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Verifiable and Nonverifiable Information in a Two-Period Agency Problem

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Abstract

I examine how a firm’s opportunity to verify information influences the joint use of verifiable and unverifiable information for incentive contracting. I employ a simple two-period agency model, in which contract frictions arise from limited liability and the potential unverifiability of the principal’s information about the agent’s action. With short-term contract, the principal benefits from both a more informative and a more conservative verification of his private information. With long-term contracts, he may prefer a less informative verification, but his preference for a conservative verification persists.

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1 Introduction

In this paper I examine how a firm’s opportunity to verify information influences the joint use of verifiable and unverifiable information for incentive contracting. Unverifiable information is of importance for managerial incentives because when firms decide on the compensation of its managers, they often make use not only of hard performance information such as accounting income or sales numbers. The board or a compensation committee in most cases also considers soft factors and subjectively assesses how exogenous factors have influenced the objective performance measures. Also on a lower hierarchical level, incentives may be provided by senior managers’ subjective assessments of their subordinates’ performance (Murphy and Oyer 2003; Gibbs et al. 2004).

Since subjective assessments are likely to be nonverifiable information, compensation committees usually are conceded substantial discretion when they determine bonus payments. As an example, consider the following statement on executive compensation at General Electric. "Except with respect to our long-term performance awards and the PSUs [Performance Share Units] granted to our CEO, both of which depend on achieving specific quantitative performance objectives, the MDCC [Management Development and Compensation Committee] does not use formulas in determining the amount and mix of compensation. Thus, the MDCC evaluates a broad range of both quantitative and qualitative factors, including reliability in delivering financial and growth targets, performance in the context of the economic environment relative to other companies, a track record of integrity, good judgment, the vision and ability to create further growth and the ability to lead others.” (General Electric Company, Proxy Statement, March 14, 2011).

While on the one hand such discretion enables the committee or the board to make use of any kind of information, it comes at the cost that ex post, a board may have an incentive to misreport its soft information in order to cut compensation payments. Theoretical studies have considered numerous ways to mitigate this credibility problem. One possible way is a bonus pool, in which a fixed amount of money is distributed among a group of employees.
Many firms employ such discretionary bonus pools. As an example, the bonus plan at FirstCity Financial Corporation is set up as follows. "The bonus plan provides that a bonus pool for the executive officers will be cumulatively funded to the extent that the net earnings to FirstCity common shareholders for calendar year 2011, prior to accrual for compensation under the bonus plan, exceed a minimum threshold of $1.00 per share [...] The executive officers, including named executive officers of FirstCity, participating in the bonus plan are eligible to receive up to 65% of the funded amount of the bonus plan [...] The remaining 35% of the funded bonus plan and any portion of the initial allocation which is not awarded according to the initial allocation as a result of a forfeiture or reduction by the Compensation Committee may be distributed at the discretion of the Compensation Committee to these certain executive officers and/or other executive officers." (Form 8-K filing of FirstCity Financial Corporation, March 21, 2011)

This example demonstrates that the size of a firm’s bonus pool is usually related to publicly observable data, but that the compensation committee may exercise considerable discretion in their decision about how to allocate the bonus pool. As the example suggests, the committee may even decide to cut the bonus pool of the beneficiaries and divert payments to another party. From a theoretical perspective, such third-party payments may be beneficial to increase incentives (MacLeod 2003, Budde 2007). Anecdotal evidence is also found at Credit Suisse, where in 2009 an amount of CHF 20 million of the executive board’s compensation pool (relating to a compensation of CHF 19.20 million for the highest paid member of this board) was diverted to fund charitable contributions.

Although third-party payments may help a principal to provide higher-powered incentives and by this means to maintain a larger share of the benefits from an agency, they nevertheless reduce the total amount to be shared. Therefore, the firm will likely try to avoid burning money, and the verification of the committee’s private information becomes of importance. Verification of the compensation committee’s information may be achieved by different means. Part of the information, such as stock prices or accounting income, will be verifiable
per se because it is publicly observable. Another part of the information will be verifiable by additional measures, such as the commissioning of consultancy firms, as it is frequently intended in compensation committee charters. And part of the committee’s information may be not verifiable at all.

In this paper, I analyze the question of whether and when the firm benefits from the verification of its subjective assessments of a manager’s performance. More specifically, I ask for properties of a verification process that the firm prefers over another alternative. I ask whether a compensation committee will always prefer to have more of its information verified, and if not, which part of its information it prefers to have verified. I analyze this question in a simple agency model with one agent and two periods, where contract frictions arise from the agent’s limited liability and, potentially, from the unverifiability of the principal’s information about the agent’s action.

In the baseline model, I represent the verification as a black box, releasing hard evidence as a noisy measure of the entirety of the principal’s information. From this model, I derive properties of a preferred verification device, stated in terms of the statistical properties of the verification process. In a second step, I ask how these properties relate to the measures potentially employed by a board or a compensation committee.

From an analytical perspective, the black box representation of verification is closely related to models that analyze the contracting usefulness of conservatism in accounting by means of a stochastic transformation of information (Gigler and Hemmer 2001, Kwon et al. 2001, Kwon 2005, Chen et al. 2007, Chen et al. 2010). As in Kwon et al (2001) and Kwon (2005), I find that a bias towards bad news may be of advantage in situations where the agent earns a rent from limited liability. Different to their models, the advantage does not only come from a reduction of the agent’s rent, but also from a reduced probability of third-party payments. Moreover, I find that in a multi-period agency, verified information may also have negative incentive effects because it affects the agent’s action choice in later periods. In that sense, my model is more similar to that of Chen et al. (2010), who study a situation where de-
cision making and control interact. The main difference to all of these papers is that I assume that the principal does not only make use of the verifiable information when he designs the compensation contract, but also employs the unverifiable part of his information to determine the manager’s compensation. That way, I bring together two strands of the literature: the one of the contracting usefulness of accounting conservatism mentioned above, and the one on the combined use of verifiable and unverifiable performance information (Baiman and Rajan 1995, Rajan and Reichelstein 2009, Ederhof 2010, Ederhof et al. 2011).

From an economic perspective, the model is related to models of costly state verification, such as Townsend (1979), Gale and Hellwig (1985), or Hart and Moore (1988). However, these authors assume that the uninformed party (a lender) may conduct a costly audit to share the informed party’s (the borrower’s) information, whereas I assume that the informed party engages in verification and that this verification is costless (at least with respect to the principal’s decision relevant cost), but noisy in the sense that the reported signal only imperfectly reflects the principal’s information. In that sense, the present model may be described as one of noisy costless state verification. Gao (2012) considers noisy verification as part of the accounting process, but defines it in a narrower way than I do. In his notion, the noisy verification technology presented in this paper would rather be considered an accounting measurement, consisting of a report and its verification. In section 5, I provide a more detailed comparison.

The remainder of this paper is structured as follows. In section 2, I describe the baseline model. In section 3, I derive the optimal short-term contract and compare verification technologies with respect to compensation cost. In section 4, I compare different verification technologies with respect to compensation cost in long-term contracts. Given the results of sections 3 and 4, I discuss the verification technology in more detail in section 5. Section 6 concludes.
2 The Baseline Model

Consider a principal (firm) who hires an agent (manager) to perform a certain task repeatedly over two periods $t = 1, 2$. In each period, the manager provides a productive effort, $a_t \in \{a^H, a^L\}$. The firm (principal) wants to implement action $a^H$ in each period. To that purpose, in each period, the principal privately observes a performance metric, $Y_t$, which can take values in $\mathcal{Y} = \{y_L, y_H\}$. The assumption of a binary outcome space is made for computational simplicity. However, since $Y_t$ is assumed to be the principal’s private information, at least the one-period contract would take the form of a bonus contract also for richer outcome spaces (cf. MacLeod 2003, Budde and Hofmann 2012), essentially aggregating information to a binary signal. Economically, $Y_t$ may be interpreted as the essence of all information on the agent’s action the principal may collect, condensed to a statement of whether the agent failed ($y_L$) or succeeded ($y_H$) in his task. The probability of a success depends on the agent’s action choice. If he chooses $a^L$, $y_H$ will be realized with probability $p^L$, and if he chooses $a^H$, he succeeds with probability $p^H$, where $p^H > p^L$ without loss of generality. I assume that $Y_1$ and $Y_2$ are stochastically independent.

