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Statistical Analysis of Absolute Transaction Price Changes of Options

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Abstract

In market microstructure theory the effect of time between consecutive transactions and trade volume on transaction price changes of exchange traded shares and options has been considered (e.g. Diamond and Verecchia (1987) and Easley and O'Hara (1987)). The goal of this paper is to investigate if these theoretical considerations can be supported by a statistical analysis of data on transaction price changes of options on shares of the Bayer AG in 1993-94. For this appropriate regression models with non linear and interaction effects are developed to study the influence of trade volume, time between trades, intrinsic value of an option at trading time and price development of the underlying share on the absolute transation price change of an option. Since price changes are measured in ticks yield count data structure, we use in a first analysis ordinary Poisson generalized linear models (GLM) ignoring the time series structure of the data. In a second analysis these Poisson GLM's are extended to allow for an additional AR(1) latent process in the mean which accounts for the time series structure. Parameter estimation in this extended model is not straight forward and we use Markov Chain Monte Carlo (MCMC) methods. The extended Poisson GLM is compared to the ordinary Poisson GLM in a Bayesian setting using the deviance information criterion (DIC) developed by Spiegelhalter et al. (2002). With regard to market microstructure theory the results of the analysis support the expected effect of time between trades on absolute option price changes but not for trade volume in this data set.

Key words: Poisson-GLM, latent process, AR, MCMC, model selection, DIC, market microstructure, transaction price changes of options

1 Introduction

Financial transaction data, sometimes also called ultra-high-frequency data, have become a new focus of applied financial market theory as markets observed at this level generate data that describe the trading and asset pricing process in a much more detailed way than well known macroeconomic data such as daily stock returns, exchange rates, etc. (e.g. Dacorogna et al. (2001) and Bauwens and Giot (2001)). From an econometric point of view ultra-high frequency financial data have two main characteristics: the irregularity of time intervals between consecutive trades and the discreteness of price changes. Sophisticated econometric models for transaction price changes which also take into account the time series structure of the data have been developed and applied to real market data with a focus on stock and bond markets. While one focus of current research is the further development of autoregressive conditional duration (ACD) models based on the work by Engle (2000), other publications (e.g. Liesenfeld and Pohlmeier (2003)) are concerned with the adequate modelling of the asset pricing process at transaction level within a count data framework. The aim of this paper is to model the relationship between absolute transaction price changes of equity options and a series of explanatory micro-variables such as the already mentioned time between consecutive transactions, trade volume and option-specific variables such as price change of the corresponding underlying or the intrinsic value of the option at the time of the trade. This will lead to a better insight into option market mechanics and option market microstructure in general. In particular we are interested if a statistical analysis supports implications from market microstructure theory. More specifically does a large time between trades and higher trading volume imply on average a larger absolute price change of the option? As a representative example of an exchange traded equity option we consider in this paper options on shares of the Bayer AG from 1993 to 1994, traded at the *Deutsche Terminbörse (DTB)* at that time. Together with the SWX Swiss Exchange the Deutsche Börse AG now jointly operates the Eurex Exchange which has replaced the DTB and is now the world's largest exchange for financial derivatives.

We use a Poisson generalized linear regression model (GLM) as major stochastic tool for the statistical analysis of the available data. We also fit a Poisson-GLM with an AR(1) latent process in the mean, which introduces autocorrelation and therefore accounts for the time series structure of the data. This class of models has been discussed and applied to biometric and econometric data for example by Zeger (1988), Chan and Ledolter (1995) and Davis et al. (1999). For parameter estimation we use Markov Chan Monte Carlo (MCMC) methods and we finally compare and assess the adequacy of the different models by using the deviance information criterion proposed by

Spiegelhalter et al. (2002).

Our modelling results indicate that the explanatory power of the variables "time between consecutive transactions" and "trade volume" is rather low in this data set with respect to the absolute transaction price changes of the options considered as variable of interest in this paper. Although we do not dispose of exact intraday data for the underlying Bayer AG share we can still observe that explanatory variables which involve the value of the underlying (such as price changes of the underlying itself or the intrinsic value of the option at the time of the trade) have an important effect on the absolute transaction price changes of the options, which is, of course, in line with basic economic considerations. Moreover, our modelling results for the data of one selected Call option indicate that autocorrelations between the absolute transaction price changes are low and a modelling approach which considers the absolute transaction price changes as independent is to be preferred. This result may, however, merely be a consequence of the comparatively low trading activity for the equity options considered in this paper and may not be generalized without further investigation.

The paper is organized as follows: Section 2 gives a short introduction to the basic economic background and terminology related to options and option markets. Section 3 describes and exploratively analyzes the available data and gives first empirical results. The ordinary Poisson-GLM and extensions of the model which incorporate an AR(1) latent process are briefly introduced in Section 4. Poisson-GLMs are fitted to data for sets of options and results are related to implications of market microstructure theory for the effect of time between trades and trade volume on absolute option price changes in Section 5. Section 6 focuses on one particular security for which an ordinary Poisson-GLM and a Poisson-GLM with an AR(1) latent process are fitted in a Bayesian model setting. The adequacy of the two models of different complexity is assessed using the deviance information criterion proposed by Spiegelhalter et al. (2002). Finally, Section 7 summarizes the results and gives conclusions.

2 Economic background

In this chapter we give a very brief introduction to the basic definitions and market mechanics of exchange-traded options. For a detailed introduction to options and other financial derivatives see for example Hull (2003).

2.1 Basic definitions

An option is a privilege sold by one party to another that offers the buyer the right, but not the obligation, to buy (*Call option*) or sell (*Put option*) an underlying asset by a certain date for an agreed-upon price. The price in the contract is called *exercise price* or *strike price*. The date in the contract is known as *expiration date* or *maturity*. Options that can be exercised at any time up to the expiration date are referred to as *American options*, while those options that can only be exercised on the expiration date itself are called *European options.* Of course, there are two sides to every option contract. Firstly, the investor who has bought the option or taken a *long position* and secondly the investor who has sold the option or taken a *short position.* The seller of the option is also called the *writer* of the option. The writer of an option receives a certain amount of cash, the option price, up-front from the buyer. After paying the up-front option price, the holder of a long position in an option does not have any potential liabilities later. The potential losses from a short position, however, are theoretically unlimited. The following table shows the payoffs of the four possible option positions at maturity of the option. S_T denotes the price of the underlying asset at maturity, K the strike price of the option.

	Call	Put
Long position	$max(S_T - K, 0)$	$max(K - S_T, 0)$
Short position	$-max(S_T - K, 0)$	$-max(K - S_T, 0)$

The *intrinsic value* of a Call option at time $t \leq T$ is defined as:

$$max(S_t - K, 0).$$

An option with intrinsic value > 0 is called *in-the-money*, an option with intrinsic value $= 0$ is called *at-the money* if the current price of the underlying is equal to the strike price and *out-of-the-money* if the current price of the underlying is below the strike price of the option. For Put options the intrinsic value is defined as $max(K - S_T, 0)$ and the in-the-money, at-the-money, out-of-the-money terminology is used analogically. In general, the present value of a European option equals the intrinsic value plus the *time value* of the option. The time value reflects the possibility of favourable movements in the price of the underlying in the future. It is of course equal to zero when the option has reached maturity. The underlying assets exchange-traded options are currently actively traded on include stocks, stock indices, foreign currencies and futures contracts.

2.2 Option market participants

Before analyzing the option trading process it will be useful to shortly introduce the main participants in option markets. There are three broad categories of traders that actively trade options: *Hedgers*, who seek to reduce risk exposure, *speculators* who explicitly wish to take a position and bet on market movements and *arbitrageurs* who try to lock in risk-free profits by entering into transactions in two or more markets simultaneously. Orders are executed by a *broker*, an individual or firm that charges commissions for executing orders submitted by an investor. In order to facilitate trading in a particular security option exchanges use *market makers.* A market maker is both a broker and a dealer willing to accept the risk of holding a particular number of shares of a particular security in order to facilitate trading in that security. Each market maker competes for customer order flow by displaying bid and ask quotations for a guaranteed number of shares. Once an order is received, the market maker will immediately sell from its own inventory, buy on its own account or seek the other side of the trade so that the trade can immediately be executed. Market makers ensure that both buy and sell orders can always be executed without any delay and therefore add liquidity to the market.

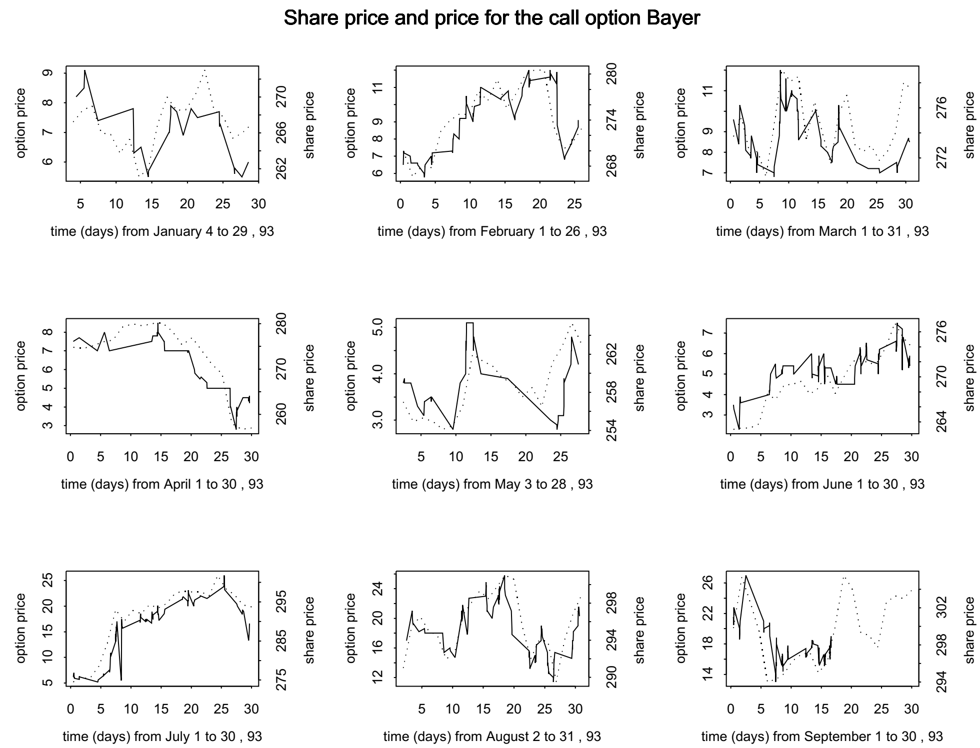

Figure 1: Bayer share $(\cdots)$ and Call option $(-)$ with strike price 280 DM and expiration month September 1993.

