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Munich Discussion Paper No. 2007-29

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Equal Sharing Rules in Partnerships

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August 2007

Abstract

Partnerships are the prevalent organizational form in many industries. Most partnerships share profits equally among the partners. Following Kandel and Lazear (1992) it is often argued that "peer pressure" mitigates the arising free-rider problem. This line of reasoning takes the equal sharing rule as exogenously given. The purpose of our paper is to show that with inequity averse partners – a behavioral assumption akin to peer pressure – the equal sharing rule arises endogenously as an *optimal solution* to the incentive problem in a partnership.

JEL Classification: D20, D86, J54

Keywords: equal sharing rule, partnerships, incentives, peer pressure, inequity aversion

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1 Introduction

Partnerships are the prevalent organizational form in many industries, such as law, accounting, investment banking, management consulting, or medicine.[1] The defining features of a partnership are (i) the joint production of output and (ii) the distribution of the profits among the partners. While there are many systems by which profits can be divided among partners, *equal* profit sharing appears to be widely used.[2] From the viewpoint of incentive theory, this is a puzzling observation. Partnerships that employ a simple equal profit sharing rule are prone to the free-rider problem. Each partner bears her full effort cost but receives only $1/N$ of the profits in a partnership of size N. However, the fact that many such partnerships are observed, suggests that free-riding is not necessarily a problem. A prominent explanation is proposed by Kandel and Lazear (1992).[3] They demonstrate that "peer pressure" can mitigate the free-riding problem. The basic insight is that peer pressure complements monetary incentives: if a partner shirks, he feels guilt or shame, or has to take social reprisals, which prevents him from shirking.

In Kandel and Lazear's analysis, however, the equal sharing rule is *assumed*, and *given* the equal sharing rule they analyze how peer pressure can overcome the free-riding problem.[4] The purpose of our paper is to show that with inequity averse partners – a behavioral assumption that nests guild or shame – the equal sharing rule can be *derived* as the *optimal solution* to the incentive problem in a partnership. Interestingly, the result on the optimality

[1]See Hansmann (1996) for a detailed account of professions where partnerships are widespread.

[2]Encinosa, Gaynor, and Rebitzer (2007) analyze data on medical group practices and report that 54.2% of small practices (3-5 physicians) have an equal sharing rule in place and that 46% of all groups in their sample fall into this (smallest) category (Table 2, p. 199). Similarly, Farrell and Scotchmer (1988) have data on law firms. They write: "The first and most straightforward system is that all members with the same seniority receive the same profit share. Since junior partners eventually become senior partners, such a system would be equal division if the firm's profitability were constant over time. Such a sharing scheme [...] is probably used by most two or three-person law firms, which account for about 2/3 of all firms (although less than half the lawyers)"(p. 293).

[3]There is also a literature that offers contractual solutions to the free-riding problem. For instance, Holmström (1982) points to the role of a budget-breaker and Miller (1997) proposes reporting schemes based on mutual monitoring.

[4]Other papers that take equal sharing rules as given in their analyses of partnerships are, e.g., Farrell and Scotchmer (1988) and Levin and Tadelis (2005).

of the equal sharing rule is *not* driven by the partners' direct preference for sharing rules that induce egalitarian outcomes. Rather, we show that equal sharing rules *maximize the incentives* of the partner who has the weakest incentives to exert effort.

In our model, we analyze partnerships in which each partner decides whether or not to contribute effort to some joint production. Effort is observable but not verifiable. Contracts can condition on output, but they cannot assign monetary payments depending on individual partners' effort choices. Consequently, the classic free-rider problem may arise. Partners can renegotiate the contract after they have made their effort decisions. We assume that partners are inequity averse; they suffer a utility loss whenever other partners receive different rents defined as monetary payoff minus effort costs.[5]

There are the following results. We first show how inequity aversion affects renegotiations. If partners are highly inequity averse, renegotiations always result in an equal division of rents independently of the original contract. Since each partner is compensated for his effort costs and receives an equal share in the joint surplus, they make efficient effort choices. In case partners are not highly inequity averse, contracts are not renegotiated if they are budget-balancing. Contracts thus determine incentives. We can now derive our main result: If there exists some contract that induces all partners to exert effort, an equal sharing rule also induces all partners to exert effort. The converse is not true.

Furthermore, given the equal sharing rule as optimal contractual solution, we can derive the condition on the degree of inequity aversion and the production technology under which all partners exert effort. If the production technology has non-increasing returns to scale, our results imply that cooperation in partnerships becomes more difficult to sustain as the group size increases. This accords to the observation that the fraction of partnerships with equal sharing rules decreases with team size.[6]

[5]There exists ample experimental evidence on the existence and prevalence of such social preferences. See Camerer (2003) and Fehr and Schmidt (2006) for a survey of the literature.

[6]In their data on medical group practices, Encinosa et al. (2007) report that the fraction of practices with equal sharing rule falls from 54.2% in practices with 3-5 physicians, to 42% in practices with 6-7 physicians, to roughly 20-30% on average for larger practices (Table 2, p. 199).