In addition to $Y_t$, in each period there is a publicly observable signal $X_t \in \mathcal{X} = \{x_L, x_H\}$. Compared to the entire information $Y_t$ observed by the principal, $X_t$ is only a noisy measure. Formally, let $q_{ij}$ be the probability that $x_j$ is observed if $y_i$ is realized. Without loss of generality, let $x_L$ and $x_H$ be labelled such that $q_{HH} > q_{LH}$ (since $q_{iL} = 1 - q_{iH}$, this implies $q_{LL} > q_{HL}$), i.e. $y_i$ is more likely to be observed when $x_i$ is realized. Borrowing from the economic literature on communication, the matrix $Q = (q_{ij})$ could be considered a communication device (Myerson 1982). But in the literature on communication, it is assumed that the informed agent discretionarily reports his information into that noisy channel, and

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1 Binary action spaces are mainly used for ease of exposition. In a single-period model, a richer action space essentially boils down to the binary case if some regularity conditions are met and only local incentive constraints are binding (Grossman and Hart 1983). In a multi-period setting, richer action spaces may allow for the implementation of signal-dependent actions in future periods (Schmitz 2005). Here, I focus on the problem of implementing a certain desired action at minimal cost, and leave the analysis of such path-dependent action profiles to future research.
truth-telling becomes an issue. In this paper, I will not address the problem of an informed principal. To delineate the model from those on communication, I will therefore refer to $Q$ as the firm’s *verification technology*. I assume that the verification technology is identical in both periods.

The verification must not be understood as purely an instrument to prove a certain piece of information, as it is assumed in Gao (2012). Rather, the verification technology here comprises all measures that deliver hard evidence of what the principal knows about the agent’s action. Thus, if $Y_t$ is everything that the principal knows and $X_t$ is what everybody knows, $Q$ determines the amount of private information the principal has. As extreme cases, the principal has no private information about the agent’s action if $q_{LL} = q_{HH} = 1$, and all information is private if $q_{LL} = q_{HL}.2$

At least two sources of uncertainty determine the noise incorporated in $Q$. Firstly, hard evidence may be misleading, i.e. it may point to a failure although the agent has succeeded in his task or vice versa. An example could be the non-attainment of a milestone which is caused by reasons privately observed by the principal. And secondly, noise is also caused by the fact that part of the principal’s private information may not be verifiable at all, or only at a prohibitively high cost.

Part of the principal’s information will be public per se, such as the firm’s stock price or its accounting income.3 Other hard evidence may be provided by arrangements of the principal, such as reports of consultancy firms employed by the compensation committee. By its decision of whether to demand such reports the committee may affect the verification technology. In Section 5, I discuss the consequences of the principal’s opportunity to at least partly design $Q$. For the baseline model, I will treat $Q$ as exogenously given.

The principal offers the agent a compensation contract in which the agent’s wage may depend on both the verifiable signal $X_t$ and the unverifiable signal $Y_t$. In order to make cred-

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2I will refer to this cases as *perfect verification* and *no verification* in section 4.

3I will not consider the fact that both of these measures may be subject to the agent’s manipulation such as earnings management. Earnings management is considered, for example, in Chen et al. (2007), or Gao (2012, 2012b).
ible use of the unverifiable information \( Y_t \), the principal specifies bonus pools for each of the possible outcomes of verifiable information, and threatens to punish the agent, if necessary, by diverting part of this amount to a third party if the unverifiable signal takes a certain unfavorable value. I consider two types of contracts: In a short-term contract, the agent’s compensation \( s_t(X_t, Y_t) \) only depends on the information observed in that particular period. In a long-term contract, the agent’s compensation \( s_t \) may depend on all signals observed up to this point of time.

The principal wants to minimize his cost of implementing \( a_t = a^H \) in each period. The agent is risk-neutral and has limited liability. His utility is

\[
u_1(s_1, s_2, a_1, a_2) = \sum_{t=1}^{2} s_t - c_t(a_t)
\]

before the first period and \( u_2(s_2, a_2) = s_2 - c_2(a_2) \) at the beginning of the second period, where \( c_t(a_t) \) is his cost of effort in period \( t \). For ease of notation, I normalize the cost of low effort to zero, \( c_t^L = 0 \), and assume that the cost of high effort is \( c_t(a^H) = c^H \) in both periods. From the agent’s outside options, his reservation utilities are

\[
U_1^R = u_1^R + u_2^R \text{ at the beginning of the first period and } U_2^R = u_2^R \text{ at the beginning of the second period.}
\]

By assuming that \( u_1^R = u_2^R = u^R \), I ensure that under short-term contracts, the two periods are identical.

To account for the fact that \( Y_t \) may be only partly verifiable, I focus on bonus pool contracts, as it is done in Rajan and Reichelstein (2009). The principal commits to a total payments \( w \) that he distributes to the agent and a third party, such as a charity. Since the total payment is observable, the principal ex post has no incentive to misreport the agent’s performance. In short-term contracts, a bonus pool \( w_t(X_t) \) is set up for each period and each realization of \( X_t \), whereas in the long-term contract a single bonus pool \( w(X_1, X_2) \) may be specified for the hole planning horizon, or the bonus of the second period may depend on the verifiable measures of both periods.

The firm cannot be sold to the manager because of his limited wealth. More formally, I assume that payments have to exceed a minimum level \( s^{\text{min}} \) in each period. If the liability constraint is binding, an incentive problem arises even with a risk-neutral agent and completely verifiable information (Innes 1990, Kim 1997, and Demougine and Fluet 1998). By adding the
assumption of partly unverifiable information, I am able to study the tradeoff between rents received by the agent (as in the model with verifiable information and limited liability) and the losses incurred by the principal via third-party payments (as are already present in the model of unverifiable information with unlimited liability).

3 Verification in Short-Term Contracts

If short-term contracts are applied, the principal’s problem to minimize his expected payment $E[w_t(X_t)]$ in period $t$ is similar to that analyzed by Rajan and Reichelstein (2009). Different to my analysis, Rajan and Reichelstein assume (i) risk aversion on side of the agent, (ii) stochastically independent measures $X_t$ and $Y_t$, and (iii) unlimited liability. Risk aversion would change my results under short-term contracts only quantitatively, but my assumption that $X_t$ is a noisy measure of $Y_t$ enables me to analyze how the relation of these two measures affects the principal’s compensation cost. I do this for both cases of a binding and non-binding liability.

To do so, consider the principal’s cost minimization program to implement action $a^H$ in period $t$ with a short-term contract:

$$\min_{\{s_t, w_t\}} E[w_t(X_t) \mid a^H] = (1 - p^H) \sum_{j=L,H} q^H_{Lj} w_t(x_j) + p^H \sum_{j=L,H} q^H_{Hj} w_t(x_j) \quad (1)$$

subject to:

$$\begin{align*}
(1 - p^H) \sum_{j=L,H} q_{Lj} s_t(y_L, x_j) + p^H \sum_{j=L,H} q_{Hj} s_t(y_H, x_j) - c^H & \geq u^R \quad (2) \\
(1 - p^H) \sum_{j=L,H} q_{Lj} s_t(y_L, x_j) + p^H \sum_{j=L,H} q_{Hj} s_t(y_H, x_j) - c^H & \geq 0 \quad (3) \\
q_{Lj} s_t(y_L, x_j) + p^L \sum_{j=L,H} q_{Hj} s_t(y_H, x_j) & \leq w_t(x_j) \quad i, j = L, H \quad (4) \\
s_t(y_i, x_j) & \geq s^{\min} \quad i, j = L, H. \quad (5)
\end{align*}$$

The principal seeks to minimize his expected compensation payment (1). Inequality (2) is...
the participation constraint, ensuring that the agent is willing to sign the contract. (3) is the incentive constraint that guarantees that the agent prefers the desired action $a^H$ over $a^L$. The budget constraints (4) state that the agent’s compensation must not exceed the size of the bonus pool, and the liability constraints make sure that compensations do not fall short of the minimum wage $s_{\text{min}}$.

I am mainly interested in how the noise parameters $q_{ij}$ of the verification technology affect the principal’s cost of compensation. To analyze this question, in a first step I describe the optimal contract, from which the expected cost of compensation can be derived in closed form. Based on this, in a second step I compare the expected compensation cost of different verification technologies.

To describe the optimal contract, I distinguish two cases. If the agent’s liability level $s_{\text{min}}$ is low enough, the incentive constraint (3) will not bind in the optimal contract, and the principal can obtain his first-best solution:

**Lemma 1** If the agent’s liability constraint (5) is not binding, the optimal short-term contract has the following properties:

1. Compensations and bonus pools fulfil $w_j = s_{Lj} = s_{Hj}, j = L, H$

2. Expected compensation cost is $E[w(X_t)] = u^R + c^H$

Due to the agent’s risk neutrality, no incentive problem arises as long as the agent’s liability constraint is not binding and the verifiable measure contains at least some piece of information about the agent’s action. In order to implement $a^H$, the agent’s compensation under $x^H$ only has to exceed his compensation under $x_L$ by a certain amount. The absolute level of pay can be adjusted as to meet the agent’s participation constraint, and no agency cost is incurred. Since no improvements to this short-term solution can be achieved by a long-term contract, I will no longer consider this case in the remainder of this paper, and focus on the case where the agent earns a rent from limited liability.
**Assumption 1** The agent’s liability level is such that he earns a strictly positive rent in the optimal contract.

