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THE IMPACT OF SOME INVESTMENT FUNCTIONS IN A KALDORIAN GROWTH MODEL

by

Ekkehart Schlicht

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THE IMPACT OF SOME INVESTMENT FUNCTIONS IN A KALDORIAN GROWTH MODEL[1)]

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Ekkehart Schlicht

Fachbereich Wirtschaftswissenschaft, Universität Regensburg, Regensburg, Germany

1. Introduction

It is often claimed that 'the really important difference between the [neoclassical and the Cambridge theories of growth and distribution lies] on the determination of investment in the economy. Does savings with proper government intervention determine investment, that is, is investment sufficiently elastic that it can be made equal to desired savings, or does the distribution of income adjusts savings to investment?'[2)] It seems, therefore, that the long run impact of alternative investment behaviour is of considerable importance, and it is into this I wish to explore.

It is generally recognized that the problem of the determination of investment in all growth models is extraordinarily demanding[3)], and I apologize for not being able to propose a satisfactory theory of investment presently. My point is a different one. I will try to show in the following that alternative assumptions about investment behaviour are capable of leading to crucially divergent results.

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1) paper presented to the Winter Symposium of the Econometric Society, Budapest, January 26th-28th, 1974. I would like to thank W.VOGT for helpful conversations.

2) STIGLITZ/UZAWA p. 313

3) BLISS p.1, KALDOR (1961) p.210 f.

The impact of alternative investment behaviour can be analyzed rather transparently in Kaldorian growth models, since these models are always containing an investment function which needs only to be replaced by the respective alternatives we wish to study. It would be more difficult to incorporate suitable investment functions into neoclassical growth models[1], although I feel that one could frame the following considerations neoclassically, too.

There seems to be some disagreement pertaining to Kaldorian growth models. As STIGLITZ and UZAWA put it, 'neoclassicists accuse the Cambridge school of *ad hoc*-ery'[2], and this reproach is certainly not unfounded regarding the technical progress function in Kaldorian models. But the corresponding neoclassical *deus ex machina*, i.e. the innovation possibility frontier, can be legitimately attacked on the same ground[3]. On the other hand, the Kaldorian theory is much simpler concerning the treatment of technical progress. It is not necessary to introduce those shifting parameters of the production function called efficiency of capital and of labour, respectively, which are so difficult to interpret[4], and to postulate an invariant relationship between the corresponding airy growth rates. On the contrary, Harrod neutrality comes out quite naturally, even without mentioning it.

The points which I want to raise can be discussed most simply in a malleable capital framework as used by KALDOR (1957,1961). The vintage approach, as outlined in KALDOR/MIRRLEES would make things very difficult indeed, and this would perhaps overshadow the main concern. I want to keep things simple.

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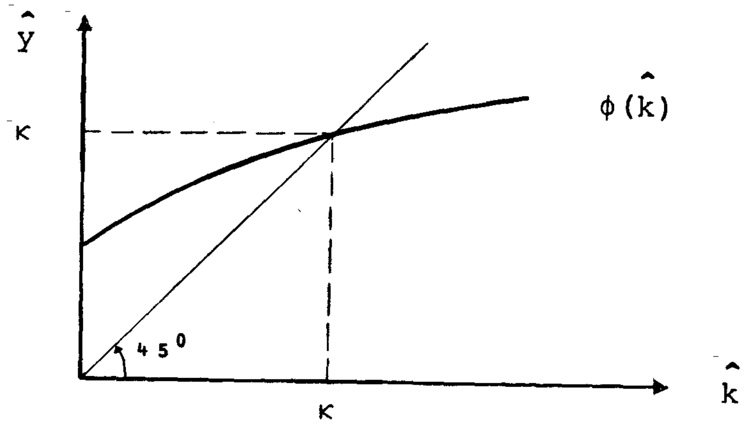
1) see on this the important paper by BLISS. For the so-called KEYNES-WICKSELL-models, see e.g. FISCHER.
2) p. 310 n.3
3) STIGLITZ/UZAWA p. 121 f.
4) SAMUELSON p.355

2. The Model Framework

2.1. The Technical Progress Function

Denote output by Y, capital by K and labour by N. $y = Y/N$ is labour productivity, and $k = K/N$ is capital invested per man. KALDOR assumes that the rate of capital deepening $\hat{k}$ determines the growth of productivity $\hat{y}$[1]. This is described by the technical progress function (Figure):

(1 $\quad \hat{y} = \phi(\hat{k}), \quad \phi(o)>o, \ \phi'>o, \ \phi''<O, \ \phi(\kappa)=\kappa$ for some $\kappa>O$



In other words, if capital invested per man remains constant through time, there will be some increases in productivity. Increasing investment per man allows for a higher productivity growth, but with decreasing returns. Furthermore, if the rate of capital deepening exceeds a certain critical value κ, the induced rate of productivity growth will be smaller than the rate of capital deepening.

