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Schlicht, Ekkehart:

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Munich Discussion Paper No. 2013-15

Department of Economics  
University of Munich

Volkswirtschaftliche Fakultät  
Ludwig-Maximilians-Universität München

Online at <https://doi.org/10.5282/ubm/epub.17249>

# Unexpected Consequences of Ricardian Expectations – Erratum

Ekkehart Schlicht\*

## Abstract

This note identifies a severe mistake in my article “Unexpected Consequences of Ricardian Expectations” that appeared in this journal in the July 2013 issue.

*Keywords:* Barro-Ricardo equivalence, Ricardian equivalence, fiscal policy, debt, taxation, rational expectations, Ricardian expectations, Barro expectations

*Journal of Economic Literature Classification:* E2, E62, H62

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\*Ekkehart Schlicht ([schlicht@lmu.de](mailto:schlicht@lmu.de)) is professor emeritus of economics at the Ludwig-Maximilians-Universität Munich, Germany.

## 1 Introduction

Earlier this year I published an article entitled “Unexpected Consequences of Ricardian Expectations“ in this journal (Schlicht, 2013). While the mathematics was correct, the conclusion I drew from it is wrong. I wrote in particular:

*... the present value of the households' lifetime income has increased by switching from the pay-as-you-go regime to the debt regime. The Barro expectations held by the subjects are not fulfilled. The necessity for tax increases, that they believed to be unavoidable, never arises. The households engage in precautionary savings in order to finance tax increases that need never occur. ... As the value of their lifetime income stream has increased, they could have afforded higher expenditure, with more consumption and more investment.*

This reasoning is mistaken. The savings are not precautionary savings, but arise because households correctly anticipate tax increases and want to realize optimal savings – keeping their consumption stream flat in face of anticipated increasing taxation. Because of these savings, they receive additional income from interest. These interest payments lead to increased lifetime income, *but such an increase would not occur without these savings*. As a consequence, these savings could not be spent without harming lifetime income, as I have wrongly suggested. As an earlier referee put it (and I did not understand), disposable income is endogenous to the problem (Anonymous, 2012).

I have convinced myself in the meanwhile that, with optimizing household behavior, the Ricardian equivalence thesis is correct under quite general conditions. Actually I should have seen that from earlier critical referee reports but did not. Without any claim for originality I would explain this as follows in terms of the household's optimization strategy, rather than budget constraints.

Consider a representative household – one of many identical households. The time path of wages the household receives is  $w_t$ , the time path of interest is  $r_t$ , and the time path of taxation is  $z_t$ . All these time paths are taken as given by the household.

By deciding about consumption and accumulation, the household makes a decision about the time path of its assets  $a_t$ . These assets include shares in

firms and government bonds. Gross income of the household comprises wage income  $w_t$  and interest income  $r_t a_t$ . The disposable income is gross income minus taxes:  $w_t + r_t a_t - z_t$ . This is used for consumption  $c_t$  and savings. Savings take the form of asset accumulation  $\dot{a}_t$ . So we have

$$w_t + r_t a_t - z_t = c_t + \dot{a}_t.$$

The household (or dynasty) chooses the time path of asset accumulation such that its discounted utility

$$U = \int_0^{\infty} e^{-\theta t} u(c_t) dt$$

is maximized, where  $u(\cdot)$  is its utility function and  $\theta > 0$  is the rate of time preference.

Assume that, for given time paths of  $w$ ,  $r$ , and  $z$  and for an initial stock of assets  $a_0$ , the time path  $a_t$  maximizes discounted utility. As discounted utility can be written as

$$U = \int_0^{\infty} e^{-\theta t} u(w_t + r_t a_t - z_t - \dot{a}_t) dt,$$

the maximization of  $U$  by a suitable choice of  $a$  is a standard problem in the calculus of variations (Bolza, 1960, 22). If  $a_t$  is optimal, it must satisfy the Euler equation

$$\frac{\partial}{\partial a_t} e^{-\theta t} u(w_t + r_t a_t - z_t - \dot{a}_t) = \frac{d}{dt} \frac{\partial}{\partial \dot{a}_t} e^{-\theta t} u(w_t + r_t a_t - z_t - \dot{a}_t)$$

which reduces to

$$e^{-\theta t} u' r_t = -\frac{d}{dt} e^{-\theta t} u'$$

and finally to

$$u'(r_t - \theta) = u''(\dot{w}_t + \dot{r}_t a_t + r_t \dot{a}_t - \dot{z}_t - \ddot{a}_t). \quad (1)$$

This is the Euler equation in the presence of government debt. It must be satisfied by the optimal time path of assets  $a_t$ . Under standard assumptions and together with boundary values  $a_0$  and  $a_\infty = 0$ , the differential equation (1) has a unique solution  $a_t$ . In the following we assume that this is the case.