A sufficient condition for Assumption 1 to be met is that $s_{\text{min}} = u^R = 0$. In this case, the optimal contract takes the following form:

**Lemma 2** If Assumption 1 is met, the optimal short-term contract has the following properties:

1. Compensations $s_t(Y_t, X_t)$ and bonuses $w_t(X_t)$ are

\[
s_t(y_L, x_L) = s_t(y_L, x_H) = s_t(y_H, x_L) = s_{\text{min}},
\]

\[
s_t(y_H, x_H) = s_{\text{min}} + \frac{c^H}{q_{HH}(p^H - p^L)} \text{ and }
\]

\[
w_t(x_L) = s_{\text{min}},
\]

\[
w_t(x_H) = s_{\text{min}} + \frac{c^H}{q_{HH}(p^H - p^L)}.
\]

2. Expected compensation cost is

\[
E[w_t(X_t)] = s_{\text{min}} + \frac{p^H c^H}{p^H - p^L} \cdot \frac{p^H q_{HH} + (1 - p^H)q_{LH}}{p^H q_{HH}}.
\] (6)

Only the minimum wage $s_{\text{min}}$ will be paid under $y_L$ because under limited liability, any bonus that is paid under bad news in Milgrom’s (1981) notion is harmful since it only hinders incentives (Budde and Hofmann 2012). With the present binary action space, all outcomes which are more likely under the undesired action $a_L$ convey bad news. These are $(y_L, x_L)$ and $(y_L, x_H)$. Under good news $y_H$, the bonus $w_t(x_L)$ or $w_t(x_H)$ is paid out in full. The fact that nevertheless only $s_{\text{min}}$ is paid under $(y_H, x_L)$ is due to the agent’s risk neutrality and the principal’s tension to minimize third-party payments. Since the agent is risk neutral, he is indifferent to lotteries of wages with the same expected value. Thus, he does not bother the exact amount $s_t(y_H, x_L)$ and $s_t(y_H, x_H)$ as long as the expected value $q_{HL}s_{HL} + q_{HH}s_{HH}$ is identical. Therefore, the principal chooses $s_t(y_H, x_L) = w_t(x_L)$ and $s_t(y_H, x_H) = w_t(x_H)$ as to minimize third-party payments. This is done by paying a positive bonus only for the value of $X_t$ for which it is more likely that the value $y_H$ is realized and no third-party payments are due. Under the assumptions on the verification technology $Q$, this realization is $x_H$. 

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The expected compensation cost in (6) is well interpreted in terms of the accuracy of the verification of the principal’s information $Y_t$ by the verification technology $Q$: the first two terms in (6) are the expected cost of compensation the principal would incur if the principal’s information $Y_t$ were perfectly observable. Therefore, the last factor in (6) quantifies the compensation cost under noisy verification as compared to that under noiseless verification. A closer look shows that this ratio is reciprocal to the posterior probability of $x_H$, given $y_H$. Thus, the clearer $X_t$ indicates whether the agent succeeded in his task, the lower the compensation cost. Quite general, this can formally be stated in terms of Blackwell’s (1953) comparison of experiments:

**Proposition 1** Let there be two alternative verifiable signals $X_t \in \{x_L, x_H\}$ and $Z_t \in \{z_L, z_H\}$ with verification technologies $Q^x$ and $Q^z$. If Assumption 1 is met, compensation cost in a short-term contract is lower under $X_t$ than under $Z_t$ if $X_t$ is more informative about $Y_t$ than $Z_t$ in Blackwell’s sense, i.e. if there exists a stochastic matrix $B$ such that $Q^z = Q^x B$.

Proposition 1 is well in line with results on the comparison of information systems in a standard agency model with verifiable information (Gjesdal 1982, Kim 1995). If more precise (verifiable) information is available, expected compensation cost can be reduced. Similar to these studies, however, Proposition 1 only provides a sufficient condition for one verifiable signal to dominate another in terms of expected compensation cost. For a reduction of the latter, only the posterior with respect to the “good news” outcome $y_H$ is crucial. The posterior of the “bad news” outcome $y_H$ is without any effect on compensation cost. It is therefore straightforward to ask whether a biased verification that buys a reduction in type 1 errors at the cost of increased type 2 errors may be beneficial. Kwon et al. (2001) and Kwon (2005) introduce such biased verification as an example of conservative accounting. They assume that $X_t$ is derived from a real-valued signal with two distinct overlapping distributions under $y_L$ and $y_H$, where $x_H$ is obtained when the signal exceeds a certain threshold level. Kwon (2005) shows that a higher threshold (a more conservative accounting in his notion) results in
a higher value of both $q_{HH}/q_{LH}$ and $q_{HL}/q_{LL}$, implying that the probability of a type 1 error (indicating a success via $X_t$ although the agent has failed in his task) is decreased, whereas the probability of a type 2 error (indicating a failure although the agent has succeeded in his task) is increased. As can be seen from (6), such a bias decreases compensation cost in the present model:

**Proposition 2** Let there be two alternative verifiable signals $X_t \in \{x_L, x_H\}$ and $Z_t \in \{z_L, z_H\}$ derived from $Y_t$ via verification technologies $Q^x$ and $Q^z$. If Assumption 1 is met, compensation cost in a short-term contract is lower under $X_t$ than under $Z_t$ if $X_t$ is more biased towards $y_L$ in the sense that $q_{xHH}/q_{xLH} > q_{zHH}/q_{zLH}$ and $q_{xLL}/q_{xHL} < q_{zLL}/q_{zHL}$.

The most important point in Proposition 2 is that an increased type 2 error does not hinder a reduction of expected compensation cost as long as the type 1 error is reduced. Formally, the reason is that in the expected compensation cost (6), only the posterior of the favorable verifiable outcome $y_H$ matters. This result is similar to those in Kwon et al. (2001) and Kwon (2005), who both analyze an agency model where only the verifiable measure is used for compensation purposes. The reason for the result, however, is more diverse in my model. In Kwon et al. (2001) and Kwon (2005), only the type 1 error matters because due to the binding liability constraint, no further punishment under the unfavorable outcome $x_L$ is possible, and therefore the informational content of $x_L$, measured in terms of the posterior of $y_H$ compared to its prior, does not affect compensation cost. In my setting, a variation of payments under $x_L$ would still be possible by distinguishing $s_t(y_L, x_L)$ and $s_t(y_H, x_L)$. This, however, would come at the cost of third-party payments. As Lemma 2 shows, such variation is only applied under the favorable outcome $x_H$. Therefore, biased information in the present model reduces compensation cost not only by allowing for more effective incentives via $s_t(y_L, x_H)$ and $s_t(y_H, x_H)$, but also by reducing the probability of third-party payments under $(y_L, x_H)$.

Proposition 2 provokes the question of whether the verification bias should be extended to infinity if the principal could influence which pieces of information are verified. In the
setting of Kwon (2005), where accounting conservatism is determined by a threshold for a raw signal with an unbounded support (the real line), the benefits from “more conservative” accounting (Kwon 2005, Proposition 2) indeed imply that the principal would like to shift the threshold ad infinitum, and that his most preferred accounting system does not exist. In contrast, if the threshold is set for a signal with two distinct distributions on overlapping, but bounded supports, like it is assumed in Kwon et al. (2001), the principal would like to shift the distribution only up to the point where “bad news” $y_H$ can be precluded. In my model, this would result in a verification technology with $q_{LH} = 0$, which would drive the type 1 error to zero as long as $q_{HH} > 0$. Then, the agent truly has succeeded in his task if $x_H$ is observed, and the principal does not have to make any third-party payments. Expected compensation cost is as if the principal’s subjective information $Y_t$ were verifiable, no matter the informational content of $x_L$.

Such extremely biased verification technologies, where the probability of a type 1 error is zero, whereas a type 2 error is strictly larger than zero, have been considered in the accounting literature as a simplified model of conservatism (Bagnoli and Watts 2005). In section 4, I will also use this type of biased verification for the comparison of long-term contracts. For brevity, I will denote this verification technology as “perfectly biased verification”. Corollary 1 summarizes my above considerations for this verification technology.

**Corollary 1** If Assumption 1 is met, the expected compensation cost in a short-term contract are identical under perfect verification and perfectly biased verification.

To summarize, my analysis of short-term contracts adds to the literature on the usefulness of conservative accounting for incentive contracting by offering third-party payments as an additional rationale in favour of biased verification. More generally, it substantiates previous findings that under limited liability, biased verification may be beneficial.
4 Verification in Long-Term Contracts

Over the two-period planning horizon, the principal in general can make use of more complex contractual agreements in which the agent’s second-period compensation may not only depend on his current performance, but also on the outcomes of the first period. Due to my assumption that $Y_1$ and $Y_2$ are stochastically independent, such recourse does not add any information on the agent’s action. However, it allows to pool incentives of the two periods, and by this means to potentially reduce both the agent’s rent and the payments to the third party.

