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Anticompetitive Vertical Merger Waves

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Abstract

We develop a model of vertical merger waves leading to input foreclosure. When all upstream firms become vertically integrated, the input price can increase substantially above marginal cost despite Bertrand competition in the input market. Input foreclosure is easiest to sustain when upstream market shares are the most asymmetric (monopoly-like equilibria) or the most symmetric (collusive-like equilibria). In addition, these equilibria are more likely when (i) mergers generate strong synergies; (ii) price discrimination in the input market is not allowed; (iii) contracts are public; whereas (iv) the impact of upstream and downstream industry concentration is ambiguous.

1 Introduction

This paper develops a theory of anticompetitive vertical merger waves. Consider, as a motivating example, the satellite navigation industry in 2007. The upstream market is

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the market for navigable digital map databases, where only Tele Atlas and Navteq are active. At the downstream level, firms embed digital maps in the devices they manufacture in order to provide their customers with navigation solutions. Downstream firms include portable navigation device manufacturers such as TomTom, and manufacturers of mobile handsets that incorporate navigation possibilities, such as Nokia. In October 2007, TomTom notified American and European competition authorities that it would acquire Tele Atlas; four months later Nokia responded by announcing its planned acquisition of Navteq. At the time, the main concern in the industry was that the upstream market would then end up being supplied by two vertically integrated firms.[1]

While competition authorities cleared these mergers without conditions,[2] we argue that such mergers can have severe anticompetitive effects.[3] In our model there are initially M upstream firms and $N > M$ downstream firms. The game starts with a merger stage in which downstream firms can acquire upstream firms. Next, upstream firms (integrated or not) compete in prices to sell a homogeneous input to the remaining unintegrated downstream firms. Finally, downstream firms (integrated or not) compete in prices with differentiated products. If fewer than M mergers have taken place, the standard Bertrand logic applies and upstream competition drives the upstream price down to marginal cost.

The Bertrand logic no longer applies when all upstream firms are vertically in-

[1]See 'MLex Comment: TomTom, Nokia map deals raise parallel vertical competition issues', 23 October 2007: "*At the end of the day, whether either of these two transactions runs into trouble will be down to whether competitor device suppliers to both TomTom and Nokia baulk at the idea of having to purchase maps from their rivals.*"

[2]These merger cases were handled very differently on each side of the Atlantic. While the European Commission conducted in-depth investigations (see EC COMP M.4854 *TomTom/Tele Atlas* and COMP M.4942 *Nokia/Navteq*) and used these mergers to showcase its new non-horizontal mergers guidelines, the United States Department of Justice approved both of them within the 15-day waiting period prescribed for cash tender offers.

[3]Another example is the mobile telephony industry. The upstream market is the market for operating systems. The downstream market is the market for mobile handsets. Three of the four main players (Apple, Blackberry, and Google, after it completed its acquisition of Motorola in 2012) are currently vertically integrated. If the Microsoft-Nokia merger (announced in September 2013) receives clearance from competition authorities, then there will be no unintegrated upstream competitor left. The Premdor/Masonite merger documented in Riordan (2008) is another example of a vertical merger which eliminates the last unintegrated upstream producer.

tegrated. We obtain *monopoly-like equilibria* in which one vertically integrated firm sells the input to all unintegrated downstream firms at the monopoly upstream price – an outcome similar to Ordover, Saloner and Salop (1990)'s foreclosure outcome, but obtained without exogenous upstream commitment. We also obtain *collusive-like equilibria* in which all vertically integrated firms sell the input at the same price above marginal cost and share the upstream market symmetrically – an outcome similar to the collusive outcome of Nocke and White (2007), but obtained without repeated interactions.

These partial foreclosure outcomes can be sustained in equilibrium for the following reasons. A vertically integrated firm which sells the input to downstream firms has incentives to increase its downstream price even if it loses some downstream consumers. This is because some of these consumers will end up purchasing from unintegrated downstream firms, thereby raising upstream profits. It follows that vertically integrated firms which control high upstream market shares tend to set high downstream prices. Now, if a vertically integrated firm starts stealing upstream market shares from its integrated rivals, then these rivals cut their downstream prices, since they now have lower upstream market shares and therefore less incentive to be soft. As a result, expanding upstream market shares is not necessarily profitable, because the additional upstream profits may not be enough to compensate for the cost of facing more aggressive competitors in the downstream market.

The existence of input foreclosure equilibria in the M-merger subgame generates complementarities between vertical mergers. If no other merger takes place, then incentives to merge are weak, because the merger would not affect the upstream market outcome. By contrast, if $M - 1$ mergers have taken place, then an additional merger will move the upstream market away from marginal cost pricing, thereby making this last merger highly profitable. This leads to an equilibrium wave of vertical mergers in which every upstream firm integrates with a downstream firm, and the remaining unintegrated downstream firms obtain the input at a high price.

The model allows us to derive policy implications for merger control. When competition authorities decide whether to clear a vertical merger, they often compare its potential foreclosure effects with the efficiency gains it may generate. We show that vertical mergers which generate strong synergies are also more conducive to input foreclosure. An implication of this result is that the optimal decision of a competition authority is non-monotonic in the strength of synergies. We also analyze the impact of the upstream and downstream market structures on the vertically integrated firms' ability to foreclose. We show that a more competitive downstream market tends to make foreclosure easier to sustain, and that, surprisingly, a less concentrated upstream market can make input foreclosure more likely.

The scope of vertical contracting is yet another important determinant of input foreclosure. First, we show that our monopoly-like and collusive-like equilibria exist whether upstream tariffs are linear or two-part, whether third-degree price discrimination in the input market is allowed or banned, and whether upstream offers are publicly observed or secret. Second, in order to assess the impact of vertical contracting on input foreclosure, we compare the equilibrium outcomes across these different market settings. We show that vertical integration is less conducive to input foreclosure when two-part tariffs are used, when third-degree price discrimination is allowed, and when upstream offers are secret.

The anticompetitive effects of vertical mergers have received much attention in the literature. The traditional vertical foreclosure theory, which was widely accepted by antitrust practitioners until the end of the 1960s, was seriously challenged by Chicago school authors in the 1970s (Posner, 1976; Bork, 1978). A more recent strategic approach, initiated by Ordover, Saloner and Salop (1990), has established conditions under which vertical integration can relax competition. The main message conveyed in this strand of literature is that vertical mergers can lead to input foreclosure because upstream competition is softer between integrated and unintegrated firms than only among unintegrated firms. However, this is based on specific assumptions, including extra commitment power for vertically integrated firms (Ordover, Saloner and Salop, 1990; Reiffen, 1992), choice of input specification (Choi and Yi, 2000), switching costs (Chen, 2001), tacit collusion (Nocke and White, 2007; Normann, 2009), exclusive dealing (Chen and Riordan, 2007), information leakages (Allain, Chambolle and Rey, 2011).[4] We show that even in the absence of such assumptions, vertical merger waves that eliminate all unintegrated upstream firms can have severe anticompetitive effects. This is because upstream competition between vertically integrated firms only,

[4]Other contributions include Salinger (1988) who considers Cournot competition in both markets, and the strand of literature initiated by Hart and Tirole (1990) which analyzes the consequences of upstream secret offers, focusing mainly on the commitment problem faced by an upstream monopolist.

a market structure the literature has surprisingly overlooked, can be ineffective.[5]

The rest of the paper is organized as follows. We describe the model in Section 2, and solve it in Section 3. We discuss competition policy, industry concentration and welfare in Section 4. Our results on the scope of vertical contracting are presented in Section 5. Section 6 concludes. The proofs of results involving general demand functions are contained in Appendix A. Results involving linear demands are proven in a separate technical appendix (Hombert, Pouyet and Schutz, 2013).

2 Model

2.1 Setup

We consider a vertically related industry with $M \geq 2$ identical upstream firms, U_1, U_2, ..., U_M, and $N \geq M+1$ symmetric downstream firms, D_1, D_2, ..., D_N. The upstream firms produce a homogeneous input at constant marginal cost m and sell it to the downstream firms. The downstream firms can also obtain the input from an alternative source at constant marginal cost $\overline{m} > m$.[6] The downstream firms transform the intermediate input into a differentiated final product on a one-to-one basis at a constant unit cost, which we normalize to zero.

Downstream firms will be allowed to merge with upstream producers. When D_k merges with U_i, it produces the intermediate input in-house at unit cost m, its downstream unit transformation cost drops by $\delta \in [0, m]$, and its downstream marginal cost therefore becomes $m - \delta$. We say that mergers involve synergies if $\delta > 0$.

The demand for D_k's product is $q_k = q(p_k, \boldsymbol{p_{-k}})$, where p_k denotes D_k's price, $\boldsymbol{p_{-k}}$ denotes the vector of prices charged by D_k's rivals,[7] and function $q(.,.)$ is twice continuously differentiable. The demand addressed to a firm is decreasing in its own price ($\partial q_k/\partial p_k \leq 0$ with a strict inequality whenever $q_k > 0$) and increasing in its competitors' prices ($\partial q_k/\partial p_{k'} \geq 0$, $k \neq k'$, with a strict inequality whenever $q_k,\ q_{k'} > 0$).

The model has three stages. Stage 1 is the merger stage. First, all N downstream firms bid simultaneously to acquire U_1, and U_1 decides which bid to accept, if any.

[5]Bourreau, Hombert, Pouyet and Schutz (2011) present a special case of our model with an exogenous market structure with three firms and focus on the tradeoff between complete foreclosure and partial foreclosure.

[6]The alternative source can come from a competitive fringe of less efficient upstream firms.

[7]We use bold fonts to denote vectors.

Next, the remaining unintegrated downstream firms bid simultaneously to acquire U_2. This process goes on up to U_M. Firms cannot merge horizontally, and downstream firms cannot acquire more than one upstream firm. Without loss of generality, we relabel firms as follows at the end of stage 1: if K vertical mergers have taken place, then for all $1 \leq i \leq K$, U_i is acquired by D_i to form integrated firm $U_i - D_i$, while $U_{K+1}, \ldots, U_M$, and $D_{K+1}, \ldots, D_N$ remain unintegrated.

In the second stage, each upstream firm (integrated or not) $U_i(-D_i)$ announces the price $w_i \geq m$ at which it is willing to sell the input to any unintegrated downstream firm.[8] Next, each downstream firm privately observes a non-payoff relevant random variable θ_k. Those random variables are independently and uniformly distributed on some interval of the real line. Unintegrated downstream firms will use these random variables to randomize over their supplier choices, which will allow us to ignore integer constraints on upstream market shares.

In the third stage, downstream firms (integrated or not) set their prices and, at the same time, each unintegrated downstream firm chooses its upstream supplier.[9] We denote D_k's choice of upstream supplier by $U_{s_k}(-D_{s_k}$ if it is integrated), $s_k \in \{0, \ldots, M\}$, with the convention that U_0 refers to the alternative source of input and that $w_0 \equiv \overline{m}$. Next, downstream demands are realized, unintegrated downstream firms order the amount of input needed to supply their consumers, and make payments to their suppliers.[10]

We look for perfect Bayesian equilibria in pure strategies.[11]

[8]Upstream prices are public, discrimination is not possible, only linear tariffs are used, and below-cost pricing is not allowed. We relax these assumptions in Section 5.

[9]To streamline the analysis, vertically integrated firms are not allowed to buy the input in the upstream market. It is easy to show that they would have no incentives to do so.

[10]The assumption that downstream pricing decisions and upstream supplier choices are made simultaneously simplifies the analysis by ensuring that unintegrated downstream firms always buy the input from the cheapest supplier. In Section 5.1.2 we show that our results still obtain if the choice of upstream supplier is made before downstream competition.