The present paper is closely related to a growing literature that studies the impact of social preferences in moral hazard problems with multiple agents.[7] These articles find that inequity aversion can improve incentives since agents work harder to avoid suffering from unfavorable inequality. For example, Itoh (2004), Demougin and Fluet (2006), and Rey Biel (2007) show that optimal contracts might actually generate inequality to capitalize on this positive incentive effect. In contrast, we demonstrate that sharing rules that minimize inequality maximize incentives in partnerships if partners are inequity averse.

The remaining paper is organized as follows. Section 2 presents the model. Section 3 describes the influence of inequity aversion on renegotiation and derives the optimality of equal sharing rules. Section 4 discusses implications of our results for the efficiency of effort provision and the size of partnerships. Section 5 concludes. All formal proof are in the Appendix.

2 The Model

Joint Production, Effort, and Information

Consider a partnership of N identical agents (partners) who can produce joint output x. Let $\mathcal{N} = \{1, 2, .., N\}$ denote the set of agents in a partnership of size N. Each agent i chooses an effort contribution $e_i \in \{0, 1\}$ to joint production. Individual effort choices are not verifiable. Effort e_i causes costs $c(e_i)$, where $c(1) = c > 0$ and $c(0) = 0$. Thus, an agent "works" if she chooses high effort and "shirks" if she chooses low effort. Let $e = \langle e_i, e_{-i} \rangle$ be an effort vector consisting of agent i's effort e_i and the vector e_{-i} of all other agents' effort choices. Joint output is a deterministic function x of the number of agents working. It does not depend on the identity of the agents that either work or shirk. Thus, output reveals the number of agents that work not whether a particular agent worked or shirked. Let $x(K)$ denote joint output if K agents work. Define $\Delta x(K) = x(K) - x(K - 1)$ as the marginal contribution of the K-th working agent. Output is observable and verifiable, and it is sold at a price normalized to unity. Thus, the revenue is $x(K)$ when K agents work.

[7] For articles where an agent compares herself with the principal, see Glazer and Dur (2007) or Englmaier and Wambach (2005). For further articles that study social comparisons among agents, see for example Demougin, Fluet, and Helm (2006) or Neilson and Stowe (2005).

Contracts

Though we will allow for renegotiation, the relationship between the agents is initially governed by a formal contract. A *contract* S is a vector valued function that specifies how the revenue generated by the agents is distributed among the agents. In a partnership, agents work closely together and thus know who puts in effort and who does not. Hence, we assume that agents can observe the other agents' effort decisions but that this information is not verifiable to a court. A contract can thus condition the distribution of the revenue only on joint output (but not on individual agents' effort choices). For each number K, the number of agents that work, a contract S specifies a vector $S(K)$ that consists of the individual monetary payoffs $s_i(K)$ for each agent $i \in \mathcal{N}$. Define $y(K) = \sum s_i(K)$ as the sum of monetary payoffs that is allocated to the agents, and $\Delta y(K) = y(K) - y(K-1)$ as the change in this aggregate payment if K agents rather than $K-1$ agents work. Money can be "burned" but a contract cannot distribute more than the entire output. Further, we assume limited liability so that all payments must be non-negative. This implies $y(K) \leq x(K)$ and $s_i(K) \geq 0$ for all i.

The following definitions are used frequently. A contract is called "budget-balancing at K" if $y(K) = x(K)$. It is "budget-balancing" if it is budget-balancing at all $K \in \{0, 1, ..., N\}$. Further, a contract is "equal at K" if $s_i(K) = s_j(K)$ for all i, $j \in \mathcal{N}$ so that all agents get the same monetary payoff in case K agents work. It is "equal at the top" if it is equal at $K \in \{N-1, N\}$ and thus all agents get the same monetary payoff in case all agents or all but one agent work.

Inequity Aversion and Agents' Utility Functions

There exists ample empirical evidence that individuals contribute costly effort to joint production in partnerships although this is inconsistent with the neo-classical model of rational behavior. Contributions might be driven by social sanctions of other members of the partnership, by internalized social norms, or by guilty feeling when cheating letting the others down. To model such cooperative behavior we refer to the theory of inequity aversion as proposed Fehr and Schmidt (1999).[8] Agents have the following utility function. Within a partnership of N agents consider an effort vector e with K agents working with corresponding vector

[8]For a detailed discussion of social preferences see Camerer (2003) or Fehr and Schmidt (2006).

$S(K)$ of monetary payoffs. First, define agent i's *rent* as her monetary payoff net of effort cost

$$u_i(e, S(K)) = s_i(K) - c(e_i). \tag{1}$$

In accordance with classic equity theory agents incorporate effort costs and compare rents.[9] We can now define an agent's preferences as follows.

Assumption 1 *Within a partnership of N agents consider an effort vector e with K agents working with corresponding vector $S(K)$ of monetary payoffs. Then let*

$$\begin{aligned} v_i(e, S(K)) = u_i(e, S(K)) \\ - \alpha \frac{1}{N-1} \sum_{j=1,\, j \neq i}^{N} \max\left[u_j(e, S(K)) - u_i(e, S(K)), 0\right] \\ - \beta \frac{1}{N-1} \sum_{j=1,\, j \neq i}^{N} \max\left[u_i(e, S(K)) - u_j(e, S(K)), 0\right] \end{aligned}$$

denote agent i's utility.

