Czado, Rudolph:

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Application of Survival Analysis Methods to Long Term Care Insurance

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Abstract

With the introduction of compulsory long term care (LTC) insurance in Germany in 1995, a large claims portfolio with a significant proportion of censored observations became available. In first part of this paper we present an analysis of part of this portfolio using the Cox proportional hazard model (Cox (1972)) to estimate transition intensities. It is shown that this approach allows the inclusion of censored observations as well as the inclusion of time dependent risk factors such as time spent in LTC. This is in contrast to the more commonly used Poisson regression with graduation approach (see for example Renshaw and Haberman (1995), where censored observations and time dependent risk factors are ignored. In the second part we show how these estimated transition intensities can be used in a multiple state Markov process (see Haberman and Pitacco (1999)) to calculate premiums for LTC insurance plans.

Keywords: Cox Proportional Hazard, Survival Analysis, long term care insurance, multiple state Markov model

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1 Introduction

Due to increasing life expectancy long term care (LTC) insurance becomes more and more important in all industrialized countries. In Germany this type of insurance forms part of the social security system since 1995. For the insurers this has meant that all provider of health assurance were obliged to underwrite this new type of business without any medical exam. The consequence of this compulsory characteristic was, that from the beginning insurers had to manage a large claims portfolio, which normally does not occur due to the underwriting selection.

In this paper, we analyze part of the LTC-claims portfolio of a German health insurance. Our main goal is to assess the influence of factors like severeness of a disease, gender and type of care (i.e. care at home or in a nursing home) on the survival curve of the observed claims. For this purpose we utilize the semiparametric proportional hazard model proposed by Cox (1972) in 1972 for our data. Cox assumes that the hazard function $\lambda(t)$ of the survival curve can be modeled as a product of a general, all observations underlying baseline hazard function $\lambda_0(t)$ and a covariate dependent factor.

Renshaw (1988) investigated excess mortality with a similar approach. The main difference between the approach taken in this paper and his approach is that Renshaw identifies the likelihood of the Cox proportional hazard model with that of a Poisson regression with offsets depending on the unknown underlying baseline hazard. Estimation of these unknown offsets is facilitated by using life tables, a process Renshaw calls graduation. In contrast, we estimate the baseline hazard rate directly from the observed data using Breslow's estimator (Breslow 1974) of the baseline hazard rate. This has the advantage that we can use all available data, even the censored data, while Renshaw has to model claims durations to construct the required life tables. With help of the counting process interpretation of the Cox proportional hazard model, we are able to assess the quality of our model using martingale based residuals (see Therneau, Grambsch, and Fleming (1990)).

Using the estimated hazard rates of the Cox proportional hazard model as transition intensities in a multiple state Markov model, we are now able to fit a multiple state insurance model. In the work of Renshaw and Haberman (1995) a similar model was used for health insurance. There the main interest was the estimation of recovery rates of insured persons using above mentioned graduation techniques.

The paper is organized as follows: Section 2 gives the necessary foundations for the Cox proportional hazard model. The estimation of required transition intensities using the Cox proportional hazard model for a large data set from the German compulsory long term insurance program is given in Section 3. Section 4 introduces a multiple state space model and necessary actuarial setup to calculate premiums for specified long term care plan. The paper closes with a discussion and summary section.

2 Cox Proportional Hazard Model

As an important tool in survival analysis for modeling the dependency of the survival time on covariates serves the proportional hazard model, proposed by Cox (1972). This semiparametric approach assumes that the hazard function for the random life time T

$$\lambda(t) := \lim_{dt \to 0} \frac{P(t \leq T < t + dt)|T \geq t)}{dt} \tag{2.1}$$

is the product of a baseline hazard $\lambda_0(t)$ and a specific, covariate-dependent scaling factor, which enters in the form

$$\lambda(t|Z = z) = \lambda_0(t) \exp(\beta^t z), \tag{2.2}$$

where $z \in \mathbb{R}^p$ denotes the observed covariate vector and $\beta \in \mathbb{R}^p$ the unknown regression coefficient. Assume that T_i denotes the life time of subject $i, i = 1, \cdots, n$. Since we allow for censoring we actually observe

$$Y_i = (min(T_i, C_i), \delta_i), i = 1, \cdots, n$$

where

$$\delta_i = \begin{cases} 1 & T_i \leq C_i \\ 0 & \text{otherwise} \end{cases},$$

is the censoring indicator for the subject specific censoring time C_i. In the case of no ties among the observed ordered death times $t_1 < \cdots < t_k$, let R_j denote the set of subjects, which have survived until t_j-. Under the proportional hazard assumption (2.2) the partial likelihood

$$\begin{aligned} L_p(\beta) &:= \prod_{j=1}^{k} \frac{P(i \text{ dies at time } t_j | i \text{ survives until } t_j-)}{P(\text{ a death in } R_j \text{ at time } t_j)} \\ &= \prod_{j=1}^{k} \frac{\exp(\beta^t z_i)}{\sum_{l \in R_j} \exp(\beta^t z_l)} \end{aligned}$$

is independent of the unknown baseline hazard function and can be maximized to yield an estimate of β. We denote this estimate with $\hat{\beta}$. Cox (1975) claimed that the partial likelihood contains most of the information about β, which has been supported by empirical work in small samples by Efron (1977) and Oakes (1977). In the case of ties among the observed death times the partial likelihood can be adjusted (see Klein and Moeschberger (1997)). Various authors (see e.g. Andersen and Gill (1982)) considered the original model of Cox in the context of counting processes and they were able to prove asymptotic consistency and normality of the partial likelihood estimator under regularity conditions.

In addition, the intensity process $\lambda(t)$ is sufficient for determining the survival function $S(t) := P(T > t) = 1 - F(t)$ ($F(t)$ is the distribution function) since following the relationship

$$S(t) = \exp\left(-\int_0^t \lambda(t)\, dt\right) \tag{2.3}$$

holds.

To estimate the underlying baseline hazard $\lambda_0(t)$ we use the Breslow estimator (Breslow 1974) $\hat{\Lambda}_0(t)$ for the cumulative baseline hazard rate $\Lambda_0(t) = \int_0^t \lambda(s)\, ds$ which is defined as

$$\hat{\Lambda}_0(t) := \sum_{t_i \leq t} \frac{d_i}{\sum_{j \in R_i} \exp(\hat{\beta}^t Z_j(t_i))}, \tag{2.4}$$

where d_i the number of events in t_i and R_i the risk-set at time t_i.

3 Data Analysis for Compulsory Long Term Care Insurance

In 1995 the German government introduced compulsory long term care (LTC) insurance. This required part of the German welfare system paid benefits for home care since April 1, 1995. From July 1, 1996, the benefits were extended to care in a nursing home as well. The LTC-claimants receive their benefits according to a three-level-system, which represents a scale for the severity of a claim. The definition of the levels is given as follows

- **Level 1**: The LTC-claimant needs at least 90 minutes help per day to manage his/her activities of daily living (like going to bed, washing, eating, ...)
- **Level 2**: The LTC-claimant needs at least 180 minutes help per day to manage his/her activities of daily living
- **Level 3**: The LTC-claimant needs at least 300 minutes help per day to manage his/her activities of daily living

				Home			Nursing Home		
	cens.	dead	recov.	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
Home Level 1	1012	279	28	–	444	75	118	71	34
Home Level 2	877	598	1	47	–	296	9	208	59
Home Level 3	308	632	2	2	20	–	0	4	87
Nursing Home Level 1	248	85	3	11	1	0	–	108	26
Nursing Home Level 2	451	263	2	1	6	1	7	–	116
Nursing Home Level3	376	437	0	1	1	6	2	9	–

Table 1: Number of observed transitions in the LTC data

The basis for our statistical inference on LTC was a large data set from a German private insurance company. The data was recorded between April 1, 1995 and December 31, 1998. It contained data on 5603 recipients of benefits from the compulsory LTC insurance. 3511 (2092) recipients were female (male). For each recipient, the data contained information about age, gender, claim severity (Level 1-3), care required (at home/in a nursing home) and disease causing LTC. In addition, transition times between care levels, care required (home to nursing home and vice versa) and between states of LTC claimant (claiming LTC to healthy and LTC claiming to death). There was no information available considering the transitions from healthy to LTC claiming and healthy to death. Table 1 gives the observed transitions. Note that a single person can have several transitions. The transition to death is denoted by dead, while a censored transition was recorded when the person remained in a particular LTC level until December 31, 1998. It can be seen that censoring cannot be ignored and the estimation method utilized for this data set has to be able to account for this. Recovery from LTC is a very rare event.