Since an analysis of the long-term contract under a general verification device $Q$ would become very subtle, I focus on three special cases to study the impact of the precision and the bias of the verification technology: (i) the case where $Q$ allows to perfectly infer $Y_t$ from $X_t$ (perfect verification, subsection 4.1), (ii) the case where $X_t$ contains no information on $Y_t$ (no verification, section 4.2), and (iii) the case where $Q$ is biased towards $x_L$ such that $x_H$ unequivocally implies $y_H$, but the same does not hold for $x_L$ and $y_L$ (perfectly biased verification, section 4.3).

4.1 Perfect Verification

The principal’s subjective information $Y_t$ can perfectly be inferred from the observable signal $X_t$ if $q_{LL} = q_{HH} = 1$. Then, $Y_t$ provides no additional information and can be neglected in the contract. Thus, $s_1(Y_1, X_1) = w_1(X_1)$ and $s_2(Y_1, X_1, Y_1, X_2) = w_2(X_1, X_2)$ for all $(Y_1, Y_2)$ in the optimal long-term contract and no third-party payments are due. The probabilities $\Pr\{x_H \mid a^k\} = \Pr\{y_H \mid a^k\} = p^k$, $k = L, H$ of a high outcome are identical to the success probabilities. The problem becomes a repeated agency model with common knowledge about technology and preferences, in which short-term contracts are sufficient to replicate the optimal long-term contract (Fudenberg et al 1990). The structure of the optimal contract is therefore as follows:

Lemma 3 If $q_{LL} = q_{HH} = 1$ and assumption 1 is met, the optimal contract has the following
1. Compensations and bonus pools in both $t = 1, 2$ can be fixed as

$$s_t(y_L, x_L) = w_t(x_L) = s_{\text{min}} \text{ and } s_t(y_H, x_H) = w_t(x_H) = s_{\text{min}} + \frac{c^H}{p^H - p^L}.$$ 

2. Expected total compensation cost is

$$E[w_1 + w_2] = 2 \left( s_{\text{min}} + \frac{p^H c^H}{p^H - p^L} \right). \quad (7)$$

Lemma 3 states that the optimal long-term contract can be composed of two subsequent short-term contracts. The opportunity of second-period contracts with memory does not provide any improvements because in order to provide second-period incentives, both under $X_1 = x_L$ and $X_1 = x_H$ the same wage spread $w_2(X_1, x_H) - w_2(X_1, x_L)$ has to be offered. Since $X_1$ is publicly observable, the principal cannot save rents via “pooling” or “reusing” incentives by, for example, rewarding the agent only when he has succeeded in both periods. If the principal were able to block communication such that both $X_1$ and $X_2$ would become observable only after the second period, he could do better by choosing compensations $w_2(x_H, x_H) = s_{\text{min}} + c_H / ((p^H)^2 - (p_L)^2)$ and $s_{\text{min}}$ otherwise. With $X_1$ observable after the first period, however, if the agent has failed, under a pooled incentive scheme he has no more incentive to choose $a^H$ in the second period because he no longer can receive the reward.

Since in Section 3 I have already found that with short-term contracts, the principal can achieve the same result as under perfect verification also under perfectly biased verification, I have a first result concerning the comparison of different verification technologies in long-term contracts:

**Corollary 2** With long-term contracts, expected compensation cost with perfectly biased verification is not higher than under perfect verification.

As under short-term contracts, Corollary 2 provides only weak arguments for biased accounting. In section 4.3, I analyze whether there are conditions under which the principal
strictly prefers biased verification over perfect verification.

4.2 No Verification

If the verification technology is absolutely uninformative ($q_{LL} = q_{HH}$), incentives have to be provided exclusively based on unverifiable information. Then, the situation is as in Fuchs (2007), who considers a multi-period contracting model with purely subjective information on the agent’s action (and some publicly observable information which is unrelated to the agent’s action). Fuchs considers both finite and infinite planning horizons and unlimited liability of the agent. Nevertheless, his arguments on communication apply to my model as well. Communication is important here because in general, based on his private information the principal could send messages to the agent and a third party. Since these messages are observable, the agent’s compensation could be based on them. However, in the Perfect Bayesian Equilibrium of the game, the principal will not send any message until the terminal date of the planning horizon. The reason is that since the principal’s messages are cheap talk, they are merely a matter of principal’s commitment. But as the principal only sends messages and pays wages, there is nothing to commit except compensation. With regard to compensation payments, however, any message would only add incentives constraints to the principal’s optimization problem. Therefore, the optimal long-term contract can be written in the form that the principal withholds all subjective information until the terminal date, when performance-related payments are done (Fuchs 2007, Lemma 1).

I can apply this result and focus on contracts where in the first period, only the minimum wage $s_{min}$ is paid, regardless of the agent’s success, and all performance-related payments are done at the end of the second period. Moreover, I may drop the verifiable signal $X_t$ for ease of notation because it only adds noise and provides no information on $a_t$. Thus, incentives are provided from a bonus pool $w_2$ by second-period compensation payments $s_2(Y_1, Y_2)$ which are based on the principal’s observation of both periods’ unverifiable information. Since the agent in this case receives no feedback on his success before he chooses $a_2$, his dynamic action
choice problem essentially becomes a static multi-task problem.

For a situation without liability limits, Fuchs (2007, proposition 3) describes the optimal contract for \( T \) periods as one where a bonus is always paid, except the case where the agent failed in all periods. This result is a straightforward extension of MacLeod’s (2003) wage compression result for the one-period model: to minimize third-party payments, the principal extends the scope of bonus payments to the maximum and punishes only the worst performance. If the agent does not get any feedback, the same rationale applies to the multi-period model, and the punishment essentially can be reused for the provision of incentives.

Under limited liability, in contrast, not only third-party payments, but also the agent’s rent from limited liability matter. In a one-period model, this leads to the optimality of a bonus contract that rewards all outcomes that convey good news about the agent’s action in the sense that they are more likely under the desired action \( a^H \) than under the alternative action \( a^L \), and pays the minimum wage in all other cases (Budde and Hofmann 2012). In the two-period model, this good news/bad news classification is less clear because the agent has to prefer the desired action \((a^H, a^H)\) to all alternatives \((a^L, a^H), (a^H, a^L)\) and \((a^L, a^L)\). For medium performances \((y_L, y_H)\) of \((y_H, y_L)\), it is not obvious whether they are good news or bad news, and the structure of the optimal contract will depend on the success probabilities:

**Lemma 4** If \( q_{LL} = q_{HL} \) and Assumption 1 is met, the optimal long-term contract can be written in the following form:

1. **First-period compensations and bonus pools** are \( s_1(Y_1) = w_1(Y_1) = s_{min} \) for all \( Y_1 \)

2. **Second-period compensations** \( s_2(Y_1, Y_2) \) and **bonus pools** \( w_2 \) are as follows:

   (a) If \( p^H \leq 0.5 \), they are
      \[
      s_2(y_L, y_L) = s_{min},
      s_2(y_L, y_H) = s_2(y_H, y_L) = s_2(y_H, y_H) = w_2 = s_{min} + \frac{c^H}{(p^H - p^L)(1 - p^H)}.
      \]

   (b) If \( 0.5 < p^H \leq 1 - p^L \), they are
\( s_2(y_L, y_L) = s_{\min}, \quad s_2(y_L, y_H) = s_2(y_H, y_L) = s_{\min} + \frac{c^H}{p^H - p^L} \),
\( s_2(y_H, y_H) = w_2 = s_{\min} + \frac{2c^H}{p^H - p^L} \).

(c) If \( p^H > 1 - p^L \), they are
\( s_2(y_L, y_L) = s_2(y_L, y_H) = s_2(y_H, y_L) = s_{\min}, \)
\( s_2(y_H, y_H) = w_2 = s_{\min} + \frac{2c^H}{(p^H)^2 - (p^L)^2} \).

3. Expected total compensation cost is

\[
E[w_1 + w_2] = \begin{cases} 
2s_{\min} + \frac{c^H}{(p^H - p^L)(1 - p^H)} & \text{if } p^H < 0.5 \\
2s_{\min} + \frac{2c^H}{p^H - p^L} & \text{if } 0.5 \leq p^H \leq 1 - p^L \\
2s_{\min} + \frac{2c^H}{(p^H)^2 - (p^L)^2} & \text{if } p^H > 1 - p^L 
\end{cases}
\] (8)

The structure of the optimal contract is merely a matter of the tradeoff between liability rents and third-party payments. “Pooling” or “reusing” incentives by rewarding only a success in both periods enables the principal to cut the agent’s rents from his limited liability. On the other hand, it increases the probability of third-party payments. Whether, and if, to which extent the principal will reuse incentives therefore crucially depends on the success probability \( p^H \) which determines the probability of third-party payments, and the informational content of \( Y_t \), which mainly determines the agent’s rent from limited liability.