Different debt policies will affect the time series  $z_t$  in (1) and will affect the optimal accumulation of assets  $a_t$ . We describe the government's debt policy as follows. Let  $G_t$  denotes the (given) time series of government spending. The government has to pay for this spending and serve the public debt  $D_t$  at a rate of interest  $r_t$ . So it has to finance  $G_t + r_t D_t$  by taxes  $Z_t$  and new debt  $\dot{D}_t$ , and we obtain the government's budget constraint

$$G_t + r_t D_t = Z_t + \dot{D}_t \quad (2)$$

This can be written in per capita terms a

$$g_t + r_t d_t = z_t + \dot{d}_t + \gamma_t$$

with  $g_t = \frac{G_t}{N_t}$  denoting per capita government spending,  $d_t = \frac{D_t}{N_t}$  per capita government debt,  $z_t = \frac{Z_t}{N_t}$  per capita taxes, and  $\gamma_t = \frac{\dot{N}_t}{N_t}$  the time path of the rate of population growth. By selecting a fiscal policy, the government selects a time series of new per capita debt  $\dot{d}_t$  which implies a time series of per capita debt according to  $d_t = \int_0^t \dot{d}_\tau d\tau + d_0$ . For such a policy, the implied per capita taxation is

$$z_t = g_t + r_t d_t - \dot{d}_t - \gamma_t. \quad (3)$$

Its change over time is

$$\dot{z}_t = \dot{g}_t + \dot{r}_t d_t + r_t \dot{d}_t - \ddot{d}_t - \dot{\gamma}_t.$$

Inserting this into the Euler equation yields

$$u'(r_t - \theta) = u''(\dot{w}_t + \dot{r}_t a_t + r_t \dot{a}_t - \dot{z}_t - \ddot{a}_t).$$

$$u'(r_t - \theta) = u''(\dot{w}_t + \dot{r}_t a_t + r_t \dot{a}_t - \dot{g}_t - \dot{r}_t d_t - r_t \dot{d}_t + \ddot{d}_t + \dot{\gamma}_t - \ddot{a}_t).$$

As the assets comprise per capita capital  $k_t$  and per capita debt  $d_t$  we can replace  $a_t$  by  $k_t + d_t$ ,  $\dot{a}_t$  by  $\dot{k}_t + \dot{d}_t$ , and  $\ddot{a}_t$  by  $\ddot{k}_t + \ddot{d}_t$ . This gives

$$u'(r_t - \theta) = u''(\dot{w}_t + \dot{r}_t(k_t + d_t) + r_t(\dot{k}_t + \dot{d}_t) - \dot{g}_t - \dot{r}_t d_t - r_t \dot{d}_t + \ddot{d}_t + \dot{\gamma}_t - (\ddot{k}_t + \ddot{d}_t))$$

and finally

$$u'(r_t - \theta) = u''(\dot{w}_t + \dot{r}_t k_t + r_t \dot{k}_t - \dot{g}_t + \dot{\gamma}_t - \ddot{k}_t).$$

This is the Euler equation that would be obtained from (1) for the case that the government runs a balanced budget and debt  $d_t$  is zero for all  $t$ . Further, the derivatives  $u'$  and  $u''$  are taken at  $c_t = w_t + r_t a_t - z_t - \dot{a}_t$ . With (3) and  $a_t = k_t + d_t$  this reduces to  $c_t = w_t + r_t k_t - g_t + \gamma_t - \dot{k}_t$  which is, again, independent of the government's debt policy. This proves the Ricardian equivalence for the general case of arbitrary time paths of the population, wages, and interest, and regardless of any production function or theory of factor prices.

So I was completely wrong regarding this matter. I thought to provide an internal criticism of the Ricardian equivalence thesis, thereby closing a gap between theory and observation, but I failed. The gap remains. I can only offer my apologies to the readers, the editors, and the referees for any unnecessary work and inconvenience caused by my mistake.

## References

- Anonymous (2012). Referee Report on "Unexpected Consequences of Ricardian Expectations" by Ekkehart Schlicht. URL [http://www.economics-ejournal.org/economics/discussionpapers/2012-13/comment.2012-04-13.1011252334/at\\_download/file](http://www.economics-ejournal.org/economics/discussionpapers/2012-13/comment.2012-04-13.1011252334/at_download/file).
- Bolza, O. (1960). *Lectures on the Calculus of Variations*. Chelsea, second edition. URL <http://archive.org/download/lecturesonthecal029607mbp/lecturesonthecal029607mbp.pdf>.
- Schlicht, E. (2013). Unexpected Consequences of Ricardian Expectations. *Metroeconomica*, 64(3): 498–512. ISSN 1467-999X. DOI 10.1111/meca.12017. URL <http://dx.doi.org/10.1111/meca.12017>.