The data analysis will proceed in two steps. First we have a look at the survival of LTC claimant, i.e. we will consider the state transitions from LTC-claimance to death. In the second step, we will consider the transitions between different types of care (i.e. transitions between care levels) as well as transitions between care at home and care in a nursing home.

3.1 Analysis of the Survival of LTC Claimants

We fit the Cox proportional hazard model (2.2) for modeling the survival of benefit recipients, where we considered the covariates

$$
\begin{aligned}
Z_{\text{Sex}} &:= \begin{cases} 1 & \text{female} \\ 0 & \text{male} \end{cases} \\
Z_{\text{Age}}(t) &:= \text{Age of claimant when a state transition occurs at time t} \\
Z_{\text{Level i}}(t) &:= \begin{cases} 1 & \text{care at level i at time } t \\ 0 & \text{otherwise} \end{cases}, i = 2, 3 \\
Z_{\text{nh}}(t) &:= \begin{cases} 1 & \text{care in a nursing home at time } t \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

Initially, we fitted a model with all possible interactions of the above covariates. We used Akaike's (Akaike 1973) information criterion (AIC) to filter out covariates with highly significant influence on the transition intensities. This criterion considers the partial log-likelihood as evidence of significance and is defined as

$$
\text{AIC} := -2\log(L_p(\hat{\beta})) + np,
$$

where $\log(L_p(\hat{\beta}))$ is the partial log-likelihood evaluated at the estimated coefficient vector $\hat{\beta}$, p the number of covariates in the model and n an integer. The choice of n depends on whether one likes to fit a conservative model that includes only covariates with highly significant terms, or a model that includes more covariates with less important terms. In our case, we have chosen $n = 2$.

After eliminating covariates with no significance, we get a covariate dependent representation of the hazard rate as follows

$$
\begin{aligned}
\lambda(t) &= \lambda_0(t)\exp[\beta_1\, Z_{\text{Age}}(t) + \beta_2\, Z_{\text{Sex}} + \beta_3\, Z_{\text{nh}}(t) + \beta_4\, Z_{\text{Level2}}(t) \\
&+ \beta_5\, Z_{\text{Level3}}(t) + \beta_6\, Z_{\text{Age}}(t) \times Z_{\text{Sex}} + \beta_7\, Z_{\text{Age}}(t) \times Z_{\text{nh}}(t) \\
&+ \beta_8\, Z_{\text{Sex}} \times Z_{\text{nh}}(t) + \beta_9\, Z_{\text{nh}}(t) \times Z_{\text{Level2}}(t) \\
&+ \beta_{10}\, Z_{\text{nh}}(t) \times Z_{\text{Level3}}(t)]. \qquad (3.1)
\end{aligned}
$$

In Table 2 we give the estimated regression coefficients $\hat{\beta}_i$ and its estimated standard error $se(\hat{\beta}_i)$, the estimated multiplier $\exp(\hat{\beta}_i)$, the Wald statistic $W_i = \frac{\hat{\beta}_i}{se(\hat{\beta}_i)}$ for testing $H_0 : \beta_i = 0$ and the corresponding p-value (e.g. Rao (1965)).

Covariate	$\hat{\beta}$	$\exp(\hat{\beta})$	$se(\hat{\beta})$	W	p-value
Z_{Age}	0.02421	1.025	0.0025	9.78	$< 10^{-15}$
Z_{Sex}	0.3202	1.377	0.2811	-1.47	$2.5 \cdot 10^{-1}$
Z_{nh}	-0.5969	0.551	0.4063	1.14	$1.4 \cdot 10^{-1}$
Z_{Level2}	0.8180	2.266	0.0729	11.22	$< 10^{-15}$
Z_{Level3}	1.7553	5.785	0.0734	23.91	$< 10^{-15}$
$Z_{\text{Age}} \times Z_{\text{Sex}}$	-0.0065	0.993	0.0035	-1.89	$5.9 \cdot 10^{-2}$
$Z_{\text{Age}} \times Z_{\text{nh}}$	0.0162	1.016	0.0047	3.45	$5.7 \cdot 10^{-4}$
$Z_{\text{Sex}} \times Z_{\text{nh}}$	-0.4405	0.644	0.0989	-4.46	$8.4 \cdot 10^{-6}$
$Z_{\text{nh}} \times Z_{\text{Level2}}$	-0.3320	0.717	0.1446	-2.30	$2.2 \cdot 10^{-2}$
$Z_{\text{nh}} \times Z_{\text{Level3}}$	-0.6753	0.509	0.1391	-4.86	$1.2 \cdot 10^{-6}$

Table 2: Estimated Regression Coefficients in Model (3.1)

We want now to assess the fit of Model (3.1). This will be done for quantitative and qualitative covariates using separate graphical diagnostic tools. For quantitative covariates martingale residuals $\hat{M}$ will be utilized. These are defined for each observation as follows

$$\hat{M}_i(t) := N_i(t) - \int_0^t Y_i(s) \exp(\hat{\beta}^t Z_i(s)) \, d\hat{\Lambda}_0(s),$$

where $N_i(t)$ is the counting process of the observed events up to time t for individual i. $Y_i(t)$ denotes the indicator function defined as

$$Y_i(t) = \begin{cases} 1 & \text{subject } i \text{ under observation at time } t \\ 0 & \text{otherwise} \end{cases} .$$

Here, $\Lambda_0(t)$ denotes the cumulative baseline hazard rate, $Z_i(t)$ the covariate vector at time t and finally $\hat{\beta}$ the estimated regression coefficient. The martingale residuals can be interpreted to be the difference between observed events and expected events due to the model. If the model is correct each residual is a martingale. They are used to check whether a quantitative covariate follows the proportional hazard model. If proportional hazard assumption (2.2) is correct, a

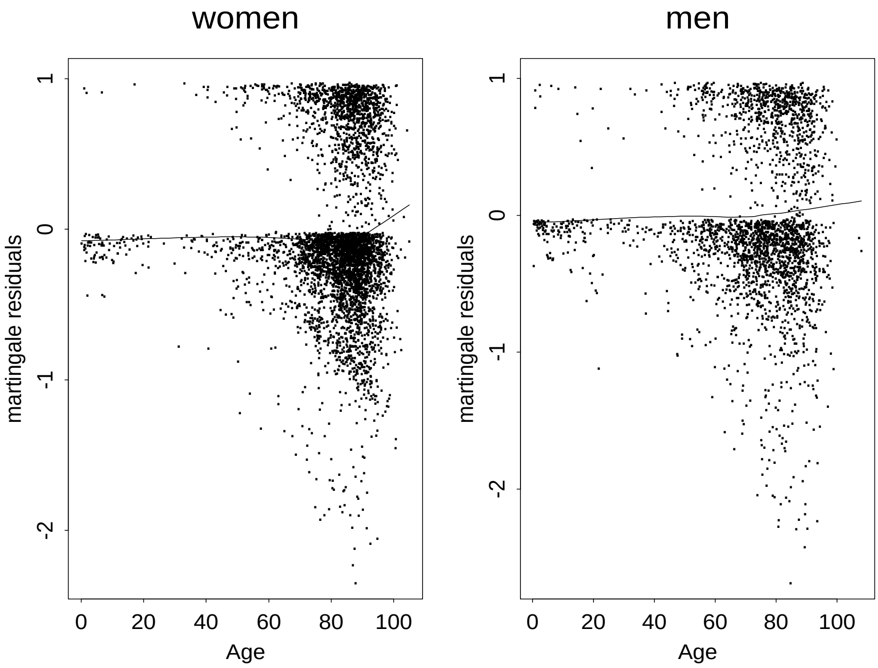

Figure 1: Martingale residuals (grouped by gender)

smooth of the residuals plotted against the value of the quantitative covariate should be constant 0. For further details on the application of this type of residuals, see Therneau, Grambsch, and Fleming (1990). Figure 1 gives the martingale residuals, separate for women and men against the quantative covariate Z_{Age}. We can observe that until the age of approximately 80 years the plot follows roughly the constant null line, whereas for higher ages the line is increasing for women as well as for men. This shows that there are more deaths observed than expected from the model. Therefore we add two new covariates $Z_{\theta_{\mathrm{f}}}$ and $Z_{\theta_{\mathrm{m}}}$ which are defined as follows

$$Z_{\theta_{\mathrm{f}}}(t) = \begin{cases} 1 & \text{if } Z_{\mathrm{Age}}(t) \geq \theta_{\mathrm{f}} \text{ and } Z_{\mathrm{Sex}} = 1 \\ 0 & \text{otherwise} \end{cases},$$