3 Data description and explorative data analysis

3.1 Data description and the variable of interest

As a representative example for option transaction data we consider in this paper American-style options on shares of the Bayer AG. The available data set contains both call and put options together with the corresponding strike price, time of maturity, trading time (up to hundredth of a second) and trading volume (traded lots in contracts) of each trade. The major drawback of the data is the lack of detailed intraday data for the underlying Bayer AG shares. Prices of the Bayer AG shares are only available on a daily basis, so that the determination of option-specific explanatory variables as mentioned in Section 1 is limited. One option contract consists of the right to buy/sell 50 shares. Note however, that the option prices are always quoted for the purchase or sale of 1 unit of the underlying, i.e. for the purchase or sale of 1 Bayer AG share. During the sampling period the smallest possible incremental change of the option price was set to 0.10 DM.

The smallest possible incremental price change of a certain security is commonly called *tick size.* Thus, in our data example 1 tick equals 0.10 DM.

In a first step we investigate empirically the relationship between the Bayer stock price and the price of options on the Bayer share. Figure 1 shows the price development of a Call option on the Bayer share with strike price 280 DM and expiration month September 1993. We have chosen the strike price of 280 DM for this purpose since it was the most actively traded option within the corresponding option series. Figure 1 shows a strong relationship between the value of the Bayer

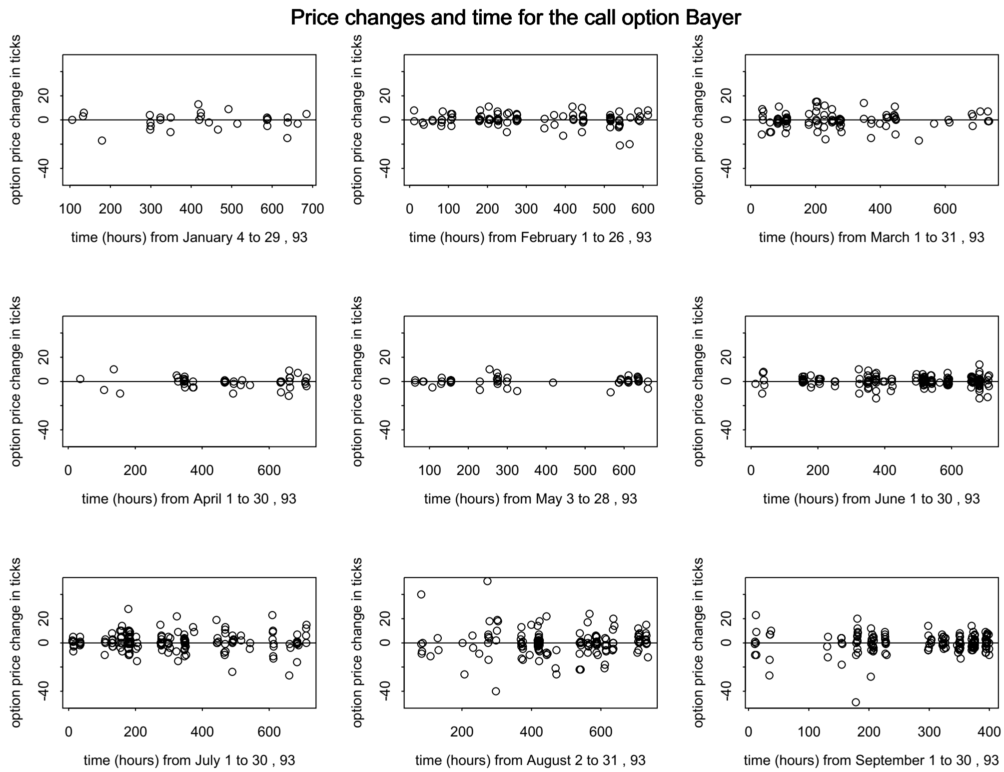

Figure 2: Price changes of the Call option with strike price 280 DM and expiration month Sept. 1993.

stock and the value of the Call option as expected. Furthermore one can observe that the number of trades of the Call option increases as one gets closer to the expiration date of the option. For example, only 30 transactions were observed in January 93 in contrast to 146-170 transactions per month from June to September 93.

Our primary goal in this paper is to relate transaction price changes of options to other marks of the trading process. In order to get a first impression of the structure of these price changes we plot the price changes of the Call option on the Bayer share with a strike price of 280 DM and expiration month September 1993 against time in Figure 2. Most option price changes are between - 20 to 20 ticks, but a few larger values of more than 50 ticks especially in the last 3 trading months of this Call option can be observed. When comparing the price changes of the option to the price changes of the underlying share, one can see that relative price changes of the option are much higher than those of the underlying, i.e. option price changes have much more financial impact than those of the underlying. This well-known phenomenon is commonly called *leverage effect* of the option. Histograms of the transaction price changes are reasonably symmetric around a peak at 0, which gives some justification to consider absolute price changes as the variable of primary interest for further modelling.

3.2 Relationship between absolute option price changes and time between consecutive transactions

We now consider again the absolute price changes of the Call option on the Bayer share with strike price 280 DM and expiration month September 1993 and plot them against time between consecutive transactions for each month in 1993 up to maturity of the option in Figure 3. We also draw the regression line based on a simple linear least squares regression for each plot.

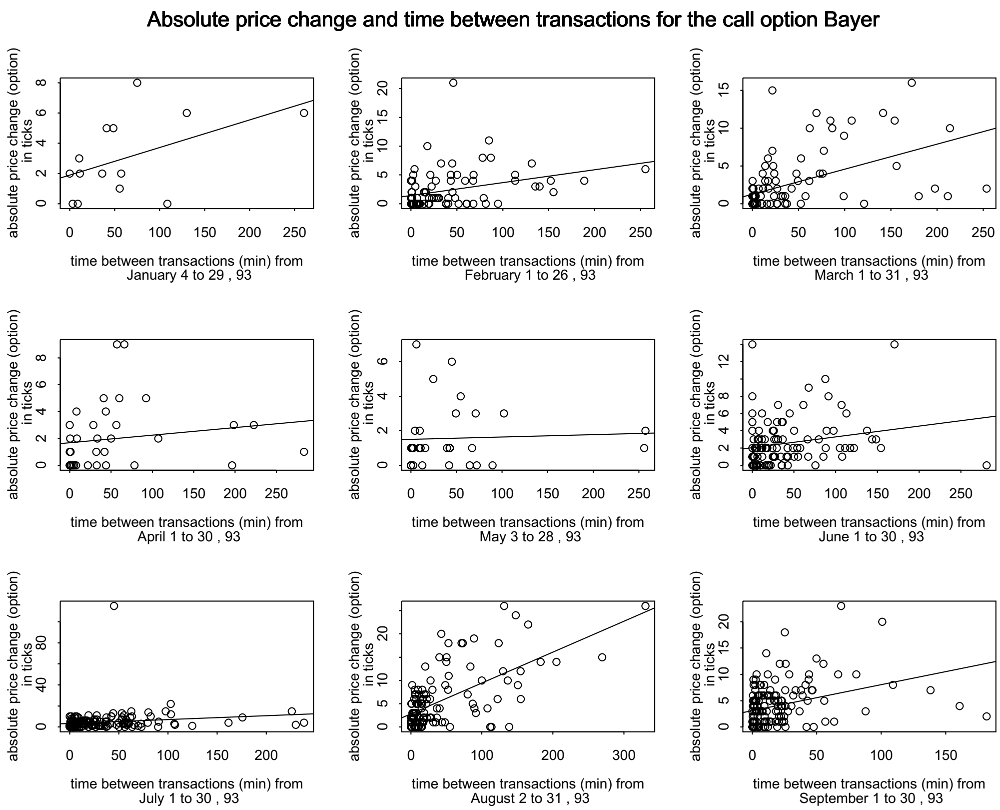

Figure 3: Relationship between absolute option price changes of the Call option with strike price 280 DM and expiration month 1993 and time between consecutive transactions.

The positive slopes of the regression lines indicate a positive relationship between absolute option price changes and time between consecutive transactions, which means that the longer the time between two consecutive transaction the larger the expected absolute option price change. Note that the time between transactions can be considered as an indicator of the speed of the market.

3.3 Relationship between option price changes and stock price changes

As already mentioned earlier the major drawback of the available data is the lack of intraday prices of the underlying Bayer share. So, when comparing the price of the underlying at a certain option transaction time with the price of the underlying at the time of the previous option transaction this will necessarily yield a difference of 0 unless the two transactions did not occur on the same day. This drawback in mind we construct a vector with the price changes of the underlying share which corresponds to the vector of the price changes of the considered option. Of course, the underlying price change vector will contain mainly zeros. One can expect that in general there is a very strong relationship between the price change of the underlying and the price change of the option. Just consider the case of a Call option which is in-the-money. If the value of the underlying increases e.g. by 0.50 DM, the intrinsic value of the option increases by the same amount. As the value of the option can generally be calculated as intrinsic value + time value, one would expect the option price to rise approximately 0.50 DM in this example.