If the success probability \( p^H \) under the desired action \( a^H \) is rather low \( (p^H \leq 0.5) \), it is too expensive to pool incentives and pay a bonus only if the agent has succeeded in both periods because under such a contract third-party payments would occur with a rather high probability of at least \( 1 - 0.5^2 = 0.75 \). The optimal contract therefore has a structure similar to that derived by Fuchs (2007), where a bonus is refused only under the worst performance that the agent failed in both periods.

If, as the other extreme, \( p^H \) is higher than 0.5 and \( y^H \) is more informative about \( a^H \) than \( y_L \) is about \( a^L \) (this is the case if \( p^H > 1 - p^L \)), the benefits from saving rents by pooled incentives clearly outweigh the cost of higher expected third-party payments, and a bonus is
paid only if the agent succeeds in both periods. The structure of this contract is identical to the one that is offered if communication could be blocked for a verifiable signal (see the remark in subsection 4.1).

And last, in the intermediate situation that third-party payments are not very likely ($p_H > 0.5$) and $y_L$ is more informative about $a_L$ than $y_H$ is about $a_H$ (this is the case if $p_H \leq 1 - p_L$), the principal will pool incentives less aggressively and also reward the agent if he has succeeded once, but with only half of the bonus that is paid for two successes.

A comparison of expected compensation cost in situations with perfect verification and no verification gives a somewhat surprising result:

**Proposition 3** If Assumption 1 is met, expected compensation cost is lower (higher) under no verification than under perfect verification if $p_L \geq (<) \frac{1}{p_H} - p_H$.

As long as $y_L$ is more informative about $a_L$ than $y_H$ is about $a_H$ (this is the case if $p_H \leq 1 - p_L$), the structure of the optimal contract without verification is quite similar to that under perfect information because a bonus is paid for all but the worst performance. The usefulness of verification then directly follows from the fact that under perfect verification, no third-party payments are due. In contrast, if $y_H$ is more informative about $a_H$ than $y_L$ is about $a_L$ (this is the case if $p_H > 1 - p_L$), the optimal contract without verification pools rewards by paying a bonus only for the very best outcome. Since this cannot be done under perfect verification, the usefulness of verification depends on the tradeoff between saving rents with unverifiable information and saving third-party payments with verifiable information. It turns out that perfect verification is useful if $p_H$ is below a certain threshold, i.e., if third-party payments are sufficiently likely with unverifiable information.

Under a focus on verifiable information, proposition 3 may be seen as another contradiction to Holmström’s (1979) informativeness result: If $p_H$ is high enough, the principal prefers to have no verifiable information. By a comparison to proposition 1, the result is also in contrast to the short-term contract, where the principal prefers to have more verifiable information.
although third-party payments allow for the provision of incentives even if a piece of information is not verified. The informational advantage of private information therefore solely arises from the principal’s opportunity to reuse rewards, which is possible because the agent receives no early feedback on his first-period success.

4.3 Perfectly Biased Verification

The fact that depending on the success probabilities, the principal may prefer either perfect verification or no verification provokes the question whether there are situations where he prefers selective verification of his information. Biased verification may help to better trade-off the benefits from saving rents by reusing rewards and saving third-party payments by the verification of the principal’s private information. An inspection of Lemma 2 suggests how a biased verification technology enables the principle to cherry-pick the advantages of both regimes: in the short-term contract, third-party payments are only due if the verifiable measure $X_t$ has taken its favorable realization $x_H$ although the agent has failed in his task. Therefore, third-party payments can be precluded if $q_{LH} = 0$, i.e. if the verification technology is perfectly biased (Corollary 1). I have already stated in Corollary 2 that this enables the principal to reproduce the solution under perfect verification, at least in terms of expected compensation cost. The question therefore is merely whether the remaining noise of the biased verification technology, given by $q_{HH}$ and $q_{LH} = 1 - q_{HH}$, may then serve as an instrument to reuse rewards in the long-term contract, and to strictly improve the short-term solution. To exemplify the benefits from biased verification, I therefore focus on the case of perfectly biased verification as defined in section 3 ($q_{LH} = 0$ and $q_{HH} \in ]0, 1[$). In this case, biased verification is preferred if it is not too precise:

**Proposition 4** Suppose assumption 1 is met. Then, expected compensation cost under biased verification is strictly less than under perfect verification if $q_{HH} < (2p^H - 1)/(p^H)^2$. 

20
Perfectly biased verification is not strictly preferred to perfect verification if the success probability is low \( (p^H \leq 0.5) \). The reason again is that third-party payments are too likely under this condition because as Lemma 4 already showed, reusing rewards requires the introduction of additional third-party payments. If \( p^H > 0.5 \), however, the short-term contract can be improved if \( q_{HH} \) is below a certain threshold. A minimum uncertainty about the agent’s first-period success is necessary to maintain second-period incentives also by a reused reward.

Proposition 4 is a strong result in favour of biased verification because it states that a certain amount of imprecision in detecting \( y^H \) is required for a biased verification to be strictly preferred to perfect verification. This is different to the results of Proposition 1 and 2 for the short-term contract, stating that both more informative and more biased verification help to reduce expected compensation cost. The uncertainty is needed to maintain the opportunity of a high reward offered for a first-period success \( (y^H) \) also for the agent whose verified first-period performance is bad \( (x_L) \). Thus, with long-term contracts, the principal prefers a verification bias which is bought at the cost of strictly less informational content. In a model like that of Kwon et al. (2001), the principal would therefore like to shift the threshold even beyond the point where bad news are uniquely detected.

The last point provokes the question of how much uncertainty about \( Y_t \) under the unfavourable outcome \( x_L \) the principal prefers. A complete answer to this question is not straightforward. But from the above results, at least some general predictions are possible. If \( p > 0.5 \) and \( p^L \leq 1/p^H - p^H \), the principal strictly prefers perfectly biased verification to perfect verification and perfect verification to no verification. Therefore, expected compensation cost in this case is not monotonic in \( q_{HH} \), and there exist(s) certain level(s) of \( q_{HH} \in]0, 1[ \) for which compensation cost is minimized. For \( p^L > 1/p^H - p^H \), from the above results the principal may even prefer to have no information. However, simulations suggest that there always exists a perfectly biased verification technology that the principal prefers to no verification.
5 Discussion

Up to this point, I have treated the verification technology as a black box, completely determined by the matrix $Q$ which stochastically maps the principal’s private information $Y_t$ onto the publicly observable signal $X_t$. In corporate practice, verification will be processed by a number of different instruments.

Probably the most important of these instruments is the accounting system that by financial statements and other reports, such as mandatory filings, inevitably delivers information that can be used in a formal contract. The informational content of these data, however, at least partly depends on the rules of accounting, and how the firm exercises these rules. In this respect, accounting conservatism has long intrigued researchers, and has become an increasingly popular theme over the last decade. Broadly speaking, conservatism is defined as the asymmetric recognition of profits and losses, in its purest form requiring to anticipate all losses, but to anticipate no profits (Watts 2003). Conservatism has long been an integral part of accounting rules, as it is still in the German Commercial Code (§252 HGB, only in part superseded by the accounting law modernization act (BilMoG) in 2009 via the fair valuation rule of §253 HGB). Recently, however, both US and international standard setters have emphasized the fair value principle of accounting and eliminated conservatism as a general accounting principle from their conceptual frameworks in 2010 (IASB 2010).

Since verification in my model consists of more than accounting measurement, I will not derive empirical implications of my results here. Rather, I will discuss implications of conservatism in my model. To that purpose, one may consider the principal’s information $Y_t$ as a random draw from a set of data, part of which is admissible for recognition in the financial statements, and part of which is not. This representation is in line with the analysis of Gao (2012, 2012b), who models an accounting rule as consisting of three parts: the set of admissible transaction characteristics, the verification requirement, and the evidence threshold. Gao (2012) studies consequences of conservatism in the verification requirement, and Gao (2012b) focuses on the evidence threshold. In the present model, the verification requirement
and the evidence threshold may be captured by a technology $\hat{Q}$ as introduced in section 2. The degree of conservatism may then be constituted by the number of admissible transaction characteristics, as compared to those which are not admissible. A formalization of this idea could be provided by a slight modification of the characterization offered in Ewert and Wagenhofer (2011), who assume that with a certain probability ($1 - \pi$, say), no accounting signal is produced. Conservatism then can be characterized by the fact whether the realization of “no accounting signal” is pooled with $x_L$ or $x_H$. To introduce a notion of increased conservatism into this model, one could characterize conservatism by the set of admissible transaction characteristics, assuming that the probability that no accounting signal is produced is different for $y_L$ and $y_H$. The probabilities $\pi_L$ and $\pi_H$ of admissible data under $y_L$ and $y_H$ would then characterize the accounting requirements for admissible transaction characteristics. As a consequence, the verification technology would consists of $\pi = (\pi_L, \pi_H)$ and $\hat{Q}$. The black box characterization $Q$ would then reflect the degree of conservatism caused by the set of admissible data, no matter whether the outcome “no accounting signal” is pooled with the accounting signal $x_H$ or the signal $x_L$. It is easily proved that both an increase in $\pi_L$ and a decrease in $\pi_H$ lead to a stronger bias towards $x_L$, as defined in section 3. In my model, the principal would therefore in many cases benefit from a restriction of the set of admissible data conveying good news, and an expansion of the set of admissible data conveying bad news.