The parameters α and β measure the importance of inequity concerns for the agents. As Fehr and Schmidt (1999) we assume that an agent suffers a utility loss if she receives a rent different than other agents, but suffers more from inequity if it is not in her favor, $\alpha \geq \beta$ and $1 > \beta \geq 0$. We normalize the agents' utility to zero if they decide not to work in a partnership.

Sequence of Actions, Renegotiation, and Equilibrium

We want to characterize contracts that induce all agents in a partnership of given size to exert effort. In partnerships renegotiations are likely because there is no outside ownership. Renegotiation should be expected to influence ex-post payoffs, which in turn determine ex-ante effort incentives. For example, agents could initially agree on a contract that divides output evenly if all agents work but "burns" the entire output if at least one agent shirks. Since every agent's effort decision is pivotal, all agents have incentives to work. However, Holmström (1982) shows that such a contract is not renegotiation-proof. Once an agent has

[9] According to equity theory individuals want a fair relationship between inputs (in our model effort) and outputs (in our model monetary payoffs). Agents thus account for effort costs in their social comparisons. See Festinger (1957) and Adams (1963).

shirked all agents can agree to equally divide what ought to be burnt. Because all agents profit from redistribution, renegotiation renders the initial contract not credible.

We capture renegotiation in the following way. Consider a partnership of size N with initial contract S. There is the following sequence of actions. First, agents simultaneously make their effort choice. Second, output is observed. If for example K agents work, the initial contract S endows agents with legal claims on monetary payoffs as summarized by the monetary payoff vector $S(K)$. The contract thereby sets the stage for renegotiations. Third, agents might renegotiate the monetary payoff vector $S(K)$. Finally, payoffs are realized.

Instead of modeling the renegotiation process explicitly we make the following assumptions. For once, agents anticipate that certain monetary payoff vectors and thus certain contracts are renegotiation-proof in the following sense. A monetary payoff vector $S(K)$ is "renegotiation-proof" if and only if there exists no $S'(K)$ that strictly increases the utility of at least one agent without reducing the utility of at least one other agent. A contract S is "renegotiation-proof" if and only if for all $K \in \{0, 1, ..., N\}$ the monetary payoff vector $S(K)$ is renegotiation-proof for all effort vectors e with K agents working. Note that this definition distinguishes between renegotiation-proof monetary payoff vectors $S(K)$ and renegotiation-proof contracts S. To preclude renegotiation as far as possible we restrict attention to contracts that are renegotiation-proof whenever possible. Yet in the following section we show that if agents are highly inequity averse (to be made precise below), no contract is renegotiation proof. To pin down the result of renegotiation in this case we make the following assumption.

Assumption 2 *Consider an effort vector e with K agents working. If the monetary payoff vector $S(K)$ is not renegotiation-proof, renegotiation results in a Pareto-efficient renegotiation-proof payoff vector $S'(K)$.*

We can now define what constitutes equilibrium effort choices. In our reduced form game (renegotiation is not modeled explicitly) a worker's pure strategy is simply an effort choice. A Nash-equilibrium then consists of effort choices for all agents $i \in \mathcal{N}$ that are mutually optimal given an initial contract S and the anticipated renegotiations.

3 Optimality of Equal Sharing Rules

Inequity Aversion and Renegotiation

Before we proof the optimality of equal sharing rules, we have to analyze the impact of inequity aversion on renegotiations. This is captured in the following proposition. All formal proofs can be found in the appendix.

Proposition 1 (Renegotiation) *Consider an effort vector e with K agents working.*

1. *If $\beta < (N-1)/N$, then $S(K)$ is renegotiation-proof if and only if it is budget-balancing.*
2. *If $\beta \geq (N-1)/N$, then $S(K)$ is renegotiation-proof if and only if it is budget-balancing and $u_i(e, S) = u_j(e, S)$ for all agents $i, j \in \mathcal{N}$.*

This result is based on the following intuition. Suppose $S(K)$ is not budget-balancing. Consider the following new allocation: Take the part of the output that ought to be burned and divide it equally among the agents. This increases the monetary payoff of all agents without changing their relative standing. All agents thus agree. In the ensuing analysis we thus restrict attention to contracts that are budget-balancing.

Further, if agents are not highly inequity averse, $\beta < (N-1)/N$, then every budget-balancing contract is renegotiation-proof. In this case agents do not agree to a reduction in their monetary payoffs even if the redistribution decreases inequity by increasing the monetary payoffs of the agents that are worse off. Any meaningful redistribution of a budget-balancing monetary payoff vector requires that the monetary payoff of at least one agent is reduced. This agent vetoes any renegotiations.

But if agents are highly inequity averse, $\beta \geq (N-1)/N$, a budget-balancing monetary payoff vector $S(K)$ need not be renegotiation-proof. In this case agents are so keen on diminishing inequity amongst themselves so that they hand over some of their monetary payoff to agents being worse off. $S(K)$ is thus renegotiation-proof if and only if it is budget-balancing and all agents receive the same rent irrespective of their effort choice. Thus, a renegotiation-proof contract S must condition the vector of monetary payoffs not only on the number of agents working but on the entire effort vector e. This is unfeasible as individual effort choices are not contractible. Contrary to an individual monetary payoff vector $S(K)$, a contract S can thus never be renegotiation-proof if agents are highly inequity averse.