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1) KALDOR (1957,1961). Regarding notation: Let $x(t)$ be a real function of time t. We write $\dot{x}$ for $dx(t)/dt$ and $\hat{x}$ for $\dot{x}/x$.

It should be noted that the concept of capital as used in neoclassical models differs slightly from the present use: In neoclassical models, K is interpreted as an index of the aggregate capital stock. Resources which have been devoted to the production of technical progress are excluded. In contrast, the Kaldorian notion of capital refers to *all* investment expenditures which have been made, even those made for research[1]. There is no distinction between the 'capital stock' and the 'level of technique', as a consequence, and it is maintained that such a distinction is not very useful since there is not much sense in distinguishing between outlays for machinery and expenditure for the development of its construction. Instead it is assumed from the beginning that investment expenditure is distributed optimally among its different uses.

Define

(2) $$x := y/k = Y/K$$

as the output-capital ratio implied by y and k. The technical progress function can be interpreted as determining the changes of the parameters of the Leontief production function

(3 $$Y = \min \{ y \cdot N, x \cdot K \}$$

over time which are dependent upon capital deepening[2]:

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1) depreciation excluded

2) $\hat{y}$ can be interpreted as the rate of labour-augmenting technical progress and $\hat{x}$ as the rate of capital-augmenting technical progress, cf. SOLQW et al.p. 82. Technical progress is Hicks-neutral for $\hat{k}$=o and Harrod-neutral for $\hat{k}=\hat{K}$, etc. Eliminating $\hat{k}$ from (4) gives the innovation possibility curve as derived by KENNEDY, p. 547 n.1, but this obscures perhaps the Kaldorian view that it is the rate of capital deepening which determines the bias and there is no choice of the bias independent of capital deepening.

$$(4) \qquad \hat{y} = \phi(\hat{k}) \qquad \hat{x} = \phi(\hat{k}) - \hat{k}$$

2.2. The Working of the Model in the Simplest Case

In order to get some feeling of how the model works, it is perhaps appropriate to discuss the models' behaviour in the simplest possible case[1].

We assume that investment $\dot{K}$ is a constant fraction s of output Y.

$$(5) \qquad \dot{K} = s \cdot Y \,, \qquad 0 < s < 1$$

Concerning population growth we will assume here and throughout the paper exponential growth with rate n.

$$(6) \qquad \hat{N} = n \,, \qquad n \geq 0$$

From k = K/N and (2),(5),(6) we find

$$(7) \qquad \hat{k}(x) = s \cdot x - n$$

if full employment is assumed. Inserting (7) into (4) gives a differential equation for x

$$(8) \qquad \hat{x} = \phi(\hat{k}(x)) - \hat{k}(x)$$

1) The working of the model has not always been clear. McCALLUM for instance has conjectured a tendency to instability in Kaldorian growth models recently. I hope to elucidate in the following that this conjecture is not very well founded. On the stability of the 1957-model, see CHAMPERNOWNE.

Since $\hat{k}(x)$ is an increasing function of x , and since $\hat{x} \gtrless 0$ if $\hat{k}(x) \gtrless \kappa$,(8) has the unique stable solution $\bar{x}$ defined by $\hat{k}(\bar{x}) = \kappa$. *In the long run, a constant saving ratio implies that productivity grows with rate κ, the rate of capital deepening is κ, and the output-capital ratio will remain constant.*

1.3. Interaction of Saving and Investment

To assume investment to be a constant fraction of income is very plain indeed, and KALDORs view of how the level of investment is determined is not as simple as that. He assumes that there is a saving function with a savings ratio dependent on profits' share in the economy, and an investment function which might be dependent on profits, too. Changes in profit margins will equalize planned savings and planned investment, and thereby the actual share of profits and actual saving and investment is determined. We have to go into this in the following. But from the foregoing analysis at least this can be said: *If the rate of capital deepening, which results from the interaction of saving and investment, can be written as an increasing function of the output-capital ratio, the same result as in the last section will be obtained, and the average savings ratio will eventually remain constant*[1].