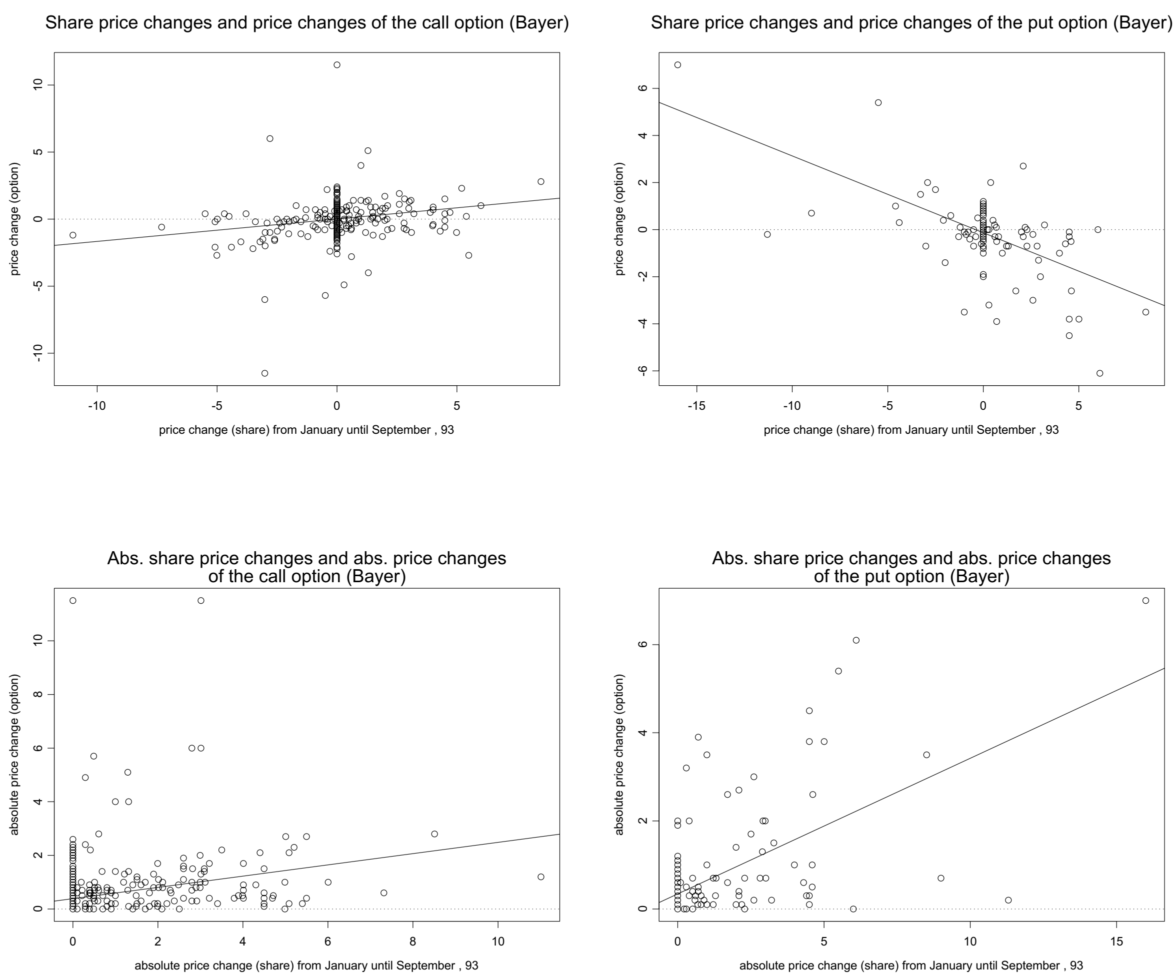

Figure 4: Relationship between option and share price changes (first row) and relationship between absolute option and absolute share price changes (second row) for the Call and Put option on the Bayer share with strike price 280 DM and expiration month Sept. 1993 respectively.

Despite the lack of more detailed data for the underlying Bayer shares, one can still see a clear positive relationship between the price changes of the Call option and the price changes of the underlying and a clear negative relationship between the corresponding Put option price changes and the price changes of the underlying Bayer share in the first row of Figure 4. Again, we draw the regression lines in order to better illustrate these relationships. Figure 4 also shows the relationship between the absolute option price changes and the absolute share price changes (second row).

3.4 Relationship between absolute option price changes and trade volume

In Figure 5 we show the relationship between the absolute transaction price changes of the Call option with strike price 280 DM and expiration month September 1993 and the volume of the corresponding transaction (traded lots in contracts). The negative slope of the regression lines indicates that in general the absolute transaction price changes become smaller when the trade volume increases.

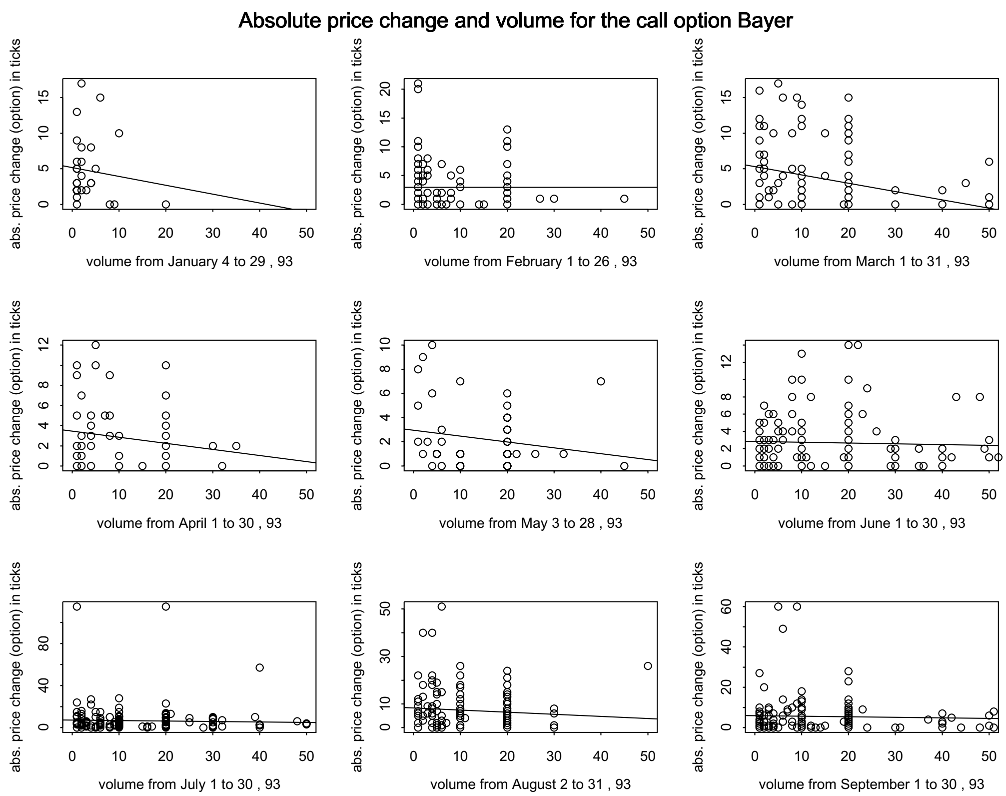

Figure 5: Relationship between absolute price changes of the Call option on the Bayer share with strike price 280 DM and expiration month September 1993 and trade volume.

3.5 Relationship between absolute option price changes and intrinsic value of the option

We have already defined the intrinsic value of an option in Section 2. However, when we consider the intrinsic value as a possible explanatory variable for our variable of interest we will use this term in a slightly different way. We define the variable "intrinsic value" as $IV_t := S_t - K$ where again S_t denotes the price of the underlying Bayer share at time t and K denotes the strike price of the option. Due to the lack of detailed intraday data for the Bayer share the intrinsic value as previously defined can only be determined as a daily value and must therefore be considered as an approximation of the real intraday value at the time of each trade.

As our variable IV_t is not restricted to non-negative values it can also specify how far an option

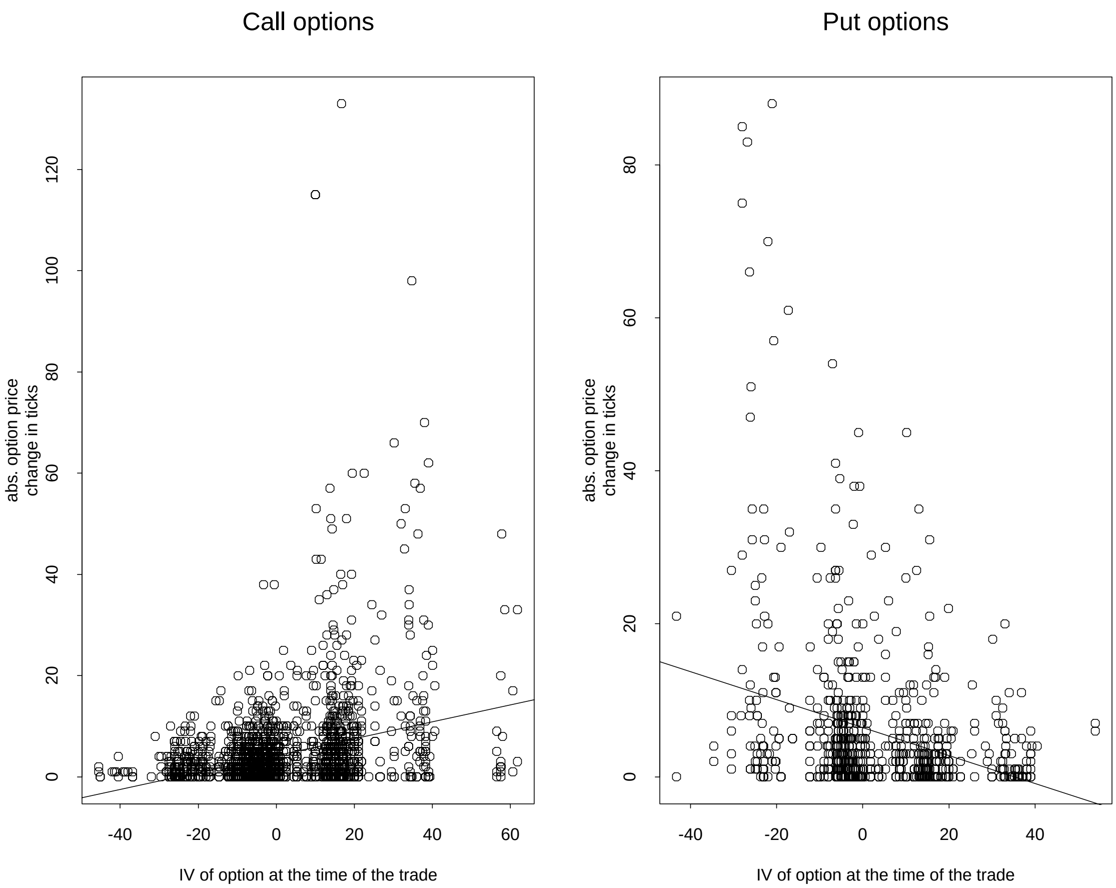

Figure 6: Relationship between absolute price changes of a set of Call and Put options on the Bayer share with expiration month September 1993 and "intrinsic value" of the options between January 1993 and September 1993.

is out-of-the money and therefore provides more detailed information than the common definition of the intrinsic value of an option stated in Section 2. We define IV_t in this way for both Call options and Put options which somehow simplifies the data handling but, of course, has to be remembered when it comes to economic interpretations. In the following we will not consider single options but a whole set of options (options of the same type with the same expiration date but with different strike prices and possibly a different life-span) during a certain period of time, up to maturity of the options. We successively write the observations of the different options into one vector. This approach broadens the data base and it is therefore easier to analyze the relationship between possible explanatory variables such as the previously defined "intrinsic value" and the variable of interest. In Figure 6 we show the relationship between the absolute option price changes and the "intrinsic value" as previously defined for a set of Call options and for a set of Put options respectively. The regression lines indicate that the more the options are in-the-money in the proper economic sense (for both Call and Put options), the larger the absolute option price changes. In addition to the expectation of the absolute option price changes, it can also be observed that their variance increases the more the options are in-the-money.