as well as

$$Z_{\theta_{\mathrm{m}}}(t) = \begin{cases} 1 & \text{if } Z_{\mathrm{Age}}(t) \geq \theta_{\mathrm{m}} \text{ and } Z_{\mathrm{Sex}} = 0 \\ 0 & \text{otherwise} \end{cases}$$

and maximize the partial log likelihood for the proportional hazard model

$$\begin{aligned} \lambda(t) &= \lambda_0(t) \exp[\beta_1 Z_{\mathrm{Age}}(t) + \beta_2 Z_{\mathrm{Sex}} + \beta_3 Z_{\mathrm{Stat}}(t) + \beta_4 Z_{\mathrm{level2}}(t) \\ &+ \beta_5 Z_{\mathrm{level3}}(t) + \beta_6 Z_{\mathrm{Age}}(t) \times Z_{\mathrm{Sex}} + \beta_7 Z_{\mathrm{Age}}(t) \times Z_{\mathrm{Stat}}(t) \\ &+ \beta_8 Z_{\mathrm{Sex}} \times Z_{\mathrm{nh}}(t) + \beta_9 Z_{\mathrm{nh}}(t) \times Z_{\mathrm{level2}}(t) \end{aligned}$$

$$+ \quad \beta_{10}\, Z_{\mathrm{nh}}(t) \times Z_{\mathrm{level3}}(t) + \beta_{11}\, Z_{\theta_{\mathrm{f}}}(t) + \beta_{12}\, Z_{\theta_{\mathrm{m}}}(t)] \tag{3.2}$$

with respect to the variables θ_{f} and θ_{m}. A description of this approach can be found in Klein and Moeschberger (1997) pp. 334 - 336. We find, that the partial likelihood has a global maximum at $\theta_{\mathrm{f}} = 91$ and $\theta_{\mathrm{m}} = 75.25$ years. The fit of Model (3.2) with $\theta_{\mathrm{f}} = 91$ and $\theta_{\mathrm{m}} = 75.25$ is presented in Table 3.

Covariate	$\hat{\beta}$	$\exp(\hat{\beta})$	$se(\hat{\beta})$	W	p-value
Z_{Age}	0.0279	1.028	0.0036	7.68	$1.6 \cdot 10^{-14}$
Z_{Sex}	0.7637	2.146	0.3202	2.39.	$1.7 \cdot 10^{-2}$
Z_{nh}	-0.3724	0.689	0.4035	-0.92	$3.6 \cdot 10^{-1}$
Z_{Level2}	0.8101	2.248	0.0730	11.10	$< 10^{-15}$
Z_{Level3}	1.7521	5.767	0.0734	23.87	$< 10^{-15}$
$Z_{\theta_{\mathrm{f}}}$	0.2146	1.239	0.0737	2.91	$3.6 \cdot 10^{-3}$
$Z_{\theta_{\mathrm{m}}}$	-0.1401	0.869	0.1041	-1.34	$1.8 \cdot 10^{-1}$
$Z_{\mathrm{Age}} \times Z_{\mathrm{Sex}}$	-0.0138	0.986	0.0045	-3.08	$2.1 \cdot 10^{-3}$
$Z_{\mathrm{Age}} \times Z_{\mathrm{nh}}$	0.0134	1.014	0.0047	2.87	$4.1 \cdot 10^{-3}$
$Z_{\mathrm{Sex}} \times Z_{\mathrm{nh}}$	-0.4374	0.646	0.0993	-4.40	$4.4 \cdot 10^{-5}$
$Z_{\mathrm{nh}} \times Z_{\mathrm{Level2}}$	-0.3254	0.722	0.1447	-2.25	$2.5 \cdot 10^{-2}$
$Z_{\mathrm{nh}} \times Z_{\mathrm{Level3}}$	-0.6733	0.510	0.1391	-4.84	$1.3 \cdot 10^{-6}$

Table 3: Estimated Regression Coefficients in Model (3.2) with $\theta_{\mathrm{f}} = 91$ and $\theta_{\mathrm{m}} = 75.25$

We assess now the model adequacy of qualitative covariates. Here, we use the so called *Andersen plots*. The basic idea of this method is to consider the interesting component Z_1 of the covariate vector $Z = (Z_1, Z_2^t)^t$ ($Z_2 \in \mathbb{R}^{p-1}$ is the vector of the remaining $p-1$ components of the covariate vector). We assume Z_1 takes a finite number of values (the values are w.l.o.g $\{1, ..., K\}$). To check the proportional hazard assumption (2.2), we group our data with respect to the values of the covariate of interest Z_1 and estimate the corresponding baseline-hazard rates $\Lambda_{0i}(t) \quad i = 1, ..., K$ separately for each group. At each time, when an event of interest happens $t_1 < ... < t_k$ we plot the cumulative hazard rate $\Lambda_{0i}(t)$ (x-axis) against the cumulative hazard rate of another group $\Lambda_{0j}(t)$. If the proportional hazard assumption holds, the plot should follow a straight line. In Figures 2 to 4 these plots are examined for the qualitative covariates Z_{Sex}, Z_{nh}, Z_{Level2} and Z_{Level3}, respectively.

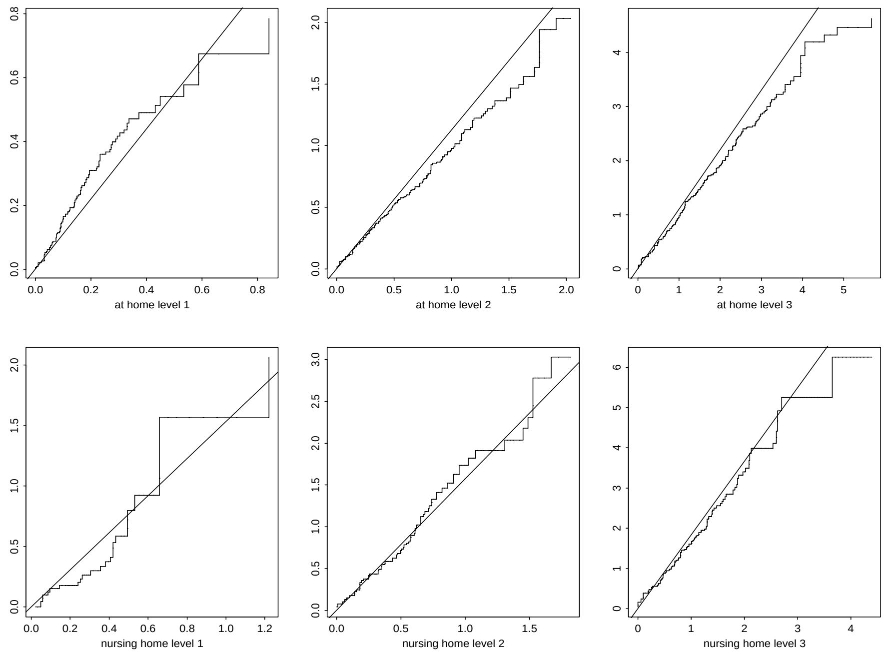

Figure 2: Andersen Plots grouped by Gender, x-axis: female, y-axis: male

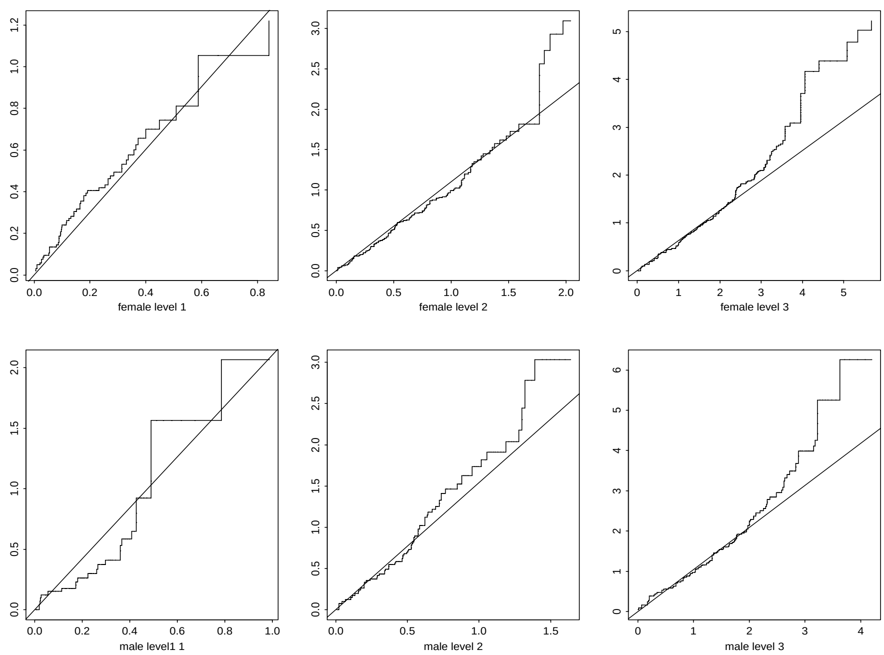

Figure 3: Andersen Plots grouped by Type of Care, x-axis: at home, y-axis: in a nursing home