[11]We cannot use subgame-perfect equilibrium because the θ_k's are private information. Since the θ_k's are not payoff-relevant and since they are realized at the last stage of the game, perfect Bayesian equilibrium is needed only to impose sequential rationality.

2.2 Equilibrium of stage 3

We solve the game by backward induction and start with stage 3. Denote by $\boldsymbol{w} = (w_0, \ldots, w_M)$ the vector of upstream offers and assume K mergers have taken place. The profit of unintegrated downstream firm D_k is

$$\pi_k = (p_k - w_{s_k})\, q(p_k, \boldsymbol{p_{-k}}). \tag{1}$$

The profit of integrated firm $U_i - D_i$ is given by

$$\pi_i = (p_i - m + \delta)\, q(p_i, \boldsymbol{p_{-i}}) + (w_i - m) \sum_{k:\ s_k = i} q(p_k, \boldsymbol{p_{-k}}),$$

where the first term is the profit obtained in the downstream market and the second term is the profit earned from selling the input to unintegrated downstream firms D_k such that $s_k = i$.

We restrict attention to equilibria in which downstream firms do not condition their prices on the realization of random variables θ_k's, i.e., firms do not randomize on prices. A strategy for unintegrated downstream firm D_k is a pair $(p_k(\boldsymbol{w}), s_k(\boldsymbol{w}, \theta_k))$. The strategy of vertically integrated firm $U_i - D_i$ can be written as $p_i(\boldsymbol{w})$. From now on, we drop argument $\boldsymbol{w}$ to simplify notations. The expected payoff of $U_i - D_i$ for a given strategy profile $(\boldsymbol{p}, \boldsymbol{s})$ is then equal to:

$$E(\pi_i) = (p_i - m + \delta)\, q(p_i, \boldsymbol{p_{-i}}) + (w_i - m)\, E\left(\sum_{k:\ s_k(\theta_k) = i} q(p_k, \boldsymbol{p_{-k}}) \right). \tag{2}$$

An equilibrium of stage 3 is a pair $(\boldsymbol{p}, \boldsymbol{s}(.))$ such that every integrated firm $U_i - D_i$ maximizes its expected profit (2) in p_i given $(\boldsymbol{p_{-i}}, \boldsymbol{s}(.))$, and every unintegrated downstream firm D_k maximizes its profit (1) in p_k and $s_k(\theta_k)$ given $(\boldsymbol{p_{-k}}, \boldsymbol{s_{-k}}(.))$ for every realization of random variable θ_k. Consider first the upstream supplier choice strategy of D_k. Given $(\boldsymbol{p}, \boldsymbol{s_{-k}}(.))$, $s_k(.)$ is sequentially rational if and only if for every realization of θ_k, $s_k(\theta_k) \in \arg\min_{0 \leq i \leq M} w_i$, i.e., if and only if D_k chooses (one of) the cheapest offer(s).

Next, we turn our attention to downstream pricing strategies. For any profile of sequentially rational supplier choices $\boldsymbol{s}(.)$, we assume that firms' best responses in prices are unique and defined by first-order conditions ($\partial \pi_k / \partial p_k = 0$), that prices are strategic complements (for all $k \neq k'$, $\partial^2 \pi_k / \partial p_k \partial p_{k'} \geq 0$), and that there exists a unique profile of downstream prices $\boldsymbol{p^s}$ such that $(\boldsymbol{p^s}, \boldsymbol{s})$ is a Bayes-Nash equilibrium of stage 3. Notice that, when several upstream firms (integrated or not) are offering the lowest upstream price, $\min(\boldsymbol{w}) = \min_{0 \leq i \leq M}\{w_i\}$, there are multiple equilibria in stage 3, since any distribution of the upstream demand between these upstream firms can be sustained in equilibrium.

To streamline the exposition, we adopt the following (partial) selection criterion. When several input suppliers offer $\min(\boldsymbol{w})$, and when at least one of these suppliers is vertically integrated, firms play a Nash equilibrium of stage 3 in which no downstream firm purchases from an unintegrated upstream firm. In Section 5.1.1, we motivate this selection criterion, and show that the main message of the paper would be preserved without it.

Throughout the paper, we assume that a firm's equilibrium profit is a decreasing function of its marginal cost, which means that the direct effect of a cost increase dominates the indirect ones. Finally, we assume that $\overline{m}$ is a relevant outside option: whatever the market structure, an unintegrated downstream firm earns positive profits if it buys the intermediate input at a price lower than or equal to $\overline{m}$.

2.3 The Bertrand outcome

We define the Bertrand outcome (in the K-merger subgame) as the situation in which all downstream firms, integrated or not, receive the input at marginal cost and set the corresponding downstream equilibrium prices. It follows from equations (1) and (2) that this profile of downstream prices does not depend on who supplies whom in the upstream market, since upstream profits are all zero.

Lemma 1. *After $K \in \{0, \dots, M\}$ mergers have taken place, the Bertrand outcome is always an equilibrium. If $K < M$, then the Bertrand outcome is the only equilibrium.*

Therefore, competition in the upstream market drives the input price down to marginal cost as long as at least one unintegrated upstream producer is present.

3 Merger Waves

From now on, we consider the M-merger subgame, and look for *partial foreclosure equilibria*, i.e., equilibria in which the input is priced above cost.

3.1 Preliminaries

For $1 \leq i \leq N$, we denote by P_i, Q_i and Π_i the equilibrium expected downstream price, demand and profit of D_i ($U_i - D_i$ if this firm is vertically integrated), respectively. For a given profile of upstream offers $\boldsymbol{w}$, there exists a continuum of equilibria of stage 3 in which the integrated firms offering $w = \min(\boldsymbol{w})$ share the upstream market. Fix one such equilibrium. Then, we define $\alpha_i \equiv \frac{1}{N-M}\sum_{k=M+1}^{N} Pr(s_k(\theta_k) = i)$, $i = 1, \ldots, M$, we call α_i the upstream market share of $U_i - D_i$, and we denote by $\boldsymbol{\alpha}$ the vector of upstream market shares. The following lemma states that it is enough to know the input price and the upstream market shares to calculate equilibrium prices, quantities and profits:

Lemma 2. *In the M-merger subgame, when the input price is w, at the unique equilibrium with supplier choices $\boldsymbol{s}(.)$:*

- *For integrated firm $U_i - D_i$ ($1 \leq i \leq M$), P_i, Q_i and Π_i can be written as $P(\alpha_i, \boldsymbol{\alpha_{-i}}, w)$, $Q(\alpha_i, \boldsymbol{\alpha_{-i}}, w)$ and $\Pi(\alpha_i, \boldsymbol{\alpha_{-i}}, w)$. These functions are invariant to permutations of $\boldsymbol{\alpha_{-i}}$.*
- *For downstream firm D_k ($M + 1 \leq k \leq N$), P_k, Q_k and Π_k can be written as $P_d(\boldsymbol{\alpha}, w)$, $Q_d(\boldsymbol{\alpha}, w)$ and $\Pi_d(\boldsymbol{\alpha}, w)$. These functions are invariant to permutations of $\boldsymbol{\alpha}$.*

Therefore, the equilibrium profit of $U_i - D_i$ is given by:

$$\Pi(\alpha_i, \boldsymbol{\alpha_{-i}}, w) = (P(\alpha_i, \boldsymbol{\alpha_{-i}}, w) - m + \delta)\, Q(\alpha_i, \boldsymbol{\alpha_{-i}}, w) + \alpha_i(N - M)(w - m)Q_d(\boldsymbol{\alpha}, w),$$

while D_k earns $\Pi_d(\boldsymbol{\alpha}, w) = (P_d(\boldsymbol{\alpha}, w) - w)Q_d(\boldsymbol{\alpha}, w)$.

The following notations will be useful to characterize equilibria. We let $\mathbf{0}$, $\mathbf{1}$ and $\boldsymbol{1/M}$ the (M–1)-tuples $(0, \ldots, 0)$, $(1, 0, \ldots, 0)$ and $(1/M, \ldots, 1/M)$, respectively. For $1 \leq Z \leq Y \leq M$, we define $S_Y(Z)$ as the set of feasible equilibrium market shares in an industry with Y integrated firms, when only the first Z firms offer the cheapest input price:

$$S_Y(Z) = \left\{ \boldsymbol{\alpha} \in [0,1]^Y : \sum_{i=1}^{Z} \alpha_i = 1, \text{ and } \alpha_i = 0 \; \forall i > Z \right\}.$$

It will be useful to keep in mind that, when exactly Z firms are offering the cheapest price, given a feasible profile of market shares $\boldsymbol{\alpha}$, there exists a permutation of $\boldsymbol{\alpha}$ which belongs to $S_M(Z)$.

3.2 Monopoly-like equilibria

In this section, we look for equilibria in which only one vertically integrated firm makes an upstream offer. Suppose $U_i - D_i$ supplies the entire upstream market at a price $w > m$. $U_i - D_i$'s first-order condition on the downstream market is given by:

$$q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w - m)\sum_{k=M+1}^{N}\frac{\partial q_k}{\partial p_i} = 0. \tag{3}$$

Let $j \neq i$ in $\{1, \ldots, M\}$. The first-order condition of integrated firm $U_j - D_j$, which does not supply the upstream market, is:

$$q_j + (p_j - m + \delta)\frac{\partial q_j}{\partial p_j} = 0. \tag{4}$$

Since the last term in the right-hand side of equation (3) is positive, $U_i - D_i$ has more incentives to increase its downstream price than $U_j - D_j$. Intuitively, when $U_i - D_i$ increases its downstream price, some of the consumers it loses in the final market start buying from unintegrated downstream firms. These downstream firms therefore need to purchase more input, which eventually increases $U_i - D_i$'s profit in the upstream market. It follows that, in equilibrium, $U_i - D_i$ charges a higher downstream price than $U_j - D_j$. $U_j - D_j$ benefits from $U_i - D_i$'s being a soft downstream competitor, and therefore, by revealed profitability, earns a larger downstream profit than $U_i - D_i$. We summarize these insights in the following lemma:

Lemma 3. *If $w > m$, then:*

$$\begin{aligned} P(1, \mathbf{0}, w) &> P(0, \mathbf{1}, w), \\ (P(1, \mathbf{0}, w) - m + \delta)\, Q(1, \mathbf{0}, w) &< (P(0, \mathbf{1}, w) - m + \delta)\, Q(0, \mathbf{1}, w). \end{aligned}$$

Now, consider the incentives of $U_j - D_j$ to expand its upstream market share. More precisely, we check whether $U_j - D_j$ wants to set its upstream price at $w_j = w - \epsilon$ so as to take over the upstream market. Undercutting brings in profits from the upstream market. But on the other hand, $U_j - D_j$'s *downstream* profit jumps downward, since $U_i - D_i$ no longer has incentives to be a soft downstream competitor. The decision to undercut therefore trades off the *upstream profit effect* against the loss of the *softening* *effect*. The change in profit if $U_j - D_j$ undercuts is equal to:

$$\Pi(1,\mathbf{0},w) - \Pi(0,\mathbf{1},w) = \underbrace{(N-M)(w-m)Q_d(1,\mathbf{0},w)}_{\text{Upstream profit effect } (>0)}$$
$$+ \underbrace{\left[(P(1,\mathbf{0},w) - m + \delta)\, Q(1,\mathbf{0},w) - (P(0,\mathbf{1},w) - m + \delta)\, Q(0,\mathbf{1},w)\right]}_{\text{Softening effect } (<0 \text{ by Lemma 3})}$$

If the softening effect dominates the upstream profit effect, then $U_j - D_j$ does not undercut.