To summarize: We want contracts to be renegotiation-proof (as far as possible) and thus limit attention to budget-balancing contracts. If agents are highly inequity averse, no contract is renegotiation proof. We then assume that renegotiation results in a renegotiation-proof distribution of monetary payoffs and thus in a budget-balancing monetary payoff vector that equalizes all agents' rents.

Optimal Contracts

We can now present our main result. We derive optimal contracts that provide all agents with incentives to work. Although there is usually no unique optimal contract, the following proposition shows that a contract that is equal at the top is always optimal.

Proposition 2 (Equal Sharing Rule) *Suppose there exists a budget-balancing contract S with aggregate payments $y(N)$ and $y(N-1)$ that induces all agents to work. Then there exists a budget-balancing contract S' with identical aggregate payments that is equal at the top and induces all agents to work.*

If agents are highly inequity averse, $\beta \geq (N-1)/N$, incentives are essentially determined by renegotiation. Contracts are then irrelevant and Proposition 2 is trivially satisfied. In the more interesting case agents are not highly inequity averse, $\beta < (N-1)/N$, and contracts determine incentives. For an illustration of Proposition 2 consider a contract that is not equal at the top. Then there exists an agent, say agent i, who gets the lowest monetary payoff if all agents work. Since all agents incur the same effort costs if all agents work, agent i then holds the lowest rank - the lowest relative position - with respect to her rent. Consider the following changes in the contract. Agent i's monetary payoff is increased in case $N-1$ and in case N agents work. These changes satisfy the following properties. First, what is given to agent i is taken from the others so that the monetary payoff vector remains budget-balancing. Second, agent i's incentives are held constant.

This change in contract has the following incentive effects. Agents suffer more from being worse off than from being better off than others. Therefore, the lower the rank of an agent the higher the utility gain from increasing her monetary payoff. By choice of agent i her rank cannot be lower if she is the only agent shirking as compared to the situation in which everybody works - in the latter case she already holds the lowest possible rank. To keep her incentives unchanged, her monetary payoff need never be increased by a larger amount if all agents work than if only one agent shirks. This has the following implication for the incentives of the other agents. Due to budget-balance the monetary payoff of all other agents decreases weakly more if one agent shirks than if all agents work. As in the considered case agents are not highly inequity averse and hence enjoy having a higher monetary payoff, their incentives to work are never harmed but potentially improved. Thus, the proposed change renders the contract more equal without harming incentives or altering aggregate payments. Iterated application of this procedure finally results in a contract that is equal at the top.

Importance of Inequity Aversion for Equal Sharing Rules

It is key to understand that Proposition 2 derives the optimality of equal sharing rules by only referring to agents' incentives to exert effort. Optimal contracts are equal at the top not because inequity averse agents have a preference for equal sharing rules (that minimize inequality). Instead, a contract that is equal at the top maximizes the incentive of the agent who has the weakest incentive to work. In this sense the contract "maximizes minimum incentives."

In contrast to the case with inequity aversion, maximizing minimum incentives does not generate a trend towards equal sharing rules if agents are selfish. Selfish agents' effort choices depend only on the resulting changes in their effort costs and changes in their monetary payoffs. Therefore, maximizing minimum incentives only requires that the changes in monetary payoffs (if one agent shirks) are the same for all agents. Thus the change in aggregate monetary payoffs must be divided equally. Making the level of payoffs more equal has no effect on incentives. This does not hold true if agents are inequity averse, as such agents also take into account how shirking affects their rank.

As illustration consider a team of three agents with production technology $x(2) = 6$ and $x(3) = 12$. A contract specifying $S(2) = (1, 2, 3)$ and $S(3) = (3, 4, 5)$ provides selfish agents with equal effort incentives as shirking reduces each agent's monetary payoff by two. Making this contract equal at the top does not affect incentives. However, suppose agents are inequity averse and $c = 3$. If all agents work, they get rents $(0, 1, 2)$. If agent 1 shirks, the agents get rents $(1, -1, 0)$. Agent 1's incentives to work are thus $(3/2)\,\beta - 1 - (3/2)\,\alpha$. Consider a contract that is equal at the top so that $S'(2) = (2, 2, 2)$ and $S'(3) = (4, 4, 4)$. Agent 1's incentives to work are then $(3/2)\,\beta - 1$. Making the contract equal at the top strictly improves incentives. Therefore, inequity aversion offers an explanation for why equal sharing rules are often used in partnerships.

4 Further Results

Efficient Production in Partnerships

The impact of inequity aversion on incentives can now be easily derived. If agents are highly inequity averse, there is the following result.

Corollary 1 *Suppose $\beta \geq (N-1)/N$ and consider a partnership of size N. If and only if*

$$\Delta\, x(N) \geq c,$$

then all agents working forms a Nash-equilibrium.