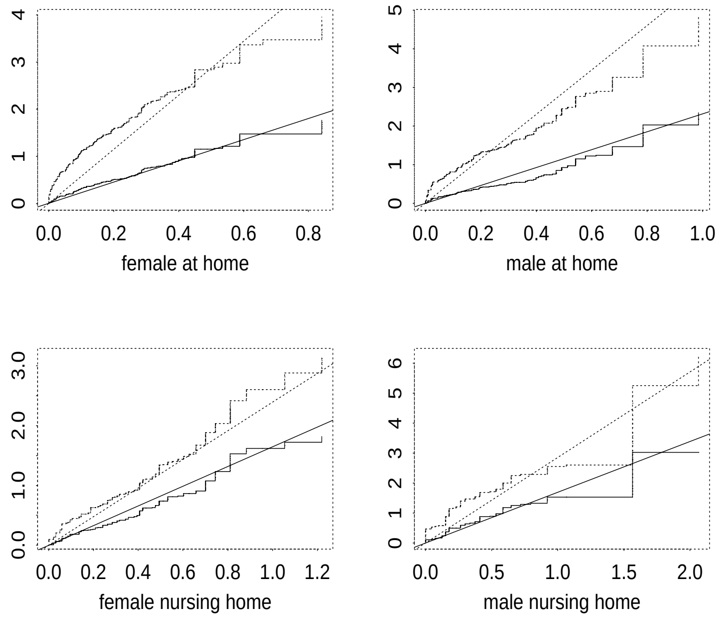

Figure 4: Andersen Plots grouped by Level of Care, x-axis: Level 1, y-axis: Level 2, Level 3 (dotted line)

From these Figures we see that especially the covariate $Z_{\text{Level 3}}$ does not satisfy the proportional hazard assumption; in the first years after the begin of LTC-claimance the relative risk of dying for a person is higher than in later years. This means, that the value of the regression coefficient is changing with time. This means we need to estimate a unknown regression function $\beta(t)$. A first approach is to fit step functions for $\beta(t)$, which corresponds to fitting a piecewise constant model in disjunct time intervals. As a further simplification we now estimate separate coefficients for times $T \leq t_0$ and $T > t_0$ where t_0 will be optimized. For example, if we consider the covariate $Z_{\text{Level3}(t)}$ we introduce a new covariate

$$Z_{t_0}(t) = \begin{cases} 1 & \text{if } t \leq t_0 \\ 0 & \text{if } t > t_0 \end{cases}$$

The corresponding hazard function has now the following representation

$$\lambda(t) = \lambda_0(t) \exp[\gamma Z_{\text{Level3}(t)} \times Z_{t_0}(t) + \beta^t Z(t)],$$

where $Z(t)$ is the covariate vector of Model (3.2) and $\gamma \in \mathbb{R}, \beta \in \mathbb{R}^{12}$ the are unknown regression coefficients to be estimated.

For each of the 12 covariates $Z_i(t)$ in Model (3.2), we now find an optimal t_0 by considering the partial log likelihood when the covariate $Z_i(t) \times Z_{t_0}$ is added as a function of t_0 and finding the t_0 value which maximizes this partial log likelihood. In Figure 5 the partial log likelihood as a function of t_0 is shown. From this we see, that there is a strong improvement in the partial log likelihood for the covariates for age, as well as for Level 2 and Level 3. From Table 4 we see that all these improvements are highly significant.

Covariate	Maximum t_0	LL (Model for $T \leq t_0$, $T > t_0$)	LL (Model(3.2))	p-value
Z_{Age}	400	-15829.59	-15874.79	$< 10^{-15}$
Z_{Level2}	75	-15866.69	-15874.79	$5.6 \cdot 10^{-5}$
Z_{Level3}	125	-15845.82	-15874.79	$2.7 \cdot 10^{-14}$

Table 4: Maximum of the Log Likelihood for Separate Estimation of the Covariates for Age, Level 1 and Level 2 (grouped by duration of LTC $T \leq t_0$ and $T > t_0$)

These results motivate us to add the covariates $Z_{75}(t)$, $Z_{125}(t)$ and $Z_{400}(t)$ to the Model (3.2) which are defined by

$$Z_{t_0}(t) = \begin{cases} 1 & t \leq t_0, \ \ t_0 = 75, \ 125, \ 400 \\ 0 & \text{otherwise} \end{cases}$$

and to consider their interaction with the corresponding covariates for Age, Level 2 and Level 3. For this model the log likelihood is improved to the value of 15801.4. The estimated regression coefficients for this model can be found in Table 5. These coefficients will be used in the calculations in later sections.

3.2 Analysis of the State Transitions between Care at Home and Care in a Nursing Home

Until now, we only considered the event "death of a LTC claimant" and its dependency on various risk factors. In this section we extend the model and consider a multiple state Markov model for the three states: care at home, care in a nursing home, death. In a first step we estimate the transition intensities λ_{ij}, $i \in 1,2,3$, $j \in 1,2,3$ (see Appendix) corresponding to the states that are illustrated in Figure 6.

Using the final model of the previous section, we can estimate the transition intensities $\lambda_{13}(t)$ ($\lambda_{23}(t)$) by setting $Z_{nh}(t) = 0 (Z_{nh}(t) = 1)$. Therefore, we are now interested in estimating $\lambda_{12}(t)$

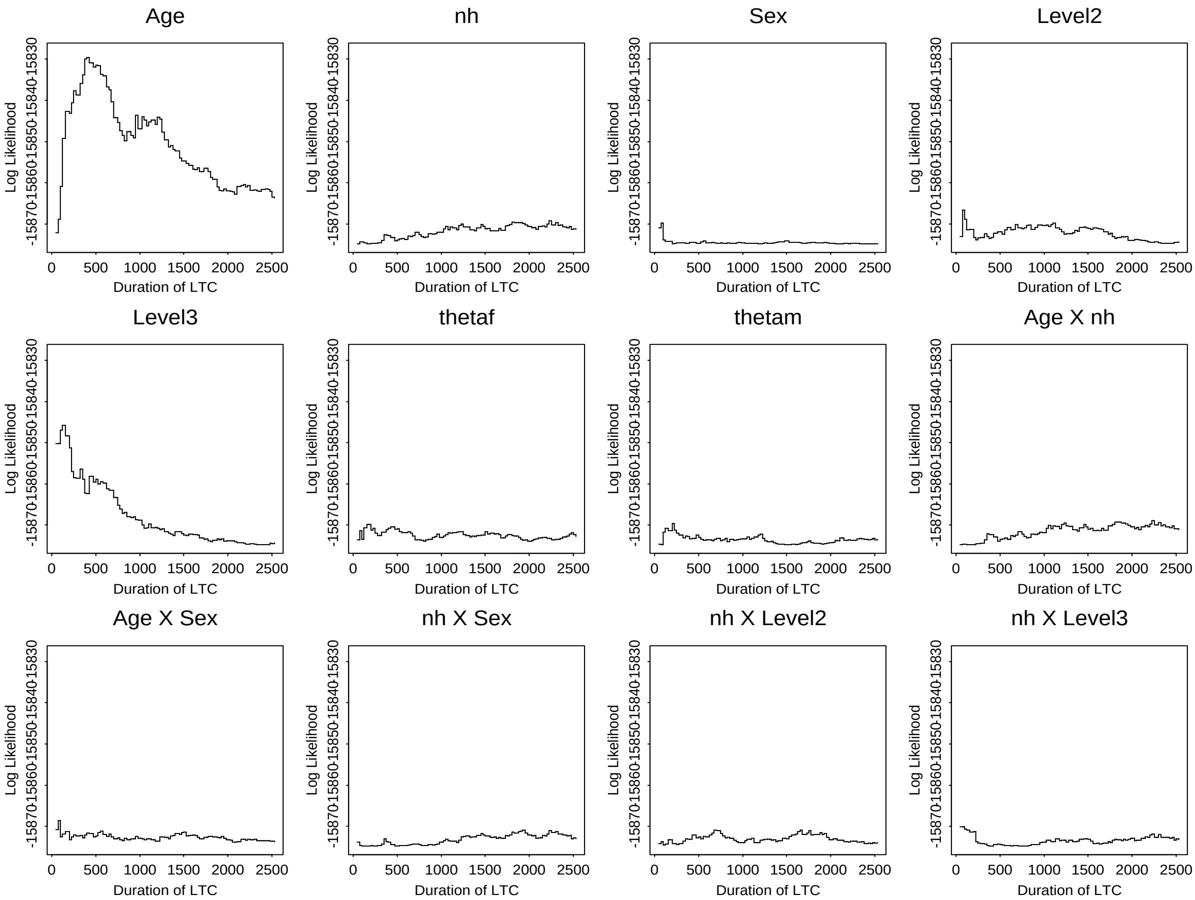

Figure 5: Partial log likelihood as a function of t_0 for each Covariate $Z_i(t)$ of Model (3.2) with added Covariate $Z_i(t) \times Z_{t_0}(t)$