1.4. Savings Behaviour

A real wage rate w implies the rate of profits

$$(9) \qquad r = \frac{Y - w \cdot N}{K} = x \cdot \{1 - \frac{w}{y}\}$$

1) The savings function and the investment function in KALDORs 1957 model together with the short run stability condition (16) discussed below imply just that. Thus CHAMPERNOWNEs stability result is easily generalized for the case of a nonlinear technical progress function.

since r is what remains of the output per unit of capital employed, after wage costs per unit of capital have been deduced, and profits share is

$$(10) \qquad \pi = \frac{r \cdot K}{Y} = \frac{r}{x}$$

We assume the savings ratio to be an increasing function of this quantity.

$$(11) \qquad s = s(\pi), \quad 0 < s' \leq s < 1 \quad \text{for all} \quad \pi \in (0,1)$$

i.e. total savings S is

$$(12) \qquad S = s(\pi) \cdot Y$$

Behind (11) lies the assumption that there is saved more out of profits than out of wages, partly because 'capitalists' save more than 'wage earners', partly because some fraction of total profits will be retained by the firms and will not constitute a part of the income of profit earners. Since retained profits are all saved by definition, this contributes to the positive association between the share of profits and the average propensity to save.[1)]

1.4. The Determination of the Share of Profits

The share of profits is determined through the interaction of the savings function and the investment function. The investment functions I wish to study will be specified later,

1) See KALDOR (1956) p.95 and (1966) p.316 f. KALDOR writes $s(\pi) = s_K \cdot \pi + s_N \cdot (1-\pi)$, $0 \leq s_N < s_K \leq 1$, which is a special case of (11).
See also STIGLITZ/UZAWA p.310f.

but assume for the moment a given investment behaviour for the purpose of clarifying the distribution mechanism[1]. Assume for example that investment is determined by the rate of profit r. Since $r = \pi \cdot x$, investment is a function of profit's share π and other variables.

$$(13) \qquad I = I(\pi, \ldots)$$

At full employment, excess demand in the commodity market relative to output Y is a function of pofits share and other variables, too:

$$(14) \qquad e(\pi, \ldots) = \frac{I-S}{Y} = \frac{1}{Y} \cdot I(\pi, \ldots.) - s(\pi)$$

Now the Keynesian view would be that changes in output equate planned savings to planned investment. In the Kaldorian distribution mechanism this constitutes just the first step, since it is assumed that the price level reactsratherquickly to disequilibria in the commodity market.Because it is assumed that wages are changing more slowly, the behaviour of the price level governs the behaviour of the share of profits. If there is unemployment, the share of profits declines, since there is excess supply in the commodity market. If there is excess demand, the share of profits increases. (All this may take place around an inflationary trend.) The mechanism can be formalized as

$$(15) \qquad \dot{\pi} = \alpha \cdot e(\pi, \ldots \quad \text{for some } \alpha > 0$$

Now it is assumed that the speed of adjustment α is very high as compared to the time-scale of our growth model. This means that as long as the adjustment mechanism (15) has a stable equilibrium solution π_o, i.e. as long as

1) The following is a somewhat personal interpretation of some remarks by KALDOR (1957) p. and (1968) p.197 ff.

$$(16) \qquad e(\pi, \ldots) \gtreqless 0 \Longleftrightarrow \pi \lesseqgtr \pi_0 \ , \ \pi_0 \in (0,1)$$

we can assume that the share of profits is π_0. Inserting this into the investment function gives actual investment.

2. A Neoclassical Approach

2.1. Cost Minimization[1)]

If entrepreneurs behave as price takers, they will try to minimize unit costs. They think that they can hire as much labour and as much capital goods as they like at the ruling prices. Let the real wage rate be w and denote the real market rate of interest by ρ [2)]. At a given moment of time, unit costs

$$17) \qquad c = \frac{w}{y} + \frac{\rho}{x}$$

Entrepreneurs will try to minimize unit costs for the next period, i.e. they will try to minimize the expected change $\dot{c}$ in unit costs by a suitable rate of capital deepening $\hat{k}$:

$$(18) \qquad \dot{c} = -\frac{w}{y} \cdot \hat{y} - \frac{\rho}{x} \cdot \hat{x} + \frac{\dot{w}}{y} + \frac{\dot{\rho}}{x} = \min_{\hat{k}}!$$

Since $\dot{w}$ and $\dot{\rho}$ are considered as exogeneous by the individual entrepreneur, in view of (2),(4) the problem reduces to

$$c \cdot \phi(\hat{k}) - \frac{\rho}{x} \cdot \hat{k} = \max_{\hat{k}}!$$

1) This section owes much to parts of von WEIZSÄCKERs analysis inchapter 3 of his book.
2) that is, the difference between the market rate of interest and the expected rise in prices.

