{CN} - \underline{v}\right] \end{aligned} \tag{A.30}$$

The buyer prefers the first strategy over the second if and only if

$$\begin{aligned} & I^{CR} - \frac{1}{2}(I^{CR})^2 - I^{CN} + \frac{1}{2}(I^{CN})^2 \geq \\ & \quad (1-\pi_1)\left[(1-\alpha)(v^* + I^{CR} - \underline{v}) - (v^* + I^{CN} - \underline{v}) + \alpha(1+\lambda)^2(c^* - \underline{c})\right] \\ \Leftrightarrow \quad & I^{CR} - \frac{1}{2}(I^{CR})^2 - I^{CN} + \frac{1}{2}(I^{CN})^2(1-\pi_1)(1-\alpha)I^{CR} + (1-\pi_1)I^{CN} \geq \\ & \quad (1-\pi_1)\left[\alpha(1+\lambda)^2(c^* - \underline{c}) - \alpha(v^* - \underline{v})\right] \\ \Leftrightarrow \quad & (\pi_1 + \alpha(1-\pi_1))I^{CR} - \frac{1}{2}(I^{CR})^2 - \pi_1 I^{CN} + \frac{1}{2}(I^{CN})^2 \geq \\ & \quad (1-\pi_1)\left[\alpha(1+\lambda)^2(c^* - \underline{c}) - \alpha(v^* - \underline{v})\right] \\ \Leftrightarrow \quad & \frac{1}{2}\left(\pi_1 + \alpha(1-\pi_1)\right)^2 - \frac{1}{2}\pi_1^2 + \alpha(1-\pi_1)(v^* - \underline{v}) \geq (1-\pi_1)\left[\alpha(1+\lambda)^2(c^* - \underline{c})\right] \\ \Leftrightarrow \quad & \frac{\frac{1}{2}\left(\pi_1^2 + 2\alpha\pi_1(1-\pi_1) + \alpha^2(1-\pi_1)^2 - \pi_1^2\right) + \alpha(1-\pi_1)(v^* - \underline{v})}{(1-\pi_1)\alpha(c^* - \underline{c})} \geq (1+\lambda)^2 \\ \Leftrightarrow \quad & \frac{v^* - \underline{v} + \pi_1 + \frac{\alpha}{2}(1-\pi_1)}{c^* - \underline{c}} \geq (1+\lambda)^2 \\ \Leftrightarrow \quad & \lambda \leq \sqrt{\frac{v^* - \underline{v} + \pi_1 + \frac{\alpha}{2}(1-\pi_1)}{c^* - \underline{c}}} - 1 \equiv \bar{\lambda} \end{aligned} \tag{A.31}$$

Note that $\bar{\lambda}(I^{CN}) < \bar{\lambda} < \bar{\lambda}(I^{CR})$. Thus if $\lambda \leq \bar{\lambda}$ the buyer prefers to invest $I^{CR}$ and there will be renegotiation if $\theta \neq \theta_1$, while if $\lambda > \bar{\lambda}$ the buyer invests $I^{CN}$ and there will be no renegotiation if $\theta \neq \theta_1$.

(1) Suppose $\lambda \leq \bar{\lambda}$. If the parties write a specific performance contract the buyer invests $I^{CR} = \pi_1 + \alpha(1-\pi_1)$ and his expected utility is

$$EU^B = v^* + I^{CR} - \frac{1}{2}(I^{CR})^2 - \bar{p} - (1-\pi_1)\left[(1-\alpha)(v^* + I^{CR} - \bar{v}) + \alpha(1+\lambda)^2(c^* - \underline{c})\right] \tag{A.32}$$

while the expected utility of the seller is

$$EU^S = \bar{p} - c^* + (1-\pi_1)\left[\frac{1-\alpha}{1+\lambda}(v^* + ICR - \underline{v}) + \alpha(1+\lambda)(c^* - \underline{c}) - \lambda(c^* - \underline{c})\right] \tag{A.33}$$