4 Regression models for count response data

Due to the count data structure of absolute option transaction price changes measured in ticks, a natural starting point for a regression analysis is the Poisson regression model. Since the Poisson regression model is a special case of a generalized linear model, its statistical properties are well-known and parameter estimation can easily be handled (see for example McCullagh and Nelder (1989)). Moreover, the interpretation of the modelling results is straightforward. Note however, that the Poisson distribution allows only one single parameter to estimate both the mean and the variance, which may be a very restrictive approach for some real data problems.

In a second step, we also take into account the time series structure of the transaction price changes by considering a Poisson-GLM with an AR(1) latent process in the mean. The unobservable latent process introduces autocorrelation into the model and therefore accounts for its time series structure. Parameter estimation in this class of models is, however, much more complicated than in ordinary Poisson-GLMs. In the literature several estimation techniques have been proposed. In this paper we will use the WinBUGS software (freely available on the BUGS-project website http://www.mrc-bsu.cam.ac.uk/bugs/welcome.shtml) which estimates the parameters of the model with Markov Chain Monte Carlo (MCMC) methods. MCMC methods can be used in Bayesian Inference. In contrast to the classical approach, model parameters are considered to be random variables in Bayesian Inference. They are distributed according to a *prior distribution* which summarizes all a-priori knowledge about the parameters obtained for example by previous experiments or experts' assumptions. Then, given a random sample, the knowledge about the model parameters can be updated by using the new sample and by calculating the conditional distribution of the parameters given the new sample. This conditional distribution is commonly called *posterior distribution*. MCMC methods can be used to obtain an approximate sample from a complex posterior distribution by recording values of a simulated Markov Chain with a limiting distribution which is equal to the posterior distribution of the model parameters. The mean of these values generated by the Markov Chain can then be used as estimate for the model parameters. For a detailed introduction to MCMC estimation see for example Gamerman (1997) or Gilks et al. (1996). We will assess convergence of the MCMC simulation with the method proposed by Gelman and Rubin (1992).

Before we build and fit specific models in the following sections, we give a very brief introduction to Poisson-GLMs in general and to Poisson-GLMs with an AR(1) latent process in the mean.

4.1 The Poisson generalized linear model

As already mentioned earlier, the Poisson regression model is a special case of the generalized linear model (GLM). In a GLM, given the covariates $\boldsymbol{x}_i$, the response variables y_i, $i = 1, \ldots, n$ are independent and identically distributed with a density of the form

$$f(y; \theta, \phi) = \exp\left\{\frac{\theta y - b(\theta)}{a(\phi)} + c(y, \phi)\right\}, \tag{4.1}$$

which is (for a known ϕ) a density from the exponential family. θ is a location parameter (not necessarily the mean) and ϕ is a dispersion parameter. The probability density function of the Poisson distribution with mean λ

$$f_{poi}(y) = \exp\{-\lambda + y \log \lambda - \log y!\}$$

can be rewritten in the form (4.1) with $a(\phi) = 1$, $\theta = \log \lambda$, $b(\theta) = \exp(\theta) = \exp(\log \lambda) = \lambda$, and $c(y, \phi) = -\log y!$. The function which links the regressors to the parameter(s) of the distribution of the variable of interest is called *link function.* In the case of the Poisson regression model the canonical link function is the log-link, i.e. $\eta_i := \boldsymbol{x_i^t \beta} = \log(\lambda_i)$ for $i = 1, .., n$. Therefore, the conditional mean of y_i given the covariates $\boldsymbol{x_i}$ is given by

$$E(Y_i \mid \boldsymbol{x_i}) = \exp(\eta_i) = \exp(\boldsymbol{x_i^t \beta}),\ i = 1, .., n.$$

One important property of the Poisson distribution is equidispersion, i.e.

$$\lambda_i = E(Y_i \mid \boldsymbol{x_i}) = Var(Y_i \mid \boldsymbol{x_i}).$$

If in a data set the variance exceeds the mean, we say that the data shows *overdispersion.* We speak of *underdispersion*, if the variance is lower than the mean. For a detailed discussion of Poisson regression models and of count data regression in general see Cameron and Trivedi (1998) or Winkelmann (1997).

4.2 Poisson-GLMs with an AR(1) latent process in the mean

This class of models can be considered as an extension of the previously described Poisson-GLM when the observations Y_t, $t = 1, ..., T$, come from a time series and are unlikely to be independent. The general model framework as considered for example by Chan and Ledolter (1995) is given by:

$$Y_t \sim Poi(\lambda_t), \qquad \lambda_t = exp(u_t + \mathbf{x}_t^t \boldsymbol{\beta}), \tag{4.2}$$

$$u_t = \rho \cdot u_{t-1} + \varepsilon_t, \qquad \varepsilon_t \overset{\text{i.i.d}}{\sim} N(0, \sigma_\varepsilon^2)$$

where the observations Y_t conditional on $\mathbf{x}_t$ and on the latent variables u_t are independent for $t = 1, .., T$. Since $\{u_t\}$ is an ordinary AR(1) process, it follows that

$$E(u_t) = 0, \qquad \sigma_u^2 := Var(u_t) = \frac{\sigma_\varepsilon^2}{1-\rho^2},$$

$$\gamma_u(k) := Cov(u_t, u_{t+k}) = \rho^k \sigma_u^2.$$

Thus, in this model $u_t \sim N(0, \sigma_u^2)$ and using the common formulas for the expectation and variance of the log-normal distribution it follows that

$$\begin{aligned} E(exp(u_t)) &= exp(\frac{\sigma_u^2}{2}) \\ Var(exp(u_t)) &= exp(\sigma_u^2)(exp(\sigma_u^2) - 1). \end{aligned}$$

Using these results it is easy to see that

$$E(Y_t|\mathbf{x}_t) = E(E(Y_t|u_t, \mathbf{x}_t)) = E(exp(u_t)) \cdot exp(\mathbf{x}_t^t \boldsymbol{\beta}) = exp\left(\frac{\sigma_u^2}{2} + \mathbf{x}_t^t \boldsymbol{\beta}\right). \tag{4.3}$$

Using the abbreviation $\nu_t := exp(\mathbf{x}_t^t \boldsymbol{\beta})$ the variance of Y_t conditional on the regressors (but not conditional on the latent process) can be calculated as:

$$\begin{aligned} Var(Y_t|\mathbf{x}_t) &= E(Var(Y_t|\mathbf{x}_t, u_t)) + Var(E(Y_t|\mathbf{x}_t, u_t)) \\ &= exp(\frac{\sigma_u^2}{2}) \cdot \nu_t + Var(exp(u_t) \cdot \nu_t) \\ &= exp(\frac{\sigma_u^2}{2}) \cdot \nu_t + Var(exp(u_t)) \cdot \nu_t^2 \\ &= exp(\frac{\sigma_u^2}{2}) \cdot \nu_t + exp(\sigma_u^2)(exp(\sigma_u^2) - 1) \cdot \nu_t^2. \end{aligned} \tag{4.4}$$

Since $Var(Y_t|\mathbf{x}_t) > E(Y_t|\mathbf{x}_t)$ if $\sigma_u^2 > 0$, the model allows for overdispersion. Furthermore, for $k = 1, 2, ...$ the autocovariance between Y_t and Y_{t+k} conditional only on the regressors can be calculated as :

$$Cov(Y_t, Y_{t+k}|\mathbf{x}_t, \mathbf{x}_{t+k}) = \nu_t \cdot \nu_{t+k} \cdot exp(\sigma_u^2) \cdot (exp(\rho^k \sigma_u^2) - 1). \tag{4.5}$$

Note that if $\gamma_u(k) \neq 0$ it follows that $Cor(Y_t, Y_{t+k}|\mathbf{x}_t, \mathbf{x}_{t+k}) \neq 0$, i.e. the latent process induces autocorrelation into Y_t. Since $\gamma_u(k) = 0$ only if $\rho = 0$, autocorrelation in Y_t given the regressors is present if $\rho \neq 0$.

These results are also stated in Zeger (1988) and Davis et al. (1999) for a slightly different parameterization of the latent process. Parameter estimation of Model (4.2) is not straightforward and computer intensive. Chan and Ledolter (1995) use a Monte Carlo Expectation Maximization (MCEM) algorithm while Zeger (1988) proposes estimating equations based on a time series analogue of quasi-likelihood methods. We will carry out parameter estimation using the WinBUGS software for MCMC estimation.

5 A Poisson-GLM for the absolute transaction price changes of sets of Call and Put options

When building a regression model the regressors that enter into the model have to be determined, possibly after transformations and possibly taking into account interaction effects between the regressors. In order to get a better impression of the significance of the explanatory variables already considered in the previous section we show the following plots, which illustrate the empirical effects of the explanatory variables on the variable of interest for the set of Call options with expiration month September 1993, considered between January and September 1993 (Figure 7), and for the set of corresponding Put options (Figure 8). For these plots the explanatory variables were divided into their 10 % quantiles and for each quantile the empirical log-mean of the variable of interest (the absolute option price changes) was calculated separately. These plots once again confirm the assumptions about the relationships between the explanatory variables and the absolute option price changes which we have already stated in the previous section. Note that in the case of the absolute price changes of the underlying share only values $\neq 0$ were considered and divided into 20% quantiles.