For this outcome to be an equilibrium, $U_i - D_i$ should not be willing to change its upstream price either. We denote by $w_m \equiv \arg\max_{w \leq \overline{m}} \Pi(1,\mathbf{0},w)$ the monopoly upstream price.

Lemma 4. *w_m exists, and it is strictly larger than m.*

Lemma 4 states that monopoly power generates a positive markup in the input market. w_m is only constrained by the alternative source of input to be no larger than $\overline{m}$, and we assume for simplicity that this price is unique. It is straightforward to check that, if other integrated firms stay out of the market, then $U_i - D_i$ is better off offering w_m rather than letting the alternative source of input supply the upstream market.

We define a monopoly-like outcome as a situation in which one vertically integrated firm sets w_m and the other integrated firms make no upstream offers.[12] Lemmas 3 and 4 imply that such an outcome may be sustained at the equilibrium of the upstream competition subgame.

Proposition 1. *When M mergers have taken place, there is a monopoly-like equilibrium if and only if*

$$\Pi(1,\mathbf{0},w_m) \leq \Pi(0,\mathbf{1},w_m). \tag{5}$$

When the softening effect is strong enough so that condition (5) holds, the outcome in which $M-1$ integrated firms exit the upstream market, granting a monopoly position to the remaining integrated firm, is an equilibrium. In Section 4, we use a linear demand specification to map condition (5) into fundamental parameters of the model, such as the strength of synergies, downstream product differentiation, and the number of upstream and downstream firms. We perform the same exercise for the other equilibria identified in the present section.

[12] By 'make no upstream offers', we mean that the other integrated firms offer input prices above $\overline{m}$.

Proposition 1 gives a simple and novel foundation to the classical analysis of Ordover, Saloner and Salop (1990), in which a vertically integrated firm commits to exit the upstream market in order to let its upstream rival charge the monopoly price. We show that no commitment is actually needed when upstream rivals are integrated, provided that the softening effect is strong enough. Chen (2001) already noticed that an integrated firm tends to set higher downstream prices when it supplies input to downstream rivals. He shows that if, in addition, there are upstream cost asymmetries and costs of switching input suppliers, then an unintegrated upstream firm is unable to undercut the integrated firm on the upstream market. In contrast, we show that when several integrated firms are competing against each other, integrated firms are *able* to undercut, but they are not *willing* to do so.

3.3 Symmetric collusive-like equilibria

While the previous section derived an existence condition for the most asymmetric equilibria, the present section looks for the most symmetric ones. More precisely, we look for *symmetric collusive-like equilibria*, in which integrated firms all set the same input price $w > m$, and get the same market share $1/M$. We start from this symmetric situation and, as before, investigate the incentives of a vertically integrated firm, call it $U_i - D_i$, to expand its upstream market share. Suppose for instance that $U_i - D_i$'s market share increases from $1/M$ to $1/M + x$, $x > 0$, while $U_j - D_j$'s market share decreases from $1/M$ to $1/M - x$:

Lemma 5.

$$\left.\frac{dP_i}{dx}\right|_{x=0} = -\left.\frac{dP_j}{dx}\right|_{x=0} > 0 \quad and \quad \left.\frac{dP_k}{dx}\right|_{x=0} = 0 \ (k \neq i, j \ in \ \{1, \ldots, N\}).$$

The softening effect is still at work: when $U_i - D_i$ expands its upstream market share, it has incentives to increase its downstream price so as to protect its upstream sales. Conversely, since $U_j - D_j$'s market share shrinks, this firm cuts its downstream price. Totally differentiating $U_i - D_i$'s profit with respect to x, using Lemma 5 and the envelope theorem, we get:

$$\left.\frac{d\Pi_i}{dx}\right|_{x=0} = \underbrace{(N-M)(w-m)\left. Q_d\right|_{x=0}}_{\text{Upstream profit effect (>0)}} + \underbrace{\left.\frac{dP_j}{dx}\right|_{x=0} \frac{\partial \pi_i}{\partial p_j}}_{\text{Softening effect (<0 by Lemma 5)}},$$

where $\partial\pi_i/\partial p_j$ is evaluated at the equilibrium price vector when all market shares are equal to $1/M$. As in the previous section, when $U_i - D_i$ expands its upstream market share, it benefits from a positive upstream profit effect, but it loses (part of) the softening effect. Symmetrically, if $U_i - D_i$ deviates to reduce its market shares ($x < 0$), it benefits from a stronger softening effect, but gives up upstream profits.

In the above paragraph, we have considered infinitesimal variations of $U_i - D_i$'s market share. In fact, $U_i - D_i$ can only do two things: undercut, so as to take over the entire upstream market, or exit the upstream market altogether.[13] Collusive-like equilibria exist when none of these deviations is profitable:

Proposition 2. *When M mergers have taken place, there exists a symmetric collusive-like equilibrium at price $w > m$ if and only if*

$$\Pi(\frac{1}{M}, \frac{\mathbf{1}}{\boldsymbol{M}}, w) \geq \max\left\{\max_{\tilde{w}\leq w}\Pi(1,\mathbf{0},\tilde{w}),\ \min_{\boldsymbol{\beta}\in S_{M\text{-}1}(M\text{-}1)}\Pi(0,\boldsymbol{\beta},w)\right\}. \quad (6)$$

The first term on the right-hand side of (6) states that undercutting the input price w should not be profitable. To understand the second term of the maximum, remember that if an integrated firm deviates and exits the input market, then unintegrated downstream firms select a supplier among the $M - 1$ other integrated firms, and any distribution of the upstream demand between those integrated firms can be sustained at the equilibrium of stage 3 following the deviation. The second term on the right-hand side of (6) states that there must exist an equilibrium of stage 3 in which the deviator's profit does not increase.

In a symmetric collusive-like equilibrium, all integrated firms set the same input price above cost, and share the upstream market equally, as in models of collusion with repeated interactions. Nocke and White (2007) obtain similar upstream outcomes in a repeated game framework with a market structure close to our model's.[14] Proposition 2 says that these outcomes can actually be sustained in a one-shot game when all upstream firms are vertically integrated. This happens when the softening effect is

[13] Other strategic effects start kicking in when deviations are not infinitesimal. In particular, other firms might change their downstream prices too, but this does not affect the key tradeoff: the decision to expand or contract upstream market shares trades off the upstream profit effect against the softening effect.

[14] However, Nocke and White (2007)'s downstream outcome is different from ours, since they focus on equilibria in which overall industry profit is maximized.

strong enough, so that integrated firms do not want to undercut, but not too strong, so that integrated firms are not willing to exit.

3.4 Equilibria: Complete characterization

Since any distribution of the upstream demand between integrated firms offering w is feasible, *collusive-like* outcomes do not have to be symmetric. Consider a collusive-like outcome in which $Z \in \{2, \dots, M\}$ integrated firms offer $w = \min(\boldsymbol{w}) > m$, and assume without loss of generality that the distribution of market shares is given by $\boldsymbol{\alpha} \in S_M(Z)$. Then, it is straightforward to extend Proposition 2 to show that this outcome can be sustained in equilibrium if and only if

$$\min_{1 \leq i \leq M} \Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w) \geq \max \left\{ \max_{\tilde{w} \leq w} \Pi(1, \mathbf{0}, \tilde{w}), \min_{\boldsymbol{\beta} \in S_{M\text{-}1}(Z\text{-}1)} \Pi(0, \boldsymbol{\beta}, w) \right\}. \tag{7}$$

As before, undercutting and exit decisions trade off the softening effect and the upstream profit effect.

Equilibrium multiplicity in stage 3 generates equilibrium multiplicity in stage 2. However, if integrated firms' profit function is quasi-concave in the market shares, then, among collusive-like equilibria, the symmetric ones are the easiest to sustain:

Lemma 6. *Assume that, for all $w > m$, $(\alpha_i, \boldsymbol{\alpha}_{-i}) \mapsto \Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ is quasi-concave. If there is a collusive-like equilibrium at upstream price $w > m$, then there is also a symmetric collusive-like equilibrium at upstream price w.*

To see the intuition, assume $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ is quasi-concave in $(\alpha_i, \boldsymbol{\alpha}_{-i})$, and start from an asymmetric collusive-like outcome. Then, making market shares more symmetric raises the profit of the firm which earns the least. This therefore lowers the deviation incentives of the firm which is the most likely to deviate. The quasi-concavity condition sounds natural in an environment with convex preferences, and symmetric and constant unit costs. We will show later on that it is satisfied when demand is linear.

Lemma 6 implies that, when looking for a partial foreclosure equilibrium other than the monopoly-like one, it is enough to focus on the symmetric collusive-like one. This concludes the equilibrium characterization in stage 2:

Proposition 3. *After M mergers, there exist no other equilibria than monopoly-like, collusive-like and Bertrand equilibria. If condition (5) is not satisfied, condition (6) is not satisfied for any $w > m$, and $\Pi(.,.,w)$ is quasi-concave for all $w > m$, then the Bertrand outcome is the only equilibrium.*

3.5 Outcome of the merger game and equilibrium bids

Combining Lemma 1 and Propositions 1, 2 and 3, we obtain the following result:

Proposition 4. *Assume* $\Pi(.,.,w)$ *is quasi-concave for all* $w > m$*. There exists an equilibrium with a merger wave and partial foreclosure in the input market if and only if condition* (5) *is satisfied or condition* (6) *is satisfied for some* $w > m$*.*
Moreover, when $\delta = 0$ *(resp.* $\delta > 0$*), there is also an equilibrium with no merger (resp. a merger wave) and the Bertrand outcome on the upstream market.*

When the existence condition for monopoly-like or symmetric collusive-like equilibria is satisfied, a vertical merger raises the joint profits of the merging parties: firms merge to implement a partial foreclosure equilibrium and, when $\delta > 0$, to benefit from efficiency gains.

Case $\delta = 0$ illustrates the fact that vertical mergers are strategic complements. If the Bertrand outcome is expected to arise in every subgame of the upstream competition stage, the absence of synergies implies that unintegrated downstream firms and integrated firms earn the same profit in every subgame. As a result, downstream firms have no incentives to integrate backward, and there always exists an equilibrium with no merger and the Bertrand outcome on the upstream market. Conversely, when firms expect partial foreclosure to take place in M-merger subgames, a wave of mergers occurs for purely anticompetitive reasons. The M-th merger is profitable only if the first $M-1$ upstream firms have merged before. By the same token, the first merger is profitable only because the merging parties anticipate that it will be followed by $M-1$ counter-mergers.[15]

We conclude this section by discussing which firms are likely to gain or to lose from a vertical merger wave leading up to partial foreclosure. The analysis is tedious in the general case, because equilibrium bids depend on which equilibrium is selected in each of the M-merger subgames. To simplify, we focus on the most symmetric case, in which a symmetric collusive-like equilibrium at price $w > m$ arises in all M-merger subgames. In equilibrium, all winning bids are equal to $\Pi(1/M, \mathbf{1/M}, w) - \Pi_d(1/M, \mathbf{1/M}, w)$. The owners of downstream firms end up with net payoff $\Pi_d(1/M, \mathbf{1/M}, w)$, whereas the ini-

[15]On top of the complementarity induced by the foreclosure effect, synergies may introduce some substitutability. This kind of substitutability arises if a firm is less willing to pay for a marginal cost reduction when its rivals already have a low marginal cost, as in Athey and Schmutzler (2001).

tial owners of upstream firms end up with payoff $\Pi(1/M, \mathbf{1/M}, w) - \Pi_d(1/M, \mathbf{1/M}, w)$. Therefore, upstream firms' owners clearly gain from the merger wave, whereas *all* downstream firms' owners suffer from it. The reason is that the sequence of auctions which takes place in stage 1 involves negative externalities between buyers: when a downstream firm integrates backward, other unintegrated downstream firms suffer both because of synergies and partial foreclosure. The result that all downstream firms' owners suffer from the merger wave does not depend on the particular bargaining structure we are assuming: if we allow instead the upstream firms to bid to acquire the downstream firms, then it is possible to show that the equilibrium payoffs are the same as when downstream firms bid.