Thus, total expected social surplus is

$$\begin{aligned} ES^{CR} = {} & v^* - c^* + I^{CR} - \frac{1}{2}(I^{CR})^2 \\ & - (1-\pi_1)\left[\frac{\lambda(1-\alpha)}{1+\lambda}(v^* + I^{CR} - \underline{v}) + (\alpha\lambda(1+\lambda) + \lambda)(c^* - \underline{c})\right] \end{aligned} \tag{A.34}$$

If the parties rely on the allocation of asset ownership the buyer invests $I^A = \beta + \alpha(1-\beta)$ and total social surplus is

$$S^A = v^* - c^* + I^A - \frac{1}{2}(I^A)^2 \tag{A.35}$$

Thus, $S^A \geq ES^{CR}$ if and only if

$$\begin{aligned} & I^A - \frac{1}{2}(I^A)^2 \geq \\ & I^{CR} - \frac{1}{2}(I^{CR})^2 - (1-\pi_1)\left[\frac{\lambda(1-\alpha)}{1+\lambda}(v^* + I^{CR} - \underline{v}) + (\alpha\lambda(1+\lambda) + \lambda)(c^* - \underline{c})\right] \end{aligned} \tag{A.36}$$

which is equivalent to (35). It is straightforward to check that $\Phi'(\lambda) > 0$.

(2) Suppose that $\lambda > \bar{\lambda}$. If the parties write a specific performance contract the buyer invests $I^{CN} = \pi_1$, there will be no renegotiation, and his expected utility is

$$EU^B = \pi_1(v^* + I^{CN}) + (1-\pi_1)\underline{v} - \bar{p} - \frac{1}{2}(I^{CN})^2 \tag{A.37}$$

while the expected utility of the seller is

$$EU^S = \bar{p} - \pi_1 c^* - (1-\pi_1)\underline{c} \tag{A.38}$$

so total expected social surplus is

$$ES^{CN} = \pi_1[v^* - c^*] + (1-\pi_1)(\underline{v} - \underline{c}) + \pi_1 I^{CN} - \frac{1}{2}(I^{CN})^2 \tag{A.39}$$

If the parties rely on the allocation of asset ownership total social surplus is given by (A.35). Thus, $S^A \geq ES^{CN}$ if and only if

$$I^A - \frac{1}{2}(I^A)^2 \geq \pi_1 I^{CN} - \frac{1}{2}(I^{CN})^2 - (1-\pi_1)(v^* - c^* - \underline{v} + \underline{c}) \tag{A.40}$$

which is equivalent to (36).

□

*Proof of Lemma 1.* First, suppose that the parties write a specific performance contract $(\bar{x}, \bar{p})$. With probability one the realized state of the world is such that $x^*(\theta) \neq \bar{x}$. Thus, by Proposition 2 there is scope for renegotiation if and only if there exists a $p$ such that

$$\frac{v^* - \underline{v}}{1+\lambda} \geq p - \bar{p} \geq (1+\lambda)(c^* - \underline{c}) \tag{A.41}$$

Such a price $p$ exists if and only if

$$v^* - \underline{v} \geq (1+\lambda)^2(c^* - \underline{c}) \iff \lambda \leq \sqrt{\frac{v^* - \underline{v}}{c^* - \underline{c}}} - 1 \equiv \bar{\lambda}^S \tag{A.42}$$

If $\lambda < \bar{\lambda}^S$ the parties renegotiate and trade the service $x^*(\theta)$ at price

$$\hat{p}^S = \bar{p} + \frac{1-\alpha}{1+\lambda}[v^* - \underline{v}] + \alpha(1+\lambda)[c^* - \underline{c}] \tag{A.43}$$

In this case the buyer's utility is $U^B = v^* - \hat{p}^S - \lambda[\hat{p}^S - \bar{p}]$ while the seller's utility is $U^S = \hat{p}^S - c^* - \lambda[c^* - \underline{c}]$. If $\lambda > \bar{\lambda}^S$ there is no renegotiation and payoffs are $U^B = \underline{v} - p$ and $U^S = p - \underline{c}$. Thus, the total surplus generated by a specific performance contract is given by

$$S^S = \begin{cases} v^* - c^* - \lambda(1 + \alpha(1+\lambda))[c^* - \underline{c}] - \frac{\lambda(1-\alpha)}{1+\lambda}[v^* - \underline{v}] & \text{if } \lambda \leq \bar{\lambda}^S \\ \underline{v} - \underline{c} & \text{if } \lambda > \bar{\lambda}^S \end{cases} \tag{A.44}$$