In order to statistically quantify the influence of the explanatory variables on the absolute option transaction price changes we fit a first Poisson-GLM where all explanatory variables enter into the model without transformations and without interaction effects. Table 5 summarizes the results of the modelling for two sets of Call options and two sets of Put options.

Call options

	str. month 09	
	93	94
trade volume	$--$	$--$
time betw. transactions	$++$	$++$
abs. share price change	$++$	$++$
IV of option	$++$	$++$

Put options

	str. month 09	
	93	94
trade volume	$-$	$+$
time betw. transactions	$++$	$++$
abs. share price change	$++$	$++$
IV of option	$--$	$--$

Table 1: Influence of regressors: +/- represents a weak ($2 \leq |t - value| \leq 6$) positive/negative influence and ++/-- a strong ($|t - value| > 6$) positive/negative one.

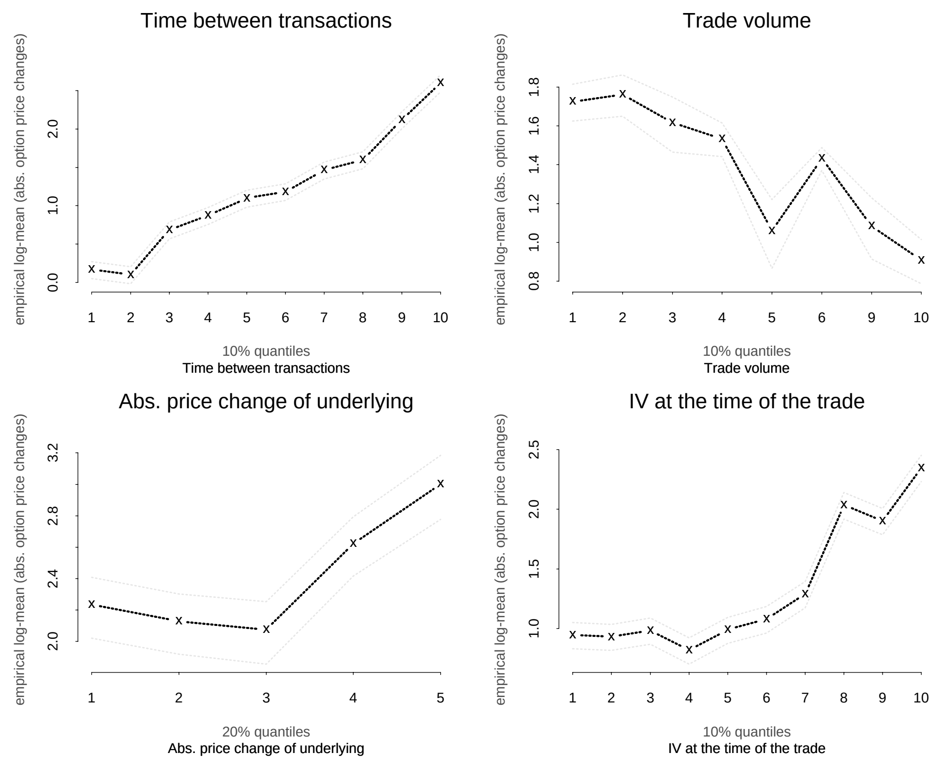

Figure 7: Main effects for abs.transaction price changes of a set of Call options on the Bayer share with maturity Sept. 1993 (with 90%-confidence intervals).

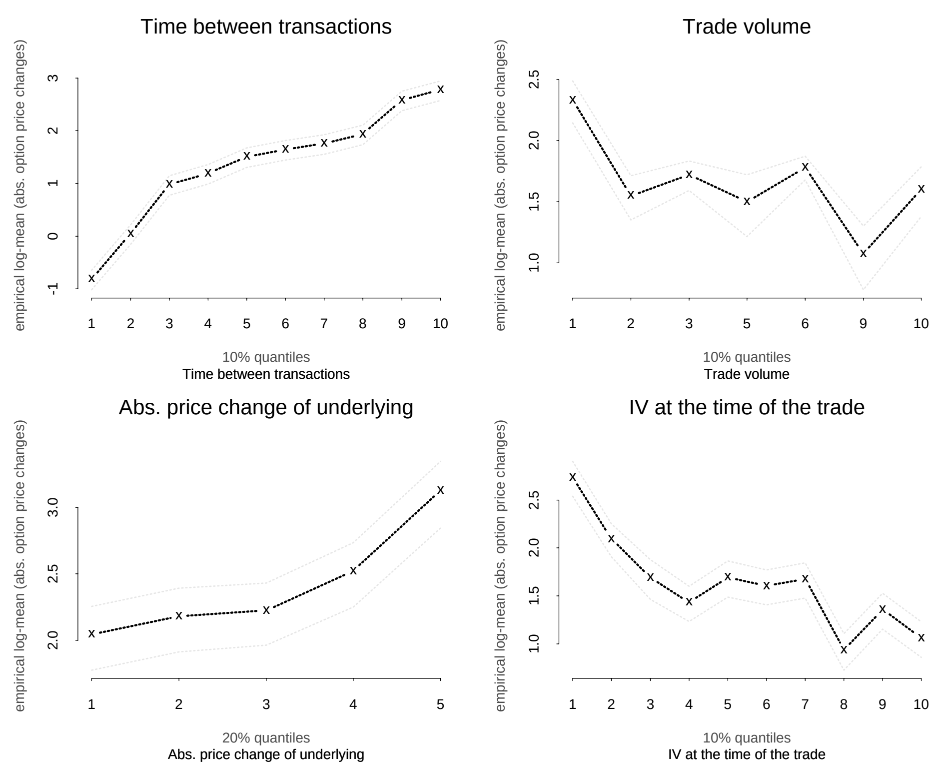

Figure 8: Main effects for abs.transaction price changes of a set of Put options on the Bayer share with maturity Sept. 1993 (with 90%-confidence intervals).

As a next step we consider possible transformations of the regressors in order to improve the model fit. Figures 7 and 8 already indicate that it may be better to transform some of the covariates, e.g. by taking the logarithm, before they enter into the model as regressors.
With the four data sets already used in Table 1 we investigate log-, sqrt- and polynomial transformations of the covariates. This is done by building simple Poisson-GLMs with only one covariate, e.g. the intrinsic value. For each data set and for each explanatory variable we fit four models of the form

$$Y_t \sim Poi(\mu_t)$$

with

$$\begin{array}{ll} 1. & \mu_t = exp(\beta_0 + x_t\beta_1) \\ 2. & \mu_t = exp(\beta_0 + log(x_t)\beta_1) \\ 3. & \mu_t = exp(\beta_0 + \sqrt{x_t}\beta_1) \\ 4. & \mu_t = exp(\beta_0 + x_t\beta_1 + x_t^2\beta_2) \end{array}$$

where Y_t denotes the t-th absolute option price change and x_t denotes the t-th observation of one particular explanatory variable. In the case of the logarithm it has to be ensured that $x_t > 0$ for all t. This can be done by adding a large enough constant c to x_t for all t. Proceeding in the same way, it can also be assured that $x_t \geq 0$ in the case of the square root transformation. We finally evaluate the transformations by comparing residual deviances of these models. In general it can be observed that log(time between transactions), log(trade volume), sqrt(abs. share price change) and a polynomial of degree 2 for the intrinsic value lead to the best models with regard to residual deviances. From now on, we will use these transformations of the explanatory variables for the modelling of the absolute option price changes.
As a further step in the model building process we take into account possible interaction effects between the regressors. With four possible explanatory variables we have to investigate $\binom{4}{2} = 6$ possible interaction effects. We use the *step*-function in S-Plus in order to determine significant regressors and significant interaction effects between the regressors. In this way we obtain the final model for each data set. Table 2 gives an overview of the covariates that enter into each model. Although some important similarities in the models for the four different data sets can be observed one can hardly expect to obtain completely identical models for the different data sets, considered between January and September of the corresponding year. We use the S-Plus notation "A : B" to denote the interaction between the covariates A and B. For example, "1) : 2)" denotes the interaction between log(time between transactions) and log(trade volume).

In the following we will further discuss and interpret the model for the transaction price changes of the set of Call options with expiration month September 1993. Table 3 shows the results of the maximum-likelihood estimation for the GLM with the absolute transaction price changes of the set of Call options on the Bayer share with expiration month September 1993 as response variable

Significant regressors after stepwise covariate selection				
	Call 09/93	Call 09/94	Put 09/93	Put 09/94
Intercept	××	××	××	××
1) log(time betw. transactions)	××	××	××	××
2) log(trade volume)	××	××	××	×
3) poly(intrinsic value,2)	××	××	××	××
4) sqrt(abs. share price change)	××	0	0	××
1) : 2)	××	××	0	×
1) : 3)	×	××	×	××
1) : 4)	0	××	0	×
2) : 3)	0	××	×	0
2) : 4)	×	0	××	0
3) : 4)	×	×	×	×

Table 2: Significant (×) and highly significant (××) regressors after stepwise regressor selection for four data sets of option transaction price changes.

and the significant main and interaction effects according to Column 2 in Table 2 as regressors. Yet, in a model with interaction effects it is not straightforward to interpret estimation results. For this purpose we draw the following plots (Figure 9) where each plot shows the expected absolute transaction price change of the option as a function of two of the explanatory variables on a linear scale between their 10% and 90% quantiles with all other explanatory variables set to their median values.