If agents are highly inequity averse, then renegotiation ensures that all agents get equal rents. Each agent thus knows that she will be compensated for the incurred effort cost and in addition receive a share $1/N$ of the generated surplus distributed to the agents. If N agents work, the surplus is the agents' aggregate monetary payoff minus the sum of their effort costs, $y(N) - N\,c$. Each agent thus has incentives to exert effort if and only if her effort costs are smaller than the resulting increase in aggregate payment. If agents are highly inequity averse, ex-post renegotiation solves the free-rider problem.

If agents are not highly inequity averse, budget-balancing contracts are not renegotiated and directly determine incentives. By Proposition 2 we can restrict attention to contracts that are equal at the top. Therefore, it is possible to derive the precise conditions under which all agents working can form a Nash-equilibrium.

Corollary 2 *Suppose $\beta < (N-1)/N$ and consider a partnership of size N. If and only if*

$$\frac{\Delta\, x(N)}{N} \geq (1-\beta)\, c,$$

then all agents working forms a Nash-equilibrium.

Corollary 2 is based on the following argument. Take a contract that is equal at the top. If an agent shirks whereas all other agents work, her monetary payoff is reduced by her share $\Delta x(N)/N$ in the reduction of the aggregate payment to all agents. She saves c on effort costs. Since the agent is inequity averse she suffers βc from cheating the other agents. Thus, an agent has no incentive to shirk if and only if $\Delta x(N)/N \geq (1-\beta)\,c$.

The above results imply that if all agents working is not efficient, it is not implementable. Thus, inequity aversion can never support cooperation in inefficiently large partnerships. Further, inequity aversion facilitates cooperation: There exist situations in which all agents working is implementable if and only if agents are inequity averse. The condition for exerting effort depends on the agents' degree of inequity aversion. If agents are highly inequity averse, $\beta \geq (N-1)/N$, they anticipate renegotiation and thus have an incentive to maximize joint surplus. If agents are not highly inequity averse, $\beta < (N-1)/N$, they know that there will be no renegotiation. They are thus not interested in the joint surplus. Yet if an inequity averse agent shirks whereas all other agents work, she incurs a utility loss from being better off than all other agents. If this "shame for cheating" outweighs the - potential - increase in her rent, the agent abstains from shirking. Putting it differently, inequity averse agents overcome the team production problem if "compassion" or "shame for cheating" is large enough. It is this behavioral trait of "feeling bad" when cheating the others that creates incentives to exert effort. In contrast a selfish agent does not bear these behavioral costs, which makes cooperation more difficult to sustain.

Inequity Aversion and Size of Partnerships

Proposition 2 shows that one can restrict attention to equal sharing rules if all agents are to exert effort. Corollary 1 and 2 then imply that the minimum level of inequity aversion required to sustain cooperation increases with the size N of the firm if $\Delta\, x(N)/N$ decreases. The present paper therefore can explain why small partnerships often work well in reality whereas larger ones frequently suffer from free-riding. Further, inequity aversion increases the maximum supportable team size as it facilitates cooperation.

At least since Ward (1958) it is well known in the literature that equal sharing rules restrict the maximum size of partnerships as existing partnerships are only willing to accept a new member if the latter increases average profitability. Yet absent inequity aversion it is not clear why new team members have to be given an equal share in total profits. In fact, in law or consultancy firms senior partners usually get higher shares of the joint profit as compared to junior partners. The following numerical example shows that inequity aversion restricts how unequal a contract can treat partners if all agents are to work hard.

Consider a firm consisting of two agents, agent 1 and 2. If both agents exert effort, they each incur effort costs $c = 1$ but produce joint output $x(2) = 10$. Suppose both divide revenue evenly, and that both agents working forms a Nash-equilibrium. Thus, both get a rent of 4 if firm size is 2. However, the firm has the opportunity to expand and employ an agent 3. If all three agents work, output increases to $x(3) = 13$. Agent 3 has effort costs $c = 1$. As the increase of 3 in joint output exceeds the effort costs of 1 it is efficient to expand and employ the agent. Suppose all agents are selfish. Then an unequal contract with $S(3) = (5, 5, 3)$ and $S(4, 4, 2)$ is budget-balancing and provides efficient effort incentives for all. Since the rents of agents 1 and 2 are unchanged, they allow agent 3 to join their partnership. However, suppose agents are inequity averse. Agent 3 then receives utility $2 - 2\alpha$ if she workers, whereas she gets utility $2 - \alpha$ if she shirks. By shirking she can thus reduce her suffering from inequity aversion, and the above contract no longer provides sufficient effort incentives. Yet giving agent 3 a larger share in the joint profit reduces the rents of agents 1 and 2. Thus, they no longer allow agent 3 to join their partnership.

5 Conclusion

This paper shows how incentive provision in partnerships is affected if agents are inequity averse in the sense of Fehr and Schmidt (1999). Optimal contracts accounting for inequity aversion involve simple, budget-balancing, and equal sharing rules. These optimal contracts maximize all agents' effort incentives; they are optimal not because agents have a preference for equal sharing rules. Moreover, inequity aversion can provide all agents with sufficient incentives to work in cases where this is unfeasible if agents only care for their own monetary payoff and effort costs. Our results are arise since guilt, shame, or social sanctions reduces a shirking agent's utility precisely in those cases in which she actually shirks. As contracts cannot condition on an agent's effort decision, they cannot afflict the above punishment with equal precision. Thus, inequity aversion facilitates incentive provision. The conditions under which inequity aversion permits cooperation amongst the agents depend on the size of the team. They usually become less restrictive with decreasing size of the team, which fits the common observation that small teams often work well whereas larger ones suffer from free-riding. Summarizing, the present paper shows that inequity aversion and the associated incentive effects could offer a fruitful new perspective on the internal organization of firms.