4 Competition Policy, Concentration, and Welfare

In this section, we add more structure to the downstream industry by assuming that demands are linear. This allows us to better understand and compare the existence conditions for monopoly-like and collusive-like equilibria (Section 4.1), to derive comparative statics on the determinants of partial foreclosure (Section 4.2), and to discuss the welfare effect of vertical mergers (Section 4.3). We use Shubik and Levitan (1980)'s usual linear demand system:

Example 1. *A unit mass of identical consumers have utility function*

$$U = q_0 + \sum_{k=1}^{\overline{N}} q_k - \frac{1}{2}\left(\sum_{k=1}^{\overline{N}} q_k\right)^2 - \frac{\overline{N}}{2(1+\gamma)}\left(\sum_{k=1}^{\overline{N}} q_k^2 - \frac{1}{\overline{N}}(\sum_{k=1}^{\overline{N}} q_k)^2\right), \tag{8}$$

where q_0 is consumption of the numeraire, q_k is consumption of D_k's product, $\gamma > 0$ parameterizes the degree of differentiation between final products, and $\overline{N} \geq N$ is the number of varieties of the final product.

The demands derived from utility function (8) can be written as:

$$q(p_k, \boldsymbol{p_{-k}}) = \frac{1+\gamma}{\frac{N}{\overline{N}}+\gamma}\frac{1}{N}\left(1 - p_k - \gamma\frac{N}{\overline{N}}\left(p_k - \frac{\sum_{k'=1}^{N} p_{k'}}{N}\right)\right).$$

γ parameterizes the degree of differentiation between final products. Products become homogeneous as γ approaches ∞, and independent as γ approaches 0. $\overline{N}$ is the number of varieties of the final good. N varieties are sold by the downstream firms while the other $\overline{N} - N$ are not available to consumers. Allowing the *potential* number of varieties to differ from the *actual* number of varieties will be helpful, as this will allow us to perform comparative statics on the number of downstream firms without arbitrarily changing consumers' preferences.

4.1 Monopoly-like versus symmetric collusive-like equilibria

In Example 1, all the assumptions made in Section 2.2 are satisfied. We show in the technical appendix that the profit function $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$ does not depend on $\boldsymbol{\alpha}_{-i}$, and that it is strictly concave in α_i. We will therefore omit argument $\boldsymbol{\alpha}_{-i}$ in the remainder of this section. Proposition 4 applies and we have necessary and sufficient conditions for the existence of equilibria with a merger wave leading up to partial foreclosure:[16]

Proposition 5. *In Example 1, there exist three thresholds δ_m, $\underline{\delta}_c$, and $\overline{\delta}_c$, such that:*[17]

(i) There is an equilibrium with M mergers and a monopoly-like outcome if and only if $\delta \geq \delta_m$.

(ii) There is an equilibrium with M mergers and a symmetric collusive-like outcome if and only if $\underline{\delta}_c \leq \delta < \overline{\delta}_c$. In this case, the set of prices which can be sustained in a symmetric collusive-like equilibrium is an interval.

(iii) If $\delta < \underline{\delta}_c$, then a merger wave never leads to partial foreclosure.

Moreover, $\underline{\delta}_c \leq \delta_m < \overline{\delta}_c$.

The cutoffs defined in Proposition 5 reflect the tradeoff between the upstream profit effect and the softening effect. Consider first the condition for monopoly-like equilibria: monopoly-like equilibria exist when synergies are strong enough. Intuitively, as the cost differential between unintegrated and integrated firms widens, the market shares of the former decline and profits in the upstream market shrink. The magnitude of the softening effect, which works at the margin and reflects the willingness of upstream suppliers to raise their upstream demand, is not directly affected. Because undercutting

[16] To avoid the proliferation of cases, we assume that δ is not too high, so that the unconstrained maximization problem $\max_w \Pi(1, w)$ has an interior solution. We also assume that $\overline{m}$ is high enough, so that it does not constrain the monopoly upstream price. We make similar assumptions to prove all propositions involving linear demands.

[17] These thresholds are functions of parameters (M, N, γ). While $\overline{\delta}_c$ is always strictly positive, $\underline{\delta}_c$ and δ_m can be equal to zero. This is more likely to happen when γ is high and when M and N are not too high. When this is the case, our partial foreclosure equilibria exist even in the absence of synergies.

decisions trade off the upstream profit effect with the softening effect, it becomes more and more attractive to stay out of the market as δ increases.

Consider now the existence condition for collusive-like equilibria. The two thresholds on δ come from the two terms in the right-hand side of non-deviation condition (6). First, the upstream profit effect should not be too strong compared to the softening effect to make undercutting not profitable. This arises when synergies are strong enough. Second, the softening effect should not be too strong compared to the upstream profit effect to make exit not profitable. This arises when synergies are not too strong. Proposition 5 also shows that there exist a continuum of symmetric collusive-like equilibria parameterized by the input price. This comes from the fact that equilibrium condition (6) is an inequality. Therefore, if it holds strictly for a given w, then, by continuity, it is also satisfied in a neighborhood of w.

The fact that $\underline{\delta}_c \leq \delta_m < \overline{\delta}_c$ follows from the quasi-concavity of Π. To see this, suppose that the monopoly-like equilibrium condition is just satisfied, $\delta = \delta_m$, which implies that the no-undercut condition is binding, $\Pi(1, w_m) = \Pi(0, w_m)$. Quasi-concavity implies that

$$\Pi(1/M, w_m) > \min\{\Pi(1, w_m), \Pi(0, w_m)\} = \Pi(1, w_m) = \Pi(0, w_m).$$

Therefore, there also exists a symmetric collusive-like equilibrium with input price w_m. From this, we can conclude that collusive-like equilibria are easier to sustain when δ is intermediate, whereas monopoly-like equilibria are easier to sustain when δ is large.

4.2 Competition policy: Determinants of partial foreclosure

In this section we study the impact of downstream product differentiation and of upstream and downstream industry concentration on the emergence of an equilibrium vertical merger wave leading up to partial foreclosure. Since Proposition 5 shows that partial foreclosure equilibria arise if and only if $\delta \geq \underline{\delta}_c$, the problem boils down to analyzing the behavior of $\underline{\delta}_c(\gamma, M, N)$ as a function of γ, M and N. Results in this subsection are derived using numerical simulations.

Product differentiation. First, we show that industries with competitive downstream markets (high γ) tend to have non-competitive upstream markets:

Result 1. *In Example 1, $\gamma \mapsto \underline{\delta}_c(\gamma, M, N)$ is (weakly) decreasing.*[18]

[18] More precisely: either $\underline{\delta}_c$ is strictly decreasing on $(0, \infty)$, or there exists $\bar{\gamma}$ such that $\underline{\delta}_c$ is strictly

Intuitively, when the substitutability between final products is strong, an integrated firm which supplies (part of) the upstream market is reluctant to set too low of a downstream price since this would strongly contract its upstream profit. The other integrated firms benefit from a substantial softening effect and, as a result, are not willing to undercut in the upstream market. The reverse holds when downstream products are strongly differentiated.

In its non-horizontal merger guidelines (EC, 2007), the European Commission argues that vertically integrated firms have less incentive to foreclose when pre-merger downstream margins are low, because integrated firms would not find it profitable to forego upstream revenues to preserve low downstream profits.[19] The Commission also emphasizes that, when assessing the potential anti-competitive effect of a vertical merger, the competition authority should distinguish the integrated firms' *ability* to foreclose from their *incentives* to foreclose. Our model does focus on the ability to foreclose. It shows that, if pre-merger downstream margins are low because final products are close substitutes, then integrated firms are better able to sustain input foreclosure in equilibrium.

Upstream concentration. Contrary to the conventional wisdom, the upstream market is not necessarily more competitive when more firms compete in this market:

Result 2. *In Example 1, $M \mapsto \underline{\delta}_c(M, N, \gamma)$ is (i) decreasing when γ is low, (ii) increasing when γ is high and N is small, (iii) hump-shaped when γ and N are high.*

The reason is that more upstream firms at the beginning of the game translates into fewer unintegrated downstream firms in the M-merger subgame. Therefore, fewer firms need to buy the input in the upstream market, which weakens both the upstream profit effect and the softening effect. Depending on which effect is most affected, a higher upstream concentration may or may not make the upstream market more competitive.

Downstream concentration. The impact of downstream concentration is ambiguous as well:

Result 3. *In Example 1, $N \mapsto \underline{\delta}_c(M, N, \gamma)$ is (i) decreasing when $M \geq 4$ or when*

decreasing on $(0, \bar{\gamma}]$ and equal to zero on $[\bar{\gamma}, \infty)$.

[19] Inderst and Valletti (2011) question the EC's reasoning. They argue that low downstream margins are indicative of closely substitutable final products and that, in this situation, the integrated firms' incentives to raise their rivals' costs are strong.

$M < 4$ and γ is low, (ii) U-shaped when $M = 2$ and γ is intermediate, or when $M = 3$ and γ is high, (iii) increasing when $M = 2$ and γ is high.

Intuitively, an increase in the number of downstream firms strengthens both the softening effect and the upstream profit effect, and downstream concentration may or may not make the upstream market more competitive.

4.3 Competition policy: Welfare

To discuss the welfare impact of vertical mergers, we define the following market performance measure. We fix $\lambda \in [0, 1]$ and define market performance as $W(\lambda) =$ (Consumer surplus) + $\lambda \times$ (Industry profit). Notice that $W(0)$ is consumer surplus, and $W(1)$ is social welfare.

The first $M - 1$ mergers improve market performance when there are synergies ($\delta > 0$) and leave performance unaffected when there are no synergies ($\delta = 0$). The welfare effect of the last merger of the wave depends on the outcome in the upstream market. If the upstream market remains supplied at marginal cost, then the M-th merger also improves market performance. By contrast, when input foreclosure arises in the M-merger subgame, there is a tradeoff between efficiency gains and anticompetitive effects. From an antitrust perspective, it is therefore the last merger of the wave that calls for scrutiny.