Parameter	Est.	Std. error	t-value
β_1 (Intercept)	1.297	0.035	37.31
β_2 (log(Trade volume))	-0.175	0.014	-13.01
β_3 (log(Time betw. transactions))	0.185	0.023	7.97
β_4 (Intrinsic value)	0.031	1.5e-03	20.55
β_5 ((Intrinsic value)^2)	3.2e-04	4.4e-05	-6.88
β_6 (sqrt(Abs. share pr. change))	0.262	0.044	6.02
β_7 (log(Trade volume):log(Time betw. transactions))	0.074	9.4e-03	7.83
β_8 (Intr. value:sqrt(Abs. share pr. change))	5.8e-03	1.7e-03	3.41
β_9 ((Intr. value)^2:sqrt(Abs. share pr. change))	7.0e-05	4.9e-05	1.44
β_{10} (Intr. value:log(Time betw. transactions))	9.9e-04	8.8e-04	1.12
β_{11} ((Intr. value)^2:log(Time betw. transactions))	-8.5e-05	2.3e-05	-3.36
β_{12} (log(Trade volume):sqrt(Abs. share pr. change))	-0.054	0.018	-3.04

Table 3: Summary of estimation results of the Poisson-GLM for the transaction price changes of the set of Call options on the Bayer share with maturity September 1993, considered between January and September 1993.

First of all, the plots in Figure 9 confirm the results of the influence of each of our explanatory variables on the absolute transaction price changes of the options which we have already given in Table 1. Due to basic economic considerations it is obvious that a high absolute price change of the underlying Bayer share will lead to a high absolute price change of the options on that share. It is not a surprise either that, generally speaking, the larger $S_t - K$ of a Call option (i.e. the higher the intrinsic value of the Call option as defined in the previous section) the higher the absolute transaction price changes. As an example imagine a clearly out-of-the-money option shortly before maturity. The value of this option will be close to zero and although there may still be some trading activity for that option large absolute price movements can not be expected any more. Contrarily, the price changes of a clearly in-the-money option shortly before maturity should approximately equal the absolute price changes of the underlying. Economic interpretations of the relationship between transaction rates (i.e. time between transactions) and transaction price changes have been discussed in the literature in the context of market microstructure theory (see for example O'Hara (1995)). For our Bayer option data example we have observed that low transaction rates (i.e. longer times between transactions) are associated with larger absolute transaction price changes. A possible explanation for this is given by Diamond and Verecchia (1987). Provided that short-selling is not feasible, they consider longer times between transactions as a possible signal for bad news in the market. The absence of a short selling mechanism in this case prevents informed market participants from profiting by exploiting the information through corresponding transactions. As soon as the news have been spread in the market the price of the next transaction will reflect the bad news and a large absolute price change is likely. We can finally observe in Figure 9 that, in general, there is a negative relationship between trade volume and absolute transaction price changes which confirms our results in Table 1, but is in contrast to a common theory (see for example Easley and O'Hara (1987)) that large volumes per transaction indicate additional news in the market and are likely to be associated with large absolute price changes. According to this theory, given that they want to trade, informed traders prefer to trade larger amounts at any given price to profit from their current informational advantage. As a result, large absolute price changes associated with large trade volumes are likely. For a more detailed discussion of the relationship between transaction price changes, trade volume and times between transactions in the context of market microstructure theory see O'Hara (1995), Chapter 6.

Another observation which takes into account the interaction effects between the explanatory variables is the fact that, in general, the higher the intrinsic value of the option the larger the impact of the other explanatory variables on the expected transaction price changes of the option. This can be seen in the plots of the right column of Figure 9. Particularly high absolute transaction price changes can be expected if the transaction rate is low and, at the same time, the option is clearly in-the-money.

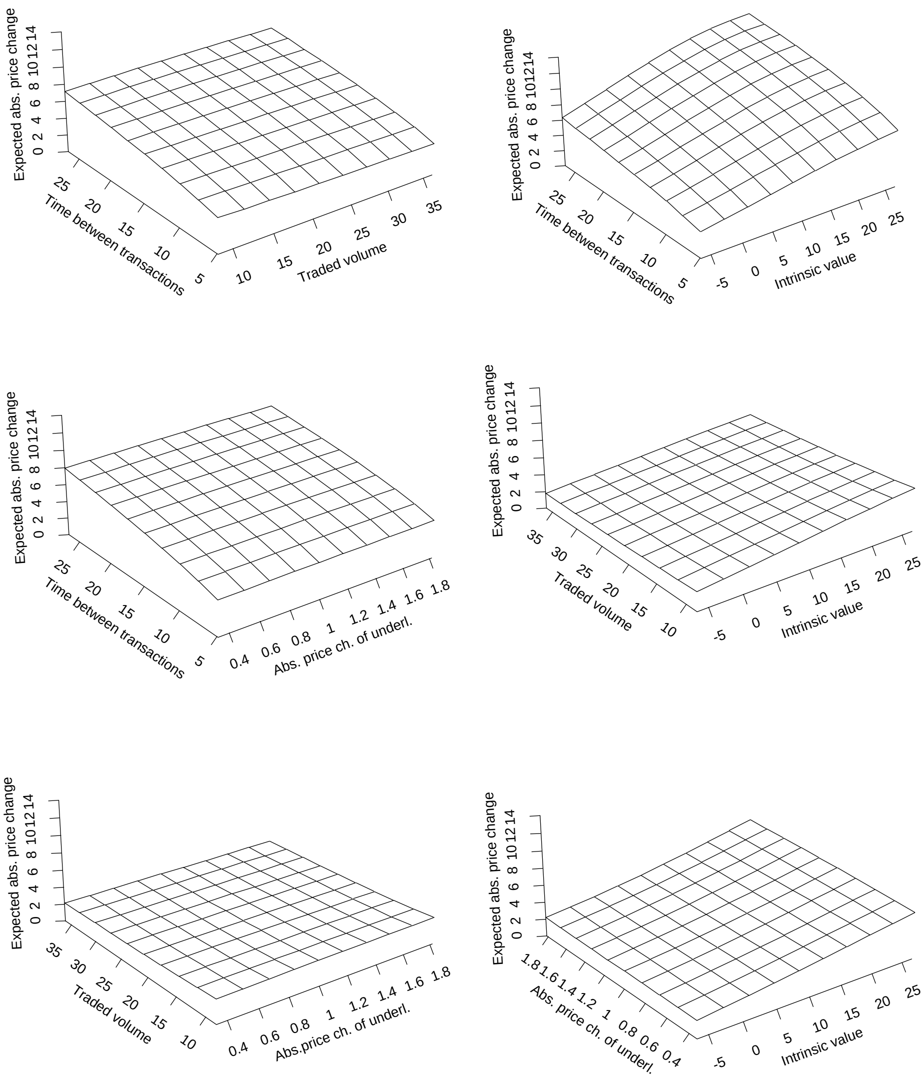

Figure 9: Fitted regression surfaces of the Poisson-GLM for the absolute transaction price changes of the set of Call options on the Bayer share with maturity Sept. 1993 when two regressors vary and the remaining regressors are set to their median values.

6 Regression modelling of the absolute transaction price changes of a single security

We finally take into account the time series structure of the data and fit a Poisson-GLM with an AR(1)-latent process in the mean as introduced in Section 4.2 to our data. For this purpose we can not consider whole sets of options together, but have to focus on one particular security whose absolute transaction price changes are the time series of interest.

As a representative example we consider again between January and September 1993 the Call option on the Bayer share with strike price 280 DM and expiration month September 1993. Since considering only one particular option reduces the data base (the total number of observations for this security is 992), we also reduce the complexity of the regressors and only take into account the traded volume, the time between transactions, the intrinsic value at the time of the trade and the absolute price change of the underlying Bayer share without any transformations and without interaction terms. This also makes the interpretations of the estimation results more straightforward. Parameter estimation is carried out by the WinBUGS software with Markov Chain Monte Carlo (MCMC) methods. Table 4 shows the estimation results of a MCMC simulation from 3 parallel chains (with different starting values) for each parameter where after an initial burn-in of 2000 iterations a further 3000 iterations are recorded in each chain.

Parameter	mean	std.err.	2.5%	median	97.5%
β_1 (Intercept)	0.669	0.063	0.545	0.670	0.793
β_2 (Trade volume)	-0.004	0.002	-0.008	-0.004	-0.001
β_3 (Time betw. trans.)	0.011	0.003	0.005	0.011	0.017
β_4 (Intrinsic value)	0.031	0.004	0.024	0.031	0.038
β_5 (Abs. share pr. ch.)	0.226	0.044	0.138	0.226	0.311
ρ	0.190	0.050	0.091	0.191	0.288
σ^2_ε	1.111	0.085	0.958	1.107	1.286

Table 4: MCMC sampling results for the parameters of the Poisson-GLM with an AR(1)-latent process applied to the Call option on the Bayer share with strike price 280 DM and expiration month Sept. 1993 based on the last 3000 iterations in each chain.

The trajectories of the simulated chains and a consideration of the corresponding Gelman-Rubin convergence statistics indicate that convergence is reached very quickly, which justifies the choice of the burn-in. Autocorrelations within the chains are reasonably small, so that there is no need to thin the recorded iterations. Figure 10 shows the posterior density estimates for the components of the parameter vector $\boldsymbol{\beta}$, Figure 6 shows the trajectories, the posterior density estimates and the Gelman-Rubin convergence statistics for the parameters ρ and σ^2_ε.