But accounting is not the only verification device the principal may employ. As it was stated in the introduction, compensation committees frequently mandate consultants to provide additional information that is relevant for compensation purposes. Since in contrast to accounting there are no legal rules determining which pieces of information the consultant shall prepare, the principal in this respect has considerable discretion, and may cherry-pick which information to verify. The consequences of such selective verification depend on the time at which the principal decides on the verification. The model allows for predictions only in case that the principal can commit to his verification activities before he observes his private
If such commitment is possible, my results of Proposition 2 and Proposition 4 suggest that the principal will obviously be interested in the verification of bad news. His propensity to verify good news, in contrast, is less clear. Proposition 4 states that a certain minimum amount of uncertainty is needed, implying that the principal will prefer not all good news to be verified. Whether he will engage in further activities to verify his information will therefore depend on how likely the verification by the accounting system is.

6 Conclusion

In this paper I have analyzed the role of verification for the joint use of verifiable and unverifiable information. My analysis shows that for short-term arrangements, previous findings for contracts exclusively based on verifiable information are reinforced. To reduce the agent’s rent from limited liability and third-party payments, the principal prefers both more informative and more conservative verification. In a long-term contract for two periods, in contrast, the principal may prefer a less informative verification. He particularly benefits from verification technologies that are biased towards bad news because they preserve bonus opportunities and by this means enable the principal to reuse rewards in the long-term contract.

A verification bias may result from at least two reasons. First, the accounting system may be conservative in the sense that the requirements for transaction characteristics to be admissible for financial reporting are more demanding for good news than they are for bad news. Beside this exogenous reason, a bias may also arise endogenously from the principal’s propensity to verify only bad news.

\footnote{Otherwise, after the first period the principal would, in order to reduce his compensation cost, engage in verification only for that outcome for which the smaller bonus pool was stipulated. Thus, different to my assumption, \( Q \) would be endogenous.}
Appendix - Proofs

Proof of Lemma 1: Obvious from the incentive constraint (3).

Proof of Lemma 2:
The lagrangian to problem (1)-(5) is

\[ L = -(1 - p^H) \sum_{j=L,H} q_{Lj}^H w_t(x_j) - p^H \sum_{j=L,H} q_{Lj}^H w_t(x_j) \]

\[ + \lambda \left( (1 - p^H) \sum_{j=L,H} q_{Lj} s_t(y_L, x_j) + p^H \sum_{j=L,H} q_{Hj} s_t(y_H, x_j) - c^H - u^R_t \right) \]

\[ + \mu \left( (p^H - p^L) \left( \sum_{j=L,H} q_{Hj} s_t(y_H, x_j) - \sum_{j=L,H} q_{Lj} s_t(y_L, x_j) \right) - c^H \right) \]

\[ - \sum_{i=L,H} \sum_{j=L,H} \nu_{ij} \left[ s_t(y_i, x_j) - w_t(x_j) \right] \]

\[ + \sum_{i=L,H} \sum_{j=L,H} \eta_{ij} \left[ s_t(y_i, x_j) - s^{\min} \right]. \] (9)

From (9), the first-order conditions

\[ \frac{\partial L}{\partial s_t(y_L, x_j)} = \lambda (1 - p^H) q_{Lj} - \mu (p^H - p^L) q_{Lj} - \nu_{Lj} + \eta_{Lj} = 0 \] (10)

and

\[ \frac{\partial L}{\partial s_t(y_H, x_j)} = \lambda p^H q_{Hj} + \mu (p^H - p^L) q_{Hj} - \nu_{Lj} + \eta_{Lj} = 0 \] (11)

can be derived. Since the agent earns a rent from limited liability, the participation constraint (2) will be slack. Thus \( \lambda = 0 \) and the first term in both (10) and (11) vanishes. Moreover, \( \mu > 0 \) holds because otherwise the principal would do best offering a flat wage, which cannot implement \( a^H \). Since \( p^H > p^L \) by assumption, (10) therefore can only be met if \( \eta_{Lj} > 0 \), and (11) can only be fulfilled if \( \nu_{Hj} > 0 \). Thus, \( s_t(y_L, x_L) = s_t(y_L, x_H) = s^{\min} \), \( s_t(y_H, x_L) = w_t(x_L) \) and \( s_t(y_H, x_H) = w_t(x_H) \).
Thus, problem (1)-(5) can be restated as

\[
\min_{w_t} \mathbb{E}[w(X_t) \mid a^H] = (1 - p^H) \sum_{j=L,H} q_{Lj}^H w_t(x_j) + p^H \sum_{j=L,H} q_{Lj}^H w_t(x_j)
\]

s.t. \[
p^H( q_{HL}w_t(x_L) + q_{HH}w_t(x_H)) - c^H \geq p^L (q_{HL}w_t(x_L) + q_{HH}w_t(x_H))
\]

\[
w_L, w_H \geq s_{min}
\]

Here, \(s_t(y_i, x_j)\) have been substituted from the above result. Furthermore, I have dropped the participation constraint (because it is not binding) and the budget constraint (because the \(s_t(y_i, x_j)\) have been substituted). From the Lagrangian to this problem, the first-order conditions

\[
\frac{\partial L}{\partial w_t(x_L)} = -(1 - p^H)q_{LL} - p^H q_{HL} + \mu (p^H - p^L) q_{HL} + \eta_L = 0
\]

and

\[
\frac{\partial L}{\partial w_t(x_H)} = -(1 - p^H)q_{LH} - p^H q_{HH} + \mu (p^H - p^L) q_{HH} + \eta_H = 0
\]

can be derived. Now suppose that both \(w_t(x_L)\) and \(w_t(x_H)\) are larger than \(s_{min}\). Then, \(\eta_L = \eta_H = 0\) and (15) and (16) can be stated as

\[
\mu = \frac{(1 - p^H)q_{LL} + p^H q_{HL}}{(p^H - p^L)q_{HL}} \quad \text{and} \quad \mu = \frac{(1 - p^H)q_{LH} + p^H q_{HH}}{(p^H - p^L)q_{HH}}
\]

or

\[
\mu = \frac{(1 - p^H)q_{LL} + p^H q_{HL}}{(p^H - p^L)q_{HL}} \quad \text{and} \quad \mu = \frac{(1 - p^H)q_{LH} + p^H q_{HH}}{(p^H - p^L)q_{HH}}
\]

Expansion by \((p^H - p^L)/p^H\) yields

\[
\frac{(1 - p^H)q_{LL} + p^H q_{HL}}{p^H q_{HL}} = \frac{(1 - p^H)q_{LH} + p^H q_{HH}}{p^H q_{HH}}
\]

i.e., the principal will set both \(w_t(x_L)\) and \(w_t(x_H)\) larger than \(s_{min}\) only if the posterior prob-
ability of $y^H$ is equal under both $x_L$ and $x_H$, which cannot be the case for $q_{HH} > q_{LH}$ and $p^H \in ]0, 1[$. Therefore, the liability constraint will only be slack for the value of $X_t$ with the higher posterior, which is $x_H$, and $w_t(x_L) = s_{min} = 0$. $w_t(x_H)$ then can be computed from the (binding) incentive constraint.

**Proof of Proposition 1:** If $Q^x$ is more informative than $Q^z$ in Blackwell’s sense, then $q_{LL}^x/q_{HL}^x \geq q_{LL}^z/q_{HL}^z$ and $q_{HH}^x/q_{LH}^x \geq q_{HH}^z/q_{LH}^z$ (Kwon 2005, p. 1629). From the latter relation, it follows that the expected compensation cost in (6) is weakly smaller under $Q^x$.

**Proof of Proposition 2:** If $Q^x$ is more biased in the sense that $q_{LL}^x/q_{HL}^x < q_{LL}^z/q_{HL}^z$ and $q_{HH}^x/q_{LH}^x > q_{HH}^z/q_{LH}^z$, it directly follows from the latter relation that the expected compensation cost in (6) is strictly smaller under $Q^x$.

**Proof of Lemma 3:** Since all requirements formulated in Fudenberg et al. (1990) are fulfilled, the optimal contract can be composed of two short-term contracts. The optimal short-term contract was derived in lemma 2. Substitution of $q_{LL} = q_{HH} = 1$ and $q_{LH} = q_{HL} = 0$ yields compensations, bonus pools and expected compensation cost.