We illustrate this tradeoff in the special case $M = 2$ and $N = 3$. We compare $W(\lambda)$ at the unique equilibrium outcome of the one-merger subgame (the Bertrand outcome), and at the equilibrium outcome of the two-merger subgame. We adopt the following equilibrium selection in the two-merger subgame: the monopoly-like equilibrium is selected when it exists, otherwise the Bertrand equilibrium is selected.[20]

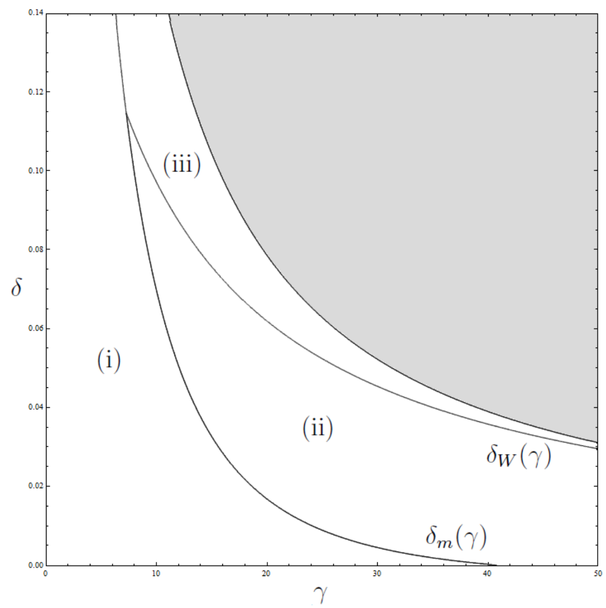
Proposition 6. *There exists γ_1 and δ_W such that the second merger degrades market performance if and only if $\gamma > \gamma_1$ and $\delta \in [\delta_m, \delta_W)$.*[21]

As shown in Figure 1, the optimal policy response to the second merger is quite different from the simple rule-of-thumb, whereby the competition authority is more favorable towards a vertical merger when synergies are stronger. When γ is interme-

[20] As shown in the technical appendix, results are similar with the following alternative equilibrium selection: the symmetric collusive-like equilibrium with the highest upstream price is selected when collusive-like equilibria exist, otherwise the Bertrand equilibrium is played.

[21] As in Proposition 5, δ_W and δ_m are functions of γ and λ, and γ_1 is a function of λ.

Figure 1: Welfare effect of the last merger ($M = 2$, $N = 3$, $\lambda = 0.5$)



Note: In area (i) the second merger leads to the Bertrand outcome and improves market performance; in area (ii) it leads to a monopoly-like outcome and reduces market performance; and in area (iii) it leads to a monopoly-like outcome and improves market performance. In the shaded area, δ is so high that the monopoly upstream price is no longer interior; we rule out these cases by assumption.

diate, the competition authority should clear the merger when $0 < \delta < \delta_m$, challenge it when $\delta \in [\delta_m, \delta_W)$, and clear it again when $\delta \geq \delta_W$. So the optimal merger policy is non-monotonic in δ. This follows from the fact that, while larger efficiency gains improve welfare *for a given outcome in the input market*, they also increase the likelihood of input foreclosure. This highlights that foreclosure and efficiency effects are intertwined and should be considered jointly when investigating the competitive effects of a vertical merger.

5 Price Discrimination, Non-Linear Pricing and Secret Offers

This section assesses how the scope of vertical contracting affects vertical foreclosure. We show that vertical integration is less conducive to input foreclosure under upstream price discrimination (Section 5.2), two-part tariffs (Section 5.3) and secret offers (Section 5.4). Section 5.1 contains technical preliminaries, which the reader should feel free to skip.

5.1 Technical assumptions

5.1.1 Equilibrium selection in stage 3

Throughout the paper, we have maintained the assumption that, when several firms offer the lowest upstream price, and when at least one of these firms is vertically integrated, no downstream firm purchases from an unintegrated upstream firm. One way to motivate our selection criterion is to allow downstream firms to pre-commit *ex ante* to their supplier choices, as in Chen (2001). Consider the following modification of our timing: in stage 2, after input prices have been set, each downstream firm elects one upstream supplier. In stage 3, after downstream prices have been set, each downstream firm is allowed to switch to another supplier if it pays a fixed cost ε. Then, we can show that, as ε goes to zero, the equilibria of this family of auxiliary games converge towards equilibria of our original game which satisfy our equilibrium selection criterion. The reason is that downstream firms want to pre-commit to purchase from integrated firms so as to make them softer competitors in the downstream market.

Without this equilibrium selection, the Bertrand outcome may not be the only equilibrium of stage 2 when fewer than M mergers have taken place. To see the intuition, consider the $M = 3$ and $N = 5$ case, assume two mergers have taken place, and start from an equilibrium candidate in which the three upstream firms offer the same input price $w > m$, and each of these firms supplies exactly one downstream firm. Then, it could be that the integrated firms want neither to exit nor to undercut as in a collusive-like equilibrium. The unintegrated upstream firm may not want to undercut, because if it did so, then integrated firms would become more aggressive on the downstream market, and this would reduce the input demand coming from the downstream firm it already supplies.

While we have not been able to construct such equilibria, we cannot rule them out either. If they exist, then there can be equilibria of the whole game with fewer than M (anticompetitive) mergers. In this case, anticompetitive vertical integration still takes place because of the tradeoff between the softening effect and the upstream profit effect, and the main message of our paper is preserved.[22]

5.1.2 Sequential timing

Suppose now that unintegrated downstream firms choose their input supplier (in stage 2.5) after upstream prices have been set (in stage 2) but before downstream competition takes place (in stage 3). We also assume that unintegrated downstream firms have access to a public randomization device: downstream firms commonly observe the realization of a random variable θ between stages 2 and 2.5.

Then, supplier choices made in stage 2.5 have an impact on equilibrium downstream prices in the continuation subgame. Because of this, the choices of upstream suppliers become a strategic game between unintegrated downstream firms, and some market share distributions may not be equilibria of the supplier choice subgame. This complicates the analysis, but we are still able to solve the model when demands are linear: in our technical appendix, we show that Proposition 5 still holds under sequential timing if we replace threshold $\underline{\delta}_c$ by $\underline{\delta}_c^t$, where $\underline{\delta}_c \leq \underline{\delta}_c^t < \overline{\delta}_c$.[23]

5.1.3 Below-cost pricing

Let us now relax the assumption that upstream firms cannot set input prices below marginal cost. All the equilibria we have characterized so far remain equilibria. Starting from an equilibrium in which the input price is no smaller than marginal cost, no firm has incentives to cut its price below marginal cost, since this firm would then start making losses, and all downstream prices would fall down by strategic complementarity.

New equilibria may pop up too. If $w < m$, then the upstream profit effect is negative, the softening effect is positive, and lower upstream market shares do not necessarily lead to higher overall profits. Therefore, (7) may still hold, and there may exist equilibria with negative upstream markups.[24] However, these equilibria

[22] A similar remark applies to the extensions laid out in Sections 5.1.2 – 5.4. In those extensions, the Bertrand outcome may not be the only equilibrium in subgames with fewer than M mergers, because of the tradeoff between the softening effect and the upstream profit effect.

[23] The analysis of monopoly-like equilibria is unaffected. When M divides $N - M$, symmetric collusive-like equilibria can be implemented, and $\underline{\delta}_c^t$ is the same as in Section 4. Otherwise, a symmetric distribution of market shares is not feasible, even with a randomization device, since upstream suppliers are known before downstream competition takes place. In this case $\underline{\delta}_c^t$ is higher than with the original timing. See our technical appendix for details.

[24] When demand is linear, it is straightforward to adapt the proof of Proposition 5 to show that

are always Pareto-dominated by the Bertrand equilibrium from the point of view of upstream players, and they would raise antitrust concerns in any country where below-cost pricing is forbidden.

5.2 Discrimination

Next, we extend Proposition 5 to a setting with third-degree price discrimination in the input market:

Proposition 7. *Assume upstream producers can price-discriminate in the input market. In Example 1:*

(i) There exists an equilibrium with M mergers and a monopoly-like outcome in the upstream market if and only if $N \leq N^d$ and $\delta \geq \delta_m^d$, where $\delta_m^d \geq \delta_m$ and $N^d \geq M+4$.

Moreover, the monopoly upstream price is the same as under non-discrimination.

(ii) There exists an equilibrium with M mergers and a symmetric collusive-like outcome in the upstream market if $\delta \in [\underline{\delta}_c^d, \overline{\delta}_c]$, where $\underline{\delta}_c \leq \underline{\delta}_c^d \leq \overline{\delta}_c$.

Moreover, for all δ, if an input price can be sustained in a symmetric collusive-like equilibrium under discrimination, then it can also be sustained under non-discrimination.

Therefore, partial foreclosure equilibria are more difficult to sustain when upstream price discrimination is allowed. This is because, under discrimination, integrated firms can cut their prices selectively when they deviate from a partial foreclosure equilibrium, which raises the maximum deviation profit they can attain. This suggests that allowing price discrimination in input markets can actually make these markets more competitive.

5.3 Two-part tariff competition

Assume that firms compete in two-part tariffs on the upstream market, and denote by (w_i, T_i) the contract offered by U_i. We allow the variable part w_i to take any value, but we restrict the analysis to non-negative fixed parts: $T_i \geq 0$.[25] We also assume

such equilibria exist if and only if $\delta > \overline{\delta}_c$.

[25] If upstream offers are non-exclusive, i.e., if a downstream firm is allowed to accept several upstream offers, then negative fixed fees cannot survive in equilibrium (see Chen, 2001). Schutz (2012) shows that, if upstream offers are exclusive and negative fixed-fees are allowed, then the no-merger subgame

that upstream suppliers are chosen before downstream competition takes place as in Section 5.1.2.[26]

As explained in Section 5.1.2, when upstream suppliers are chosen before stage 3, the choices of upstream suppliers become a strategic game between downstream firms. We sidestep this difficulty by focusing first on the $N = M + 1$ case, so that there is only one unintegrated downstream firm left after a merger wave. In M-merger subgames, we denote by $\Pi_d(0, \mathbf{0}, \overline{m})$ the profit of the unintegrated downstream firm when it buys the input from the alternative source at price $\overline{m}$. Assume that $\Pi(1, \mathbf{0}, w)$ and $\Pi(1, \mathbf{0}, w) + \Pi_d(1, \mathbf{0}, w)$ are strictly quasi-concave in w.

Then, the monopoly upstream offer, (w_m^{tp}, T_m^{tp}), which solves

$$\max_{(w,T)} \Pi(1, \mathbf{0}, w) + T \text{ subject to } \Pi_d(1, \mathbf{0}, w) - T \geq \Pi_d(0, \mathbf{0}, \overline{m}) \text{ and } T \geq 0,$$

exists and is unique, and we can prove the following lemma:

Lemma 7. *$m < w_m^{tp} \leq w_m$.*

Two-part tariffs alleviate double-marginalization ($w_m^{tp} \leq w_m$), but not completely so ($w_m^{tp} > m$). Intuitively, the upstream supplier wants to increase the marginal cost of the unintegrated downstream firm to reduce the cannibalization of its own downstream sales, and to soften downstream competition as in Bonanno and Vickers (1988).

We define a monopoly-like outcome under two-part pricing as a situation in which the unintegrated downstream firm accepts a contract with a variable part equal to w_m^{tp}. Since $w_m^{tp} > m$, the softening effect is still at work, and the integrated firms which do not supply the upstream market earn higher downstream profits than the upstream supplier. Those firms may therefore not be willing to take over the upstream market:

Proposition 8. *In the $N = M + 1$ case, when firms compete in two-part tariffs, there exists an equilibrium with M mergers and a monopoly-like outcome if and only if*

does not have an equilibrium.

[26]If we were to stick to our original timing, we would face the following problem. Assume U_i offers a low variable part and a high fixed part, whereas U_j offers a high w and a low T. Then, a downstream firm's optimal choice of supplier would depend on the downstream price it sets at the same time. If it sets a low downstream price, then the demand it receives is high, incentives to minimize marginal cost are strong, and the downstream firm should pick U_i's offer. Conversely, if it sets a high price, then it should go for U_j's offer. The fact that a downstream firm's marginal cost can depend on its downstream price may make the best response in downstream price discontinuous, which jeopardizes equilibrium existence in stage 3.

$\Pi(1, \mathbf{0}, w_m^{tp}) \leq \Pi(0, \mathbf{1}, w_m^{tp})$.

In Example 1, this condition is equivalent to $\delta \geq \delta_m^{tp}$, where $\delta_m^{tp} > \delta_m$.