The estimation results for the autocorrelation coefficient of the latent process ρ imply that there

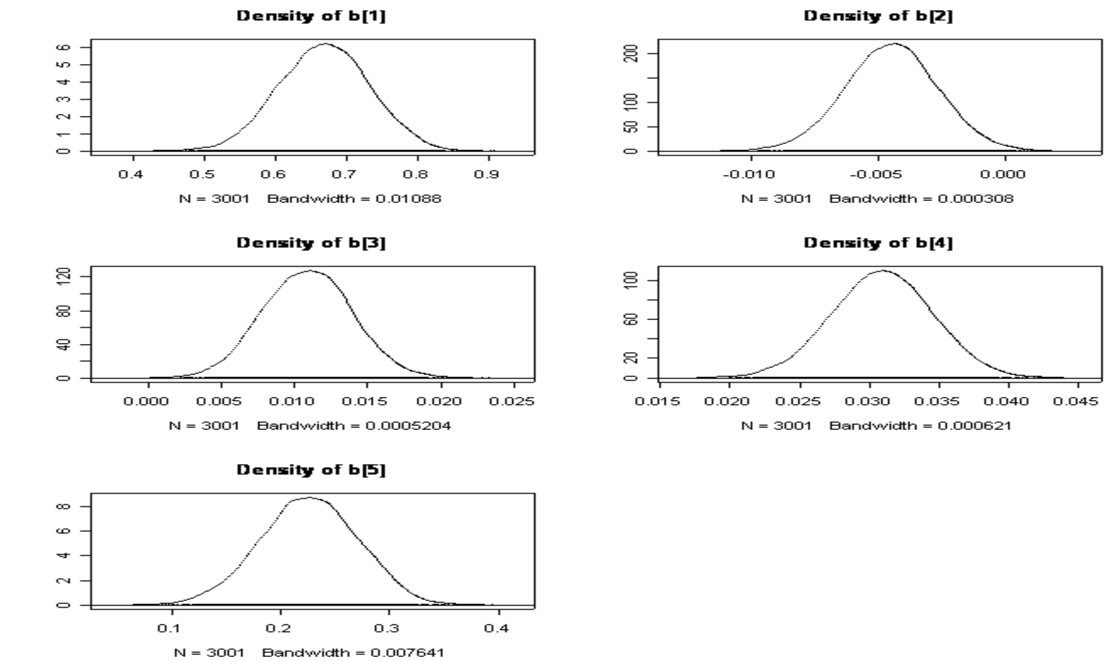

Figure 10: Posterior density estimates for $\beta_1, ..., \beta_5$ (b[1],...,b[5]) in the Poisson-GLM with an AR(1) latent process based on the last 3000 iterations in each chain.

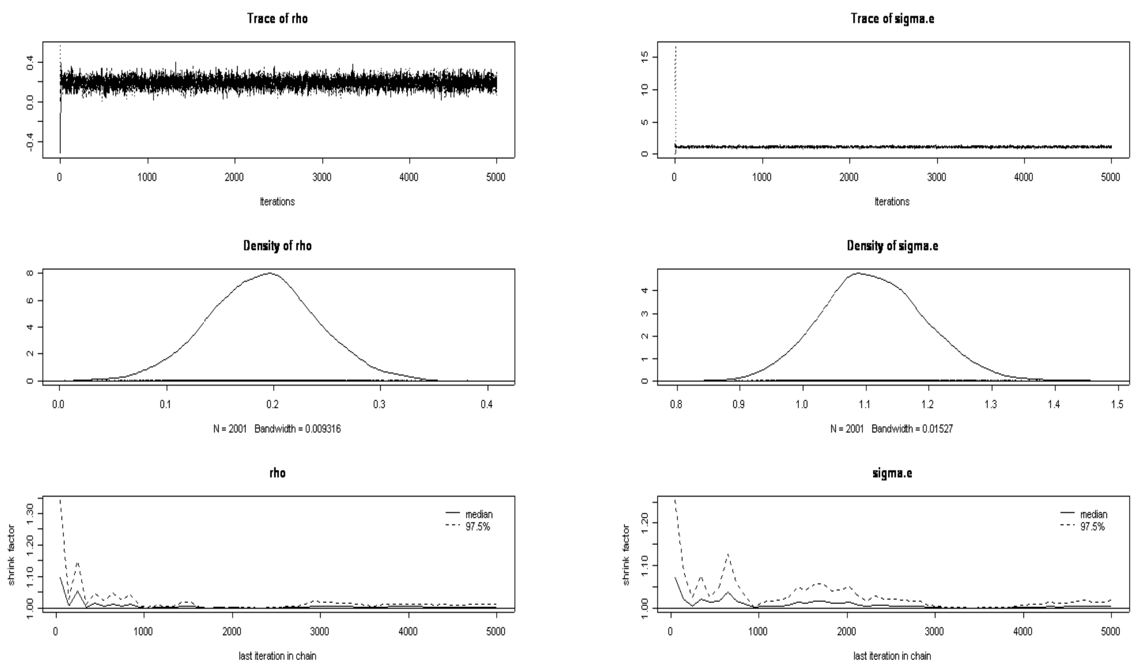

Figure 11: MCMC simulation, posterior density estimate and Gelman-Rubin convergence diagnostics for ρ (left column) and σ_ε^2 (right column) in the Poisson-GLM with an AR(1) latent process.

is indeed statistical evidence for some autocorrelation in the latent process. The posterior mean estimate of the variance $\hat{\sigma}_u^2$ of the latent variables u_t is equal to 1.16. This implies that the latent process has a strong impact on the expectation of the variable of interest given the regressors. In our MCMC simulation this can be seen by considering the 992 posterior mean estimates of u_t, $t = 1, ..., 992$, which lie within the interval $[-1.97, 3.73]$, while those for $\mathbf{x}_t^t\boldsymbol{\beta}$ lie within the smaller interval $[-0.77, 3.38]$. The lowest lower bound of all 992 95%-credible intervals of $\hat{u}_t$, $t = 1, ..., 992$, is -3.34 while the largest upper bound of these credible intervals has the value 3.95. For the quantity $\mathbf{x}_t^t\hat{\boldsymbol{\beta}}$, which measures the influence of the explanatory variables, these values are -1.91 and 4.17 respectively. It can be concluded that the effect of the latent variables on the absolute option price changes is somewhat larger than the effect of the explanatory variables. The expectation, variance and covariance of the variable of interest Y_t (the absolute transaction price changes) conditional on the regressors $\mathbf{x}_t$ can be estimated according to (4.3), (4.4) and (4.5):

$$\widehat{E(Y_t|\mathbf{x}_t)} = e^{0.5\hat{\sigma}_u^2} \cdot e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}} = 1.79 \cdot e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}}$$

$$\begin{aligned}\widehat{Var}(Y_t|\mathbf{x}_t) &= e^{0.5\hat{\sigma}_u^2} \cdot e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}} + e^{\hat{\sigma}_u^2} \cdot \left(e^{\hat{\sigma}_u^2} - 1\right) \cdot \left(e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}}\right)^2 \\ &= 1.79 \cdot e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}} + 6.99 \cdot \left(e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}}\right)^2\end{aligned}$$

$$\begin{aligned}\widehat{Cov}(Y_t, Y_{t+k}|\mathbf{x}_t, \mathbf{x}_{t+k}) &= e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}} \cdot e^{\mathbf{x}_{t+k}^t\hat{\boldsymbol{\beta}}} \cdot e^{\hat{\sigma}_u^2} \cdot \left(e^{\hat{\rho}^k \cdot \hat{\sigma}_u^2} - 1\right) \\ &= e^{\mathbf{x}_t^t\hat{\boldsymbol{\beta}}} \cdot e^{\mathbf{x}_{t+k}^t\hat{\boldsymbol{\beta}}} \cdot 3.19 \cdot \left(e^{0.19^k \cdot 1.16} - 1\right).\end{aligned}$$

Note that the estimated overdispersion is large while autocovariances and, consequently, autocorrelations in the absolute transaction price change process fall off rather quickly. Naturally, the question arises whether the incorporation of the latent process into the mean of the Poisson-GLM leads to a better fit than an ordinary Poisson-GLM. In order to answer this question we fit an ordinary Poisson-GLM to the same data. Table 5 gives the results of this estimation, carried out in S-Plus with maximum-likelihood methods. It can be observed that concerning the relationship between the explanatory variables and the absolute transaction price changes of the option, the modelling results of both the Poisson-GLM with an AR(1) latent process and the ordinary Poisson-GLM given in Table 4 and Table 5 respectively are in line with the results already discussed on the basis of broader data and more complex regressors in the previous sections. In Figure 12 we illustrate comparatively the way the ordinary Poisson-GLM of Table 5 and the Poisson-GLM with an AR(1) latent process of Table 4 estimate the influence of each explanatory variable on the absolute option price changes. In each plot, one of the explanatory variables is again shown on a linear, untransformed scale between the 10%- and 90%-quantiles. With all other explanatory variables set to their median values, the expected absolute option price change according to each model is calculated at ten equidistant grid points. It can be observed that the expected absolute transaction price changes of the option are slightly larger in the Poisson-GLM with the AR(1) latent process for almost all values of the explanatory variables.

Parameter	Estimate	Std.err.	t-value
β_1 (Intercept)	1.2781	0.025	50.52
β_2 (Trade volume)	-0.0084	0.001	-7.45
β_3 (Time betw. trans.)	0.0080	0.001	10.97
β_4 (Intrinsic value)	0.0342	0.001	27.89
β_5 (Abs. share pr. ch.)	0.2095	0.010	20.56

Table 5: Estimation results of an ordinary Poisson-GLM fit to the data of the Call option on the Bayer share with strike price 280 DM and expiration month Sept. 1993.