Denote the inverse function of ϕ'by γ

(20) $\gamma(. := \phi'^{-1}(.)$, hence $\gamma' < 0$

then write

$$\hat{k} = \gamma\left(\frac{\rho/x}{c}\right)$$

is the unique solution of (19) if there exists a solution at all, but this will be taken for granted. According to (21), *the slope of the technical progress function at the cost minimizing rate of capital deepening equals the proportion of capital costs in total costs.*

A 'Neoclassical' Investment Function

To assume price taking behaviour and cost minimization is rather neoclassical indeed. It puts great emphasis on the rôle of factor prices concerning the choice of technique: The cost minimizing rate of capital deepening is determined by (21 The simplest way to derive an investment function therefrom is to assume full employment of labour:

$$I = \dot{K} = \frac{d}{dt}(k \cdot N) = K \cdot \{ \gamma\left(\frac{\rho/x}{c}\right) + n \}$$

With regard to ρ we assume that the rate of profit includes a constant risk premium $\sigma \geq 0$ and that competition in the capital market assures always

$$r - \sigma = \rho$$

(We will encounter the difference $\sigma = r - \rho$ repeatedly, and we will always call it 'risk premium' for convenience, even in those cases where it is unrelated to risk.)

(9),(17), and (23) allow us to write[1]

$$\frac{\rho/x}{c} = \frac{\pi\cdot x-\sigma}{x-\sigma}$$

the excess demand function (14) assumes the form

$$e = \frac{1}{x}\cdot\left[\gamma\left(\frac{\pi\cdot x-\sigma}{x-\sigma}\right)+ n - s(\pi)\cdot x\right]$$

which clearly satisfies the short-run stability condition (16) of the Kaldorian adjustment mechanism: An increase in profit margins increases savings and reduces investment since the associated increase in the real rate of interest slows down capital deepening. Thus the adjustment mechanism generates a share of profits π_o which just equilibrates the commodity market and is defined by the condition $e = 0$ as a function of x[2]

(26) $$\pi_o = \pi(x)$$

The resulting actual rate of investment gives rise to an actual rate of capital deepening

$$\hat{k}(x) = \gamma\left(\frac{\pi(x)\cdot x - \sigma}{x - \sigma}\right)$$

From 11 ,(20),(24),(25) it follows that $\hat{k}(x)$ is increasing in x:

(28) $$\frac{d\hat{k}}{dx} > 0$$

Now we can go back to our earlier analysis of sections 1.2. and 1.3. If we substitute (27) for (7), we get the result that the model converges towards a growth equilibrium where productivity rises with rate κ and the output-capital ratio as well as the average saving propensity remain constant. But in view of our

1)Since $\sigma/x \leq \pi$ for economic reasons, we have $x - \sigma > 0$.
2)We assume a solution to $e=0$ to exist.

analysis of cost minimization, we can even say more: *In the long run, the slope of the technical progress function at κ determines the proportion of capital costs in total costs. If the real market rate of interest equals the rate of profits (i.e. if $\sigma = 0$), profits' share is eventually determined by the slope of the technical progress function at κ.* The latter result is very neoclassical in several respects, but particularily in that distribution is determined by the character of technical progress alone[1].

Some more results are obtained from the long run conditions

$$s(\pi) \cdot x = \kappa + n \qquad \frac{\pi \cdot x - \sigma}{x - \sigma} = \phi'(\kappa)$$

(29) implies particularily: The higher the risk premium σ, the higher will be the share of profits in the long run and the smaller will be the output-capital ratio[2]. π will always be somewhat above $\phi'(\kappa)$ according to the risk premium required.

3. On Two Themes by KALECKI

Imperfect Competition

The quite different Kaleckian 'degree of monopoly' approach leds to rather similar results, but these are resulting from a somewhat reversed causality[3].

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1) that is, $\phi'(\kappa) = \pi$, and ϕ' is indeed the Kaldorian analogue to the elasticity of output with respect to capital, as can readily be seen from

$$\phi' = \frac{d\dot{y}}{d\dot{k}} \; \frac{k}{y}$$

2) Differentiation of (29) gives

$$\text{sign} \; \frac{dx}{d\sigma} = \text{sign} \left((1-\pi) \cdot \frac{\sigma}{x} - (1-\frac{\sigma}{x}) \cdot \frac{s}{s'} \right) < 0$$

since $1 > \pi > \frac{\sigma}{x}$, $\frac{s}{s'} \geq 1$ by (9), (10), (11), (23)

3) KALECKI (1954), chapters 1 and 2. See also von WEIZSÄCKER, chapter 3.

Assume that raw material costs are proportionate to the wage bill. Then prime costs, which are the sum of wages and raw material outlays, are proportionate to wage costs. The imperfection of the product market determines the degree of monopoly and thereby the mark-up on prime costs. As a consequence, prices will be a fixed multiple of wage costs per unit, and since the price level is taken as unity, this implies that real wage costs per unit (w/y) are fixed. In other words, the share of profits $\pi = 1 - w/y$ is determined by the degree of monopoly.

$$\pi = \bar{\pi} = \text{constant}$$

In this case, clearly the Kaldorian stability mechanism (15) cannot work. Instead, we have to assume that wages are fixed exogeneously, for instance by collective bargaining. Prices are determined by mark-up pricing therefrom.