and $\lambda_{21}(t)$. For this we define the following counting processes

$$\begin{aligned} N_{12}(t) &= \#\text{ transitions from at home to nursing home until time } t \\ N_{21}(t) &= \#\text{ transitions from nursing home to at home until time } t \end{aligned}$$

and fit a proportional hazard model for the transition intensities using the remaining covariates Z_{Sex}, $Z_{\text{Age}}(t)$, $Z_{\text{Level2}}(t)$ and $Z_{\text{Level3}}(t)$. Again, we fitted first a model with all possible interactions and then we took Akaike's information criterion to reduce the model to highly significant covariates.

For the transition intensities $\lambda_{12}(t)$ our final model is given by

$$\begin{aligned} \lambda_{12}(t) &= \lambda_{120}(t)\exp[\beta_1\, Z_{\text{Age}}(t) + \beta_2\, Z_{\text{Sex}} + \beta_3\, Z_{\text{Level 2}}(t) \\ &+ \beta_4\, Z_{\text{Level 3}}(t)]. \end{aligned} \tag{3.3}$$

Covariate	$\hat{\beta}$	$\exp(\hat{\beta})$	$se(\hat{\beta})$	W	p-value
Z_{Age}	0.0401	1.041	0.0041	9.65	$< 10^{-15}$
$Z_{\text{Age}} \times Z_{400}$	-0.0327	0.968	0.0035	-9.47	$< 10^{-15}$
Z_{Sex}	0.7986	2.222	0.3214	2.49	0.01
Z_{nh}	-0.0531	0.948	0.4004	-0.13	0.89
Z_{Level2}	0.8011	2.228	0.0733	10.93	$< 10^{-15}$
$Z_{\text{Level2}} \times Z_{75}$	-0.8080	0.446	0.4982	-1.62	0.10
Z_{Level3}	1.6420	5.165	0.0752	21.83	$< 10^{-15}$
$Z_{\text{Level3}} \times Z_{125}$	1.135	3.113	0.1828	6.21	$5.2 \cdot 10^{-10}$
$Z_{\theta_{\text{f}}}$	0.1815	1.199	0.0743	2.443	0.02
$Z_{\theta_{\text{m}}}$	-0.1206	0.886	0.1052	-1.15	0.25
$Z_{\text{Age}} \times Z_{\text{Sex}}$	-0.0141	0.986	0.0045	-3.13	$1.8 \cdot 10^{-3}$
$Z_{\text{Age}} \times Z_{\text{nh}}$	0.0093	1.009	0.0047	1.99	0.05
$Z_{\text{Sex}} \times Z_{\text{nh}}$	-0.4257	0.653	0.0996	-4.27	$1.9 \cdot 10^{-5}$
$Z_{\text{nh}} \times Z_{\text{Level2}}$	-0.3420	0.710	0.1448	-2.36	0.02
$Z_{\text{nh}} \times Z_{\text{Level3}}$	-0.6533	0.520	0.1393	-4.69	$2.7 \cdot 10^{-6}$

Table 5: Estimated Coefficients in Model (3.2) with Separate Estimated Coefficients for $T \leq t_0$ and $T > t_0$ for Age and Level of Care

The corresponding estimated regression coefficients are presented in Table 6.

For transitions from care in a nursing home to care at home none of the covariates together with possible interactions exhibit significant influence. A reason for this is the extrem low number of transitions between these states. Therefore we decided to ignore the corresponding transition intensities (i.e. to set $\lambda_{21}(t) = 0 \; \forall t$) and to consider the state "care in a nursing home" as a stricty transient state, from which one can only enter in the state death. This allows us to calculate analytically the transition probabilities in the model by using the Kolmogorow equations (5.1) given in the Appendix.

3.3 Analysis of the State Transitions between Levels of Care

Similarly, we now investigate transitions between levels of care. In Table 7 we display the corresponding observed transitions. Also in this context we note, that there were many more tran-

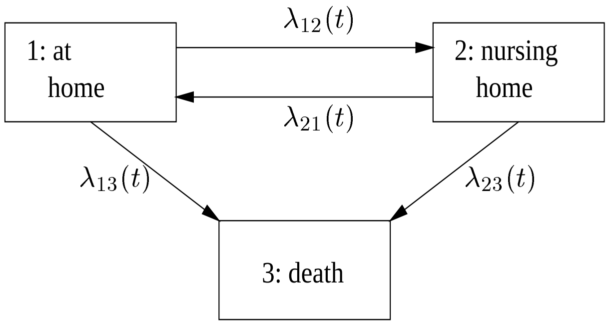

Figure 6: Markov Model with States Care at Home, Care in Nursing Home and Death

Covariate	$\hat{\beta}$	$\exp(\hat{\beta})$	$se(\hat{\beta})$	W	p-value
Z_{Age}	0.0305	1.03	0.0038	7.99	$1.2 \cdot 10^{-15}$
Z_{Sex}	0.3842	1.47	0.0956	4.02	$5.9 \cdot 10^{-5}$
Z_{Level2}	0.3191	1.38	0.0912	3.50	$4.6 \cdot 10^{-4}$
Z_{Level3}	0.1661	1.18	0.1279	1.30	$1.9 \cdot 10^{-1}$

Table 6: Estimated Regression Coefficients for the Transition Intensity $\lambda_{12}(t)$ as given by Model (3.3)

sitions to a worse level and only few observation, where the level improved. This motivated us to build our model as shown in Figure 7. Again we fit a proportional hazard model for the

	Level 1	Level 2	Level 3
Level 1	–	624	135
Level 2	64	–	472
Level 3	5	34	–

Table 7: Number of Observed Transitions between Levels of Care

transition intensities λ_{12}, λ_{23} and λ_{13} allowing for covariates and possible interactions. With respect to transitions between levels of care, we also observed that the need of LTC in most cases grows (we note a low number of transitions to better levels) and therefore we assume again that the transitions intensities λ_{32}, λ_{31} and λ_{21} are zero. For transitions between Level 1 and Level 2 the best fitting model is given by

$$\begin{aligned}\lambda_{12}(t) &= \lambda_{120}(t)\exp(\beta_1\, Z_{\text{Age}}(t) + \beta_2\, Z_{\text{Sex}} + \beta_3 Z_{\text{Stat}}(t) \\ &+ \beta_4\, Z_{\text{Age}}(t) \times Z_{\text{Sex}} + \beta_5\, Z_{\text{Age}}(t) \times Z_{\text{nh}}(t)\end{aligned}$$

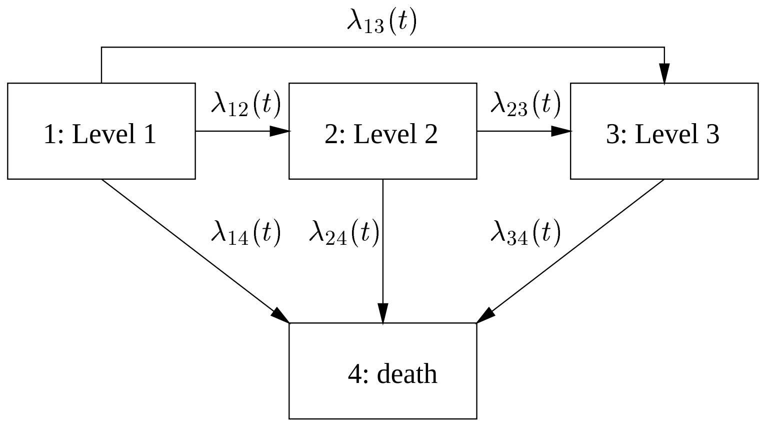

Figure 7: Markov Model with States Level 1, Level 2, Level 3 and Death

$$+ \quad \beta_6 \, Z_{\text{Sex}} \times Z_{\text{nh}}(t) + \beta_7 \, Z_{\text{Age}}(t) \times Z_{\text{Sex}} \times Z_{\text{nh}}(t)) \tag{3.4}$$