Compared to linear tariff competition, the monopoly-like outcome is both less harmful to consumers ($w_m^{tp} \leq w_m$, which leads to lower downstream prices) and more difficult to sustain ($\delta_m^{tp} > \delta_m$) under two-part pricing. The intuition for $\delta_m^{tp} > \delta_m$ is that when w becomes very large, the upstream demand and therefore the upstream profit shrink to zero. By continuity, it follows that the softening effect dominates when w is large. Since $w_m^{tp} \leq w_m$, the softening effect is more likely to dominate under linear pricing than under two-part pricing.

In the $N = M + 1$ case, there is only one unintegrated downstream firm left in M-merger subgames, and since we assume upstream suppliers are chosen in stage 2.5, we cannot use a private randomization device to get rid of integer constraints. To investigate the robustness of collusive-like equilibria to two-part pricing, we solve the model in another special case, with $M = 2$, $N = 4$ and linear demands, which takes care of integer constraints:

Proposition 9. *Assume $M = 2$ and $N = 4$. In Example 1, when firms compete in two-part tariffs, there exists an equilibrium with two mergers and a collusive-like outcome in the upstream market if $\delta \in [\underline{\delta}_c^{tp}, \overline{\delta}_c^{tp}]$.*

5.4 Secret offers

We modify the timing and the information structure as follows. At the beginning of stage 2, upstream firms offer secret, linear and discriminatory contracts to the downstream firms. Next, each downstream firm decides which offer to accept, if any. In stage 3, acceptance decisions are publicly observed (i.e., everybody knows who purchases from whom, but not on which terms), and downstream firms set their prices simultaneously.[27]

We look for monopoly-like equilibria in the $N = M+1$ case; collusive-like equilibria and the general case will be discussed later. The first step is to define the monopoly upstream price under secret offers. Suppose $U_i - D_i$ supplies D_{M+1} at price w, but all other integrated firms believe the upstream price is w^b. Those integrated firms set

[27]We allow upstream firms to third-degree price discriminate as in Section 5.2, since non-discriminatory and secret offers would be *de facto* observed by all downstream firms. We also use the sequential timing introduced in Section 5.1.2.

the downstream price they would charge under public offers when $U_i - D_i$ supplies the upstream market at price w^b: $P(0, \mathbf{1}, w^b)$. In this branch of the game tree, everything works as if $U_i - D_i$ and D_{M+1} were playing a two-player game with common knowledge of the upstream price (w) and of the prices set by other integrated firms ($P(0, \mathbf{1}, w^b)$). We assume that this game has a unique Nash equilibrium, which determines the downstream prices of $U_i - D_i$ and D_{M+1}. By strategic complementarity, these equilibrium prices are increasing in $P(0, \mathbf{1}, w^b)$. We assume the equilibrium quantities of $U_i - D_i$ and D_{M+1} are also increasing in $P(0, \mathbf{1}, w^b)$, which means as usual that direct effects dominate indirect ones. Denote by $\Pi^s(1, \mathbf{0}, w, w^b)$ and $\Pi^s_d(1, \mathbf{0}, w, w^b)$ the upstream supplier's and the downstream firm's equilibrium profits. We assume that $\Pi^s(1, \mathbf{0}, w, w^b)$ and $\Pi(1, \mathbf{0}, w)$ are strictly quasi-concave in w.

w^s_m is a monopoly upstream price under secret offers if and only if $U_i - D_i$ indeed wants to set w^s_m when other integrated firms believe the upstream price is w^s_m. Formally, $w^s_m = \arg\max_w \Pi^s(1, \mathbf{0}, w, w^s_m)$ subject to $\Pi^s_d(1, \mathbf{0}, w, w^s_m) \geq \Pi_d(0, \mathbf{0}, \overline{m})$.

Lemma 8. *There exists a monopoly upstream price under secret offers. Any monopoly upstream price under secret offers belongs to the interval* $(m, w_m]$.

To streamline the analysis, we assume that w^s_m is unique, and that $\overline{m}$ is not too high, which ensures that $\Pi(1, \mathbf{0}, w^s_m) \geq \Pi(0, \mathbf{0}, \overline{m})$, i.e., $U_i - D_i$ prefers supplying the market at w^s_m rather than letting D_{M+1} purchase from the alternative source. The intuition for $w^s_m \leq w_m$ is that, under public offers, when $U_i - D_i$ cuts its upstream price, other integrated firms understand that both $U_i - D_i$ and D_{M+1} will become more aggressive on the downstream market. By strategic complementarity, those other integrated firms lower their downstream prices too, which hurts $U_i - D_i$. Under private contracting, those firms do not observe the deviation, and this mechanism therefore disappears.[28]

As usual, we define a monopoly-like outcome as a situation in which $U_i - D_i$ offers w^s_m, and other integrated firms make not upstream offer. When investigating whether undercutting is profitable for, say, $U_j - D_j$, we need to specify how other integrated firms update their beliefs if they find out that $U_j - D_j$ has become the upstream sup-

[28]This is reminiscent of the opportunism problem identified by Hart and Tirole (1990), O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and Rey and Tirole (2007) in that, starting from the optimal public contract, the upstream supplier has incentives to offer a secret 'sweetheart deal' to the downstream firm to increase their profits at the expense of other firms in the industry.

plier. Since the perfect Bayesian equilibrium concept does not put any restrictions on such out-of-equilibrium beliefs, it is easy to construct beliefs which would ruthlessly 'punish' $U_j - D_j$'s deviation. We refine these out-of-equilibrium beliefs using forward induction.[29] The idea is that, when firms observe that D_{M+1} takes an out-of-equilibrium action, they should not perceive this as an involuntary tremble, but rather as a consequence of D_{M+1}'s optimizing behavior. In turn, D_{M+1}'s deviation should come from the fact that $U_j - D_j$ also deviated, and was also trying to maximize its profit.

The implications of this concept in terms of beliefs formation are the following. Assume that $U_j - D_j$ deviates by offering w_j, that D_{M+1} accepts this offer, and that other integrated firms believe that $U_j - D_j$ offered w_j^b to D_{M+1}. Then, $U_j - D_j$ earns a profit of $\Pi^s(1, \mathbf{0}, w_j, w_j^b)$. Under forward induction, the other integrated firms expect $U_j - D_j$ to maximize its deviation profit: beliefs are consistent with forward induction if and only if $w_j^b \in \arg\max_{w_j} \Pi^s(1, \mathbf{0}, w_j, w_j^b)$ subject to $\Pi_d^s(1, \mathbf{0}, w_j, w_j^b) \geq \Pi_d^s(1, \mathbf{0}, w_m^s, w_m^s)$. Therefore, $w_j^b = w_m^s$. It follows that there exists a monopoly-like equilibrium with beliefs consistent with forward induction if and only if $\Pi(1, \mathbf{0}, w_m^s) \leq \Pi(0, \mathbf{1}, w_m^s)$.

In subgames with fewer than M mergers, we show that the Bertrand outcome is an equilibrium in passive beliefs. In terms of behavior, passive beliefs have the following (appealing) implications: (a) a downstream firm never accepts an upward deviation, and (b) when a downstream firm receives a deviating offer below marginal cost, it always accepts this offer and cuts its downstream price. It is easy to see that the Bertrand outcome would also be an equilibrium with any beliefs system generating these two properties.

Proposition 10. *In the $N = M + 1$ case, when upstream offers are secret, there exists an equilibrium with M mergers and a monopoly-like outcome if and only if $\Pi(1, \mathbf{0}, w_m^s) \leq \Pi(0, \mathbf{1}, w_m^s)$. In Example 1, this condition is equivalent to $\delta \geq \delta_m^s$, where $\delta_m^s > \delta_m$.*

Under secret offers, monopoly-like equilibria are less harmful to consumers than under public offers ($w_m^s \leq w_m$, which leads to lower downstream prices). As explained in Section 5.3, this implies that they are also less likely to arise ($\delta_m^s > \delta_m$).

[29]See Fudenberg and Tirole (1991) for a discussion of forward induction. McAfee and Schwartz (1994) apply forward induction to define wary beliefs in a vertical relations model with an upstream bottleneck. See also Rey and Vergé (2004) for a thorough treatment of wary beliefs.

Extending Proposition 10 to the general $N \geq M + 1$ case is difficult. In the M-merger subgame, when a downstream firm receives an unexpected offer, it updates its beliefs about the offers made to other downstream firms. Starting from a monopoly-like outcome, a downstream firm which receives an out-of-equilibrium offer from $U_j - D_j$ must form beliefs about the number of other downstream firms to which $U_j - D_j$ made offers and about the prices of these other unexpected offers. We have not been able to refine these beliefs using forward induction. For the same reasons, it is difficult to establish the robustness of collusive-like equilibria to secret offers. Nevertheless, we prove the following proposition, which provides a necessary and sufficient condition for symmetric collusive-like equilibria in passive beliefs to exist when $M = 2$ and $N = 4$:

Proposition 11. *Assume $M = 2$ and $N = 4$. In Example 1, in the two-merger subgame, there exist symmetric collusive-like equilibria in passive beliefs if and only if $\delta \in [\underline{\delta}_c^s, \overline{\delta}_c^s)$. Moreover, when this condition is satisfied, the set of input prices which can be sustained in a symmetric collusive-like equilibrium in passive beliefs is an interval.*

6 Conclusion

The main message conveyed in this paper is that upstream competition between vertically integrated firms can be much softer than competition between vertically integrated firms and upstream firms, or than competition between upstream firms only. The reason lies in the softening effect, which links changes in the upstream market shares of vertically integrated firms to changes in downstream pricing strategies. The softening effect may induce a vertical merger wave, which effectively eliminates all unintegrated upstream firms and leads to the partial foreclosure of the remaining unintegrated downstream firms.

In our model, if there are initially more upstream firms than downstream firms, or if fewer than M mergers take place, then vertical mergers do not lead to input foreclosure. This results from the homogeneous input assumption. We conjecture that things would be smoother and the competitive pressure coming from unintegrated upstream firms would not be as stringent if the input were differentiated.[30] Following a merger wave which does not lead to the complete forward integration of the upstream industry, unintegrated upstream firms would no longer be able or willing to take over

[30]Product differentiation in input markets is known to be difficult to model in a tractable way. See Inderst and Valletti (2011) and Reisinger and Schnitzer (2012) for recent contributions on this topic.

the upstream market when prices are above costs.[31] Integrated firms would still be reluctant to steal upstream market shares from their integrated rivals, fearing that these rivals would then become more aggressive in the downstream market. The main message of our paper would survive, and, in fact, become smoother: a vertical merger wave, by increasing the proportion of vertically integrated firms competing in the input market, leads to higher input prices.

A Proofs

A.1 Proof of Lemma 1

Existence of the Bertrand equilibrium is standard. Now, assume that $K < M$ mergers have taken place, and let us prove that the Bertrand outcome is the only equilibrium. Suppose that the input is supplied at a price $w > m$. If $K = 0$, then an unintegrated upstream firm can profitably deviate by setting $w - \varepsilon$ as in the textbook Bertrand model. If $K > 0$, given our equilibrium selection in stage 3, either the upstream market is supplied by unintegrated upstream firms only, or it is supplied by vertically integrated firms only. In the latter case, an unintegrated upstream firm can profitably deviate by setting $w - \varepsilon$. In the former case, we claim that a vertically integrated firm, call it $U_i - D_i$ can profitably deviate by matching price w. If $U_i - D_i$ does not deviate, then its first-order condition is given by:

$$q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} = 0.$$

If it matches w, and becomes the sole input supplier, its first-order condition becomes:

$$q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w - m)\sum_{k=K+1}^{N}\frac{\partial q_k}{\partial p_i} = 0.$$

Since the last term in the right-hand side is positive, $U_i - D_i$'s first-order condition shifts upward when it matches w. It follows from a supermodularity argument (see Vives, 1999, p.35) that all downstream prices go up when $U_i - D_i$ matches. Therefore, $U_i - D_i$ wants to match so as to soften downstream competition and to make positive upstream profits.