If π is constant, we know from (11) that the aggregate savings ratio will remain constant. Equilibrium in the commodity market and full employment would require a rate of capital deepening

$$\hat{k} = s(\bar{\pi}) \cdot x - n$$

If (31) can be assumed to hold, we are back again to our simplest case, but how does (31) come about?

It could be produced by changes in the real market rate of interest like that: The cost minimizing choice of technique obviously implies

$$(32) \qquad \hat{k} = \gamma\left(\frac{\rho/x}{\rho/x + w/y}\right)$$

Now assume $\gamma > s \cdot x - n$. There will be excess demand in the commodity market and excess demand for loanable funds.

This will drive the money rate of interest upward, the real rate will follow, and this finally establishes equilibrium in the commodity market. For $\gamma < s \cdot x - n$, the mechanism works the other way round. (Since mark up pricing is assumed, all this leaves the rate of inflation unaffected. Changes in the rate of inflation cannot cause the real rate of interest to move in the other direction.)

In the long run, the result is very similar to the previous one, but the 'risk premium' is determined by the share of profits rather than the other way round, and profits share is determined by the 'degree of monopoly' primarily rather than by the character of technical progress. This has policy implications of course.

3.2. The Principle of Increasing Risk

If one assumes, in a neoclassical vein, price taking behaviour of the individual firm and a constant returns to scale technology, the output level remains undetermined by profit maximization, and thus the level of investment is left undetermined, too. Nothing in this will be altered for the economy as a whole if the individual firms are subject to decreasing returns, since in that case it is the number of firms which remains undetermined[1)]. Indeed, the neoclassical way of telling the story says us how to determine the choice of technique, but the level of investment is simply *assumed* to be such as to guarantee full employment.

A simple way out and towards a theory on the determination of the level of investment is to argue that the excess of the profit rate over the real market rate of interest gives the inducement to invest. This view, that it is σ which governs

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1) BLISS p.3, KALECKI (1937) p.441 f, 1954) p.91

the volume of investment, has been introduced by KALECKI and is underlying the approach of the so-called KEYNES-WICKSELL-models, too[1].

Assume

$$\hat{K} \qquad \psi(\sigma) \qquad\qquad \psi' > 0$$

The positive slope of this function can be motivated by KALECKIs 'principle of increasing risk'[2]; The larger the amount invested, the higher will be the risk incurred by the investor, and the higher will be the necessary risk premium. In addition, firms have to raise capital in imperfect capital markets. The greater the funds they want to raise, the higher will be the returns they have to offer in order to attract also those investors who are rather pessimistic concerning the firm's future, and so it is they who need a risk premium also.

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1) On KEYNES-WICKSELL models, see e.g. FISCHER. In this context it should be noted that the adjustment cost approach to the derivation of the flexible accelerator, which gives rise to investment functions like (33) in KEYNES-WICKSELL models, is not appropriate in our analytical framework, since increasing average costs of new investment are embodied in the convex shape of the technical progress function already. On the adjustment cost approach, see GOULD, however.

2) See KALECKI (1937),(1954) ch. 8 and 9, STEINDL, v. WEIZSÄCKER chapter 3. The following states KALECKIs position in a very abriged if not amputated form since I do not wish to complicate things by distinguishing between entrepreneurial capital and borrowed capital. Although this is a rather central point in KALECKI, I do not feel that it is particularily relevant in the present context. Furthermore, I do not want to distinguish here between the actual rate of profit and the prospective rate of profit as KALECKI and KALDOR (1961) do, but this distinction will be implicit in the analysis of the following chapter.

Still another motivation for (33) is to assume that firms are making investment decisions dependent on the 'pay off period' of investment, which obviously varies with σ. See von WEIZSÄCKER for a detailed discussion of this approach.

In view of the cost minimizing condition (21) which involves the real market rate of interest, one might think that ρ governs the choice of technique and $\sigma=r-\rho$ determines the volume of investment in this theory. But as KALECKI (1937) has argued, this need not be so. If capital is scarce, entrepreneurs will maximize the return on capital rather than minimize costs[1]. Instead of (18) we will have to maximize

$$\frac{d}{dt}\left[x - \rho - w\cdot\frac{x}{y}\right] = x\cdot\left[\pi\cdot\hat{x} + (1-\pi)\cdot\hat{y}\right] - \dot{\rho} - \dot{w}\cdot\frac{x}{y}$$

with respect to $\hat{k}$ which gives the solution

(35) $$\hat{k} = \gamma(\pi)$$

Since $\pi = r\cdot x$, the rate of profit (and not the rate of interest) determines the choice of technique.