Covariate	$\hat{\beta}$	$\exp(\hat{\beta})$	$se(\hat{\beta})$	W	p-value
Z_{Age}	0.0143	1.01	0.0047	3.06	0.002
Z_{Sex}	-0.8544	0.43	0.5588	-1.53	0.130
Z_{nh}	-10.7437	$2.6 \cdot 10^{-5}$	3.5743	-3.01	0.03
$Z_{\text{Age}} \times Z_{\text{Sex}}$	0.0099	1.01	0.0069	1.42	0.150
$Z_{\text{Age}} \times Z_{\text{nh}}$	0.1221	1.13	0.0407	3.00	0.003
$Z_{\text{Sex}} \times Z_{\text{nh}}$	11.7833	$1.31 \cdot 10^{5}$	3.7654	3.13	0.002
$Z_{\text{Age}} \times Z_{\text{Sex}} \times Z_{\text{nh}}$	-0.1323	0.88	0.0429	-3.08	0.002

Table 8: Estimated Regression Coefficients for $\lambda_{12}(t)$ given by (3.4)

From Table 8 we notice the large interactions between the covariates. For the transition intensities $\lambda_{13}(t)$ and $\lambda_{23}(t)$ the model selection procedure finds a model that was only depending on the covariates Z_{Sex} and Z_{Age}, i.e. that corresponding hazard functions had following representation:

$$\lambda_{13}(t) = \lambda_{130}(t) \exp[\beta_1 \, Z_{\text{Age}}(t) + \beta_2 Z_{\text{Sex}}], \tag{3.5}$$

$$\lambda_{23}(t) = \lambda_{230}(t) \exp[\beta_1 \, Z_{\text{Age}}(t) + \beta_2 \, Z_{\text{Sex}}]. \tag{3.6}$$

The corresponding estimated regression coefficients are given in Tables 9 and 10

With this set of covariate dependent hazard rates, we were now able to fit actuarial models.

Covariate	$\hat{\beta}$	$\exp(\hat{\beta})$	$se(\hat{\beta})$	W	p-value
Z_{Age}	0.035	1.04	0.0087	3.98	$7 \cdot 10^{-5}$
Z_{Sex}	-0.368	0.70	0.1873	-1.97	0.05

Table 9: Estimated Regression Coefficients for $\lambda_{13}(t)$ given by (3.5)

Covariate	$\hat{\beta}$	$\exp(\hat{\beta})$	$se(\hat{\beta})$	W	p-value
Z_{Age}	0.035	1.04	0.0087	4.06	$4.8 \cdot 10^{-5}$
Z_{Sex}	-0.359	0.70	0.1865	-1.92	0.06

Table 10: Estimated Regression Coefficients for $\lambda_{23}(t)$ given by (3.6)

4 Actuarial Application

4.1 Model Description

In this section we apply the results of the data analysis presented in the previous section to a multiple state insurance model. We follow the approach taken by Haberman and Pitacco (1999) and we sketch now the necessary setup. The development of an insured risk is modeled as a time continuous Markov process $\mathcal{S} : \mathcal{T} \times \Omega \mapsto \{1, ..., n\}$ with finite state space $Z = \{1, ..., n\}$. Here $\mathcal{T} = [0, \tau)$ denotes the interval from policy begin to policy end. The insurance process is modeled by a random cash-flow function between the insurer and the insured.

Definition 4.1 (Cash flow functions) *For the insurance process S we define following cash flow functions*

(i) $p_i(t)$: A continuous premium, payed by the insured while the risk is in state i.

(ii) $b_i(t)$: A continuous annuity payed by the insurer while the risk in state i

(iii) $c_{ij}(t)$: A lump sum paid by the insurer at time t because a state transition from state i to state j occurs

(iv) $d_i(t_0)$: A lump sum paid by the insurer because the risk is in state i at time t_0

Equiped with an adequate interest structure we can now define random present values. We suppose the force of interest δ to be constant and set $v = e^{-\delta}$.

Definition 4.2 (Present values) *For the cash flow functions given in 4.1 (i) - (iv) we define following present values:*

(i) The present value of a continuous premium p_j to be paid in $[u, u + du]$ at time $t < u$ is:

$$Y_t^{p_j}(u, u + du) := v^{u-t} I(S(u) = j) p_j(u)\, du.$$

The present value for the interval $[u_1, u_2)$ mit $t \leq u_1 < u_2$ is:

$$Y_t^{p_j}(u_1, u_2) := \int_{u_1}^{u_2} v^{u-t} I(S(u) = j) p_j(u)\, du.$$

For a continuous annuity b_j at time t for the interval $[u_1, u_2)$

$$Y_t^{b_j}(u_1, u_2) = \int_{u_1}^{u_2} v^{u-t} I(S(u) = j) b_j(u)\, du.$$

(ii) The present value of a lump sum c_{ij} when a transition from state i to state j occurs at time u is

$$Y_t^{c_{ij}}(u) = v^{u-t} I(S(u-) = i \wedge S(u) = j).$$

(iii) Finally we define the present value of a single payment $d_j(t_0)$, payed by the insurer if the risk is in state j at time t_0 as

$$Y_t^{d_j}(t_0) = v^{u-t} I(S(u) = j) d_j(t_0).$$

Finally, we have to define actuarial values, which provide the basis for calculating premiums and reserves.

Definition 4.3 (Actuarial Values) *Actuarial values are expected present values. Let us suppose that at time t the insured risk is in state i, then the actuarial values are given as conditional expectations of the present values, i.e. we are interested in $E[Y_t(u)|S(t) = i]$ for lump sum payment and $E[Y_t(u, u + dt)|S(t) = i]$ for annuities. Therefore, we define:*

(i) The actuarial value of a continuous premium p_j for the infinitesimal interval $[u, u + du)$ is given by

$$E[Y_t^{p_j}(u, u + du)|S(t) = i] = v^{u-t} P_{ij}(t, u) p_j(u)\, du.$$

For the interval $[u_1, u_2)$ the actuarial value is

$$E[Y_t^{p_j}(u_1, u_2)|S(t) = i] = \int_{u_1}^{u_2} v^{u-t} P_{ij}(t, u) p_j\, du.$$

(ii) Similiarly for a continuous annuity $b_j(t)$, the actuarial value is

$$E[Y_t^{b_j}(u_1,u_2)|S(t)=i] = \int_{u_1}^{u_2} v^{u-t} P_{ij}(t,u) d_j \, du.$$

(iii) For a lump sum $c_{jk}(u)$, payed by the insurer because a transition of the insured risk from state j to state k occurs, the actuarial value is

$$E[Y_t^{c_{jk}}(u)|S(t)=i] = v^{u-t} P_{ij}(t,u)\lambda_{jk}(u)c_{jk}(u)\, du$$

for the interval $[u_1,u_2)$ the actuarial value is

$$E[Y_t^{c_{jk}}(u_1,u_2)|S(t)=i] = \int_{u_1}^{u_2} v^{u-t} P_{ij}(t,u)c_{jk}(u)\lambda_{jk}(u)\, du$$

(iv) Finally for a lump sum $d_j(t_0)$, payed by the insurer because the insured risk is in state j at t_0 the actuarial value is

$$E[Y^{d_j}(t_0)|S(t)=i] = v^{t_0-t} P_{ij}(t,t_0) d_j(t_0)$$

The calculation of premiums is now based on the equivalence principle, i.e. the expected amount of premiums has to equal the expected amount of benefits. For the cash flow between insurer and insured this means, that at policy begin ($t=0$) the actuarial value of all benefits has to be the same as the actuarial value of the premiums. This equivalence principle can now described as follows:

Definition 4.4 *For the insurance process $S(t)$ with policy end at τ we define*

$$\begin{aligned}
\mathcal{B}_i(t,\tau) \quad &:= \quad \int_t^\tau v^{u-t}\left[\sum_{j\in\mathcal{S}} P_{ij}(t,u)b_j(u)\right] du \\
&+ \quad \int_t^\tau v^{u-t}\left[\sum_{j\in\mathcal{S}}\sum_{k\neq j} P_{ij}(t,u)\lambda_{jk}(u)c_{jk}(u)\right] du \\
&+ \quad \sum_{u:u\geq t} v^{u-t}\left[\sum_{j\in\mathcal{S}} P_{ij}(t,u)d_j(u)\right]
\end{aligned}$$

to be the sum of all expected benefits at time t, given $S(t)=i$ and similarly

$$\mathcal{P}_i(t,\tau) = \int_t^\tau v^{u-t}\left[\sum_{j\in\mathcal{S}} P_{ij}(t,u)p_j(u)\right] du$$

to be the expected value of all premiums at time t, given $S(t)=i$.