Appendix

Proof of Proposition 1

A) Budget-balance is necessary for a vector $S(K)$ of monetary payoffs to be renegotiation-proof independently of the level of inequity aversion. Consider a $S(K)$ with $y(k) < x(K)$ for K agents working yielding output $x(K)$. Then $S'(K)$ with $s'_i(K) = s_i(K) + [x(K) - y(K)]/N$ for all $i \in \mathcal{N}$ increases the monetary payoff for all agents by an identical, strictly positive amount while keeping the inequity between the agents unchanged. Therefore, all agents are strictly better off under $S'(K)$, and $S(K)$ is not renegotiation-proof.

B) Budget-balance is also sufficient for $S(K)$ to be renegotiation-proof if $\beta < (N-1)/N$. Given a budget-balancing $S(K)$ consider any other $S'(K)$ with different monetary payoffs. Then there exists an agent i with $s'_i(K) < s_i(K)$. If $\beta < (N-1)/N$, each agent's utility is strictly increasing in her monetary payoff even if the money taken away from her is given to those agents with lower utility thus decreasing inequity. Therefore, at least agent i does not agree to $S'(K)$, and $S(K)$ is renegotiation-proof.

C) If $\beta \geq (N-1)/N$ then given an effort vector e with K agents working, a monetary payoff vector $S(K)$ is renegotiation-proof only if $u_i(e, S(K)) = u_j(e, S(K))$ for all $i, j \in \mathcal{N}$ and $S(K)$ is budget-balancing. Suppose $S(K)$ is budget-balancing but there exist $i, j \in \mathcal{N}$ with $u_i(e, S(K)) > u_j(e, S(K))$. Define $\mathcal{A} = \{i \in \mathcal{N} : u_i(e, S(K)) \geq u_j(e, S(K)) \, \forall \, j \in \mathcal{N}\}$ as the set of agents with the highest utility, and $\mathcal{A}^C = \mathcal{N} \setminus \mathcal{A}$ as its complement. Denote by $\#\mathcal{A}$ the cardinality of $\mathcal{A}$. Consider another $S'(K)$ with new monetary payoffs $s'_i(K) = s_i(K) - \epsilon$ for all $i \in \mathcal{A}$ whereas $s'_j(K) = s_j(K) + \epsilon \cdot (\#\mathcal{A}/\#\mathcal{A}^C)$ for all $j \in \mathcal{A}^C$. Thus, no money is burnt and $S'(K)$ is budget-balancing. Choose $\epsilon > 0$ sufficiently small so that for all $i \in \mathcal{A}, j \in \mathcal{A}^C$

we keep $u_i(e, S'(K)) \geq u_j(e, S'(K))$. We can now check whether $S'(K)$ is accepted by all agents. All agents $j \in \mathcal{A}^C$ receive higher monetary payoffs. Since for these agents payoffs increase equally, suffering from inequity with respect to all agents in $\mathcal{A}^C$ remains unchanged. However, the suffering with respect to all agents $i \in \mathcal{A}$ is reduced. Thus, all agents $j \in \mathcal{A}^C$ prefer $S'(K)$ to $S(K)$. Equally, for all agents $i \in \mathcal{A}$ utility is changed by

$$v_i(e, S'(K)) - v_i(e, S(K)) = -\epsilon + \beta \frac{1}{N-1} \sum_{j \in \mathcal{A}^C} \left[\epsilon + \epsilon \cdot \frac{\#\mathcal{A}}{\#\mathcal{A}^C} \right] = \epsilon \cdot \left[\beta \frac{N}{N-1} - 1 \right] \geq 0.$$

All agents $i \in \mathcal{A}$ thus weakly prefer $S'(K)$ to $S(K)$, and $S(K)$ is not renegotiation-proof. Therefore, $u_i(e, S(K)) = u_j(e, S(K))$ for all $i, j \in \mathcal{N}$ and budget-balance is necessary for a contract to be renegotiation-proof.

D) If $\beta \geq (N-1)/N$, budget-balance and, given e with K agents working, $u_i(e, S) = u_j(e, S)$ for all $i, j \in \mathcal{N}$ is also sufficient for a contract to be renegotiation-proof. If this condition is satisfied, any changes in monetary payoffs implied by another $S'(K)$ reduce the monetary payoff of at least one agent. Let i be the agent whose payoff is reduced by the most. Then $u_i(e, S'(K)) < u_i(e, S(K))$ and $u_i(e, S'(K)) \leq u_j(e, S'(K))$ for all $j \in \mathcal{N}$. Agent i's rent is reduced while in addition she now suffers from inequity with respect to some other agents. As she prefers $S(K)$ to $S'(K)$, the monetary payoff vector $S(K)$ is renegotiation-proof. *Q.E.D.*

Proof of Proposition 2

A) If agents are sufficiently inequity averse, $\beta \geq (N-1)/N$, the ex-post distribution of monetary payoffs is determined by renegotiation. Incentives depend on the anticipated ex-post distribution of monetary payoffs. As the latter is independent of the initial contract S, replacing any initial contract S with any other contract S' being equal at the top and with the same $y(K)$ for $K \in \{N-1, N\}$ does not change incentives and Proposition 2 is trivially satisfied.