In order that the decisions made according to (35) concerning the choice of technique be compatible with the volume of investment as determined by (33), the real market rate of interest has to be fixed appropriately. If this is possible, we arrive at the same result as in the 'neoclassical' case for $\sigma = 0$, since then (21) and (35) are identical[2].

But it is highly improbable that the model will meet the short run stability condition (16) since if there is excess demand in the commodity market, this will raise profit margins and thereby increase investment demand even more. In addition, the rise in the price level lowers *ceteris paribus* the real market rate of interest, which increases investment demand

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1) This view, that the real rate of interest does not affect the choice of technique, seems to be implicit in the KEYNES-WICKSELL models also.
2) KALDORs analysis of this problem seems to be erroneous, see KALDOR (1961) p.211. Even if one deletes (35) and writes K as a function of r alone (i.e. for fixed ρ), the resulting rate of capital deepening will be a function of x alone, and the analysis of section 1.3. will be applicable.

also. In order to stabilize the model one had to introduce a monetary mechanism which makes the money rate of interest to react very violently to disequilibria in the commodity market. Indeed, changes in the money rate of interest had to offset both the changes in profit margins and inflation. I doubt that this is possible even under a monetarist rule in realistic circumstances. Perhaps one has to pilot the economy manually by means of the market rate of interest and by controlling profit margins *via* effective demand. Nevertheless, if a given stabilization policy succeeds, we know the outcome.[1) So I leave this section as it is.

4 A Keynesian Approach

4.1. The Corfu Investment Function

I have gone into all this in order to contour some patterns of investment behaviour yielding very similar results. But perhaps everything works the other way round, perhaps reality is Keynesian[2)].

Take, as a convenient starting point, the investment function proposed by KALDOR at the Corfu Conference of the I.E.A.[3)]

$$\hat{k}(t+\theta) = \phi^{-1}\left(\hat{y}(t)\right) + \mu \cdot \dot{x}(t) \qquad \theta > 0\ ,\ \mu > 0 \tag{36}$$

There is an investment lag θ. Firms think that demand will

1) Things will become rather complicated, if (21) instead of (35) is assumed, since then changes in the money rate of interest will steer both the choice of technique and the level of investment . Save by mere coincidence, one of these quantities will be steered wrong.
2) The adjective 'Keynesian' refers to the stress on demand expectations rather than th KEYNES' original investment theory, which is a short run theory and cannot be discussed in the present context. See KEYNES ch. 11, however.
3) KALDOR(1961). Some misprints have been corrected in KALDOR (1970). I do not discuss KALDORs first investment function here. It seems not to be very appropriate since an exogeneously fixed accelerator is presupposed, but in our model the capital-output ratio 1/x varies endogeneously. This seems to have been recognized by KALDOR, (1961) p.212 n.

increase at the present rate $\hat{y}$, and in order to meet this expected demand increase they had to plan a rate of capital deepening $\phi^{-1}(\hat{y})$. But the actual rate which is planned according to (35) is modified according to whether the output capital ratio rises or declines. The argument behind that is that increases in the output-capital ratio are reflecting increases in the competitiveness of new investment as compared to old investment. Thus the difference between the prospective rate of profits on new investment and the actual rate of profits will be increasing if $\dot{x}$ increases[1)]. If this difference is very high, firms will invest more than is necessary to meet expected demand since they will try to increase their respective market shares.

To sum up, the first term in (36) is Keynesian: It gives the amount of investment which is necessary to attain the planned increase in output according to expected demand, and the second term gives the cost inducement to invest.

If (36) is incorporated into the model, we get a mixed difference-differential equation which is too troublesome to analyze. It will be sufficient for our purposes to assume that the investment lag θ is sufficiently short as compared with the time scale of our model as to allow for the TAYLOR approximation

(37) $$\hat{k}(t+\theta) \sim \hat{k}(t) + \theta \cdot \dot{\hat{k}}(t)$$

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1 KALDOR (1961) p. 216, states this a little differently. Denote by r^+, π^+ and x^+ the prospective rate of profit, the expected profit margin and the expected output capital ratio respectively. Clearly we have $r^+ = \pi^+ \cdot x^+$. Assume $\pi^+ = \pi$ and $x^+ = x + \nu \cdot \dot{x}$, implying $r^+ - r = \nu \cdot \pi \cdot \dot{x}$. If $\nu \cdot \pi$ remains constant, we get (36), but one cannot assume this, even if π^+ were based on past values of π. Nevertheless (36) might serve as a first step.