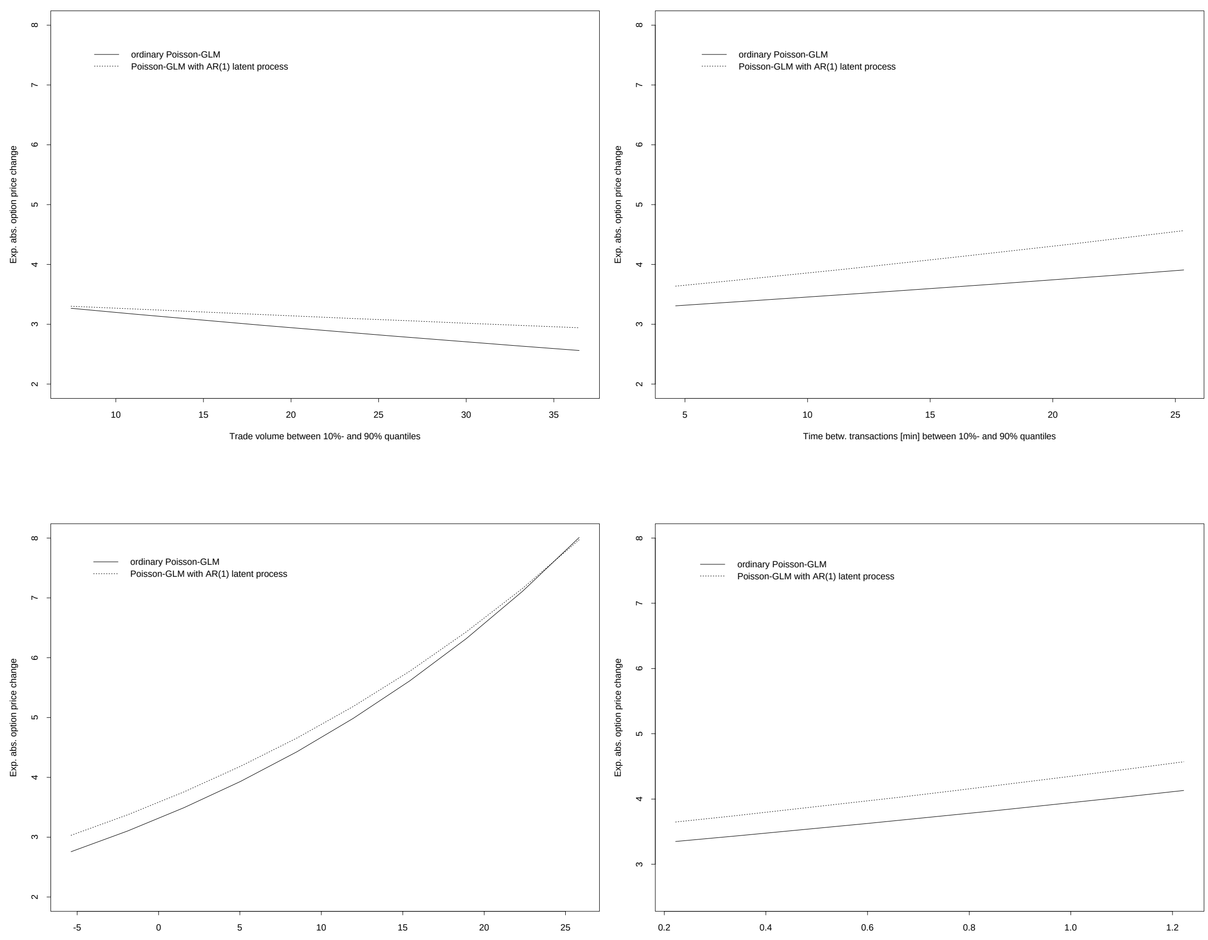

Figure 12: Comparative illustration of the estimated influence of the explanatory variables on the absolute transaction price changes of the Call option on the Bayer share with strike price 280 DM and expiration month September 1993.

In order to get a first informal impression of the goodness of fit of the previously discussed models we compare the observations of the variable of interest to the fitted values of the models. We consider the mode $\lfloor \hat{\lambda}_t \rfloor$ of the estimated $Poi(\hat{\lambda}_t)$ distribution as fitted value $\hat{y}_t$ for the variable of interest Y_t and illustrate the accuracy of the fit of the two different models by calculating for both models the quantity

$$SEM := \sum_{t=1}^{992} |\hat{y}_t - y_t|^2$$

where residuals are squared and added for $t = 1, .., 992$. In the ordinary Poisson-GLM the calculation of SEM yields a value of 54021 while in the Poisson-GLM with the AR(1) latent process SEM has the significantly smaller value 992. These results strongly indicate that the fit of the Poisson-GLM with the AR(1) latent process is much better than the fit of the ordinary Poisson-GLM. This informal approach, however, neglects the model complexity, which should be traded off against the goodness of fit when comparing models.

A more formal way to assess the goodness of fit of these models is the consideration of deviances. The deviance of an ordinary Poisson-GLM is defined by (see for example McCullagh and Nelder (1989)):

$$D(\mathbf{y}, \hat{\boldsymbol{\lambda}}) := -2 \cdot [l(\mathbf{y}, \hat{\boldsymbol{\lambda}}) - l(\mathbf{y}, \mathbf{y})] \tag{6.6}$$

where $Y_t \sim Poi(\lambda_t)$, $\lambda_t = exp(\mathbf{x}_t^t \boldsymbol{\beta})$, independent for $t = 1, 2, ..., T$. Here l denotes the log-likelihood of the Poisson-GLM and $\hat{\boldsymbol{\lambda}}$ is the corresponding maximum-likelihood estimate. Basically, the deviance can be considered as a function of the normalized log-likelihood of the model with the log-likelihood of the saturated model as the normalizing constant. In the Poisson-GLM of Table 5 the residual deviance is 5631 on 987 degrees of freedom compared to a Null deviance (the deviance of a model with the Intercept as only regressor) of 7242 on 991 degrees of freedom. This yields a p-value of 0 for a partial deviance test with the null-hypothesis $H_0: \ \beta_2 = ... = \beta_5 = 0$. Spiegelhalter et al. (2002) extend the concept of residual deviances to Bayesian models and define a Bayesian deviance. For the Poisson-GLM and for the Poisson-GLM with the AR(1) latent process considered in this paper the Bayesian deviance as defined by Spiegelhalter et al. (2002) is given by:

$$D(\boldsymbol{\lambda}) := -2 \cdot log[p(\mathbf{y}|\boldsymbol{\lambda})] + 2 \cdot log[p(\mathbf{y}|\mathbf{y})] \tag{6.7}$$

where $p(\mathbf{y}|\boldsymbol{\lambda})$ denotes the probability function of the data given the vector of mean parameters $\boldsymbol{\lambda}$. Note that (6.6) and (6.7) are equal in the Poisson-GLM if $\boldsymbol{\lambda} = \hat{\boldsymbol{\lambda}}$. Spiegelhalter et al. (2002) consider the mean deviance

$$\overline{D(\boldsymbol{\lambda})} := \frac{1}{R-B} \sum_{r=B}^{R} D(\boldsymbol{\lambda}^{(r)}),$$

where $\boldsymbol{\lambda}^{(r)}$ denotes the r-th MCMC iterate of $\boldsymbol{\lambda}$, as a Bayesian measure of fit and the quantity

$$p_D := \overline{D(\boldsymbol{\lambda})} - D(\overline{\boldsymbol{\lambda}}),$$

the mean deviance minus the deviance of the means, as the "effective number of parameters" of the Bayesian model. Here, $\overline{\boldsymbol{\lambda}} := \frac{1}{R-B}\sum_{r=B}^{R}\boldsymbol{\lambda}^{(r)}$ denotes the posterior mean estimate of $\boldsymbol{\lambda}$. As a criterion for the comparison of two Bayesian models they finally suggest the *deviance information criterion* which is given by:

$$DIC := D(\overline{\boldsymbol{\lambda}}) + 2p_D = \overline{D(\boldsymbol{\lambda})} + p_D. \tag{6.8}$$

Thus, the DIC is the Bayesian measure of fit $\overline{D(\boldsymbol{\lambda})}$, penalized by an additional complexity term p_D. A certain model is, as a general rule, to be preferred to another model, if its DIC value is lower.

In our fit of the Poisson-GLM with an AR(1) latent process we obtain a value of 5388 for the Bayesian deviance $\overline{D(\boldsymbol{\lambda})}$ and a value of 532 for $D(\overline{\boldsymbol{\lambda}})$, which yields a value of 4856 for the effective number of parameters in the model and a DIC value of 10244.

Of course, a direct comparison of Bayesian and non-Bayesian deviances is not possible. We therefore reconsider the ordinary Poisson-GLM in a Bayesian setting, choose the same priors for $\boldsymbol{\beta}$ as in the model with the AR(1) latent process and run 3 parallel chains for each component of the parameter vector $\boldsymbol{\beta}$. The simulated chains converge very quickly, so that it is again sufficient to record 5000 iterations and consider the first 2000 as burn-in in each chain. The posterior mean estimates of the components of $\boldsymbol{\beta}$ are identical to the maximum-likelihood estimates in Table 5. We obtain a Bayesian deviance $\overline{D(\boldsymbol{\lambda})}$ of 5636 and a plug-in deviance $D(\overline{\boldsymbol{\lambda}})$ of 5630 in this model. Thus, the "effective number of parameters" p_D is equal to 6 and the DIC value is 5642 for the ordinary Poisson-GLM considered in a Baysian setting.

The plug-in deviance confirms the previously discussed informal result that the fit of the ordinary Poisson-GLM is clearly worse than the fit of the Poisson-GLM with an AR(1) latent process. The latter model, however, has a very high value of effective parameters which can be interpreted as a high degree of over-fitting. This could be expected since the low value of the estimate $\hat{\rho}$ and the high value of $\hat{\sigma}_u^2$ nearly result in independent, normally distributed latent variables u_t for $t = 1, ..., 992$. This leads to a high DIC value and finally to the conclusion that, according to the DIC, the ordinary Poisson-GLM is the "better" of the two considered models.

7 Conclusions

In this paper we have investigated the influence of a series of explanatory variables on absolute transaction price changes of options on shares of the Bayer AG. For this purpose we have fitted Poisson regression models to the available data and we have related the results to implications of market microstructure theory. We have come to the conclusion that longer times between transactions are in general associated with larger absolute transaction price changes of the options. The influence of the trade volume on the absolute option price changes, however, is not perfectly clear in our data, although it could be observed that in most cases smaller trade volumes are associated with larger transaction price changes. In any case, it can be concluded that the explanatory power of the explanatory variables "time between consecutive transactions" and "trade volume" is rather low with respect to the absolute transaction price changes of the options.

Furthermore we have observed that option-specific explanatory variables such as the intrinsic value of the option at the time of the trade as well as the absolute price changes of the underlying share have a significant influence on the absolute transaction price changes of the options, which is in line with basic economic considerations. For the option-specific explanatory variables mentioned before similar studies with detailed intraday data for the underlying should be carried out in order to confirm the results obtained in this paper.

For the modelling of the absolute transaction price changes of one particular Call option in a Bayesian model setting we have seen that the incorporation of an AR(1) latent process into the mean of the ordinary Poisson-GLM, which accounts for both overdispersion and an autoregressive structure of the absolute transaction price changes, leads to a significantly better fit. In our example, however, according to the deviance information criterion, which trades off model complexity with goodness-of-fit, the ordinary Poisson-GLM is the "better" model due to the high complexity of the Poisson-GLM with the AR(1) latent process.

A simultaneous consideration of the transaction price processes of corresponding Put and Call options (e.g. in a bivariate autoregressive framework for the price change processes) could be a further field of research.

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