B) For the remainder assume $\beta < (N-1)/N$ so that budget-balancing contracts are not renegotiated and directly determine incentives. First, we show that any budget-balancing contract S giving some agents unequal payoffs $s_i(N) \neq s_j(N)$ for some $i, j \in \mathcal{N}$ can be transformed into a budget-balancing contract S' with equal monetary payoffs $s'_i(N) = s'_j(N)$

for all $i, j \in \mathcal{N}$ without impairing incentives.

Consider a budget-balancing contract S where $s_i(N) \neq s_j(N)$ for some $i, j \in \mathcal{N}$. Define $\mathcal{B} = \{i \in \mathcal{N} : s_i(N) \leq s_j(N) \forall j \in \mathcal{N}\}$ as the set of agents with the lowest monetary payoff if all agents work. $\mathcal{B}$ is non-empty and a strict subset of $\mathcal{N}$. Define $\mathcal{C}$ as the subset of agents from $\mathcal{B}$ who have the lowest monetary payoff in case one agent shirks so that $\mathcal{C} = \{i \in \mathcal{B} : s_i(N-1) \leq s_j(N-1) \forall j \in \mathcal{N}\}$. Note that $\mathcal{C}$ can be empty. For any agent $i \in \mathcal{N}$ define $\mathcal{H}_i = \{j \in \mathcal{N} : s_j(N-1) - c > s_i(N-1)\}$ as the set of agents with a strictly higher monetary payoff net of effort costs than agent i if agent i shirks and all other agents work. Correspondingly, define $\mathcal{L}_i = \mathcal{H}_i^C = \mathcal{N} \setminus \mathcal{H}_i$. Finally, let $\langle e_i, e^*_{-i} \rangle$ be an effort vector e where all agents apart from agent i work, and agent i chooses effort $e_i \in \{0, 1\}$.

Consider the following transformation of contract S resulting in contract S':

1. Whenever more than one agent shirks, contract S' and S are identical, $S'(K) = S(K)$ for all $K \in \{0, 1, .., N-2\}$.

2. If one or no agent shirks, monetary payoffs of all agents $i \in \mathcal{B}$ are increased, $s'_i(N-1) = s_i(N-1) + \epsilon(N-1)$ and $s'_i(N) = s_i(N) + \epsilon(N)$, where $\epsilon(N)$ and $\epsilon(N-1)$ are strictly positive constants.

3. If one or no agent shirks, monetary payoffs of all agents $j \in \mathcal{B}^C = \mathcal{N} \setminus \mathcal{B}$ are reduced, $s'_j(N-1) = s_j(N-1) - \gamma\, \epsilon(N-1)$ and $s'_j(N) = s_j(N) - \gamma\, \epsilon(N)$, where $\gamma = \#\mathcal{B}/\#\mathcal{B}^C$. Thus, what is given to the agents in $\mathcal{B}$ is taken from the agents in $\mathcal{B}^C$ so that $y'(N-1) = y(N-1)$ and $y'(N) = y(N)$, and S' is again budget-balancing.

4. $\epsilon(N)$ and $\epsilon(N-1)$ are chosen so that incentives for all agents $i \in \mathcal{C}$ to work if all other agents work remain constant. The consequence of this property is explained below.

5. $\epsilon(N)$ and $\epsilon(N-1)$ are chosen as large as possible but sufficiently small so that the rank order of the agents is preserved in the following sense. For all $i \in \mathcal{B}$, $j \in \mathcal{B}^C$, whenever $s_j(N) > s_i(N)$ then $s'_j(N) \geq s'_i(N)$. Further, if $s_j(N-1) - c > s_i(N-1)$ then $s'_j(N-1) - c \geq s'_i(N-1)$. Finally, if $s_j(N-1) > s_i(N-1) - c$ then $s'_j(N-1) \geq s'_i(N-1) - c$. Thus, whenever according to the initial contract S an agent $j \in \mathcal{B}^C$ is strictly better off than an agent $i \in \mathcal{B}$ if all agents work, if only agent i shirks or if only

agent j shirks, then according to the new contract S' she is not strictly worse off in the corresponding situation.