**Proof of Lemma 4:** From Lemma 1 in Fuchs (2007) it follows that all bonus payments can be made in the second period. This proves part 1 of the lemma. To prove part 2, I restrict myself to the analysis of symmetric payment schemes in which $s_2(y_L, y_H) = s_2(y_H, y_L)$. With respect to compensation cost, this is without loss of generality because without interim information of the agent both periods are identical. For ease of notation, denote $s_2(y_L, y_H) = s_L$, $s_2(y_L, y_H) = s_2(y_H, y_H) = s_M$ and $s_2(x_H, x_H) = s_H$. Then, the principal’s optimization
problem of choosing second-period compensations and bonus pool can be written as:

\[
\begin{align*}
\min_{\{s_L, w_2\}} & \quad w_2 \\
\text{s.t.} & \quad s^\text{min} + (1 - p^H)^2 s_L + 2(1 - p^H)p^H s_M + (p^H)^2 s_H - 2c^H \geq U^R_1 \\
& \quad (1 - p^H)^2 s_L + 2(1 - p^H)p^H s_M + (p^H)^2 s_M - 2c^H \geq (19) \\
& \quad (1 - p^H)(1 - p^L)s_L + ((1 - p^H)p^L + p^H(1 - p^L))s_M + p^H p^L s_H - c^H \geq (20) \\
& \quad (1 - p^L)^2 s_L + 2(1 - p^L)p^L s_M + (p^L)^2 s_H \\
& \quad s_L, s_M, s_H \leq w_2 \\
& \quad s_L, s_M, s_H \geq s^\text{min}
\end{align*}
\]

Only the first-period participation constraint (18) has to be considered because the agent receives no interim information. (19)-(20) are the incentive constraints ensuring that the agent will prefer the desired action \((a^H, a^H)\) over the alternative actions \((a^H, a^L), (a^L, a^H)\) and \((a^L, a^L)\). At least one of these incentive constraint will be binding because otherwise the principal would do best paying a flat wage, which cannot implement \(a^H\). (21) and (22) are the budget constraints and the liability constraints. Let \(\lambda, \mu_1, \mu_2, \nu_i\) and \(\eta_i\) for \(i = L, M, H\) denote the multipliers of the constraints. If the agent earns a rent from limited liability, the participation constraint (18) will be slack and \(\lambda = 0\). Then, the first-order condition with respect to \(s_L\) is

\[
\frac{\partial L}{\partial s_L} = \mu_1(p^L - p^H)(1 - p^H) + \mu_2 ((1 - p^H)^2 - (1 - p^L)^2) - \nu_L + \eta_L = 0
\]

The multipliers of both \(\mu_1\) and \(\mu_2\) are strictly negative. Since at least one incentive constraint is binding, \(\mu_1 = \mu_2 = 0\) is impossible and \(\eta_L > 0\) must hold to fulfil the condition. Thus, \(s_L = s^\text{min}\).
To determine \( s_M \) and \( s_H \), consider the first-order conditions

\[
\frac{\partial L}{\partial s_M} = \mu_1 \left( 2(1 - p^H)p^H - (1 - p^H)p^L - (1 - p^L)p^H \right) + \mu_2 \left( 2(1 - p^H)p^H - 2(1 - p^L)p^L \right) - \nu_M + \eta_M = 0 \tag{23}
\]

and

\[
\frac{\partial L}{\partial s_H} = \mu_1 (p^L - p^H)p^H + \mu_2 \left( (p^H)^2 - (p^L)^2 \right) - \nu_L + \eta_L = 0. \tag{24}
\]

First consider the case that \( p < 0.5 \) and suppose that only the one-period incentive constraint (19) is binding. Then, \( \mu_2 = 0 \), and (23) can only be fulfilled for \( \nu_M > 0 \) because the multiplier of \( \mu_1 \) is strictly larger than 0 for \( p < 0.5 \). Similarly, \( \nu_H > 0 \) must hold in order to fulfil (24). Thus, \( s_M = s_H = w_2 \) in this case. \( w_2 = s_{\text{min}} + c_H / ((p^H - p^L)(1 - p^H)) \) is derived from the one-period incentive constraint (19). Substitution of \( s_M \) and \( s_H \) in (20) proves that the two-period incentive constraint is indeed slack.

Next consider the case \( p^H > 1 - p^L \), which implies \( p^H > 0.5 \) since \( p^H > p^L \). Suppose only the two-period incentive constraint (20) is binding. Then, \( \mu_1 = 0 \) and (23) can only be fulfilled if \( \eta_M > 0 \) because the multiplier of \( \mu_2 \) is strictly negative for \( p^H > 1 - p^L \). Conversely, (24) can only be fulfilled if \( \eta_H > 0 \) because here the multiplier of \( \mu_2 \) is strictly positive. Thus, \( s_M = s_{\text{min}} \) and \( s_H = w_2 = s_{\text{min}} + 2c_H / ((p^H)^2 - (p^L)^2) \), the value of which is derived from the binding incentive constraint (20). Substitution of \( s_M \) and \( s_H \) in (19) proves that the one-period incentive constraint is indeed slack.

For the remaining case \( 0.5 \leq p^H \leq 1 - p^L \), both (19) and (20) are binding. Solving these two equalities for \( s_M \) and \( s_H \) yields \( s_M = c_H / (p^H - p^L) \) and \( s_H = 2c_H / (p^H - p^L) \). \( w_2 = s_H \) follows from \( s_H = \max\{s_L, s_M, s_H\} \). This completes the derivation of optimal compensations and bonus pools. Total compensation cost can be computed as \( w_1 + w_2 \).

**Proof of Proposition 3:** Comparing the expected compensation cost of (7) and (8), I get
that

\[ 2 \left( s_{\text{min}} + \frac{p^H c^H}{p^H - p^L} \right) < 2s_{\text{min}} + \frac{c^H}{(p^H - p^L)(1 - p^H)} \]

and

\[ 2 \left( s_{\text{min}} + \frac{p^H c^H}{p^H - p^L} \right) < 2s_{\text{min}} + \frac{2c^H}{p^H - p^L} \]

for all \( p^L < p^H < 1 \), whereas

\[ 2 \left( s_{\text{min}} + \frac{p^H c^H}{p^H - p^L} \right) < (\geq)2s_{\text{min}} + \frac{2c^H}{(p^H)^2 - (p^L)^2} \]

holds for \( p^L < (\geq)1/p^H + p^H \).

**Proof of Proposition 4:**

With noisy verification, the principal’s problem is to fix compensations \( s_1(Y_1, X_1) \) and \( s_2(Y_1, X_1, Y_2, X_2) \) and bonus pools \( w_1(X_1) \) and \( w_2(X_1, X_2) \) as to solve the following cost minimization problem:
max \quad E[w_1 + w_2 \mid (a^H, a^H)] \\
\text{s.t} \quad E[s_1 + s_2 \mid (a^H, a^H)] - 2c^H \geq U_1^R \quad (25) \\
E[s_2 \mid (a^H, a^H)] - c^H \geq U_2^R \quad (26) \\
E[s_1 + s_2 \mid (a^H, a^H)] - 2c^H \geq E[s_1 + s_2 \mid (a^L, a^H)] - c^H \quad (27) \\
E[s_1 + s_2 \mid (a^H, a^H)] - 2c^H \geq E[s_1 + s_2 \mid (a^H, a^L)] - c^H \quad (28) \\
E[s_1 + s_2 \mid (a^H, a^H)] - 2c^H \geq E[s_1 + s_2 \mid (a^L, a^L)] \quad (29) \\
E[s_2 \mid (a^H, a^H), X_1 = x_L] - c^H \geq E[s_2 \mid (a^H, a^L), X_1 = x_L] \quad (30) \\
E[s_2 \mid (a^H, a^H), X_1 = x_H] - c^H \geq E[s_2 \mid (a^H, a^L), X_1 = x_H] \quad (31) \\
s_1(Y_1, X_1) \leq w_1(X_1), \quad \forall Y_1, X_1 \quad (32) \\
s_2(Y_1, X_1, Y_2, X_2) \leq w_2(X_1, X_2) \quad \forall Y_1, X_1, Y_2, X_2 \quad (33) \\
s_1(Y_1, X_1) \geq s_{\min}^1, \quad s_2(Y_1, X_1, Y_2, X_2) \geq s_{\min}^2 \quad \forall Y_1, X_1, Y_2, X_2 \quad (34)

The inequalities (26) and (27) are the agent’s participation constraints for the first and the second period. If Assumption 1 is met, they will both not be binding in the optimal contract. (28)-(30) are the incentive constraints for the first period, ensuring that the agent from an ex ante perspective prefers the desired action \((a^H, a^H)\) over the alternative plans \((a^L, a^H)\), \((a^H, a^L)\), and \((a^L, a^L)\). (31) and (32) are the incentive constraints for the second period, ensuring that the agent prefers \(a^H\) over \(a^L\) in the second period, regardless whether \(X_1 = x_L\) or \(X_1 = x_H\) has been observed. (33) and (34) are the budget and liability constraints.