To simplify further we write $\hat{x}$ instead of $\dot{x}$ [1]
Thus we get:

$$\hat{k}(t) + \theta \cdot \dot{\hat{k}}(t) = \phi^{-1}(\hat{y}(t)) + \mu \cdot \hat{x}(t)$$

which reduces, in view of (1 , to

$$\dot{\hat{k}}(t) = \nu \cdot \hat{x}(t) \qquad \nu = \mu/\theta$$

i.e. the *change* of the rate of capital deepening is governed by the change of the output-capital ratio. Together with (4) we obtain the differential equation

$$(40) \qquad \dot{\hat{k}}(t) = \nu \cdot \left(\phi(\hat{k}) - \hat{k}\right)$$

which has the unique stable equilibrium solution $\hat{k} = \kappa$. Since there is an investment lag, investment ~~is fixed~~ at any point of time independently of the share of profits. Therefore the Kaldorian stability mechanism (15),(16) works again and the share of profits is determined by the equation $s(\pi) \cdot x = \hat{k} + n$, where x results from $\hat{x} = \phi(\hat{k}) - \hat{k}$ and its initial value.

If entrepreneurs behave according to the investment function (39), the rate of capital deepening and the rate of productivity growth will approach κ *, the output-capital ratio will approach a constant* $\bar{x}$ *which is dependent on initial conditions, and the share of profits will approach* $\pi = s^{-1}\left((\kappa+n)/\bar{x}\right)$.

This result is in marked contrast to the other results previously arrived at since income distribution will be dependent on initial conditions and *transitory changes in the rate of capital deepening will have lasting effects on income distribution*. For instance, if the economy is in a growth equilibrium with $\hat{k} = \kappa$ and $\hat{k}$ is increased by a disturbance of the system,

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1 this is merely technical. It is easily checked that it does not affect our main conclusion.

$\bar{x}$ will decline and the share of profits will approach a higher level and will remain there.

But how to reconcile all this with cost minimization? There is no problem of course if initial conditions are such as to guarantee $\pi \geq \gamma^{-1}(\kappa)$. In this case, the real market rate of interest will assure equilibrium as described in section 3.1.[1)]

4.2. Another 'Keynesian' Investment Function

In this section I will try to rear another Keynesian investment function from KALDORs ideas underlying (36).

Consider first the Keynesian term. Firms invest in order to meet expected growth in demand, call it λ, and they form their expectations by exponential smoothing[2)]:

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1) One has to assume again that the impact of inflation on the real market rate of interest does not generate a Wicksellian cumulative process, for instance by assuming that the long run expectations are responding quite slowly to transitory changes in the rate of inflation.

2) This is rational if firms think $\hat{y}$ to be generated by the stochastic model

(') $\quad \hat{y}(t) = \eta(t) + u(t)$

('') $\quad \eta(t) = \eta(t-1) + \varepsilon(t)$

where $u(t)$ and $\varepsilon(t)$ are normally distributed disturbances with mean zero and variances v_u, v_ε, respectively. Then the (unbiased) maximum likelihood forecast for $\hat{y}(t+1)$ is given by

(''') $\quad \lambda(t+1) = \lambda(t) + \beta \cdot \{\hat{y}(t) - \lambda(t)\}$

where β is a function of v_u and v_ε. (see SCHLICHT). (''') is the discrete counterpart of (42). The model ('),('') seems to be rather acceptable in that firms think that long run growth rates η are changing rather slowly according to (''), but actual growth in demand is subject to short-run disturbances u, as in (').

(41) $$\lambda = \beta \cdot \int_{-\infty}^{t} e^{-\beta(t-\tau)} \hat{y}(t-\tau)\, d\tau \qquad \beta > 0$$

or

(42) $$\dot{\lambda} = \beta \cdot \hat{y}$$

In the following , the demand inducement to invest will be taken as $\phi^{-1}(\lambda)$.

In modification of the second term in (36), the cost inducement to invest, I assume that firms are minimizing unit costs according to (21), that the capital market is cleared by changes in ρ as discussed in section 3.1., and I assume above all that it is the change in unit costs brought forth by changes in x and y what matters. From (18) we see that the reduction in unit costs caused by changes in x and y is

(43) $$\frac{w}{y}\cdot\hat{y} + \frac{\rho}{x}\cdot\hat{x}$$

If (21 holds, this quantity is equal to

(44) $$\left(1 - \frac{\sigma}{x}\right)\cdot\left(\phi(\hat{k}) - \phi'(\hat{k})\cdot\hat{k}\right)$$

which is decreasing in $\hat{k}$ and increasing in x.