[31] In the same vein, if upstream firms were capacity-constrained, then a small number of unintegrated upstream firms would not be able to disrupt a foreclosure equilibrium.

A.2 Proof of Lemma 2

Fix an input price vector $\boldsymbol{w}$ and a profile of supplier choices $(\theta_k \mapsto s_k(\theta_k))_{M+1\leq k\leq N}$ consistent with sequential rationality. Let $i \leq M$ and $j \geq M+1$. Since all downstream firms end up purchasing at price $w = \min(\boldsymbol{w})$, and since there exists a unique profile of equilibrium downstream prices associated with this profile of supplier choices, all unintegrated downstream firms set the same price. It follows that, for all $k \geq M+1$, $q_k = q_j$ and $\partial q_k/\partial p_i = \partial q_j/\partial p_i$, where the functions are evaluated at the equilibrium price vector. $U_i - D_i$'s first-order condition is given by:

$$\begin{aligned} 0 &= q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w-m)\sum_{k=M+1}^{N} E[\mathbb{1}_{s_k(\theta_k)=i}]\frac{\partial q_k}{\partial p_i}, \\ &= q_i + (p_i - m + \delta)\frac{\partial q_i}{\partial p_i} + (w-m)\alpha_i(N-M)\frac{\partial q_j}{\partial p_i}. \end{aligned}$$

D_k's first-order condition is given by:

$$q_k + (p_k - w)\frac{\partial q_k}{\partial p_k} = 0.$$

It follows that the equilibrium downstream prices and quantities depend only on w and $\boldsymbol{\alpha}$. By symmetry between integrated firms, $U_i - D_i$'s equilibrium downstream price can be written as $P(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$, where P is invariant to permutations of $\boldsymbol{\alpha}_{-i}$. By the same token, a similar property holds for integrated firms' equilibrium quantities, and for downstream firms' equilibrium prices, quantities and profits.

$U_i - D_i$'s profit is equal to

$$\pi_i = (p_i - m + \delta)q_i + (w-m)\alpha_i(N-M)q_j.$$

Therefore, the equilibrium profit of $U_i - D_i$ only depends on w and $\boldsymbol{\alpha}$. Symmetry implies again that this profit can be written as $\Pi(\alpha_i, \boldsymbol{\alpha}_{-i}, w)$, and is invariant to permutations of $\boldsymbol{\alpha}_{-i}$.

A.3 Proof of Lemma 3

Denote by $BR_1(p)$ (reps. $BR_0(p)$) the best response of $U_i - D_i$ (resp. $U_j - D_j$) to $U_j - D_j$'s (resp. $U_i - D_i$'s) downstream price. We omit the other arguments of these functions to simplify the notations. The first-order conditions (3) and (4) indicate that

$BR_1(p) > BR_0(p)$, and we know that these functions are increasing in p by strategic complementarity. Assume by contradiction that $P(1,\mathbf{0},w) \leq P(0,\mathbf{1},w)$. Then,

$$P(1,\mathbf{0},w) = BR_1(P(0,\mathbf{1},w)) > BR_0(P(0,\mathbf{1},w)) \geq BR_0(P(1,\mathbf{0},w)) = P(0,\mathbf{1},w).$$

Contradiction! The second part of the lemma follows from revealed profitability.

A.4 Proof of Lemma 4

We have already proven in Section A.1 that $\Pi(1,\mathbf{0},w) < \Pi(1,\mathbf{0},m)$ for $w < m$. Therefore, the maximization problem becomes $\max_{w\in[m,\overline{m}]} \Pi(1,\mathbf{0},w)$. Since $[m,\overline{m}]$ is compact and $\Pi(1,\mathbf{0},.)$ is continuous, w_m exists.

Now, we claim that $\partial\Pi(1,\mathbf{0},m)/\partial w > 0$. Denote by $U_i - D_i$ the upstream supplier. Using the envelope theorem, we get:

$$\frac{\partial \Pi(1,\mathbf{0},m)}{\partial w} = (P_i - m + \delta)\left(\sum_{1\leq k\leq N, k\neq i} \frac{\partial q_i}{\partial p_k}\frac{\partial P_k}{\partial w}\right) + (N-M)Q_d(1,\mathbf{0},m) > 0,$$

since, by supermodularity, the downstream prices are increasing in w. We conclude that $w_m > m$. Notice finally that, if all integrated firms offer prices above $\overline{m}$, then one integrated firm can profitably deviate by matching $\overline{m}$. When it does so, all downstream prices go up, and the deviator starts making upstream profits.

A.5 Proof of Lemma 5

Let $\tilde{\boldsymbol{P}}(x)$ (resp. $\hat{\boldsymbol{P}}(x)$) the equilibrium downstream price vector when $\alpha_i = 1/M + x$, $\alpha_j = 1/M - x$ (resp. $\alpha_i = 1/M - x$, $\alpha_j = 1/M + x$), and $\alpha_{i'} = 1/M$ for $i' \neq i, j$ in $\{1,\ldots,M\}$. Then, for all x, $\tilde{\boldsymbol{P}}(x) = \hat{\boldsymbol{P}}(-x)$.

If $k \neq i, j$ in $\{1,\ldots,N\}$, then, by symmetry, $\tilde{P}_k(x) = \hat{P}_k(x)$ for all x. It follows that $\tilde{P}_k(x) = \tilde{P}_k(-x)$ for all x, and therefore, that $\tilde{P}'_k(0) = 0$. Besides, by symmetry, $\tilde{P}_i(x) = \hat{P}_j(x) = \tilde{P}_j(-x)$. Therefore, $\tilde{P}'_i(0) = -\tilde{P}'_j(0)$. Totally differentiating $U_i - D_i$'s first-order condition at point $x = 0$, and using the fact that $\tilde{P}'_k(0) = 0$ for $k \neq i, j$, we get:

$$\begin{aligned} 0 &= \frac{\partial^2 \pi_i}{\partial p_i^2}\tilde{P}'_i(0) + \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\tilde{P}'_j(0) + (w-m)(N-M)\frac{\partial q_d}{\partial p_i}, \\ &= \tilde{P}'_i(0)\left(\frac{\partial^2 \pi_i}{\partial p_i^2} - \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}\right) + (w-m)(N-M)\frac{\partial q_d}{\partial p_i}. \end{aligned}$$

The second term on the right-hand side is positive by substitutability. The terms in factor of $\tilde{P}_i'(0)$ are negative due to $U_i - D_i$'s second-order condition and to strategic complementarity. Therefore, $\tilde{P}_i'(0) > 0$.

A.6 Proof of Lemma 6

Suppose there exists a collusive-like equilibrium with an input price $w > m$ offered by $Z \in \{2, \dots, M\}$ integrated firms, and a profile of upstream market shares $\boldsymbol{\alpha} \in S_M(Z)$. Define $\boldsymbol{\beta}_{-i} = (\alpha_{i+1}, \dots, \alpha_M, \alpha_1, \dots, \alpha_{i-1})$. Then,

$$\begin{aligned} \Pi(\frac{1}{M}, \dots, \frac{1}{M}, w) &= \Pi(\frac{\sum_{i=1}^M \alpha_i}{M}, \dots, \frac{\sum_{i=1}^M \alpha_i}{M}, w) = \Pi(\sum_{i=1}^M \frac{1}{M}(\alpha_i, \boldsymbol{\beta}_{-i}), w), \\ &\geq \min_{1 \leq i \leq M} \Pi(\alpha_i, \boldsymbol{\alpha_{-i}}, w) \text{ by quasi-concavity and Lemma 2}, \\ &\geq \max\left\{ \max_{\tilde{w} \leq w} \Pi(1, \mathbf{0}, \tilde{w}), \min_{\boldsymbol{\beta} \in S_{M\text{-}1}(Z\text{-}1)} \Pi(0, \boldsymbol{\beta}, w) \right\}, \\ &\geq \max\left\{ \max_{\tilde{w} \leq w} \Pi(1, \mathbf{0}, \tilde{w}), \min_{\boldsymbol{\beta} \in S_{M\text{-}1}(M\text{-}1)} \Pi(0, \boldsymbol{\beta}, w) \right\}, \end{aligned}$$

where the penultimate inequality follows from condition (7), and the last inequality follows from the fact that $S_{M-1}(Z-1) \subseteq S_{M-1}(M-1)$.

A.7 Proof of Proposition 3

We still need to show that there is no equilibrium of the M-merger subgame in which only one integrated firm offers the lowest upstream price w, with $w > m$ and $w \neq w_m$. Assume by contradiction that such an equilibrium exists and call the upstream supplier $U_i - D_i$. If the other integrated firms make no upstream offer or make offers above w_m, then $U_i - D_i$ has a profitable deviation: set w_m.

Otherwise, we denote the second lowest upstream price by $w' = \min(\boldsymbol{w}_{-i})$. If $U_i - D_i$ withdraws its offer, then the upstream market is supplied with some market shares $\boldsymbol{\alpha}_{-i}$ by the integrated firms which set price w'. $U_i - D_i$ earns $\Pi(0, \boldsymbol{\alpha}_{-i}, w')$, which is no smaller than $\Pi(0, \mathbf{1}, w')$ by quasi-concavity. So, if this deviation is not profitable, then $\Pi(1, \mathbf{0}, w) \geq \Pi(0, \mathbf{1}, w')$. By supermodularity, $\Pi(0, \mathbf{1}, w') > \Pi(0, \mathbf{1}, w)$, and $U_j - D_j$ $(j \neq i)$ can profitably deviate by setting $w - \varepsilon$: contradiction!

A.8 Proof of Proposition 4

Assume condition (6) holds for some $w > 0$. In all M-merger subgame, we select the symmetric collusive-like equilibrium at price w. Then, the following profile of bidding strategies is an equilibrium: in all periods of stage 1, all unintegrated downstream firms bid $\Pi(1/M, \mathbf{1/M}, w) - \Pi_d(1/M, \mathbf{1/M}, w) > 0$. In this equilibrium, a merger wave leads to partial foreclosure. Now, assume condition (5) holds. In all M-merger subgames, we select the monopoly-like equilibrium in which U_1 supplies the upstream market. Then, the following profile of bidding strategies is an equilibrium: all unintegrated downstream firms bid $\Pi(1, \mathbf{0}, w_m) - \Pi_d(1, \mathbf{0}, w_m) > 0$ for U_1 and $\Pi(0, \mathbf{1}, w_m) - \Pi_d(1, \mathbf{0}, w_m) > 0$ for all the other unintegrated upstream firms. Again, we have an equilibrium merger wave which leads to foreclosure. Conversely, if neither condition (5) nor condition (6) is satisfied, then no merger wave can lead to foreclosure.

Next, assume $\delta = 0$. We select the Bertrand equilibrium in all subgames. Then, all downstream firms, integrated or not, earn the same profit no matter what the outcome of the merger game is. Therefore, there is an equilibrium in which all downstream firms bid zero, no merger takes place, and the Bertrand outcome arises on the upstream market. If $\delta > 0$, then we still select the Bertrand equilibrium in all subgames. Since $\delta > 0$, $\Pi(1, \mathbf{0}, m) > \Pi_d(1, \mathbf{0}, m)$, and the profile of bidding strategies in which all unintegrated downstream firms bid $\Pi(1, \mathbf{0}, m) - \Pi_d(1, \mathbf{0}, m)$ is an equilibrium. On the equilibrium path, a merger wave leads to the Bertrand outcome.