Due to the complexity of this optimization problem, I will not try to characterize the structure of the optimal contract. Rather, I will use it to derive conditions under which it is possible to improve the optimal contract under perfect information without violation of the constraints (26)-(34).

From Lemma 3 the optimal two-period contract under perfect information can be repli-
cated by two subsequent short-term contracts. From Corollary 1, this contract has the same expected compensation cost as the optimal short-term contract under perfectly biased information, as described in Lemma 2. Due to my assumption of the agent’s utility function, this contract can be replicated by a contract where all bonus payments are postponed to the second period. This contract would fix \( s_1(Y_1, X_1) = s_{\text{min}} \) for all \((Y_1, X_1)\) for the first period, and for the second period, compensation would be

\[
s_2(Y_1, X_1, Y_2, X_2) = \begin{cases} 
  s_{\text{min}} + \frac{c}{q_{HH}(p^H - p^L)} & \text{if } (Y_t, X_t) = (y_H, x_H) \text{ for one } t = 1, 2 \\
  s_{\text{min}} + \frac{2c}{q_{HH}(p^H - p^L)} & \text{if } (Y_t, X_t) = (y_H, x_H) \text{ for both } t = 1, 2 \\
  s_{\text{min}} & \text{else}
\end{cases}
\]

Thus, the bonuses from the short-term contract of the first period are simply transferred to the second period, yielding a scheme where one bonus is paid if the agent has succeeded in delivering \((y_H, x_H)\) in one period, and twice the bonus is paid if he succeeded in both periods.

It is noteworthy that by this kind of deferred compensation no pooling effect has yet been realized. All incentive constraints (28)-(32) are binding under this contract. But a pooling effect can be realized if this contract is further compressed. This is possible because from an ex ante perspective, the agent bothers second-period incentives only in expected terms. Thus, instead of paying \( B = c/(q_{HH}(p^H - p^L)) \) as a bonus for first-period success in all cases where \((Y_1, X_1) = (y_H, x_H)\), the principal could likewise offer to pay \( B/(q_{HH}(p^H - p^L)) = c/(q_{HH}(p^H - p^L))^2 \) only if both \((Y_1, X_1) = (y_H, x_H)\) and \((Y_2, X_2) = (y_H, x_H)\). The second-period compensation then would be

\[
s_2(Y_1, X_1, Y_2, X_2) = \begin{cases} 
  s_{\text{min}} + \frac{c}{q_{HH}(p^H - p^L)} & \text{if } (Y_1, X_1) \neq (y_H, x_H) \text{ and } (Y_2, X_2) = (y_H, x_H) \\
  s_{\text{min}} + \frac{c(1 + q_{HH})^H}{(q_{HH}(p^H - p^L))^2} & \text{if } (Y_1, X_1) = (y_H, x_H) \text{ and } (Y_2, X_2) = (y_H, x_H) \\
  0 & \text{else}
\end{cases}
\]  

(35)
Compensation cost under this contract are the same as under the initial contract, but now the first-period incentive constraints (28) and (30) as well as the second-period incentive constraint (32) are slack. This can be used to improve the contract. To that purpose, I only consider variations of those compensations $s_2(y_1, X_1, y_H, x_H)$ for which a positive bonus is paid under the compressed payments scheme (35). For notational convenience, denote $s_2(y_i, x_j, y_H, x_H) = s_{ij}$ and variations of this as $\delta s_2(y_i, x_j, y_H, x_H) = \delta_{ij}$.

The remaining incentive constraints are (28) and (32). The first-period incentive constraint (28) can then be written as

$$s^{\text{min}} + p^H q_{HH} \left[ (1 - p^H)q_{LL}s_{LL} + q_{LH}s_{LH} \right] + p^H(q_{HL}s_{HL} + q_{HH}s_{HH}) - 2c^H \geq s^{\text{min}} + p^H q_{HH} \left[ (1 - p^L)(q_{LL}s_{LL} + q_{LH}s_{LH}) + p^L(q_{HL}s_{HL} + q_{HH}s_{HH}) \right] - c^H$$

and simplifies to

$$p^H q_{HH}(p^H - p^L) \left[ -(q_{LL}s_{LL} + q_{LH}s_{LH}) + (q_{HL}s_{HL} + q_{HH}s_{HH}) \right] \geq c^H. \quad (36)$$

The second-period incentive constraint (28) is the one for $X_1 = x_L$ and writes as

$$s^{\text{min}} + p^H q_{HH} \left[ \frac{(1 - p^H)q_{LL}s_{LL} + p^H q_{HL}s_{HL}}{(1 - p^H)q_{LL} + p^H q_{HL}} \right] \geq s^{\text{min}} + p^L q_{HH} \left[ \frac{(1 - p^H)q_{LL}s_{LL} + p^H q_{HL}s_{HL}}{(1 - p^H)q_{LL} + p^H q_{HL}} \right]. \quad (37)$$

The fractions in (37) are the conditional probabilities of $Y_1 = y_L$ and $Y_1 = y_H$, given $X_1 = x_L$. The constraint simplifies to

$$\frac{(p^H - p^L)q_{HH}}{(1 - p^H)q_{LL} + p^H q_{HL}} \left[ (1 - p^H)q_{LL}s_{LL} + p^H q_{HL}s_{HL} \right] \geq c^H \quad (38)$$
Since (36) and (38) are both binding, any variation $\delta$ has to fulfil

$$p^H q_{HH} (p^H - p^L) \left[ - (q_{LL} \delta_{LL} + q_{LH} \delta_{LH}) + (q_{HL} \delta_{HL} + q_{HH} \delta_{HH}) \right] \geq 0 \quad (39)$$

and

$$\frac{(p^H - p^L) q_{HH}}{(1 - p^H) q_{LL} + p^H q_{HL}} \left[ (1 - p^H) q_{LL} \delta_{LL} + p^H q_{HL} \delta_{HL} \right] \geq 0 \quad (40)$$

to keep the constraints fulfilled. The expected compensation under the compressed contract is

$$E[w_1 + w_2 \mid (a^H, a^H)] = 2s_{\text{min}}$$

$$+ p^H q_{HH} \left[ (1 - p^H) q_{LL} + p^H q_{HL} \right] s_{HL} \left[ (1 - p^H) q_{LL} + p^H q_{HH} \right] s_{HH}.$$ 

It is decreased if

$$p^H q_{HH} \left[ (1 - p^H) q_{LL} + p^H q_{HL} \right] \delta_{HL} + (1 - p^H) q_{LL} + p^H q_{HH} \delta_{HH} < 0, \quad (41)$$

provided that the relations $s_{LL} \leq s_{HL}$ and $s_{LH} \leq s_{HH}$ remain valid. Since in the contract (35) it holds that $s_{LH} < s_{HH}$, for small variation this only requires

$$\delta_{LL} \leq \delta_{HL}. \quad (42)$$

The contract (35) can be improved if there exists a variation that fulfills (39), (40), (41) and (42). Since $p^H > p^L$ and $p^H \in [0, 1]$ by assumption, the multiplier in front of the brackets in (39), (40), (41) can be dropped as long as $q_{HH} > 0$. Since $q_{LL} = 1$ and $q_{LH} = 0$ for perfectly biased verification, this is the case if $X_i$ is not completely uninformative. After substitution
of \( q_{LL} = 1 \) and \( q_{LH} = 0 \), the system of inequalities can therefore be written as

\[
\begin{align*}
-\delta_{LL} + q_{HL}\delta_{HL} + q_{HH}\delta_{HH} & \geq 0 \\
(1 - p^H)\delta_{LL} + p^H q_{HL}\delta_{HL} & \geq 0 \\
-\delta_{LL} + \delta_{HL} & \geq 0 \\
((1 - p^H) + p^H q_{HL})\delta_{HL} + p^H q_{HH}\delta_{HH} & < 0
\end{align*}
\]

By Farkas’ lemma, such a variation exists if the system

\[
\begin{align*}
-u_1 + (1 - p^H)u_2 - u_3 &= 0 \\
qu_{HL}u_1 + p^H q_{HL}u_2 + u_3 &= (1 - p^H) + p^H q_{HL} \\
qu_{HH}u_1 &= p^H q_{HH}
\end{align*}
\]

has no non-negative solution \((u_1, u_2, u_3)\). The solution to (43)-(45) is

\[
\begin{align*}
u_1 &= p^H, \quad u_2 = \frac{1}{1 - p^H q_{HH}}, \quad u_3 = \frac{1 - 2p^H + (p^H)^2 q_{HH}}{1 - p^H q_{HH}}
\end{align*}
\]

\(u_1\) and \(u_2\) are both nonnegative because \(p^H, q_{HH} \in ]0, 1]\], but \(u_3 \geq 0\) only if \(q_{HH} \geq (2p^H - 1)/(p^H)^2\). Therefore, if \(q_{HH} < (2p^H - 1)/(p^H)^2\), a strict improvement of the contract (35) is possible.
References