Suppose now that the parties write an authority contract with price $\bar{p}$ that gives the buyer the right to choose any $x \in X$ as he sees fit. Thus, without renegotiation the buyer would order the seller to deliver $\tilde{x}(\theta) \neq x^*(\theta)$. There is scope for renegotiation if there exists a price $p$ such that

$$(1+\lambda)(\tilde{v} - v^*) \leq \bar{p} - p \leq \frac{\tilde{c} - c^*}{1+\lambda} \tag{A.45}$$

Such a price $p$ exists if and only if

$$\tilde{v} - v^* \leq \frac{\tilde{c} - c^*}{(1+\lambda)^2} \iff \lambda \leq \sqrt{\frac{\tilde{c} - c^*}{\tilde{v} - v^*}} - 1 \equiv \bar{\lambda}^A \tag{A.46}$$

If $\lambda < \bar{\lambda}^A$ the parties renegotiate and trade the service $x^*(\theta)$ at price

$$\hat{p}^A = \bar{p} - (1-\alpha)(1+\lambda)[\tilde{v} - v^*] - \frac{\alpha}{1+\lambda}[\tilde{c} - c^*] \tag{A.47}$$

In this case the buyer's utility is $U^B = v^* - \hat{p}^A - \lambda[\tilde{v} - v^*]$ while the seller's utility is $U^S = \hat{p}^A - c^* - \lambda[\bar{p} - \hat{p}^A]$. If $\lambda > \bar{\lambda}^A$ there is no renegotiation and payoffs are $U^B = \tilde{v} - p$ and $U^S = p - \tilde{c}$. Thus, the total surplus generated by an authority contract is given by

$$S^S = \begin{cases} v^* - c^* - \lambda(1 + (1-\alpha)(1+\lambda))[\tilde{v} - v^*] - \frac{\lambda\alpha}{1+\lambda}[\tilde{c} - c^*] & \text{if } \lambda \leq \bar{\lambda}^A \\ \tilde{v} - \tilde{c} & \text{if } \lambda > \bar{\lambda}^A \end{cases} \tag{A.48}$$

□

*Proof of Proposition 7.* Comparing the efficient social surplus $S^* = v^* - c^*$ to the social surplus generated by an authority contract, the efficiency loss of the authority contract is given by

$$S^* - S^A(\lambda) = \begin{cases} \lambda(1 + (1-\alpha)(1+\lambda))[\tilde{v} - v^*] + \frac{\lambda\alpha}{1+\lambda}[\tilde{c} - c^*] & \text{if } \lambda \leq \bar{\lambda}^A \\ (v^* - c^*) - (\tilde{v} - \tilde{c}) & \text{if } \lambda > \bar{\lambda}^A \end{cases} \tag{A.49}$$

Similarly, comparing the efficient social surplus $S^*$ to the social surplus generated by a specific performance contract, the efficiency loss of the specific performance contract is given by

$$S^* - S^S(\lambda) = \begin{cases} \lambda(1 + \alpha(1+\lambda))[c^* - \underline{c}] + \frac{\lambda(1-\alpha)}{1+\lambda}[v^* - \underline{v}] & \text{if } \lambda \leq \bar{\lambda}^S \\ (v^* - c^*) - (\underline{v} - \underline{c}) & \text{if } \lambda > \bar{\lambda}^S \end{cases} \tag{A.50}$$

The proposition follows directly from comparing the efficiency losses of the two contracts in the different cases. □

*Proof of Proposition 8.* By Proposition 1 the renegotiation set is independent of $\bar{p}$ and by Proposition 3 the renegotiation outcome and the renegotiation markup $\hat{p} - \bar{p}$ is also independent of $\bar{p}$. Thus, no matter which $\bar{p}(\sigma)$ is in place at the renegotiation stage, the renegotiation outcome is always the same. □

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