A.9 Proof of Lemma 7

Denote by $\hat{w}_m^{tp}$ the unique solution of maximization problem $\max_w \Pi(1, \mathbf{0}, w) + \Pi_d(1, \mathbf{0}, w)$. Assume first that $\Pi_d(1, \mathbf{0}, \hat{w}_m^{tp}) \leq \Pi_d(0, \mathbf{0}, \overline{m})$. Then, it follows from the quasi-concavity of the joint profit and from the definition of w_m that $(w_m^{tp}, T_m^{tp}) = (w_m, 0)$, which implies that the two inequalities in the statement of Lemma 7 are satisfied.

Conversely, assume that $\Pi_d(1, \mathbf{0}, \hat{w}_m^{tp}) > \Pi_d(0, \mathbf{0}, \overline{m})$. Then, the monopoly contract is $w_m^{tp} = \hat{w}_m^{tp}$, and $T_m^{tp} = \Pi_d(1, \mathbf{0}, \hat{w}_m^{tp}) - \Pi_d(0, \mathbf{0}, \overline{m})$. Applying the envelope theorem twice, we get:

$$\frac{\partial(\Pi(1, \mathbf{0}, m) + \Pi_d(1, \mathbf{0}, m))}{\partial w} = (P_i - m + \delta) \sum_{j \neq i} \frac{\partial q_i}{\partial p_j} \frac{\partial P_j}{\partial w} + (P_{M+1} - m) \sum_{j \neq M+1} \frac{\partial q_k}{\partial p_j} \frac{\partial P_j}{\partial w},$$

which is strictly positive since $\partial P_j / \partial w > 0$ for all j. By quasi-concavity, it follows that $\hat{w}_m^{tp} > m$. Now, notice that

$$\frac{\partial \Pi(1, \mathbf{0}, \hat{w}_m^{tp})}{\partial w} > \frac{\partial(\Pi(1, \mathbf{0}, \hat{w}_m^{tp}) + \Pi_d(1, \mathbf{0}, \hat{w}_m^{tp}))}{\partial w} = 0,$$

where the inequality follows from $\partial\Pi_d(1,\mathbf{0},w)/\partial w < 0$. Therefore, since $\Pi(1,\mathbf{0},w)$ is strictly quasi-concave in w, $w_m > w_m^{tp}$.

A.10 Proof of Proposition 8

First, we prove that the Bertrand outcome is an equilibrium in subgames with fewer than M mergers. Assume that all upstream firms, integrated or not, offer $(m, 0)$. Suppose first that U_i (which may or may not be integrated) deviates upward, and offers (w, T), with $w > m$ and $T \geq 0$. Then, no downstream firm will accept this offer since, by assumption, the equilibrium profit of a downstream firm is decreasing in the variable part of its contract.

Now, assume that U_i offers (w, T) with $w < m$, and that this deviation attracts a set S of unintegrated downstream firms. Let $D_j \in S$. Denote by $\boldsymbol{p}^S$ the equilibrium downstream price vector when all firms in S (and only in S) accept the deviating offer. Denote also by $\boldsymbol{p}^{S\backslash\{j\}}$ the equilibrium downstream price vector when the deviating offer is only accepted by firms in $S\backslash\{j\}$. When D_j accepts the deviating offer, its best-response function shifts down. By supermodularity, all downstream prices decrease, i.e., $\boldsymbol{p}^{S\backslash\{j\}} > \boldsymbol{p}^S$. Clearly, $p_j^S \geq m$, otherwise the joint profit of U_i and of the firms in S would be negative. A necessary condition for the deviating offer to be accepted by D_j in an equilibrium of the continuation subgame is that

$$(p_j^S - w)q(p_j^S, \boldsymbol{p}_{-j}^S) - T \geq (p_j^{S\backslash\{j\}} - m)q(p_j^{S\backslash\{j\}}, \boldsymbol{p}_{-j}^{S\backslash\{j\}}).$$

Rearranging terms, we get:

$$\begin{aligned}
(w-m)q(p_j^S, \boldsymbol{p}_{-j}^S) + T &\leq (p_j^S - m)q(p_j^S, \boldsymbol{p}_{-j}^S) - (p_j^{S\backslash\{j\}} - m)q(p_j^{S\backslash\{j\}}, \boldsymbol{p}_{-j}^{S\backslash\{j\}}),\\
&\leq (p_j^S - m)q(p_j^S, \boldsymbol{p}_{-j}^{S\backslash\{j\}}) - (p_j^{S\backslash\{j\}} - m)q(p_j^{S\backslash\{j\}}, \boldsymbol{p}_{-j}^{S\backslash\{j\}}),\\
&< 0, \text{ by definition of } p_j^{S\backslash\{j\}}.
\end{aligned}$$

Therefore, U_i makes negative profits and the deviation is not profitable. By the same token, a vertically integrated firm has no incentive to deviate downward, and the Bertrand outcome is an equilibrium in all subgames with fewer than M mergers.

Next, we look for monopoly-like equilibria in the M-merger subgame. Assume there exists a monopoly-like equilibrium. Then, the firms which do not supply the upstream market should not be willing to undercut: $\Pi(0,\mathbf{1},w_m^{tp}) \geq \Pi(1,\mathbf{0},w_m^{tp}) + T_m^{tp} \geq \Pi(1,\mathbf{0},w_m^{tp})$, since $T_m^{tp} \geq 0$.

Conversely, suppose that $\Pi(1, \mathbf{0}, w_m^{tp}) \leq \Pi(0, \mathbf{1}, w_m^{tp})$. We distinguish two cases. Assume first that $\Pi(1, \mathbf{0}, w_m^{tp}) + T_m^{tp} \leq \Pi(0, \mathbf{1}, w_m^{tp})$. Then, the monopoly-like outcome in which $U_1 - D_1$ offers contract (w_m^{tp}, T_m^{tp}) and other integrated firms do not make any offer is an equilibrium. Second, assume that $\Pi(1, \mathbf{0}, w_m^{tp}) \leq \Pi(0, \mathbf{1}, w_m^{tp}) < \Pi(1, \mathbf{0}, w_m^{tp}) + T_m^{tp}$. Then, the monopoly-like outcome in which all integrated firms offer contract $(w_m^{tp}, \Pi(0, \mathbf{1}, w_m^{tp}) - \Pi(1, \mathbf{0}, w_m^{tp}))$ and the unintegrated downstream firm accepts $U_1 - D_1$'s contract is an equilibrium.

A.11 Proof of Lemma 8

Fix some w^b and let $\bar{w}(w^b)$ (resp. $\bar{w}$) such that $\Pi_d^s(1, \mathbf{0}, w, w^b) \geq \Pi_d(0, \mathbf{0}, \overline{m})$ (resp. $\Pi_d(1, \mathbf{0}, w) \geq \Pi_d(0, \mathbf{0}, \overline{m})$) if and only if $w \leq \bar{w}(w^b)$ (resp. $w \leq \bar{w}$). By strict quasi-concavity, $\hat{w}(w^b) = \arg\max_{w \leq \bar{w}(w^b)} \Pi^s(1, \mathbf{0}, w, w^b)$ exists and it is unique. Let $f(w^b) = \hat{w}(w^b) - w^b$, and notice that f is continuous. $\tilde{w}$ is a monopoly upstream price under secret offers if and only if it is a zero of f. Below we show that $f(w^b) > 0$ for all $w^b \leq m$ and that $f(w^b) < 0$ for all $w^b > w_m$, which proves Lemma 8.

Claim 1: $f(w^b) > 0$ for all $w^b \leq m$.

Let $w^b \leq m$. Clearly, $\Pi_d^s(1, \mathbf{0}, w^b, w^b) > \Pi_d(1, \mathbf{0}, \overline{m}) > \Pi_d(0, \mathbf{0}, \overline{m})$, so $w^b < \bar{w}(w^b)$. Next, we show that the upstream supplier, call it $U_i - D_i$, has incentives to slightly increase w, starting from $w = w^b$. Using the envelope theorem:

$$\left.\frac{\partial \Pi^s(1, \mathbf{0}, w, w^b)}{\partial w}\right|_{w=w^b} = \left[(p_i - m + \delta)\frac{\partial q_i}{\partial p_{M+1}} + (w^b - m)\frac{\partial q_{M+1}}{\partial p_{M+1}}\right]\frac{\partial p_{M+1}}{\partial w} + q_{M+1}.$$

The first and third terms on the right-hand side are positive, and, since $w^b \leq m$, the second term is non-negative. Therefore, by quasi-concavity, $f(w^b) > 0$.

Claim 2: $f(w^b) > 0$ for all $w^b > w_m$.

Assume first that $w_m = \bar{w}$. If $w^b > \bar{w}$, then $\Pi_d(1, \mathbf{0}, w^b) < \Pi_d(0, \mathbf{0}, \overline{m})$. Therefore, $\bar{w}(w^b) < w^b$, and $f(w^b) < 0$.

Next, assume $w_m < \bar{w}$. First, we show that, when $\tilde{w} > m$, $\left.\frac{\partial \Pi(1, \mathbf{0}, w)}{\partial w}\right|_{w=\tilde{w}} > \left.\frac{\partial \Pi^s(1, \mathbf{0}, w, w^b)}{\partial w}\right|_{w=w^b=\tilde{w}}$. Since $\Pi(1, \mathbf{0}, w) = \Pi^s(1, \mathbf{0}, w, w)$ for all w,

$$\left.\frac{\partial \Pi(1, \mathbf{0}, w)}{\partial w}\right|_{w=\tilde{w}} = \left.\frac{\partial \Pi^s(1, \mathbf{0}, w, w^b)}{\partial w}\right|_{w=w^b=\tilde{w}} + \left.\frac{\partial \Pi^s(1, \mathbf{0}, w, w^b)}{\partial w^b}\right|_{w=w^b=\tilde{w}}.$$

Therefore, all we need to show is that $\left.\frac{\partial \Pi^s(1, \mathbf{0}, w, w^b)}{\partial w^b}\right|_{w=w^b=\tilde{w}} > 0$. This is true, since the equilibrium prices of $U_i - D_i$ and D_{M+1} are increasing in w^b (by supermodularity), and so are their equilibrium demands (by assumption).

By quasi-concavity, for all $w^b \geq w_m$, $\left.\frac{\partial \Pi(1,\mathbf{0},w)}{\partial w}\right|_{w=w^b} \leq 0$. It follows that $\left.\frac{\partial \Pi^s(1,\mathbf{0},w,w^b)}{\partial w}\right|_{w=w^b}$ is negative, which implies that $\hat{w}(w^b) < w^b$ and that $f(w^b) < 0$.

A.12 Proof of Proposition 10

We show that the Bertrand outcome is an equilibrium in passive beliefs in subgames with fewer than M mergers. Assume that all upstream firms, integrated or not, offer the input at marginal cost to all downstream firms on the equilibrium path. An unintegrated downstream firm which receives an upward deviation would reject this offer (because a firm's profit is decreasing in its marginal cost) and would not change its downstream price (due to passive beliefs). Therefore, there is no point in deviating upward. By the same token, if an unintegrated downstream receives a downward deviation, then it accepts it and cuts its downstream price. Ensuing upstream losses and tougher downstream competition make such a deviation unprofitable for upstream firms, integrated or not.

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