We will now show that incentives are not impaired in this process. Given the above transformation only the inequity between agents $i \in \mathcal{B}$ with respect to agents $j \in \mathcal{B}^C$ changes. The change in incentives for all agents $i \in \mathcal{C}$ is thus given by

$$\epsilon(N) \cdot \left[1 + (1+\gamma)\frac{\alpha \# \mathcal{B}^C}{N-1}\right] - \epsilon(N-1) \cdot \left[1 + (1+\gamma) \cdot \left(\frac{\alpha \#(\mathcal{H}_i \cap \mathcal{B}^C)}{N-1} - \frac{\beta \#(\mathcal{L}_i \cap \mathcal{B}^C)}{N-1}\right)\right].$$

As we are in the case where agents are not sufficiently inequity averse to agree to a reduction in their monetary payoff in the course of potential renegotiations, agent i's overall utility $v_i(e, S(K))$ is strictly increasing in her monetary payoff even if favorable inequity thus increases. More formally, as $\beta < (N-1)/N$ and $\gamma = \#\mathcal{B}/\mathcal{B}^C$, $\epsilon(N-1)$ is multiplied with a strictly positive factor in the above expression. By choice of the set $\mathcal{B}$, agents $i \in \mathcal{B}$ have the lowest possible rank when all agents, including themselves, are working. Thus, these agents can only improve in their rank by shirking. As some agents $j \in \mathcal{B}^C$ may then be in $\mathcal{L}_i$ (and thus not in $\mathcal{H}_i$), we must have $\#(\mathcal{H}_i \cap \mathcal{B}^C) \leq \#\mathcal{B}^C$ and $\#(\mathcal{L}_i \cap \mathcal{B}^C) \geq 0$. The marginal impact of an increase in monetary payoff depends negatively on an agent's rank: the lower the rank, the more unfavorable inequity is reduced, and the higher the marginal increase in utility. Due to the argument above, an increase in the monetary payoff if all agents work has a higher impact on utility than an increase in monetary payoff if one agent shirks. As $\epsilon(N)$ and $\epsilon(N-1)$ are chosen so that the above change in incentives is zero, we get

$$\epsilon(N) \leq \epsilon(N-1)$$

as $\epsilon(N)$ is multiplied with a larger factor than $\epsilon(N-1)$.

Consider now the incentives for any agent $i \in \mathcal{B}\backslash\mathcal{C}$ whenever this set is non-empty. Compared to any agent $j \in \mathcal{C}$ we have $s_i(N-1) \geq s_j(N-1)$ by definition of $\mathcal{C}$ and consequently $\#(H_i \cap \mathcal{B}^C) \leq \#(H_j \cap \mathcal{B}^C)$ and $\#(L_i \cap \mathcal{B}^C) \geq \#(L_j \cap \mathcal{B}^C)$. Thus, agents $i \in \mathcal{B} \setminus \mathcal{C}$ will in general improve their rank by more when shirking than agents $j \in \mathcal{C}$. Since the marginal impact of the increase $\epsilon(N-1)$ in monetary payoff if one agent shirks is lower, the incentive to work hard if all other agents work hard is at least preserved for any agent $i \in \mathcal{B} \setminus \mathcal{C}$ as it is at least as large as for any agent $j \in \mathcal{C}$.

Finally, consider any agent $i \in \mathcal{B}^C$, whose change in incentives is given by

$$-\epsilon(N)\left[\gamma-(1+\gamma)\frac{\beta\#\mathcal{B}}{N-1}\right]+\epsilon(N-1)\left[\gamma+(1+\gamma)\left(\frac{\alpha\#(\mathcal{H}_i\cap\mathcal{B})}{N-1}-\frac{\beta\#(\mathcal{L}_i\cap\mathcal{B})}{N-1}\right)\right].$$

Again, the second factor of the above expression must be strictly positive as $\gamma = \#\mathcal{B}/\#\mathcal{B}^C$ and $\beta < (N-1)/N$. Since $\#(\mathcal{H}_i \cap \mathcal{B}) \geq 0$ and $0 \leq \#(\mathcal{L}_i \cap \mathcal{B}) \leq \#\mathcal{B}$, the above expression is at least weakly positive as $\epsilon(N-1) \geq \epsilon(N)$, and all agents $i \in \mathcal{B}^C$ keep their incentives to work hard. Summarizing, the above transformation of the contract S does not harm incentives. Iterated application of this transformation eventually results in a contract S' with $s'_i(N) = s'_j(N)\,\forall\, i, j \in \mathcal{N}$.

C) However, after the above transformations S is not yet necessarily equal at the top as there might exist $s_i(N-1) \neq s_j(N-1)$ for at least some $i, j \in \mathcal{N}$. In this case define $\mathcal{D} = \{i \in \mathcal{N} : s_i(N-1) \geq s_j(N-1) \forall j \in \mathcal{N}\}$ as the set of agents with the highest monetary payoff if one agent shirks. $\mathcal{D}$ is non-empty and a strict subset of $\mathcal{N}$. As the contract is equal if all agents work we have $s_i(N) = s_j(N)$ for all $i, j \in \mathcal{N}$ and all agents get the same utility if all agents work. As agents $i \in \mathcal{D}$ get the highest monetary payoff if one agent shirks, and as their utility is increasing in their monetary payoff as $\beta < (N-1)/N$, these agents have the minimum incentive to work if all other agents work. It is then possible to find a budget-balancing decrease of monetary payoffs $s_i(N-1)$ for $i \in \mathcal{D}$ and increase of $s_j(N-1)$ for $i \in \mathcal{D}^C$ that decrease work incentives for workers in $\mathcal{D}$ but increases work incentives for the other workers. Thus, minimum incentives to work are increased. Iteration of such changes in the monetary payoff vector in case one agent shirks eventually results in a contract that is equal at the top. *Q.E.D.*

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