Now, if we were to write this term instead of KALDORs $\dot{x}$, we would get an undesired simultaneity: Expectations are causing current decisions on $\hat{k}$ which determine those expectations in turn. This seems to be not very meaningful from an economic point of view . I assume instead that firms guess productivity changes according to their demand expectations λ : Firms expect cost reductions to be

(45) $$\left(1 - \frac{\sigma}{x}\right) \cdot \left(\lambda - \phi'\left(\phi^{-1}(\lambda)\right) \cdot \phi^{-1}(\lambda)\right)$$

instead of (44), and hence the cost inducement to invest can be written as

$$\varepsilon = \varepsilon(\lambda, x) \quad , \quad \varepsilon_\lambda < 0 \; , \; \varepsilon_x > 0$$

(Of course (46) is sufficiently general as to be deducible from other arguments as those underlying (45).)

Now the development of the model can be described completely.

$$\hat{k} = \phi^{-1}(\lambda + \varepsilon(\lambda, x)$$

$$\dot{\lambda} = \beta \cdot \left(\phi(\hat{k}) - \lambda \right)$$

$$\hat{x} = \phi(\hat{k}) - \hat{k}$$

(47) are the Kaldorian ideas in a different guise. These three equations can be reduced to two:

(50) $$\dot{\lambda} = \beta \cdot \{ \qquad \}$$

$$\hat{x} = \left(\phi\left(\phi^{-1}(\lambda + \varepsilon(\lambda, x)\right) - \phi^{-1}(\lambda) - \varepsilon(\lambda, x) \right)$$

Substitute into (47),(48) according to

$$z = \ln x \; , \quad \dot{z} = \hat{x} \; , \quad x = e^z$$

and write down the Jacobian

$$(53)\quad J = \begin{bmatrix} \frac{\partial\dot{\lambda}}{\partial\lambda} & \frac{\partial\dot{\lambda}}{\partial z} \\ \frac{\partial\dot{z}}{\partial\lambda} & \frac{\partial\dot{z}}{\partial z} \end{bmatrix} = \begin{bmatrix} \beta\cdot\phi'\cdot\varepsilon_\lambda & \beta\cdot\phi'\cdot\varepsilon_x\cdot x \\ -(1-\phi')\cdot(\varepsilon_\lambda + \frac{1}{\phi}) & 1-\phi')\cdot\varepsilon_x\cdot x \end{bmatrix}$$

It is easy to check that the OLECH conditions[1] for global stability :

$$(54)\qquad \operatorname{tr} J < 0 \qquad \det J > 0 \qquad \frac{\partial\dot{\lambda}}{\partial\lambda}\cdot\frac{\partial\dot{z}}{\partial z} \neq 0$$

are fulfilled as long as the weak additional assumption

$$(54)\qquad \phi' <$$

is introduced. This means that, as long as there exists an equilibrium solution to (50),(51) at all, it will be unique and globally stable. Thus we get a quite different result than in the Corfu case: *The long run output capital ratio will approach an equilibrium value which is independent of initial conditions. The rate of productivity growth and the rate of capital deepening will approach* κ *and the share of profits as well as the average savings propensity will approach an equilibrium value independent of initial conditions.*

4.3. The Cost Inducement to Invest

Thus we have arrived at a by now familiar result, but this result is crucially dependent on the assumption that the cost-

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1) see GARCIA p.541

inducement to invest is a function of x. If we replace (46) by

$$(56) \qquad \varepsilon = \varepsilon(\lambda) \qquad \varepsilon_\lambda < 0$$

the system (47)-(49) reduces to

$$(57) \qquad \dot{\lambda} = \beta \cdot \left[\phi\left(\phi^{-1}(\lambda) + \varepsilon(\lambda)\right) - \lambda \right]$$

which is clearly stable, but the resulting x and hence the resulting income distribution remains dependent on the initial conditions as in the Corfu case, where the influence of x is absent, too.

It is perhaps appropriate to mention still another point. One could consider it as a 'pure' Keynesian case if all investment is demand induced and cost-inducement is zero. Assume this. Assume $\varepsilon \equiv 0$ in (57). This implies $\dot{\lambda} = 0$ and hence $\hat{k} = 0$: A given rate of capital deepening will be maintained forever. Any given growth rate $\hat{y}$ will be maintained because it is expected to remain and this expectation is fulfilled. This implies of course that profits share will continuously decline for $\hat{k} < \kappa$ and increase for $\hat{k} > \kappa$, respectively. All this gives perhaps a new twist to the old puzzle whether it is growth which generates profits or whether it is profits which promote growth: Both profits and growth might be generated by the mere momentum of inertia inherent in the economy.

5. Conclusion

I am unable to put together the many open ends of the above tangle. What I have tried to show is that there are some puzzling problems relating to investment in a growth context, some of them being of considerable import indeed. But even if one arrives at the conclusion that it is solely promising to deal with those problems discussed above in a much broader context, say of cyclical growth or involving monetary phenomena, the above might have supported this conviction and might have been not entirely futile therefore.

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