This allows us to describe the equivalence principle formally

Definition 4.5 (equivalence principle) *For an insured risk with policy end at τ and initial state $S(0) = 1$ the equivalence principle is satisfied if and only if*

$$\mathcal{P}_1(0,\tau) = \mathcal{B}_1(0,\tau) \tag{4.7}$$

This principle does not yet define a fixed premium amount, because it only has to be satisfied at policy begin. There exist many examples in life assurance, where premiums are growing for higher ages due to the increasing risk of death. To get constant premium for the whole policy duration we have to build up reserves (i.e. in the first years the insured has to pay higher premium that will bee used later to cover the risk in the last years). For Equation (4.7) this implies that at each time $t \leq \tau$, $\mathcal{P}_1(t,\tau) \leq \mathcal{B}_1(t,\tau)$ has to be satisfied.

The difference between the actuarial values of premiums and benefits is called the prospective reserve at time t

$$\overline{V}_i(t,\tau) = \mathcal{B}_1(t,\tau) - \mathcal{P}_1(t,\tau).$$

With these preparations and the estimation of the transition intensities, we are now able to calculate premiums for a multiple state model in long term care insurance. The two models which we considered already in the previous are shown in Figures 8 and 9. In the first model we focus our attention to transitions between the different types of care (at home - nursing home). The transitions surrounded by the dotted line can be estimated from the data, while there is no information available in the data about the transitions $1 \to 2$, $1 \to 3$ and $1 \to 4$. Therefore, we have to construct the remaining intensities from other data sources. The mortality rates for healthy individuals we derived from the Bavarian Lifetable 1986 - 1988, incidence rates for LTC (i.e. transition intensities from healthy to LTC claimance) we took from a table published by the Custodial Insurance, Japan (see for example Rudolph (2000), Appendix C.1). This might be problematic, since the definition of LTC varies in the different countries. To calculate transition probabilities of hazard rates for the model shown in Figure 8 we have to solve the following Kolmogorov differential equations for the transition probabilities $P_{ij}(z,t)$ (see (5.1) in the Appendix)):

$$\begin{aligned}
\frac{d}{dt}P_{11}(z,t) &= -P_{11}(z,t)(\lambda_{12}(t) + \lambda_{13}(t) + \lambda_{14}(t)),\\
\frac{d}{dt}P_{12}(z,t) &= P_{11}(z,t)\lambda_{12}(t) - P_{12}(z,t)(\lambda_{23}(t) + \lambda_{24}(t)),\\
\frac{d}{dt}P_{13}(z,t) &= P_{11}(z,t)\lambda_{13}(t) + P_{12}(z,t)\lambda_{23}(t) - P_{13}(z,t)\lambda_{34}(t),
\end{aligned}$$

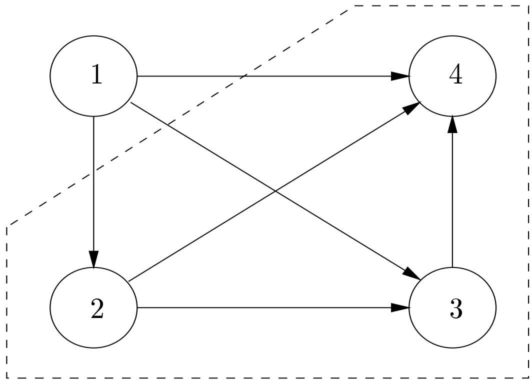

Figure 8: State transitions in LTC insurance - type of care

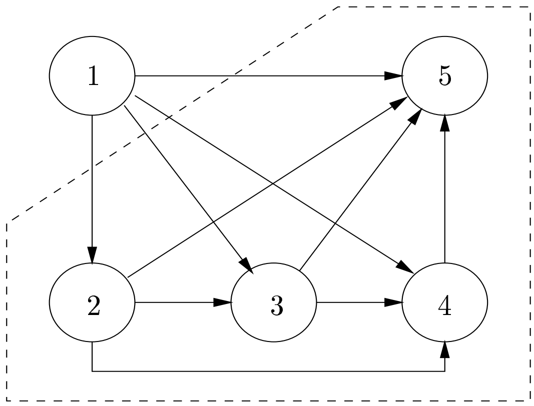

Figure 9: State transitions in LTC insurance - level of care

$$\frac{d}{dt}P_{14}(z,t) \quad = \quad P_{11}(z,t)\lambda_{14}(t) + P_{12}(z,t)\lambda_{24}(t) + P_{13}(z,t)\lambda_{34}(t), \tag{4.8}$$

with solution

$$\begin{aligned}
P_{11}(z,t) &= \exp\left(-\int_z^t [\lambda_{12}(u) + \lambda_{13}(u) + \lambda_{14}(u)]\,du\right), \\
P_{22}(z,t) &= \exp\left(-\int_z^t [\lambda_{23}(u) + \lambda_{24}(u)]\,du\right), \\
P_{33}(z,t) &= \exp\left(-\int_z^t \lambda_{34}(u)\,du\right), \\
P_{34}(z,t) &= 1 - P_{33}(z,t), \\
P_{23}(z,t) &= \int_z^t P_{22}(z,u)\lambda_{23}(u)P_{33}(u,t)\,du, \\
P_{24}(z,t) &= 1 - P_{22}(z,t) - P_{23}(z,t), \\
P_{12}(z,t) &= \int_z^t P_{11}(z,u)\lambda_{12}(u)P_{22}(u,t)\,du, \\
P_{13}(z,t) &= \int_z^t [P_{11}(z,u)\lambda_{13} + P_{12}(z,u)\lambda_{23}(u)P_{33}(u,t)]\,du,
\end{aligned}$$

$$P_{14}(z,t) \quad = \quad 1 - P_{11}(z,t) - P_{12}(z,t) - P_{13}(z,t). \tag{4.9}$$

For the model shown in Figure 9 a similar system of differential equations had to be solved.

Since in life insurance premium calculation is based on 1-year transition rates we have to derive a set of 1-year transition probabilities from our estimated transition intensities.

For every combination of LTC-duration d and age x we calculate one year transition rates from state i to state j. Let $p_{i,j}(x,d)$ denote the one year transition probability from level of care i to level of care j of a x-year old person who has been a LTC-claimant for d years:

$$p_{i,j}(d,x) = P(S(d+1) = j | S(d) = i, Z_{\text{Age}} = x)$$

We use now the estimated transition intensities given in Section 3 and the fact that for a survival function F with discrete hazard rates λ_{t_i} the following holds:

$$P(t_1 \leq T < t_2 | T \geq t_2) = \frac{F(t_1) - F(t_2)}{F(t_1)} = 1 - \prod_{t_1 \leq t_i < t_2} (1 - \lambda_{t_i}) = \sum_{t_1 \leq t_i < t_2} \lambda_{t_i} \prod_{t_1 \leq t_k < t_i} (1 - \lambda_{t_k}).$$

Therefore we can estimate the one-year transition probabilities as

$$p_{ij}(d,x) = \sum_{d \leq t_k < d+1} \hat{\lambda}_{ij}(t_k, Z_{\text{Age}} = x) \prod_{d \leq t_l < t_k} (1 - \lambda_{ij}(t_l, Z_{\text{Age}} = x)), \tag{4.10}$$

where $\hat{\lambda}_{i,j}(t)$ are the estimated transition rates for LTC-claimants.

In order to simplify the notation in the following we will denote the transition probabilities only in dependency on the insurance duration n. For the model shown in Figure 8, it follows that

$$p_{ij}(n) = P(S(n+1) = j | S(n) = 1) \;\; i \in \{1,2,3,4\}, \;\; j \in \{2,3,4\}$$

and $p_{ij}(n) = 0$ for $j \leq i$. Note that these transition intensities still depend on age, gender, level of care (model shown in Figure 8), respectively type of care (model shown in Figure 9). For example in the model that distinguishes between care in a nursing home and care at home, we have for the transition probability from State 2 (care at home) to State 4 (death) for an x-year old female person:

$$\begin{aligned} p_{2,4}(d,x) \quad = \quad & \sum_{d \leq t_k < d+1} \hat{\lambda}_{24}(t_k, Z_{\text{Age}} = x, Z_{\text{nh}} = 0, Z_{\text{Sex}} = 1) \\ & \prod_{d \leq t_l < t_k} (1 - \hat{\lambda}_{24}(t_l, Z_{\text{Age}} = x, Z_{\text{nh}} = 0, Z_{\text{Sex}} = 1)). \end{aligned}$$

With (4.10) we are now able to determine easily the actuarial values for this model. Let $\mathcal{B}_{1,c_{1j}}(0)$, $j \in \{2,3\}$ denote the actuarial value at the beginning of insurance contract for an individual with entry age x where a lump sum c_{1j} is payable at the moment when an active live changes to "care at home" (State 2) or to "care in a nursing home" (State 3). Let ω denoting the actuarial end-age, i.e. for the random variable life time T we define $P(T > \omega) := 0$. Recall that $P_{ij}(n,m) = P(S(m) = j|S(n) = i)$ for $n \leq m$, which can be calculated by using a discretized version of (4.9). In particular, the following holds

$$\mathcal{B}_{1,c_{1j}}(0) = \sum_{i=0}^{\omega-x-1} P_{11}(0,i)p_{1j}(i)v^i c_{1j}$$

In this formula we consider the probability of an insured person for a transition to state j in year i $P_{11}(0,i)p_{1j}(i)$ and multiply this probability with the discounted actuarial value of the benefit $v^i c_{1j}$. Building the sum over all years i gives us the expected value of the payments.

Similarly for a annuity b_j, payable while an insured person is in state j, $j \in \{2,3\}$ we have

$$\mathcal{B}_{1,b_j} = \sum_{i=0}^{\omega-x-1} P_{1j}(0,i)v^i b_j.$$

Here we consider the probability of a person of being in state j after i years $P_{1j}(0,i)$ and multiply this value with the discounted actuarial value of the annuity b_j. Finally, the actuarial value of the expected total payment to be payed by the insurance is given by $\sum_{j=2}^{3}(B_{1,b_j}(0) + B_{1,c_{1j}}(0))$ which has to be equal to $P_{1,\pi}$ by the equivalence principle. This determines the annual premium π. The actuarial value of the payments for a single annual premium π is given by

$$P_{1,\pi}(0) = \sum_{i=0}^{\omega-x-1} P_{11}(0,i)v^i \pi.$$

The same approach can be used to calculate the actuarial values for the model presented in Figure 9. For further details of the calculation see Rudolph (2000).

4.2 Calculation of Premiums

In this section we calculate premiums for the Long Term Care-plan "PET" sold by a German health insurer. For this purpose a computer program in C was written, that calculated the appropriate premiums based on the estimated intensities of the Cox proportional hazard model presented earlier together with the actuarial model given above. Input parameters were annuities, whose values could vary depending on the level and type of care required. In the plan "PET", an insured person can contract a fixed amount that serves as a daily cash allowance. In case of LTC which requires care at home, the individual receives a certain percentage of this allowance. In particular, the individual is payed 25 % of the agreed allowance in Level 1, 50 % of the agreed allowance in Level 2 and 75 % of the agreed allowance in Level 3. For care in a nursing home, an insured individual receives 100 % of the agreed allowance.

Since the data does not include any information with respect to the transition intensities from active to disabled and from active to death, we used the life table for Bavarian males and females (1986-1988), as well as the LTC incidence rates of custodial insurance, Japan. Since these incidence probabilities are commercial rates and therefore subject to high loading, a direct comparison of the calculated premium rates and the ones offered by the German health insurer is not very reasonable (see Table 11). However, we see that the premium rates based on the Cox proportional hazard model behave similar with respect to age and the proportion between the genders to the premium rates offered by the German health insurer.

5 Discussion

In this paper we have shown, that not only graduation techniques, but also the direct application of the proportional hazard model is an appropriate tool for estimating hazard intensities in actuarial models. The advantage of this approach is in addition to the inclusion of information contained in censored observations the availability of a number of graphical and analytical methods for controlling the model. With splitting observation time into several disjoint intervals and estimating coefficients for each of those intervals separately, we achieved a significant improvement over the basic model. This improvement would have been difficult to achieve with the Poisson approach followed by Renshaw and Haberman (1995). In future works, efforts will be put in developing models in which time continuous regression coefficients, perhaps modeled as polynomial splines, could be included. Since for the data used we also have information about the diagnosis which leads to LTC, we also want to include this information.

have seen that not only age, but also the time spent in LTC, as well as other factors like severeness of the claim and type of care offered have significant influence on the survival of a patient.

	Premium based on Cox		Premium offered by	
	Proportional Hazard Model		German Health Insurer	
Age	Female	Male	Female	Male
20	2.81	2.31	2.12	1.70
25	3.50	2.89	2.92	2.33
30	4.41	3.65	3.90	3.10
35	5.63	4.67	5.05	4.01
40	7.28	6.06	6.44	5.13
45	9.48	7.91	8.16	6.52
50	12.49	10.42	10.39	8.36
55	16.67	13.93	13.32	10.86
60	22.56	18.90	17.31	14.40
65	30.98	26.04	22.01	18.84
70	42.91	36.40	29.04	25.71

Table 11: Comparison of monthly payable premiums for DM 10 daily allowance calculated for the plan "PET" of a German Health Insurer. Left columns: Premiums calculated using the Markov Model shown in Figure 8, Right columns: Premiums offered by a German Health Insurer.

Appendix: Time-continous Markov Chains with Countable State Space

To obtain an actuarial basis for premium calculation we need some basic concepts of the theory of time-continuous Markov chains with numerable state space. A description of actuarial methods in the context of Markov Chains can be found in Chapter 1 of Haberman and Pitacco (1999). Let $S : \mathcal{T} \times \Omega \mapsto \mathcal{Z}$ be a time continuous Markov chain, where $\mathcal{T} := [0, \infty)$ is the parameter space and $\mathcal{Z} \subset \mathbb{N}$ a countable state space. Consider now the transition probabilities

$$P_{ij}(z,t) := P(X(t) = j | X(z) = i)$$

defined for all $z \leq t$ and $t, z \in \mathcal{T}$ and $i, j \in \mathcal{Z}$. The corresponding transition intensities are given by

$$\lambda_{ij}(t) = \lim_{dt \to 0} \frac{P_{ij}(t, t+dt)}{dt}.$$

Furthermore we define with

$$\lambda_i(t) := \sum_{j \neq i} \lambda_j(t)$$

the intensity for leaving state i. Important relationships between transition probabilities and transition intensities are given by the forward Kolmogorov-differential equations (e.g. Karlin and Taylor (1981), Chapter 14):

$$\frac{d}{dt} P_{ij}(z,t) = \sum_{k:k \neq j} P_{ik}(z,t)\lambda_{kj}(t) - P_{ij}(z,t)\lambda_j(t) \tag{5.1}$$

References

Akaike, H. (1973). Information Theory and a Extension of the Maximum Likelihood Principle. *2nd International Symposium of Information Theory and Control*, 267–281.

Andersen, P. K. and R. D. Gill (1982). Cox's regression modell for counting processes. *The Annals of Statistics 10*, 1100–1120.

Breslow, N. (1974). Covariance analysis of censored survival data. *Biometrics 30*, 89–99.

Cox, D. (1975). Partial likelihood. *Biometrika 62*, 269–276.

Cox, D. R. (1972). Regression models and life-tables. *Journal of the Royal Stat. Society B 34*, 187–220.

Efron, B. (1977). The efficiency of cox's likelihood function for censored data. *Journal of the American Statistican Association 72*, 557–565.

Haberman, S. and E. Pitacco (1999). *Actuarial Models for Disability Insurance*. Chapman & Hall/CRC.

Karlin, S. and H. Taylor (1981). *A second course in stochastic processes*. New York: Academic Press.

Klein, J. P. and M. L. Moeschberger (1997). *Survival Analysis*. New York: Springer.

Oakes, D. (1977). The asymptotic information in censored survival data. *Biometrika 64*, 441–448.

Rao, C. R. (1965). *Linear Statistical Inference and Its Applications*. New York: Wiley.

Renshaw, A. E. (1988). Modelling excess mortality using GLIM. *Journal of the Institute of Actuaries 115*, 299–315.

Renshaw, A. E. and S. Haberman (1995). On the graduation associated with a multiple stae model in permanent health insurance. *Journal of the Institute of Actuaries 17*, 1–17.

Rudolph, F. (2000). Anwendungen der Überlebenzeitanalyse in der Pflegeversicherung. Diplomarbeit, Technische Universität München.

Therneau, T. M., P. M. Grambsch, and T. R. Fleming (1990). Martingale-based residuals for survival models. *Biometrika 1*, 147–160.